TEST OF PC AND PCT INVARIANCE IN DECAY PROCESSES

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INTRODUCTION

1. In 1956 Lee and Yang [1] questioned the invariance of physical processes with respect to the operation P.

P is defined (see, for example [2], Sec. 7) as the operation of changing the sign of spatial coordinates or momenta of all particles in the given physical system: x_i, y_i, z_i are replaced by $-x_i$, $-y_i$, $-z_i$ (i is the particle number) or $\mathbf{p}_i$ is replaced by $-\mathbf{p}_i$ (if the state of the system is given in momentum representation). Spin projections are left unchanged. All other variables that may be needed to characterize the state of each individual particle (charge, isotopic spin projection, etc) are unaffected by the operation P (by definition of P).

The process of transition of the system from some state Ψ_0 into some other state Φ is said to be invariant with respect to P if the probability amplitude for the transition from Ψ_0 to Φ is equal to the probability for the transition from $P\Psi_0$ to $P\Phi$, where $P\Psi_0$ is the state obtained from the state Ψ_0 on application of the operation P.

Noninvariance with respect to P has indeed been established in a number of well known experiments (*see, for example, the review* [3]).

This fact could be interpreted as evidence that empty space (the physical vacuum) has definite spirality so that inverted space differs from the noninverted. Such an interpretation is based on the following considerations [4].

If space is homogeneous in all directions then it is easy to verify that the geometrical operation of inversion $\mathbf{x} \to -\mathbf{x}$ commutes with the geometrical operation of displacement (shift) in time. Therefore if P provides a representation of the operation $\mathbf{x} \to -\mathbf{x}$ (if the operation P adequately describes the change in the state of the inverted physical system compared to the noninverted), then the operator P should commute with the operator $U(t, t_0)$—the operator of evolution of the isolated physical system in time (representing the geometrical operation of a shift in time by the amount $t - t_0$): $UP = PU$. Indeed, if the operators P and U pretend to be adequate (true) representations of the corresponding geometrical operations then they should satisfy the same commutation relations as do the original operations. It follows from $UP = PU$ or $P^{-1}UP = U$ that $\langle\Phi, U\Psi_0\rangle$—the probability amplitude for the transition from Ψ_0 into Φ—is equal to $\langle P\Phi, UP\Psi_0\rangle$, i.e., that all physical processes should be invariant with respect to P.

However another interpretation of the experiments on "parity nonconservation" is also possible: they only provide evidence that the above defined operation P is not an adequate (true) representation of the geometrical operation $\mathbf{x} \to -\mathbf{x}$. Space is not "right" or "left' and therefore there should exist another operator that provides a representation of $\mathbf{x} \to -\mathbf{x}$, and commutes with U. Already in 1952 Wick, Wightman and Wigner [5] showed that the operator PC also provides a representation of $\mathbf{x} \to -\mathbf{x}$, where C is the charge conjugation operator.*

This point of view can also be expressed in terms of the concept of "pseudoscalar charge" [6]: upon spatial inversion of a particle its electric charge changes sign so that a charged particle does not have a well-defined P parity [5]. Since the operation PC also involves a change of the sign of the lepton and baryon numbers and of strangeness, these characteristics are also pseudoscalar.

One may give one more formulation [7,8]: the mirror image of a particle is its antiparticle and not the particle itself.

If the operator PC provides a representation of $\mathbf{x} \to -\mathbf{x}$ then all physical processes should be invariant under PC. This hypothesis of "conservation of combined parity" reestablishes the equivalence of right and left in the vacuum, explains at the same time the experiments on "parity nonconservation," and—which is no less important—insures "parity conservation" in strong interactions when augmented by the hypothesis that the strong interactions are also invariant under the operation C.† One may then intro-

*Usually C is defined as the operator that replaces the particle by its antiparticle; one then must make additional statements as to what one means by the antiparticle of such particles as the π or K mesons. More directly C may be defined as the operator which changes the electric charge, baryon number, lepton number and strangeness of each particle into its opposite (in the sense of the algebraic change of the sign in front of the corresponding characteristic). The signs of the momenta, and in general all the mechanical characteristics of the particle (mass, spin), are left unchanged by C.

†According to the definition of invariance of a process (given above with P-invariance as an example) this means, for example, that the angular distribution of the particles, their polarization, and other mechanical characteristics of the process $a + b \to c + d$ (the collision of particle a with particle b giving rise to the appearance of particles c and d) should be the same as those of the corresponding antiparticles in the charge-conjugate process $\tilde{a} + \tilde{b} \to \tilde{c} + \tilde{d}$. It should be noted that it would have been sufficient for the interpretation of each individual experiment on "parity nonconservation"

duce the conventional concept of intrinsic spatial P-parity for the particles involved in the given process. For particles that are not self charge conjugate this concept is of limited applicability (although of wider applicability than the concept of isobaric spin).

2. The hypothesis of PC invariance is sufficient to explain the experiments on "parity nonconservation," it is not however a necessary consequence of these experiments. Let us pose the problem of how to test for such invariance. If it were to turn out that this invariance is absent one would have to either look for a different representation of $\mathbf{x} \rightarrow -\mathbf{x}$, or admit the possibility of "spirality" for the physical vacuum. Having posed the problem as being that of experimental clarification of the properties of space we must not base our proposals for appropriate experiments on some preconceived notions about the form of admissible types of interactions (for example, the notion that interactions must be local). In particular one must not replace the study of PC invariance by the study of invariance under the Wigner time reversal T (for a definition of T see below, Sec. 1) by resorting to the Luders-Pauli theorem [2,9] which is valid only for a certain class of theories (see item 3, below).

This means that we must compare the characteristics of a given process with the characteristics of the charge conjugate process.

We restrict ourselves to the testing of PC invariance for decay processes only (for "weak interactions"). For "parity conserving" interactions the test of PC reduces to the test of C invariance. C invariance has been solidly established for electromagnetic interactions. It is not expected that it should be violated in strong interactions either. The most accessible experiments in that direction are, apparently, the experiments on the interactions of antiprotons with protons [10,11]. Some of these experiments have already been carried out [12,13]. We have to give up the discussion of processes charge conjugate to β decay. The most accessible of them, the β decay of the antineutron, cannot be studied at this time because of the small number of antineutrons and their long lifetime.

The decays of $\pi^{\pm}$, $\mu^{\pm}$, K mesons and also antihyperons have already been observed. The corresponding experiments will therefore be discussed with the aim of summarizing the known data on PC invariance and of indicating additional possible experiments.

3. It turns out that a majority of these experiments may be looked upon as a test of PCT invariance. In this paper these experiments will be discussed from that point of view, and also from the point of view of distinguishing the experimental consequences of PC and PCT invariances.

It is worthwhile to emphasize the distinction between the concept of PCT invariance and the PCT theorem (or the Luders-Pauli theorem). PCT invariance means equality of the amplitudes for the transition from the state Ψ into the state Φ and from the state Φ' to the state Ψ', where Ψ' and Φ' are the PCT-inverted states Ψ and Φ (see Sec. 1, below). One may pose the question of experimental verification of the hypothesis of PCT invariance of physical processes, analogously to the question of verification of P, PC or T invariance.

The Luders-Pauli theorem asserts that invariance with respect to the operation PCT is the consequence of certain general postulates [2,9], that do not include the assumption of invariance under any inversion, i.e., that for a certain class of theories PCT invariance is a property of that class, and not a consequence of direct assumptions about the inversion properties of space and particles. In view of the existence of this theorem a test of PCT invariance is at the same time a test of the validity of the premises on which this theorem is based.

A majority of the experiments to be discussed has been previously suggested in a number of papers among which one should particularly note the papers of Stapp [14] and Luders and Zumino [15] devoted especially to the test of the PCT theorem.

4. The method for the experimental verification of the invariance or noninvariance of some process with respect to some operation is based on the following logic. We assume that the invariance in question holds; starting from that assumption we find the consequences for observable quantities and then test these consequences experimentally.

Some of the consequences of the invariances under discussion for concrete processes may be rather simply obtained directly from the definition of the corresponding operation (see, for example, the last footnote), without making use of any mathematical apparatus. However it is rather hard to obtain in this way most of the consequences, particularly since aside from the invariances being discussed one must simultaneously take into account invariance with respect to displacements and rotations of space (i.e., the conservation of the total momentum and angular momentum and relativistic invariance). This invariance is assumed throughout this paper. It must be taken into account because certain symmetries in an experiment (in angular distribution etc), which at first glance seem to arise from the invariances under discussion, may already follow from these conservation laws. We will therefore discuss most of the experiments on the basis of a theory that takes into account these conservation laws, and also the unitarity of the transition amplitude U (the so called general, or phenomenolog-

to assume that the decaying particle has no well defined intrinsic parity (i.e., can be represented as a superposition of even and odd eigenfunctions of the operator P), without connecting this fact with the existence of charge. Then homogeneity of space does not forbid the observed asymmetries, however it becomes difficult to explain "parity conservation" in strong interactions.

ical, reaction theory). We shall use the theory in the form presented in the papers [16-19].*

This approach consists then of a test of PC and PCT invariances in a pure form, without the help of additional special assumptions such as for example the local nature of the interactions, or some analyticity properties of the transition amplitude.

1. TRANSITION AMPLITUDE. POLARIZATION VECTOR. SELECTION RULES OF PC, T, AND PCT INVARIANCES

1. We are interested in the transitions of one or two free particles into some final state (resulting from the decay of a particle or from the interaction of two particles) that also consists of free particles. The probability amplitude for the presence at the instant of time t of the desired final state Φ in the evolved initial state $\Psi(t)$ is equal to $\langle\Phi, \Psi(t)\rangle$, where $\Psi(t) = U(t, t_0)\Psi_0$. Ψ_0 is the given initial state (at the instant of time t_0), and $U(t, t_0)$ is the evolution in time operator.†

Thus the probability amplitude for a pion of momentum $\mathbf{p}_0$ to decay into a muon of momentum $\mathbf{p}_1$ and spin projection n_1, and a neutrino of momentum $\mathbf{p}_2$ and spin projection n_2 is equal to the matrix element of U between the wave functions of the corresponding states of the free particles

$$\langle\Phi, U\Psi_0\rangle = (|\mathbf{p}_1 n_1 \mathbf{p}_2 n_2\rangle, U|\mathbf{p}_0\rangle).$$

All that we know about the matrix U is what follows from the conservation laws. The latter impose definite restrictions on U, which may be obtained from the commutation of U with the operators for displacements and rotations in space, as well as with the operators for other possible transformations: inversions, charge conjugation, and so on, if invariance with respect to these transformations is assumed.‡

In this manner the phenomenological theory consists in the ability to describe the free particle states and in knowing how to draw conclusions from the invariance of U with respect to certain transformations (for details see Chap. VI in [22]).

The initial and final states are given by stating what are the particles in these states together with the momentum $\mathbf{p}$ and the spin projection n of each particle. Such a description will be relativistic if given the wave function $|\mathbf{p}, n\rangle$ of the state in one Lorentz frame we can find the wave function of this state in another frame of reference. The formulas necessary for this follow from the theory of representations of the inhomogeneous Lorentz group (see, for example, [23]).

The conservation law of the total momentum is expressed mathematically in the diagonality of U in the total momentum indices. Thus, the above mentioned element $(|\mathbf{p}_1 n_1 \mathbf{p}_2 n_2\rangle, U|\mathbf{p}_0\rangle)$ should be of the form

$$(|\mathbf{p}_1 n_1 \mathbf{p}_2 n_2\rangle, U|\mathbf{p}_0\rangle) = (|\mathbf{P}\mathbf{p} n_1 n_2\rangle, U|\mathbf{p}_0\rangle)\,\delta(\mathbf{P}-\mathbf{p}_0). \quad (1.1)$$

We have introduced in place of $\mathbf{p}_1$ and $\mathbf{p}_2$ the variables of the total momentum $\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2$ and the relative momentum $\mathbf{p} = (\mathbf{p}_1 - \mathbf{p}_2)/2$. In what follows the indices $\mathbf{P}$ will be omitted (it will be assumed that we are in the barycentric system of the reaction, where $\mathbf{P} = \mathbf{p}_0 = 0$), so that among the indices of the elements of U only relative momenta will appear. For the restrictions imposed on U by the law of conservation of the total momentum see Sec. 4 and Appendix A.

2. In what follows we shall need the concept of the polarization vector for a particle. Since we shall not deal with particles of spin higher than 1/2 it is not necessary to introduce more complicated concepts (namely polarization tensors). For the neutrino the polarization concept is altogether unnecessary,* so it will be sufficient to define the polarization vector for particles with nonzero rest mass.

We define the state of a free particle with spin in the following manner: 1) we indicate the momentum $\mathbf{p}$ of the particle (for example in the barycentric frame of the reaction); 2) we indicate its spin state in the Lorentz frame in which the particle is at rest. Let $\langle|n\rangle$, $n = -1/2, +1/2$ denote the wave functions of

*Aside from being relativistic and allowing for the discussion of the neutrino, it is distinguished by its simplicity (in the sense of less awkward formulas) even in comparison with the better known formulation of Wolfenstein.[20] The latter, even in its relativistic modification (see Stapp[21]), is inconvenient for a majority of the decay processes discussed.

†In scattering processes $U(\infty, -\infty)$ is called the S matrix. In decay processes one should take instead of the S matrix $U(T, 0)$; the time count should start at the moment of production of the unstable particle. As T one should take a time many times larger than the lifetime of the decaying particle (but shorter than the lifetime of its decay products).

‡Processes occurring in an isolated physical system which has been as a whole displaced, rotated, etc., in space should proceed in the same way as in the untransformed system. Namely, if $\Psi = U\Psi_0$ is the result of the evolution of a certain state Ψ_0, and if the result of the evolution of the displaced, rotated, etc., state $D\Psi_0$ is $\Psi_D = UD\Psi_0$, then this expression indicates "just as well" that Ψ_D differs from Ψ only by its distribution (orientation) in space. Namely, for all transformation except Lorentz transformations we have $\Psi_D = D\Psi$. But if $\Psi_D = D\Psi$ then we have $UD\Psi_0 = DU\Psi_0$, and since this is true for any state Ψ_0 it follows that $UD = DU$, i.e., U commutes with the corresponding operator for the transformation in question.

In the case of Lorentz transformations L, Ψ_L turns out to also be displaced in space in comparison with $L\Psi$. As a result of somewhat more complicated considerations one finds that although $[L,U]$ does not vanish it is equal to a known operator.

*The state of a two-component neutrino is fully characterized by the single specification of its momentum. In the case of a four-component neutrino one could introduce the polarization concept (for reference see the papers [16,17]), however for the time being the neutrino polarization is not accessible to experimental measurements.

a particle with definite spin projections n in that frame. We define in the usual manner the density matrix

$$\varrho_{n_1, n_2} = \sum_n \alpha_n \langle n_1 | n \rangle \langle n_2 | n \rangle^*, \qquad (1.2)$$

where α_n are the weights of individual pure states $\langle | n \rangle$. For an unpolarized state $\alpha_{-1/2} = \alpha_{1/2}$. With appropriate normalization $\mathrm{Sp}\,\rho = 1$.

The polarization vector will be defined in the Lorentz frame where the particle is at rest as twice the average value of the total angular momentum operator **M**, equal to the spin operator (since the orbital angular momentum is zero in this frame):

$$P_k = 2\,\mathrm{Sp}\, M_k \varrho = \mathrm{Sp}\, \sigma_k \varrho \quad (k = x,\ y,\ z), \qquad (1.3)$$

where σ_k are the Pauli matrices. For the connection between this and other possible definitions see, for example, [24]. The question of how does the polarization vector defined in this manner transform under Lorentz transformations has been studied in the work [18], item 3. In what follows we propose the measurement of those components of the polarization for which these special transformations are irrelevant. The measurement of the polarization vector is performed by the usual methods (for example, from the difference in the right-left secondary scatterings).

3. PC inversion. Let $|\mathbf{p} n \epsilon\rangle$ be the wave function of a state of some particle of definite momentum **p** and spin projection n on some fixed axis. ϵ denotes the sign of the charge, lepton number, baryon number, strangeness. The action of PC is defined as the product of the operations P and C:

$$PC\,|\mathbf{p} n \varepsilon\rangle = |-\mathbf{p},\ n,\ -\varepsilon\rangle\, e^{i\eta(\varepsilon)}. \qquad (1.4)$$

It can be shown that the possible phase factor exp $i\eta$ does not depend on **p** and n, but may be different for different particles (analogously to the intrinsic P parity of a particle).*

Let $(|\mathbf{p}, n_1, n_2\rangle,\ U|M\rangle)$ be the amplitude for the transition $a \to 1 + 2$ (the decay of particle a with spin projection M into two particles 1 and 2 with relative momentum **p** and spin projections n_1 and n_2). By definition of PC invariance the amplitude for the transition $a \to 1 + 2$ should be equal to the amplitude for the transition from the PC-inverted initial state [the antiparticle $\tilde{a}$ with the same spin projection M in accordance with Eq. (1.4)] into the PC-inverted state $\tilde{1} + \tilde{2}$:

$$(|\mathbf{p} n_1 n_2\rangle,\ U|M\rangle) = (PC\,|\mathbf{p} n_1 n_2\rangle,\ UPC\,|M\rangle)$$
$$= e^{i(\eta_a - \eta_1 - \eta_2)}\ (|-\mathbf{p} n_1 n_2\rangle,\ \tilde{U}\,|M\rangle). \qquad (1.5)$$

The tilde ~ over U in Eq. (1.5) means that the element in question refers to the process $a \to \tilde{1} + \tilde{2}$.

*This follows from the commutation properties of P and C with the generators of three dimensional rotations M_k (the total angular momentum operator) and Lorentz transformations (see, for example, [4]).

Thus PC invariance relates the amplitudes of mutually charge conjugate processes.

The probability for the decay of the unpolarized particle a* into the channel 1 + 2 is proportional to the expression

$$\sum_M \sum_{n_1 n_2} \int d^3p\,|(|\mathbf{p} n_1 n_2\rangle,\ U\,|M\rangle)|^2.$$

In view of Eq. (1.5) this expression is equal to

$$\sum_M \sum_{n_1 n_2} d^3p\,|(|-\mathbf{p} n_1 n_2\rangle,\ \tilde{U}\,|M\rangle)|^2,$$

i.e., to the expression for the probability of the decay $\tilde{a} \to \tilde{1} + \tilde{2}$. It therefore follows that the total decay probabilities (through all channels) of a and $\tilde{a}$ are equal, i.e. that their lifetimes are equal.

In Appendix B are obtained consequences of PC invariance also for decays of the type $a \to 1 + 2 + 3$ in the case when the particle a (and correspondingly $\tilde{a}$) is polarized or when the polarization of the decay products is measured. It will be shown on the examples of pion and muon decays how certain consequences may be deduced by elementary means.

4. Wigner time reversal. The invariance with respect to the Wigner time reversal T is formulated differently from the other invariances. If T invariance holds then the amplitude for the transition from the state Ψ_0 into the state Φ_f should be equal to the probability for the transition from the state $T\Phi_f$ into the state $T\Psi_0$. In the case of reaction of the type $a + b \to c + d$ this means the equality of the amplitudes of the direct reaction and the inverse reaction $c + d \to a + b$, in which the initial state is the T-inverted (see below) final state of the direct reaction, and the final state is the T-inverted initial state of the direct reaction. For more details see Secs. 21 and 25 of the book [22], and also Chap. I, Sec. 5 of the review [9]. All that has been said can be expressed in terms of the elements of the U matrix as follows:

$$\langle \Phi_f,\ U(t, 0)\,\Psi_0\rangle = \langle T\Psi_0,\ U(t, 0)\,T\Phi_f\rangle, \qquad (1.6)$$

where T-inversion is defined by the relation (see [22,25])

$$T\,|\mathbf{p} n \varepsilon\rangle = |-\mathbf{p},\ -n,\ \varepsilon\rangle\,(-1)^n e^{i\xi}. \qquad (1.7)$$

As in Eq. (1.4) ξ does not depend on the momentum **p** and the spin projection n. Under T inversion the spin projections n onto a fixed axis of quantization change sign. For the decay $a \to 1 + 2$ we obtain from Eqs. (1.6) and (1.7)

$$(|\mathbf{p} n_1 n_2\rangle,\ U\,|M\rangle)$$
$$= e^{i(\xi_a - \xi_1 - \xi_2)}\,(|-M\rangle,\ U\,|-\mathbf{p},\ -n_1, -n_2\rangle). \qquad (1.8)$$

*It can be shown (see Appendix B) that the total decay probability into any given channel does not depend on the state of polarization of the particle; therefore the remark that the particle is unpolarized is irrelevant.

The relation (1.8) connects the elements of the transition amplitude of the direct $a \to 1 + 2$ and the inverse $1 + 2 \to a$ reactions. The latter is practically inexistent (in contrast to reactions of the type $a + b \to c + d$) and therefore from the Eq. (1.8) alone no selection rules result for decay processes.

Let us show that if the products of the decay $a \to 1 + 2$ interact weakly, then the unitarity of the U matrix, i.e., $U^+U = 1$ (see Sec. 19 in [22]), gives rise together with Eq. (1.8) to a certain selection rule for the process $a \to 1 + 2$.

Let us formally introduce the transition matrix R, defining it by the relation $U = 1 + iR$. For transitions involving a change in the state, U and iR coincide. Let us rewrite $1 = U^+U$ in the form $1 = U^+(1 + iR)$ or $1 - U^+ = iU^+R$, or $R^+ = U^+R$. Let us take matrix elements of this last equality between the states $|f\rangle$ — the final state and $|a\rangle$ —the initial state:

$$\langle f|R^+|a\rangle = \langle f|U^+R|a\rangle \equiv \sum_n \langle f|U^+|n\rangle\langle n|R|a\rangle$$
$$\equiv \langle f|R|a\rangle - i\sum_n \langle f|R^+|n\rangle\langle n|R|a\rangle. \quad (1.9)$$

In the calculation of the matrix product the summation $\sum_n$ is taken over all possible states n into which the particle a can decay. If these states can transform into each other only via weak interactions then one may ignore the term $i\sum_n\langle f|R^+|n\rangle \times \langle n|R|a\rangle$. Indeed, let us consider the decay $\pi \to \mu + \nu$. The weakness of the interaction between the decay products means that the matrix elements $\langle \mu\nu|R|\mu\nu\rangle$ and $\langle \mu\nu|R|e\nu\rangle$ of the processes $\mu + \nu \to \mu + \nu$ and $e + \nu \to \mu + \nu$ are very small (the symbols μ, e, ν denote the particle type, its spin projection and momentum). Starting from the number 10^{-44} cm^2 for the cross section for neutrino (from a pile) interactions with nucleons, the universality of the Fermi four-fermion weak interaction and the possible increase of the cross section with energy that follows from that interaction form [26], one may give the estimate of 10^{-10} for the order of magnitude of this element.

And thus, to an accuracy of better than a millionth of one per cent, Eq. (1.9) transforms into the statement of hermiticity for the R matrix

$$\langle f|R^+|a\rangle = \langle f|R|a\rangle \text{ or } \langle a|R|f\rangle = \langle f|R|a\rangle^*. \quad (1.10)$$

This relation (usually referred to as the principle of detailed balance) also connects matrix elements of the direct and inverse processes, like Eq. (1.8). Therefore Eqs. (1.8) and (1.10) combined give rise to a selection principle for the process $a \to 1 + 2$ itself

$$(|\mathbf{p}n_1n_2\rangle, R|M\rangle)$$
$$= e^{i(\xi_a-\xi_1-\xi_2)}(|-\mathbf{p}, -n_1, -n_2\rangle, R|-M\rangle)^*. \quad (1.11)$$

This selection rule has no simple experimental consequences. To test T invariance one must have polarized particles a and study the polarization of the decay products (see Table I in Sec. 2).

5. PCT inversion. PCT invariance means the equality (apart from a phase factor) of the amplitudes for the direct process and the process in which the initial and final states are interchanged, the particles are replaced by their corresponding antiparticles, and their spin projections have their signs changed [the definition of PCT inversion is obtained by combining Eqs. (1.4) and (1.7)].

For decay processes in the case of weak interaction between the decay products the selection rule that follows from PCT invariance is obtained by combining Eqs. (1.5) and (1.11) and has the following form for the process $a \to 1 + 2$

$$(|\mathbf{p}n_1n_2\rangle, U|M\rangle) = e^{i\alpha}(|\mathbf{p}, -n_1, -n_2\rangle, \tilde{U}|-M\rangle)^*. \quad (1.12)$$

The phase α is composed of the phases η and ξ. From here, exactly as in the case of PC invariance, follows the equality of the weights of mutually charge conjugate decay channels a and $\tilde{a}$ [14,9] and the equality of the lifetimes of a and $\tilde{a}$ [27,28]. We note that the last equality is also obtained in the case of strong interactions among the decay products. This case is discussed in Appendix B.

2. DECAYS OF PIONS AND MUONS

The experimentally established equality of the lifetimes of the π^+ and π^-, and the μ^+ and μ^- (see [29], Chap. 2) is one of the simplest consequences of PC and PCT invariances and the first indication of the validity of at least one of these invariances [27,14]. The next simplest consequence should be the identity of the angular distributions of the charged decay products in the decays of π^+ and π^- and unpolarized μ^+ and μ^-. However the isotropy of all these distributions follows already from invariance with respect to three dimensional rotations (so that this identity has no relation to any tests of PC or PCT invariances). Indeed, a pion at rest or an unpolarized muon at rest does not give rise to any preferred direction in space. In the final state the direction of the momentum of the charged particle is measured. The amplitudes (and probabilities) for transitions from an arbitrarily rotated initial state (which will be fully equivalent to the original unrotated state) into the correspondingly rotated final state must therefore be equal, but such a final state corresponds to an unrotated final state characterized by a different direction of the momentum of the charged particle (it is, of course, assumed that the decaying particle or its decay products can be treated as an isolated physical system, not subject to the action of any external fields). Consequently transitions into states with different momentum directions are equally probable, i.e., the angular distribution must be isotropic.*

Let us consider some more complicated experiments.

1. First of all we note that as a consequence of rotational invariance the muons in the decay $\pi \to \mu + \nu$ and the electrons in the decay $\mu \to e + \nu + \tilde{\nu}$ of unpolarized muons can only be longitudinally polarized (the directions of emission of the neutrino in $\mu \to e + \nu + \tilde{\nu}$ are not detected).† Indeed, suppose that there is some nonvanishing component of the polarization $\mathbf{P}_\perp$, perpendicular to the momentum $\mathbf{p}$ of the muon or the electron. By arguing as before we conclude that we should obtain a quantity of the same magnitude for the component obtained from $\mathbf{P}_\perp$ by rotating that vector about p, including therefore also the component in the opposite direction, i.e., we conclude that the particle should at the same time have the component $\mathbf{P}_\perp$ as well as $-\mathbf{P}_\perp$; this means that its resultant perpendicular polarization component vanishes.

Let us show that if the pion decay at rest is PC invariant then the directions of the longitudinal polarizations of the decay μ^+ and μ^- are opposite. In Fig. 1a is shown a μ^- emitted in the direction $\mathbf{p}$. The vector shown along with the momentum vector p (and parallel to it) represents the meson polarization. After P inversion we obtain Fig. 1b (we recall that the polarization is a pseudovector) and if this is followed by C inversion we obtain Fig. 1c. As can be seen the polarization vector of the μ^+ comes out antiparallel to its momentum. For convenience of comparison with the initial Fig. 1a one may perform a rotation by 180° in the plane of the paper (see Fig. 1d). The requirement of PC invariance (together with rotational invariance) means that whatever number of π^- decays into the state shown in Fig. 1a, the same number of π^+ decays into the state shown in Fig. 1d.

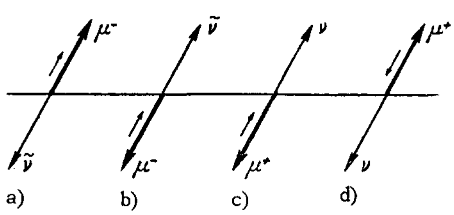

FIG. 1

In precisely the same way one shows that if the longitudinal component of e^- from the decay of μ^- is parallel to the momentum of the e^-, then as a consequence of PC invariance the longitudinal component of e^+ from the decay $\mu^+ \to e^+ + \nu + \tilde{\nu}$ should be antiparallel to the momentum of the e^+. One can convince oneself that this assertion is also valid if the $\mu^\pm$ have polarizations acquired in the process of the $\pi^\pm$ decays (see also Appendix B).

In passing into the laboratory frame of reference, wherein the pion is in motion, the momentum of the decay muon changes direction whereas its polarization does not.* Therefore in the laboratory frame it will not be along the momentum. However, as before, the polarization vector of the μ^- will be directed oppositely to the direction of the polarization vector of the μ^+ if the directions of emission of the μ^- and μ^+ form the same angle with the direction of flight of the pion. In particular, the longitudinal components of such μ^- and μ^+ will be antiparallel also in the laboratory frame.

In Appendix B it is shown that all the discussed relations between the polarizations of μ^+ and μ^- (or e^+ and e^-) also follow from PCT invariance. Below we shall give a summary of the remaining consequences of PC and PCT invariances (see Table I). They are particular cases of Eqs. (B.4) and (B.5), Appendix B.

2. As a consequence of the existence of a nonlongitudinal component for the muon polarization in the laboratory frame the angular distribution F of the electrons produced in μ decay depends not only on the angle θ between the momenta $\mathbf{p}_{\mu\,\mathrm{lab}}$ and $\mathbf{p}_e$ (electron momentum in the barycentric frame of μ decay), but also on the angle φ between the vectors $\mathbf{p}_{\mu\,\mathrm{lab}} \times \mathbf{p}_e$ and $\mathbf{p}_{\pi\,\mathrm{lab}} \times \mathbf{p}_{\mu\,\mathrm{lab}}$. It can be shown that as a result of rotational invariance the angular distribution $F(\theta, \varphi)$ must be an even function of φ.

If the μ^- and μ^+ were produced in the decay of π^- and π^+ respectively (and did not become depolarized) then it follows from PC invariance that $F^-(\theta, \varphi) = F^+(\theta, -\varphi)$, and from PCT invariance that $F^-(\theta, \varphi) = F^+(\theta, \varphi)$. Since F is an even function of φ it follows from either PC or PCT invariance that the angular distributions of e^- and e^+ are equal.

3. For the experiments discussed so far, T invariance leads to no conclusions (see Table I) as can be established starting from Eq. (B.12). For this reason PC and PCT invariances led to the same consequences. Differences between them arise only in most complicated experiments: those in which electron polarization is observed in decays of polarized muons.

If PC invariance holds then e^+ polarization in the cascade $\pi^+ \to \mu^+ + \nu$, $\mu^+ \to e^+ + \nu + \tilde{\nu}$ is related to

*One may also argue as follows: in the initial state there exists no direction with respect to which one could measure the direction of emission of the charged particle.

†In the case of a two-component neutrino the muons from the decay $\pi \to \mu + \nu$ are completely polarized along the momentum. Computations using the four-fermion interaction theory with a two-component neutrino and a V − A or V + A type of interaction give for the longitudinal polarization of electrons from the $\mu \to e + \nu + \tilde{\nu}$ decay a number close to 100%.[30-33] A similar number is obtained under the hypothesis of nonobservability of the sign of the mass.[32,34]

*More precisely, it is slightly rotated.[18,23] However, this slight rotation of relativistic origin does not change the conclusions.

Table I. π and μ Decays

Quantities measured	PC	PCT	T
Muon polarization vectors from the $\pi \to \mu + \nu$ decay	$\mathbf{P}_\mu^+ = -\mathbf{P}_\mu^-$	$\mathbf{P}_\mu^+ = -\mathbf{P}_\mu^-$	—
Longitudinal components of the e^+ and e^- polarizations in the cascades $\pi^\pm \to \mu^\pm \to e^\pm$	$P_\parallel^+ = -P_\parallel^-$	$P_\parallel^+ = -P_\parallel^-$	—
Angular distributions of e^+ and e^- in the cascades $\pi^\pm \to \mu^\pm \to e^\pm$	$F^+(\theta,\varphi) = F^-(\theta,\varphi)$	$F^+(\theta,\varphi) = F^-(\theta,\varphi)$	—
Perpendicular components of the e^+ and e^- polarizations in the cascades $\pi^\pm \to \mu^\pm \to e^\pm$	$P_\perp^+ = P_\perp^-$	$P_\perp^+ = -P_\perp^-$	$P_\perp = 0$

the e^- polarization in the cascade $\pi^- \to \mu^- + \tilde{\nu}$, $\mu^- \to e^- + \nu + \tilde{\nu}$ as follows (see Appendix B):

$$\begin{aligned} P_z^+(\theta, \varphi) &= -P_z^-(\theta, -\varphi) = -P_z^-(\theta, \varphi), \\ P_x^+(\theta, \varphi) &= -P_x^-(\theta, -\varphi), \\ P_y^+(\theta, \varphi) &= P_y^-(\theta, -\varphi). \end{aligned} \tag{2.1}$$

Here P_z is the projection of the polarization vector onto the direction of the momentum $\mathbf{p}_e$ of the electron in the barycentric frame of the μ decay, P_y is the projection onto the direction $\mathbf{p}_{\mu\,\mathrm{lab}} \times \mathbf{p}_e$, i.e., the direction perpendicular to $\mathbf{p}_{\mu\,\mathrm{lab}}$ and to the electron momentum in equally well the barycentric frame of the μ decay or the laboratory frame. The x axis is chosen so as to form with the above defined z and y axes a righthanded orthogonal coordinate system.

If instead the π and μ decays are invariant under PCT then

$$\left.\begin{aligned} P_z^+(\theta, \varphi) &= -P_z^-(\theta, \varphi), \\ P_x^+(\theta, \varphi) &= -P_x^-(\theta, \varphi), \\ P_y^+(\theta, \varphi) &= -P_y^-(\theta, \varphi), \end{aligned}\right\} \tag{2.2}$$

i.e., $\mathbf{P}^-(\theta, \varphi) = -\mathbf{P}^+(\theta, \varphi)$.

Since the electron polarization component $P_\parallel(\theta, \varphi)$, parallel to the electron momentum $\mathbf{p}_{e\,\mathrm{lab}}$, is a linear combination of the components $\mathbf{P}_z$ and $\mathbf{P}_x$ it follows that in the case of PC invariance $P_\parallel^+(\theta, \varphi) = -P_\parallel^-(\theta, -\varphi)$, and in the case of PCT invariance $P_\parallel^+(\theta, \varphi) = -P_\parallel^-(\theta, \varphi)$. The component $P_\perp$ perpendicular to $\mathbf{p}_{\mu\,\mathrm{lab}}$ and $\mathbf{p}_{e\,\mathrm{lab}}$ is equal to P_y.

In Table I are listed the relations between the components $P_\parallel(\theta, \varphi)$ and $P_\perp(\theta, \varphi)$ integrated over φ, denoted by $P_\parallel$ and $P_\perp$. Of course if T invariance holds the distinction between PC and PCT disappears: in that case $P_x(\theta, \varphi)$ is an even function of φ and $P_y(\theta, \varphi) = P_\perp(\theta, \varphi)$ is an odd function so that, in particular, $P_\perp = \int_0^{2\pi} d\varphi P_\perp(\theta, \varphi) = 0$.* The even nature of $P_z(\theta, \varphi)$ follows already from rotational invariance.

4. Experiment. The longitudinal components of the e^+ and e^- polarizations were measured in the papers [38] and [39]. Let us note that $P_\parallel^+$ and $P_\parallel^-$ have opposite signs both in the case of unpolarized muons and in the case of muons with the polarization acquired in the π decay process (see item 1). The polarization of the electrons was measured by means of absorption of their bremsstrahlung in magnetized iron. The coefficient of absorption of the γ quanta depends in this case on their polarization, which in turn is uniquely related to the electron polarization. The signs of the longitudinal components of the e^+ and e^- polarizations turned out to be opposite and, within the experimental errors (which were rather large), equal in magnitude.*

The attempt to measure the polarization of cosmic μ^+ and μ^-, undertaken by Alikhanov, Lyubimov et al [41], has not as yet yielded a definite conclusion on the relation of the signs and magnitudes of the polarizations (although within the experimental errors the

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*Many authors have noted that a measurement of $P_\perp$ would test T invariance. Hori et al[32] start from a parity violating Fermi interaction of the most general form; the neutrino is taken to be a four-component neutrino, see also[35]. In the papers [36,37] it is shown that the large forward-backward asymmetry, as well as the experimentally determined large longitudinal polarization of the electrons[38] from μ decay (close to a 100%), make it unlikely that $P_\perp$ should be large even if T invariance does not hold.

*Green and Hurst[40] have remarked that since the forward-backward asymmetry in the $\pi \to \mu \to e$ cascade is not a pseudoscalar quantity, its presence, strictly speaking, does not prove P noninvariance. It could be explained by the introduction of muon parity doublets (two μ^+ and two μ^- with opposite parities). The above described experiment eliminates this hypothesis: it is not capable of explaining the observed difference in the number of γ quanta absorbed for opposite directions of the magnetic field.

In addition this experiment provides direct evidence for C noninvariance in the decay $\mu \to e + \nu + \tilde{\nu}$; if it were charge conjugation invariant then the signs of $P_\parallel^+$ and $P_\parallel^-$ would be the same.[14] One usually concludes that C invariance is not valid by making use of the Luders-Pauli theorem[27]: since the forward-backward asymmetry proves also PT noninvariance (the same consequences follow from PT invariance as from P invariance for all experiments except those of item 3) this means that there is no C invariance either.

results are not in contradiction with PC and PCT invariances).

It should be noted that in recent experiments with μ^- mesons obtained from accelerators [42], the precision with which the magnitude of the μ^- longitudinal polarization is measured has been substantially increased. However the corresponding experiments with μ^+ mesons have not as yet been carried out.

The large depolarization of μ^- mesons in matter prior to decay makes it impossible to test for the equality of the magnitudes of the forward-backward asymmetries in the $\pi^\pm \to \mu^\pm \to e^\pm$ decays (experiment of item 2). The existing measurements (see [29], Chap. 7.10) confirm only that the signs of these asymmetries are the same.

The perpendicular component of the polarization $P_\perp$ has not been measured so far even for positrons.

5. As we have seen, the symmetries tested so far (the equality of lifetimes and $P_\parallel^+ = -P_\parallel^-$) are consequences of either PC or PCT invariances. One might ask, what can be properly considered as having been tested (see editor's remarks to the paper [43])?

In the first row of Table II are written out the four possibilities: there is neither PC nor PCT invariance, there is no PC but there is PCT invariance, etc. In the second row we write the experimental consequences of each possibility in the form of the answer to the question: if one possibility or another is assumed, do the relationships between the polarizations and angular distributions of charge conjugate processes, written out in the first three rows of Table I, hold or not?

It is seen that if experiment answers this question with a "no" then there is neither PC nor PCT invariance. If the answer is "yes," as is the case at this time (it would perhaps be better to say that experiment is not in contradiction with the "yes" answer), then there remain three possibilities. If PCT invariance is considered to be more fundamental from a theoretical point of view (i.e., its violation is expected in the last place), then the experimental "yes" should not be interpreted as a confirmation of PC invariance. If one has no preconceived theoretical viewpoint then the objective consequence of the answer "yes" is simply the establishment of the validity of one of the three possibilities of Table II.

Table II

PC, PCT	No, no	No, yes	Yes, no	Yes, yes
Experiment	No	Yes	Yes	Yes

In order to establish which of the possibilities in reality occurs it is necessary to measure experimentally the nonlongitudinal components $P_\perp = P_y$ or P_x, as can be seen from Eqs. (2.1) and (2.2).

If it turns out that $P_\perp^+ = P_\perp^- = 0$ then both invariances are valid. If instead $P_\perp^+$ and $P_\perp^-$ do not vanish (but are equal in magnitude) then either PC or PCT invariance is valid, depending on the relative sign of $P_\perp^+$ and $P_\perp^-$.

The measurement of $P_x(\theta, \varphi)$ [with the aim of establishing whether the relation $P_x^+(\theta, \varphi) = -P_x^-(\theta, -\varphi)$ or the relation $P_x^+(\theta, \varphi) = -P_x^-(\theta, \varphi)$ is valid, see Eqs. (2.1) and (2.2)] requires tedious transformations from the laboratory frame into the barycentric frame of the muon with relativistic spin rotation taken into account [18,23].

3. DECAYS OF K MESONS

1. First of all we note the fact of the equality of the lifetimes of the K^+ and K^- and the approximate equality of the weights of the corresponding decay channels, which has been established for the channels $K \to \mu + \nu, \to \pi + \pi, \to \pi + \pi + \pi$ [44,49]. This is the first and so far the only evidence in favor of PC and PCT invariance of $K^\pm$ decays.

Let us note that if there existed strong transitions $\pi + \pi \to \pi + \pi + \pi$, then the equality of the weights of the channels $K^+ \to \pi^+ + \pi^0$ and $K^- \to \pi^- + \pi^0$, as well as $K^+ \to \pi^+ + \pi^+ + \pi^-$ and $K^- \to \pi^- + \pi^- + \pi^+$, would not follow from PCT invariance (see [15] and also Appendix B). However such transitions are forbidden by the generalized Furry theorem (see [2], Sec. 17).

Let us discuss the possibilities of more detailed tests of PC and PCT invariances in $K^\pm$ decays.

2. The channel $K \to \mu + \nu$ has the largest branching ratio. Since all the experimental data agree best with the assumption of zero spin for the K meson, with respect to the decay $K \to \mu + \nu$ one can repeat everything that has been said with respect to $\pi \to \mu + \nu$ (and then followed by $\mu \to e + \nu + \tilde{\nu}$, see Table I). Experimentally only the angular asymmetry in the cascade $K^+ \to \mu^+ \to e^+$ has been measured [45].

3. Among the pionic channels one can discuss only the $K_{\pi 3}$ decays. The zero spin of the K meson gives rise to isotropy in both $K_{\pi 2}^+$ and $K_{\pi 2}^-$ decays.

From PC invariance follows the equality of the angular and momentum distributions in the decays $K^+ \to \pi^+ + \pi^+ + \pi^-$ and $K^- \to \pi^- + \pi^- + \pi^+$, and also $K^+ \to \pi^+ + \pi^0 + \pi^0$ and $K^- \to \pi^- + \pi^0 + \pi^0$:

$$F^+(p^{(1)}, \theta_2) = F^-(p^{(1)}, \theta_2) \tag{3.1}$$

(see Appendix B, Sec. 1). With the help of Eq. (A.1) one verifies easily that the angular distribution depends only on θ_2 [the function $D^J(-\pi, \theta, \pi - \varphi)$ is equal to unity for $J = 0$]. θ_2 is defined as the angle between the vectors $\mathbf{p}^{(1)}$ and $\mathbf{p}^{(2)}$, where $\mathbf{p}^{(1)}$ is the momentum of the π^- (in the barycentric frame of the reaction) in the case of $K^+ \to \pi^+ + \pi^+ + \pi^-$ and the momentum of the π^+ in the case of $K^- \to \pi^- + \pi^- + \pi^+$, and $\mathbf{p}^{(2)}$ is the relative momentum of the two π^+ or the two π^- respectively in the Lorentz frame in which their total momentum vanishes (see Appendix A).

Equation (3.1) does not follow from PCT invariance [15]. If the matrix elements for the transition $K^+ \to \pi^+ + \pi^+ + \pi^-$ were known (i.e., if the function $F^+(p^{(1)}, \theta_2)$ were known], as well as the matrix elements of the reactions $\pi^+ + \pi^+ + \pi^- \to \pi^+ + \pi^+ + \pi^-$ and $\to \pi^+ + \pi^0 + \pi^0$, then there would follow from PCT invariance a definite form for the function $F^-(p^{(1)}, \theta_2)$ (see Sec. 4, item 4 below), which could be compared with the experimental distribution. The relation (3.1) has not been tested as yet because there exist sufficient statistics only for the τ^+ decay [46].

4. In principle the $K_{\mu 3}$ and K_{e3} decays provide many opportunities for testing PC and PCT invariances. However the branching ratios of these channels are small and only few events of $K_{\mu 3}$ and K_{e3} have been observed [44]. We shall therefore not discuss the corresponding experiments. By considering all possible special cases of the relations (B.8) and (B.15) it is not hard to construct a table similar to Table I. In essence all these experiments have been proposed by various authors (mainly for a different reason—to determine the type of the weak interactions). With respect to angular distributions see, for example, [47]; the measurement of the polarization component of μ or e, perpendicular to the plane of the decays $K \to \mu + \nu + \pi$ or $K \to e + \nu + \pi$ was proposed in [48]; for a complete set of experiments for these decays see the paper [50]. Let us remind the reader that for our purposes it is necessary that similar experiments be performed with both K^+ and K^- decays.

5. The interpretation of the decays of neutral K mesons from the point of view of testing PC and PCT invariances presents definite difficulties.

In the decays strangeness is not conserved, therefore the distinction between K^0 and $\widetilde{K}^0$ is lost: a free K^0 can exhibit some time after production the properties of a $\widetilde{K}^0$ in a strong interaction with matter (as a consequence of the possible transition $K^0 \to \pi + \pi \to \widetilde{K}^0$).

Experimentally one observes decays of neutral K mesons with two sharply different lifetimes (about 10^{-10} and 10^{-7} sec [44]).

In most proposals for testing PC invariance (see, for example, [51] and also [52]) the Gell-Mann and Pais hypothesis [53] is assumed, according to which, among other things, the longlived K^0 meson possesses a well-defined charge (more precisely, PC) parity with respect to decay processes. Certain selection rules follow from this hypothesis and from the assumed invariance under PC of the decay processes. Should these rules turn out not to be valid one should, before arriving at the conclusion of PC noninvariance, make sure about the less fundamental Gell-Mann and Pais hypothesis.

In view of this situation we limit ourselves to the discussion of two experimental facts involving the decays of the longlived neutral K meson, which will be denoted by K_l^0 (for longlived).

1) Equality of the number of π^+ and π^- mesons in K_l^0 decays. The main channels of K_l^0 decay are

$$K_l^0 \to e^+ + \pi^- + \nu \text{ and } K_l^0 \to \mu^+ + \pi^- + \nu \qquad (3.2)$$

and the charge conjugate channels. The total number of π^- mesons from channels (3.2) should be equal to the number of all π^+ mesons from the charge conjugate channels if: a) the K_l^0 has well-defined PC parity (or PCT parity, since the T operation has no effect on the wave function of a spinless particle at rest), i.e., it is the same as the K_2^0 in the notation of Gell-Mann and Pais; b) the above mentioned decays are invariant with respect to PC [51] or PCT [27,14]. In latest experiments [54] the ratio of the total number of π^+ mesons detected in K_l^0 decays to the total number of π^- mesons turned out to be equal to 0.90 ± 0.18, i.e., not in contradiction with assumptions a) and b). Should this ratio be different from unity (as was the case in earlier papers, see [44] p. 262) then first of all one should question assumption a).

2) Absence of decays $K_l^0 \to \pi + \pi$. It has been shown by Weinberg [55] that when the existence of two lifetimes for neutral K mesons is taken into account and if it is assumed that their decays are PCT invariant then from the absence of the decays $K_l^0 \to \pi^+ + \pi^-$ and $\to \pi^0 + \pi^0$ there follow, in particular, relations of the type of Eq. (B.14), characteristic of T invariance (see Dalitz [56], Chap. 5). Consequently the absence of $K_l^0 \to \pi + \pi$ establishes T invariance or, since PCT invariance was assumed anyway, PC invariance. In Weinberg's proof the Gell-Mann and Pais hypothesis is not utilized.

4. THE CASCADE $\widetilde{p} + p \to \Lambda + \widetilde{\Lambda}$, $\Lambda \to p + \pi$, $\widetilde{\Lambda} \to \widetilde{p} + \widetilde{\pi}$

This cascade is considered not only as an example of a possible experiment in which the relative polarizations of the hyperon and antihyperon are known [57]. The reaction $\widetilde{p} + p \to \Lambda + \widetilde{\Lambda}$ has given so far the largest number of antihyperons, and indeed antilambdas, seen. For the decays of Λ and $\widetilde{\Lambda}$ it is possible to realize a complete set of experiments for testing PC invariance, i.e., a set such that from it PC invariance necessarily follows (i.e., such that it follows from it that all of the relations of the type (1.5) are satisfied).

1. In order to indicate such a complete set we need certain results of the general reaction theory.

We shall specify the state of a free particle by indicating its momentum and the projection m of its spin onto the direction of the momentum (helicity). In Sec. 1 we discussed only the consequences of conservation of the total momentum (not counting the discussion of PC, T and PCT invariances). The conservation law of the total angular momentum imposes additional restrictions on the matrix elements $(|pm_1m_2\rangle U|M\rangle \equiv \langle pm_1m_2|U|M\rangle$ [see Sec. 1, item 1, in particular Eq. (1.1)] describing the decay of particle a with spin J and projection M into two particles, 1 and 2, with spins s_1 and s_2 and projections m_1 and m_2 (onto the direction of their relative momentum **p**). It can be shown [16,17,59] that these elements are of the form

$$\langle \mathbf{p}m_1m_2|U|M\rangle = \sqrt{\frac{2J+1}{4\pi}}\, D^J_{m_1+m_2,M}(-\pi,\vartheta,\pi-\varphi) \times \langle m_1m_2|U^J|\rangle; \quad (4.1)$$

where ϑ and φ are the spherical angles of **p** with respect to a certain triplet of axes O_a (with respect to which also the projections M are referred). $|\mathbf{p}|$ is expressed in terms of the rest masses of the particles a, 1 and 2. $D^J_{m,n}(\varphi_2, \theta, \varphi_1)$ are known functions, that reduce for the special cases of $m = 0$ or $n = 0$ to spherical harmonics (see Appendix D). The angles $-\pi, \vartheta, \pi-\varphi$ are the Euler angles $\varphi_2, \theta, \varphi_1$ of the rotation* of the axes O_a into coincidence with the triplet of axes $O_1: z_1 \parallel \mathbf{p}, y_1 \parallel z_a \times \mathbf{p}$ (O_1 is precisely that triplet of axes with respect to which the projections m_1 and m_2 are referred). The relation (4.1) means that the ϑ and φ dependence of $\langle pm_1m_2|U|M\rangle$ is completely known. This is the fact that represents the consequences of the conservation law of the total angular momentum.

In the case of the decay $\Lambda \to p + \pi$ we have

$$\langle m|R(\vartheta,\varphi)|M\rangle = \sqrt{\frac{1}{2\pi}}\, D^{1/2}_{m,M}(-\pi,\vartheta,\pi-\varphi)\langle m|R^{1/2}|\rangle. \quad (4.2)$$

We have introduced the matrix $R = (U-1)/i$, the rest of the notation is obvious. Since $m = \pm 1/2$, the decay $\Lambda \to p + \pi$ is fully described by the two complex elements $\langle -1/2|R^{1/2}|\rangle \equiv R_-$ and $\langle +1/2|R^{1/2}|\rangle \equiv R_+$ [59]. It is usual in the literature to take for the parameters describing the decay $\Lambda \to p + \pi$ the s and p amplitudes R_0 and R_1, i.e., the elements of the transition matrix labeled by the values 0 and 1 of the orbital angular momentum of the decay products [60]. The two descriptions are related by

$$R_- = \frac{1}{\sqrt{2}}(R_0+R_1), \qquad R_+ = \frac{1}{\sqrt{2}}(R_0-R_1). \quad (4.3)$$

In terms of the elements $\langle m|R(\vartheta,\varphi)|M\rangle$ the selection rules of PC invariance have the form

$$\langle m|R(\vartheta,\varphi)|M\rangle = \varkappa(-1)^{m-M}\langle -m|\tilde{R}(\vartheta,-\varphi)|-M\rangle. \quad (4.4)$$

*Definition of the Euler angles (when the coordinate system is rotated): 1) φ_1 is the angle of rotation about the z axis. At that angle the y axis goes into the y′ axis; 2) θ is the angle of rotation about the y′ axis. At that z goes into z′; 3) φ_2 is the angle of rotation about the z′ axis. All rotations are clockwise (when looking from the end of the axis about which one is rotating). It is understood that the axes form right-handed triplets.

The derivation of Eq. (4.4) is analogous to the derivation of Eq. (B.4) of Appendix B. With the help of Eqs. (4.1) and (D.7) Eq. (4.4) may be rewritten in the form of the following relations between the elements $\langle m|R^{1/2}|\rangle$:

$$\langle m|R^{1/2}|\rangle = \varkappa\langle -m|\tilde{R}^{1/2}|\rangle, \quad m = -1/2, +1/2 \quad (4.5)$$

(where κ is some phase factor, see Appendix B).

In terms of the elements R_l, $l = 0,1$ we have correspondingly

$$R_l = \varkappa(-1)^l\tilde{R}_l. \quad (4.6)$$

In a complete set of experiments designed to test PC invariance there should be included experiments that test the validity of these two complex relations (or of the corresponding four real relations, see below).

It is clear that first of all we must have Λ and $\tilde{\Lambda}$ with known polarizations. We then may hope to be able to obtain and compare the elements R and $\tilde{R}$ by measuring and comparing the angular distributions and polarizations of the protons and antiprotons in the decays.

2. Suppose that the reaction

$$\tilde{p} + p \to Y + \tilde{Y} + N\pi \quad (N = 0, 1, 2, \ldots) \quad (4.7)$$

is invariant under P. Then the polarization vectors of Y and $\tilde{Y}$ will be perpendicular to the momentum $\mathbf{p}_a$ of the incident antiproton and the momentum **p** of the hyperon Y if one integrates over all variables connected with the mesons. It has been shown by Chou Kuang-Chao [57] that the polarization vectors of Y and $\tilde{Y}$ (which on the average are emitted in opposite directions after the above mentioned integration) are equal in magnitude and either both parallel or antiparallel to the vector $\mathbf{p}_a \times \mathbf{p}$. If the polarization does not vanish upon integration over all angles of emission of the hyperon, then the average (perpendicular) polarization of Y is equal to the average polarization of $\tilde{Y}$.

On transforming into the laboratory frame the perpendicular component is unchanged [21,23] and consequently the same relation between the polarizations of Y and $\tilde{Y}$ holds also in the laboratory frame.

3. By making use of Eq. (4.2) and the formulas of Appendix A we can obtain expressions for the angular distribution and polarization of the decay products in the decay $\Lambda \to p + \pi^-$ (or $\tilde{\Lambda} \to \tilde{p} + \pi^+$) in terms of the elements $\langle m|R^{1/2}|\rangle$.

We choose the z axis of quantization for the spin projections M of the hyperon (antihyperon) along the direction of the momentum of the hyperon $\mathbf{p}_\Lambda$ (antihyperon $\mathbf{p}_{\tilde{\Lambda}}$) in the laboratory frame. The y axis is taken, for the Λ as well as for the $\tilde{\Lambda}$, parallel to $\mathbf{p}_a \times \mathbf{p}_\Lambda$, i.e., along (or opposite to) the polarization vector of the Λ or $\tilde{\Lambda}$ (see above). In this coordinate system the polarization vectors of the Λ and the $\tilde{\Lambda}$ have the projections

$$P_z = P_x = 0, \quad P_y = P^\Lambda. \quad (4.8)$$

We give only the results of the calculations. The angular distribution of the protons from the decay $\Lambda \to p + \pi$ is given by

$$F(\vartheta, \varphi) = \frac{1}{4\pi}\{|R_-|^2 + |R_+|^2\}\left\{1 - \frac{|R_-|^2 - |R_+|^2}{|R_-|^2 + |R_+|^2} P^\Lambda \sin\vartheta \sin\varphi\right\} \sim 1 - \alpha P^\Lambda \cos(\widehat{\mathbf{p}_p \mathbf{P}^\Lambda}), \qquad (4.9)$$

where ϑ and φ are the spherical angles of the momentum $\mathbf{p}_p$ of the proton referred to the above described axes. The asymmetry coefficient α is more familiar in the form $\alpha = 2\,\mathrm{Re}\,R_0R_1^*/(|R_0|^2 + |R_1|^2)$ [60]. Upon integrating Eq. (4.9) over ϑ and φ we find that the probability for the Λ to decay via the channel $p + \pi^-$ is equal to $|R_-|^2 + |R_+|^2$. We find analogously that the probability for the $\widetilde{\Lambda}$ to decay via the channel $\widetilde{p} + \pi^+$ is equal to $|\widetilde{R}_-|^2 + |\widetilde{R}_+|^2$, which in view of Eq. (4.5) is equal to $|R_+|^2 + |R_-|^2$. Since the probabilities for the corresponding channels are equal so should be the lifetimes. The measured lifetimes of Λ and $\widetilde{\Lambda}$ are respectively equal to [44,58]:

$$(2.7 \pm 0.1)\cdot 10^{-10}\ \text{sec and}\ \left(2.8\,{}^{+1.1}_{-0.7}\right)\cdot 10^{-10}\ \text{sec}.$$

It further follows from Eq. (4.5) that the coefficients of angular asymmetry for Λ and $\widetilde{\Lambda}$ are of opposite sign: $\alpha = -\widetilde{\alpha}$. Since the Λ and $\widetilde{\Lambda}$ are polarized in the same way [if they are taken from the reaction (4.7)] this means that $F(\vartheta, \varphi) = \widetilde{F}(\vartheta, -\varphi)$, or that as a consequence of PC invariance the up-down asymmetry $A = (U - D)/(U + D)$ has opposite sign for Λ and $\widetilde{\Lambda}$. (U stands for the number of decay protons emitted into the upper hemisphere with respect to the reaction plane; "up" is defined in the same way for Λ and $\widetilde{\Lambda}$, namely as the direction of $\mathbf{p}_a \times \mathbf{p}_\Lambda$.)

The component of the polarization of the decay protons perpendicular to $\mathbf{p}_\Lambda$ and $\mathbf{p}_p$ is given by

$$P_\perp^p(\vartheta, \varphi) F(\theta, \varphi) = \frac{P^\Lambda}{2\pi}(\mathrm{Re}\,R_-R_+^* \cos\varphi + \mathrm{Im}\,R_-R_+^* \cos\theta \sin\varphi); \qquad (4.10)$$

$P_\perp^p$ is calculated in the barycentric frame of the decay. Since it is the component in the direction of $\mathbf{p}_\Lambda \times \mathbf{p}_p$, it can also be defined in the laboratory frame as the component perpendicular to the momentum of the Λ and the decay proton [because $(\mathbf{p}_\Lambda \times \mathbf{p}_p) \parallel (\mathbf{p}_\Lambda \times \mathbf{p}_{p,\mathrm{lab}})$]. It is this possibility of calculating such an invariant component that influenced our choice of the coordinate system zyx.

Let us compare the average perpendicular components of the polarization of the decay protons and antiprotons in the cascade

$$\widetilde{p} + p \to \Lambda + \widetilde{\Lambda} + \ldots,\quad \Lambda \to p + \pi^-,\quad \widetilde{\Lambda} \to \widetilde{p} + \pi^+, \qquad (4.11)$$

emitted into hemisphere $-\pi/2 \le \varphi \le \pi/2$ (to be called left):

$$P_{\perp l}^p = \int_{-\pi/2}^{+\pi/2} d\varphi \int_0^\pi \sin\vartheta\, d\vartheta P_\perp^p(\vartheta, \varphi) = \frac{2P^\Lambda}{\pi}\mathrm{Re}\,R_-R_+^* = P_{\perp l}^{\widetilde{p}}; \qquad (4.12)$$

we have used here Eq. (4.5). We remind the reader that the z axes for Λ and $\widetilde{\Lambda}$ are different.

The same relation holds for the averages over the right hemisphere. Since $P_{\perp l} = -P_{\perp r}$ one may include in the statistics all protons (antiprotons) provided that in the averaging the polarization of the protons in the right hemisphere is taken with the opposite sign:

$$P_{\perp l}^p - P_{\perp r}^p = P_{\perp l}^{\widetilde{p}} - P_{\perp r}^{\widetilde{p}}. \qquad (4.13)$$

For the perpendicular components averaged over the region $0 \le \varphi \le \pi$, $0 \le \vartheta \le \pi/2$ (emission up and forward, denoted by the abbreviation uf) we find

$$P_{\perp \mathrm{uf}}^p = \int_0^\pi d\varphi \int_0^{\pi/2} \sin\vartheta\, d\vartheta P_\perp^p(\vartheta, \varphi) = \frac{P^\Lambda}{2\pi}\mathrm{Im}\,R_-R_+^* = -P_{\perp \mathrm{uf}}^{\widetilde{p}}. \qquad (4.14)$$

One also has the relation, analogous to Eq. (4.13),

$$P_{\perp \mathrm{uf}}^p - P_{\perp \mathrm{ub}}^p - P_{\perp \mathrm{df}}^p + P_{\perp \mathrm{db}}^p = -(P_{\perp \mathrm{uf}}^{\widetilde{p}} - P_{\perp \mathrm{ub}}^{\widetilde{p}} - P_{\perp \mathrm{df}}^{\widetilde{p}} + P_{\perp \mathrm{db}}^{\widetilde{p}}) \qquad (4.15)$$

(the subscripts uf, ub, df and db denote respectively up and forward, up and backward, down and forward, down and backward).

The above illustrates the manner in which one obtains from PC invariance, i.e., from the equalities (4.5), relations between angular distributions and polarizations of the products of the decay of particle and antiparticle. These relations are derived in a general form in Appendix B.

3. Let us suppose that the equality of lifetimes, the relation $\alpha = -\widetilde{\alpha}$ and the relations (4.13) and (4.15) have been experimentally confirmed (Table III). Do Eqs. (4.5) follow from these relations? It turns out that the system of equations

$$\left.\begin{aligned} |R_-|^2 + |R_+|^2 &= |\widetilde{R}_-|^2 + |\widetilde{R}_+|^2, \\ |R_-|^2 - |R_+|^2 &= -|\widetilde{R}_-|^2 + |\widetilde{R}_+|^2, \\ \mathrm{Im}\,R_-R_+^* &= \mp\mathrm{Im}\,\widetilde{R}_-\widetilde{R}_+^*, \\ \mathrm{Re}\,R_-R_+^* &= \pm\mathrm{Re}\,\widetilde{R}_-\widetilde{R}_+^* \end{aligned}\right\} \qquad (4.16)$$

indeed has Eq. (4.5) as its solution regardless of the sign choice in the last two equations. The $\pm$ signs are put down to include the case when the sign of the antiproton polarization $P_\perp^{\widetilde{p}}$ is not determined (see the paper [61]). The sign of the polarization of the proton is also unnecessary for our purposes.

4. T and PCT invariances. The intereactions between the products of the Λ decay belong to the class of strong interactions. However the corresponding phase shifts of the πN interaction at ~ 30 MeV are all less than $10°$ [62]. If this interaction is completely ignored then it follows from T invariance and the hermiticity of the R matrix that (see Appendix B, item 2):

$$\langle m|R^{1/2}|\rangle = \Xi_0 \langle |R^{1/2}|m\rangle = \Xi_0 \langle m|R^{1/2}|\rangle^*, \qquad (4.17)$$

Table III. The Decays Λ and $\tilde{\Lambda}$

Quantities measured	PC	PCT	T
Up-down asymmetry $A = (U - D)/(U + D)$ for Λ and $\tilde{\Lambda}$	$A = -\tilde{A}$ or $\alpha = -\tilde{\alpha}$	$A \cong -\tilde{A}$	—
Polarization component of the decay proton (antiproton) perpendicular to $\mathbf{p}_\Lambda(\mathbf{p}_{\tilde{\Lambda}})$ and to $\mathbf{p}_p(\mathbf{p}_{\tilde{p}})$	$P^p_{\perp l} = P^{\tilde{p}}_{\perp l}$	$P^p_{\perp l} \cong P^{\tilde{p}}_{\perp l}$	—
	$P^p_{\perp \mathrm{uf}} = -P^{\tilde{p}}_{\perp \mathrm{uf}}$	$P^p_{\perp \mathrm{uf}} \cong + P^{\tilde{p}}_{\perp \mathrm{uf}}$	$P^p_{\perp \mathrm{uf}} \cong 0 \cong P^{\tilde{p}}_{\perp \mathrm{uf}}$

hence $\mathrm{Im}\, R_- R_+^* = 0$. Therefore the right and left sides of Eqs. (4.14) and (4.15) should vanish (see Table III).

By combining Eqs. (4.5) and (4.17) one obtains the "selection rule" for PCT invariance:

$$\langle m \,|\, R^{1/2} \,|\rangle = K\Xi_0 \langle -m \,|\, \tilde{R}^{1/2} \,|\rangle^*. \qquad (4.18)$$

From here follows $\mathrm{Re}\, R_- R_+^* = \mathrm{Re}\, \tilde{R}_+^* \tilde{R}_-$, i.e., the same relation as from Eq. (4.5), but $\mathrm{Im}\, R_- R_+^* = \mathrm{Im}\, \tilde{R}_+^* \tilde{R}_- = +\mathrm{Im}\, \tilde{R}_- \tilde{R}_+^*$. This means that we can obtain the corresponding consequences of PCT invariance by changing in the right side of Eq. (4.15) the $-$ sign to a $+$ sign (see Table III).

In principle PCT invariance may be tested also without neglect of the πN interaction. To that end it is necessary to perform a complete experiment for Λ and determine R_+ and R_-. Then with the help of the relations [see Eq. (B.14)]

$$K^* \langle -m \,|\, \tilde{R}^{1/2} \,|\rangle^* = \sum_n \langle m \,|\, U^+ \,|\, n\rangle \langle n \,|\, R^{1/2} |\rangle, \qquad (4.19)$$

which replace Eq. (4.18), one may find $\tilde{R}_-$ and $\tilde{R}_+$ and compute the results to be expected from experiments with $\tilde{\Lambda}$ (angular distribution and polarization of the antiprotons). We note again that one should not obtain an exact equality for the weights of the channels $\Lambda \to p + \pi^-$, $\tilde{\Lambda} \to \tilde{p} + \pi^+$, and also $\Lambda \to n + \pi^0$, $\tilde{\Lambda} \to \tilde{n} + \pi^0$, but rather a definite relation between them.

In the simpler case of the decay $\Sigma^- \to n + \pi^-$, where there is only one channel, Chou Kuang-Chao[63] obtained the following expression for the ratio of the asymmetry coefficients:

$$\frac{\tilde{\alpha}}{\alpha} = -\frac{\cos(\Delta_s - \Delta_p - (\delta_s - \delta_p))}{\cos(\Delta_s - \Delta_p + (\delta_s - \delta_p))}, \qquad (4.20)$$

where δ_S and δ_p are the phase shifts of the $\pi^- + n \to \pi^- + n$ scattering at 120 MeV ($\delta_S - \delta_p = -17°$) and the phase shift Δ_S is defined as the difference of the phase of the element $\langle l = 0 \,|\, R \,|\, \Sigma^- \rangle$ and δ_S:

$$\langle l = 0 \,|\, R \,|\Xi^-\rangle = |\langle l = 0 \,|\, R \,|\, \Sigma^-\rangle| e^{i(\Delta_s + \delta_s)}.$$

Δ_p ($l = 1$) is defined analogously.

If the decay $\Sigma^- \to n + \pi^-$ is invariant under T then $\Delta_S = \Delta_p = 0$, and $\tilde{\alpha}/\alpha = -1$ (as in the case of PC invariance, since invariance under PCT has been assumed). If however it should, for example, turn out that $\Delta_S - \Delta_p \sim 90°$, then the ratio (4.20) can turn out to even be positive. The decay will not be invariant under PCT if the value of $\tilde{\alpha}$ calculated from Eq. (4.20) should turn out to be different from the measured value.

CONCLUSIONS

Existing experiments are not in contradiction with the assumption of PC or PCT invariance of the processes under consideration. Consequently additional special experiments are needed if we wish to know which of the invariances, PC or PCT (or both together), is valid in reality.* These are experiments of the same nature as experiments needed to test T invariance (see Tables I and III). One must measure perpendicular components of the polarization vectors of decay particles. The difficulties of such experiments are compounded for our purposes by the fact that they must be performed also with negatively charged particles, which are either depolarized or captured by matter (μ^- or K^- mesons). We note that at the moment T invariance has been more or less reliably established only for the β decay of the neutron [56,64].

Let us list the experiments whose execution would be desirable from the point of view of testing and distinguishing PC and PCT invariances. (A majority of these experiments has been proposed in the papers [14,15,71].)

1) The measurement of the polarization of $\mu^\pm$ in $\pi^\pm$ $K^\pm_{\mu 2}$ decays, see Secs. 2 and 3.

2) The measurement of the forward-backward asymmetry in $\pi^- \to \mu^- \to e^-$ and $K^- \to \mu^- \to e^-$ cascades.

3) The measurement of the perpendicular component of e^+ and e^- polarizations in the $\pi \to \mu \to e$ cascade (at that one must insure that the polarization

*Strictly speaking, the existing experiments (on π and μ decays, for example) cannot disprove the assertion: "PC invariance does not hold, but PCT invariance does" (or: "PCT invariance does not hold, but PC invariance does").

of the μ is preserved, see Sec. 2), and also the $K_{\mu 2} \to \mu \to e$ cascade.

4) The measurement of the angular distributions in the decays $K_{3\pi}^{\pm}$, $K_{\mu 3}^{\pm}$ and $K_{e3}^{\pm}$. In the latter two it is desirable to also measure the polarization of the μ and e.

5) A more precise measurement of the equality of lifetimes, and also measurements of the branching ratios in the decays, of hyperons and antihyperons. The measurement of angular distributions and polarizations of their decay products.

APPENDIX

A. RELATIVISTIC PHENOMENOLOGICAL REACTION THEORY

1. In Sec. 4 we gave for decays of the type $a \to 1 + 2$ the Eq. (4.1) which follows from the conservation law of the total angular momentum. The method of specifying the state of a free particle, utilized in Sec. 4, by indicating its momentum and the spin projection m onto the direction of the momentum (helicity) is applicable to both particles with nonvanishing and vanishing rest mass [16,17]. Therefore Eq. (4.1) may also be applied to the decay $\pi \to \mu + \nu$ (for a "two-component neutrino" its projection m_2, and consequently the spin projection of the muon m_1, can take on only one value).

In the case of the decay $a \to 1 + 2 + 3$ [19]

$$\langle \mathbf{p}^{(1)}\mathbf{p}^{(2)} m_1 m_2 m_3 | U | M \rangle = \frac{1}{4\pi}\sqrt{2J+1}\sum_{j,m} D^{j}_{m_3+m_2,\, m}(-\pi,\ \vartheta_2,\ \pi-\varphi_2)\sqrt{2j+1}\, D^{J}_{m+m_1,\, M}(-\pi,\ \vartheta,\ \pi-\varphi_1)\langle m_1 m_2 m_3 j m p^{(1)} | U^J | \rangle. \qquad \text{(A.1)}$$

The relative momenta $\mathbf{p}^{(1)}$ and $\mathbf{p}^{(2)}$ have here special definitions [65]: $\mathbf{p}^{(1)}$ is the total momentum of particles 2 and 3 in the Lorentz barycentric frame K_{123} of the three particles 1, 2, 3; it is equal to $-\mathbf{p}_1$, i.e., it is antiparallel to the momentum of particle 1 in K_{123}; $\mathbf{p}^{(2)}$ is the momentum of particle 2 in the Lorentz frame K_{23} in which the total momentum of 2 and 3 is equal to zero. The expression for $\mathbf{p}^{(2)}$ in terms of $\mathbf{p}_2$ and $\mathbf{p}_3$ of particles 2 and 3 in K_{123} is given in the papers [19] and [66]; ϑ_1 and φ_1 denote the spherical angles of $\mathbf{p}^{(1)}$ with respect to a certain triplet of axes O_a, with respect to which the spin state of the decaying particle has been quantized (projection M). ϑ_2 and φ_2 are the spherical angles of $\mathbf{p}^{(2)}$ with respect to a triplet of axes 0_1 defined as follows: $z_1 \parallel \mathbf{p}^{(1)}$, $y_1 \parallel (z_a \times \mathbf{p}^{(1)})$, where z_a is the third axis of O_a. The projections m_1 are quantized with respect to the axes O_1, and m_2 and m_3 are the spin projections onto $\mathbf{p}^{(2)}$.

2. For the purposes of formulating the selection rules due to PC, T and PCT invariance in terms of the observed quantities it turns out to be convenient to introduce the so called cyclic projections of the defined in Sec. 1 polarization vector $\mathbf{P}$ and the operator $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$:

$$P_{-1} = \frac{1}{\sqrt{2}}(P_x + iP_y) = \mathrm{Sp}(\sigma_{-1}\varrho), \quad P_0 = P_z; \qquad \text{(A.2)}$$

$$P_{+1} = -\frac{1}{\sqrt{2}}(P_x - iP_y)/\sqrt{2} = -P^*_{-1};$$

$$\sigma_{-1} = \begin{pmatrix} 0 & \sqrt{2} \\ 0 & 0 \end{pmatrix}, \quad \sigma_0 = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma_{+1} = \begin{pmatrix} 0 & 0 \\ -\sqrt{2} & 0 \end{pmatrix}. \qquad \text{(A.3)}$$

The density matrix ρ may now be expressed in the form

$$\varrho = {}^1/_2 \dot{I} + {}^1/_2 \sum_{\tau=-1}^{+1} (\sigma_\tau^t P_\tau) \qquad \text{(A.4)}$$

(t denotes transposition). Throughout the following, the projection $P_z \equiv P_0$ denotes the projection of the polarization vector onto the particle momentum.

4. If the wave function Ψ' of the decay products is expressed in the form $\Psi' = U\Psi_0$, then their density matrix can be expressed in terms of the density matrix ρ_0 of the initial state and U as follows (see [20], Chap. 4): $\rho' = U\rho_0 U^+$.

Let us introduce into this expression in place of the density matrix of each particle the quantities $\mathrm{Sp}\,\rho$ and P_τ ($\tau = -1, 0, +1$). The result of the computations (see [20], Chap. 4) will be given in terms of one formula containing both the expression $F = \mathrm{Sp}\,UU^+/2$ for the angular distribution of the decay products of an unpolarized particle, as well as the more complicated expressions for their polarizations in the decay of a polarized particle:

$$\varrho'(\vartheta_1, \varphi_1, p^{(1)}; \vartheta_2, \varphi_2 \ldots; q_1\tau_1, q_2\tau_2 \ldots) = \sum_{q_a\tau_a} (q_1\tau_1 q_2\tau_2 \ldots | W(\vartheta_1, \varphi_1, p^{(1)}; \vartheta_2, \varphi_2 \ldots) | q_a\tau_a)\, \varrho_0(q_a\tau_a), \qquad \text{(A.5)}$$

$$(q_1\tau_1 q_2\tau_2 \ldots | W(\vartheta_1, \varphi_1 \ldots) | q_a\tau_a) = \frac{1}{2}\mathrm{Sp}\,\sigma_{\tau_1}^{q_1}\sigma_{\tau_2}^{q_2} \ldots R\sigma_{\tau_a}^{tq_a}R^t. \qquad \text{(A.6)}$$

For the expanded version of Eq. (A.6) see Appendix B. σ_τ^q for a particle of spin 1/2 denotes $\left(\begin{smallmatrix}1&0\\0&1\end{smallmatrix}\right)$ for $q = 0 = \tau$ and $\sigma_\tau^1 = \sigma_\tau$, $\tau = -1, 0, +1$; for a particle of spin 0 $\sigma_\tau^q = 0$ for $q \neq 0$ and $\sigma_0^0 = 1$. In place of the U matrix we have introduced the transition matrix R, see Sec. 1.

<u>Remaining notation</u>: $\rho_0(0, 0) = 1$ denotes the number of decaying particles and $\rho_0(1, \tau) = P_\tau$ the cyclic projections of their polarization vector. $\rho^1(\vartheta_1, \varphi_1, p^{(1)}; \vartheta_2\varphi_2 \ldots; 00; 00)$ denotes the angular distribution $F(\vartheta_1, \varphi_1, p^{(1)}; \vartheta_2\varphi_2 \ldots)$ of the reaction products (and also their distribution in the magnitude of the relative momenta [65]). $\rho'(\vartheta_1, \varphi_1, p^{(1)}; \vartheta_2\varphi_2 \ldots; 1\tau; 00\ldots)/F(\vartheta_1, \varphi_1, p^{(1)}; \vartheta_2\varphi_2 \ldots)$ denotes the cyclic projection of the polarization vector of the decay produce 1, emitted in the direction ϑ_1, φ_1 (with momentum $p^{(1)}$), while the other decay products were emitted in the directions characterized by the angles ϑ_2, φ_2, etc. These projections are referred to the triplet of axes 0_1, defined above in item 1.

B. SELECTION RULES ARISING FROM PC, T, PCT INVARIANCES

1. PC inversion. If the process $a \to 1 + 2 + 3$ is invariant under PC then (see Sec. 1, item 3)

$$(PC\,|\,\mathbf{p}^{(1)}\mathbf{p}^{(2)}m_1m_2m_3\rangle,\ RPC\,|\,\mathbf{p}_aM\rangle) = (|\,\mathbf{p}^{(1)}\mathbf{p}^{(2)}m_1m_2m_3\rangle,\ R\,|\,\mathbf{p}_aM\rangle). \tag{B.1}$$

Under spatial inversion spin projections are unchanged, however our z quantization axis for m is parallel to the direction of the momentum which becomes reversed under spatial inversion. The y quantization axis for m remains unchanged under spatial inversion. Indeed, for the initial state the only possible preferred direction, in addition to that of the direction z_a of the momentum of a in the laboratory frame, could be the direction of polarization; but the polarization is a pseudovector and it does not change under spatial inversion. The axis y_1 (see Sec. 1) also remains unchanged: $y_1 \parallel (\mathbf{p}_a \times \mathbf{p}^{(1)}) \parallel (-\mathbf{p}_a \times -\mathbf{p}^{(1)})$. Analogously $y_2' \parallel y_2$ (for the quantization of m_2 and m_3). Let us denote respectively by O_a', O_1', O_2' the "inverted" triplets of axes that have the z axis inverted, but not the y axis. On the left side of Eq. (B.1) we must refer the spin projections to these axes in order to preserve also after inversion their definition as projections onto the momentum direction. We note that the O′ differ from the O only by a rotation by a 180° about the y axis. The wave function Ψ_m of a state with definite projection m onto the old z axis is given in terms of the spin functions $\Phi_{m'}$ with definite projections m′ onto the new z axis by

$$\Psi_m = \sum_{m'=-s}^{+s} \Phi_{m'} D^s_{m',m}(0,\pi,0) = (-1)^{s+m}\Phi_{-m}. \tag{B.2}$$

Therefore

$$PC\,|\,\mathbf{p}^{(1)}\mathbf{p}^{(2)}m_1m_2m_3\rangle,\ RPC\,|\,\mathbf{p}_aM\rangle) = (-1)^{m_1+m_2+m_3-M}K \times (|-\mathbf{p}^{(1)},-\mathbf{p}^{(2)},-m_1,-m_2,-m_3\rangle,\ \widetilde{R}\,|-\mathbf{p}_a,-M\rangle). \tag{B.3}$$

The sign ~ over R means that the matrix element on the right side describes the process involving the corresponding antiparticles. K is an inessential to the following discussion product of phase factors $\exp i\eta$ and $(-1)^S$.

Equation (A.1) means that the elements of U or R do not depend on the directions $\mathbf{z}_a$, $\mathbf{p}^{(1)}$ and $\mathbf{p}^{(2)}$ individually, but only on their relative orientation. More precisely, they depend on the Euler angles $\{-\pi, \vartheta_1, \pi-\varphi_1\}$ and $\{-\pi, \vartheta_2, \pi-\varphi_2\}$ which specify the orientation of O_1 with respect to O_a and of O_2 with respect to O_1 (see Sec. 4, item 1; details in the paper [67]). It is easy to show (for example by making a spatial model) that if that orientation was originally given by the angles $\vartheta_1, \varphi_1; \vartheta_2, \varphi_2$, then the orientation of O_1' with respect to O_a' and of O_2' with respect to O_1' is given by the angles $\vartheta_1, -\varphi_1; \vartheta_2, -\varphi_2$ respectively.

And so from Eqs. (B.3) and (B.1) we obtain the relation

$$\langle m_1m_2m_3\,|\,R(\vartheta_1,\varphi_1,p^{(1)};\ \vartheta_2,\varphi_2)\,|\,M\rangle = K(-1)^{m_1+m_2+m_3-M} \times \langle -m_1,-m_2,-m_3\,|\,\widetilde{R}(\vartheta_1,-\varphi_1,p^{(1)};\ \vartheta_2,-\varphi_2)\,|\,-M\rangle. \tag{B.4}$$

Let us obtain the resultant selection rule in terms of the coefficients W; see Eq. (A.6).

We write out Eq. (A.6) in detail:

$$(q_1\tau_1q_2\tau_2\ldots\,|\,W(\vartheta_1,\varphi_1,\ldots)\,|\,q_a\tau_a) = {}^1/_2\sum(\sigma^{q_1}_{\tau_1})_{m_1',m_1}(\sigma^{q_2}_{\tau_2})_{m_2'm_2}\ldots\langle m_1m_2\ldots\,|\,R(\vartheta_1,\varphi_1\ldots)\,|\,M\rangle\langle m_1'm_2'\ldots\,|\,R(\vartheta_1\varphi_1\ldots)\,|\,M'\rangle^*(\sigma^{q_a}_{\tau_a})_{M,M'}; \tag{B.5}$$

$\sum$ denotes summation over all repeated indices. Let us substitute in place of the elements R in Eq. (B.5) the right hand sides of Eq. (B.4), which are equal to them:

$$= {}^1/_2\sum(\sigma^{q_1}_{\tau_1})_{m_1',m_1}\ldots(-1)^{m_1+m_2+\ldots-M}\langle -m_1,-m_2\ldots\,|\,\widetilde{R}(\vartheta_1,-\varphi_1,\ldots)\,|\,-M\rangle(-1)^{-m_1'-m_2'-\ldots+M'}\langle -m_1',-m_2',\ldots\,|\,\widetilde{R}(\vartheta_1,-\varphi_1,\ldots)\,|\,M'\rangle^*(\sigma^{q_a}_{\tau_a})_{M,M'}. \tag{B.6}$$

Starting from the definitions of the matrices σ^q_τ (see Appendix A) and the expressions (A.3) for σ_{-1}, σ_0, σ_{+1}, one can show that the σ^q_τ have the property

$$(\sigma^q_\tau)_{m',m} = (-1)^q(\sigma^q_{-\tau})_{-m',-m}. \tag{B.7}$$

We introduce into Eq. (B.6) in place of $(\sigma^q_\tau)_{m',m}$ the right side of Eq. (B.7) and relabel the summation indices by setting m equal to $-\widetilde{m}$. The sum is not affected by this: since m runs through the values $-1/2$ and $+1/2$, this simply corresponds to a rearrangement of the terms in the sum. Noting also that $(-1)^{m-m'} = (-1)^\tau$ for the nonvanishing elements of σ^q_τ we may extend Eq. (B.6) to:

$$= {}^1/_2\sum(-1)^{q_1+\tau_1}(\sigma^{q_1}_{-\tau_1})_{\widetilde{m}_1',\widetilde{m}_1'}(-1)^{q_2+\tau_2}(\sigma^{q_2}_{-\tau_2})_{\widetilde{m}_2',\widetilde{m}_2}\ldots\langle\widetilde{m}_1\ldots\,|\,\widetilde{R}(\vartheta_1,-\varphi_1,\ldots)\,|\,\widetilde{M}\rangle\langle\widetilde{m}_1'\ldots\,|\,\widetilde{R}(\vartheta_1,-\varphi_1\ldots)\,|\,\widetilde{M}'\rangle^*(-1)^{q_a+\tau_a}(\sigma^{q_a}_{-\tau_a})_{\widetilde{M},\widetilde{M}'}. \tag{B.7'}$$

Comparing the resultant expression with the definition of the W coefficient, Eq. (B.5), we see that it is equal to

$$(-1)^{q_1+q_2+\ldots+q_a}(-1)^{\tau_1+\tau_2+\ldots+\tau_a}(q_1,-\tau_1,q_2,-\tau_2\ldots\,|\,W(\vartheta_1,-\varphi_1\ldots)\,|\,q_a-\tau_a),$$

i.e., that we obtain the relation

$$(q_1\tau_1q_2\tau_2q_3\tau_3\,|\,W((\vartheta_1,\varphi_1,p^{(1)};\ \vartheta_2,\varphi_2)\,|\,q_a\tau_a) = (-1)^{q_1+q_2+q_3+q_a}(-1)^{\tau_1+\tau_2+\tau_3+\tau_a} \times (q_1-\tau_1q_2-\tau_2q_3-\tau_3\,|\,\widetilde{W}(\vartheta_1,-\varphi_1,p^{(1)};\ \vartheta_2,-\varphi_2)\,|\,q_a-\tau_a). \tag{B.8}$$

It differs from the obtained in [68] selection rule due to P invariance only in the fact that on the right side of Eq. (B.8) stand the W coefficients for the charge conjugate process.

The corresponding to Eq. (B.8) relation for the decay $a \to 1 + 2$ is obtained by removing from Eq. (B.8) the indices q_3, τ_3 and $p^{(1)}$, ϑ_2, φ_2.

The simplest consequence of Eq. (B.8) is obtained when all q and τ are equal to zero: the angular distributions in the decays of unpolarized particle and antiparticle are equal. Since the integral of the angular distribution over all angles is independent of the particle polarization,* the simplest consequence of Eq. (B.8) also leads to the equality of particle and antiparticle lifetimes and weights of corresponding (mutually charge conjugate) decay channels.

2. Wigner time reversal. Analogously to the preceding we obtain from the relations (1.6), (1.7) and (B.2) (as before, the m projections have to be reexpressed in terms of the "inverted" triplets of axes) the following expression for the decay $a \to 1 + 2 + \ldots$

$$\langle m_1 m_2 \ldots | R(\vartheta_1, \varphi_1 \ldots) | M\rangle = \Xi \langle M | R(\vartheta_1, -\varphi_1 \ldots) | m_1 m_2 \ldots\rangle. \tag{B.9}$$

Ξ is defined analogously to K in Eqs. (B.3) and (B.4).

It can be shown with the help of Eqs. (4.1) and (A.1) that in the $m_1 m_2 JM$ representation Eq. (B.9) becomes the relation

$$\langle m_1 m_2 \ldots | R^J | M\rangle = \Xi_0 \langle M | R^J | m_1 m_2 \ldots\rangle. \tag{B.10}$$

If the phase factors are chosen in such a way that $\Xi_0 = 1$ (Ξ_0 is not equal to Ξ), then Eq. (B.10) means that the matrix R^J is symmetric as a consequence of T invariance.

If the decay products interact weakly then $R^+ = R$ (see Sec. 1, item 4); in that case Eq. (B.9) may be rewritten in the form

$$\langle m_1 m_2 \ldots | R(\vartheta_1 \varphi_1 \ldots) | M\rangle = \Xi \langle m_1 m_2 \ldots | R(\vartheta_1, -\varphi_1 \ldots) | M\rangle^*, \tag{B.11}$$

from which follows the selection rule:

$$(q_1 \tau_1 \ldots | W(\vartheta_1, \varphi_1 p^{(1)}; \vartheta_2, \varphi_2 \ldots) | q_a \tau_a) = (-1)^{\tau_1 + \ldots + \tau_a} (q_1, -\tau_1 \ldots | W(\vartheta_1, -\varphi_1, p^{(1)}; \vartheta_2, -\varphi_2 \ldots) | q_a - \tau_a). \tag{B.12}$$

Its proof is analogous to that given above in the derivation of Eq. (B.8). The differences: after having used Eq. (B.11) one must take instead of Eq. (B.7) the relation

$$(\sigma^q_\tau)_{m', m} = (-1)^\tau (\sigma^q_{-\tau})_{m, m'}. \tag{B.13}$$

In addition, the replacement $m \to -\widetilde{m}$ is not needed.

Let us analyze the case when the decay products interact strongly. It was shown in Sec. 1, item 4, that by the states n in Eq. (1.9) one understands only those states (into which the particle a can decay) that can go over into the states f as a result of strong interactions.

Since $\langle f | R^+ | a\rangle \equiv \langle a | R | f\rangle^*$, it follows that Eq. (1.9) connects matrix elements of the direct $a \to n$ (including $a \to f$) and inverse reactions, just like the relations (B.9) or (B.10). According to Eq. (B.10) (if $\Xi_0 = 1$) we have $\langle f | R^+ | a\rangle = \langle f | R | a\rangle^*$, which together with Eq. (1.9) gives rise in the general case to the following equation for the matrix elements of the processes $a \to n$ (cf. [63]):

$$\langle f | R | a\rangle^* = \sum_n \langle f | U^+ | n\rangle \langle n | R | a\rangle. \tag{B.14}$$

Of course, all the corresponding elements of the strong processes (phase shifts) must be known.

We want to emphasize that they can be obtained by studying separately scattering processes and other strong interactions. For example, for the decay $\Sigma^- \to \pi^- + n$ it is sufficient to know the phase shifts of $\pi^- + n \to \pi^- + n$ scattering (see, for example, [62]). Indeed, since $U^+(t, 0) = U^{-1}(t, 0) = U(0, t)$ it follows that Eq. (1.9) is in fact of the form $R^+(t, 0) = U(0, t) R(t, 0)$. $U(0, t)$ accomplishes the backward evolution of the decay products from the instant of time t to the instant $t = 0$. They first converge (as free particles, if the radius r_0 of their strong interaction is finite); at the instant of decay τ they are confined in a small volume and interact; by the time $t = 0$ they have diverged again and may be considered free if the radius r_0 is finite and their relative velocity v is not very small, so that $r_0/v \ll \tau$. Numerically the elements $U(0, t)$ are equal to the elements $U^+(t, 0)$, which can be considered equal to the elements $U^+(t, -t)$ [or even $U^+(\infty, -\infty)$], since in both cases the initial and final states are free particle states, and the reaction probabilities per unit time calculated with the help of $U(t, 0)$ or $U(t, -t)$ practically coincide if $r_0/v \ll \tau$.

3. The selection rule that follows from PCT invariance in the case of weak interacting decay products can be obtained by combining Eqs. (B.8) and (B.12):

$$(q_1 \tau_1 \ldots | W(\vartheta_1, \varphi_1 p^{(1)}; \vartheta_2, \varphi_2 \ldots) | q_a \tau_a) = (-1)^{q_1 + \ldots + q_a} (q_1 \tau_1 \ldots | \widetilde{W}(\vartheta_1, \varphi_1, p^{(1)}; \vartheta_2, \varphi_2 \ldots) | q_a \tau_a). \tag{B.15}$$

As from Eq. (B.8), for all q and τ equal to zero one obtains from here the equality of the angular distributions of the decay products of a and $\widetilde{a}$, equality of their lifetimes [27,28] and equality of the weights of the corresponding decay channels [14,9]. Let us show that the equality of the total decay probabilities (lifetimes) also follows in the case of strongly interacting decay products.

For the total decay probability $\widetilde{\lambda}_{total}$ of the antiparticle we have the expression

$$\widetilde{\lambda}_{total} = Tr\widetilde{\varrho}' = Tr\widetilde{R}\varrho_0\widetilde{R}^\dagger = Tr\,(U^\dagger R)^* \varrho_0 (U^\dagger R)^{*\dagger}$$
$$= \sum_f \sum_{n', n''} \langle f | U^{\dagger *} | n'\rangle \langle n' | R^* | a\rangle \langle a | R^{*\dagger} | n''\rangle \langle n'' | U^* | f\rangle, \tag{B.16}$$

where Tr or $\sum_f$ denotes the summation over all chan-

*Starting from Eqs. (A.6), (A.1) and (D.2) one can show that

$$\iint d\cos\vartheta_1\, d\varphi_1 \iint d\cos\vartheta_2\, d\varphi_2\, (000000 | W(\vartheta_1, \varphi_1 \ldots) | q_a \tau_a) \sim \delta_{q_a, 0}\delta_{\tau_a, 0}.$$

nels (and in each channel integration over all momentum variables and summation over all spin variables). We have

$$\sum_f \langle f \mid U^{+*} \mid n' \rangle \langle n'' \mid U^* \mid f \rangle = \delta_{n', n''},$$

if the states f exhaust all states that can be reached from the states n′, n″, i.e., in the final analysis from the state a. It therefore follows from Eq. (B.16) that

$$\widetilde{\lambda}_{\text{total}} = TrR^* \varrho_0 R^{*+} = \lambda_{\text{total}};$$

ρ_0 is the density matrix describing an arbitrary pure state a of the unstable particle.

In the same manner we obtain the equality of the weights of the channels $a \to f_0$ and $\widetilde{a} \to \widetilde{f}_0$, if the states f_0 of the selected channel cannot go into the remaining states f via strong interactions. In that case the sum $\sum_{f_0} \langle f_0 \mid U^+ \mid n' \rangle \langle n'' \mid U \mid f_0 \rangle$ is nearly zero if n′ and n″ do not coincide with any of the states f_0. Further

$$\sum_{f_0} \langle f_0 \mid U^+ \mid f_0' \rangle \langle f_0'' \mid U \mid f_0 \rangle = \sum_f \langle f \mid U^+ \mid f_0' \rangle \langle f_0'' \mid U \mid f \rangle = \delta_{f_0', f_0''},$$

since $\langle f_0' \mid U \mid f \rangle \cong 0$, if f does not coincide with f_0'.

C. PC AND PCT INVARIANCE IN THE $\pi \to \mu \to e$ CASCADE

We have seen that in simple special cases Eqs. (B.8) and (B.15) gave rise directly to consequences of PC and PCT invariance on experimentally observable quantities. It is also easy to see that when $q_1 = 1$, $\tau_1 = 0$, while all remaining q and τ vanish, then from Eq. (B.8) as well as from Eq. (B.15) follows the result obtained in Sec. 2: the longitudinal components of the μ^+ and μ^- polarizations from the decay of pions at rest have opposite signs.

We shall now illustrate on the example of the $\pi \to \mu \to e$ cascade how all the remaining consequences follow from the selection rules (B.8) and (B.15), i.e., the relations between the angular distributions and polarizations of particles produced in mutually charge conjugate processes.

As was established in Sec. 2, the relative spin states of μ^+ and μ^- are known if these muons come from the decays of π^+ and π^- mesons. It is only necessary to take into account the fact that in the laboratory frame the polarization vector of the muon will also possess a nonlongitudinal component (see Sec. 2, item 1). We choose the following triplet of axes for the decay $\mu \to e + \nu + \widetilde{\nu}$: axis $\mathbf{z}_a \parallel \mathbf{p}_{\mu\text{lab}}$, and axis $\mathbf{y}_a \parallel (\mathbf{p}_{\pi\text{lab}} \times \mathbf{p}_{\mu\text{lab}})$.

Then the nonlongitudinal component will be directed along the axis x_a, and the projections of the polarization vectors of μ^+ and μ^- emitted at the same angle with respect to the direction $\mathbf{p}_{\pi\text{lab}}$ will be related as follows:

$$P_{z_a}^+ = -P_{z_a}^-, \quad P_{x_a}^+ = -P_{x_a}^-, \quad P_{y_a}^+ = P_{y_a}^- = 0. \quad \text{(C.1)}$$

It will be convenient to have the relations corresponding to Eq. (C.1) for the cyclic projections of **P**. Since $P_y = 0$ we have two equivalent expressions

$$P_\nu^+ = -P_\nu^-, \quad \nu = -1, 0, +1, \quad \text{(C.2)}$$

$$P_\nu^+ = (-1)^{1+\nu} P_{-\nu}^-, \quad \nu = -1, 0, +1. \quad \text{(C.2′)}$$

The general formula for the angular distribution and polarization of the electrons from the decay $\mu^+ \to e^+ + \nu + \widetilde{\nu}$ is obtained from Eq. (A.5) by integration over the angles of emission of the neutrino (i.e., over the angles ϑ_2, φ_2; see Appendix A):

$$\varrho'(\vartheta, \varphi; q\tau 0000) = (q\tau 0000 \mid W(\vartheta, \varphi) \mid 00)\, \varrho_0(0, 0) + \sum_\nu (q\tau 0000 \mid W(\vartheta, \varphi) \mid 1\nu)\, P_\nu^+. \quad \text{(C.3)}$$

The four zeros represent summation over the neutrino polarizations; in the following they will be omitted. ϑ is the angle between $\mathbf{p}_{\mu\text{lab}}$ and the momentum $\mathbf{p}^{(1)}$ of the electron in the barycentric frame of the muon. Let us note that the first term on the right side of Eq. (B.3) is independent of ϑ, φ and does not vanish only if $\tau = 0$.

If the μ decay is invariant under PC then in view of Eqs. (B.8) and (C.2′) the equality (C.3) gives rise to the relation:

$$\begin{aligned}\varrho'(\vartheta, \varphi; q\tau) &= (-1)^{q+\tau} (q-\tau \mid \widetilde{W} \mid 00)\cdot 1 \\ &+ \sum_\nu (-1)^{q+\tau} (q-\tau \mid \widetilde{W}(\vartheta, -\varphi) \mid 1, -\nu)\, (-1)^{1+\nu} (-1)^{1+\nu} P_{-\nu}^- \\ &= (-1)^{q+\tau} \widetilde{\varrho}'(\vartheta, -\varphi; q, -\tau);\end{aligned} \quad \text{(C.4)}$$

and for PCT invariance there follows with the help of Eq. (C.2)

$$\varrho'(\vartheta, \varphi; q\tau) = (-1)^q (q\tau \mid \widetilde{W} \mid 00)\cdot 1 + \sum_\nu (-1)^q (q\tau \mid \widetilde{W}(\vartheta, \varphi) \mid 1\nu)\, (-1)^1 (-P_\nu^-) = (-1)^q \widetilde{\varrho}'(\vartheta, \varphi; q\tau). \quad \text{(C.5)}$$

Particular consequences of Eqs. (C.4) and (C.5) were enumerated in items 1, 2 and 3 of Sec. 2:

1) For unpolarized muons it follows from either PC or PCT invariance that the longitudinal polarization components of e^+ and e^- have opposite signs: $P_{\parallel}^+ = -P_{\parallel}^-$. The fact that in this case $P_{\parallel}$ is independent of the angles and that the remaining polarization components vanish has already been noted above.

2) It can be shown that as a consequence of rotational invariance the angular distribution of electrons from μ decay must be an even function of φ: $F(\vartheta, \varphi) = F(\vartheta, -\varphi)$. Hence if the μ^- and μ^+ were produced in the decays of π^- and π^+ respectively (and did not become depolarized) then both PC and PCT invariance lead to equality of the angular distributions of e^+ and e^-: $F^+(\vartheta, \varphi) = F^-(\vartheta, \varphi)$.[14]

3) If Eqs. (C.4) and (C.5) for $q = 1$, $\tau = -1, 0, +1$ are written out for conventional Cartesian components then one obtains respectively Eqs. (2.1) and (2.2).

D. THE FUNCTIONS $D^j_{mn}(\Phi_2, \Theta, \Phi_1)$

$$D^j_{mn}(\Phi_2, \theta, \Phi_1) = e^{-im\Phi_2} d^j_{mn}(\theta) e^{-in\Phi_1}. \tag{D.1}$$

For general expressions and recurrence relations for the functions $d^j_{mn}(\theta)$ see the papers [17] and [69]. These papers also contain references to additional literature, which may be supplemented by § 7 of the book [70], which introduces the function P^j_{mn}: $d^j_{mn} = i^{n-m} P^j_{mn}$.

It should be noted that the matrix of the functions P^l_{mn} for $l = 1/2, 1, 2$, given on p. 95 of [70] is incorrect. It disagrees with Eq. (22) of Sec. 7 and, apparently, refers to the function $(-1)^{m-n} P^l_{mn}$.

1. Orthogonality relations:

$$\int_0^\pi d^{j_1}_{mn}(\theta)\, d^{j_2}_{mn}(\theta) \sin\theta\, d\theta = \frac{2\delta_{j_1, j_2}}{2j_1+1}, \tag{D.2}$$

$$\sum_m d^j_{mn_1}(\theta)\, d^j_{mn_2}(\theta) = \delta_{n_1, n_2}. \tag{D.3}$$

2. Decomposition of product of two D functions:

$$D^a_{\alpha,\alpha'}(\varphi_2, \theta, \varphi_1)\, D^b_{\beta,\beta'}(\varphi_2, \theta, \varphi_1) = \sum_c D^c_{\gamma,\gamma'}(\varphi_2, \theta, \varphi_1)\, C^{c\gamma}_{a\alpha b\beta} C^{c\gamma'}_{a\alpha' b\beta'}. \tag{D.4}$$

$$\sum_{\alpha', \beta'} D^a_{\alpha,\alpha'}(\varphi_2, \theta, \varphi_1)\, D^b_{\beta,\beta'}(\varphi_2, \theta, \varphi_1)\, C^{c\gamma'}_{a\alpha' b\beta'} = D^c_{\gamma,\gamma'}(\varphi_2, \theta, \varphi_1)\, C^{c\gamma}_{a\alpha b\beta}. \tag{D.5}$$

The $C^{c\gamma}_{a\alpha b\beta}$ denote Clebsch-Gordan coefficients, see for example [69].

3. Certain symmetry properties:

$$D^{j*}_{mn}(\varphi_2, \theta, \varphi_1) = (-1)^{m-n} D^j_{-m,-n}(\varphi_2, \theta, \varphi_1), \tag{D.6}$$

$$d^j_{mn}(\theta) = (-1)^{m-n} d^j_{n,m}(\theta) = (-1)^{m-n} d^j_{-m,-n}(\theta), \tag{D.7}$$

$$d^j_{m,n}(\theta) = (-1)^{j-n} d^j_{-m,n}(\pi-\vartheta) = (-1)^{j+m} d^j_{m,-n}(\pi-\vartheta), \tag{D.8}$$

$$D^{j*}_{mn}(\varphi_2, \theta, \varphi_1) = [D^j(\varphi_2, \theta, \varphi_1)]^{-1}_{nm} = D^i_{nm}(\pi-\varphi_1, \theta, -\pi-\varphi_2). \tag{D.9}$$

4. Relation to the not normalized Legendre polynomials and the spherical harmonics (normalized):

$$D^j_{0,0}(\varphi_2, \theta, \varphi_1) = d^j_{0,0}(\theta) = P_j(\cos\theta), \tag{D.10}$$

$$D^j_{0,n}(\varphi_2, \theta, \varphi_1) = \sqrt{\frac{4\pi}{2j+1}}\, Y_{j,n}(\theta, \pi-\varphi_1), \tag{D.11}$$

$$D^j_{m,0}(\varphi_2, \theta, \varphi_1) = \sqrt{\frac{4\pi}{2j+1}}\, (-1)^m Y_{j,m}(\theta, \pi-\varphi_2). \tag{D.12}$$

5. Special values:

$$d^j_{m,n}(0, \pi, 0) = (-1)^{j+m} \delta_{m,-n}, \tag{D.13}$$

$$d^j_{m,n}(90^\circ) = (-1)^{j-n} d^j_{-m,n}(90^\circ) = (-1)^{j+m} d^j_{m,-n}(90^\circ) \tag{D.14}$$

$$d^{1/2}_{m,n}(\theta) = \begin{array}{c|cc} m \backslash n & -1/2 & +1/2 \\ \hline -1/2 & \cos\theta/2 & \sin\theta/2 \\ +1/2 & -\sin\theta/2 & \cos\theta/2 \end{array},$$

$$d^1_{m,n}(\theta) = \begin{vmatrix} \frac{1+\cos\theta}{2} & \frac{\sin\theta}{\sqrt{2}} & \frac{1-\cos\theta}{2} \\ -\frac{\sin\theta}{\sqrt{2}} & \cos\theta & \frac{\sin\theta}{\sqrt{2}} \\ \frac{1-\cos\theta}{2} & -\frac{\sin\theta}{\sqrt{2}} & \frac{1+\cos\theta}{2} \end{vmatrix} \tag{D.15}$$

Note added in proof (to Sec. 3). By now the equality of the number of π^+ and π^- mesons produced in the decays of long-lived K^0 mesons has been confirmed in a larger experimental sample: in addition to [54] see also D. Luers et al, Phys. Rev. Lett. 7, 255 (1961). This is in agreement if not with PC then with PCT invariance of these decays[27] (see Sec. 3, item 5). Since in the cited papers also the absence of two-meson decays of the longlived K^0_l has been established accurate to 0.3%, we may conclude on the basis of Weinberg's theorem[56, 55] that the totality of these data is in agreement with the assumption of PC invariance of the decays of longlived K^0. So far this is the only argument in favor of the validity of precisely PC invariance in the decays of strange particles.

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