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## EXTENSIVE AIR SHOWERS OF COSMIC RADIATION\*

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### INTRODUCTION

THE very name—extensive air showers—is sufficiently descriptive of the phenomenological aspects of the phenomenon arising when cosmic-radiation particles of ultrahigh energy ( $E_0 > 10^{13}$  eV) pass through the atmosphere. The ability of cosmic-ray particles to produce groups of simultaneously appearing particles was disclosed as long ago as in 1929 by the observations of D. V. Skobel'tsyn.<sup>[1]</sup> Among 20 cloud-chamber photographs containing tracks of cosmic-ray particles he observed three photographs with two particles and one photograph with three particles. The existence of air showers, encompassing areas amounting to a thousand square meters and more, was disclosed in 1938 by the experiments of P. Auger and R. Maze<sup>[2]</sup> and independently by W. Kolhörster and his co-workers.<sup>[3]</sup>

The discovery of cascades consisting of a large number of charged particles coincided with the appearance of the electron-photon cascade theory.<sup>[4-6]</sup> This enabled H. Euler<sup>[7]</sup> to identify the extensive air showers with electron-photon cascades produced in the atmosphere by primary electrons of ultrahigh energy. This point of view regarding the nature of extensive

air showers was universally accepted for many years. However, in 1942 Skobel'tsyn<sup>[8]</sup> proposed, upon analyzing data on the lateral distribution of particles in extensive air showers, that two distributions are superimposed. One distribution corresponds to the electron-photon cascade theory. The other, which is broader, is connected with the interference of additional meson mechanisms, which lead to the presence of penetrating particles on the periphery of the shower. The subsequent experiments, carried out essentially by a group of Soviet physicists under the guidance of Skobel'tsyn, were devoted to a clarification of shower properties that are difficult to explain from the point of view of the pure electron-photon cascade theory. Among such shower properties one must include, along with the lateral distribution of the charged particles, also the character of the absorption of the showers in the lower part of the atmosphere and the presence of penetrating particles in the shower. The change in intensity of extensive air showers of different primary energy with the variation in the observation height was a problem solved with sufficient reliability by the electron-photon theory,<sup>[9]</sup> so that a thorough analysis of the discrepancy between theoretical predictions and experiment was of considerable interest. The presence of penetrating particles in extensive air showers was indicated by the results of the first shower observations with the aid of counters.<sup>[9]</sup> A clear-cut confirmation of the presence of penetrating

\*The present article is dedicated to D. V. Skobel'tsyn on the occasion of his 70th birthday. An outline of Skobel'tsyn's scientific and public activity is published in this issue.

particles in showers was obtained by Doudin<sup>[10]</sup> with the aid of a cloud chamber. However, disparities in the results of various experiments and errors in the quantitative estimates permitted the treatment of the penetrating particles as a secondary phenomenon. Experiments<sup>[11]</sup> carried out in 1944–1948 on Pamir (elevation 3860 meters above sea level) and in Moscow have shown that the difficulties arising when an attempt is made to describe extensive air showers with the aid of the ordinary electron-photon cascade theory are fundamental in character. This result was reflected in a new point of view concerning the nature of extensive air showers, formulated by G. T. Zatsepin<sup>[12,13]</sup> in 1948. According to this new point of view: 1) extensive air showers are generated by primary cosmic-ray nucleons; 2) the first act is a nuclear interaction whose secondary products form the soft, penetrating, and nuclear-active components of extensive air showers; 3) secondary nuclear-active particles, colliding with nuclei of the air atoms, form in turn all the shower components, and this leads to the nuclear-cascade process.

This scheme of the formation and development of extensive air showers explained qualitatively the hitherto accumulated experimental data, and at the same time eliminated the assumption that the primary cosmic radiation contains a noticeable number of electrons (or photons) of ultrahigh energy, a fact that at that time no longer agreed with the experimental data on primary cosmic rays at lower energies. Furthermore, that time coincided with the discovery of electron-nuclear showers in cosmic rays<sup>[14]</sup> and the first observations of  $\pi^\pm$  mesons.<sup>[15]</sup> All this taken as a whole transformed cosmic-ray physics into a branch of nuclear physics, and the study of extensive air showers turned into a method for investigating nuclear interactions at ultrahigh energies.

Some deductions concerning the processes that occur at ultrahigh energies were drawn at the very start of this new stage in extensive air shower research. One was the conclusion<sup>[16]</sup> that the effective cross section for the inelastic interaction between the nucleons and the atomic nuclei does not decrease with increasing colliding-particle energy up to  $E_0 \approx 10^{17}$  eV. Another result concerns the angular distribution of the secondary particles in nuclear-interaction events at energies of  $10^{14}$ – $10^{15}$  eV. It was found that the lateral distribution of the electrons near the shower axis unavoidably leads to the conclusion that the emission of the secondary particles, which carry away the overwhelming part of the primary-nucleon energy, is essentially anisotropic.<sup>[13,17]</sup> In particular, one of the first statistical models<sup>[18]</sup> of multiple particle production was in clear contradiction with the lateral distribution of the electrons near the axis of extensive air showers.

The study of extensive air showers is also important to the theory of the origin of cosmic rays, since there is at present no other approach to the investigation of primary cosmic radiation with energy  $E_0 > 10^{15}$  eV, other than observing the extensive showers produced by ultrahigh-energy primaries. Direct registration of such primary particles is practically impossible, owing to their exceeding rarity. Extensive air showers, which propagate over large distances away from the primary-particle trajectory, greatly increase the effective registration area. Modern arrays for individual study of extensive air showers, using shower-particle scintillation detectors with a total area of several dozen square meters, have an effective gathering area on the order of  $10 \text{ km}^2$  and register events connected with the appearance of primaries of energy  $10^{17}$ – $10^{19}$  eV. Extensive air showers produced by primaries with energy  $\sim 10^{17}$  eV were observed by Skobel'tsyn with the aid of shower-particle detectors separated by large distances as long ago as in 1946.<sup>[19]</sup>

However, both the investigation of extensive air showers from the point of view of the elementary processes occurring at ultrahigh energies, and the study of showers in connection with the theory of the origin of cosmic rays, are still far from being complete. Moreover, both trends of the investigations are closely intertwined: when the experimental data on extensive air showers are compared with the proposed patterns of shower development in the atmosphere it is essential to know the composition of the primary cosmic radiation in the corresponding energy interval. On the other hand, to determine the composition and energy spectrum of the primary cosmic radiation we need information on the processes that occur during the development and formation of the shower in the atmosphere.

In the sections that follow we report the main experimental data accumulated during the last decade in observations of extensive air showers in the lower part of the atmosphere.

## 1. METHOD OF COMPREHENSIVE STUDY OF EXTENSIVE AIR SHOWERS OF COSMIC RADIATION

The new point of view concerning the nature of extensive air showers necessitated a change in the formulation of the experiments. Whereas the earlier experimental researches were as a rule confined to a simultaneous measurement of one or two parameters, it now became necessary to obtain where possible complete quantitative data on the extensive showers caused by primaries of different energy. The number of shower particles at the observation level, being the easiest to determine experimentally, was chosen as a parameter related to the primary-particle energy

and characterizing each registered shower. Another feature of the new methodological approach was the tendency not to use during the course of the measurements any assumptions concerning the structure and composition of the shower. As will be shown later on, this condition was satisfied, for the only obligatory requirement in the determination of the shower axis and of the total number of charged particles was that the particle flux density decrease symmetrically with increasing distance from the shower axis.

The first experiments of this type were performed by a group of the Pamir expedition of the Physics Institute of the U.S.S.R. Academy of Sciences in 1952 at 3860 meters above sea level.<sup>[20]</sup> The installation consisted of approximately 1500 hodoscopic counters and a small cloud chamber. Later on measurements with similar arrays were carried out both on mountains and at sea level, and the counters were supplemented with large cloud chambers,<sup>[21]</sup> diffusion chambers,<sup>[22]</sup> and ionization chambers,<sup>[23]</sup> and the number of simultaneously connected counters reached several thousand. The gist of the new method of investigating extensive air showers can be understood from a description, for example, of the first comprehensive array. A characteristic feature of this and subsequent comprehensive arrays for the investigation of extensive showers is the distribution, over an area on the order of 100 m<sup>2</sup>, of a large number of hodoscopic-counter groups, intended for the measurement of the particle flux density (Fig. 1). Groups 1–19 consisted of 24 counters 24 cm<sup>2</sup> in area, groups 20–41 contained each 24 counters 100 cm<sup>2</sup> in area, while groups 42–50 contained each 12 counters with 330 cm<sup>2</sup> area. The hodoscopic system was examined whenever an extensive air shower with a particle flux density exceeding a certain value struck

the system. When the shower axis passed near the center of the array, the coordinates of the axis could be determined without making any assumptions except that the shower is symmetrical. The effective area for the determination of the axis amounted to about 60 m<sup>2</sup>. The selection and analysis of the events of interest to us reduced by a factor 5–10 the number of showers directly registered by the apparatus, for in most of the showers recorded the axis was not close to the center of the array. The combination of a comprehensive recording system along with subsequent selection of the necessary events is indeed the principal feature of the new method for investigating extensive air showers. Wide variations of the event selection conditions make it possible to regard the data processing as a direct continuation of the experiment, allowing even some changes in the initial setup of the physical problem.

The location of the shower axis can be determined by various means, but essentially the matter reduces to locating the center of symmetry of the shower-particle flux density. This operation can be determined by direct comparison of the particle flux density over various hodoscopic groups; the variation of the particle flux density in different directions can be examined by averaging data obtained by several groups of counters, as is illustrated in Fig. 2. Either mathematically rigorous statistical reduction or an analog computer can be used. Finally, an electronic computer can be used to find the probability distribution of the passage of the axis in different points of the measurement plane for the actually observed number of counters operating in each group, varying the form of the lateral-distribution function. This yields not only the

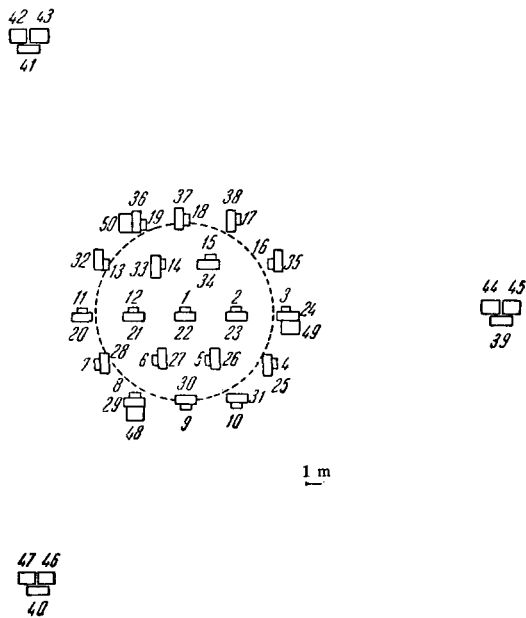


FIG. 1

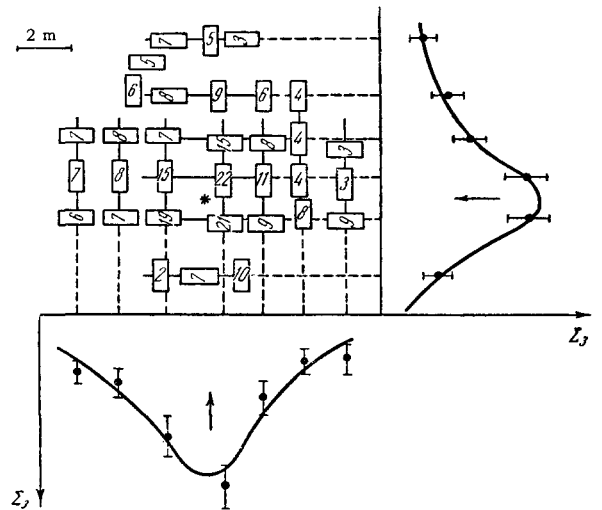


FIG. 2. Determination of the location of the passage of the shower axis. The rectangles denote groups of hodoscope counters. The numbers correspond to the number of counters operated in a given group. The results of summation over three groups, lying on one straight line, are shown on the plots. The asterisk denotes the location of the shower axis.

most probable location of the axis, but also the accuracy with which it is determined for each separate shower. However, for all the variety of methods, the mean error in the determination of the axis location remains practically the same, provided the number of hodoscopic groups whose readings are included in the processing remains constant. The error amounts to approximately half the distance between counter groups, depending on the form of the lateral distribution function.

The next problem after finding the shower axis is to determine the total number of particles in a given shower. By the same token, the energy of the primary particle producing the shower is estimated. To determine the total number of particles in an individual shower it is necessary to know the lateral distribution of the particles in the same shower. In principle, the lateral distribution function could be measured for each shower; to do this it is necessary to know in addition to the particle flux density at the center also the particle flux density at large distances (20, 40, 60, 100, 200, and 300 meters) from the central counter groups. In the comprehensive array shown in Fig. 1, there were in addition to the center counters also six groups located approximately 20 meters from the center and one group located 60 meters away. Therefore the lateral distribution for each registered shower could be obtained directly from the experimental data only in the interval of 1–60 meters from the axis. An example of such a distribution is shown in Fig. 3. The determination of the total number of charged particles from such a distribution in an individual shower already entails some additional assumptions regarding the particle flux distribution function on the periphery of the shower and the hypothesis that its form is not subject to large fluctuations. The lateral distribution function on the periphery of the shower was investigated in many of the earlier researches,<sup>[24]</sup> so that the form of the extrapolation was not subject to any doubt. Knowing the total number of particles and the location of the axis in each of the selected extensive air showers, and having the readings of the other recording devices in the same showers, it was possible

to investigate various shower components. The character of the information obtained (statistical data averaged over a group of showers with a given particle number, or quantitative information for each selected shower) depends on the capabilities of the recording apparatus employed. In some cases the apparatus itself has low registration efficiency, such as neutron counters or a cloud chamber for the observation of the nuclear-active component of extensive showers. The information obtained is in that case of a clearly statistical character. In other cases the statistical character of the data (the possibility of obtaining only the average or probable value of a quantity for a considered group of showers) is due to the small area of the recording apparatus, so that the observed particles do not strike the sensitive area of the array in each selected case. This happens particularly frequently in observations on the periphery of the shower, when frequently even the electron flux density for a chosen group of showers can be estimated only "in the mean."<sup>[25]</sup> Finally, in many cases it is possible to obtain perfectly definite data for each selected shower (for example, in measuring the energy of the electron-photon component of a shower core with the aid of large-area ionization chambers<sup>[26]</sup>). Naturally, further averaging of the obtained data is not excluded here. The possibility of obtaining simultaneously several parameters for individual cases when extensive air showers strike the array is, if the statistics of the events are sufficiently complete, a very important advantage of the method of large comprehensive arrays with subsequent selection. Such data are particularly valuable for the physical interpretation of the experiment.

When using large comprehensive arrays with subsequent selection for the investigation of some particular shower characteristic, with the selection criterion defined beforehand, the inevitable question arises of whether it would be more advantageous to get rid of the prolonged subsequent data reduction by selecting corresponding events during the course of the measurements, using electronic circuitry. This suggests the comparison with the methods using coincidences of pulses in counters and ionization chambers, which have been in successful use for cosmic-ray research for two decades, and in which the measurement results are expressed in counts obtained in different registration channels. Such a measurement procedure is attractive because it eliminates a tremendous volume of statistical experimental data reduction. On the other hand, however, to set up such an experiment it is necessary to formulate in clear-cut fashion the conditions under which the investigated parameter should be registered. At the start of the series of experiments described here it was deemed useful to dispense with the quantitative data then available on extensive air showers. Accordingly, the measurement procedure with the aid of comprehensive arrays with

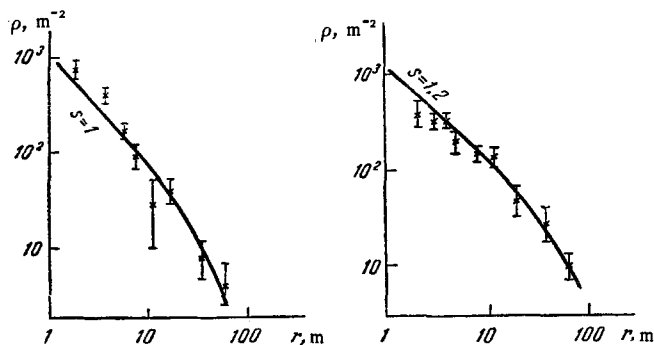


FIG. 3. Lateral distribution of electrons in individual showers.

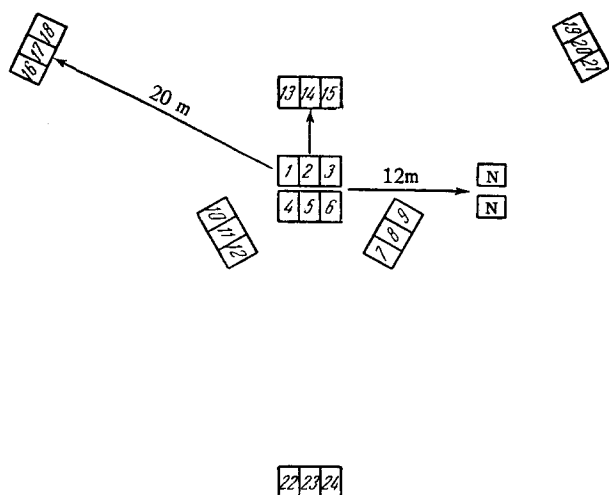


FIG. 4. Plan of array with "rigid selection." Counter groups 7-15 were located at a distance of 6 meters. N - neutron detectors.

subsequent selection seemed to be the more suitable research method. Later on, as experimental data were accumulated, it became possible to formulate unambiguously before the start of the measurements the criteria for selecting the necessary events, for example, extensive showers with a specified number of particles and axis location, and to effect the selection itself automatically by electronic means.

The plan of such an array (with "rigid selection") is shown in Fig. 4.<sup>[27]</sup> The array was assembled to measure the dependence of the number of nuclear-active particles on the total number of charged particles in a shower at an altitude of 3333 meters above sea level in the spring of 1961. The nuclear-active particles were registered with the aid of neutron counters embedded in paraffin. To increase the probability of interaction of the nuclear-active particle in the neutron detector, additional lead absorbers were used. The selection of extensive air showers with a specified number of particles and with the axis passing near the center of the array was made by a coincidence and anticoincidence combination in 24 groups of counters of specified area  $\sigma$ . For a shower to be registered it was required that there occur six-fold coincidence of the discharges in the central groups (1-6) and anti-coincidence with the pulse from the triple discharge coincidences in any of the triplets of counter groups (7-9, 10-12, 13-15, 16-18, 19-21, 22-24), circularly arranged at distances of 6 and 20 meters. The condition that there be no triple coincidences in any of the peripheral counter groups greatly reduces the probability of registering showers whose axes crossed the plane of observation far away from the center of the array. By the same token, the particle-number interval is noticeably narrowed down in the registered showers and shifts towards the lower limit. The calculated density distribution of the axis location for the showers registered by the ar-

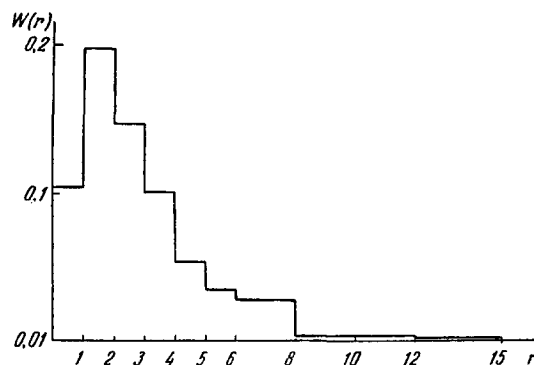


FIG. 5. Probability of registering the shower axis at different distances from the center of the array shown in Fig. 4. Abscissas - distance in meters.

ray is shown in Fig. 5. Comparing the intensity of the observed showers with the relative number of events when a pulse is produced in some neutron detector simultaneously with the shower, we can calculate the average value of the flux density of the nuclear-active component at a specified distance from the axis in the given group of showers. Variation of the area of the selecting counters  $\sigma$  leads to a suitably-directed change in the interval of investigated showers. In the series of measurements performed,<sup>[27]</sup> six different areas of shower-selection groups were used simultaneously, so that data could be obtained on showers in the particle-number interval from  $10^3$  to  $10^7$ . The results of the measurements were recorded in numerical form at the output of more than 40 recording channels of the electronic equipment.

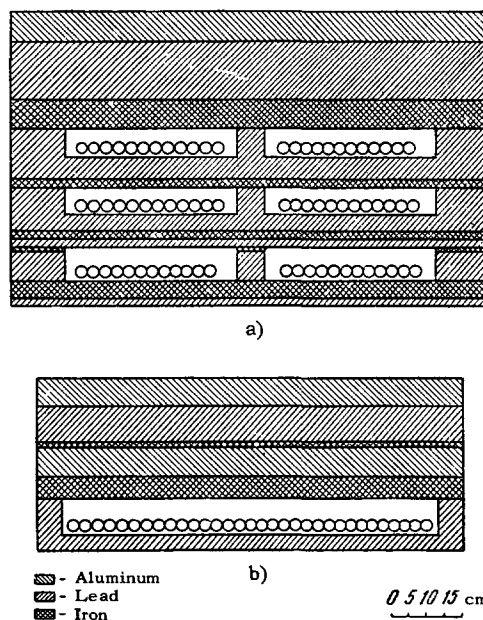


FIG. 6. Hodoscopic detectors for penetrating particles. a) Detector for distinguishing  $\mu$  mesons and nuclear-active particles; b) detector for observation of  $\mu$  mesons away from the shower axis. o - counter.

It would be incorrect to contrast the method of rigid selection of registered events with the method of comprehensive arrays with subsequent selection. They should supplement each other when used in conjunction with a recording system that permits extensive automatization of the experimental data reduction.

The detectors used for various components of extensive air showers comprise the entire arsenal of instruments used for cosmic-ray observation. A characteristic of the study of the properties of extensive air showers is the use of a large number of gas-discharge counters connected with hodoscopic registration systems, the use of large plastic scintillators, the measurement of ionization under different absorbers with the aid of ionization chambers or scintillators, and the registration of flashes of Cerenkov radiation in the atmosphere. Figures 6 and 7 show a hodoscopic detector for penetrating particles and a device for the investigation of the energy composition of cores of extensive air showers.

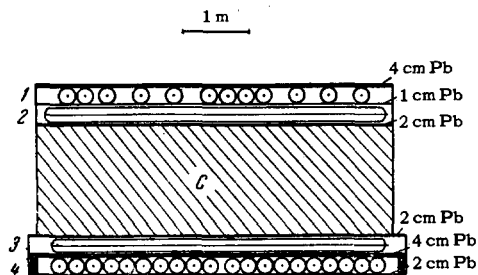


FIG. 7. Array for the measurement of the energy of the electron-photon component of the shower core and for the observation of high-energy nuclear-active particles. 1-4 - Rows of ionization chambers.

## 2. LATERAL DISTRIBUTION OF CHARGED PARTICLES IN EXTENSIVE AIR SHOWERS AT THE OBSERVATION LEVEL

The lateral distribution of the particle flux in extensive air showers of cosmic radiation is one of the oldest problems attracting the attention of experimenters. This interest is due to two perfectly different causes. First, the investigation of the lateral distribution has made it possible to obtain much information on the pattern of formation and development of the extensive shower. It is sufficient to mention here that it is precisely the analysis of the lateral distribution of the particle flux near the shower axis that has led to the refutation of the statistical model of the elementary nucleon-nucleon Fermi interaction event with isotropic scattering of the secondary particles in the system of colliding nucleons at an energy  $E_0 > 10^{12}$  eV with inelasticity coefficient equal to unity. [16,17]

Second, knowledge of the lateral distribution function of the shower particles makes it possible to determine the total number of particles at the observa-

tion level. The latter is the most important parameter in the analysis of the experimental data on extensive air showers, since the total number of particles at the observation level can serve as an estimate of the energy of the primary particle producing the shower.

In using the terms "charged" or "shower" particles, we purposely refrain from specifying the nature of these particles. As will be shown later on, in most experiments  $\geq 90\%$  of the particles registered by hodoscopic counters without any filters above them are electrons and positrons. If we confine ourselves to the central region of the shower, the fraction of the electrons among all the charged particles rises to 98%. On the periphery of the shower the relative contribution of the muons to the charged particle flux is more appreciable, a fact that must be borne in mind in the interpretation of the form of the lateral distribution function away from the shower axis. An account of the contribution of the muons to the total number of particles in estimates of the energy of the primary particle producing the shower is in principle of no significance, since conversion from the number of electrons to the energy of the primary nuclear-active particles cannot pretend to be more than 10% accurate.

a) Total number of charged particles in extensive air showers at the measurement level. As can be seen from Fig. 8, the lateral distribution of the particles depends little on the size of the shower. In experiments with comprehensive arrays, one measures with the best accuracy the particle flux density of each individual shower at distances of 2-10 meters. If we neglect the fluctuations and the weak dependence of the lateral distribution function on the number of particles in the shower, then for the shower particle-number interval  $10^4 - 2 \times 10^6$  the total number of charged particles can be estimated from the particle flux density in the 2-10 meter interval at  $N = 500r\rho(r)$  where  $r$  is the distance in meters and  $\rho(r)$  the charged particle flux density at the distance  $r$ . The simplifying assump-

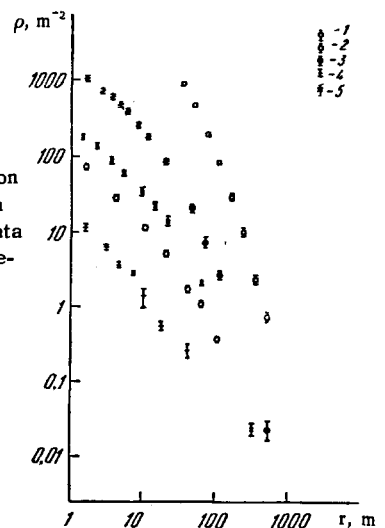


FIG. 8. Lateral distribution of flux of charged particles in extensive air showers. 1 - Data of [30],  $N = 1.5 \times 10^7$ ; 2-5 - Results of the Pamir measurements [25,26],  $N = 7.5 \times 10^4$ ,  $1.2 \times 10^6$ ,  $10^4$ , and  $2 \times 10^5$ , respectively.

tions made limit the accuracy of the estimate of the total number of particles, and the inaccuracy in the obtained number of particles is both statistical and systematic.

Let us consider the dependence of the function of the lateral distribution on the size of the shower. Since in showers observed at mountain altitudes less than 10% of the total particle flux is contained within a distance of 10 meters from the shower axis, greatest interest lies in the distances of 20–200 meters. This is clearly seen in Fig. 9, where the abscissas represent the distance from the shower axis on a logarithmic scale, and the ordinates represent the function

$$\varphi(r) = 2\pi r f(r) \frac{dr}{d \log r}.$$

The averaged experimental data on the lateral distribution of the electrons\* are taken for showers with total particle number  $N = 7.5 \times 10^4$  (curve 1)<sup>[25]</sup> and  $N = 7.5 \times 10^5$  (curve 2).<sup>[25,28]</sup>

For a better comparison of the curves with each other, the same figure shows the calculated functions  $2\pi r f^T(r) dr/d \log r$ . The theoretical distribution  $f^T(r)$  employed was the result of calculations<sup>[29]</sup> for the lateral distribution of electrons in a cascade shower with a parameter  $s = 1.2$  and  $E_0/\beta = \infty$  † and with a parameter  $s = 1$  and  $E_0/\beta = 10^5$ . The calculated curves were normalized to the experimental ones in the distance interval 1–10 meters. As can be seen from Fig. 9, the lateral distribution of showers with smaller numbers of particles is in good agreement with the calculated one with the parameter  $s = 1.2$ ; the lateral distribution of showers with a larger number of particles agrees best with the calculation for  $s = 1$  and  $E_0/\beta = 10^5$ . The difference in the lateral distribution leads to a difference in the estimates of the total number of particles. For the first group, the integration over the experimentally observed distribution leads to  $N = 7.5 \times 10^4$  in place of  $N' = 500\rho(r)r = 6.5 \times 10^4$ . For the second group we have respectively  $N = 7.5 \times 10^5$  and  $N' = 8.5 \times 10^5$ . At the present time there are no data to allow us to trace in detail the variation of the lateral distribution. The observed difference in the lateral distribution of the showers that differ in the number of particles by a mere factor of 10 cannot be extrapolated to a broad interval of primary energies. Indeed, the lateral distribution of showers with  $N = 1.5 \times 10^7$  at practically the same altitude<sup>[30]</sup> agrees with that calculated for the parameter  $s = 1$ . On the other

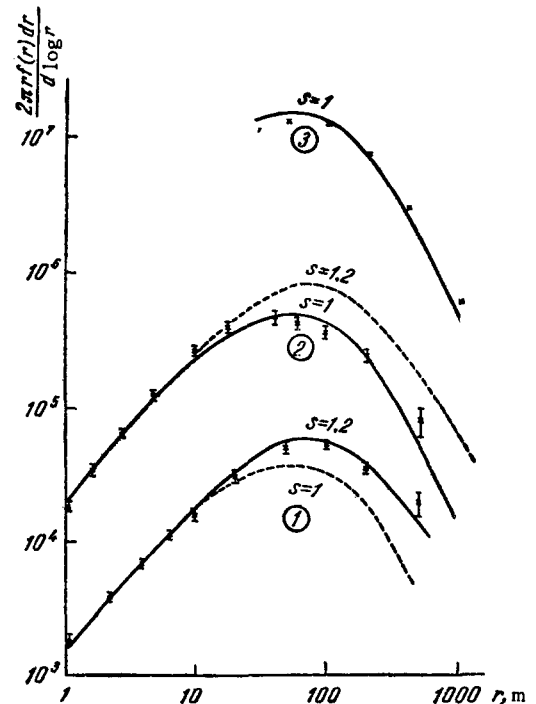


FIG. 9

hand, if the scattering angles of the  $\pi^0$  mesons in nuclear-interaction events do not influence the lateral distribution of the electrons in the shower, then, irrespective of the specific model of the development of the nuclear-cascade process in the shower, one can expect from the altitude variation of the observed number of showers with  $N < 10^5$  that the averaged lateral distribution function would agree with the calculated one<sup>[31,32]</sup> for a cascade parameter  $s = 1.2$ – $1.4$ .\*

In the central region of an extensive air shower no such change in the lateral distribution function with increasing total number of particles in the shower is observed. To the contrary, on going over from showers with  $N < 10^5$  particles to showers with  $N > 10^5$  particles there is a tendency for a weakening of the dependence of the total particle flux on the distance to the shower axis, corresponding to an increase in the cascade parameter  $s$ , if the lateral distribution of the electrons were to be determined only by the Coulomb scattering. Experimental data<sup>[28]</sup> on the distribution of the particle flux density in the central region of the shower are shown in Fig. 8.

The slope of the linear portions of the curves has been analyzed by the least-squares method. The values of the index  $n$  in the expression  $\rho(r) \sim r^{-n}$ , corresponding to a straight line on a log-log scale, are shown in the plot of Fig. 10. It can be seen that the values of  $n$  for showers with  $N > 10^5$  are smaller than

\*Showers with  $N = 2.1 \times 10^4$  particles (see Fig. 8) are also in good agreement with calculation for  $s = 1.2$ , and at any rate for  $s < 1.4$ .

\*The contribution of the muons and the electrons in equilibrium with them at distances  $r > 100$  meters has been taken into account.

†Here and throughout we use the universally employed cascade theory parameters:  $\beta$  – “critical energy,”  $s$  characterizes the energy spectrum of the particles in the cascade and by the same token is a measure of the “age” of the cascade ( $s < 1$  up to the maximum,  $s > 1$  beyond the maximum of cascade development).

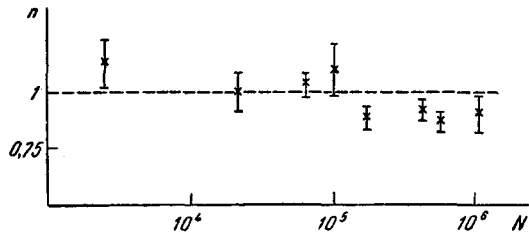


FIG. 10

unity ( $\bar{n} = 0.91 \pm 0.016$ ). At the same time, for showers with  $N < 10^5$  we have  $n > 1$ . An explanation of this difference must be sought in the structure of the cores of showers with different numbers of particles and, possibly, in differences in the energy composition of the showers.

b) Core structure of extensive air showers and energy spectrum of the electron-photon component. The lateral distribution of electrons at small distances from the shower axis, unlike the distribution of the particle fluxes at medium distances, is connected not only with the energy spectrum of the particles at the observation level, but also with the energy of the primary  $\gamma$  quanta, which make the greatest energy contribution to the electron flux in shower cores. Distances from the shower axis  $r \lesssim 2$  m were investigated at mountain altitudes for showers with  $N \approx 10^5$  particles.<sup>[33]</sup> The results of the measurements are shown in Fig. 11. The analysis was made under the assumption that all the showers are identical and consequently the averaged data correspond to the pattern in an individual shower. The neutral-pion scattering angles in the generation acts were neglected. The observed lateral distribution  $f(r \leq 1) \sim 1/\sqrt{r}$  can then be explained within the framework of the electron-photon cascade theory, if the decisive contribution to the formation of the electron-photon component observed near the shower axis is made by the low-energy neutral pions ( $E_0 = 10^{11} - 10^{12}$  eV). Although both assumptions of this analysis correspond little to really observed showers, the estimate of the neutral-pion en-

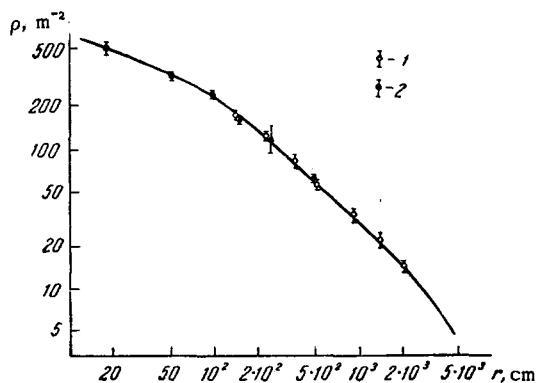


FIG. 11. Lateral distribution of electrons near the axis of an extensive air shower with  $N = 10^5$ . Data: 1 - from <sup>[20]</sup>; 2 - from <sup>[33]</sup>

ergy turns out to be correct for showers with a total of  $N = 10^4 - 10^6$  particles. This was confirmed by measurements of the energy spectrum<sup>[21,34]</sup> of the electron-photon component in the cores of extensive air showers. The experiments were performed using jointly a multi-plate cloud chamber and a large comprehensive array at an altitude of 3860 meters and with an array using the rigid selection criterion at sea level. The experiments have shown that the energy spectrum of the electron-photon component in the cores of extensive air showers is independent of the observation altitude in the lower third of the atmosphere, and apparently does not change when the total number of shower particles varies in the range  $N = 10^4 - 10^6$ . The main result of the analysis of the data was the deduction that the observed energy spectrum at  $E > 10^9$  eV (Fig. 12) can be understood if in the nuclear-cascade scheme of shower development,

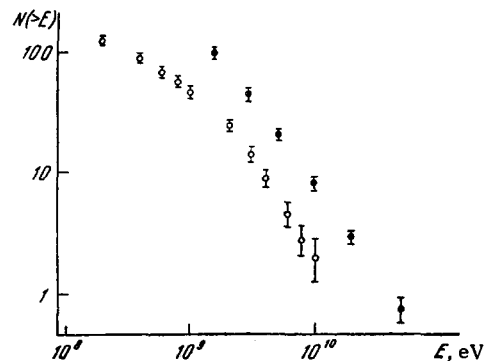


FIG. 12. Energy spectrum of electrons and photons in the cores of extensive air showers with  $N = 1.3 \times 10^5$  as given by <sup>[21,34]</sup> (○) and over the entire shower <sup>[35]</sup> (●).

in addition to taking into account the angles of emission of the neutral pions in the generation events, it is also assumed that the effective energy of the neutral pions deep in the atmosphere amounts to approximately  $10^{11}$  eV, which is much lower than the energy of the corresponding primaries. A similar result was obtained in later experiments,<sup>[35]</sup> in which the energy spectrum of the electron-photon component with energy  $E > 10^9$  eV was obtained in the entire extensive air shower (Fig. 12).

Returning to the lateral distribution of the electron-photon component in the cores of extensive air showers, it is necessary to point out another cause of the "plateau" in the distribution of the particle flux density near the shower axis: in some cases the showers have a "multicore" character. The first experimental data on the complicated structure of the cores in individual cases when extensive air showers were registered were obtained using a multi-section ionization chamber.<sup>[36]</sup> The reason for the appearance of showers with a complicated core structure may be both the presence of multiply charged nuclei in the composition of the primary cosmic radiation, and the angles of the



pion emission in nuclear interaction events near the observation levels. Consequently, the tendency noted in [23,37] towards an increase in the number of "multi-core" cases with increasing number of particles in the shower may be the consequence of both the change in the composition of the primary cosmic radiation and the change in the characteristics of the elementary act. Unfortunately, data on "multicore" showers are still very scanty. Even the distances between individual cores are known only very approximately: in showers with  $N \sim 10^5$  particles the region where individual cores are observed is limited to 2–3 meters. [37] There are also indications that with increasing size of the shower the distance between individual "cores" decreases. There is no doubt, however, that a study of such showers is an urgent problem which most probably will single out the cases when showers are produced by multiply charged nuclei of the primary cosmic radiation. [38]

c) Fluctuations of the lateral distribution of electrons in extensive air showers. Inasmuch as in observations of extensive air showers with the aid of comprehensive arrays with many hodoscopic counters it is possible to construct for each individual registered shower the lateral distribution of the electrons, the problem of measuring the fluctuations of the lateral distribution function can be reduced to a comparison of the experimental distributions with the assumed functions. The standards used for this purpose were the functions calculated from the electromagnetic cascade theory in [29]. The form of the function is uniquely related with the parameter  $s$ . The experimental data pertain to the interval of distances from the shower axis 1–100 meters. Figure 13 shows the

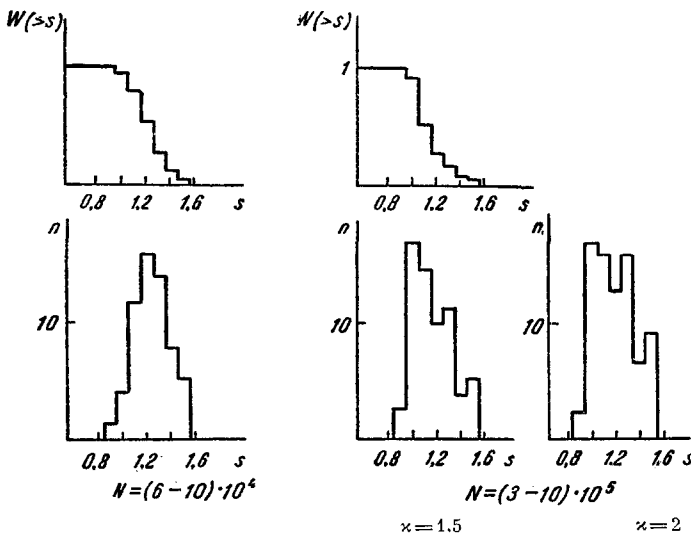


FIG. 13. Fluctuations of the lateral distribution of electrons at the observation level ( $p = 650 \text{ g/cm}^2$ ). Top – distribution over  $s$  directly observed in the experiment:  $W(>s) = \int_s^\infty \varphi^+(s, \rho_c) ds$ . Bottom – the distribution  $\varphi(s)$ .

integral distribution of the analyzed showers as a function of the parameter  $s$ , at which the theoretical distribution corresponds best to the experimentally observed one. The experimental data made it possible to determine  $s$  with an accuracy  $\Delta s = \pm 0.1$ .

Inasmuch as in classifying the showers by the total particle number use was made of the number of electrons observed near the shower axis (at distances of 1–10 meters), the distributions obtained characterize the probability of observing showers with different values of  $s$  for a specified particle flux density near the core:  $\varphi^+(s, \rho_c)$ . To interpret the experimental data it is much more useful to obtain the distribution with respect to  $s$  of extensive air showers with a specified total number of particles at the observation level,  $\varphi(s, N)$ . If we assume that the distribution about the axis depends weakly on  $N$ ,  $\varphi(s, N) \approx \varphi(s)$ , this distribution can be readily derived from the experimentally observed  $\varphi^+(s, \rho_c)$ . Here it is necessary to take into account the spectrum of the extensive air showers  $f(N) dN = A dN/N^{\kappa+1}$  and the connection between the number of particles near the core  $\rho_c$  and the total number of charged particles at the observation level  $N$  for different values of  $s$ :

$$\rho_c = a(s) N,$$

$$\varphi^+(s, \rho_c) ds d\rho_c = \varphi(s) f(N) dN ds = \varphi(s) ds f \left[ \frac{\rho_c}{a(s)} \right] \frac{dN}{d\rho_c} d\rho_c,$$

$$\varphi(s) = \frac{\varphi^+(s, \rho_c)}{f \left[ \frac{\rho_c}{a(s)} \right] \frac{dN}{d\rho_c}}.$$

The result of the recalculation of the experimentally observed distributions is shown in Fig. 13 (in the form of histograms). For the shower group with  $(6-10) \times 10^4$  particles the value of  $\kappa$  in the particle-number spectrum of the showers is taken equal to 1.5. The distribution obtained is in good agreement [the Pearson criterion  $p(\chi^2) = 0.35$ ] with a normal distribution around  $\bar{s} = 1.23 \pm 0.03$  and a half-width 0.15. For the group of showers with  $(3-10) \times 10^5$  particles the recalculation was carried out for two values of  $\kappa$ . The distributions obtained are characterized by mean values  $\bar{s} = 1.18 \pm 0.03$  and  $\bar{s} = 1.2 \pm 0.03$  and half widths 0.16 and 0.17 for  $\kappa = 1.5$  and  $\kappa = 2$ , respectively. In this case, however, the distribution cannot be reconciled with the normal distribution [the Pearson criterion is  $p(\chi^2) \leq 0.006$ ]. This means that apparently this group contains showers of various nature.

The fluctuations of the lateral distribution function in the 1–25 meter interval of distances from the shower axis was analyzed in [39] for showers with a total number of particles  $N > 10^5$  at sea level. The result of the comparison of 25 showers with the theoretical distribution [29] for different parameters  $s$  can be expressed as  $\bar{s} = 1.25 \pm 0.03$  with a distribution half width  $\sigma = +0.1, -0.2$ . The inaccuracy in the determination of the value for the individual case amounted to

$\pm 0.1$ . The correction for the influence of the selection of the registered events was not introduced.

Evidence that the observed fluctuations of the lateral distribution function are not due to the apparatus but are a reflection of the development of the shower can be found in the results of [41], pertaining to investigations carried out at 2770 meters above sea level. The authors have analyzed the frequency of appearance of "gently sloping" ( $s > 1.25$ ) and "steep" ( $s < 1.25$ ) lateral distribution functions near the axis in the case of "active" and "inactive" showers. The group of "active" showers consisted of cases when the relative number of high-energy electrons and photons ( $E \geq 10^9$  eV) exceeded the average value by three times. The "inactive" showers were classified as those in which the number of electrons and photons is  $\frac{1}{3}$  of the mean value. In the first group there were 3% of "gently sloping" distributions, 32% "steep" ones, and 65% "normal" ones ( $s \sim 1.25$ ). The group of "inactive" showers contained 39% "gently sloping" showers, 61% "normal" showers, and there were no "steep" ones. Thus, fluctuations of the form of the lateral distribution function are noticeably correlated with the energy spectrum of the electron-photon component. Fluctuations of the lateral distribution of electrons at the observation level must be expected in practically any scheme of nuclear-cascade development of the shower. They can be connected with the height of the start of the development of the shower or with the number of acts wherein a large fraction of the energy is transferred from the nuclear-active particles to the electron-photon component of the shower above the measurement level. [40] Quantitatively this problem has not yet been sufficiently fully examined.

However, comparing the experimentally observed fluctuations for showers with  $10^4$ – $10^5$  particles at the measurement level with the calculations [32] and taking into account the apparatus scatter of the values of  $s$ , we can conclude that the model of the formation of extensive air showers deep in the atmosphere, with complete transfer of energy to the electron-photon component in a single act [32] does not agree with experiment.

d) Electron-photon component and nuclear-cascade development of extensive air showers. By the time the nuclear-cascade scheme for the formation and development of extensive air showers was corroborated, three principal facts were known regarding the electron-photon component of the shower that could not be explained within the framework of the electron-photon cascade theory. The first of these three facts—the broader lateral distribution of the charged-particle flux—turned out to be connected with the lateral distribution of the muon flux in the shower; this will be considered in the next section of this review. The other two facts—the independence of the lateral distribution function of the shower particles of the size of the shower, of the height of observation, and the independence of the absorption of the showers in the at-

mosphere of the primary energy—are manifestations of one and the same aspect of the phenomenon. This can be seen from the fact that these circumstances can be reconciled with the electron-phonon theory by making one additional assumption: the height of formation of the registered showers above the observation level depends on the energy of the primary particle producing the shower, and does not depend on the height of observation. The coefficient of shower absorption in the atmosphere  $\mu$  and the coefficient of absorption of electrons in the individual cascade  $\mu_e$  are related by the equation  $\mu = \kappa\mu_e$ , where  $\kappa$  is the exponent in the particle-number spectrum of the showers  $F(>N) \sim N^{-\kappa}$ . This relation holds true in the case when there are no fluctuations in the development of the showers, and the situation becomes more aggravated by the fact that in most schemes of formation and development of showers the calculated quantity is  $\mu_e$ , while the experimentally observed quantity has so far been the value of  $\mu$ . Table I gives the values of  $1/\mu$  obtained in the lower part of the atmosphere.

From the data given in Table I we can conclude that in measurements of the absorption of extensive air showers in the atmosphere the methodological errors, which lead to differences in the absorption coefficients, have still not been eliminated. The fact that the absorption coefficient of the particles in the shower  $\mu_e \sim 1/200$  g/cm<sup>2</sup> is approximately constant over a wide range of shower particle numbers was in its time a factor which could not be explained without making use of the nuclear-cascade scheme of shower development. The main difficulty lay in the fact that such a small coefficient of particle absorption in the shower cannot be explained in any way near sea level for extensive air showers corresponding to primary nucleons with energies  $E_0 < 10^{15}$  eV. For such showers it is essential to assume that the electron-photon component is formed deep in the atmosphere by nuclear-active particles absorbed in the atmosphere exponentially, with a range  $\lambda \approx 120$  g/cm<sup>2</sup>. Moreover, from the condition that the attenuation of such showers be exponential and that the structure of the shower core be conserved near sea level we can conclude [40] that the energy of the primary nucleon at  $E_0 \approx 10^{14}$ – $10^{15}$  eV is carried deep into the atmosphere by a single energetically singled-out particle (the "leading" particle).

The lateral distribution of electrons in extensive air showers at mountain altitudes can be described with the aid of a lateral distribution function in the electron-photon shower with given parameter  $s$  in the distance interval from 2 to 300–500 meters. The experimental data at lower altitudes [46,47] agree with this result. The average value of the parameter, characterizing the form of the lateral distribution averaged over the group of showers with given total number of particles, changes very little with increasing number of particles in the shower. Indeed, at altitudes of

Table I

Method of observation	Height of observation	Number of particles in shower	$1/\mu$ , g/cm <sup>2</sup>
Change in barometric pressure [42]	Sea level	$3 \cdot 10^5$	$114 \pm 10$
		$8 \cdot 10^5$	$110 \pm 12$
		$2 \cdot 10^6$	$111 \pm 18$
		$10^7$	$106 \pm 12$
Change in barometric pressure [43]	Sea level	$10^4$ — $10^5$	$130 \pm 10$
		$10^5$ — $10^6$	$105 \pm 10$
Change in barometric pressure [44]	Sea level	$10^4$	$120 \pm 2$
		$2.5 \cdot 10^4$	$146 \pm 5$
		$3 \cdot 10^5$	$144 \pm 6$
		$2 \cdot 10^6$	$173 \pm 18$
Comparison of particle-number spectrum of showers at two altitudes [45]	Sea level and 3860 meters above sea level	$10^5$ — $10^6$	$156 \pm 22$
Distribution over the zenith angles [46]	Sea level	$1.2 \cdot 10^8$	121
		$1.8 \cdot 10^7$	109

3860—4000 meters above sea level the value of  $s$  varies from  $s = 1.25$  for showers with a total number of particles  $N \approx 10^4$  to  $s = 1$  for showers with  $N \approx 10^7$ . At sea level the value of this parameter agrees with  $s = 1.3 \pm 0.1$  for showers with  $N$  from  $10^4$  to  $10^6$ , and with further increase in the total number of particles in the shower at the measurement level the parameter  $s$  decreases to 1 at  $N \approx 10^9$ .

If we assume, without making more exact the specific mechanism of shower development, that the account of the nuclear-cascade process will make it possible to explain the independence of the absorption coefficient of the electron-photon component of the shower of the total number of particles in the shower, then the weak dependence of the experimentally observed lateral distribution function on the size of the shower can be easily explained, at least in the mean. Indeed, since in the electron-photon cascade theory both the lateral distribution and the electron absorption coefficient depend on the energy spectrum of the electrons and photons in the cascade, the weak dependence of the particle absorption in the cascade on the size of the shower implies a weak variation of the parameter  $s$ , which precisely characterizes the energy spectrum. If the absorption range of the particles in the shower amounts to  $\lambda \sim 200$  g/cm<sup>2</sup>, as follows from the altitude variation of the showers, then in accordance with the cascade theory this corresponds to a parameter value  $s = 1.2$ — $1.3$ . This value of the parameter agrees also with the lateral distribution of the electrons. It was shown in [32] that this connection between the lateral distribution and the absorption of the particles in the shower is conserved in the mean also in the case of large fluctuations in the development of the extensive air showers, when the formation of the shower is assumed to be due to one act of nuclear interaction of the nucleon with catastrophic energy losses, at an ar-

bitrary depth in the atmosphere. The mean value  $s \approx 1.2$  averaged over the group of showers is determined by the absorption range of the nucleons in the atmosphere, which is  $\lambda \approx 120$  g/cm<sup>2</sup>.

### 3. MUONS IN EXTENSIVE AIR SHOWERS

The presence of penetrating nuclear-passive particles in extensive air showers was established long before the nuclear-cascade shower development scheme was affirmed. [10] There is at present no doubt that the penetrating nuclear-passive component of the extensive air showers consists of muons and that essentially the process whereby the low-energy muons are produced is  $\pi$ - $\mu$  decay. The contribution of K mesons to the generation of muons with energy  $E_\mu > 10^{11}$  eV is not sufficiently clear. The relatively long lifetime of the  $\pi^\pm$  mesons causes the predominance to be assumed by the low-energy  $\pi^\pm$  mesons, the number of which is particularly large near the maximum of the nuclear-cascade avalanche. By the same token, the lateral and energy properties of the muon component of extensive air showers as a whole are connected with the characteristics of the nuclear collisions at low energies ( $E < 10^{12}$  eV), and with the peculiarities in the development of the nuclear-cascade avalanche with depth, and depend only indirectly on the character of the collision of the primary particle which has given rise to the extensive air shower with the nucleus of the air atom. The parameters of the nuclear interactions at energies  $E < 10^{12}$  are being investigated at the present time in much more direct measurements, so that the analysis of the data on the lateral distribution of the muons, aimed at making more precise the value of the transverse momentum of the particles produced in the interactions, which was the practice a few years back, is of no particular interest.

It is advantageous to dwell here on the following characteristics of muons in extensive air showers:

a) Lateral distribution; b) energy spectrum; c) total number of muons in showers with different numbers of electrons at the measurement level; d) fluctuations in the relative number of muons at the observational level, and e) muon beams.

The first three problems are wholly connected with the overall picture of the development of the nuclear-cascade shower in the atmosphere. Interest in this problem is connected with the hope of obtaining information on the composition of the primary cosmic radiation in the energy region  $E_0 \geq 10^{16}$  eV. It is assumed that showers due to the heavy primary nuclei develop intensely in the upper layers of the atmosphere and are therefore relatively richer in muons. Finally, the last problem relative to beams of muons in extensive air showers is usually related with new phenomena occurring in collisions of particles with energy  $E_0 > 10^{15}$  eV.

a) Lateral distribution of muons. The first information on the lateral distribution was obtained by investigating the dependence of the number of coincidences produced by the muons on the distance between the registering devices. The measurements have shown that the muons have a broader distribution in extensive air showers than the electrons. [48]

By using comprehensive arrays it became possible to investigate the lateral distribution of the muons much more completely. Figure 14 shows the data on the lateral distribution of muons in extensive air showers with a total number of particles  $10^5$  and  $7.7 \times 10^5$  at an altitude of 3860 meters above sea level. Muons with energy  $E_\mu \geq 440$  MeV were registered. [49] Data [60] for large showers at the same altitude were obtained for muons with an energy  $E_\mu \geq 220$  MeV. Data on the lateral distribution of muons at sea level are quite abundant.

Figure 15 shows the results of only the most complete investigations. [53,60,71] The accuracy attained in

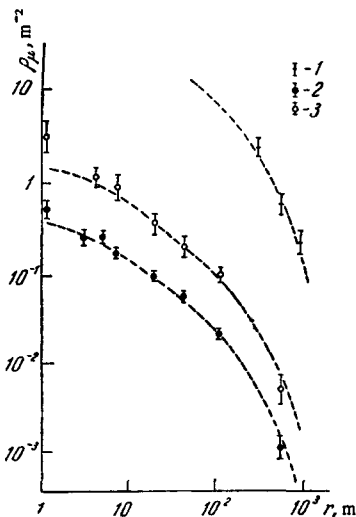
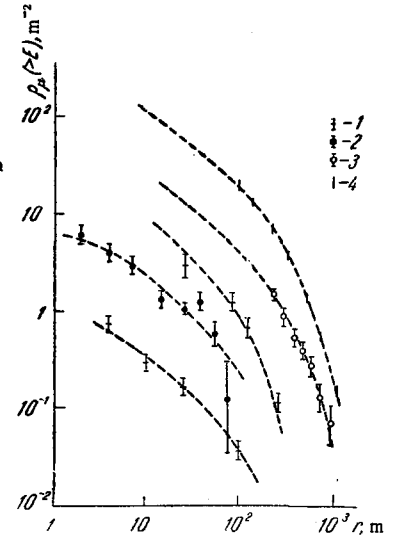


FIG. 14. Lateral distribution of flux of muons at an altitude of 3860 meters. 1 - data of [50],  $N = 6 \times 10^7$ ; 2 and 3 - data of [49] for  $N = 10^5$  and  $N = 7.7 \times 10^5$ , respectively. The dashed lines show the distribution in accordance with formula (3.1).

FIG. 15. Lateral distribution of muon flux near sea level. The dashed lines show the distribution in accordance with formula (3.1). 1 - data of [53],  $E_\mu \geq 5$  BeV,  $N = 2 \times 10^5$  and  $N = 6 \times 10^6$ ; 2 - results of [61],  $E_\mu \geq 5$  BeV,  $N = 10^6$ ; 3 - data of [71],  $N = 2 \times 10^7$ ; 4 - data of [60],  $N = 10^8$ .



most of the investigations has so far not disclosed any dependence of the type of lateral distribution on the total number of particles in the shower or on the observation height. To illustrate this fact, the experimental data are compared with a series of similar curves of the type

$$\rho_\mu(r) = A(N, E_\mu) (r+2)^{-0.7} e^{-r/r_0(E_\mu)}. \quad (3.1)$$

Here  $r$  is the distance from the shower axis in meters, and the meaning of  $A(N, E_\mu)$  will be considered later. The values for  $r_0(E_\mu)$  were chosen from those fitting best the results of the investigation of the energy spectrum and the lateral distribution of the muons in extensive air showers at sea level [51] and at the Pamir altitude [52]

$$r_0(E_\mu \geq 440 \text{ MeV}) = 300 \text{ m}, \quad r_0(E_\mu \geq 1 \text{ BeV}) = 220 \text{ m}, \\ r_0(E_\mu \geq 5 \text{ BeV}) = 100 \text{ m}.$$

The form of the approximation formula is interesting because in most experimental investigations the measurements were carried out at relatively small distances from the axis, where the muon flux density decreases more slowly with distance than  $\sim r^{-2}$ . It is possible to recalculate such data to the total number of muons in the shower, only on the basis of some a priori data concerning the form of the lateral distribution of the muons. In various works, various approximations were used. The simplest expression for the lateral distribution is the function  $\rho_\mu(r) \sim 1/r^n$ , where  $n$  is determined by experiment and is in turn a function of the distance (and of the muon energy). However, if we take the value of  $n$  observed at distances  $\sim 500$  meters from the shower axis and extrapolate this value to large distances, the integration will yield too high a number of muons.

In many investigations [53,54] the factor determining the lateral distribution function of the muons at large distances is the Gaussian function  $f(r) \sim r^\alpha \exp(-r^2/a)$ . Such a form of the lateral distribution

can be expected if multiple Coulomb scattering of the muons is regarded to be the principal cause of the deflection of the muons at large distances from the axis. In [51], on the basis of a detailed analysis of the experimental data over a wide range of distances, the following approximation formula was derived for the lateral distribution of muons of various energies:

$$Q_{\mu}(N, r, E_{\mu}) = 14.4 \left( \frac{N}{10^6} \right)^{0.75} r^{-0.75} \left( 1 + \frac{r}{320} \right)^{-2.5} \times \left( \frac{3}{E_{\mu} + 2} \right)^{0.14 r^{0.375}} \frac{51}{E_{\mu} + 50}. \quad (3.2)$$

Here  $\rho_{\mu}$  is the flux density of muons with energy  $\geq E_{\mu}$  at a distance  $r_{\mu}$  from the axis in an extensive air shower with a total number of  $N$  charged particles.

Formula (3.2) is based on direct experimental data and is applicable to the distance interval  $20 \leq r \leq 500$  meters at muon energies  $1 \leq E_{\mu} \leq 20$  BeV. Within the same limits it agrees well with relation (3.1).

A comparison of the influence of the Coulomb scattering and the deflection of the muons by the earth's magnetic field with the actually observed muon distribution in extensive air showers was carried out in [49, 50]. If we choose muons with energy  $E = 3.5$ – $4$  BeV, then the observed mean-square radius amounts to  $\bar{R} \sim 300$  meters. The Coulomb scattering of mesons produced even at an altitude of 10 kilometers leads to a mean-square radius  $\sim 50$  meters. Deflection by the earth's magnetic field yields a value of  $\sim 50$  meters, observed only in the east-west direction. Both estimates neglect the ionization losses, so that the deflections away from the axis are exaggerated, and nevertheless the total deflection due to the foregoing factors is much smaller than that experimentally observed. It is seen therefore that the third cause of the deflection, namely the transverse momenta acquired by the pions during the generation acts play a decisive role in the deflection of the muons from the shower axis.

Let us estimate the lateral distribution of the muon flux corresponding to the distribution of the transverse momenta of the secondary  $\pi^{\pm}$  mesons in the nuclear interactions. Since the energy of most particles produced in a nuclear interaction is much less than the energy of the particle causing this interaction, and the transverse momentum is apparently independent of the energy of the colliding particles, we can confine ourselves to an account of the deflections from the shower core only in the last act of the nuclear collision. The distribution of the transverse momenta has been well investigated in experiments with emulsions and cloud chambers. It can be represented in the form

$$W(p_{\perp}) dp_{\perp} \sim p_{\perp}^2 e^{-p_{\perp}^2/a} dp_{\perp}, \quad a = 0.105 \text{ BeV}/c.$$

It is seen from Fig. 16 that such a distribution is in satisfactory agreement with the experimental data. [55] Both the most probable value  $p_{\perp} = 0.21$  BeV/c, and the average value  $\bar{p}_{\perp} = 0.32$  BeV/c are in good agreement with many emulsion measurements.

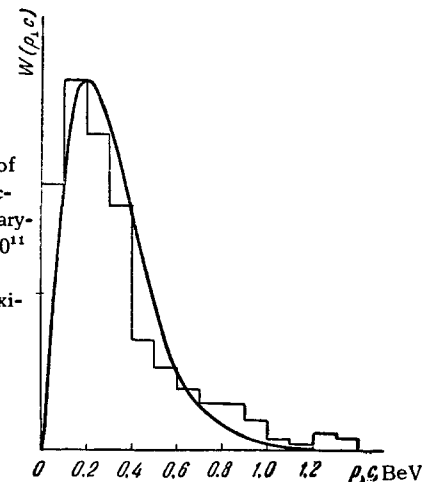


FIG. 16. Distribution of transverse momenta of secondary particles at a primary-particle energy  $E_0 = 3 \times 10^{11}$  eV. Histogram – data from [55]; smooth curve – approximation given in the text.

From the transverse-momentum distribution it is easy to obtain the lateral distribution function of the muons with an energy above a specified value  $E_m$ , by making the following assumptions: 1) the energy spectrum of the particles in the energy interval of interest to us has the form  $f(E) dE \sim E^{-2} dE$ ; 2) the distribution of the heights of muon production can be replaced by an average height  $h_{\text{eff}}$ ; 3) all particles are formed near the shower axis.

Under these assumptions, and putting for simplicity  $r/h_{\text{eff}} \ll 1$  and  $cp_{\perp}/E \ll 1$ , the lateral distribution function of the muon flux in an extensive air shower assumes the form

$$f(r, > E_m) \sim \frac{dr}{r} \left( \frac{E_m r}{ah} + 1 \right) e^{-E_m r/ah}. \quad (3.3)$$

This expression coincides in the form and in the values of the parameters with the experimentally observed distribution (3.1).

b) Energy spectrum of muons in extensive air showers. Data on the energy spectrum of muons in extensive air showers at mountain altitudes have been confined so far to the work reported in [52]. In these measurements a comprehensive array with subsequent selection by absorption in lead and in the ground was used to determine the energy spectrum of the muons in extensive air showers with various numbers of particles at different distances from the axis. A relatively small muon energy interval, from 300 MeV to 3.5 BeV, was investigated. The results are listed in Table II, in the form of values of  $\alpha$ , the exponent in the integral energy spectrum of the muons  $\rho(r, > E) \sim E^{-\alpha(r)}$ .

Taking into account the lateral distribution of the muons, we can recalculate the data given in Table II to fit the muon spectrum in the interval 0.3–3.5 BeV in the extensive air shower as a whole at the observation level. For this purpose, however, it is necessary to extrapolate the function  $\alpha(r)$  for values of  $r$  from 500 to 1000 meters. Because of the arbitrariness of such an operation, we can only state that the energy spectrum of the muons in the entire shower, in the

Table II

$\bar{r}, m \backslash N$	$1.6 \cdot 10^6$	$8 \cdot 10^5$	$4 \cdot 10^5$	$5 \cdot 10^4$	$5 \cdot 10^4$
4	$0.24 \pm 0.02$	$0.24 \pm 0.02$	$0.27 \pm 0.12$	$0.22 \pm 0.1$	$0.14 \pm 0.17$
100	—	$0.32 \pm 0.03$	—	—	—
300	$0.8 \pm 0.2$	$0.8 \pm 0.2$	$0.86 \pm 0.18$	—	—

energy interval 0.3–3.5 BeV, is close to  $N_\mu (> E_\mu) \sim E_\mu^{-1 \pm 0.2}$ . Particular attention was paid in [52] to the number of muons with energy  $E_\mu < 0.3$  BeV. Their number turned out to be small, and the energy  $E_\mu \approx 0.2$  BeV can be regarded as the lower limit of the muon spectrum at the observation level.

Several investigations were made at sea level to determine the energy spectrum of the muons in extensive air showers. In most investigations the energy of the muons was determined from their penetrating ability [56–58]; a magnetic spectrometer was used in [51]. The results of the last investigation were expressed in the form of an approximation formula, which is valid in the distance interval from 25 to 500 meters from the shower axis:

$$Q_\mu(r, > E_\mu) = g(N, r) \left[ \frac{3}{E_\mu + 2} \right]^{0.14r^{0.375}} \frac{51}{E_\mu + 50}$$

(the energy is in units of  $10^9$  eV).

For comparison with the results obtained by others, the experimental data of [51] are best represented in the form

$$Q_\mu(> E_\mu, r) \sim \frac{1}{E_\mu^{\alpha(r)}}$$

The statistical errors of the experiment in [51] permit one to note some deviations from the purely power-law expression only for average distances  $r = 25$  meters from the shower axis. The comparison is made in Table III.

Whereas Table II was a good illustration that the energy spectrum of the low-energy muons does not depend on the number of particles in the shower, Table III shows that the muon spectrum remains essentially unchanged with variation of the observation height of extensive air showers.

c) Dependence of the number of muons on the total number of particles in an extensive air shower at the

Table III

	Energy interval, BeV	25 m	80–125 m	250–475 m
Reference 51	1–20	$0.3 \pm 0.03$	$0.4 \pm 0.1$	$0.7 \pm 0.5$
Reference 56	5–10	$0.5 \pm 0.1$	$0.67 \pm 0.15$	$1.8 \pm 0.9$
Reference 52	0.3–3.5	$0.28 \pm 0.03$	$0.32 \pm 0.03$	$0.8 \pm 0.2$
Reference 57	0.5–30	$0.65 \pm 0.05^*$	—	—

In [57] neither the axis position nor the number of particles in the individual showers were determined.

observation level. The determination of the fraction of muons relative to the number of electrons in extensive air showers, caused by primary particles of different energy, is closely connected with the dependence of the muon lateral distribution function on the total number of particles. Inasmuch as this function can be regarded as independent of the total number of particles, an investigation of the dependence of the relative number of muons on the shower size reduces to measurements of the relative fraction of muons at a specified distance from the shower axis. Such measurements were started already by Cocconi [59] who varied the areas of the counters connected in the coincidence circuit. In this series of measurements it was established that the relative number of muons decreases with increasing size of the shower.

Subsequent measurements, carried out with better methods both at mountain altitudes [49, 50, 60] and at sea level [53, 61, 62] have confirmed that the fraction of muons decreases with increasing number of electrons in the shower. Although the authors of most of these references reach the conclusion that the fraction of muons decreases with increasing number of electrons in the shower in proportion to  $\sim N^{-0.25}$ , the aggregate of all the data obtained (Fig. 17) can be well reconciled with a single dependence  $N_\mu/N \sim N^{-0.19 \pm 0.03}$  for the entire interval of shower particle numbers  $10^4 \leq N \leq 10^9$ . Moreover, an extrapolation of this dependence to limitingly small showers ( $N \approx 10$ ) does not lead to absurd results. Such a dependence of the relative number of muons on the total number of particles in the shower is in good agreement with the theoretical calculations. [63]

Summarizing the data concerning the flux of muons in extensive air showers, we can express the flux density of muons with energy  $E_\mu \geq 2 \times 10^8$  eV at a distance of  $r$  meters from the axis of a shower with a total particle number  $10^3 < N < 10^8$  at a measurement level with pressure  $p \geq 500$  g/cm<sup>2</sup> in the form

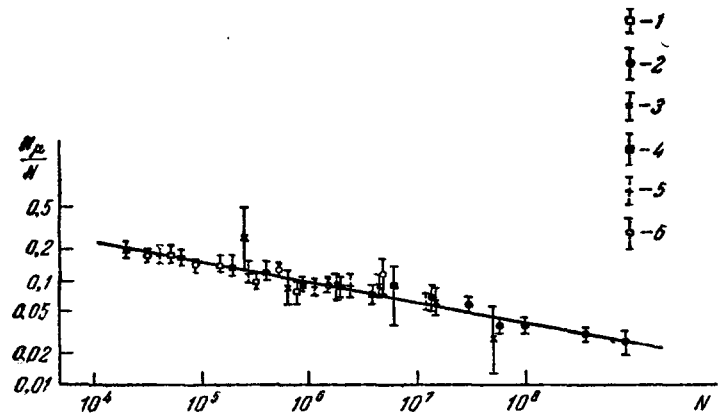
$$Q_\mu(r, E_\mu, N, p) = \frac{0.22p}{[2 \cdot 10^6(p-500) + E_\mu]^{0.25}} \times \left( \frac{N}{10^5} \right)^{-0.81} (2+r)^{-0.75} e^{-r/r_0},$$

where  $r_0 = 300$  meters for muons with energy  $E_\mu \geq 4.4 \times 10^8$  eV,  $r_0 = 220$  meters for  $E_\mu \geq 10^9$  eV, and  $r_0 = 100$  meters for  $E_\mu \geq 5 \times 10^9$  eV.

Thus, at mountain altitudes ( $p = 650$  g/cm<sup>2</sup>) extensive air showers with a total number of charged particles  $N = 10^5$  contain approximately 7300 muons with energy  $E_\mu \geq 4.4 \times 10^8$  eV. Accordingly, at sea level, in showers with the same number of charged particles the number of muons at the observation level is  $\sim 12,500$ .

d) Fluctuations in the relative number of muons at the measurement level. The difference in the paths of accumulation of electrons and muons in an extensive air shower and the relatively larger range of the muons

FIG. 17. The relative number of muons in extensive air showers with different total number of particles as given by various investigators: 1-[49]; 2-[60]; 3-[62]; 4-[61]; 5-[53]; 6-[59].



give grounds for hoping that a study of the fluctuations in the relative number of muons in the shower will yield information concerning the fluctuations in the development of showers in the atmosphere, and concerning the role of multiply-charged nuclei in the formation of extensive air showers. The simultaneous influence of both factors on the muon flux fluctuations is not subject to any doubt. Therefore a quantitative solution of the problem is possible only in conjunction with other data on the fluctuations in the development of showers deep in the atmosphere or on the composition of the primary cosmic radiation in the corresponding energy interval. Up to now, the question considered in greatest detail was that of fluctuations of the muon flux resulting from the difference in the effective cross section for the nuclear interaction between protons and the heavier nuclei of the primary cosmic radiation. [56,61] The main assumption of such calculations consisted in the fact that the muon flux at the measurement level is proportional to the total energy lost by the primary particle in the atmosphere, while the number of electrons depends on the height at which the showers are generated. For an experimental determination of the number of muons in an individual extensive air shower it is necessary to have a large number of detectors to measure the lateral distribution of the muons in the individual shower. The energy of the registered muons should be sufficiently high to enable one to neglect their absorption in the atmosphere ( $E_\mu \geq 2 \times 10^9$

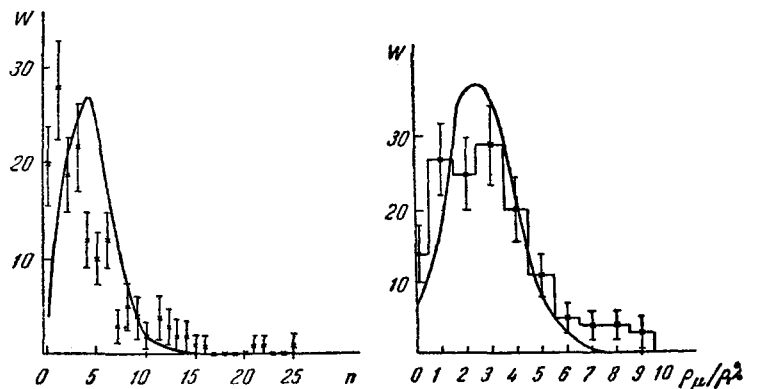
eV). In the work performed to date [53,61,64] the area and number of the detectors were insufficient for such measurements. In the experiments of [53,61] (Fig. 18), the fluctuations of the muon flux density at a definite distance were investigated in showers with a given number of particles. In interpreting the experimental data, the authors assumed that there were no fluctuations in the lateral distribution of the muons, and considered the observed distribution of the relative muon flux density to be a reflection of the fluctuations of the total number of muons in the shower. The arbitrariness of such an assumption follows also from the experimentally observed fluctuations in the form of the muon-flux lateral distribution function in the shower (Fig. 19) and from the relation between the lateral distribution of the muons and the distribution of the transverse momenta in the elementary nuclear-interaction events [relations (3.1) and (3.3)].

Comparing the parameters of the exponentials in these relations, we can obtain an estimate of the average height of observation of the muons with energy  $> E_\mu$ :

$$h_{\text{eff}} \approx \frac{r_0(> E_\mu) E_m}{a}$$

Here  $E_m > E_\mu$  owing to the ionization energy losses of the muons along the path from the point of generation to the observation level. For muons with energy  $E_\mu \lesssim 10^9$  this must be taken into account. The average

FIG. 18. Left - comparison of the expected frequency of registration of  $n$  muons (curve) with the observed frequency [53] for  $E \geq 10$  BeV,  $N = (2-4) \times 10^6$ . Right - comparison of the expected and observed [61] relative flux density of the muons with  $E \geq 5$  BeV, in percent, for showers with  $5 \times 10^5 \leq N \leq 3 \times 10^6$ .



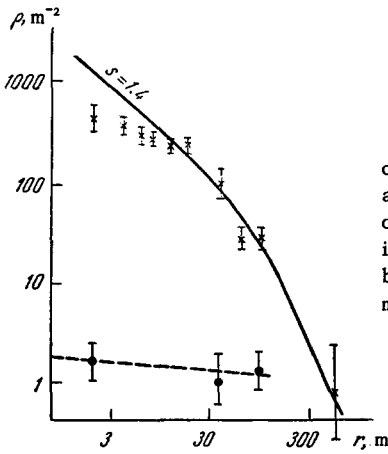


FIG. 19. Rather rare case of registration of an extensive air shower which differs radically from the average shower in the lateral distribution of both the electrons (x) and the muons (●).

height of muon production is found to be  $h_{\text{eff}} = 3$  km in the case when  $E_{\mu} \geq 440$  MeV and  $h_{\text{eff}} \approx 3.2$  km for  $E_{\mu} \geq 1$  BeV. These are the same heights above the observation level, at which the electron-photon component of the shower is effectively produced in the case of fluctuations in the development of the shower over the depth of the atmosphere. By the same token, by studying the fluctuations of the number of muons of such low energies, it is possible to speak with greater justification of the fluctuations in the energy transferred to the  $\pi^{\pm}$  and  $\pi^0$  mesons, than of the composition of the primary cosmic radiation producing the observed extensive showers.

The average height of formation of the muons increases with increasing energy of the registered muons: for muons with  $E_{\mu} > 5$  BeV we have  $h_{\text{eff}} \approx 5$  km above the observation level, while for  $E_{\mu} > 10$  BeV we have  $h_{\text{eff}} \approx 7$  km. It must be borne in mind, however, that the estimates made pertain to observations of muons at distances from the shower axis which give the greatest contribution to the number of muons with energy above that specified at the observation height. In this case one can disregard the fluctuations in the muon-flux lateral distribution function. In most measurements performed to date, the fluctuations of the muon flux density were investigated at distances  $r < r_0$ , where the muons produced at heights  $h < h_{\text{eff}}$  are more effectively registered, and apparently one cannot neglect the fluctuations in the form of the lateral distribution function.\*

e) Narrow groups of muons in extensive air showers. The first observations of narrow groups of muons in the cores of extensive air showers were made at a depth of 1600 meters water equivalent.<sup>[58]</sup> The frequency of such groups was  $\sim 2 \times 10^{-3} \text{ m}^{-2} \text{ h}^{-1}$ . Subsequent analysis has shown that it is difficult to explain the narrow muon groups without making additional assumptions concerning the processes whereby they are produced at high energies.<sup>[65]</sup> In<sup>[66,67]</sup> muon groups

were observed in cloud chambers underground. The greatest frequency of such muon beams was registered in<sup>[67]</sup> ( $\sim 6 \times 10^{-2} \text{ m}^{-2} \text{ h}^{-1}$ ). Later on, however, the same group of authors has shown that the muon groups observed by them can be attributed to statistical fluctuations in the muon flux density in extensive air showers.<sup>[68]</sup> The statistical fluctuations in the distribution of the muon flux density in extensive air showers were analyzed more thoroughly in<sup>[56]</sup>, where the position of the axis and the total number of particles in the extensive air showers, accompanying the groups of muons, and also the muon flux density near the observed muon group were determined. The reduction of the experimental data has shown that the muon groups observed away from the shower axis can be attributed entirely to lateral fluctuations in the muon flux density. On the other hand, in the cores of extensive air showers, the fluctuations of the lateral density distribution of the particle flux give a much lower frequency for the narrow muon groups than is experimentally observed (Fig. 20). The absolute intensity of such groups is estimated at  $3 \times 10^{-3} \text{ m}^{-1} \text{ h}^{-1}$ . These data which are beyond reproach with respect to the statistical analysis are not corroborated by a clear cut confirmation with the aid of a cloud chamber. The urgency of proving that the narrow muon groups actually exist is brought about by the fact that to explain the narrow groups it is necessary to resort to additional assumptions concerning the character of the particle collision processes at ultrahigh energy ( $E_0 > 10^{14}$  eV) up to the possibility of direct multiple production of muons in the collision of particles of such energy. The main difficulty in reconciling the probability of appearance of narrow groups of muons with the existing notions concerning the character of the nuclear interaction event lies in the fact that owing to the relatively long lifetime of the  $\pi^{\pm}$  and K mesons it is essential that several dozens of particles with transverse momenta

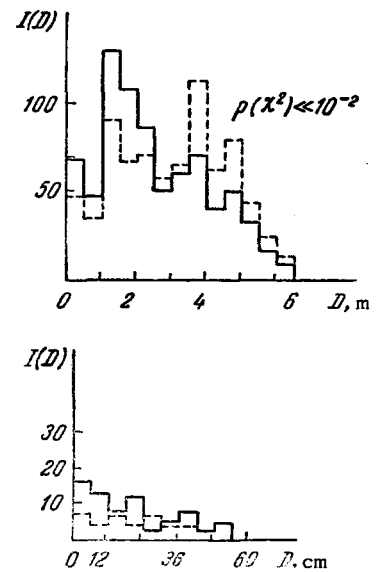


FIG. 20. Comparison of the calculated number of appearances of discharges in counters (dashed lines) a distance D apart with the experimentally observed number. The selected events were those when the shower axis passed near the hodoscopic counters.

\*This remark pertains also to reference<sup>[50]</sup>, where it was concluded from an analysis of the fluctuations that at  $E_0 > 10^{18}$  eV the primary radiation consists predominantly of protons.



$p_{\perp} < 3 \times 10^7$  eV/c be produced in a single event. The latter quantity must be compared with the average value of the transverse momentum of the secondary particles in nuclear interaction events,  $\bar{p}_{\perp} \approx 3 \times 10^8$  eV/c. The improbability of such an assumption could be lessened by assuming that pion groups with such small transverse momenta appear in some rather rare cases of collision of particles with energy  $E_0 < 10^{15}$  eV. But then the observation of muon groups unaccompanied by extensive air showers would be quite probable. Yet experiments show<sup>[56]</sup> that muon groups are encountered only when accompanied by extensive air showers with a total number  $N > 10^4$  particles at the observation level, and the probability of observing a group of muons in the interval  $10^4 < N < 10^6$  is proportional to  $W \sim N^{1.5}$ . Further detailed investigations of this phenomenon are necessary, and primarily measurements of the energy of the individual muons contained in the group. Some methods of estimating this energy, in particular the lack of noticeable absorption of such groups in thick layers of ground (1600 m w.e.) indicate that their energy is quite high ( $\sim 10^{12}$  eV).

#### 4. NUCLEAR-ACTIVE COMPONENT OF EXTENSIVE AIR SHOWERS

The detection of nuclear-active particles in the composition of extensive air showers has served in its time<sup>[69]</sup> as one of the arguments in favor of the nuclear-cascade nature of extensive air showers. At the present time there is no doubt concerning the major role of nuclear-active particles both at the initial stages of the avalanche development, and deep in the atmosphere. We shall consider here two problems: the dependence of the number of nuclear-active particles of all energies on the total number of particles in the shower, and the energy spectrum of the nuclear-active particles.

1) Number of nuclear-active particles in extensive air showers with different numbers of electrons. A determination of the total number of nuclear-active particles in a shower is possible only after an investigation of the lateral distribution of the nuclear-active component of the shower. For particles with energy  $E_{\text{nuc}} \geq 3 \times 10^9$  eV, such measurements have been made both at mountain altitudes<sup>[70]</sup> and at sea level<sup>[71]</sup>. The accuracy of these and of the subsequent measurements<sup>[72,73]</sup> makes it impossible to discern any dependence whatever in the form of the lateral distribution on the height of the point of observation or on the size of the shower. The distributions are analogous to the previously introduced expression (3.3):

$$Q_{\text{nuc}}(r) = Dr^{-n} e^{-r/r_0}. \quad (4.1)$$

In measurements<sup>[70]</sup> made at an altitude of 3860 meters above sea level, on the basis of the experimentally determined lateral distribution, a dependence was obtained for the number of nuclear-active particles on

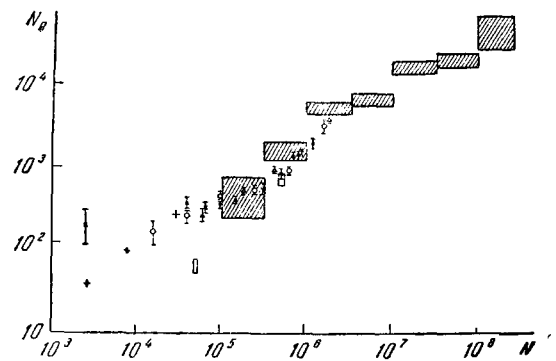


FIG. 21. Total number of nuclear-active particles in extensive air showers at the observation level, as measured by various authors. \* - Measurements of<sup>[70]</sup>;  $\circ$  - measurements of<sup>[72]</sup>; + - measurements of<sup>[27]</sup>;  $\Delta$  - measurements of<sup>[74]</sup>;  $\square$  - data of<sup>[71]</sup>;  $\text{\textbackslash}$  - reference<sup>[73]</sup>.

the total number of particles in the shower. The dependence obtained is characterized by the fact that the relative number of nuclear-active particles in showers with a total number of particles  $10^5 - 10^6$  is considerably lower than in showers with  $N < 5 \times 10^4$  particles, and this variation had a nonmonotonic character.

Figure 21 shows the results of measurements made by different workers. The data of<sup>[72]</sup>, obtained at sea level with the aid of neutron counters, have been recalculated in accordance with the lateral distribution (4.1). This makes it possible to compare the number of nuclear-active particles in a circle of identical effective radius. The experimental data of<sup>[74]</sup>, obtained with analogous recording apparatus were normalized by the authors of<sup>[72]</sup> to their own data. The results of the measurements at sea level with hodoscopic detectors of nuclear-active particles, given in<sup>[47]</sup> for a circle 22 meters in radius, were also extended to cover a larger distance in accordance with the distribution (4.1).

The work reported in<sup>[27]</sup> was carried out with neutron counters in conjunction with an array with a rigid selection criterion at an altitude 3333 meters above sea level. Finally, the data of<sup>[73]</sup>, obtained at 2740 meters with a comprehensive array for the study of extensive air showers, were normalized to the remaining data by a more complicated method. Reference<sup>[73]</sup> gives the dependence of the number of nuclear-active particles with energy  $> 2 \times 10^8$  eV on the total number of particles in the shower. The character of this dependence is completely duplicated in Fig. 21. However, the summary number of nuclear-active particles, laid off along the ordinate axis in<sup>[73]</sup>, is much smaller than in Fig. 21. It is difficult to say whether this is a technical error that crept into the publication or whether this is a result of the too narrow lateral distribution of the nuclear-active shower component assumed by the authors; however, the absolute values of the flux of particles with energy  $E_{\text{nuc}} \geq 2 \times 10^8$  eV contradicts the form of the energy spectrum and the

number of nuclear-active particles with energy  $E_{\text{nuc}} \geq 10^{11}$  eV, obtained by the authors of [73] in the same series of measurements. We have used data on the number of nuclear-active particles with energy  $E_{\text{nuc}} \geq 10^{11}$  eV for the normalization along the ordinate axis.

The aggregate of the data is in good internal agreement, with the exception of two points. The point at  $N \approx 2.5 \times 10^3$  can be disregarded not only because subsequent measurement [27] did not confirm it, but also because the selection of showers with such a number of particles was carried out in [70] only under the condition that several penetrating particles were located in the immediate vicinity ( $\lesssim 1$  meter) of the hodoscopic detector of the nuclear-active particles. This has indeed led to an appreciable increase in the relative number of nuclear-active particles in the selected showers. The causes of the underestimate of the number of nuclear-active particles at  $N \sim 5 \times 10^4$  in [47] are not clear, but this aspect is not confirmed by the results of four other investigations, two of which were carried out at the same altitude.

The dependence of the number of nuclear-active particles on the total number of electrons in an extensive air shower at the observation level has, as noted in [70], a nonmonotonic character, even if one combines the experimental data obtained by different methods at different altitudes. The number of nuclear-active particles increases slowly with increasing size of the shower in the interval  $2 \times 10^4 < N < 2 \times 10^5$ , then the increase in the number of nuclear-active particles becomes almost proportional to the increase in the number of electrons in the shower and, finally, in showers with a total of  $N \sim 10^7$  particles the relative number of nuclear-active particles again decreases. The latter, in the opinion of the authors of [73], is due to the fact that the nuclear cascade does not have time to develop to the required extent in the atmosphere above the measurement site. To explain the variation of the dependence at  $N \approx 2 \times 10^5$ , various points of view were advanced, connected: 1) with the assumption that the characteristics of the elementary act of the nuclear interaction change at an energy  $E_0 \approx 3 \times 10^4$  eV, [70, 75] 2) with the change in the composition of the primary cosmic radiation at the same energy, [76] and 3) with the different role of the unstable particles in large and small extensive showers. [77] For a final answer to this question it is necessary to accumulate further experimental data on the structure and composition of the showers in this energy interval of the primary cosmic radiation, and a more detailed analysis of the totality of the information on extensive air showers with a total  $N = 10^4 - 10^6$  particles is required.

The summary number of nuclear-active particles of all energies is determined to a considerable degree by the interaction between particles of moderate energy. However, quantitative estimates of the total number of nuclear-active particles in extensive air showers

are made difficult not so much by the uncertainty in the collision picture at ultrahigh energies, as by the lack of exhaustive data on the composition of the secondary particles that arise in multiple-production events at incident-particle energies  $E_0 < 10^{12}$  eV. In particular, it follows from the analysis of data on extensive air showers with  $N \approx 10^4$  particles, when the pattern of the initial shower-formation events is sufficiently clear, that in the interactions between nucleons or  $\pi^\pm$  mesons and nuclei of atoms of air, at incoming-particle energies  $10^{10} - 10^{12}$  eV, not less than 10% of the secondary particles are nucleons or hyperons. Otherwise it is difficult to explain the observed number of nuclear-active particles in showers with a total number of particles  $N \approx 10^4$ .

b) Energy spectrum of nuclear-active particles in extensive air showers. One of the first investigations in which the relative frequency of registration of nuclear-active particles of different energy in the composition of extensive air showers was determined, were the measurements recorded in [78]. The experiments were carried out at 3860 meters above sea level in a comprehensive array with subsequent selection of the necessary events. The energy of the nuclear-active particles was estimated from the pulses in ionization chambers under thick layers of lead ( $d > 20$  cm). Bursts equivalent to  $600 - 6 \times 10^4$  relativistic particles in the central chord of the chamber were registered. Conversion from ionization to the probable energy of the nuclear-active particles causing bursts of this magnitude strongly depends on the characteristics of the nuclear interactions of the  $\pi^\pm$  mesons and nucleons with the lead nuclei. It can be assumed that these measurements pertain to the energy interval  $10^{11} - 10^{12}$  eV. The energy distribution of the nuclear-active particles turned out to be proportional to  $\sim E_{\text{nuc}}^{-(0.9 \pm 0.2)}$  for showers with  $7 \times 10^4 < N < 7 \times 10^5$  particles.

The data of a series of later measurements are summarized in Fig. 22. The accuracy of the data is still insufficient to detect any dependence of the energy spectrum on the observation altitude. The figure shows the integrated energy distributions of nuclear-active particles for showers with total number of particles  $N = 10^4$  (lower curve),  $N = 10^5$  (middle curve), and  $N = 10^6$  (upper curve). The total number of nuclear-active particles of all energies in showers of the particular size is plotted in accordance with Fig. 21. Investigation [96] was carried out at 3860 meters with the aid of a comprehensive array with subsequent selection. The energy of the nuclear-active particles was measured with the aid of ionization chambers placed under a combination graphite and lead absorber.

The measurements of reference [93] were made at 3200 meters with the aid of a group of ionization chambers under an absorber consisting of several layers of lead and graphite. In the measurements of [81] a "scintillation calorimeter" was used, i.e., a multi-

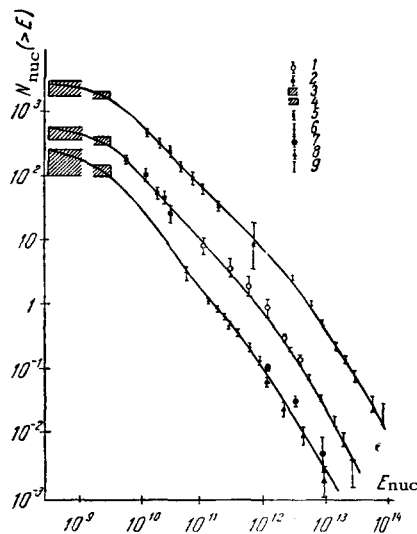


FIG. 22. Energy spectrum of nuclear-active particles in extensive air showers at the observation level. In the construction we used data of different investigations: 1—<sup>[79]</sup>; 2—<sup>[83]</sup>; 3—data of hodoscopic and neutron detectors <sup>[70,72]</sup>; 5—<sup>[73]</sup>; 6—measurements of <sup>[81]</sup>; 7—<sup>[96]</sup>; 8—<sup>[82]</sup>; 9—from work done on the array of <sup>[93]</sup>.

layer system of iron absorbers and scintillators. For showers with  $10^3$  to  $10^6$  particles in the nuclear-active particle energy interval  $10^{11}$ – $10^{12}$  eV at sea level, the spectrum can be represented in the form

$$F(>E_{\text{nuc}}) \sim \frac{1}{E_{\text{nuc}}^{1 \pm 0.1}}$$

The data were normalized to the results of <sup>[79]</sup>, since the absolute intensity given by the authors exceeds by more than five times the data of another investigation, carried out at sea level with the aid of ionization chambers. <sup>[82]</sup> Data on the energy spectrum of the nuclear-active particles in the interval  $10^{10}$ – $5 \times 10^{11}$  eV for showers with  $N = 10^6$  particles were obtained with the aid of a cloud chamber containing a large number of lead plates. These data <sup>[83]</sup> characterize well the form of the spectrum in this section, but the article gives the intensity in relative units. Therefore the position of this portion of the spectrum is set in correspondence along the ordinate axis with an interpolation between the results of <sup>[70]</sup> and <sup>[93]</sup>. Thus, seven out of nine different investigations yield results that are in good agreement without arbitrary recalculations. In general outline, the form of the energy spectrum does not depend on the number of particles in the shower, while for showers with  $N \approx 10^4$  particles the dependence of the number of nuclear-active particles on the energy, of the type  $F(>E) \sim 1/E$ , is retained up to an energy higher than the energy of the entire electron-photon component of the shower above the observation level.

Finally, a few words concerning the composition of the nuclear-active component of extensive air showers. Observation of charged and neutral particles could help

estimate the fraction of the  $\pi^\pm$  mesons among all the nuclear-active shower particles. Such investigations have been undertaken repeatedly. In the observation of nuclear-active low-energy particles ( $E \lesssim 10^{10}$  eV) the numbers of neutral and charged particles are approximately equal to  $n^0(\pi^0 + \pi^\pm) = 0.41 \pm 0.08$ . <sup>[84]</sup> Observation of particles with higher energies changes this ratio in favor of a predominance of charged particles, but owing to the closeness of the shower core and the multiplicity of the secondary particles the observation conditions become much worse. In the experiments of <sup>[73]</sup> the value obtained was  $n^0(\pi^0 + \pi^\pm) = \frac{38}{147} = 0.26 \pm 0.05$ , if we neglect 84 indeterminate events.

## 5. ENERGY FLUX CARRIED BY DIFFERENT COMPONENTS OF EXTENSIVE AIR SHOWERS

Measurement of the energy flux carried by different components of a shower is of interest both for a determination of the entire pattern of shower production and development, and for a determination of the energy of the primary inducing the shower without any arbitrary assumptions concerning the characteristics of the collisions of ultrahigh energy particles. If the purpose is to determine the energy of the primary particle giving rise to a shower with a given number of particles at the measurement level, it is necessary to measure the energy of the electron-photon component and of the nuclear-active particles at the observation level, the energy lost by the shower particles to ionization in the atmosphere up to the observation level, the energy carried by the muons below the measurement level, and finally the energy transferred to the neutrinos in the atmosphere. The energy given up to the neutrinos cannot be determined experimentally. It can be stated, however, that it does not exceed the energy obtained by the muons. There are also other energy losses, which escape quantitative observation wholly or in part, but their contribution to the total energy balance is negligible.

a) Energy of electron-photon shower component at the measurement level. The most direct method of determining the energy of the electron-photon component of the shower at the measurement level is to integrate over the energy spectrum of the electron-photon component. As was shown earlier, the energy spectrum in the shower has been investigated experimentally within the relatively narrow region of  $10^9$ – $10^{11}$  eV, while the rather appreciable portion of the spectrum from  $10^7$  to  $10^9$  eV has been poorly investigated. This leads to a serious uncertainty in the value of the total energy flux carried by the electron-photon component of the shower. The energy spectrum of the electrons and photons with energy  $> 10^9$  eV can be represented, on the basis of experiments of <sup>[35]</sup>, by  $F(>E) = 2000(E/10^9)^{-(1.5 \pm 0.1)}$ . The spectrum is normalized to a shower with  $N = 10^5$  particles. The upper limit of this spectrum does not

influence the total energy to the same extent as the character of the extrapolation to the critical electron energy in air. If we assume that the exponent of the spectrum remains the same up to  $E = 250$  MeV and that the electrons and photons have a so-called equilibrium spectrum,<sup>[85]</sup> then the total photon and electron energy is  $\mathcal{E} \geq 2.5 \times 10^{13}$  eV. If the equilibrium spectrum of the electrons and the photons is normalized to that experimentally observed at  $E = 10^9$ , then the total energy of the electron-photon component turns out to be  $\mathcal{E} = 1.7 \times 10^{13}$  eV. Upon extrapolating the observed energy spectrum from energies  $E \geq 2.5 \times 10^9$  eV to  $E > 0$ , no other normalization condition was satisfied—the total number of electrons was  $N = 10^5$ . The first extrapolation leads to  $N = 1.2 \times 10^5$ , and the second to  $N = 0.75 \times 10^5$ . Thus, the discrepancy at various extrapolations of the energy spectrum is small and the actual value of the energy of the electron-photon component amounts to  $\mathcal{E} = (2.1 \pm 0.3) \times 10^{13}$  eV for extensive showers with a total of  $N = 10^5$  particles. What are the grounds for confidence in this quantity, when 70% of the value obtained for the energy flux is connected with the extrapolated portion of the spectrum? There are two reasons for it. First, in addition to experimental data on the number of electrons and photons with energy  $E > 10^9$  eV, use was made of the experimentally observed total number of electrons in the shower ( $N = 10^5$ ). More accurately speaking, the spectrum is interpolated. Second, on the interpolated portion of the spectrum the value of the cascade parameter  $s$  changes from  $s = 1.5$  to  $s = 1$ . The lateral distribution of fluxes of electrons having approximately the same energies corresponds in the mean to a parameter  $s = 1.2-1.3$ .

This estimate of the energy of the electron-photon component is valid for both sea level and mountain altitudes, since the experimental data concerning the relative number of electrons and photons with energy  $E > 10^9$  eV at different altitudes are in agreement. There is a certain tendency towards a reduction in the relative number of high-energy electrons and photons with increasing total number of particles in the shower at the observation level.<sup>[34]</sup> However, the accuracy of the data does not permit any quantitative statements to be made concerning the dependence of the summary energy of the electron-photon component on the number of particles in the shower in the interval  $8 \times 10^3 \leq N \leq 10^6$ .

More direct measurements of the energy of the electron-photon component of the shower were obtained by observing the transition curves of absorption of the cascade in matter with large atomic number. Such data were obtained at mountain altitudes (650 g/cm<sup>2</sup><sup>[86]</sup> and 740 g/cm<sup>2</sup><sup>[87]</sup>) and at sea level.<sup>[88]</sup> The results are summarized in Table IV.

The data in the last line of the table lie in the range  $1.8 \times 10^8 N \leq \mathcal{E} \leq 2.3 \times 10^8 N$  eV, which is in good agreement with the previously obtained value of the

Table IV

	Altitude 3860 m (650 g/cm <sup>2</sup> )		Altitude 2740 m (740 g/cm <sup>2</sup> )	Sea level
	$N=10^5$	$N=3 \cdot 10^5$	$N=4 \cdot 10^5$	$N=10^5$
In circle with $r=2$ m	$2.4 \cdot 10^{12}$ eV	—	$9 \cdot 10^{12}$ eV	$1.8 \cdot 10^{12}$ eV
In circle with $r=30$ m	$1.6 \cdot 10^{13}$ eV	$4.5 \cdot 10^{13}$ eV	—	—
Over the entire shower	$2.3 \cdot 10^{13}$ eV	$6.6 \cdot 10^{13}$ eV	—	$1.8 \cdot 10^{13}$ eV

total energy flux of the electron-photon component at the observation level,  $\mathcal{E} = 2.1 \times 10^8 N$  eV.

b) Energy lost by the shower particles in the atmosphere above the observation level. Measurements of this energy became possible only owing to the Cerenkov glow of shower particles in the atmosphere. The first observations of such flashes were carried out at sea level,<sup>[89]</sup> but this phenomenon was used consistently to study extensive air showers only in<sup>[90,91]</sup>. In these experiments the lateral distribution of the intensity of the Cerenkov radiation was measured in extensive air showers with different numbers of charged particles at the observation level (Fig. 23). Inasmuch as the absorption of the Cerenkov glow in the atmosphere can be neglected ( $< 10\%$ ), it is possible to obtain the total value of the ionization lost by the particles whose velocity exceeds the velocity of light in the atmosphere by integrating the lateral distribution over all distances. For electrons which make the overwhelming contribution to the ionization of the atmosphere during the passage of the shower, this corresponds to a threshold energy of 20–100 MeV, depending on the density of the air. To take into account the energy lost to ionization by electrons with energy less than the threshold value, it is sufficient to assume that the absorption of such electrons corresponds completely to the electron-photon cascade theory, and that the development of the shower with depth of the atmosphere is known (although with obvious approximation). In observations of showers with a total of  $10^5-10^6$  particles at an altitude of approximately 4 km, the latter assumption cannot influence the results of the calculations, since the observed showers are near the maximum of their development. The agreement between the behavior of electrons with energy  $E \leq 100$  MeV in a real shower and in accordance with the cascade theory is likewise not subject to any doubt. The energy determined in this manner, consumed by the shower for ionization of the atmosphere above the measurement level, amounts to

$$\bar{\mathcal{E}}_n = 8 \cdot 10^{14} \left( \frac{N}{10^6} \right)^{0.81 \pm 0.05} \text{ eV.}$$

According to the experimental data, this relation holds true for showers with  $N = 2 \times 10^6$  particles at an altitude of 3860 meters above sea level.

For smaller altitudes the results are only qualitative. If we assume that the extensive air shower is

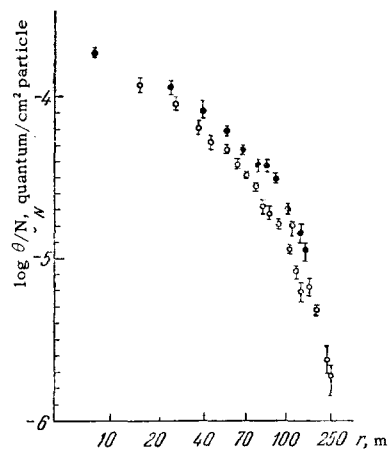


FIG. 23. Lateral distribution of the flux density of the Cerenkov glow of extensive air showers at 3860 meters above sea level. ● —  $N = 1.1 \times 10^5$ ; ○ —  $1.3 \times 10^6$ .

developed without fluctuations, then the relative magnitude of the flash of Cerenkov radiation should increase with decreasing observation altitude. The reason for this is that if the shower has passed its maximum development, the intensity of the flash of Cerenkov radiation increases with increasing path in the atmosphere, and the number of particles at the observation level decreases. The experimental results obtained on the intensity of the light flash in showers arriving at the observation level with a large zenith angle,<sup>[91]</sup> and of Cerenkov radiation in showers observed at sea level,<sup>[90]</sup> indicate that the fluctuations in the development of showers brought about by primary particles with energy  $E_0 > 10^{15}$  eV are small. However, a quantitative analysis is more difficult here, for in this case great leeway is possible in the assumption concerning the height corresponding to the maximum shower development. A shift of the maximum shower development deeper into the atmosphere leads to a relative increase in the intensity of the Cerenkov glow, for the threshold energy for the Cerenkov radiation decreases with increasing air density.

c) Energy of nuclear-active component of shower at observation level. The energy flux carried by the nuclear-active particles at the measurement level can be determined in the same way as the summary energy of the electron-photon component of the shower: either by integration over the energy spectrum of the nuclear-active particles in the shower with a given number of electrons, or from the total absorption of the energy flux in dense matter, "calorimetrically." The first approach to the problem raises two difficulties: a certain uncertainty in the form of the spectrum at energies  $E_{\text{nuc}} < 10^{10}$  eV and the relatively hard spectrum on this portion, when the number of particles with energy above the specified value becomes less than 1. It must be noted that the character of the energy spectrum of the nuclear-active particles in this portion is itself evidence of large fluctuations in the shower development. An account of particles encountered in extensive showers with a frequency of  $10^{-1}$  and  $10^{-2}$  per shower is arbitrary, for the average

energy of the nuclear-active component increases in this case at the expense of showers which are encountered in 10% or even in 1% of the cases. Integration over the energy spectrum of nuclear-active particles in extensive air showers with a total number  $N = 10^4$  particles (Fig. 22) yields for the energy flux carried by the nuclear-active component

$$\mathcal{E}_{\text{nuc}}(N = 10^4) = (1.6 \pm 0.3) \cdot 10^{12} \text{ eV.}$$

For showers with  $N = 10^5$  and  $10^6$  the energy flux is

$$\mathcal{E}_{\text{nuc}}(N = 10^5) = (7.8 \pm 1.5) \cdot 10^{12} \text{ eV}$$

$$\mathcal{E}_{\text{nuc}}(N = 10^6) = (6.3 \pm 1) \cdot 10^{13} \text{ eV.}$$

These values of the total energy for the nuclear-active component of the shower can be compared with the results of the measurements of the energy fluxes by "calorimetric" means. The limitation of such a comparison lies in the fact that measurements of the energy flux by the "calorimetric" method have been carried out only near the shower axis. Furthermore, the presence of the electron-photon component complicates the determination of the energy contained in the nuclear-active component of the shower.

Measurements at an altitude of 3860 meters were carried out for showers with  $N = 10^5 - 3 \times 10^5$  particles in a circle of radius  $r = 30$  meters.<sup>[86]</sup> The measurement result was:

$$\mathcal{E}_{\text{nuc}}(N = 10^5, r \leq 30) = (9 \pm 2) \cdot 10^{12} \text{ eV}$$

Inasmuch as we can assume, taking into account the energy spectrum of the nuclear-active particles and their distribution in the shower, that the summary energy of the nuclear-active particles outside a 30-meter circle does not exceed  $10^{12}$  eV, the result of these measurements is in good agreement with the value given above,  $(7.8 \pm 1.5) \times 10^{12}$  eV.

A large series of measurements<sup>[92]</sup> was made at sea level under conditions of an experiment that was not purely "calorimetric," but with a combined absorber made of lead and carbon. The result obtained for the energy flux in showers with total number of particles  $N = 10^4 - 10^6$  does not contradict the values already cited [ $\mathcal{E}_{\text{nuc}} \approx (0.5 - 1.0) \mathcal{E}$ ]. However, at the same time no differences are observed in the relative magnitude of the energy carried by the nuclear-active component in showers with  $N \geq 10^5$  and in showers with  $N = 10^4$ . The value  $\mathcal{E}_{\text{nuc}} = (0.5 - 1.0) \mathcal{E}$  in showers with  $N = 10^5 - 10^6$  particles exceeds the value of the energy obtained by integration of the energy spectrum of the nuclear-active particles. It is not excluded that the results of the measurement analysis yielded values that were too high, since a great uncertainty exists in the coefficient for converting from ionization under a combined filter to the energy of the nuclear-active particles interacting in this filter. It is sufficient to point out that an analysis of analogous results<sup>[93]</sup> can lead to energy flux values that are smaller by

a factor 3–5. The conversion coefficient from ionization to energy depends on the characteristics of the elementary nuclear-interaction act in the carbon and in the lead, on the form of the energy spectrum of the interacting particles, and on the energy of the nuclear-active particles. In particular, with increasing distance from the axis, a change takes place in the spectrum of the nuclear-active particles and the coefficient of conversion from the ionization to the energy of the nuclear-active particles should decrease. With decreasing size of the shower, the role of the nuclear-active particles with energy  $E_{\text{nuc}} \leq 10^{11}$  eV, for which the conversion coefficient is larger, increases. If this circumstance is not taken into account, the value obtained for the energy of the nuclear-active component of the shower may be too low.

A clear difference between the summary energy of the nuclear-active components in showers of different size appeared in the experiments<sup>[94]</sup> carried out at 3860 meters above sea level with the aid of a comprehensive array with subsequent selection. The gist of the experiment was to measure the ionization at different distances from the axis of a shower with a given number of particles using ionization chambers placed under a layer of aluminum and graphite. The layers of aluminum and graphite were chosen such that on the whole the absorber was equivalent in radiation units and in nuclear ranges to a layer of air 230 g/cm<sup>2</sup> thick. If we integrate the ionization under the absorber in showers with a fixed total number of particles over all distances,

$$N_A = \int_0^{100} i(r) \cdot 2\pi r dr,$$

we obtain by the same token the total number of electrons under the absorber, averaged over all showers with given  $N$ . This quantity can be related to the total number of particles (electrons) in the shower above the absorber, measured with the aid of the same ionization chambers, in order to eliminate corrections for transition effects. The ratio  $N_A/N$  for different values of  $N$  is shown in Fig. 24a. The value of the ratio differs greatly from that expected from the electron-photon cascade theory and changes on going from showers with  $N < 10^5$  to showers with  $N > 10^5$ . It follows, therefore, that the energy fluxes carried by the nuclear-active component differ by a factor of  $(1.7 \pm 0.2)$  in showers with a total number of particles  $10^4 < N < 10^5$  and in showers with  $10^5 < N < 10^6$ . The absolute magnitudes are determined with a great leeway owing to the uncertainty in the form of the absorption curve in dense matter, both in the interval from zero to 230 g/cm<sup>2</sup>, and deeper than 230 g/cm<sup>2</sup>. If we subtract the energy of the electron-photon component, then the summary energy of the nuclear-active particles for different  $N$  is estimated at  $\mathcal{E}_{\text{nuc}} (10^4 < N < 10^5) = (2.2 \pm 0.7) \times 10^8 N$  eV and  $\mathcal{E}_{\text{nuc}} (10^5 < N < 10^6) = (1.3 \pm 0.6) \times 10^8 N$  eV. The error takes into

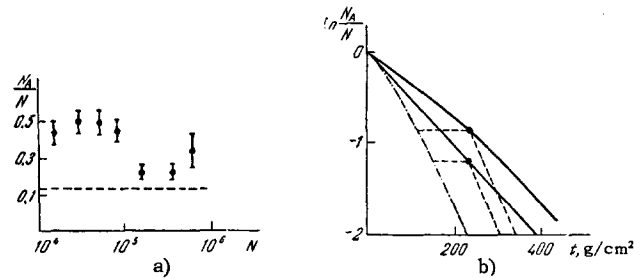


FIG. 24. a) Relative number of particles under the absorber as a function of the total number of particles in the shower at the observation level. (Dashed – expected value of ratio as given by the electron-photon cascade theory.) b) Estimate of the energy flux of the electron-photon and nuclear-active components of the shower. (Continuous curve – case of total equilibrium between the nuclear-active and electron-photon components of the shower. Dashed – case when there is no equilibrium. Dot-dash – absorption of electron-photon component incident on the absorber from the air.)

account all possible variants of the form of the absorption curve of the shower in dense matter (Fig. 24b), and must therefore be kept in mind in a comparison with estimates of the energy flux of the nuclear-active component, made by other methods earlier, but not when comparing the values obtained here with one another.

d) Energy of the muon component of the shower at the observation level. It is possible to estimate the energy of the muon component of the shower at the observation level only on the basis of the energy spectrum of the muons. Unfortunately, the experimental data on the spectrum of the shower muons at large energies are skimpy, so that the accuracy of the estimate is limited. Using the data given above on the energy spectrum of the muons at 3860 meters above sea level, we can obtain the following expression for the summary energy of the muons at the observation level:

$$\mathcal{E}_{\mu} = (2.1^{+2}_{-0.2}) \cdot 10^{14} \left( \frac{N}{10^6} \right)^{0.81} \text{ eV}$$

for  $10^4 < N < 3 \times 10^6$ .

The review<sup>[95]</sup> contains an estimate of the energy flux carried by the muons in a shower with a total number  $N = 10^6$  particles at sea level:

$$\mathcal{E}_{\mu} (N = 10^6) = 9 \cdot 10^{14} \text{ eV.}$$

This estimate appears to be too high, since it corresponds to too high a number of muons at the observation level (when  $N = 10^4$  the muons amount to 40% of all charged particles). The author of<sup>[95]</sup> suggests also that the energy spectrum of the muons does not change up to limiting high energies. This possibility is accounted for in the upper limit of the error in our estimate of the summary muon energy.

e) Fluctuations of the flux of energy carried by different components of an extensive air shower. No data on the fluctuations of the summary energy of the muon component of a shower at the measurement level have been published so far. One can merely assume that they are not smaller than the fluctuations of the total

number of muons in showers with specified total number of charged particles at the observation level (see Sec. 3d). The experimental data on the fluctuations of the energy flux carried by the remaining components of the shower will be characterized in the exposition that follows by the average relative deviation from the mean value of the energy flux carried by the given shower component at a specified total number of charged particles at the elevation of the point of observation.

$$\Delta = \frac{1}{n} \sum_i^n \left( \frac{\epsilon_i - \bar{\epsilon}}{\bar{\epsilon}} \right).$$

Here  $\bar{\epsilon}$  is the mean value of the energy flux,  $\epsilon_i$  the energy flux in the  $i$ -th shower, and  $n$  the number of cases considered. Inasmuch as the greater part of the distributions is asymmetrical with respect to  $\bar{\epsilon}$ , the summation is carried out for  $\epsilon_i > \bar{\epsilon}$  and  $\epsilon_i < \bar{\epsilon}$  separately. In the case of a normal distribution we have  $\Delta = \sigma\sqrt{2/\pi}$ , where  $\sigma$  is the mean-square deviation.

The intensity fluctuations of the Cerenkov glow accompanying an extensive air shower in the atmosphere were investigated in [91] for an observation height 3860 meters above sea level. Direct measurements yield for showers with a total number of particles  $3 \times 10^5 - 3 \times 10^6$  a distribution about the mean value corresponding to  $\Delta = \pm 0.4$ . An account of the measurement errors reduces the mean relative deviation to 0.3.

The energy of the electron-photon component at the measurement level in individual showers was determined reliably near the core of an extensive air shower. At sea level [82] in a circle of radius one meter, the average relative deviation amounts to  $\Delta = \pm 0.5$  for showers with  $N > 10^5$  particles. Measurements of the energy fluctuations of the electron-photon component at mountain altitudes (650 g/cm<sup>2</sup>) were carried out for the central part of the shower ( $r \leq 2.5$ ). The results [96] in this form can be represented as

$$\Delta = \begin{matrix} +2.5 \\ -0.6 \end{matrix} \quad \text{for } N = 10^4 - 10^5,$$

$$\Delta = \pm 0.5 \quad \text{for } N \geq 10^5.$$

The difference in the values of the fluctuations for showers with large and small numbers of particles was traced also in the nuclear-active component of the shower. In the entire shower, the fluctuations of the summary energy of the electron-photon component are considerably smaller (about half as large), since the energy carried by the electrons and photons outside the shower core remains practically the same from shower to shower. There are still no corresponding experimental data, but an estimate based on the fluctuations of the lateral distribution yields for the electron-photon component with energy  $E < 10^9$  eV/particle a value  $\Delta = \pm 0.1$ .

The energy of the nuclear-active component in individual showers was also determined only near the core. Unlike the electron-photon component, it is difficult to predict the extent to which the deviations from the mean will change if account is taken not only of the energy of the nuclear-active particles in the core of the shower, but also of the energy carried by the nuclear-active particles in the entire shower. It is natural to assume that the energy of the nuclear-active component outside the core of the shower fluctuates little relative to the number of nuclear-active particles in the shower. But so far there is no quantitative information on the fluctuations of this number relative to the total number of charged particles at the measurement level. Data on the fluctuations in the core of the shower ( $r \leq 2.5$  meters) have been obtained in [96] at 3860 meters above sea level. For showers with  $N = 10^4 - 10^5$  particles, the deviations from the mean are very large:  $\Delta = \pm 0.8$ . With increasing number of particles in the shower, the fluctuations decrease ( $\Delta = \pm 0.8$  at  $N \geq 10^5$ ). It is characteristic that the deviations from the mean value of the energy of the electron-photon component of the core do not correlate with the fluctuations of the energy carried by the nuclear-active component of the core (Fig. 25).

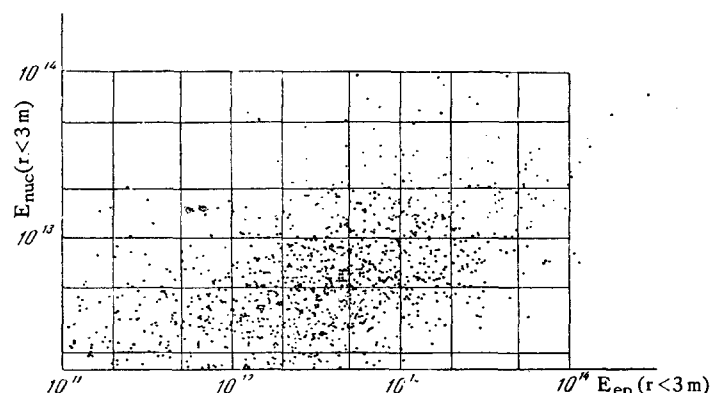


FIG. 25. Energy of the electron-photon ( $E_{ep}$ ) and of the nuclear-active ( $E_{nuc}$ ) components in cores of extensive air showers at the measurement level (3860 meters above sea level). The dots correspond to individual showers.

If we denote the ratio of the energy flux carried by the nuclear-active component of the shower core to the total energy of the electrons and photons in the same region of the  $i$ -th extensive air shower by  $\alpha$ , then  $\Delta = (\alpha_i - \bar{\alpha})/\bar{\alpha}$ , averaged over the cases  $\alpha_i > \bar{\alpha}$  and  $\alpha_i < \bar{\alpha}$  separately, will be

$$\Delta = \begin{matrix} +3 \\ -0.7 \end{matrix} \quad \text{for } N = 10^4 - 10^5,$$

$$\Delta = \pm 0.7 \quad \text{for } N \geq 10^5.$$

The relatively smaller fluctuations of the energy flux in cores of extensive air showers with total number of particles at the observation level  $N > 10^5$  may be at-



tributed to the relatively poor statistical size of the group of showers with  $N > 10^5$ . However, according to the data shown in Fig. 25, for a fixed electron-photon component of the shower core  $\mathcal{E}_c = (1.4-2.8) \times 10^{13}$  eV, which corresponds to  $2 \times 10^5 < N < 7 \times 10^5$  particles, the average relative deviation obtained for the energy of the nuclear-active particles of the shower core is also small,

$$\Delta = \pm 0.4.$$

Thus, the difference in the value of the fluctuations must be attributed to changes in the structure of the extensive air showers on going from showers with  $N < 10^5$  particles to showers with  $N > 10^5$ . The large fluctuations in showers with a small number of particles ( $N < 10^5$ ) is satisfactorily explained from the point of view of the decisive role of one energetically distinguished "leading" particle in the formation and development of extensive air showers with  $N < 10^5$ . The presence of such a particle means that the average inelasticity coefficient in collisions between nucleons and nuclei of air atoms retains a value of  $\eta \approx 0.5$  up to a nucleon energy  $E_0 \approx 3 \times 10^{14}$  eV. The reduction in the fluctuations on going over to larger showers is possibly due to a change in the shower development pattern, the vanishing of the unique energetically singled out particles.

## 6. EXTENSIVE AIR SHOWERS AND PRIMARY COSMIC RADIATION\*

Extensive air showers have been and remain the only source of our information on the primary ultra-high-energy cosmic radiation. The list of problems arising in connection with the theory of the origin of cosmic rays is quite extensive: the form of the energy spectrum of the primary cosmic radiation, searches for anisotropy in the primary radiation and in its composition, the limiting magnitude of the energy of the primary particles, the presence of electrons and  $\gamma$  quanta among the primary cosmic rays. Experimental data are not available for all the problems enumerated here, and the greater part of the answers is for the time being in the negative.

a) The particle-number spectrum of extensive air showers. To construct the particle-number spectrum one can use both direct data on the frequency of occurrence of showers with a number of particles larger than specified with a given location of the passage of the shower axis, and indirect measurements of the "density spectrum." In the latter case the particle-number spectrum of the showers can be determined if the lateral distribution function is known and varies weakly with the size of the shower. For sea level we

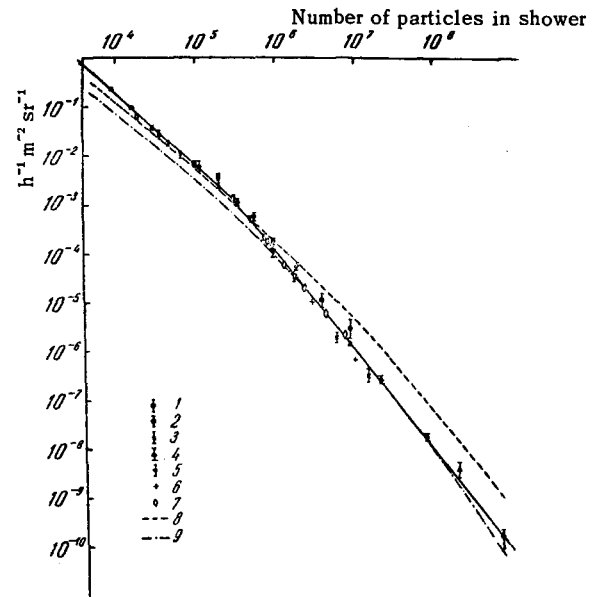


FIG. 26. Particle-number spectrum of showers at sea level as obtained by various authors. 1 - [97]; 2 - [61]; 3 - [98]; 4 - [46]; 5 - [60]; 6 - [99]; 7 - [100]; 8 - [101]; 9 - [95].

have enough data on the direct measurement of the frequency of showers with different particle numbers. The most significant data are shown in Fig. 26. The smallest showers were registered in [97] with the aid of an array with a rigid selection criterion. It was assumed there that the form of the lateral distribution function does not depend on the total number of particles in the investigated interval and that the deviations of the observed particle flux density from the mean value for a given distance from the shower axis are described by a Poisson law. In the interval  $2 \times 10^4 \leq N \leq 10^5$  there exist, in addition to the measurements indicated above, also data [98] obtained with the aid of a comprehensive array with hodoscopic counters, as well as the results of [61], obtained with the aid of scintillation counters. As can be seen, these three investigations, which differ in their procedures, are in good agreement with one another.

In the interval  $1.5 \times 10^5 < N < 8 \times 10^5$  there are only data [61, 98] which are not in good agreement with one another. The disparity between these data increases on going over to larger showers, where, however, there are results of measurements obtained by other authors. [46, 99, 100] The most accurately performed measurement [46] yielded a value which was intermediate compared with [61, 98]. The large range of showers registered in [46] was expanded in the measurements of [60] all the way to  $N \sim 10^9$ .

As can be seen from Fig. 26, the totality of the data on the particle-number spectrum of the showers can be approximated in the form of a power function

$$F(>N) = 1.9 \cdot 10^{-3} \left( \frac{N}{2.5 \cdot 10^5} \right)^{-\kappa} \text{ h}^{-1} \text{ m}^{-2} \text{ sr}^{-1},$$

The exponent  $\kappa$  changes in value at  $N = 2.5 \times 10^5$ :

\*This question was discussed in greater detail in the author's paper delivered to the All-union Conference on Cosmophysical Problems of Cosmic-ray Research (Yakutsk, 1962).



$$\begin{aligned} \kappa &= 1.44 \pm 0.03 & N < 2.5 \cdot 10^5, \\ \kappa &= 1.90 \pm 0.05 & N > 2.5 \cdot 10^5. \end{aligned}$$

The same figure shows the particle-number spectrum of showers as taken from the review<sup>[101]</sup>, where it was determined from the "density spectrum." In a later review by the same author<sup>[95]</sup> this spectrum was made more accurate, so that better agreement was obtained with the direct measurements of the particle-number spectrum of showers with  $N \sim 10^6$ . However, the agreement for  $N < 10^5$  turned out to be worse.

At the present time there are relatively few direct measurements of the intensity of showers with a specified number of particles at mountain altitudes. It must be added further that the measurements have been carried out at different altitudes and in order to compare them it is necessary to introduce the dependence of the number of registered showers on the observation altitude. The data given in Fig. 27 were obtained in the altitude interval 2770–4100 meters above sea level. The absolute intensity has been recalculated using the experimentally observed altitude variation to a level corresponding to the pressure  $p = 650 \text{ g/cm}^2$ . The data of the various investigations<sup>[73,87]</sup> carried out at one and the same high-altitude station (2770 meters above sea level) are in sharp disagreement.

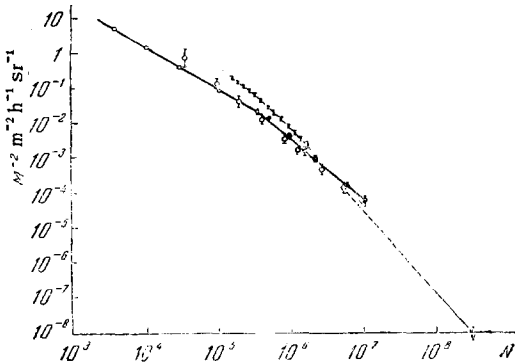


FIG. 27. Particle-number spectrum of showers at an altitude of 3860 meters above sea level from different measurements.  $\square$  – [27];  $\circ$  – [90];  $*$  – [87];  $\bullet$  – [73]; the highest energies are cited on the basis of [30].

The results of the measurements in [87] is likewise too high compared with data obtained by others. Likewise differing from the data of other investigators are the results of measurements<sup>[90]</sup> of the intensity of showers with  $N < 10^5$  particles. As in the case of the sea level, the most probable form of the spectrum is a power law with a kink at  $N = 3.5 \times 10^5$ .

$$E_{3860}(>N) = 1.9 \cdot 10^{-2} \left( \frac{N}{3.5 \cdot 10^5} \right)^{-\kappa},$$

$$\kappa = 1.3 \quad N < 3.5 \cdot 10^5,$$

$$\kappa = 1.9 \quad N > 3.5 \cdot 10^5.$$

Comparison of the intensity of extensive air showers at 3860 meters with the number of showers at sea level

leads to the following altitude variation of the number of extensive air showers in the lower third of the atmosphere:  $\sim \exp(-p/140)$  for  $N \geq 3 \times 10^5$ .

b) Primary cosmic radiation. The main difficulty arising in the determination of the energy spectrum of primary cosmic radiation from the number of observed extensive air showers lies in the uncertainty in the conversion from the number of particles in the shower to the primary-particle energy

$$E_0 = BN^\alpha.$$

Various theoretical motivations for this conversion, depending on the scheme used for the development of the extensive air shower, lead to values of the coefficient B which differ by a factor of several times.

The situation is not better when it comes to the value of the parameter  $\alpha$ .

As was shown in the preceding section, the aggregate of the experiments carried out in Pamir ( $p = 650 \text{ g/cm}^2$ ) makes it possible to calculate the entire energy expended by the primary particle in producing the shower. Thus, for showers with  $N = 10^5 - 10^6$  particles at the Pamir altitude, we can determine the average energy of the primary particles without using any arbitrary assumptions concerning the scheme of shower development or the character of the elementary processes at ultrahigh energy. If in addition to this point on the energy spectrum of the primary cosmic-ray particles we assume the geomagnetic measurement data to be reliable<sup>[102,103]</sup>, then the energy spectrum of the primary cosmic radiation can be represented in the energy interval  $10^{10} - 10^{14} \text{ eV}$  as  $\Phi(>E_0) = A(E_0/6 \times 10^{14})^{-\gamma}$ , where  $A = 1.9 \times 10^{-2} \text{ particles/h-m}^2\text{-sr}$ , and  $\gamma = 1.60 \pm 0.03$ . This spectrum is shown in Fig. 28, where the abscissas represent the energy in eV per incident particle. Data on extensive air showers do not allow as yet any quantitative conclusions to be drawn concerning the charge composition of the primary cosmic radiation.

From the form of the particle-number shower

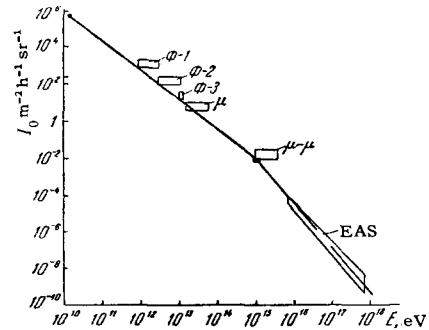


FIG. 28. Primary cosmic radiation spectrum. The abscissas are the values of the energy per incident particle. The point represents the data of [102,103], the filled rectangle – experimental data on the total energy of all the components of the extensive air shower;  $\Phi$ -1,  $\Phi$ -2 and  $\Phi$ -3 – emulsion data [104,105],  $\mu$  and  $\mu$ - $\mu$  – result of estimates on the muon intensity<sup>[58]</sup>; EAS – from [46].

spectrum (Figs. 26 and 27) it follows, in all probability, that in the interval of the primary-particle energies  $E_0 > 10^{15}$  eV the intensity of the flux of primary cosmic rays decreases with increasing energy more rapidly than in the energy region  $E_0 < 10^{14}$  eV. However, for quantitative estimates the values of the exponent in the spectrum of the primary radiation  $\Phi(>E_0) \sim E_0^\gamma$  additional assumptions must be made concerning the shower-development scheme. By way of such assumptions, the nuclear-cascade shower development scheme, which is similar in many respects to the development of an extensive air shower at primary particle energies  $E_0 \sim 10^{14}$ – $10^{15}$  eV, was used in [46]. The value of the exponent in the spectrum of the primary cosmic radiation was found to be  $\gamma = 2.1 \pm 0.15$  (Fig. 28). It can be noted here furthermore that the assumption that  $\gamma$  has a constant value of about 1.6 up to the maximum energies is equivalent to the assumption that the energy of the primary particles causing the extensive air showers of maximum size ( $N \sim 10^9$ ) is decreased by more than 20 times. It is difficult to imagine such a possibility.

Figure 28 shows also data on the intensity of the primary cosmic radiation, obtained with the aid of photographic emulsions [104,105] and by measuring the muon flux at large depths. [58] The increase in the intensity of the primary cosmic radiation, determined from the muon flux, is connected with the insufficiently complete information on the muon production processes. The error in the determination of the energy of the particles forming the jets in the emulsion is attributed to the influence of fluctuations as the intensity of the interacting particles decreases with increasing energy. [106]

By now much work has been done in many laboratories on the search for anisotropy in the primary cosmic radiation based on the intensity variations of extensive air showers, connected with sidereal time, and on searches for preferred directions of incidence of ultrahigh-energy primary cosmic-ray particles. The results of some of the investigations pointed to the presence of anisotropy. However, a clever analysis made in [107] has shown that the observed cases of deviation from isotropy cluster around a value equal to double the standard deviation from the mean observed intensity. It can be stated that as of now no noticeable anisotropy has been observed in the ultrahigh-energy primary cosmic radiation. Even for particles with energy  $E_0 \sim 5 \times 10^{18}$  eV the amplitude of the deviation from the anisotropy does not exceed 10%. [108] Nor are there as yet any indications that the upper limit of the energy spectrum of the primary particles, whose spectrum has been measured almost to  $10^{20}$  eV, is close.

## CONCLUSION

Our notions concerning extensive air showers as a nuclear-cascade process, which develops when cosmic-

ray particles of ultrahigh energy pass through the atmosphere, originated a decade and a half ago. The experimental data accumulated during that time concerning the properties and structure of extensive air showers have not only qualitatively confirmed the nuclear-cascade scheme of shower formation, but have made it possible to detail appreciably the entire picture of shower development. Now, using our information concerning the collisions of nucleons and pions with nuclei of air atoms at  $E_0 < 10^{12}$  eV, on the decay properties of elementary particles, we can describe quantitatively many characteristics of extensive air showers, and primarily the lateral distribution of various components at the observation level. However, many characteristics of the shower are difficult to predict quantitatively either because our knowledge of the nuclear interactions at particle energies  $\geq 10^{12}$  eV is insufficient, or because of the uncertainty in the picture of the first acts of shower formation by particles of primary cosmic radiation with energy  $E_0 > 10^{14}$  eV. We approach here the main problem, which from the point of view of nuclear physics faces the researchers: what can be stated concerning the elementary collisions between particles of ultrahigh energy?

The totality of the experimental data on extensive air showers with  $N < 10^5$  particles allows us to state that the main characteristics of collisions between nucleons and nuclei of the air atoms change little with increasing energy of incoming particles, up to  $E_0 \sim 10^{14}$  eV. These main characteristics are: the effective cross section for nuclear interaction, the average value of the inelasticity coefficient for the collision between nucleons and air-atom nuclei, the form of the dependence of the multiplicity of creation of secondary particles on the energy of the primary particle, and transverse momenta of the secondary particles. The primary nucleons, which as a rule lose their energy not in one but in several successive collisions with air-atom nuclei, penetrate deep into the atmosphere and represent the same energetically singled-out "leading" particle, which determines the conservation of the structure of the core of nuclei with few charged particles ( $N < 10^5$ ) down to sea level. Graphically, this shower development scheme is frequently represented in the form of an inverted Christmas tree. [10] However, the small number of nuclear ranges in the entire thickness of the atmosphere leads to large fluctuations in the shower development, and to irregularities in the "branching of the Christmas tree." Fluctuations in the value of the inelasticity coefficient in collisions between nucleons and the air-atom nuclei aggravate the situation still further, while the energy spectrum of the primary cosmic radiation, which decreases with increasing energy, brings about a situation wherein, by selecting extensive air showers with a specified number of electrons at the observation level, we give preference to the cases in which the primary nucleon, after jumping through the upper part of the atmosphere with relatively small energy loss,

gives up the greater part of its energy at altitudes which are effective for the given observation level. Therefore if we change over to a lower observation altitude and register showers which appear with the same frequency as at the higher observation altitude (and by the same token take into account, as it were, the absorption of the shower particles in the atmosphere), we get different showers having even different mean primary-particle energy.

The pattern of formation and development of the showers which are due to primary nuclei with  $Z > 1$  at energies  $E_0 < 10^{14}$  eV/nucleon differs from that described above only in smaller fluctuations in the development. Such showers average out, as it were, in the number of nucleons contained in the nucleus arriving at the top of the atmosphere. Calculations based on such a model of shower development with allowance for the fluctuations and for the complicated composition of the primary cosmic radiation, and which encompass as large a number as possible of the characteristics of extensive air showers, are possible and essential. They are essential not for a further more exact determination of the parameters of the nuclear interaction of particles with energies  $E_0 < 10^{14}$  eV, for this energy is approached in experiments involving a more direct measurement of the characteristics of the elementary acts than can be obtained by the analysis of data on extensive air showers. What is more important is, after obtaining good and all-out agreement between the calculation and the really observed showers with a total number of particles  $N < 10^5$ , to attempt to determine in detail the causes of the variations in the properties and composition of the showers produced by primary particles with energies  $E_0 \gtrsim 10^{15}$  eV.

By now, much experimental data have been accumulated indicating that the characteristics of extensive air showers with  $N > 10^5$  particles differ from those of showers with fewer particles. These include both the change in relative number of nuclear-active particles of all energies, and the change in the structure of the core of the shower, and also the sharp decrease in the fluctuations in the relative distribution in the energy among the various shower components on going to larger showers, and the appearance of narrow groups of muons, and the reduction in the fraction of the energy carried by the nuclear-active particles of the shower. These facts have been established with a different degree of reliability, some are consequences of others, but all pertain to one and the same energy interval of primary cosmic radiation. Referring to the same interval are also certain facts obtained independently of investigations of extensive air showers. Primarily, the change in energy spectrum of single electrons and photons at energies  $\sim 10^{12}$  eV, observed at airplane altitudes with the aid of emulsions interlined with lead and tungsten,<sup>[109,110]</sup> and also the increase

in the exponent of the energy spectrum of single nuclear-active particles at mountain altitudes on going over to energies  $E > 10^{13}$  eV.<sup>[111]</sup> Finally, is it an accident that practically at the same energy ( $5 \times 10^{14} - 10^{15}$  eV) a change takes place in the energy spectrum of the primary cosmic radiation? A weighty proof against such an accidental coincidence is the fact that in spite of the complicated composition of the primary cosmic radiation and the large fluctuations in the development of showers, the change in the spectrum of the primary cosmic radiation manifests itself in a narrow spectral interval of the extensive air showers with respect to the number of particles.

There are at present, apparently two possibilities of interpreting the phenomena corresponding to the primary cosmic radiation with energy  $10^{14} - 10^{15}$  eV:

1. Ignoring some experimental facts, one can attempt to explain the variations in the characteristics of the extensive air showers with a total number of charged particles  $N \sim 10^5$  as being due to the change in the energy spectrum and composition of the primary cosmic radiation in the corresponding energy interval.

2. One can assume a change in the elementary processes (or the appearance of new processes) in the collision of nucleons with energies  $10^{14} - 10^{15}$  eV.

If at the same time a) the inelasticity coefficient of the colliding nucleons increases directly in the collision act or as a result of the subsequent decays, and b) groups of electrons or photons and muons with energy  $\gtrsim 10^{12}$  eV and transverse momenta  $p_{\perp} < 10^8$  eV/c appear directly in the act or in the subsequent decay, then we can explain the entire aggregate of the facts concerning extensive air showers with total number of particles  $N > 10^5$ . If furthermore the effective cross section for inelastic collision of the nucleons and nuclei is increased by 20–30%, then the kink in the spectrum of the primary cosmic radiation can be explained as being the consequence of a change in the nuclear lifetime of the cosmic rays in the galaxy. The solution of the stated problems is the main purpose of future experiments.

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