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Meeting and Conferences

SECOND ALL-UNION SYMPOSIUM ON WAVE DIFFRACTION

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THE Second All-union Symposium on Wave Diffraction was held June 4 through June 9 at Gor'kiĭ; it was sponsored by the Commision on Acoustics of the Academy of Sciences, U.S.S.R., together with the Scientific-Research Institute for Radiophysics at the N. I. Lobachevskiĭ Gor'kiĭ State University.

Academician V. A. Fock and V. D. Kupradze, member of the Georgian Academy of Sciences, were among the 314 persons participating in the work of the symposium. Altogether, representatives of more than 30 cities of the U.S.S.R. took part in the symposium. Moscow sent the largest delegation (133 members), followed by Leningrad (66 members) and Gor'kiĭ (63 members). The majority of those taking part in the symposium were young. Despite the presence of many older scientists, the "average age" of the delegation members was only 33.

More than 120 reports and communications were read (20 more than at the first symposium, held in 1960 at Odessa).

In addition to the plenary sessions, the following section meetings were held at the symposium: asymptotic methods, mathematical problems, nonstationary problems, diffraction by various solids, the physical theory of diffraction, diffraction by a sphere, numerical methods, diffraction by a wedge, propagation in layered media, periodic wavy surfaces, wave-guide problems, simulation and propagation of radio waves along the surface of the earth, periodic lattices, propagation of waves in plasma and other media, statistical problems, diffraction by plasmas, radiation of electromagnetic waves, diffraction by gyrotropic bodies, and statistically uneven surfaces.

The symposium was opened by Professor G. D. Malyuzhinets, chairman of the diffraction section of the Commission on Acoustics of the U.S.S.R. Academy of Sciences; he presented greetings to the symposium participants from the chairman of the Commission on Acoustics, Academician N. N. Andreev, who was unable to attend.

The Acoustics Commission, said G. D. Malyuzhinets, had always considered it useful to have a joint discussion of questions dealing with diffraction, i.e., the behavior of various types of waves, by representatives of the different "wave" branches of science and technology. For many years it has been traditional for the wave-diffraction sections of the All-union Acoustics Conferences to hear reports from scientists working in the fields of electromagnetic, elastic, and hydrodynamic waves, as well as from mathematicians concerned with the development of methods for solving wave problems. In 1960, Odessa was host to a gathering unique in the history of world scientific practice: the Joint Symposium on Wave Diffraction of the Acoustics Commission, held in conjuction with the Acoustics Institute of the U.S.S.R. Academy of Sciences, and the Odessa Electrotechnical Institute for Communications. Just six months later, in April 1961, a similar symposium was held for the first time in the U.S.A. The success of the first Joint Symposium exceeded expectations. About one hundred reports were heard and discussed. The important results of the symposium included: the extension of experience and wave-investigation methods known in some branches of science and technology to other branches and raising of the scientific level of wave research in all sectors owing to the special attention devoted to problems of methodology at the symposium.

It was especially gratifying to see the way in which physicists and mathematicians drew together; this occurred as a result of joint discussions at the symposium and continued subsequently in seminars on diffraction theory organized in the mechanico-mathematical faculties of Moscow and Leningrad Universities. In view of the well-defined tendency of modern science toward expanded application of ever more precise and powerful mathematical methods, it is extremely important from the viewpoint of future diffraction investigations that physicists and mathematicians come together in this manner. Countless diffraction studies, by their specific nature, fall into some given field of physics or technology: acoustics, radio engineering, etc. At the same time, many other diffraction problems and, in particular, the majority of theoretical studies, border on mathematics and must be regarded as general scientific questions, and in particular problems of methodology. The wave-diffraction symposia were devoted to the discussion of precisely such questions. Naturally, these symposia did not exclude consideration of diffraction problems in specialized conferences, where experimental and technical details characteristic of a given field could be discussed intensively, but such topics were of less concern than the general methodological questions.

It was a stroke of good fortune that the present conference was held in Gor'kiĭ, where the tradition of the Mandel'shtam-Andronov school of the physics of oscillations, which considers oscillatory processes of any type from a single point of view, is especially strong.

The first plenary session heard G. I. Makarov's survey report "Some Problems of Diffraction and Wave Propagation." The next plenary session heard the reports of P. Ya. Ufimtsev, "Physical Theory of Diffraction," and V. D. Kupradze, "One Method of Approximate Solution for Certain Diffraction Problems." The report of P. Ya. Ufimtsev surveyed the results of an approximate solution to diffraction problems for convex ideally conducting bodies with surface discontinuities. The problems considered fall into the quasioptical region, and represent a refinement of an approximation from physical optics. The problems are based on well-known results from the mathematical theory of diffraction, but make use of self-evident physical considerations as to the nature of scattered-field formation. Special consideration was given to the scattering of a plane electromagnetic wave by a thin cylindrical conductor (dipole) of arbitrary length, and the author indicated that the solution, which allowed for multiple diffraction, was suitable not only for dipoles nearly a wavelength long or longer, but also for dipoles short in comparison with the wavelength. V. D. Kupradze discussed a new method proposed and developed by him, using as an example the diffraction of elastic oscillations in the steady state. The author assumed B_i to be a finite plane or three-dimensional region with a closed boundary S of the Lyapunov type and B_e the infinite complement of region B_i formed by the rest of space; specifying at point x in region B a time-periodic source of elastic oscillations of frequency ω, he determined the displacement (as well as stress) fields produced in B_e by the action of the source in the presence of an empty inclusion B_i in the elastic region B_e.

The solution to the problem is obtained in the form

$$u(x)=\frac{1}{4\pi}\int_S \varphi(y)\, T_y\Gamma(x,\,y,\,\omega)\, dS_y+E(x,\,x_*),\quad x\in B_e,\qquad (1)$$

where the vector $\varphi(y)$ is found from the functional equation

$$\frac{1}{4\pi}\int_S \varphi(y)\, T_y\Gamma(x,\,y,\,\omega)\, dS_y+E(x,\,x_*)=0,\quad x\in B_i;\qquad (2)$$

u(x) is the displacement vector, Tu(x) is the stress vector at point x; $\Gamma(x, y, \omega)$ is a known fundamental solution for the system (*) (more accurately, a matrix of fundamental solutions), and T_y is the stress operator taken at the point y, i.e.,

$$T_y\equiv 2\mu\frac{\partial}{\partial n_y}+\lambda n\,\mathrm{div}_y+\mu\,(n_y\times\mathrm{curl}_y)$$

(n_y is the unit normal at point y); the products that occur in the integrands of (1) and (2) are products of a vector and a matrix. Making use of the basic properties of elasto-potentials (elastic potentials) and the theory of singular integral equations, the author reported that (2) has a solution in the class of vectors that satisfy the Hoelder condition, that this solution is unique, and that when introduced into (1) it yields a vector satisfying the conditions:

1) $\mu\Delta u(x)+(\lambda+\mu)\,\mathrm{grad\,div}\,u(x)+\omega^2 u(x)=0.$

2) The limiting value of Tu(x) on S, when point x approaches x_0 on S from without (i.e., from B_e), equals zero:

$$|\,Tu(x)\,|_e=0\,|_{B_e\ni x\to x_0\in S}.$$

3) $u(x) - E(x; x_*)$ satisfies the radiation condition at infinity.

In contrast to ordinary integral equations for boundary-value problems of the elliptical type, functional equation (2) possesses special properties that makes it amenable to approximate solution by reduction to a system of algebraic equations linear in $\varphi(y_i)$. After projection upon the coordinates axes, this system becomes a system of 2N (for the plane problem) or 3N (for the three-dimensional problem) linear equations in the components of the sought vectors $\varphi_k(y_i)$, k = 1, 2, 3. When the values found for $\varphi_k(y_i)$ are introduced into (1), we obtain an approximate value for the displacement vector u(x) at an arbitrary point x within B_i. A numerical check of the method, carried out by solving boundary problems with known solutions, has established that it yields a very good approximation (for example, when the Gauss quadrature formula with 16 ordinates is used, the approximate values agree with the precise values to the sixth decimal place).

V. S. Buldyrev and I. A. Molotkov have developed a method for isolating and "Investigating the Nonanalytic Portions of a Wave field in Nonstationary Diffraction Problems." They analyzed the precise solutions to typical problems of diffraction by a transparent cylinder and sphere, which take the form of Fourier series whose coefficients contain cylindrical functions. Theorems dealing with the reconstruction of the nonanalytic portion of the function from the asymptotic expansion of its Fourier coefficients make it possible to construct converging series that describe the analytic portion of the wave field in the neighborhood of special field surfaces, which in the illuminated portion of space are the normal wave fronts, and in the shadow zone are the so-called sliding fronts. In the illuminated portion of space, the first terms in the series obtained give the geometric-optics approximation. In the neighborhood of a caustic, the nonanalytic portion of a wave field is described by associated Legendre functions. An investigation of expressions describing the nonanalytic portion of the wave field in the shadow zone made it possible to study in detail the nature of the wave-field intensification in the neighborhood of the sliding fronts. It was established that the wave field is included in the shadow zone continuously, together with its derivatives. In problems concerning the propagation of waves in a l inhomogeneous layered half-space with an analytic refractive index of refraction that rises monotonically in arbitrary fashion, the authors investigated a solution in the form of an iterated integral. The integrands are the solution to a linear differential equation with variable coefficients. By using the general properties of solutions to such equations, the authors were able to obtain for the non-analytic portion of the fields an integral representation that can be investigated by asymptotic methods. Simple functions were obtained that describe the increase in the wave field behind the sliding front, depending upon the properties of the index of refraction. Similar results were obtained in problems of diffraction by bodies bounded by coordinate surfaces S (the system of coordinates is assumed to permit separation of variables in the wave equation) with boundary conditions of the form $U|_S = 0$. The methods developed for studying the nonanalytic portions of a field can be applied successfully to diffraction problems whose solutions take the form of infinite series or improper integrals.

In recent years, antenna-synthesis problems have acquired great importance in connection with the need for creating small highly directional antennas. These problems amount to the determination of classes of radiation patterns that can be realized precisely with the aid of various types of antennas (discrete, linear, plane, etc.), and to finding the distribution of sources realizing these patterns; the calculation of distributions of sources (currents or fields) in antennas that create patterns representing sufficiently good approximations to any given patterns (that do not belong to the class of realizable patterns); the study of questions connected with antenna "superdirectivity;" the examination of optimum patterns and methods for realizing them.

In their survey report, L. D. Bakhrakh and Ya. N. Fel'd reported on the antenna patterns found feasible for antennas with linear and plane apertures, as well as for some systems of discrete radiators. Thus, for example, in the case of linear antennas, the patterns realized belong to the class of entire functions of finite degree which are absolutely square integrable along the real axis; given such a pattern, it is possible with the aid of the inverse Fourier transformation to determine uniquely the distribution of sources and the size of antennas. Similar results are also obtained for antennas with two-dimensional plane apertures, where the pattern realized determines uniquely the distribution of sources and the form of the aperture plane. Less study has been given to the systhesis of antennas with two-dimensional curved apertures and three-dimensional source distributions. When the distribution of sources providing a pattern that is a good approximation to a given arbitrary pattern is found, where in general the pattern does not belong to a realizable class, the size and shape of the radiating aperture are also specified. In this case, the solution is arrived at by employing the method of integral transformations (Fourier, Hankel, etc.), or the method of partial patterns. In the latter case, the specified pattern is approximated by a portion of a series in special functions (partial patterns) for each of which an accurate solution to the synthesis problem exists and can be found. The desired distribution of sources is obtained here as a superposition of the partial distributions corresponding to the individual partial patterns. Where the given directivity pattern and dimensions of the radiating aperture are arbitrary, rapidly changing source distributions with very large amplitudes at isolated points may be obtained. With such distributions, the system may have high reactive power, and there may be an increase in the extent of the near induction zone and, consequently, the directivity pattern may be extremely critical with respect to very small changes in source distribution. Such systems are called "super-directive" and are almost impossible to realize. Closely related to question of "superdirectivity" is the problem of creating a nearly-directional antenna that occupies minimum volume for a given bandwidth. Patterns are called optimal when they do not require for their realization "supernatural" source distributions and satisfy various practical requirements as well as possible. Methods of constructive function theory are widely used in solving such problems. Recently, there has appeared the problem of synthesizing swinging-beam antennas, in which it is necessary to obtain a given pattern for a specific beam speed; it has been necessary to synthesize antennas with special phase patterns and relatively uniform amplitude patterns, etc. In these cases, Fourier integral transformations are normally employed.

Those attending were greatly interested in the report of V. A. Fock, G. D. Malyuzhinets, and L. A. Vaĭnshteĭn, "Lateral Diffusion of Shortwaves at a Convex Cylinder." The problem of cylindrical-wave diffraction at an arbitrary convex cylinder is solved by the authors in ray (evolute) coordinates. If the radius of curvature of the cylinder on the path of the diffracted wave is sufficiently large in comparision with the wavelength, we can neglect the longitudinal diffusion of the wave amplitude and use a parabolic equation (instead of an elliptical equation). Further transformations (going over to a "natural" dimensionless variable or a bilinear transformation) separate one more small quantity that characterizes the rate of change of the curvature. When this quantity is neglected (it equals zero for a circle and for a special type of spiral whose radius of curvature is proportional to the cube of the arc measured from the focus) we obtain a standard type of parabolic equation with separable variables, which can be solved exactly with the aid of the Airy function. Analyzing the limits of applicability for this solution, the authors concluded that it was necessary to supplement the solution (near the boundary between light and shadow at large dimensionless distances from the point of tangency) with another solution, written in "natural" variables.

The report of G. N. Krylov examined methods of calculating the electromagnetic-field structure for actual antenna arrangement under steady-state and nonstationary conditions. In both cases, the antenna field (one-, two-, or three-dimensional) is constructed with the aid of the tensor Green's function, while the matrix elements that represent the tensors are interpreted as attenuation factors for the corresponding electric and magnetic field components of elementary dipoles directed along the three cartesian axes. The method proposed makes it possible to find the components field of any antenna on the basis of a standard program for the matrix elements (in the cylindrical coordinate system, there are seven independent elements in all), if we assume that the antenna currents are given. It is proposed to find a solution for the nonstationary state for the general case of stratified paths by a frequency method, with Filon's method used for integration with respect to the frequency. The accuracy of this method depends on the pulse length ΔT and the width of the frequency spectrum $\Delta\omega$; a numerical check by forward and inverse transformation showed the accuracy to equal 1% for $\Delta\omega\Delta T = 2\pi \times 10^2$. In conclusion, the author analyzed matrix elements for the steady-state and nonstationary cases for a homogeneous ground; here (clearly a unique case) it proved possible to employ a time-domain approach to the construction of the solution.

In his report, "The Application of Wiener-Hopf-Fock Type Integral Equations to Certain Diffraction Problems," V. I. Talanov surveyed studies in the theory of diffraction of waves at semi-infinite impedance structures permitting propagation of surface waves. As examples, he considered problems of surface-wave excitation by the open end of a waveguide, diffraction of waves by a surface-impedance step in shielded systems, and diffraction of waves by an impedance half-plane. In his report, "Toward a Theory of Nuclear Diffraction Processes," A. G. Sitenko discussed the fact that the interaction of nucleons and nuclei in energy regions for which the nucleon wavelength is considerably less than the nuclear radius has diffractive character. Thus, various nucleon-nucleus interactions and, in particular, inelastic processes, can be described on the basis of a diffraction method developed in analogy with the Huygens principle of optics. This diffraction method admits of generalization, making it possible to account for the semitransparent nature of the nucleus, as well as the Coulomb and spin-orbit interaction of nucleons and nuclei. By taking spin-orbit interaction into account, it may be possible to explain the polarization phenomena occurring in various nuclear processes. The optical parameters of nuclei can be connected directly with an amplitude that characterizes pair interaction between nucleons, and with the distribution of nucleons in nuclei. Thus, the imaginary part of the optical potential is expressed in terms of the nucleon-nucleon scattering amplitude and the spectral distribution of the space-time correlation function of the fluctuations in nuclear nucleon density. Such an approach enables us to allow for the effect of the Pauli principle and of interaction among nucleons in the nuclei upon the nuclear optical parameters.

Most interest was attracted by the section at which "asymptotic methods" were discussed. At present, the asymptotic treatment of diffraction problems involving stationary harmonic waves makes more and more frequent use of the ray coordinates proposed by G. D. Malyuzhinets in 1946, and the associated differential equation for transverse diffusion of the complex amplitude along the front of the propagating waves, an equation which asymptotically describes the diffraction phenomenon in the narrow sense of the word. In the case of shortwave diffraction by convex bodies, we are especially interested in finding the field in the shadow and half-shadow regions, since the very simple solution obtained with the geometric-optics approximation normally proves adequate for the illuminated region. The problem of finding an asymptotic representation of the wave field just in the shadow and half-shadow regions with the aid of the transverse diffusion equation is simplified by the fact that in this region the ray coordinates are evolute coordinates. In this connection, a difficulty appears when we attempt to avoid consideration of the diffraction field in the illuminated region, where the ray coordinates are not evolute coordinates and the transverse-diffusion equation takes a more complicated form. In his report, G. D. Malyuzhinets proposed a generalized localization principle; he had already mentioned the principle briefly at the Odessa symposium. According to the generalized localization principle, an asymptotic representation of a diffraction field in the shadow and half-shadow regions is independent of the form of the incident wave outside a narrow region containing the entire boundary of the geometric shadow. On the basis of this principle, which figures in the basic conditions of the diffraction problem, the incident-wave amplitude function can be replaced by another analytic function coinciding asymptotically with the initial function only on the boundary of the geometric shadow. As the report showed, using the example of a plane problem of diffraction by a cylinder, we can use for this function the solution to the transverse-diffusion equation in evolute coordinates, a solution that holds everywhere outside the cylinder; the form of this solution was shown for the case of a fluid cylinder of arbitrary shape. The application of the generalized localization principle and its justification are given in the report for the example of cylindrical-wave diffraction in a region that branches around a rigid circular cylinder.

I. G. Yakushkin considered the problem of "diffraction of a plane wave incident upon an infinite cone" in the direction perpendicular to its axis. The Maxwell equations for the field components in the shadow and half-shadow zones are written in ray coordinates, which are introduced for the region of space bounded by an infinite cone, in accordance with the generalized Fermat principle. A parabolic equation, solved by separation of variables, is introduced for the individual field components. The approximate field-component values obtained are quite close to the true values in the shadow and half-shadow zones at large distances from the apex of the cone. On the boundary of the illuminated zone, the solution coincides with the results obtained from a calculation by the formulas of geometric optics. In the half-shadow zone near the surface of the body, the equation reduces to the form obtained by Fock in his study of plane-wave diffraction by an arbitrary convex body. It is possible to generalize the solution to the case of arbitrary plane-wave incidence upon an infinite cone, and also to allow for the finiteness of the cone with the aid of Keller's diffraction-ray method.

B. E. Kinber and A. A. Federov showed that for identical body cross-section profiles, there is a correspondence between the solutions to the axially symmetric and two-dimensional (cylindrical) problems both for the scalar (acoustical) and arbitrarily polarized electromagnetic waves in the shadow zone. Beyond the focal focusing region, bordering on the axis of symmetry, the difference between the solutions for the axially symmetric and cylindrical problems lies in the geometric-divergence factor $1/\sqrt{h_e}$ (h_e is the Lamé factor), in the amplitude on the light-shadow boundary (this is connected with the fact that the incident field is cylindrical in one case and spherical in the second), and in the effect of source directivity (for electromagnetic waves). The solutions found for the sphere and paraboloid coincide with solutions known previously obtained with special assumptions as to the positions of the points of radiation and observation. In his report "A Local Method of Calculating the Field of an Acoustic Wave Reflected from a Boundary at Angles of Incidence Close to the Limiting Angle," B. Ya. Gel'chinskiĭ examined the problem of reflection of a stationary acoustic wave from a plane interface; the wave front (the equiphase surface) has arbitrary shape for the case $Q_{\lim} \approx \pi/2$. It is assumed that the field u_0 of the incident wave is given and is described by a ray series. The local coordinates τ, η, γ, are introduced, where τ is the eikonal of the corresponding wave, $\eta = \sin\theta_{\lim}$, and γ is the angle formed by the plane of incidence with some initial plane. If we assume that the field of the reflected (refracted) wave u can be represented in the form $u(\tau, \eta, \gamma) = U(\tau, \eta, \gamma)e^{i\omega t}$ and that the derivative $\partial u/\partial\eta$ is large, we can obtain impedance-type local boundary conditions. The reflected-wave field can be found at any point in the medium as the solution to an approximate parabolic equation with given boundary values, or with the aid of Green's formula, which is technically simpler. For the refracted wave, the local boundary conditions determine the values of the transmitted-wave field for $\eta < 1$ and the screened-wave field for $\eta > 1$. The field $U_{\lim}$ within the medium is found by a method similar to that used for the field u. Where $\eta = 1 - \epsilon$ and $\epsilon \to 0$, the refracted-wave front glides along the boundary and excites a frontal (lateral) wave. Local boundary conditions are obtained for the field U_2 of the frontal wave, in which U_2 is expressed in terms of $U_{\lim}$. The value of U_2 is next determined at any point in the medium. Using a solution constructed by a recursion method, it is possible to find the field of a wave propagating in a multilayer system with arbitrary boundaries and reflected from a plane boundary.

V. M. Babich gave "a rigorous mathematical justification for the geometric-optics approximation in the plane case." He assumed that U(x, y, k) is the solution to the problem

$$(\Delta + k^2)U = -\delta(x - x_0,\quad y - y_0),\quad \left.\frac{\partial U}{\partial n}\right|_{x,\,y\in S} = 0,$$
$$\sqrt{r}\left(\frac{\partial U}{\partial r} - ikU\right) \to 0 \text{ for } r\sqrt{x^2+y^2} \to +\infty$$

(S is a convex, sufficiently smooth closed boundary, while x_0 and y_{0n} are the coordinates of the source of oscillations), used Arsello's method to set up an integral equation for the function $U|_{xy\in S}$, whose kernel is small at large k, and solved the equation by the method of successive approximations, deriving an asymptotic formula for $U/_S$ with a remainder-term estimate that is uniform on S (as $k \to +\infty$). The asymptotic formula obtained by the author holds for the illuminated region and for the half-shadow region. If we know $U|_{x,y\in S}$, we may use Green's formula to study the behavior of U(x, y, k) as $k \to \infty$ outside S. In the illuminated region, the standard geometric-optics approximation was obtained for U(x, y, k). Deep within the shadow, these methods yield only a rough estimate $U = o(k^{-5/3})$. V. S. Buslaev considered the shortwave asymptotic approximation of the Green's function G(x, x′, k) of the external Dirichlet problem for the Helmholtz equation on a plane, and obtained a rigorous justification for the main terms of the asymptotic approximation in the illuminated and in half-shadow areas. This required an improvement in the known asymptotic formulas: he gave an asymptotic approximation Q(x, x′, k) of Green's function for any location of the points x and x′.

Yu. G. Gukasov and I. V. Sukharevskii found an asymptotic solution to the problem of diffraction of short electromagnetic waves by an ideally conducting surface covered by a thin dielectric layer or a layer of a material with finite conductance. The thickness of the layer δ is of the order of $1/k$ (k is the wave number). The solution obtained makes it possible to represent the vectors E and H as products of oscillating exponential factors and asymptotic series in powers of a small parameter. The coefficients for these series are found alternately in the layer and in air by a recursion method, each step of which consists of elementary operations. In practical calculations, it is sufficient to take just the first term.

In many problems connected short-wave diffraction it is necessary to find the asymptotic approximation (as $k \to +\infty$) for multiple integrals of the type

$$\Phi(k) = \int_C \ldots \int l^{ikF(x_1, \ldots, x_n)} f(x_1 \ldots x_n)\, dx_1 \ldots dx_n,$$

where F is a real function and C is some region in the n-dimensional space E^n. An asymptotic approximation was found by M. V. Fedoryuk for the case in which $C = E^n$ and F has only simple saddle points. He also investigated the asymptotic approximation for integrals describing the field near a caustic and taking the form

$$\Phi(k, \alpha) = \int_C \ldots \int e^{ikF(x_1, \ldots, x_n, \alpha)} f(x_1 \ldots x_n, \alpha)\, dx_1 \ldots dx_n,$$

where α is a real parameter, $|\alpha| < \delta$, $k \to +\infty$ for small $\alpha \neq 0$. The function F has two saddle points that merge at $\alpha = 0$; the functions F and f are regular in the neighborhood of the saddle points, the contour C passes through the saddle points and lies in the complex space C^n; on its boundary, $\mathrm{Re}(iF) \leq -\epsilon < 0$. V. M. Drekov used the asymptotic expansion of the integral

$$I(k) = \int_S \int F(x, y) e^{ik\Phi(x,y)}\, dx\, dy$$

for $k \gg 1$ to solve one diffraction problem in which the phase function $\Phi(x, y)$ has an extremum, but is not differentiable at some point in the region S.

The well-known asymptotic formulas of Hankel, Debye, Fock, Laguerre, et al for the Hankel function $H^{(1)}_\gamma(kr)$ can be regarded as asymptotic representations of the partial solution $e^{i\nu\varphi}H^{(1)}_\nu(kr)$ of the Helmholtz equation for a constant value of the variable φ.

Another type of formula (first proposed by G. D. Malyuzhinets at the Odessa symposium) that is conveniently used in examining diffraction in the shadow zone behind a cylinder of radius r_0 is obtained if we go from the cylindrical coordinates r and φ to the evolute coordinates ζ, η, making use of the formulas

$$r^2 = r_0^2 + (\zeta - \eta)^2, \quad r_0\varphi = \eta + r_0\gamma, \quad r_0 \tan \gamma = \zeta - \eta$$

and hold fixed in the particular solution the "tangential ray" (η = const) rather than the "radial ray" (φ = const), and then seek an asymptotic expansion for small values of the parameter $\epsilon = (1/kr_0)^{1/3}$ and finite values of the parameter $t = \epsilon(\nu - kr_0)$. The first term in this expansion yields the formula

$$e^{i\nu\varphi} H^{(1)}_\nu(kr) = \frac{\varepsilon}{i\sqrt{\pi}} e^{i(k\zeta + ty)} [e^{i\left(tz - \frac{2}{3}z^2\right)} W(t - z^2) + O(\varepsilon^2)],$$

where $x = e^2k\zeta/2$, $y = \epsilon^2k\eta/2$, $z = x - y$; $W_1(t)$ is the Airy-Fock function. In his report, I. V. Olimpiev gave a new rigorous derivation of this formula with an estimate of the constants in the remainder term, and showed an application of the formula to an analysis of the exact solution to a plane diffraction problem outside a circle $r > r_0$, for a source located on the boundary of the region $r = r_0$. He also demonstrated the possibility of using the Malyuzhinets asymptotic formula in the interior region $r < r_0$; this is important in the study of caustics.

R. G. Barantsev reported on "A Separation-of-Variables Method in the Problem of Diffraction by a Body of Arbitrary Shape." Converging and diverging radial waves are separated in the expansion of a solution to the Helmholtz equation in spherical functions on a sphere enclosing a body with arbitrary (noncoordinate) surface. Their amplitudes are expressed in the form of integrals over surfaces containing precisely the same known function q. The scattering problem appears as the problem of finding a sequence of other functionals for the same function q. The essence of the result is that in practice it is necessary to deal with a finite system of linear algebraic equations, the number of which is determined by the ratio of the body characteristic dimension to the wavelength. N. S. Smirnova reported "On the Calculation of the Principal Parts of Wave Fields in the Limiting-Ray Region in Media with Small Drops in Propagation Velocity."

The reports on "Mathematical Problems" of diffraction were concerned, in the main, with basic questions, and were directed chiefly at establishing the legitimacy of mathematical methods employed widely by physicists, frequently without proper proof. D. Z. Avazashvili reported on uniqueness and existence theorems for a solution to a problem of electromagnetic-wave diffraction by a body with arbitrary boundary and finite conductance. The results obtained were generalized for layered and multiply connected regions.

In formulating boundary problems in the theory of oscillations, it is possible to proceed from the fact that within a region G in which harmonic oscillations occur no energy should cross the boundary Γ of this region. This condition means that with harmonic oscillations of angular frequency ω, the energy flux

$$W = \frac{1}{2} \int_\Gamma \mathrm{Im}\left(\frac{p^* \frac{\partial p}{\partial n}}{\omega\varrho}\right) dS$$

leaving the region under consideration through the boundary Γ should be nonnegative; here ρ is the density of the medium and P is the pressure in the medium. The derivative is taken along the outward normal n to the surface. I. A. Urusovskiĭ formulated and communicated "Certain Uniqueness Theorems" that hold for such boundary problems in the theory of diffraction.

A difficulty also appears in representing the solution to the external Dirichlet problem for the wave equation $\Delta U + k^2 U = 0$, $U/_S = f$ using the double-layer potential: for the case in which $\lambda = k^2$ is one of the eigenvalues of the operator Δ an integral equation that is not always solvable is obtained for the potential density in the complementary interior region at zero value of the normal derivative on the boundary. The situation is the same with respect to the external Neumann problem, if its solution is sought in the form of the simple-layer potential, and $\lambda = k^2$ is an eigenvalue of the Laplacian operator in the interior region for zero value of the function on the region boundary. O. I. Panich reported on one method of overcoming this difficulty. B. R. Vaĭnberg discussed "Sommerfeld-type Conditions for Elliptical Operators of any Order."

"Nonstationary Problems" have been developed further since the first symposium. New theoretical methods have been developed, and solutions have been given for several substantial problems that meet practical needs and that also are of theoretical interest.

V. A. Afanas'ev reported on a solution to the problem of a plane wave, specified in unit step form, incident upon a wedge-shaped cut in elastic space with a rigid closure on the boundary, by reducing it to a boundary-value problem for two analytic functions interconnected on the boundary of the region, and then to a system of singular integral equations corresponding to this boundary-value problem. The author proved the existence and uniqueness of the solution. Solving the plane problem of the diffraction of an arbitrary nonstationary acoustic wave (specified by a ray expansion) by a wedge, with boundary conditions $u = 0$ or $\partial u/\partial n = 0$, A. F. Filippov considered the reflection of an arbitrary acoustic wave from a smooth boundary whose curvature has a discontinuity at some point (or for which the derivative of any order with respect to the curvature has a discontinuity), and calculated the principal part of the diffracted wave, i.e., the first term of the ray expansion in the neighborhood of the wave front. L. M. Flitman considered the problem of motion of a bulky rigid strip located on an elastic half-space, induced by a wave pulse. For a time interval so small that secondary reflections from the plate edges cannot be set up, the problem reduces to equations with constant coefficients. The equation obtained was investigated. In his report, V. A. Borovikov investigated the three-dimensional problem of diffraction by an infinite prism S having as its base an arbitrary convex polygon, for the boundary condition $U|_{S=0}$. For arbitrary positions of a point source a and observation point x, the singularities of the Green's function $\Gamma(a, x, t)$ of the wave equation were determined as functions of t for fixed a and x. Next, the author used the Fourier transform of t to determine the principal terms in the asymptotic series (as $k \to \infty$) for the Green's function $\Gamma_1(a, x, k)$ for the Helmholtz equation. It turns out that for each optical path joining points a and x, there is a corresponding contribution to the asymptotic series, beginning with terms of the order of $k^{-(3/2)n+(1/2)}$, where n is the number of kinks in the optical path. Explicit formulas were obtained, yielding the first term of the contribution to the asymptotic series as $k \to \infty$ for each optical path, as well as recurrence formulas (corresponding to the number of kinks in the optical path) for the second term. A method was developed that in principle makes it possible to find any term in the asymptotic series, and it was extended to the case with boundary condition $\partial u/\partial n|_S = 0$. S. S. Voĭt discussed the formation of unsteady long waves in a rotating bounded basin. On the assumption that a flat horizontal liquid layer bounded by a vertical plane rotates about the vertical axis and raises the liquid in the region of its free surface at the initial instant of time, the author studied the further propagation of the initial liquid crest, using the assumptions of long-wave theory. It turns out that rotation of the basin destroys symmetry in propagation of the initial crest. Kelvin-type waves propagate along the basin boundary in just one direction; they are not damped with increasing distance from the region of initial crest. These boundary crests have an amplitude that is attenuated exponentially with increasing distance from the basin boundary and, thus, they are noticeable only in direct proximity to the basin wall. As a result of the analysis, asymptotic formulas for the direct and reflected waves were obtained for the rise of the liquid surface at points distant from the initial-crest region, and the asymmetry in propagation of the initial liquid rise was established.

P. V. Krauklis showed that if a source exciting an unsteady field is located in a layer of liquid between elastic half-spaces, then in addition to that portion of the field corresponding to expansion in normal undamped waves (existing for the condition $v_0 < v_S < v_P$, where v_0 is the speed of sound in the liquid layer while v_S and v_P are, respectively, the velocity of transverse and longitudinal waves in the elastic medium), there exists a field with waveguide-type propagation, characterized by anomalous dispersion, for any relationships among v_0, v_S, and v_P. L. N. Sretenskiĭ discussed diffraction of ship-wake waves. G. I. Freĭdman showed that the problem of investigating sustained electromagnetic shock waves in waveguides with a thin layer of ferrite, where the lateral dimensions of the transmission line are sufficiently small and the configuration of the ferrite cross section sufficiently simple, reduces to linear ordinary differential equations that result from the Maxwell equations and that determine approximately, in conjunction with the nonlinear equation for the magnetization, the structure of the shock wave. The necessary condition for uniqueness of the shock-wave front structure is the same here as its stability condition in the discontinuous approximation. Using special examples, the author examined the effect on shock-wave structure of the nonstatic character of the field in a line with ideally conducting walls or the dispersion of the dominant wave (i.e., the wave whose critical frequency equals zero) in waveguides with reactive walls. It turns out that for strong shock waves, the difference between the structure of a field in a line with ideally conducting walls and the static structure has very little effect upon the parameters that determine the shock-wave structure, even when the width of the wave front is less than the distance between the walls of the waveguide. Dispersion of the dominant wave in delay lines, however, does effect the structure of strong shock waves.

Four sessions were devoted to problems of diffraction by various bodies, including the sphere and the wedge. S. M. Travinin considered wave diffraction around a half-submerged elliptical cylinder; this is connected with the problem of calculating the perturbing forces in movement of a ship through swells (for arbitrary location of the horizontal axis (x) of the cylinder relative to the direction in which the wave travels). The solution reduces to finding an auxiliary function $\psi(y, z)$ that satisfies the expressed wave equation and the boundary conditions. The function $f(y, z)$ introduced by Khaskind and connnected with ψ by the equation $-\partial f/\partial z = \partial\psi/\partial z + k\psi$ (k is the wave number) can be represented in elliptical coordinates (ξ, η) with adequate accuracy as a series of products of Bessel functions of imaginary argument with cosines of multiple arcs:

$$f(r, \eta) = \sum_{m=0}^{\infty} c_m K_{\nu_m}(k, r) \cos m\eta,$$

where $r = \frac{c}{2} e^{\xi}$, $c = \sqrt{a^2 - b^2}$.

M. G. Sukharev studied the behavior of "Ship-Wake Waves over an Uneven Periodic Bottom," on the assumption that the irregularity and pressure were of the same order of smallness. He found the velocity potential as a Laplace solution, satisfying the given boundary conditions, in the region occupied by liquid; he then expanded the solution in powers of a small parameter, obtaining in first approximation the classical problem of ship-wake waves. The second-approximation equations were solved by the method of integral transforms. V. V. Martsafeĭ studied the "Radiation of Electromagnetic Waves by an Infinite System of Flat Waveguides with Walls of Infinite Conductance." For the case in which E_{0n} or H_{0n} ($n = 1, 3, 5, \ldots$) waves are excited in phase in the waveguides, this problem can be solved by the Wiener-Hopf method. Here the field in the distant zone is expressed as a finite sum of plane waves. I. G. Petritskaya solved in linear approximation the boundary problem for determination of the velocity-vector field of air particles in a thin layer bounded by rigid cylindrical side walls between two circular plates, one of which vibrates harmonically at constant amplitude, and the other is stationary; she obtained a value for the acoustical-mechanical resistance of the air layer to the vibrations, considering the case in which the stationary plate has a single circular aperture. The resistance of the thin air layer proved to depend substantially upon the radius and location of the circular aperture. A similar problem was solved for the case in which the moving plate has a distributed vibration amplitude $\eta = \eta_0[1 - (r/a)^2]$ (r is the radius of the plate). The hydrodynamic problem of "Diffraction of Plane Sound Waves (long) by a Moving Torus with Elliptical Cross Section" was reduced by P. I. Tsoĭ to the Neumann problem (i.e., to a solution of the wave equation with given normal derivative of the velocity potential on the surface of the stationary torus); he used the velocity-potential equation in the toroidal coordinates in parametric form. M. G. Belkina, obtaining a rigorous solution to the problem of diffraction by an ideally conducting disk of an electromagnetic wave excited by an electric dipole located on the disk axis parallel to its surface, expressed the results in the form of a series in spheriodal functions. E. A. Ivanov reported on the diffraction of electromagnetic waves by two ideally-conducting infinitely thin disks of identical radius with a common vertical axis of rotation, located in a vacuum. The source of the primary field is a horizontal magnetic dipole with magnetic moment **m**. The dipole is located at a certain point between the disks on their axis of rotation. The potential functions in terms of which the components of vectors **E** and **H** of the secondary field are defined are found as series in the wave functions of an oblate spheriod. The unknown coefficients of the series are found from the infinite system of linear equations obtained from the boundary conditions for the potential functions sought on the surfaces of the disks, using the addition theorems for the wave functions. The infinite systems of equations become quasi-regular following some substitution of coefficients; their solution is found by the method of truncation.

I. N. Korbanskiĭ reported briefly on the "Radiation Resistance of a Hertz dipole Located Near an Ideally Conducting Paraboloid of Revolution." The radiation-resistance calculation was based upon a formula for the power radiated by a Hertz dipole in the presence of diffraction: $P = P_0 - (\omega/2)\,\mathrm{Im}(P^*, E)$, which follows directly from the method of induced emf's (P_0 is the power radiated by an isolated dipole, P is the dipole moment, ω is the frequency, and E is the electric field intensity of the reflected wave at the point at which the dipole is situated). To determine the electric field intensity of the reflected wave, the author solved the diffraction problem of radiation from a Hertz dipole located near a paraboloid of revolution to do this, he utilized the mathematical apparatus developed by Fock, by introducing a special form of the Hertz vector; this made it possible to write the boundary conditions on the paraboloid of revolution in a fairly simple form. The asymptotic solution for the cases in which the Hertz dipole is situated on the paraboloid axis and on its exterior surface made it possible to follow the change in the radiation resistance as the dipole is displaced with respect to the paraboloid. The report of N. A. Yablochkin dealt with "The External Electrodynamic Problem of Ideally Conducting Bodies of Complex Configuration."

In recent years, the problem of "diffraction by a wedge" has become very urgent. At the symposium, an entire group of reports was devoted to this question. In their report, G. D. Malyuzhinets and G. V. Vinel' examined plane scalar problems of field diffraction in one or several adjacent angular regions bounded or separated from each other by rays on which the boundary conditions are given. For opaque boundaries, an impedance boundary condition at $\nu = 0$ (the Leontovich condition) of the type

$$\frac{\partial U}{\partial \nu} \pm ikgU = 0 \ldots \qquad (1)$$

is used. For the case of a translucent boundary, the two following boundary conditions are given for $\nu = 0$:

$$\left.\begin{aligned} &\frac{\partial U}{\partial \nu}(+0) + ikaU(+0) - ikbU(-0) = 0, \\ &\frac{\partial U}{\partial \nu}(-0) - ikaU(-0) + ikbU(+0) = 0. \end{aligned}\right\} \qquad (2)$$

If a plate has thickness d and is made of material with a wave number $k_0 = k_n$, then when $|n| \gg 1$, the coefficients a and b are expressed in terms of parameters of the plate material (index 0) and of the surrounding medium for the acoustical and electromagnetic cases of two polarizations (the edge of the semi-infinite plate coincides with the z axis). If $\mathrm{Im}\, k_0 d \to \infty$, then $b \to 0$, $a \to g$, and the conditions (2) go over into (1). In the opposite case, $k_0 d \to 0$, where complete transparency is reached, conditions (2) go over to the conditions for continuity $U(+0) = U(-0)$ and smoothness $(\partial U/\partial \nu)(+0) = (\partial U/\partial \nu)(-0)$. For each point in the region under consideration, the fields due to the plane waves reflected from and transmitted through the translucent plates are calculated in the geometric approximation. The report considers problems that differ in the shape of the region and in the boundary conditions. In all the examples, the region and the boundary conditions are symmetric about each translucent plate. The asymmetry introduced by the form of the incident wave can be eliminated by partitioning the fields sought into sections that are odd or even relative to the translucent plates. Here all examples are reduced to the solution obtained by G. D. Malyuzhinets in 1950 to the problem of diffraction of a plane wave in an angular region $-\Phi < \varphi < \Phi$ with given boundary impedances

$$\left(\frac{1}{r}\frac{\partial v}{\partial \rho} \mp ikg_{\pm} v = 0, \quad \varphi = \pm\Phi\right).$$

The report of A. A. Tuzhilin dealt with diffraction of spherical electromagnetic waves excited by an electrical or magnetic dipole in an angular region with ideally conducting boundaries. Using a rigorous solution for this problem in the form of a Sommerfeld integral, the author obtained an asymptotic representation of the wave field both close to and far away from the edge of a wedge. In addition, there was an investigation of the transition in the limit to the case of an arbitrarily small distance from the field source to the edge. R. P. Starovoĭtova and M. S. Bobrovnikov, using the Sommerfeld integral method, obtained a rigorous solution to the problem of excitation of an impedance wedge by a filamentary magnetic current located in it vertex. An analysis using the special functions of G. D. Malyuzhinets showed that the field of the current located at the vertex of the wedge consisted of a cylindrical radiation wave and two surface waves propagating along the boundaries of the wedge from the vertex to infinity. The width of the radiation pattern at the nulls always coincides with the aperture angle of the wedge; the direction of maximum radiation is determined by the relationship of the impedances on both boundaries. The power radiated decreases as the boundary impedances rise; here there is an increase in the energy delivered by the source to the surface wave.

L. N. Lemanskiĭ and L. N. Zakhar'ev have calculated radiation patterns for sources located on the surface of an ideally conducting infinite wedge with arbitrary vertex angle. The radiation patterns were calculated for various amplitude and phase characteristics of surface radiators an arbitrary distance away from the edge of the wedge. Where the sources are far (in comparison with λ) from the wedge tip, an asymptotic formula was obtained for the radiation patterns; it yields satisfactory results (as shown by comparison with a rigorous calculation) for sources more than $2\lambda - 5\lambda$ away from an edge of the wedge.

D. P. Kouzov considered the steady-state problem of acoustic-wave diffraction in a liquid half-space from the boundaries of two eleastic plates located on the liquid surface. As we know, at low frequencies (for which the wavelength in the plate material is quite large in comparison with the plate thickness) it is possible to neglect the variation in the wave field with the lateral coordinates of the plate. A mathematical problem of this type reduces to solving the Helmholtz equation for the half-space with certain boundary conditions containing higher-order derivatives. In these boundary conditions, the plate thickness enters solely through the coefficients (together with other parameters that characterize the eleastic properties of the plates). Since longitudinal waves interact less with the liquid than the transverse waves, the author excluded from consideration processes associated with the existence of longitudinal waves in the plate material. On these assumptions, he arrived at a solution to the diffraction problem for a plane wave traveling from within a liquid in a direction perpendicular to the plate interface, and considered cases for various possible contact conditions.

The symposium participants were very much interested in the session dealing with the "Physical Theory of Diffraction." In the report of L. A. Cherches, the approximate solution to the problem of "Wave Diffraction by Bodies with Discontinuities" was considered. On the assumption that the linear dimensions of the bodies are small in comparison with the wavelength, that the edge at the point of discontinuity is wedge-shaped or of finite thickness, and that the parameters of the discontinuity periphery (radius of curvature, thickness, or angle of the edge) varies slowly, the author calculated the diffraction field at rather large distances from the edges of discontinuities. Zero boundary conditions were used at the body surface, and it was assumed that the field scattered by the edges is set up by additional "diffraction" sources located along the discontinuity peripheries, with the phase of the incident-wave field determining the phase of these secondary sources; it was also assumed that the field of an elementary diffraction source depends on distance in the same manner as the field of a spherical radiator, and that the angular characteristic corresponds to the nature of the discontinuity. The scattered field is found by summing the diffraction-source fields over all discontinuity boundaries. The most important role is played by the discontinuity sections near radiating points. The paper considered plane-wave diffraction by a disk of arbitrary profile and finite thickness with a wedge-shaped edge, by convex bodies with discontinuities, as well as by concave bodies, for which in some cases it is necessary to consider the additional effect of multiple reflections. The method employed is an original generalization of the stationary-phase method to a body with discontinuities; it makes it possible to allow for secondary scattering of waves from diffraction sources. The relationship between the method of diffraction sources reported and the Keller diffraction-ray method is the same as that existing between the stationary-phase and geometric-optics methods in reflection techniques. The scattered field is reduced to expressions identical with those obtained in the method that considers the "nonuniform" current component near the discontinuity. This serves as a check on the method employed.

B. E. Kinber discussed "Diffraction by the Open End of a Plane Sectorial Horn." The radiation from the open end of a sectorial horn is considered as the diffraction of the natural wave of an infinite horn by its edges. Here the natural wave of the infinite horn is represented as the sum of two waves, resembling Brillouin waves in a plane waveguide, and their diffraction by the horn edges as the diffraction of cylindrical waves by a wedge (since flanges may be present). The cylindrical edge wave that forms upon primary diffraction is modified in part and reflected in part into an interior plane, giving rise to edge diffraction waves of a lower order of magnitude. In another communication, the same author dealt with diffraction by an aperture in a screen for the first Schwartzschild approximation. The Kirchhoff approximation takes into account neither the orientation of the screen edges nor the polarization of the primary wave. The near field may be considered as the sum of a plane wave and two edge waves. In the far zone, this approximation produces the normal expression for the radiation pattern. Lateral radiation is formed at a distance $1/2m^2$ further away than the major lobe (m is the number of the side lobe). N. G. Bondarenko and V. I. Talanov discussed beam waveguides using mirrors of special form as phase shifters. They calculated the waveguide parameters with allowance for the diffraction and ohmic losses in the shifters, and compared them with lens-type beam waveguides. V. V. Kvartsov obtained "Integral Equations of the First Type with Regular Kernel for Current Harmonics of a Body of Revolution" for the scalar and stationary electro-magnetic case, as well as an integro-functional equation of the first type for the same quantities in the nonstationary case; he proved uniqueness for the solution to the equations obtained.

The next session of the section heard reports on "Diffraction by a Sphere." Z. A. Yanson and V. S. Buldyrev examined an elastic sphere of radius R, covered by a spherical layer of thickness $H = R_1 - R (R_1 = R + H)$. The system is excited by a concentrated rotational effect applied to the external surface of the layer (it is assumed that the wave velocity is greater in the underlying medium than in the layer). The solution is obtained by separating variables, and in spherical coordinates (r, θ, φ) takes the form

$$U(r, \theta, t) = \varphi_i \sum_{R=1}^{\infty} \varphi_c(r, t) P_c^{(1)}(\cos\theta), \qquad (1)$$

where $\varphi_e(r, t)$ are contour integrals whose integrands are meromorphic on the plane of the variable of integration. The contour integrals $\varphi(r, t)$ are calculated by residues, and the solution is then transformed by Watson's method to a sum of contour integrals on the plane (l). The contour integrals obtained are reduced by a series of manipulations to Fourier-type integrals to which the stationary-phase method is applied. As a result, the field is represented by the sum of interferring waves, each corresponding to a specific frequency component. It is of particular physical interest to study the family of disperson curves (phase- and group-velocity curves) for the interference waves obtained, and to compare them with the existing Love curves for the case of a plane layer lying at a half-space, investigated by G. I. Petrashen'. In their report, O. A. Germogenova and G. V. Rozenberg discussed scattering of a plane electromagnetic wave with a complex wave vector and complex amplitude by a sphere. They introduced the electric and magnetic potentials for which the boundary problem is solved, similar to that treated in Mie's theory. The coefficients in the expansion of the scattered-field potential in eigenfunctions of the Helmholtz equation turn out to be proportional to the corresponding Mie coefficients. As a special case, they examined dipole scattering and the way in which it depends on the wave inhomogeneity factor. The possible application of the results obtained to problems of optics and radiophysics was discussed. Yu. A. Erukhimovich and Yu. V. Pimenov obtained a new and convenient asymptotic solution to the problem by means of the Huygens-Kirchhoff method on the assumption that the density of an electric current induced on a sphere is proportional to the magnetic component of the incident wave; they compared the numerical results with data from the rigorous theory. D. S. Chernavskiĭ discussed the inelastic diffraction interaction of elementary particles.

The widespread introduction of electronic computers into theoretical investigations has again raised the question of "numerical methods" in the solution of wave-propagation and diffraction problems. An interesting report on the "Application of Integral Equations of the Second Type to Calculating the Current Distribution on a Cylinder of Finite Length" was presented by E. N. Vasil'ev and A. R. Seregina. To solve the excitation problem for a cylinder of finite length, they employed integral equations of the second type for the azimuthal harmonics of density for the electric current flowing along the cylinder surface. The integral equations were solved numerically. They considered axially symmetric excitation by radial and longitudinal dipoles, as well as excitation by a transverse slot in which the voltage depends upon the azimuthal coordinate. For the axially symmetric case, they determined the effect of cylinder length and diameter, slot width, and the shape of the end surface upon current distribution. Here they noted the existence of azimuthal-current density maxima at the narrow edge of the cylinder. The effect of source position on current-density distribution, etc., was considered. Similar current-distribution investigations were carried out for the case in which a cylinder is excited by a radial dipole and a transverse slot having a field depending on the azimuthal coordinate. N. N. Govorun found that for a thin cylindrical dipole the kernel of the approximate integral equation of the first type describing the current distribution on the surface of a body of revolution located in a lossy medium is identical with the kernel for the similar equation applying to the lossless-medium case. Extending the results of a numerical solution for the integral equation to the case of thin dipoles for various wavelengths and conductivities, he estimated the region of applicability for the quasi-stationary method of calculating the current distribution in a dipole according to a given current at the supply point. The two reports of D. M. Sazonov dealt with the arbitrary electromagnetic excitation of ideally conducting finite metal wedges and hemispheres with small electrical dimensions. A. F. Chaplin discussed the excitation by arbitrarily distributed sources of strips located on an infinite ideally conducting shield with a surface impedance varying in the transverse direction. For a strip with a constant impendance varying linearly and represented by a harmonic law, the integral equations for the distribution functions of the electrical and magnetic currents on the strip surfaces were solved numerically by the Krylov-Bogolyubov method.

The reports on "Propagation in Layered Media" deal basically with plane boundaries, chiefly of elastic layers. V. Yu. Zavadskiĭ considered Rayleigh waves propagating along the free boundary of an inhomogeneous elastic half-space with Lamé parameters increasing (or decreasing) linearly within the medium. Finding a rigorous solution, the author obtained asymptotic formulas (for large ω) for the dispersion and attenuation of these waves. In his report "Toward a Theory of Standing Waves of Finite Amplitude on the Free Interface Formed by a Heavy Liquid with Two Layers of Different Densities and Depths," Ya. I. Sekerzh-Zen'kovich considered a heavy ideal incompressible liquid consisting of two layers of different density, located one above the other. The upper layer is assumed of finite depth and the lower infinite. Lagrangian variables are used to give the complete formulation of the problem of plane standing waves on the free surface and on the interface. The author uses a small-parameter method to solve this problem to any approximation; calculations of the first two and of part of the third approximations are carried through to conclusions. An approximation formula is obtained connecting the oscillation frequency with the amplitude and wavelength, and the specific features of the radiated waves of finite amplitude, distinguishing them from the standing waves of linear theory, are established. In his communication, L. V. Iogansen presented a calculation of the conditions necessary for the appearance of the so-called "Resonance Diffraction of Acoustic Waves in Plane-Layered Systems" (considered first by the author for the electromagnetic-wave example) for the case of liquid layers playing the role of barriers with differing densities, and for solid isotropic barriers. Longitudinal and transverse waves appear simultaneously in such layers. Exponential resonant accumulation is possible if on the liquid-solid boundary there occurs simultaneously complete internal reflection for both the longitudinal and transverse waves in the solid. An expression was obtained for the characteristic resonant-diffraction length l_0, which has an order of magnitude $l_0 \approx \tan\varphi \cdot d_3 \exp(qd_2)$, where φ is the wave angle of incidence from the liquid to the solid totally reflecting layer, d_2 and d_3 are respectively the thickness of the solid totally reflecting layer and of the liquid resonator, q is the imaginary portion of the transverse-wave wave vector in the totally reflecting layer.

V. N. Krasil'nikov discussed the propagation of elastic waves from concentrated sources into a liquid half-space bounded by an elastic plane-parallel layer. For the case in which the transverse-wave propagation velocity in the layer material is greater than the speed of sound in the liquid, there will appear just one undamped surface wave; at low frequencies, it will approach a flexural wave, and at high frequency a Rayleigh wave on the solid-liquid interface. The numerical data obtained were compared with known approximate results based upon the representation of the elastic layer by a plate capable of supporting only flexural and longitudinal oscillations. It was found that longitudinal waves in the plate for the problems of the type considered do not play an independent role; the strains and stresses caused by them are relatively small, and do not form the major portion of the error associated with a transition from elastic-layer deformations to pure plate bending. In L. A. Molotkov's brief communication "On Low-frequency Oscillations in an Elastic Layer" it was noted that the use of dynamic elasticity theory made it possible to justify and refine equations for plate oscillations, and also to determine the conditions under which engineering theory is applicable. By using the action principle, it is possible to obtain equations for plate oscillations in various approximations, and to write the appropriate expressions for displacements in a layer. An analysis of these expressions in first and second approximation makes it possible to understand the nature of the deformation, and to compare the results obtained with the assumptions of Rayleigh, Timoshenko, and Uflyand.

"Diffraction at Periodic Undulating Surfaces" held the attention of those working in the fields of acoustics, optics, antennas, or interested in problems of electromagnetic-wave propagation. Although the basic fundamental problems might be considered to have been studied in this field, practical physical and engineering investigations turn up new problems all the time, and their solution provides an impetus for development and refinement of theory. A. D. Lapin solved the problem of plane-wave diffraction by a sawtooth surface with rectangular teeth. The diffraction field above the uneven surface is found as a superposition of Bragg spectra. The problem is solved by joining the fields at the boundaries of specially selected rectangular regions in which the eigenfunctions are known. An infinite system of algebraic equations with constant coefficients is obtained for the diffraction-spectra amplitudes. This system of equations is solved numerically by the reduction method for certain parameter values. G. V. Poddubnyĭ examined "Scattering of Electromagnetic Waves by a Periodic Surface" for oblique incidence of a plane electromagnetic wave by an infinite periodic surface with d and equation $x = f(y)$. The scattered field is

$$\varphi_s(y, z) = \int_L \left(G \frac{\partial \varphi_s}{\partial n} - \varphi_s \frac{\partial G}{\partial n} \right) dl,$$

where n is the outward normal to the scattering surface, and the integration path L is a single period of this surface. The function $G(y-\xi, z-\eta)$ is expressed in terms of the wave number, the angle of incidence, and the period of the scattering surface. Thus, the field can be found approximately in the far zone if the appropriate boundary conditions are given, for example, those of a sawtooth surface. Yu. M. Cherkashin reported briefly on scattering of sound by a stationary interface for two media with an abrupt, but small, relative change in the refraction index $n^2 - 1 = \Delta c/c \ll 1$. He explained that scattering of sound in the form of a periodic internal wave appearing by a poorly defined interface between two media is very similar to scattering of plane waves by a similar absolutely reflecting surface; in both cases, we obtain an identical arrangement of scattered-sound intensity maxima on an angular scattering diagram, while the pressures at the maxima differ in value. V. I. Aksenov proposed an approximate method for calculating the amplitude of an electromagnetic wave specularly reflected from a periodic irregular dielectric surface; the method is based upon the replacement of this surface by "equivalent" layered-inhomogeneous medium. If the equation for the uneven surface is given in the form $z = Z(x, y)$, then the dielectric properties of the "equivalent" medium vary along the z axis. The method is applicable where the plane electromagnetic wave is normally incident upon the irregular surface, and the wavelength λ and irregularity period Λ satisfy the inequality $\Lambda < \lambda$. For the case in which the incident-wave electric vector is perpendicular to the plane of incidence, the limitation on the angle of incidence θ_0 is removed, and the second condition is replaced by the following condition: $(1 + \sin\theta_0)\Lambda < \lambda$. The author obtained calculation formulas for determining the law obeyed by the variation in effective dielectric constant of the "equivalent" layered-inhomogeneous medium from the given form of irregularity and the known dielectric properties of the surface material; the calculation formulas yield good agreement between theory and experiment.

In the group of reports on "waveguide problems," L. A. Vaĭnshteĭn presented an interesting paper on an accurate "Theory of Contactless Plungers." He investigated contactless (reactive) plungers in a coaxial line in the form of metal cylinders, closed at one end, placed over an inner conductor. Such plungers provide almost complete reflection within a certain range of frequencies, while there is no direct contact between the inner and outer conductors of the line. The report gave both the elementary theory of such plungers, based on the telegrapher's equations and the more accurate electrodynamic theory, which takes into account diffraction at the open end of the plunger and is based upon the Wiener-Hopf-Fock method; the physical analysis for the blocking action of contactless plungers is given, along with an analyiss of the corrections used in going from the elementary theory to diffraction. The report of A. S. Il'inskiĭ and A. G. Sveshnikov deals with the application of a general calculation method for matching waveguides with substantially different cross sections to a specific problem of matching waveguides with circular and square cross sections. The essence of the general method lies in the fact that an arbitrary irregular waveguide can be represented, using an appropriate transformation of coordinates, as a waveguide with a regular side surface, but an inhomogeneous filler. The problem obtained by a method similar to Galerkin's method reduces to a boundary problem for a system of ordinary differential equations. The authors demonstrate the convergence of the approximate solution thus obtained. A numerical calculation performed on a computer for an actual problem showed that the method converges well. In her report "Transformation of Electromagnetic Waves in a Waveguide with Slowly Varying Impedance," N. P. Kerzhentseva studied the transformation of waves propagating in a multimode cylindrical waveguide with a slowly varying wall surface impedance. The solution is carried out by the cross-section method, according to which the field in each section is found as a series of natural modes for a regular waveguide with an axially constant surface impedance; the coefficients of this series are found in explicit form.

Methods of scaled and physical simulation play a considerable role in the investigation of the complex phenomena appearing in the emission and propagation of various types of waves. Problems of "Wave Simulation" were discussed in a separate session. Until recently, simulation of sound-wave propagation was chiefly employed to study the acoustic field in two-layer systems, and in connection with problems of sound scattering on an uneven surfact bounding a homogeneous medium. In addition, simulation is quite possible in investigations of the laws followed by sound propagation in such inhomogeneous media as the earth's atmosphere and the waters of the oceans and seas. In the latter case, in fact, there are both regular and statistical inhomgeneities. The simulation of inhomogeneities of fluctuation nature according to the criterion for the various statistical characteristics first requires a study of the conditions for the formation of the various fluctuations in parameters of the medium, which is quite complicated. The simulation of the regular inhomogeneities of such media as sea water, however, is quite possible. These media can be considered as layeredinhomogeneous media with vertical and horizontal sound-velocity gradients. Naturally, hydroacoustical measurements carried out by the simulation method cannot replace those carried out under natural conditions, but can add considerably to them, refine details, help in checking theory, and in addition accomplish this with little drain on material or personnel resources in comparison with measurements conducted on the ocean. Model measurements, in contrast to natural measurements, make it possible to recreate individual types of regular inhomogeneities in the natural medium in isolation, and to study their effect upon sound propagation with greater clarity. A. N. Barkhatov discussed investigations, in a special modeling tank, of sound propagation in various media of the "antiduct" type (with a constant negative vertical sound-velocity gradient; with a quasihomogeneous surface layer, in a medium with a bilinear sound-velocity distribution profile) and in an acoustical duct, and of the reflection of bounded sound packets from various layered inhomogenous media; of the effect of an uneven surface on sound propagation in the shadow region and at the surface of an acoustical duct; of the effect of internal waves on sound propagation. He checked the applicability of normal-wave theory to various cases of sound propagation in layered inhomogeneous media, clarifying the effect of undulating and rough surfaces on the sound field in layered inhomogeneous media, depending on the parameters of the rough surface and the characteristics of the media.

Although it has been more than half a century since the radiowave-prospecting method was proposed, its application has been delayed until recently owing to the fact that the theory was not worked out; the method uses the "shadow ' effect due to well-conducting bodies for locating and mapping ore deposits in the space between mine workings and the earth's surface. The basic theoretical problems are the propagation of radiowaves in layered media, diffraction at semi-infinite bodies and bodies of finite dimensions, and antenna problems. In speaking of the "Electromagnetic-Wave Diffraction Problem in the Radiowave-prospecting Method," A. D. Petrovskiĭ took note of the need for considering the field in an absorbing medium, most frequently in the zone immediately adjacent to the radiator. In order to determine the field due to an underground radiator at the earth's surface and the shielding coefficients for various strata, approximate solutions obtained with the aid of impedance boundary conditions were used. In the near zone, and in the presence of "weak" shields, these solutions are naturally unsuitable, but there is still nothing to replace them. Although it is possible with the aid of simulation to establish certain interesting relationships characteristic of radiowave diffraction in a conducting medium, the method does not permit the complete exclusion of numerical computation; in this case, for bodies of finite dimensions solutions must be obtained in the quasistatic, intermediate, and quasioptical frequency regions. It is also necessary to provide a theoretical foundation for the results of an experimental investigation into the effect of mine workings, wells, and conductors on the radiowave field in radio propecting. In speaking of a method for measuring lumped capacitances by the electrolytic-trough method, G. P. Prudkovskiĭ gave examples of resonator design. He considered a method of simulating wave fields in waveguide-type systems having an arbitrary but constant cross-section in one direction, and gave examples of resonator and periodic-system calculations. G. P. Grudinskaya, Yu. K. Kalinin, and Ya. S. Rodionov discussed "Experiments in Ground-Wave Propagation Simulation Under Laboratory Conditions." There is good agreement between theory and their data on radiowave propagation along a coast line and on the effect of the irregular surface of the earth on field strength. Using approximate boundary conditions of the impedance type, Yu. K. Kalinin obtained formulas for the coefficient of reflection from a curved surface forming one of the coordinate surfaces in an orthogonal coordinate system. He derived the pole equation for a waveguide system consisting of two coordinate surfaces, the space between which contains weakly absorbing media. The reflection coefficient and pole equation are expressed in terms of the boundary values of logarithmic derivative systems of characteristic solutions to a differential equation along the normals to the coordinate surfaces. O. G. Shamina reported on the simulation of diffraction and refraction waves.

At the session dealing with questions of "Propagation of Radiowaves Along the Earth's Surface," experimental papers were presented. V. N. Troitskiĭ reported on the fact that an experimental investigation on uhf diffraction by mountain ranges has shown that the attenuation introduced by the ranges is considerably greater than that computed from the Fresnel formula for all paths investigated with wavelengths commencing in the centimeter range and ending in the meter range. A comparison of the experimental results with the theory of diffraction by a perfectly reflecting parabolic cylinder has shown that it is impossible to explain even quantitively the anomalous difference in field-strength level for different polarizations, or certain other phenomena. It turns out that in the majority of cases the field strength for horizontal polarization was noticeably greater than with vertical polarization.

Uhf propagation owing to diffraction around mountainous obstacles far beyond of line-of-sight limits opens new possibilities for reliable multichannel radiocommunications and television in mountain regions, in view of its technical and economic advantages over ordinary troposcatter communications links. S. A. Amanov's experiments, designed to find the spatial and temporal diffraction-field distribution pattern beyond mountain ranges along various routes have shown that when there are sharp mountain peaks or crests in the propagation path, the signal level beyond the obstacle is greater and more stable than in ther absence. Over such routes, attenuation is less by tens of decibels than for propagation over a smooth portion of the earth's surface, while the diurnal and seasonal signal-level variations do not exceed 3 and 7 db, respectively. There is an interference-type relationship between field strength and the distance and height of a receiving antenna. This is explained by the presence of complicated phase relationships among the rays due to surface features on the earth and the relief of the obstacle itself. Especially sharp and frequent signal fluctuations occur in the direction perpendicular to the path. These deviations amount to 30 db. Experimental data obtained for numerous routes crossing very different forms of mountainous obstacles show that at a great distance from the obstacle the shape of the upper mountain surface and the relief near the reception point have a substantial influence on the receiving-point signal level. Thus, an intelligent utilization of mountain peaks and local features near the reception point (choice of single peaks with sharp edges, location of the reception point on moist soil and on elevations that provide direct line-of-sight between obstacles and terminal points) can produce a considerable increase in signal level and stability. On very long routes, crossing sharp rocky ranges, with high antennas (small diffraction angles) uhf propagation conditions are very nearly optical and Fresnel diffraction theory may be used in calculations; mountain peaks are approximated by wedges around which direct rays and in some cases ground-reflected rays, diffract. Experiments have supported theoretical conclusions that relatively sharp mountainous obstacles produce a considerable reduction in attenuation, reachin 50—60 db.

It is well known that approximate boundary conditions (Leontovich boundary conditions, reduced surface-impendance type conditions) are commonly employed in electromagnetic-wave propagation problems. In solving problems by such methods, the lateral wave is neglected and in this connection, we are faced with the problem of the best impedance definition according to Leontovich and Ryazin $\delta = 1/\sqrt{\epsilon' + \cos^2\psi}$ (where ϵ' is the complex dielectric constant for the earth), and if we allow for the nature of radiowave propagation along a plane boundary, it is necessary to assume

$$\delta = \sqrt{\varepsilon' - \cos^2\psi/\varepsilon'}.$$

A brief communication of G. N. Krylov and A. D. Petrovskiĭ dealt with the question of the best choice of impedance, depending upon the initial-parameter values, and explained the physical premises connected with the two determinations mentioned.

The session devoted to "Periodic Gratings," began with the report of L. A. Vaĭnshteĭn "Toward an Electrodynamic Theory of Gratings." If the spacing of an infinite plane grating formed by parallel metal cylinders of radius b located a distance l apart is small in comparison with the wavelength, the complex distribution of electromagnetic fields near the grating cannot be considered, and it is treated as a semitransparent layer $-\delta < x < \delta$, satisfying certain boundary conditions at $x = +\delta$ and $x = -\delta$. The parameters entering in these conditions for the case of circular cylinders are given in tables as functions of the ratio b/l. For cylinders of other cross-sections, they may be found with by conformal mapping. The derivation of the boundary conditions for electromagnetic waves yielded as a byproduct similar boundary conditions for acoustic fields at "hard" and "soft" gratings (where $kl \ll 1$). The value of δ in the boundary conditions can be taken arbitrarilly; thus, for example, we may let $\delta = 0$ or $\delta = b$, but we should have $k\delta \ll 1$. Special choice of δ is necessary only in the case in which the gap between neighboring cylinders is extremely small; then δ must be so chosen that the boundary conditions for the reflecting undulating surface obtained upon contace of the cylinders will be satisfied. V. V. Malin solved the problem of plane electromagnetic-wave diffraction by a grating of ribbons for the case in which the electric-field vector parallels the ribbons. The method proposed by L. Levin consists in seeking a certain singular integral equation for the sought field at the grating apertures, and then solving this equation. In the system of infinite algebraic equations finally obtained, it is necessary to consider only two equations in two unknowns (for normal incidence) in order to obtain results that hold for spacing values less than or equal to the length of the incident wave. For oblique incidence, the number of equations that must be taken into account is nearly doubled. Consideration of just one equation in one unknown yields the well-known solution to the problem in the quasi-static approximation. For the special case of half filling, consideration of three equations yields results that coincide with the precise theory (L. A. Vaĭnshteĭn) for spacing values less than or equal to two wavelengths.

In their report "Diffraction of Electromagnetic Waves by Dual Plane Metal Gratings," O. A. Tret'yakov, D. V. Khoroshun, and V. P. Shestopalov considered the case of normal incidence of plane electromagnetic E and H waves of arbitrary polarization for arbitrary spacing-wavelength ratios and any metal-ribbon width in equal-spacing and equal-slit gratings, arranged symmetrically one above the other. The reflection coefficients, as well as the diffraction-spectra amplitudes are obtained from a solution to an infinite system of linear algebraic equations. In like manner, the authors solve problems for a dual grating whose interior region is filled with a dielectric of arbitrary dielectric constant: for a p-layer equal-spacing and equal-slit gratings, for dual equal-spacing and equal-slit gratings. The case of oblique incidence of plane electromagnetic waves upon a metal-ribbon grating with spacing l and slit width d in the presence of a metal shield and a dielectric sublayer of finite thickness has been examined by A. I. Adonina and V. P. Shestopalov. They plotted the reflection and transmission coefficients and the diffraction-spectra amplitudes for various values of d/l, l/λ, α, ϵ, and a/l (a is the distance from the grating to the shield, ϵ is the dielectric constant of the medium in the region between the grating and the shield). Average boundary conditions were obtained for a grating in free space, and a grating with metal and dielectric shields; the conditions were used to find the dispersion equation for circular and helical waveguides immersed in the medium. S. A. Masalov, E. N. Podol'skiĭ, and I. E. Tarapov examined the problem of plane electromagnetic-wave incidence at an arbitrary angle to a lattice formed by infinitely thin metal strips and normal incidence on a lattice formed by metal plates of rectangular cross section. The first problem reduces to the solution of an infinite system of linear algebraic equations that can be represented as a ratio of always-converging series. The second problem is solved with the aid of Fourier series. The boundary conditions and joining conditions lead to two infinite systems of linear equations. Good convergence of solutions to the truncated systems was observed. The numerical results obtained clarified the nature of the transition from an infinitely thick grating to an infinitely thin grating on the one hand, and to a system of waveguides on the other.

The symposium participants were much interested in the session on the problems of wave propagation in plasma and diffraction at plasma and gyrotropic objects. G. I. Makarov studied the propagation of a plane electromagnetic wave in a symmetric layer of an ionized medium whose properties are described by a dielectric constant $\epsilon_m(z)$ that depends solely upon the z direction. Here he assumed that when $z \to -\infty$, a plane electromagnetic wave of the form

$$E_{x\ \mathrm{inc}}(x, z) \to E_0 e^{jkx \sin\psi - ikz \cos\psi} \quad (z \to -\infty),$$
$$E_{z\ \mathrm{inc}}(x, z) \to E_0 e^{ikx \sin\psi - ikz \cos\psi} \quad (z \to -\infty).$$

is incident upon the layer. The problem was solved by the standard-equation method; here the quantity η_0 was used as the major parameter for the problem; from the physical viewpoint, this was equivalent to requiring slow variation of the parameters in a layer of the scale of a wavelength in a vacuum. It turns out that for a layer possessing the properties described, the standard functions are Whittaker functions whose argument is found by solving a certain transcendental equation. In contrast to the solutions obtained previously for unbounded layers, there is no need for approximate joining within the layer. The reflection coefficient for the layer was found, and the way in which it depends upon the presence of the singular point $z = 0$ and the layer thickness was determined (down to zero thickness). The results obtained were generalized to the point-source case. V. V. Zheleznyakov and E. Ya. Zlotnik discussed the conversion of plasma waves into electromagnetic radiation under the conditions of an isotropic plasma; they studied the "interaction" of normal waves in a stationary plane-layered medium and in a plasma moving in accordance with the law $V = v(z)x_0$. For plasmas with a homogeneous magnetic field, they considered the conversion of ordinary waves into extraordinary waves and vice versa, a phenomenon occurring in the quasi-transverse propagation region. N. A. Kuz'min investigated the relationship of plane electromagnetic phase and group velocities in a unbounded plasma located in a magnetic field. Particle collisions were not considered. The regions and boundaries for wave existence were indicated. The results obtained for special ordinary-wave velocity relationships make it possible to explain fundamental regularities in the behavior of plane waves with variations in plasma parameters, and to select the regime needed. M. E. Gertsenshteĭn and V. I. Pustovoĭt presented a theory of "Sound-wave Propagation in Crystals in the Presence of a Direct Current." If a longitudinal electric field appears during propagation of sound waves in a crystal due to the piezoelectric properties of the crystal, the electrostriction effect in the lattice upon sound propagation along a strong constant field, to the differing movements of carriers and the lattice in an ionic cyrstal, $E = E_0 e^{i\omega t - ikx} \uparrow\uparrow K$, then $\text{div } E = -ikE \neq 0$, and a space charge pulsating at the wave frequency will appear. If, in addition there is a constant field $E_=$ within the semiconductor, causing the carriers to drift at a speed $V_=$, the alternating component of the conductance current $_\sim = q_0 V_\sim + q_\sim V_=$, where q_0 is the dc component of space-charge density, and $q_\sim$ the ac component. The term $q_0 V_\sim$ is the current due to oscillatory motion of carriers, which is normally considered in the theory of conductance. The second term $q_\sim V_=$ is the current due to orderly space-charge motion. For a certain current magnitude, the conductance becomes negative and, consequently a longitudinal wave in such a medium must be considered to be accompanied by a longitudinal electric field. In the survey "Diffraction of Partially Coherent Radiation," N. G. Denisov and L. S. Dolin examined from a single viewpoint diffraction of partially coherent (nonmonochromatic) radiation by regular objects, in connection with problems that appear in optics, radioastronomy, and the theory of wave scattering.

N. G. Denisov discussed the reception of fluctuating radiation from highly directional antennas through a layer with turbulent inhomogeneities. He computed the mean intensity at the output of the receiver device (at the lens focus) for small and large fluctuations in the radiation parameters. The effect of attenuation of the received radiation due to trubulent inhomogeneities in a layer was discussed. In the report "Diffraction of Modulated Waves by Arbitrary Inhomogeneities," V. A. Zverev discussed the fact that the diffraction of a modulated wave by an inhomogeneity leads to a change in the nature of wave modulation. This variation can be determined quantitatively by considering the phase invarient, which is the carrier phase minus the sideband phase half-sums. The magnitude of the effect turns out to depend upon the relationship between the wavelength and size of the inhomogeneity. For waves propagating in a medium containing random inhomogeneities characterized by a spatial power fluctuation spectrum $F(\kappa_1, \kappa_2, \kappa_3)$, the author computed the spatial spectrum of phase-invariant fluctuations. Here the spectrum of the phase invariant proves to be proportional to $F(\kappa_1, \kappa_2, 0)$ and contains a diffraction factor that depends upon the wave parameter at the modulating frequency. If it is possible to radiate a wave field in just one point of space by employing several modulating frequencies, it is possible to determine the spatial spectrum of the random inhomogeneities. If the random inhomogeneities occupy only a certain part of space, it is possible to determine the amount of space filled by the inhomogeneities, using the magnitude of the phase-invariant fluctuation as a function of modulating frequency. This is of interest in determining the scattering volume in receiving a signal scattered from random inhomogeneities.

The polarizability of a plasmoid located in an alternating electric field $Ee^{i\omega t}$ depends on the thermal velocity of the electrons. Since the electrons in a relatively stable plasmoid execute a finite oscillatory movement within a certain potential well, this relationship is in turn determined by the relationship between the "most probable" frequency ω_k of the electron natural oscillations and the field frequency ω. V. B. Gil'denburg examined a concrete example of field interaction with a plasmoid localized in a square potential well with a Maxwellian distribution of electrons in the blob; he showed that even in this case, where the natural oscillations were far from harmonic and showed a large spread in frequencies, the relationship of the equivalent dielectric constant of the plasmoid and the frequency in the region $\omega \approx \omega_k$ was clearly of resonance nature. The results obtained make it possible to evaluate the effect of thermal motion on the effective plasmoid scattering cross section. A. V. Gurevich presented a brief paper on diffusion in the ionosphere. M. D. Khaskind assumed that plane electromagnetic waves were incident upon an ionized meteor trail at an arbitrary angle to the trail axis, and investigated the scattered-wave field. On the basis of the polarized-currents method, he established very general integral equations for the vector scattering characteristics in the far zone. The energy relationships are obtained, and the shortwave asymptotic series is examined in detail in second approximation for trails of various structures. The longwave approximation is evaluated for a high trail linear electron density. A complete solution is obtained, chiefly in the intermediate region, with the aid of special electromagnetic potentials for an axially symmetric plasma; a system of simultaneous equations is found for the potentials, and their limiting properties are studied. The report pays especial attention to normal incidence of electromagnetic waves upon a meteor trail for two polarizations. In this case, the electromagnetic field in the plasma is described by two mutually independent equations, each determining one of the polarizations. An investigation into the reflection of normally incident electromagnetic waves from meteor trails was carried out with the aid of approximation methods from the quantum theory of scattering, which make it possible to establish the basic features of scattering in the intermediate region.

Using a diffusion equation, Yu. K. Kalinin found the electron-concentration distribution of a moving point source of ionization. He represented the source intensity by the effective meteor parameters, and used a perturbation method to find the scattering cross section of the transparent portion of the meteor trail and it lifetime. In the paper "Scattering of Monochromatic Electromagnetic Waves by Inhomogeneous Absorbing Plasmas," Yu. S. Sayasov formulated an eikonal method for an inhomogeneous medium with a complex dielectric constant, i.e., complex rays were introduced as trajectories orthagonal to the surface of the complex eikonal, and formulas were found expressing the change in polarization along these rays in terms of their curvature and torsion.

The elementary solutions thus obtained, corresponding to individual rays, were used to find the emission characteristics of conducting bodies surrounded by an absorbing medium, as well as the scattering cross sections for a plane wave of an inhomogeneous absorbing system, and in particular of a conducting body surrounded by an absorbing medium. The results are simplified when the absorption is so great that the electromagnetic energy propagates along selected rays that correspond to minimum attenuation, and also for the case in which the dielectric constant is close to unity and, consequently, the rays are nearly straight.

In their paper, V. B. Gil'denburg and I. G. Kondrat'ev discussed certain features of the reflection and refraction of electromagnetic waves by inhomogeneous plasma layers within which the dielectric constant passes through zero. As the basic task, the authors selected the problem of oblique incidence of a wave on a plane infinite layer with a dielectric constant depending upon the transverse coordinate; they used ordinary linear material equations containing dielectric constants that do not depend upon the structure of the field determined by them. Only in certain very important places are estimates given for the influence of effects associated with spatial dispersion. The values found for the reflection and transmission coefficients make it possible to estimate the absorption in the resonance regions of an inhomogeneous plasma, to find the resonances, and to show the way in which their number depends upon the nature of the inhomogeneity; it is also possible to estimate the influence of the inhomogeneity upon the diffraction characteristics of various plasma.

Reports on several different subjects were read at the "Radiation of Electromagnetic Waves" session: V. Ya. Éĭdman found the spectral and angular energy distribution for a plasma wave emitted by a charge moving in a magnetoactive plasma. G. N. Krylov calculated the "Electromagnetic Field Structure and Radiation Resistance of a Hoop-shaped Antenna" (a thin hollow cylinder of finite length), in which the current is distributed in accordance with some law; the solution was carried out by resolution into multipoles, using the apparatus of Sommerfeld integrals. This makes it possible to generalize the results obtained to the case in which the antenna is located under finitely conducting ground. The radiation resistance of the antenna coincides precisely with that obtained with the induced-emf method. B. M. Bolotovskiĭ studied the "Radiation of Charged Particles in Uniform Motion near Optical Inhomogeneities." V. P. Dokuchaev examined hydrodynamic perturbations appearing in gaseous media in which there move solid bodies with finite "frontal" resistance and dimensions less than the mean free path in the medium. He analyzed the conditions for emission of acoustic waves by moving particles, and considered the radiation resistance. Yu. P. Verbin reported "On Certain Boundary Transient Processes in the Propagation of Radiowaves along the Earth's Surface."

Reports on "Diffraction at Gyrotropic Bodies" had been presented at the first All-union Symposium on Diffraction. Intensive investigations into this field served as the basis for several new interesting communications. Yu. V. Vaĭsleĭb, investigating free symmetric waves included between an ideally conducting cylinder of finite length and a coaxial gyrotropic rod, and analyzing the characteristic equation for the case of longitudinal magnetization of the rod, evaluated the characteristics of a cylindrical resonator and amplifier with a gyrotropic rod. A. T. Fialkovskiĭ was concerned with roughly the same problem, and studied free symmetric electromagnetic waves in a cylindrical gyrotropic rod magnetized longitudinally and in the vacuum surrounding it in order to find the radiation pattern of an antenna made in the form of a gyrotropic rod of finite length. G. I. Freĭdman reported "On Electromagnetic Shock Waves with a Thin Layer of Ferrite." V. A. Permyakov discussed "Diffraction of Electromagnetic Waves on an Inhomogeneous Plasma Sphere."

Problems of reflection from "Statistically Uneven Surfaces" were discussed with great interest at the symposium. V. I. Mikhaĭlov considered scattering of electromagnetic waves by statistically homogeneous areas in the Kirchhoff approximation. Certain additional conditions that limit the applicability of M. A. Isakovich's results were investigated, and they were generalized for two limiting cases. Setting himself the problem of finding the scattered field at a statistically rough surface for a given arbitrary law of reflection "in the small," R. G. Barantsev gave a complete solution to the problem for an isotropic differentiable random surface z(x, y) and investigated various versions of simplified models, in particular normal surfaces with small fluctuations in σ_1 and quasi-Markov profiles. L. M. Yurkova and I. N. Tamoĭkina noted that the known solution in the Fraunhofer zone for the mean intensity of the field scattered from a statistically rough surface with irregularities exceeding the length of a sound wave may be extended to the Fresnel region. The scattering at angle χ is determined by only a narrow portion of the field spectrum at the rough surface near point $P = q_x/\sqrt{2q_z\sigma}$:

$$q_x = k(\cos\psi - \cos\chi), \quad q_z = -k(\sin\psi + \sin\chi);$$

σ^2 is the variance of the roughness amplitude, while ψ and χ are the angles of emission and reception. The conditions for formation of the scattering pattern of a statistically rough surface are

$$R \gg \frac{D \cdot 2\pi\sqrt{2\sigma}}{\lambda} \cot\frac{\chi - \psi}{2}$$

(D is a linear dimension of the scattering area). An experimental check was run on a model statistically rough surface (with the necessary amplitude distribution sign, amplitude variance, a definite form of the correlation function and correlation interval) made of plastic foam and permitting good simulation of reflection from a water-air interface. Using the small-perturbation method, É. P. Gulin obtained expressions for the longitudinal and transverse (in the horizontal and vertical planes) autocorrelation functions for the amplitude and phase fluctuations in a sound wave (emitted by a point source) reflected from a perfectly soft two-dimensional statistically rough surface. The spatial autocorrelation coefficient of roughness for the surface is given as a damped oscillatory function. The stationary-phase method is used in geometric approximation to isolate the rough-surface regions that are most important in scattering; the conditions are obtained for which the regions important in scattering lie in the neighborhood of a specular-reflection point on a plane surface. The author found that the spatial autocorrelation coefficients for amplitude and phase fluctuations in a sound wave reflected from an uneven surface coincide: the longitudinal and transverse (in the horizontal plane) correlation intervals for the amplitude and phase fluctuations are determined by the spatial autocorrelation intervals for the irregularities in the same directions, by the vertical coordinates of radiator and recievers, by the distance between radiator and receivers, and by the ratio of the sound-wave length to the oscillation period for the spatial autocorrelation coefficient for irregularities in the longitudinal and transverse directions, respectively. For low arrival angles, the fluctuation-coorelation interval for receivers located at different heights is considerably less than the longitudinal correlation interval. Some level of residual amplitude and phase fluctuation correlation remains until the receivers are separated by a distance comparable with the distance from the radiator.

The Second All-union Symposium on Diffraction summed up the large amount of work carried out in the Soviet Union by mathematicians, physicists, and engineers in creating new methods and developing known methods for the theoretical analysis of problems of diffraction, wave propagation in inhomogeneous media, wave emission, and similar questions. These methods have very great scientific and practical value, and will aid in the development of a variety of engineering fields in which it is necessary to deal with wave processes.

The essential role of the first and second symposia on diffraction lies in consolidating the efforts of theoreticians and experimenters working on related problems in different branches of science and technology. We should take note of the substantial benefits resulting from the exchange of knowledge on methods developed to solve problems and the results achieved among mathematicians and physicists, and specialists in acoustics, optics, radiophysics, heat engineering, geophysics, hydrodynamics, etc. The final plenary session unanimously adopted a resolution noting the achievements in diffraction studies summarized by the symposium, and calling for certain measures to be carried out in the future. It was proposed that the next symposium be held in May of 1964 at Tbilisi. It was resolved to increase the number of survey reports at sections dealing with the present state of the problem under consideration by the section. Basically, the symposium should retain its methodological nature. In conclusion, the symposium participants took note of the excellent organization of the symposium. The symposium participants were glad of the opportunity to become acquainted with the noteworthy features of such a great industrial, scientific, and cultural center as Gor'kiĭ.

Translated by W. Mitchell