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Methodological Notes

A HALL EFFECT PROBLEM (THE GALVANOMAGNETIC PARADOX)

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SOLUTION of the problem of the relation of currents and powers in Hall pickup circuits leads to an interesting result which does not appear in the literature.

The problem is to determine the current in the current leads of a Hall pickup as a function of the resistance of the circuit of the transverse electrodes, in the presence of an external magnetic field, normal to the plane of the pickup. The circuit and notation are shown in Fig. 1.

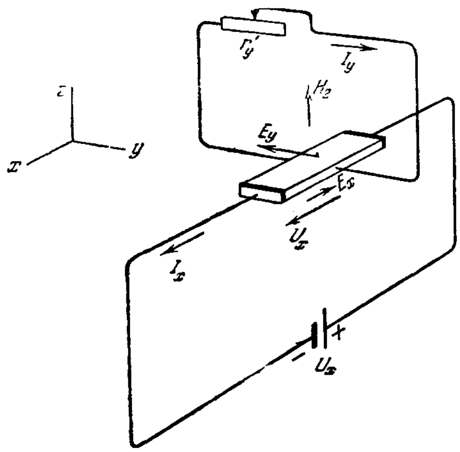

FIG. 1. U_x–emf of power supply, the internal resistance of which is small and neglected; E_x–secondary Hall emf; E_y–primary Hall emf; H_z–magnetic field intensity; I_x and I_y are the stationary currents in the power supply circuit and the Hall emf circuit, respectively; r_y' is the external resistance in the Hall circuit. The arrows indicate directions of currents and emf's.

Before giving an approximate solution of this problem we call attention to the fact that a Hall pickup in a magnetic field, as in Fig. 1, is considered as a passive four-terminal network in which the reciprocity principle is violated.[1] The solution of our problem leads to a violation of still another law which determines the properties of the passive four-terminal network. The violation is this: the change in the current and the useful power in the circuit of the transverse electrodes of the Hall pickup (the output circuit of the four-terminal network) always produces a change of opposite sign in these quantities the power supply circuit (the input circuit of the four-terminal network). Such a relation is impossible in a possive four-terminal network or in any electrical circuit with linear elements. We shall call the phenomenon which leads to this violation the galvanomagnetic "paradox."

The experimental curves of Fig. 2 illustrate the described effect; these were obtained with an InSb pickup ($\rho \approx 5 \times 10^{-2}$ ohm-cm) at $H_z = 10^4$ Oe. The measured dependence of the current in the input circuit on the resistance of the output circuit for an arbitrary circuit with linear elements is shown by the dashed line. At first glance it would seem paradoxical that a decrease of current in the circuit of the transverse electrodes always leads to an increase of current in the circuit of the current electrodes, i.e., in the power supply circuit.

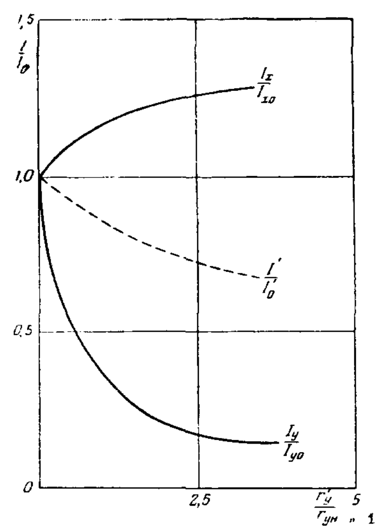

FIG. 2. Effect of the resistance r_y' on the current in the Hall pickup circuit. r_y and r_y' are the internal and external resistances in the transverse electrode circuit; I_x and I_y are the currents in the circuits x and y for $r_y' > 0$; I_{y0} and I_{x0} are the currents in the same circuits for $r_y' = 0$.

The qualitative solution of the problem is easily explained. Closing the circuit of the transverse electrodes produces in the element a current component I_y due to the Hall emf E_y parallel to the y axis. The current I_y interacts with the same magnetic field H_z and produces a secondary Hall emf E_x along the x axis.[2] Consideration of the directions of the currents, of the magnetic Lorentz forces, and of the primary and secondary Hall emf's along the y and x axes and comparison of the directions of the second-

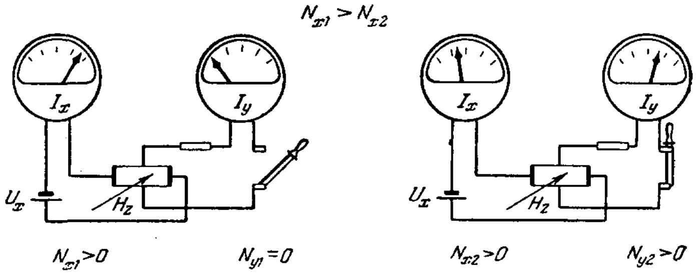

FIG. 3. Demonstration of the galvanometric paradox. Closing the circuit decreases the output power and the current in the input circuit. N_{x1} and N_{y1} are the powers in the circuits x and y for open circuit of the transverse electrode; N_{x2} and N_{y2} are the powers in the same circuits upon closing the transverse electrode circuit.

ary Hall emf and of the potential drop on the pickup generated by the current source, lead to the following rule: the secondary Hall emf is always directed counter to the external applied voltage. This rule explains the nature of the galvanomagnetic "paradox" and uniquely determines the direction of the two of the quantities I_x, E_y, and H_z when the third is given. It is valid both for electron and hole conductivity, in ac and dc circuits and in all magnetic fields. The rule serves as a basis for the determination of the additional change in the resistance of semiconductors in a magnetic field brought about by the action of the secondary Hall emf.[2]

The dependence of the current in the circuit of the current-bearing electrodes on the resistance of the external circuit of the transverse electrodes is easily found by taking it into account that the effective emf is equal to the difference of the emf of the power supply and the secondary Hall emf, and by allowing for the change in the resistance of the pick up circuits due to the magnetic field. By simple transformations we get

$$I_x = \frac{U_x}{r_{xH} + \dfrac{kR^2H_z^2}{d\,(r_{yH} + r'_y)}}.$$

Here r_{xH} is the pickup resistance along the x axis in a magnetic field of intensity H_z, r_{yH} is the same quantity along the y axis, R is the Hall coefficient, k is a correction coefficient which depends on the ratio of the sides of the pickup, and d is the dimension of the pickup in the z direction. The remaining notation is the same as Fig. 1.

Figure 3 shows how to demonstrate the galvanomagnetic "paradox." Here, the positions of the arrows on the instruments and the symbols explain the gist of the phenomenon. For successful demonstration of the experiment, it is necessary to use a pickup with high carrier mobility, for example, one made of InSb, and to measure the current with a milliammeter having minimim internal resistance. In our case we used a type MPB-46 milliammeter shunted by iron-chromium-aluminum conductor of length 10 cm and diameter 1.5 mm. In such equipment closing of the circuit of the transverse electrodes decreases in current in the power supply circuit by approximately 40% of full scale.

[1] V. P. Zhuze, Poluprovodniki v nauke i tekhnike (Semiconductors in Science and Technology), v. 1, AN SSSR 1957, p. 408.

[2] V. V. Serkov, Avtomatika i telemekhanika 23(3), 383 (1962).

Translated by R. T. Beyer