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NEUTRAL K MESONS

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1. THE TWO TYPES OF NEUTRAL K MESONS

NEUTRAL K mesons do not belong to the class of truly neutral particles: K^0 and its antiparticle $\tilde{K}^0$ have the respective strangenesses S = +1 and S = −1. Strangeness is not a strictly conserved quantity, however; K^0 and $\tilde{K}^0$ can undergo the decays $K^0 \to 2\pi$, $\tilde{K}^0 \to 2\pi$. Therefore transitions $K^0 \rightleftarrows 2\pi \rightleftarrows \tilde{K}^0$ are possible by way of virtual π-meson states. This is a special feature of the neutral K mesons, which occupy, as it were, an intermediate position between truly neutral particles (π^0 mesons, photons) and particles that differ from their antiparticles by any strictly conserved quantum numbers (electron and positron, neutron and antineutron). The difference in strangeness between K^0 and $\tilde{K}^0$ mesons has the consequence that their strangeness-conserving interactions with other particles are different. For example, $\tilde{K}^0$ mesons can be produced only in pairs along with K^0 mesons, but K^0 mesons can also be produced in pairs along with Λ^0 and Σ hyperons: therefore the threshold for production of K^0 mesons is much lower than that for the production of $\tilde{K}^0$ mesons. The only interaction between K^0 mesons of moderate energies and nucleons is that of scattering, elastic or with charge transfer: for $\tilde{K}^0$ mesons reactions with production of Λ^0 or Σ hyperons are also possible:

$$\tilde{K}^0 + p \to \Lambda^0 + \pi^+. \tag{1}$$

We may say that K^0 mesons are more easily produced, and $\tilde{K}^0$ mesons interact better. Let us consider interactions which do not conserve strangeness. It has been found experimentally that K^0 mesons can decay into two π mesons:

$$\left.\begin{aligned} K^0 &\to 2\pi^0, \\ K^0 &\to \pi^+ + \pi^-. \end{aligned}\right\} \tag{2}$$

Let us apply the charge-conjugation operator C to the initial and final states of these reactions. It is obvious that

$$\left.\begin{aligned} CK^0 &= \tilde{K}^0, \\ C(2\pi^0) &= 2\pi^0, \\ C(\pi^+ + \pi^-) &= \pi^+ + \pi^-. \end{aligned}\right\} \tag{3}$$

Thus the processes (2) are not charge-invariant. The operator C takes the K^0 meson over into its antiparticle $\tilde{K}^0$, while the right-hand sides of the reactions (2) remain unchanged. It was this contradiction that in 1955 led Gell-Mann and Pais[1] to a remarkable discovery. They introduced states which are superpositions of K^0 and $\tilde{K}^0$*:

$$\left.\begin{aligned} K_1^0 &= \frac{1}{\sqrt{2}}(K^0 + \tilde{K}^0), \\ K_2^0 &= \frac{1}{\sqrt{2}}(K^0 - \tilde{K}^0). \end{aligned}\right\} \tag{4}$$

The states K_1^0 and K_2^0 do not have definite strangeness, but correspond to eigenvalues of the change-conjugation operator:

$$\left.\begin{aligned} CK_1^0 &= \frac{1}{\sqrt{2}}(\tilde{K}^0 + K^0) = K_1^0, \\ CK_2^0 &= \frac{1}{\sqrt{2}}(\tilde{K}^0 - K^0) = -K_2^0. \end{aligned}\right\} \tag{5}$$

According to Gell-Mann and Pais the decays (2) do not go from the state K^0 (or $\tilde{K}^0$), but from the state K_1^0, which, like the π mesons that are produced, has a definite C-parity (+1). The possibility of observing the reactions (2) allows us to separate out from the state K^0, which does not have a definite value of C, the eigenstate K_1^0 of the operator C, and to study all of its properties. Similarly, by observing a reaction of decay of neutral K mesons to states odd with respect to charge (for example, the decay $K^0 \to 3\pi^0$) one can study the properties of the state K_2^0, which has the charge parity C = −1. Thus the states K_1^0 and K_2^0 appear as different particles: we shall see that they have different lifetimes and different masses. Thus according to Gell-Mann and Pais the neutral K mesons are produced in states with definite strangeness—they appear as K^0 or $\tilde{K}^0$ mesons; in the decays, on the other hand, we observe the states K_1^0 and K_2^0. The discovery of nonconservation of parity P and charge parity C in weak interactions did not alter the results of Gell-Mann and Pais. Instead of the separate conservation laws for P and C we now have their product CP as a conserved quantity. This transformation is called the combined parity, and consists of replacing particles by antiparticles and simultaneously carrying out an inversion of the space coordinates. All of the arguments that have been given remain valid, since the operator CP has the same properties (3) and (5) as the operator C.

The existence of K_1^0 and K_2^0 mesons leads to extremely curious effects, which can be observed in beams of neutral K mesons. Let us imagine a beam of K^0 mesons produced by π mesons in a target A (Fig. 1). These K^0 mesons can be represented as a superposition of the states K_1^0 and K_2^0

*Here and in what follows we use the same letter to denote a particle itself and its wave function.

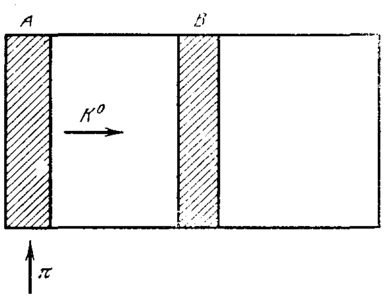

FIG. 1. Schematic arrangement of the Pais-Piccioni experiment.

$$K^0 = \frac{1}{\sqrt{2}} (K_1^0 + K_2^0)$$

[this relation follows from the equations (4)]. Since the lifetime of the K_2^0 meson is much longer than that of the K_1^0 meson (see below), the composition of the beam will change as it moves farther away from the target A. At a sufficiently large distance from the target A the beam will be practically entirely composed of K_2^0 mesons. But the pure K_2^0 state contains both the K^0 state and the $\tilde{K}^0$ state in equal proportions:

$$K_2^0 = \frac{1}{\sqrt{2}} (K^0 - \tilde{K}^0).$$

In other words, at sufficient distances from the point of production of the K^0 mesons an admixture of $\tilde{K}^0$ mesons appears in the beam: $K^0 \rightarrow \tilde{K}^0$. The presence of $\tilde{K}^0$ mesons can be detected, for example, by the occurrence of the reaction (1), which is characteristic of $\tilde{K}^0$ mesons. To observe this reaction a target B is placed in the beam. This sort of experiment was proposed by Pais and Piccioni in 1955.[2] Figure 2 shows the process as observed in a propane bubble chamber, and the transition $K^0 \rightarrow \tilde{K}^0$ can be clearly seen. The K^0 was produced by transfer of charge from a K^+ to a carbon nucleus; at some distance from the point where the K^0 is produced a reaction caused by $\tilde{K}^0$ is observed:

$$\tilde{K}^0 + P \rightarrow \Lambda^0 + \pi^+.$$

Similarly, if a beam of K_2^0 mesons is sent through matter, then owing to the difference in the ways the K^0 and $\tilde{K}^0$ mesons that make up the beam interact with the nucleons the composition of the beam will change. When the beam emerges from the target one can observe in it K_1^0 mesons which previously were not present in it. These remarkable properties of mutual interconversion are characteristic of neutral K mesons only, and are caused by the possibility of $K^0 \rightarrow \tilde{K}^0$ transitions.

The question may arise as to why one does not consider superpositions analogous to K_1^0 and K_2^0 for other neutral particles, for example

$$n_1 = \frac{1}{\sqrt{2}} (n + \tilde{n}) \quad \text{and} \quad n_2 = \frac{1}{\sqrt{2}} (n - \tilde{n}),$$

where n and $\tilde{n}$ are the wave functions of neutron and antineutron: unlike the states n and $\tilde{n}$, the states n_1 and n_2 do not have definite values of the baryon number, but like K_1^0 and K_2^0 they have definite values of the combined parity. The point is that the introduction of such states has no meaning, since there are no interactions that would lead to the decay of n_1 or n_2 into states with definite values of CP, for example into π mesons. Otherwise, as for the neutral K mesons, transitions $n \rightarrow \tilde{n}$ would be possible, which is absolutely forbidden by the law of conservation of baryon charge. Superpositions analogous to K_1^0 and K_2^0 of course are also meaningless for truly neutral particles, since for them particle and antiparticle are identical.

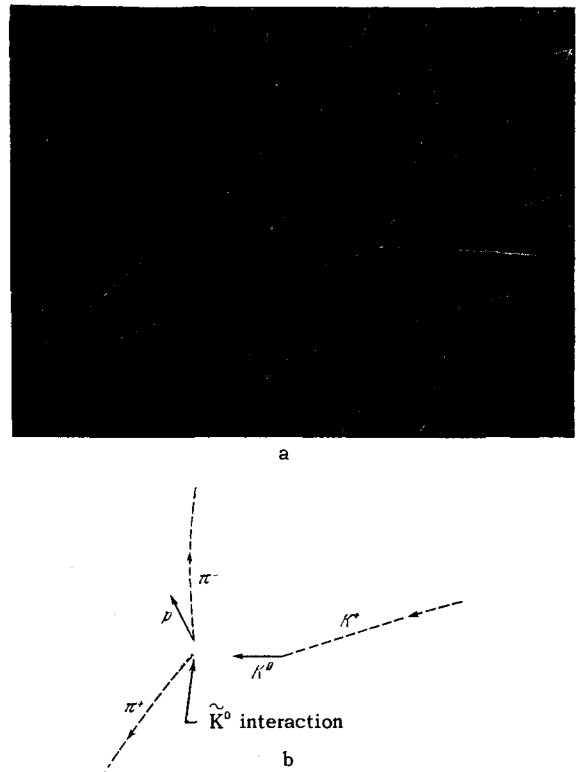

FIG. 2.a) Photograph of the process $K^0 \rightarrow \tilde{K}^0$, observed in a propane bubble chamber. The K^0 was produced from a K^+ by charge transfer to a carbon nucleus. The $\tilde{K}^0$ is revealed by the reaction $\tilde{K}^0 + p \rightarrow \Lambda + \pi^+$ (from [9]): b) diagram of the process seen in Fig. 2 a.

2. THE LIFETIMES OF K_1^0 AND K_2^0 MESONS

The difference between the lifetimes of K_1^0 and K_2^0 mesons is due to the fact that the decay processes available for these particles are different. Let us consider the possible π-meson decays of neutral K mesons:

$$K \rightarrow \begin{cases} \pi^+ + \pi^-, \\ 2\pi^0, \\ 3\pi^0, \\ \pi^+ + \pi^- + \pi^0. \end{cases}$$

The combined parity of the states $(2\pi^0)$ and $(\pi^+ + \pi^-)$ is positive [see Eq. (3)]. The state $(3\pi^0)$ has negative combined parity. In fact, owing to the fact that the π^0 mesons are identical the orbital state of the system $(3\pi^0)$ must be even: the CP parity of each π^0 meson is negative: $CP(\pi^0) = -\pi^0$. Therefore $CP(3\pi^0) = -3\pi^0$. The CP parity of the system $(\pi^+\pi^-\pi^0)$ is $(-1)^{l+1}$, where l is the orbital angular momentum of the π^0 meson relative to the center of mass of the system $(\pi^+\pi^-)$. This can be seen as follows. The CP parity of the π^0 meson is negative. The CP parity of the system $(\pi^+\pi^-)$ is always positive, since $P_{\pi^+\pi^-} = (-1)^L$, $C_{\pi^+\pi^-} = (-1)^L$, and $CP_{\pi^+\pi^-} = (-1)^{2L} = 1$. Here L is the relative angular momentum of the system $\pi^+\pi^-$ (Fig. 3). Thus the combined parity of the system $(\pi^+\pi^-\pi^0)$ is completely determined by the value of l: $CP_{\pi^+\pi^-\pi^0} = (-1)^{l+1}$. Obviously $l = L$, since the spin of the K meson is zero. We now write out the π-meson decays allowed for $K_1^0(CP = 1)$ and for K_2^0 $(CP = -1)$:

$$K_1^0 \to \begin{cases} 2\pi^0, \\ \pi^+ + \pi^-, \\ \pi^+ + \pi^- + \pi^0, \quad l = L = 1,\ 3,\ \ldots, \end{cases}$$

$$K_2^0 \to \begin{cases} 3\pi^0, \\ \pi^+ + \pi^- + \pi^0, \quad l = L = 0,\ 2,\ 4,\ \ldots \end{cases}$$

It follows that two-particle decay $(2\pi^0$ or $\pi^+ + \pi^-)$ can occur only from the state K_1^0: the K_2^0 meson can decay only into three π mesons. Therefore the lifetime of the K_1^0 meson against π-meson decay must be much shorter than that of the K_2^0 meson, since the phase volume for three-particle decay is much smaller than that for two-particle decay. The lifetimes $\tau(K_1^0)$ and $\tau(K_2^0)$ of these mesons against all possible decays depend also on the probabilities of leptonic decays,

$$K^0 \to e^+ + \nu + \pi^-, \qquad K^0 \to \mu^+ + \nu + \pi^-,$$
$$\widetilde{K}^0 \to e^- + \widetilde{\nu} + \pi^+, \qquad \widetilde{K}^0 \to \mu^- + \widetilde{\nu} + \pi^+.$$

A characteristic feature of these decays is that the final states do not have definite values of CP [for example, $CP(e^+ + \nu + \pi^-) = e^- + \widetilde{\nu} + \pi^+$], and consequently they are equally accessible for K_1^0 and K_2^0. The presence of the leptonic decays does not lead to equalization of the lifetimes of K_1^0 and K_2^0 mesons, however, because the probabilities of these decays are relatively small. According to the measurements of Luers and others [3] the ratio of the probability $\Gamma_2(L^\pm)$ for leptonic decays of K_2^0-to the probability $\Gamma_2(+-0)$ for the decay $K_2^0 \to \pi^+ + \pi^- + \pi^0$ is

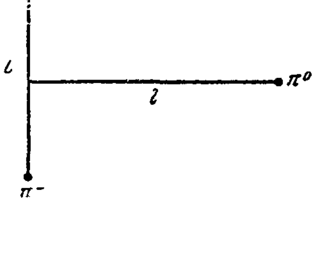

FIG. 3. Notation scheme for the orbital angular momenta of the system $(\pi^+\pi^-\pi^0)$ (see text).

$$\frac{\Gamma_2(L^\pm)}{\Gamma_2(+-0)} = 6.5 \pm 1.0. \qquad (6)$$

The lifetime of the K_1^0 meson has been determined experimentally in the bubble chamber from the flight distance (distance from point of production to point of decay) of neutral K mesons that decay into two π mesons. The value of τ_1 found from the results of several researches is [4]

$$\tau(K_1^0) = (1.00 \pm 0.04) \cdot 10^{-10} \text{ sec.}$$

Measurements of the lifetime of the K_2^0 meson can be made by studying three-particle decays of neutral K mesons at distances from the point of production which are large enough so that the K_1^0 component has completely decayed.* This is the way the most reliable values of τ_2 have been obtained. In the work of Alexander and others [5] measurements of τ_2 were made in a 180-centimeter hydrogen bubble chamber. Neutral K mesons were produced in the chamber itself by the action of π^- mesons: $\pi^- + p \to \Lambda + K^0$ $(P_{\pi^-} = 1.03$ BeV/c). In this work only those decays of K mesons were registered which were accompanied by an associated decay $\Lambda \to p + \pi^-$ visible in the chamber. The resulting possibility of a detailed kinematic analysis made possible a reliable selection of cases which were actually decays of neutral K mesons, and also an accurate knowledge of the "age" of the decaying K meson. For the determination of τ_2 cases were selected in which K mesons decayed with times of flight $3.4 \times 10^{-10} \le t \le 20.0 \times 10^{-10}$ sec (in the rest system of the K meson). In this way cases of K_1^0 decay were practically excluded. The value of τ_2 found in this work was

$$\tau(K_2^0) = \left(7.5 \begin{smallmatrix} +2.8 \\ -1.6 \end{smallmatrix}\right) \cdot 10^{-8} \text{ sec.}$$

This value of τ_2 is in good agreement with the value

$$\tau(K_2^0) = \left(8.1 \begin{smallmatrix} +3.3 \\ -2.4 \end{smallmatrix}\right) \cdot 10^{-8} \text{ sec,}$$

obtained in [6] by the method of direct measurement of the dependence of the number of K_2^0 decays on the distance traversed. Crawford and others [7] obtained the somewhat smaller value

$$\tau(K_2^0) = \left(3.6 \begin{smallmatrix} +1.4 \\ -1.0 \end{smallmatrix}\right) 10^{-8} \text{ sec.}$$

It must be remarked, however, that in this work the hydrogen bubble chamber used in the determination of τ was a small one (25 cm); as in [5], the K^0 mesons were produced in the chamber itself by the reaction $\pi^- + p \to \Lambda + K^0$. Thus the small size of the bubble chamber did not permit spatial separation of the K_1^0 and K_2^0 decays. Therefore in the determination of τ_2

*The intensity of the K_2^0 beam produced in this way in the latest experiments (L. B. Leĭpuner and others, preprint 1963) was $(2.5 \pm 0.6) \cdot 10^{-3}$ particle per square centimeter per pulse.

from the observed cases of three-particle decay of K mesons it was necessary also to know the ratio $\Gamma_1(L)/\Gamma_2(L)$ of the probabilities $\Gamma(L^\pm)$ of leptonic* decays of K_1^0 and K_2^0 mesons. Crawford et al.[7] assumed that $\Gamma_1(L)/\Gamma_2(L) = 1$. Their assumption of the value $\Gamma_1(L)/\Gamma_2(L) = 1$ is not in contradiction with the value

$$\frac{\Gamma_1(L)}{\Gamma_2(L)} = 3.5 \begin{matrix} +4 \\ -3 \end{matrix}$$

found in their work, and moreover follows from the Sakata model of the universal weak interaction. In the most recent papers,[5,8] however, there are indications that $\Gamma_1/\Gamma_2 = 9$ (with an error of the order of the quantity itself). These last results are very important in themselves, since they contradict the Sakata model; these problems will be considered in more detail in Sec. 7; at present we only point out that if we assume in accordance with the experimental data that the ratio is $\Gamma_1/\Gamma_2 = 9$, then the experiment of Crawford and others[7] leads to the value $\tau(K_2^0) = (8.2 \pm 3) \times 10^{-8}$ sec. The values of τ_2 obtained in [5,6] do not depend on any assumptions and can differ from the true value of τ_2 only owing to experimental errors.

If we assume that the values of $\tau(K_2^0)$ found in [5,6] are the most reliable, the ratio of the lifetimes of K_1^0 and K_2^0 mesons is found to be

$$\frac{\tau_2}{\tau_1} = \frac{7.8 \cdot 10^{-8}}{1.0 \cdot 10^{-10}} = 780.$$

3. THE DIFFERENCE OF THE MASSES OF K_1^0 AND K_2^0 MESONS

The possibility of the transition $K^0 \to \tilde{K}^0$ necessarily leads to a difference between the masses of K_1^0 and K_2^0. In fact, the variation with time of the states K^0 and $\tilde{K}^0$ can be written (if we neglect the decays of these particles)

$$-i\frac{\partial K^0}{\partial t} = mK^0 + \delta\tilde{K}^0,$$

$$-i\frac{\partial \tilde{K}^0}{\partial t} = m\tilde{K}^0 + \delta K^0,$$

where δ is the matrix element for the transition $K^0 \rightleftarrows \tilde{K}^0$. Using the expressions for the states K_1^0 and K_2^0 in terms of K^0 and $\tilde{K}^0$, we easily get

$$-i\frac{\partial K_1^0}{\partial t} = (m+\delta)K_1^0, \qquad -i\frac{\partial K_2^0}{\partial t} = (m-\delta)K_2^0,$$

i.e., the difference of the masses of K_1^0 and K_2^0 mesons is $\Delta m = 2\delta$. It is seen from this that $\Delta m = 0$ only for $\delta = 0$, that is, if the transition $K^0 \rightleftarrows \tilde{K}^0$ is forbidden. The difference of the masses of K_1^0 and K_2^0 mesons

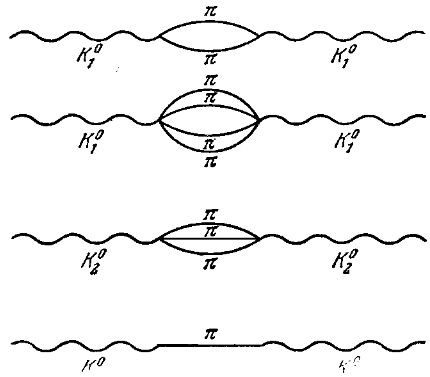

FIG. 4. Virtual states possible for K_1^0 and K_2^0 mesons.

can also be understood if we recall that the CP parities of these particles are different. Consequently the virtual states in which K_1^0 and K_2^0 mesons spend part of their time are also different: the virtual states accessible to the K_1^0 mesons are those with an even number of π mesons, and these states are not accessible to the K_2^0 meson (Fig. 4). This must lead to a difference between the masses. It is impossible, however, to calculate the magnitude of the mass difference Δm, or even to determine its sign, since at present there is no apparatus for calculating virtual interactions of strongly interacting particles. A qualitative estimate of the magnitude of Δm (but not of its sign) can be made from a consideration of the diagrams of Fig. 4. Obviously each conversion $K \to \pi$ is proportional to the weak-interaction constant G Since the diagrams have two vertices, we have $\Delta m \sim AG^2$. From dimensional considerations the constant A must be proportional to μ^5, where μ has the dimensions of mass. Taking $\mu = m_\pi$, we find $\Delta m \sim 10^{-5}$ eV; for $\mu = m_K$, we get $\Delta m = 10^{-2}$ eV. The quantity Δm is usually expressed in units $\hbar/c^2\tau_1 = 6 \times 10^{-6}$ eV, where τ_1 is the lifetime of the K_1^0 meson. The experimental determination of Δm (see below) has given the value $\Delta m \approx 10^{-5}$ eV $= 10^{-38}$ g. The determination of such a small mass difference is a brilliant example of the use of the wave properties of particles in a macroscopic experiment.*

Let us consider the idea of this experiment. Suppose that at the initial time $t = 0$ (near the target) there is a pure K^0 state. Let us denote by $K(t)$ the state of the system in question—the beam of neutral K mesons at the arbitrary time t (i.e., at various distances from the place where they are produced). Then for $t = 0$ we write $K(t)$ in the form $K(0) = K^0 = 2^{-1/2}(K_1^0 + K_2^0)$. At the time t we obviously have

*The quantity τ_2 is mainly determined by the leptonic decays of the K_2^0 mesons. The nonleptonic decays of K_2^0 can be taken into account with adequate accuracy by using the data [Eq. (6)] of Luers.[3]

*The experimentally determined value $\Delta m \sim 10^{-5}$ eV indicates that there is no interaction with $\Delta S = 2$, since with such an interaction the amplitude for the transition $K \to \tilde{K}$ would be proportional to G, not to G^2, and the magnitude of the mass difference would be $\Delta m \sim 10^{-2}$ eV.

$$K(t)=\frac{1}{\sqrt{2}}\left(K_1^0 e^{iE_1t-\frac{t}{2\tau_1\gamma}}+K_2^0 e^{iE_2t-\frac{t}{2\tau_2\gamma}}\right).$$

Here E_1 and E_2 are the respective energies of the K_1^0 and K_2^0 mesons, τ_1 and τ_2 are their lifetimes, and γ is the Lorentz factor of the beam of neutral K mesons. Using the expressions for K_1^0 and K_2^0

$$K_1^0=\frac{1}{\sqrt{2}}(K^0+\widetilde{K}^0) \quad \text{и} \quad K_2^0=\frac{1}{\sqrt{2}}(K^0-\widetilde{K}^0),$$

we find

$$K(t)=\frac{1}{2}\left[K^0\left(e^{iE_1t-\frac{t}{2\tau_1\gamma}}+e^{iE_2t-\frac{t}{2\tau_2\gamma}}\right)+\widetilde{K}^0\left(e^{iE_1t-\frac{t}{2\tau_1\gamma}}-e^{iE_2t-\frac{t}{2\tau_2\gamma}}\right)\right].$$

Thus the probabilities for observing the states K^0 and $\widetilde{K}^0$ in the beam at the time t are

$$\left.\begin{aligned}W(K^0)&=\left|e^{iE_1t-\frac{t}{2\tau_1\gamma}}+e^{iE_2t-\frac{t}{2\tau_2\gamma}}\right|^2\\&=\frac{1}{4}\left[e^{-\frac{t}{\tau_1\gamma}}+e^{-\frac{t}{\tau_2\gamma}}+2e^{-\frac{\tau_1+\tau_2}{2\tau_1\tau_2\gamma}t}\cos(\delta t)\right],\\W(\widetilde{K}^0)&=\left|e^{iE_1t-\frac{t}{2\tau_1\gamma}}-e^{iE_2t-\frac{t}{2\tau_2\gamma}}\right|^2\\&=\frac{1}{4}\left[e^{-\frac{t}{\tau_1\gamma}}+e^{-\frac{t}{\tau_2\gamma}}-2e^{-\frac{\tau_1+\tau_2}{2\tau_1\tau_2\gamma}t}\cos(\delta t)\right].\end{aligned}\right\} \quad (7)$$

Here $\delta = E_1 - E_2 \simeq m\Delta m/E$. The time dependences (7) of the probabilities $W(K^0)$ and $W(\widetilde{K}^0)$ are shown in Fig. 5. Figure 6 shows the time dependence $W(\widetilde{K}^0)$ obtained with various values of Δm. It can be seen from Figs. 5 and 6 that the possibility of $K^0 \to \widetilde{K}^0$ transitions leads to characteristic oscillations of the intensities $W(K^0)$ and $W(\widetilde{K}^0)$ as a beam of neutral K mesons travels through vacuum. The observation of these oscillations provides a possibility for determining the mass difference Δm of K_1^0 and K_2^0 mesons to a very high degree of accuracy. As can be seen from Fig. 6, one possible way to determine Δm is to measure the number of $\widetilde{K}^0$ mesons in a beam originally composed of K^0 mesons as a function of the "age" of the K mesons. One can identify the $\widetilde{K}^0$ mesons from the reaction (1) which is characteristic of these particles. Such an experiment was first done by Camerini and others [9] in 1960. Subsequently [10] these authors, using the same method, obtained a somewhat more precise value of Δm. In the experiments of Camerini and others [9,10] the K^0 mesons were produced in a propane bubble chamber in a charge-transfer reaction of K^+ mesons, $K^+ + n \to K^0 + P$. Also photographs were selected in which the K meson produced in this reaction caused an interaction in the K^0 state, i.e., gave rise to a Λ or Σ hyperon:

$$\widetilde{K}^0+P \begin{cases}\nearrow \Lambda+\pi^+,\\ \searrow \Sigma^+ +\pi^0.\end{cases} \quad (8)$$

A typical photograph of such an event is shown in Fig. 2. Measurements of times of flight of the neutral K mesons from the point of production (i.e., from where the K^+ meson disappeared) to the point of production of a hyperon by the $\widetilde{K}^0$ meson were used to construct the distribution function $I_{\widetilde{K}^0}(t)$. This quantity $I_{\widetilde{K}^0}(t)$, the number of $\widetilde{K}^0$ mesons interacting during the time Δt, bears the following relation to $W_{\widetilde{K}^0}(t)$:

$$I_{\widetilde{K}^0}(t)\,\Delta t=W_{\widetilde{K}^0}(t)\,\sigma n\,\frac{p}{m}\,\Delta t, \quad (9)$$

where n is the number of nuclei per unit volume, σ is the cross section for interaction of $\widetilde{K}^0$ mesons with these nuclei, and p is the momentum of the K mesons. $\Delta x = (p/m)\Delta t$ is the distance traversed by the $\widetilde{K}^0$ mesons in the time Δt; $\sigma n \Delta x$ is the probability for a $\widetilde{K}^0$ meson to undergo interaction in the time Δt. In [9,10] the reactions (8) of interaction of $\widetilde{K}^0$ mesons were observed in a propane (C_3H_8) bubble chamber; the relation (9) is then written in the form

$$I_{\widetilde{K}^0}(t)\,\Delta t=W_{\widetilde{K}^0}(t)\,[3\sigma_C(p)+8\sigma_H(p)]\,\frac{N_0\varrho}{A}\,\frac{p}{m}\,\Delta t. \quad (10)$$

Here $\sigma_C(p)$ and $\sigma_H(p)$ are the cross sections for the reactions (8) on carbon and hydrogen nuclei, N_0 is Avogadro's number, ρ is the density of the propane, and A is the molecular weight of propane. Figure 7 shows the experimental distribution $I_{K^0}(t)$ as found in [10], and also theoretical distributions calculated from Eq. (10) for the values $\Delta m = 1.5\hbar/\tau_1$ and $\Delta m = 0.75\hbar/\tau_1$. It can be seen from this figure that the curve of $I_{\widetilde{K}^0}(t)$ for the value $\Delta m = 1.5\hbar/\tau_1$ is in good agreement with experiment; Fig. 8 shows the corresponding plausibility function $P(\Delta m)$. From the function $P(\Delta m)$ shown in Fig. 8 it follows that the experi-

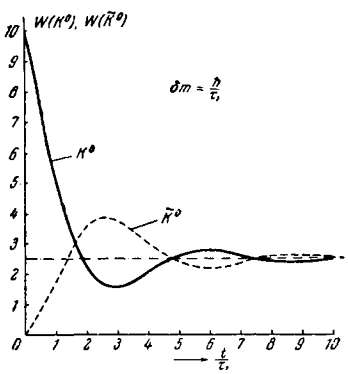

FIG. 5. Time dependences of the intensities $W(K^0)$ and $W(\widetilde{K}^0)$ in a beam which is a pure K^0 state at time t = 0.

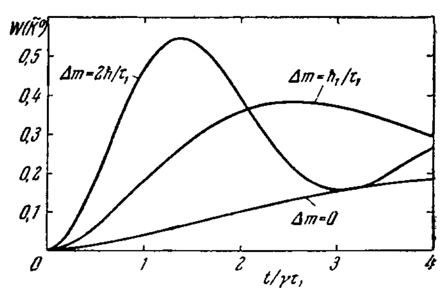

FIG. 6. The time dependence $W_{\widetilde{K}^0}(t)$ in a beam initially composed of K^0 mesons, calculated for various values of Δm.

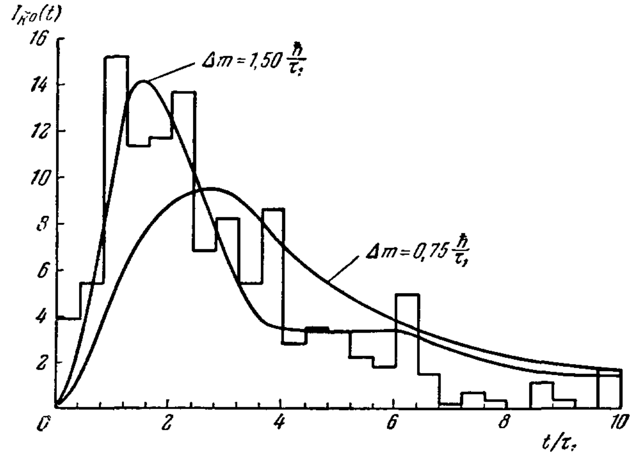

FIG. 7. Distributions of $I_{\tilde{K}^0}(t)$ as measured in the experiment of Camerini and others[10]. The curves are the theoretical distributions of $I_{\tilde{K}^0}(t)$ for the values $\Delta m = 1.5\,\hbar/\tau$ and $\Delta m = 0.75\,\hbar/\tau$.

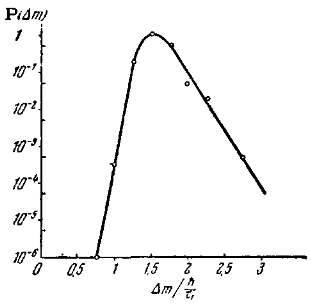

FIG. 8. The function $P(\Delta m)$ for the agreement between the experimental distribution $I_{\tilde{K}^0}(t)$ (see Fig. 7) and the theoretical distribution of Eq. (9) for various values of m (according to the data of[10]).

mental distribution $I_{\tilde{K}^0}(t)$ found in this work corresponds to the value

$$\Delta m = (1.5 \pm 0.2)\frac{\hbar}{\tau_1} = (0.9 \pm 0.12)\cdot 10^{-5}\text{ eV}.$$

A similar experiment was made in 1961 by Fitch and others.[11] Their result ($\Delta m = (1.9 \pm 0.3)\hbar/\tau_1$) agrees within the limits of error with the value of Δm found in [10].

Another method for determining the quantity Δm, which is also based on the use of the wave properties of beams of neutral K mesons, will be considered in the next section.

4. THE GENERATION OF K_1^0 MESONS IN A BEAM OF K_2^0 MESONS

It follows from the difference between the lifetimes of K_1^0 and K_2^0 mesons that at a sufficiently large distance from the place where neutral K mesons are produced the K_1^0 component has decayed completely and the beam will consist of K_2^0 mesons only. If, however, we send the K_2^0 mesons through matter, an admixture of K_1^0 mesons appears again in the beam. In fact, the pure K_2^0 state contains K^0 and $\tilde{K}^0$ components in equal amounts:

$$K_2^0 = \frac{1}{\sqrt{2}}(K^0 + \tilde{K}^0).$$

In passing through matter more of the $\tilde{K}^0$ component "drops out" of the beam, because of its stronger interaction. The resulting change of composition of the beam is equivalent to the appearance of an admixture of K_1^0 mesons. The K_1^0 mesons that appear in this way move in exactly the same direction as the primary beam of K_2^0 mesons. The interaction of the K mesons with matter also leads to the appearance of "different" K_1^0 mesons, whose direction of motion does not coincide with that of the primary beam. These K_1^0 mesons are the K_1^0 component of the elastically scattered K^0 and $\tilde{K}^0$ mesons which "compose" the primary K_2^0 beam. Since K^0 and $\tilde{K}^0$ mesons are scattered by nuclei (and by nucleons) with different amplitudes, it is obvious that the scattered wave must contain K_1^0 mesons. The scattering of the K^0 and $\tilde{K}^0$ mesons which leads to the generation of K_1^0 particles at angles $\theta > 0°$ can occur owing to two processes: diffraction scattering by nuclei, and elastic scattering by individual nucleons. In what follows we shall consider these processes separately. The process of production of K_1^0 mesons in the direction of the beam is coherent, in the sense that the total effect from the interactions with the different nuclei or nucleons in the matter is described not by adding up the corresponding intensities, but by adding the amplitudes. Therefore the amplitude for the generation of K_1^0 mesons in this process is proportional to the number N of nucleons in unit volume, and the corresponding probability is proportional to N^2. The regeneration of K_1^0 mesons owing to diffraction scattering of K^0 and $\tilde{K}^0$ by nuclei and the elastic scattering by individual nucleons is not a coherent process. The intensity of the K_1^0 mesons generated in this way will be proportional to the first power of N. The pronounced difference between the angular distributions of the coherent and incoherent processes of regeneration of K_1^0 mesons makes it experimentally possible to distinguish them reliably. We shall show later how a comparison of the probabilities of these two processes can be used to determine the magnitude of the mass difference Δm between K_1^0 and K_2^0 mesons. At present we only point out a factor which makes the description of the process of generation of K_1^0 mesons in the passage of K_2^0 mesons through matter a complicated one as compared with the description of the process of generation of $\tilde{K}^0$ mesons in a beam of K^0 mesons, as given in Sec. 3. The complication is due to the fact that whereas the generation of $\tilde{K}^0$ in a K^0 beam occurs in vacuum owing to the more rapid decay of K_1^0 as compared with K_2^0 and to the mass difference of K_1^0 and K_2^0, the generation of K_1^0 in a K_2^0 beam occurs as the beam passes through matter, so that besides the reactions which conserve the combined parity (decay of K_1^0 and K_2^0) there are also processes which conserve strangeness (interactions of the K^0 and $\tilde{K}^0$ mesons with matter).

We proceed to a detailed consideration of the processes of coherent and incoherent production of K_1^0 mesons in the passage of K_2^0 mesons through matter.

The coherent generation of K_1^0 mesons is due to the change of composition of the beam owing to the decay of the K mesons and their interaction with matter. Let us first consider the general case of change of composition of a beam which initially consists of an arbitrary superposition $\psi(K_1^0, K_2^0)$ of the states K_1^0 and K_2^0. We denote by $\psi(x)$ the wave function of the system $(K_1^0 K_2^0)$ as a function of the distance x which the beam of neutral K mesons has travelled through the matter. $\psi(x)$ can be represented as a superposition of K^0 and $\tilde{K}^0$ states, or of K_1^0 and K_2^0 states:

$$\left.\begin{aligned} \psi(x) &= a(x) K^0 + \bar{a}(x) \tilde{K}^0, \\ \psi(x) &= a_1(x) K_1^0 + a_2(x) K_2^0; \end{aligned}\right\} \tag{11}$$

it follows from Eq. (4) that

$$\left.\begin{aligned} a_1(x) &= \frac{1}{\sqrt{2}} [a(x) + \bar{a}(x)], \\ a_2(x) &= \frac{1}{\sqrt{2}} [a(x) - \bar{a}(x)]. \end{aligned}\right\} \tag{12}$$

We shall find the differential equations that describe the changes of the amplitudes a_1 and a_2 for the various values of x. In the absence of interaction with matter the composition of the beam will change only owing to the decay of the K_1^0 and K_2^0 mesons. Accordingly the expressions for the change of the amplitudes a_1 and a_2 with time can be written

$$\frac{da_1}{dt} = -i\left(E_1 - \frac{i}{2\gamma\tau_1}\right) a_1$$

or

$$\frac{da_1}{dx} = -\frac{i}{v}\left(E_1 - \frac{i}{2\gamma\tau_1}\right) a_1, \tag{13}$$

since dx = v dt. Similarly

$$\frac{da_2}{dx} = -\frac{i}{v}\left(E_2 - \frac{i}{2\gamma\tau_2}\right) a_2.$$

Here v and E are the speed and energy of the K mesons, and $\gamma\tau$ is the lifetime in the laboratory coordinate system.

Let us consider the changes of the amplitudes a_1 and a_2 "along the beam" owing to the interaction of the K mesons with matter. Since the reactions in which the K mesons interact (scattering and absorption) occur with conservation of strangeness, the eigenfunctions in these processes before the interaction are the states K^0 and $\tilde{K}^0$. The changes of the amplitudes a and $\bar{a}$ corresponding to these states [see Eq. (11)] can be written in the well known way [14] in terms of complex indices of refraction expressed in terms of the forward scattering amplitudes:

$$n = 1 + \frac{2\pi N f(0)}{k^2}$$

for the K^0 mesons and

$$\bar{n} = 1 + \frac{2\pi N \bar{f}(0)}{k^2}$$

for the $\tilde{K}^0$ mesons. Here f(0) and $\bar{f}(0)$ are the values of the scattering amplitudes of the K^0 and $\tilde{K}^0$ mesons at the angle $\theta = 0$, N is the number of nuclei per unit volume, and k is the momentum of a K meson. Then the changes of the amplitudes a(x) and $\bar{a}(x)$ can be written:

$$\frac{da}{dx} = ikna, \qquad \frac{d\bar{a}}{dx} = ik\bar{n}\bar{a}.$$

The corresponding changes of the amplitudes a_1 and a_2 can be obtained by using the relations (12):

$$\left.\begin{aligned} \frac{da_1}{dx} &= \frac{i}{2}(n+\bar{n}) k a_1 + \frac{i}{2}(n-\bar{n}) k a_2, \\ \frac{da_2}{dx} &= -\frac{i}{2}(n-\bar{n}) k a_1 + \frac{i}{2}(n+\bar{n}) k a_2. \end{aligned}\right\} \tag{14}$$

Adding the expressions (13) and (14) for da_1/dx and da_2/dx, we get

$$\left.\begin{aligned} \frac{da_1}{dx} &= ia_1\left[\frac{1}{2}(n+\bar{n})k - \frac{1}{v}\left(E_1 - \frac{i}{2\gamma\tau_1}\right)\right] + \frac{i}{2}k(n-\bar{n}) a_2, \\ \frac{da_2}{dx} &= -\frac{i}{2}k(n-\bar{n}) a_1 + ia_2\left[\frac{1}{2}(n+\bar{n})k - \frac{1}{v}\left(E_2 - \frac{i}{2\gamma\tau_2}\right)\right]. \end{aligned}\right\} \tag{15}$$

These equations describe the changes of the function $\psi(x)$ along the beam, i.e., owing to coherent processes. The equations (15) can be solved for arbitrary initial conditions, that is, for any values of a_1 and a_2 at x = 0. The solutions so obtained determine the composition of a beam of neutral K mesons moving in the original direction after passing through the distance x in the matter.

Let us now consider the case of interest to us, in which the initial beam is in the pure K_2^0 state [$a_1(0) = 0$, $a_2(0) = 1$]. We are interested in the amplitude a_1 (x = L), which determines the number of coherently generated K_1^0 mesons after passage of the beam through a plate of thickness x = L. The solution of (15) can be decidedly simplified in the case of a plate which is thin enough so that the mean free path against interaction of the K mesons is much larger than L. Under these conditions the probability of generation of K_1^0 mesons is relatively small, and also we can practically neglect the interaction with the matter of the K_1^0 mesons that are produced. It must be pointed out that the case of a "thick" plate cannot be realized in practice because of the small lifetime of K_1^0 mesons. This can be seen from the following numbers: the mean free path of the K_1^0 meson against decay is about 3 cm, whereas the free path against the interaction is ~ 40 cm (in carbon). Therefore the K_1^0 mesons will decay rather than interact. It follows that the term $(i/2) \times k(n - \bar{n}) a_1$ in the expression (15) for da_2/dx can be omitted. In solving the equations (15) we can also set $\tau_2 = \infty$ (see Sec. 2). In this approximation the amplitude $a_1(x)$ which satisfies (15) is of the form

$$a_1(x) = \frac{vk(n-\bar{n})}{2\delta - \frac{i}{\gamma\tau_1}} e^{i\left(\frac{n+\bar{n}}{2}k - \frac{E}{v}\right)x}\left[1 - e^{-\left(\frac{i\delta}{v} + \frac{1}{2\gamma\tau_1 v}\right)x}\right]. \tag{16}$$

Here $\delta = E_1 - E_2 \approx m\Delta m/E$. The number of coherently generated K_1^0 mesons in the beam after passage through a plate of thickness L is given by the expression

$$I(K_1^0) = |a_1(L)|^2 = \frac{\gamma^2\tau_1^2 v^2 k^2 (n-\bar{n})^2}{4(\delta\gamma\tau_1)^2+1} \left| e^{\frac{i\delta L}{v}} - e^{-\frac{L}{2\gamma v\tau_1}} \right|^2. \quad (17)$$

Let us introduce the dimensionless quantities

$$l = \frac{L}{v\gamma\tau_1} = \frac{L}{\Lambda}, \qquad \Delta = (m_1 - m_2)\tau_1 = \frac{E\delta}{m}\tau_1. \quad (18)$$

In these variables the intensity $I(K_1^0)$ can be written in the form

$$I(K_1^0) = \frac{\left(\frac{2\pi}{k}\right)^2 N^2\Lambda^2 |f(0)-\bar{f}(0)|^2}{1+4\Delta^2} \left| e^{i\Delta l} - e^{-\frac{l}{2}} \right|^2. \quad (19)$$

It can be seen from Eq. (19) that the intensity $I(K_1^0)$ is proportional to N^2, i.e., to the square of the nuclear density of the matter, as must be the case for a coherent process. The relation (19) describes the intensity of the K_1^0 mesons generated coherently in the passage of a beam of K_2^0 mesons through matter, and thus completely solves the problem we had set for ourselves. Let us now go on to the treatment of the incoherent generation of K_1^0 mesons.

The incoherent generation of K_1^0 mesons occurs owing to two processes: the diffraction scattering of K^0 and $\tilde{K}^0$ by nuclei and the elastic scattering by individual nucleons. The K_1^0 mesons so produced move in the direction of the scattered K^0 and $\tilde{K}^0$ waves, and thus can be distinguished experimentally from the coherently produced K_1^0 mesons, whose direction of motion coincides with that of the primary beam. Figure 9 shows the angular distribution $F(\cos\theta)$ of the K_1^0 mesons generated by K_2^0 mesons in passing through an iron plate.[13] The distribution $F(\cos\theta)$ shows a distinct peak at $\theta = 0$, which is formed by the coherently produced K_1^0 mesons. At somewhat larger angles $F(\cos\theta)$ consists mainly of K_1^0 mesons which arise in the process of diffraction scattering of the K^0 and $\tilde{K}^0$ waves by nuclei. Finally, at angles $\theta > 10°$ the main contribution to $F(\cos\theta)$ is that of the K_1^0 mesons that arise in the process of elastic scattering of the K^0 and $\tilde{K}^0$ waves by individual nucleons. Let us consider the process of diffraction generation of K_1^0 mesons at small angles. The cross sections for diffraction scattering of K^0 and $\tilde{K}^0$ mesons by nuclei are described by the respective amplitudes $f(0)$ and $\bar{f}(0)$. Since the primary beam was in the K_2^0 state,

$$K_2^0 = \frac{1}{\sqrt{2}}(K^0 - \tilde{K}^0),$$

the diffraction-scattered wave can obviously be represented in the form

$$K_{\text{diff}} = \frac{1}{\sqrt{2}}\left\{f(0)\frac{1}{\sqrt{2}}(K_1^0+K_2^0) - \bar{f}(0)\frac{1}{\sqrt{2}}(K_1^0-K_2^0)\right\}$$
$$= \frac{1}{2}\{[f(0)-\bar{f}(0)]K_1^0 + [f(0)+\bar{f}(0)]K_2^0\}.$$

Thus the amplitude of the diffraction-generated K_1^0 mesons is

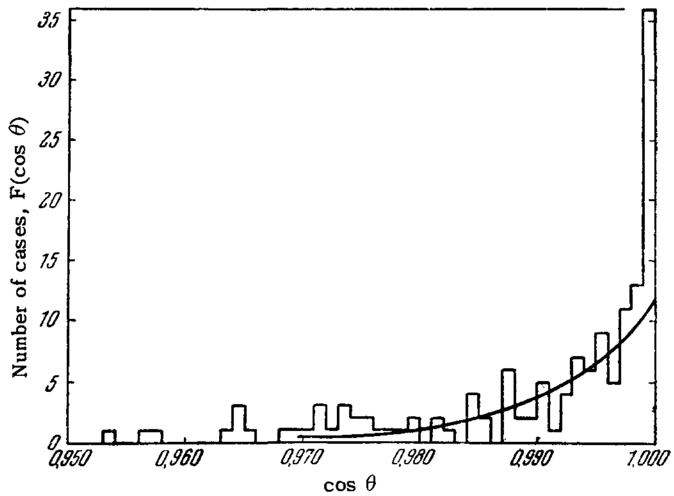

FIG. 9. Angular distribution of the K_1^0 mesons generated in a beam of K_2^0 mesons in its passage through an iron plate, in the experiment of Good and others.[13] The data plotted give the sum of the distributions $F(\cos\theta)$ for plates of thicknesses 3.6 and 14.5 cm. The smooth curve corresponds to the distribution $F_{\text{diff}}(\cos\theta)$ of the diffraction-generated K_1^0 mesons, with normalization adjusted to the experimental data.

$$f_{21} = \frac{1}{2}[f(0) - \bar{f}(0)].$$

The amplitude f_{21} determines the number of K_1^0 mesons produced in the layer dx and passing on through the thickness $L - x$ in the direction of the incident beam:

$$d\left(\frac{dI_{\text{diff}}(K_1^0)}{d\Omega}\right) = |f_{21}|^2 N e^{-\frac{L-x}{v\gamma\tau_1}} dx. \quad (20)$$

Here N is the nuclear density of the matter and Ω is the solid angle. Integrating the expression (20) over x and using the notations (18), we get

$$\frac{dI_{\text{diff}}(K_1^0)}{d\Omega} = |f_{21}|^2 N\Lambda(1 - e^{-l}). \quad (21)$$

The incoherent generation of K_1^0 mesons owing to elastic scattering of K^0 and $\tilde{K}^0$ mesons by individual nucleons has a realtively broad angular distribution and makes no appreciable contribution to the number of K_1^0 mesons moving in directions at small angles $(\theta \approx 0°)$ with the beam.

The study of the coherent generation of K_1^0 mesons in a beam of K_2^0 particles can be used to determine the mass difference of K_1^0 and K_2^0 mesons. This follows directly from the relation (19). It is also seen from Eq. (19) that the intensity of the coherently generated K_1^0 mesons depends on the amplitudes $f(0)$ and $\bar{f}(0)$, which at present are not very accurately known. We can eliminate the unknown amplitudes f and $\bar{f}$ if we take the ratio of the intensities of the coherent and incoherent processes of production of K_1^0 mesons at small angles $(\theta \approx 0°)$ with the primary beam. From Eqs. (19) and (21) it follows that this ratio is given by

$$R = \frac{4N\Lambda\left(\frac{2\pi}{k}\right)^2}{(1+4\Delta^2)(1-e^{-l})} \left| e^{i\Delta l} - e^{-\frac{l}{2}} \right|^2. \quad (22)$$

For a plate thickness L much larger than the life-path $\Lambda = v\gamma t$ of the K_1^0 meson, $l = L/\Lambda \to \infty$, the formula (22) takes a particularly simple and perspicuous form

$$R(l \to \infty) = \frac{16\pi^2 N\Lambda\lambda^2}{1+4\Delta^2}. \qquad (22a)$$

This way of determining Δm was proposed in 1958 by M. Good.[12] The experiment was done in 1961 by R. Good and others.[13] In this experiment a well collimated beam of K_2^0 mesons passed through a bubble chamber, inside which an iron (or lead) plate was placed in the path of the K_2^0 mesons (Fig. 10). The angular distribution $F(\cos\theta)$ of the K_1^0 mesons produced in this plate and registered by decays into two mesons is shown in Fig. 9. This distribution $F(\cos\theta)$ clearly shows a peak at angles $\theta = 0°$ formed by the coherently generated K_1^0 mesons. At somewhat larger angles $F(\cos\theta)$ is made up mainly of K_1^0 mesons produced owing to diffraction scattering of K^0 and $\tilde{K}^0$ by nuclei. Finally, at angles $\theta > 10°$ the main contribution to $F(\cos\theta)$ is that of K_1^0 mesons that arise in processes of elastic scattering of K^0 and $\tilde{K}^0$ waves by individual nucleons. It can be seen from Fig. 9 that the ratio R for the coherently produced and diffraction-produced K_1^0 mesons can be rather reliably determined. Equating the ratio R_{exp} found in this way to the theoretical value (22), the authors were able to obtain the value

$$\Delta m = \left(0.84\,{}^{+0.29}_{-0.22}\right)\frac{\hbar}{\tau_1}.$$

The values of Δm obtained in this and other researches are shown in Table I.

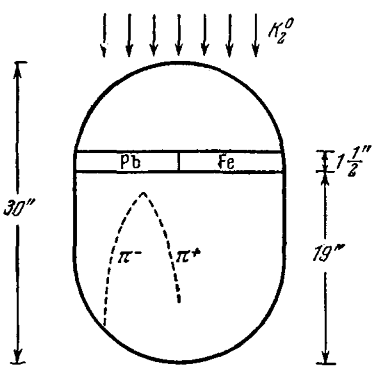

FIG. 10. Schematic representation of the propane bubble chamber in the experiment of Good and others.[13]

Table I. Values of the mass difference Δm of K_1^0 and K_2^0 mesons found from the data of various papers

Method	Δm in units $\hbar/\tau = 6\times10^{-6}$ eV	Reference
The distribution W (t) [see Eq. (7)]	1.9±0.3	11
	1.5±0.2	10
Coherent generation of K_1^0 in a K_2^0 beam	$0.84\,{}^{+0.29}_{-0.22}$	13

5. WHICH IS HEAVIER: K_1^0 OR K_2^0?

The present theory of weak interactions does not give any answer to the question: which is heavier, the K_1^0 meson or the K_2^0 meson? In the experiments on the difference between the masses of K_1^0 and K_2^0 which we have discussed it was possible to determine only the absolute value of Δm. For the determination of the sign of Δm other experiments have been proposed, which are also based on the use of the wave properties of beams of neutral K mesons. As yet, however, there are no experimental results. One such proposed experiment is as follows.[15] A beam of K_2^0 mesons is sent through two thin plates A and B, which are made of different substances and separated by the distance $x = vt$, where v is the speed of the K_2^0 mesons. As the result of passage through plate A there appears in the beam an admixture of K_1^0 mesons which have been coherently produced and are moving in the same direction:

$$K_2^0 \to a_2 K_2^0 + a_1 K_1^0. \qquad (23)$$

The dependence of the amplitudes a_1 and a_2 on the thickness of the plate is described by Eq. (15). For the case of thin plates, for which the admixture of K_1^0 mesons that arises in passage through the plate is small, the relation (23) can be written approximately in the form

$$K_2^0 \to K_2^0 + a_1 K_1^0 \qquad (a_1 \ll 1). \qquad (24)$$

The expression for the amplitude a_1 has been obtained earlier [see Eq. (16)]. It follows from Eq. (16) that for thin plates ($kL \ll 1$, where L is the thickness of the plate) the amplitude $a_1(L)$ of the K_1^0 mesons coherently produced in plate A is given by

$$a_1(L) \approx \frac{vkL(n_A - \bar{n}_A)}{2} \equiv \alpha_A e^{i\varphi_A}.$$

Here α_A and φ_A are real numbers which characterize the properties of plate A (we recall that the indices of refraction n and $\bar{n}$ are complex numbers). At the position of plate B the amplitude of the K_1^0 mesons produced in plate A is written

$$\alpha_A e^{i\varphi_A} e^{im_1 t - \frac{t}{2\tau_1}}, \qquad (25)$$

where t is the time of flight of the K mesons through the distance $x = vt$ between plates A and B. In plate B the K_2^0 mesons of the beam* generate a new "portion" of coherent K_1^0 mesons with the amplitude

$$\alpha_B e^{i\varphi_B} e^{im_2 t - \frac{t}{2\tau_2}}. \qquad (26)$$

Thus the amplitude a_1 of state K_1^0 beyond plate B is the sum of the amplitudes (25) and (26):

*Because the thickness L is small the composition of the beam changes only slightly in passing through plates A and B, and is still mainly K_2^0 mesons.

$$a_1 = a_A e^{i\varphi_A} e^{im_1 t - \frac{t}{2\tau_1}} + a_B e^{i\varphi_B} e^{im_2 t - \frac{t}{2\tau_2}}.$$

The intensity $I(K_1^0)$ of the K_1^0 mesons beyond the second plate is determined by the square of the absolute value of the amplitude a_1:

$$I(K_1^0) = |a_1|^2 = a_A^2 e^{-\frac{t}{\tau_1}} + a_B^2 e^{-\frac{t}{\tau_2}} + 2a_A a_B e^{-\frac{t}{2\tau_1} - \frac{t}{2\tau_2}} \cos(\Delta\varphi + \Delta m \cdot t). \quad (27)$$

Here $\Delta\varphi = \varphi_A - \varphi_B$ and $\Delta m = m_1 - m_2$. By observing the oscillations of $I(K_1^0)$ described by the term $\sim \cos(\Delta\varphi + \Delta m \cdot t)$ in the expression (27) as a function of the distance $x = vt$ between the plates, one can in principle find the sign of Δm. To do so, however, one must know the quantity $\Delta\varphi = \varphi_A - \varphi_B$. The phases φ_A and φ_B must be determined independently, for example from experiments on the scattering of K^0 and $\widetilde{K}^0$ mesons by the nuclei of which plates A and B are composed.

Another method for determining the sign of the mass difference Δm has been proposed by U. Camerini and others.[16] Figure 11 illustrates the idea of this experiment. A beam of neutral K mesons, which is initially in the pure K^0 state, undergoes scattering by a nucleus (or a nucleon) after the time t'. Among the scattered neutral K mesons one selects the cases of decay into two π mesons, i.e., one selects the K_1^0 component. The dependence of the intensity of the K_1^0 component on the time t' is different for the cases $m_2 > m_1$ and $m_2 < m_1$, where m_1 and m_2 are the masses of K_1^0 and K_2^0 mesons. In fact, the amplitude of the beam of neutral K mesons at the point of scattering "2" (see Fig. 11) can be written

$$K(t') = \frac{1}{\sqrt{2}} \left[\frac{K^0 + \widetilde{K}^0}{\sqrt{2}} e^{im_1 t' - \frac{t'}{2\tau_1}} + \frac{K^0 - \widetilde{K}^0}{\sqrt{2}} e^{im_2 t' - \frac{t'}{2\tau_2}} \right].$$

In the process of scattering at the point "2" the amplitude $K(t')$ is changed in the following way:

$$K(t') \to K(t')_{\rm scat} = \frac{1}{2} [fK^0 (e^{im_1 t' - \frac{t'}{2\tau_1}} + e^{im_2 t' - \frac{t'}{2\tau_2}}) + \bar{f}\widetilde{K}^0 (e^{im_1 t' - \frac{t'}{2\tau_1}} - e^{im_2 t' - \frac{t'}{2\tau_2}})],$$

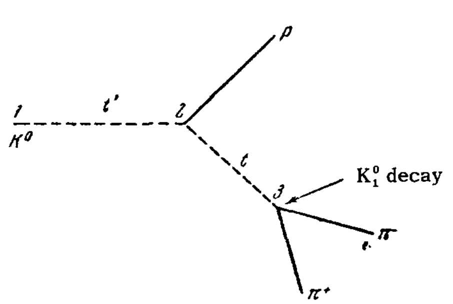

FIG. 11. Scheme of experiment proposed by Camerini and others[16] for the determination of the sign of the mass difference of K_1^0 and K_2^0 mesons.

where f and $\bar{f}$ are the respective scattering amplitudes of the K^0 and $\widetilde{K}^0$ components. From this we can easily find the intensity $I(K_1^0)$ of the K_1^0 component at the point of the decay "3"

$$I(K_1^0) = \frac{1}{8} e^{-\frac{t}{\tau_1}} \{ (|f|^2 + |\bar{f}|^2)(e^{-\frac{t'}{\tau_1}} + e^{-\frac{t'}{\tau_2}}) + 2(|f|^2 - |\bar{f}|^2) e^{-\frac{\tau_1+\tau_2}{2\tau_1\tau_2} t'} \cos(\Delta m t') + 2\,\mathrm{Re}(f\bar{f}^*)(e^{-\frac{t'}{\tau_1}} - e^{-\frac{t'}{\tau_2}}) + 4\,\mathrm{Im}(f\bar{f}^*) e^{-\frac{\tau_1+\tau_2}{2\tau_1\tau_2} t'} \sin(m_2 - m_1) t' \}.$$

The expression for $I(K_1^0)$ depends on $\sin(m_2 - m_1)t'$, and thus is different for the cases $\Delta m = m_2 - m_1 > 0$ and $\Delta m < 0$. At the same time, it can be seen from the expression for $I(K_1^0)$ that to determine the sign of Δm it is necessary to know the amplitudes f and $\bar{f}$, which at present are not well enough known. It is interesting to point out that $I(K_1^0)$ has only an exponential dependence on the time t: the oscillating terms in the expression for $I(K_1^0)$ are functions of the time t' only. A picture taken with a propane bubble chamber of an event in which a K^0 meson is scattered by a proton and then undergoes $K \to 2\pi$ decay is shown in Fig. 12.

6. THE WAVE PROPERTIES OF A SYSTEM OF NEUTRAL K MESONS

Interesting features of the wave properties of the system $K^0\widetilde{K}^0$ have been considered by V. I. Ogievetskiĭ, É. O. Okonov, and M. I. Podgoretskiĭ.[17] A $K^0\widetilde{K}^0$ pair can be produced, for example, in the reaction $\pi^- + p \to K^0 + \widetilde{K}^0 + n$. The system $K^0\widetilde{K}^0$ has the definite strangeness $S = 0$, and also the definite value of the combined parity $CP = +1$. This last is because under charge conjugation C a boson-antiboson system acquires the factor $(-1)^l$: this same factor also appears as the result of space inversion P. Thus the CP transformation leads to multiplication of the wave function of the K^0K^0 system by $(-1)^{2l} = +1$.

The decay of the system $K^0\widetilde{K}^0$ can occur according to the schemes $K_1^0K_1^0$, $K_2^0K_2^0$, or $K_1^0K_2^0$. Because they are composed of identical particles, the systems $K_1^0K_1^0$ and $K_2^0K_2^0$ always transform with an even value of the orbital angular momentum l. Therefore the combined parity of such pairs is always +1. The system $K_1^0K_2^0$ can transform with any value of l, and therefore its combined parity is $CP = (-1)^{l+1}$. From this follows the extremely interesting situation in which the decay scheme ($K_1^0K_1^0$, $K_2^0K_2^0$, or $K_1^0K_2^0$) is a detector of the parity of the orbital angular momentum of the system $K^0\widetilde{K}^0$. For example, if the $K^0\widetilde{K}^0$ pair is produced near threshold, so that $l = 0$, the decay can occur only according to the schemes $K_1^0K_1^0$ or $K_2^0K_2^0$, but not according to the scheme $K_1^0K_2^0$. The paper cited gives a detailed study of the change with time of the composition of a beam of two neutral K mesons which began as $K^0\widetilde{K}^0$ pairs. In such beams there arise characteristic

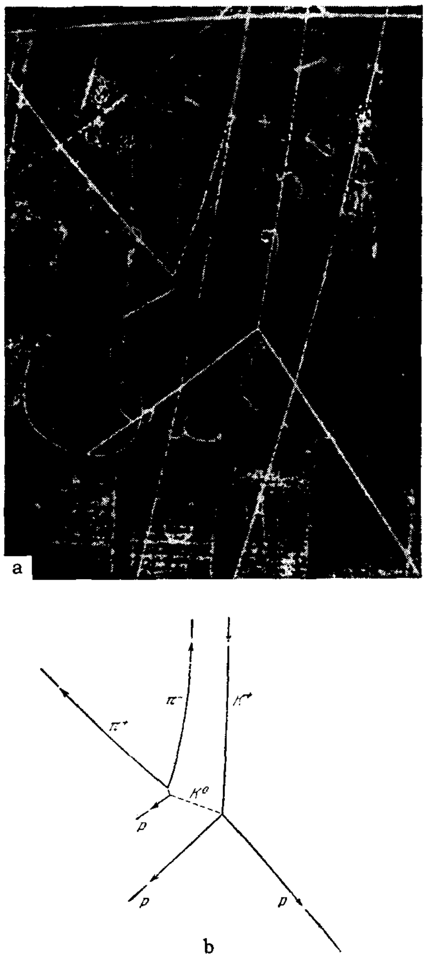

FIG. 12. a) Photograph showing scattering of a K^0 meson by a proton and its subsequent 2π decay. The time between the production of the K^0 meson and its scattering is 1.1×10^{-10} sec. (from[16]) b) Diagram of the process shown in Fig. 12 a.

beats, whose period does not depend solely on the mass difference Δm of K_1^0 and K_2^0 mesons. By studying such beats one can in principle determine the sign of Δm. We do not give a more complete description of the results obtained in [17] because there are no corresponding experimental data.

7. LEPTONIC DECAYS OF NEUTRAL K MESONS. THE $\Delta S = \Delta Q$ RULE

The leptonic decays of neutral K mesons

$$K \begin{cases} \nearrow \pi^{\pm} + e^{\mp} + \nu, & (28a) \\ \searrow \pi^{\pm} + \mu^{\mp} + \nu & (28b) \end{cases}$$

lead to possibilities for observing specific interference effects. Let us use the following symbols for the amplitudes for leptonic decays of K^0 mesons*

$$(K^0 / \pi^- e^+ \nu) = a, \qquad (K^0 / \pi^+ e^- \nu) = b.$$

Then the amplitudes for the charge-conjugate processes of decay of $\widetilde{K}^0$ mesons are

$$(\widetilde{K}^0 / \pi^+ e^- \nu) = a^*, \qquad (\widetilde{K}^0 / \pi^- e^+ \nu) = b^*,$$

where a^* and b^* are the complex conjugates of the quantities a and b. If we make the usual assumption that the combined parity CP is conserved in the decays of K mesons, the amplitudes a and b must be real. Taking a and b to be real and using the relations (4), we get the following expressions for the amplitudes of the leptonic decays of K_1^0 and K_2^0 mesons:

$$\left. \begin{aligned} (K_1^0 / \pi^+ e^- \nu) &= (K_1^0 / \pi^- e^+ \nu) = \frac{1}{\sqrt{2}}(a+b), \\ (K_2^0 / \pi^+ e^- \nu) &= -(K_2^0 / \pi^- e^+ \nu) = \frac{1}{\sqrt{2}}(b-a). \end{aligned} \right\} \quad (29)$$

Let us now consider the probabilities W for the decay processes $K \to \pi^+ + e^- + \nu$ and $K \to \pi^- + e^+ + \nu$ in a beam of neutral K mesons which is initially (at $t = 0$) in the pure K^0 state:

$$K^0 = \frac{1}{\sqrt{2}}(K_1^0 + K_2^0).$$

From the relations (4) and (29) it follows that

$$\left. \begin{aligned} W(\pi^+ + e^- + \nu) &= \frac{1}{4}\left|(a+b)\, e^{-iE_1 t - \frac{t}{2\gamma\tau_1}} + (b-a)\, e^{-iE_2 t - \frac{t}{2\gamma\tau_2}}\right|^2 \\ &= \frac{1}{4}\{(a+b)^2 e^{-\frac{t}{\gamma\tau_1}} + (a-b)^2 e^{-\frac{t}{\gamma\tau_2}} + 2(b^2-a^2)\, e^{\frac{\tau_1+\tau_2}{2\tau_1\tau_2\gamma}t} \cos(\delta t)\}, \\ W(\pi^- + e^+ + \nu) &= \\ &= \frac{1}{4}\{(a+b)^2 e^{-\frac{t}{\gamma\tau_1}} + (a-b)^2 e^{-\frac{t}{\gamma\tau_2}} - 2(b^2-a^2)\, e^{\frac{\tau_1+\tau_2}{2\tau_1\tau_2\gamma}t} \cos(\delta t)\}, \\ \delta &= E_1 - E_2 \approx \frac{m\Delta m}{E}. \end{aligned} \right\} \quad (30)$$

It can be seen from Eq. (30) that the expressions for $W(\pi^+ + e^- + \nu)$ and $W(\pi^- + e^+ + \nu)$ contain not only exponentially decreasing terms but also oscillating terms which depend on Δm.

The relations (30) can be used for the experimental determination of the ratio b/a (for a known value of Δm). Theory (the Sakata model of the weak interactions) predicts $b = 0$. The equation $b = 0$ follows from the application to the processes

$$K^0 \to \pi^+ + e^- + \nu \text{ and } \widetilde{K}^0 \to \pi^- + e^+ + \nu$$

of the rule $\Delta S = \Delta Q$,* which follows from the fact that in the Sakata model there are no interactions with $\Delta T_3 = 3/2$ for the strongly interacting particles. It can be seen from Eq. (30) that the $\Delta S = \Delta Q$ rule, i.e., that $b = 0$, leads to the equation $W(\pi^+ + e^- + \nu) = 0$ at $t = 0$, where there is a pure K^0 state. Furthermore, as the "age" of the K mesons increases the probability of the decay $K \to \pi^+ + e^- + \nu$ will vary in accord-

*For definiteness we are considering the "electronic" decays of neutral K mesons. Everything that will be said, except the absolute magnitudes of the amplitudes a and b, also applies to the "μ-mesic" decays $K \to \pi^{\pm} + \mu^{\mp} + \nu$.

*The rule $\Delta S = \Delta Q$ means that in processes occasioned by weak interactions the change of strangeness of the strongly interacting particles is equal to the change of their charge.

ance with the value of Δm. Another effect of the vanishing of the amplitude b is that the probabilities $\Gamma(L)$ of leptonic decays of K_1^0 and K_2^0 mesons are equal: $\Gamma_1(L^\pm) = \Gamma_2(L^\pm)$. This can be seen from Eq. (29), which gives immediately

$$\Gamma_1(L^\pm) = \frac{1}{2}|a+b|^2, \qquad \Gamma_2(L^\pm) = \frac{1}{2}|a-b|^2.$$

The hypothesis that $\Gamma_1(L) = \Gamma_2(L)$ has been used by Crawford and others[7] in the determination of the quantity τ_2 (see Sec. 2). As has already been pointed out in Sec. 2, however, a number of experiments have given indications that the Sakata model does not apply to the decays of K mesons; it has been observed that $\Gamma_1(L) \neq \Gamma_2(L)$, i.e., the amplitude b is not zero. Improvements in the accuracy of these results are of very great importance for the construction of a theory of the weak interactions. Let us consider these experiments in somewhat more detail. In [8] a separated beam of K^+ mesons was sent through a 75-cm propane bubble chamber, in which observations were made on the "V-forks" from the leptonic decays (28a) of neutral K mesons produced in the charge-transfer reaction $K^+ + n \to K^0 + p$. Thus the initial state (at $t = 0$) in this experiment was a pure K^0 wave, and the probabilities of the observed leptonic decays $K \to \pi^+ + e^- + \nu$ and $K \to \pi^- + e^+ + \nu$ must be given by Eq. (30). Using the value of Δm found in other experiments,[10,11,13] the authors came to the conclusion that their observed distributions in time $W_{exp}(\pi^+ + e^- + \nu)$ and $W_{exp}(\pi^- + e^+ + \nu)$ [see Eq. (30)] are consistent with the values

$$\frac{b}{a} = 0.55^{+0.08}_{-0.12} \text{ and } \frac{\Gamma_1(L)}{\Gamma_2(L)} = 12^{+8}_{-6}.$$

The leptonic decays of neutral K mesons have also been studied by Alexander and others[5] in a 180-cm hydrogen bubble chamber. The ratio they found, $\Gamma_1(L)/\Gamma_2(L) = 6.6^{+6}_{-4}$, corresponds to

$$\frac{b}{a} \text{ or } \frac{a}{b} = 0.44^{+0.12}_{-0.20}.$$

The small number of cases observed did not justify a preference for either of these two possibilities; the value

$$\frac{b}{a} = 0.44^{+0.12}_{-0.20}$$

is in good agreement with the result of Fly and others[8]

$$\frac{b}{a} = 0{,}55^{+0.08}_{-0.12}.$$

The fact that the amplitude b is not zero means that the interaction that leads to the leptonic decays of K mesons allows transitions with $\Delta T_3 = 3/2$, which are incompatible with the Sakata model of the weak interaction. The experiments[5,8] that have been mentioned, which indicate violation of the rule $\Delta S = \Delta Q$, decidedly complicate our ideas about weak interactions, which had hitherto fit so well into the framework of the Sakata model. Owing to this it is a matter of very great interest to get further confirmation of the experimental results obtained in [5,8], and improved accuracy in the values.

8. THE ISOTOPIC PROPERTIES OF NEUTRAL K MESONS

The K^0 meson forms an isotopic doublet with the K^+ meson, and the $\tilde{K}^0$ meson forms an isotopic doublet with the K^- meson. Thus the isotopic spin of the K meson is $T_K = 1/2$. Isotopic spin is not conserved in the decay of strange particles: theory (the Sakata model of the weak interactions) predicts only quite definite rules for the change of the isotopic spin in the π-meson decays of K mesons: $\Delta T = 1/2, 3/2$. We can affirm, however, from all of the experimental data on the decay of strange particles, that the amplitudes for transitions with $\Delta T = 3/2$ are much smaller than those for transitions with $\Delta T = 1/2$. Let us consider what consequences follow from the application of the rule $\Delta T = 1/2$ to the decays of neutral K mesons. First of all we must call attention to the fact that the lifetimes of charged and neutral K mesons against 2π decay are very different:

$$\tau(K_1^0 \to 2\pi) = 1\cdot 10^{-10} \text{ sec},$$
$$\tau(K^+ \to 2\pi) = 5\cdot 10^{-8} \text{ sec},$$

and this is in excellent agreement with the rule $\Delta T = 1/2$. In fact, the π mesons from the decay $K^+ \to \pi^+ + \pi^0$ can only be in a state with $T = 2$. It is easily verified that the states with $T = 0, 1$ are forbidden. Since the isotopic spin of the K meson is $T_K = 1/2$, this reaction can go only with $\Delta T = 3/2$. On the other hand the decay $K_1^0 \to 2\pi$ can occur with $\Delta T = 1/2$, and this means that the probability of this process is larger by a factor 500. Applying the rule $\Delta T = 1/2$ to the processes $K_1^0 \to \pi^+ + \pi^-$ and $K_1^0 \to 2\pi^0$, we can calculate their relative probability

$$R = \frac{\Gamma(K_1^0 \to \pi^+ + \pi^-)}{\Gamma(K_1^0 \to \pi^+ + \pi^-) + \Gamma(K_1^0 \to 2\pi^0)} = \frac{1}{3}.$$

The experimental value of R is $R_{exp} = 0.30 \pm 0.05$. Applying the rule $\Delta T = 1/2$ to the decays of K^+ and K^0 mesons into three π mesons and using the principle of isotopic invariance, we get

$$\Gamma(K_2^0 \to 3\pi^0) : \Gamma(K_2^0 \to \pi^+ + \pi^- + \pi^0) = 2,$$
$$\Gamma(K_2^0 \to 3\pi) : \Gamma(K^+ \to 3\pi) = 1{,}2.$$

In finding these ratios we have also taken into account the difference of the masses of $\pi^\pm$ and π^0 mesons, which leads to some change of the respective phase volumes. The use of the rule $\Delta T = 1/2$ enables us to get from the experimentally known lifetimes of K^+ mesons against decay into various channels predictions of the probabilities of the various types of decay of the K_2^0 meson:

$$K_2^0 \to 3\pi_0, \qquad \Gamma = 4.6\cdot 10^6 \text{ sec}^{-1},$$
$$K_2^0 \to \pi^+ + \pi^- + \pi^0, \qquad \Gamma = 2.3\cdot 10^6 \text{ sec}^{-1},$$

$$K_2^0 \to e^+ + \nu + \pi^-, \qquad \Gamma = 3.4 \cdot 10^6 \ \text{sec}^{-1},$$
$$K_2^0 \to e^- + \tilde{\nu} + \pi^+, \qquad \Gamma = 3.4 \cdot 10^6 \ \text{sec}^{-1},$$
$$K_2^0 \to \mu^+ + \nu + \pi^-, \qquad \Gamma = 3.3 \cdot 10^6 \ \text{sec}^{-1},$$
$$K_2^0 \to \mu^- + \tilde{\nu} + \pi^+, \qquad \Gamma = 3.3 \cdot 10^6 \ \text{sec}^{-1},$$

From this we get as the total probability of decay of the K_2^0 meson $\Gamma(K_2^0) = 20.3 \times 10^6 \ \text{sec}^{-1}$, which corresponds to the lifetime $\tau(K_2^0) = 5 \times 10^{-8}$ sec. This value of $\tau(K_2^0)$ is close to that found experimentally. The ratio predicted (for $\Delta T = 1/2$)

$$\frac{\Gamma_2(\pi^+ + \pi^- + \pi^0)}{\Gamma_2(L^\pm)} = \frac{2.3 \cdot 10^6}{13.4 \cdot 10^6} = 0.17$$

is also in good agreement with the experimental value[3]

$$\frac{\Gamma_2(\pi^+ + \pi^- + \pi^0)}{\Gamma_2(L^\pm)} = 0.16 \pm 0.02.$$

The probabilities of leptonic decays of K_2^0 mesons have been measured in a number of researches.[5,7,8] The results obtained are shown in Table II. In [5,18] the leptonic decays of neutral K mesons were studied at rather large distances from the place where the mesons were produced: that is, the pure K_2^0 state was selected experimentally. In the experiment of Crawford and others[7] it was impossible to distinguish the K_1^0 and K_2^0 decays experimentally (see also Sec. 2): the value of Γ_2 shown in Table II was obtained in that paper on the assumption that $\Gamma_1(L^\pm) = \Gamma_2(L^\pm)$. A comparison of the data of Table II with the predictions of the Sakata theory of the weak interactions shows that within the limits of experimental error these results are not in contradiction with the theory. At the same time, the obtaining of more accurate values of the probabilities of the various branches in the decay of K mesons is now of very great importance in connection with the experimental indications of a violation of the Sakata model in the decays of neutral K mesons, which were described in Sec. 7.

The reader will also find discussions of various questions connected with neutral K mesons in [19-21].

Table II. Experimental probabilities $\Gamma_2(L)$ of leptonic decays of K_2^0 mesons

Decay	Probability Γ_2, sec^{-1}	Reference
$K_2^0 \to e^\pm + \pi^\mp + \nu$	$(9.3 \pm 2.5) \cdot 10^6$	5
$K_2^0 \to \mu^\pm + \pi^\mp + \nu$	$(20.4^{+7.2}_{-5.6}) \cdot 10^6$	7
$K_2^0 \to e^\pm + \pi^\mp + \nu$	$(6.2 \pm 2.0) \cdot 10^6$	18
$K_2^0 \to \mu^\pm + \pi^\mp + \nu$	$(5.6 \pm 3.0) \cdot 10^6$	18

[1] A. Pais and M. Gell-Mann, Phys. Rev. **97**, 1387 (1955).

[2] A. Pais and O. Piccioni, Phys. Rev. **100**, 1487 (1955).

[3] Luers, Mittra, Willis, and Yamamoto, Phys. Rev. Letts. **7**, 255 (1961).

[4] W. H. Barkas and A. Rosenfeld, Proceedings of the 1960 Annual International Conference on High-energy Physics at Rochester, N.Y., p. 877.

[5] Alexander, Almeida, and Crawford, Phys. Rev. Letts. **9**, 69 (1962).

[6] Bardon, Lande, Lederman, and Chinowsky, Ann. Phys. (New York) **5**, 156 (1958).

[7] Crawford, Cresti, Douglass, Good, Kalbfleisch, Stevenson, and Ticho, Phys. Rev. Letts. **2**, 361 (1959).

[8] Fly, Powell, White, Baldo-Ceolin, Calimani, Campalilo, Fabbri, Farini, Filippi, Huzita, Miari, Camerini, Fry, and Natali, Phys. Rev. Letts. **8**, 132 (1962).

[9] Birge, Fly, Powell, Huzita, Fry, Gaides, Natali, Willman, and Camerini, loc. cit. [4], page 601.

[10] Camerini, Fry, Gaides, Huzita, Natali, Willman, Birge, Fly, Powell, and White, Phys. Rev. **128**, 362 (1962).

[11] Fitch, Pironé, and Perkins, Nuovo cimento **22**, 1160 (1961).

[12] M. L. Good, Phys. Rev. **110**, 550 (1958).

[13] Good, Matsen, Müller, Piccioni, Powell, White, Fowler, and Birge, Phys. Rev. **124**, 1223 (1961).

[14] M. Lax, Revs. Modern Phys. **23**, 287 (1951).

[15] I. Yu. Kobzarev and L. B. Okun', JETP **39**, 605 (1960), Soviet Phys. JETP **12**, 426 (1960).

[16] Camerini, Fry, and Gaides, Nuovo cimento **28**, 1096 (1963).

[17] Ogievetskiĭ, Okonov, and Podgoretskiĭ, JETP **43**, 720 (1962), Soviet Phys. JETP **16**, 511 (1962).

[18] Neagu, Okonov, Petrov, Rozanova, and Rusakov, JETP **40**, 1618 (1961), Soviet Phys. JETP **13**, 1138 (1961).

[19] L. M. Lederman, Rendiconti della Scuola internazionale di fisica "Enrico Fermi," Bologna, 1960, p. 365.

[20] I. Yu. Kobzarev and S. G. Matinyan, in Collection: Voprosy teorii sil'nykh i slabykh vzaimodeĭstviĭ (Questions in the Theory of Strong and Weak Interactions), Erevan, AN ArmSSR, 1962, pp. 175, 186.

[21] R. H. Dalitz, Strong Interaction Physics and the Strange Particles, Bombay, 1961 (Preprint).

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