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DIFFUSION OF CHARGED PARTICLES IN A PLASMA IN A MAGNETIC FIELD

V. E. GOLANT

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In recent years a great deal of theoretical and experimental work has been devoted to the problem of diffusion of plasma in a magnetic field. The interest in this subject is stimulated primarily by the ever widening investigation of containment and heating of a plasma in a magnetic field, carried out in connection with the problem of controlled thermonuclear fusion (cf. [1-3]); this subject is also of interest in connection with research on astrophysical plasma research. [4-5]

A number of books on plasma physics[1-3,6-12] treat various aspects of the theory of diffusion in a magnetic field and present certain experimental results. However, a systematic survey of the present state of research in diffusion does not exist. The present work has been undertaken to fill this gap; we present the theory of diffusion in a stable plasma and review the important experimental research on diffusion in a magnetic field.*

I. THEORY OF COLLISIONAL DIFFUSION

The effect of a magnetic field on diffusion of charged particles in a gas and on their motion in an electric field was noted as far back as the early work of Townsend. [14] In this and later work[14-15] an approximation method based on mean free paths was used to analyze the directed motion of charged particles in a natural gas. A more detailed analysis of transport processes involving charged particles in a neutral gas, which was based on the solution of the kinetic equation and the averaged equations of motion, was given in [16-21]. The results of these investigations were used to treat ambipolar diffusion in a weakly ionized gas. [20,21,22]

A great deal of work has been devoted to the investigation of transverse transport processes in a magnetic field in a fully ionized gas. These transport processes were investigated by means of the averaged equations of motion for the charged particles, [6,23-25] by means of the kinetic equation, [26-34] and by methods based on analysis of the displacement of individual particles. [35-39] In this work the collective Coulomb interactions of the charged particles were, in most cases, treated as independent two-body collisions with a maximum interaction range equal to the Debye radius. A special investigation using methods of quantum field theory demonstrated the validity of this analysis in treating transport processes. [40,41]

In a number of papers[42-45] diffusion across a magnetic field in a highly ionized gas has been analyzed under conditions for which collisions between charged particles and neutral particles as well as charged particles with each other were both important.

We shall not dwell here on the various approaches used in the theoretical analysis of transport processes in a magnetic field. To obtain general relations giving the diffusion rate arising from density gradients in arbitrary magnetic fields we shall make use of the simplest method, that based on the solution of the approximate equations of motion for the charged particles. We shall also consider transverse diffusion in strong magnetic fields in detail and, on the basis of an analysis of particle displacements caused by collisions, obtain expressions for the diffusion fluxes. In order to avoid complications in the presentation we shall neglect diffusion caused by temperature gradients (thermal diffusion).

1. Diffusion Theory Based on the Particle Equations of Motion

1. Averaged equations of motion. The averaged equation of motion for the charged particles can be written as follows:

*It should be noted that many experiments have revealed the existence of various forms of plasma instabilities. Problems concerning the theory of stability are not considered in the present review (cf. [13]).

$$m_\alpha\left[\frac{\partial \mathbf{u}_\alpha}{\partial t}+(\mathbf{u}_\alpha\nabla)\mathbf{u}_\alpha\right]=Z_\alpha e\mathbf{E}+Z_\alpha e\frac{[\mathbf{u}_\alpha\mathbf{H}]}{c}-\frac{\nabla p_\alpha}{n_\alpha}+m_\alpha\frac{\delta \mathbf{u}_\alpha}{\delta t}. \quad (1.1)^*$$

Here, $Z_\alpha e$ is the charge, m_α is the mass, $\mathbf{u}_\alpha$ is the mean (directed) velocity, n_α is the density, p_α is the pressure, $m_\alpha \frac{\delta u_\alpha}{\delta t}$ is the momentum change due to collisions (friction force) (all quantities refer to a particle of type α)† and $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic fields. For a Maxwellian distribution of random particle velocities the pressure p_α is given by

$$p_\alpha = n_\alpha T_\alpha \quad (1.2)$$

(T_α is the temperature in energy units).

As is well-known, (1.1) can be derived from the kinetic equation (cf. [6]). In the general case the last term $m_\alpha \frac{\delta u_\alpha}{\delta t}$ can be determined only by integration of the kinetic equation. The quantity $m_\alpha \frac{\delta u_\alpha}{\delta t}$ can depend on the magnetic field as well as the densities and temperatures of the various particles. As an approximation, however, the friction force can be given by the relation[6,23,24]

$$m_\alpha \frac{\delta \mathbf{u}_\alpha}{\delta t} = -m_\alpha \sum_{(\beta)} \nu_{\alpha\beta}(\mathbf{u}_\alpha - \mathbf{u}_\beta). \quad (1.3)$$

Each of the terms on the right side determines the averaged change in the momentum of a particle of type α per unit time resulting from collisions with particles of type β. It is reasonable to assume that the friction force due to collision between particles of type α and β is proportional to the relative velocity of these particles. The quantity $\nu_{\alpha\beta}$ is an effective collision frequency. As a consequence of momentum conservation in the collisions the following relation holds between $\nu_{\alpha\beta}$ and $\nu_{\beta\alpha}$:

$$n_\alpha m_\alpha \nu_{\alpha\beta} = n_\beta m_\beta \nu_{\beta\alpha}. \quad (1.4)$$

In what follows we shall be concerned with stationary or quasi-stationary processes, in which case

$$\left|\frac{\partial \mathbf{u}_\alpha}{\partial t}\right| \ll \left|\frac{\delta \mathbf{u}_\alpha}{\delta t}\right| \quad (1.5)$$

so that we can neglect the first term on the left side of the transport equation. Further, we assume that all perturbations, i.e., gradients in density and electric field, are small so that the quadratic terms $(\mathbf{u}_\alpha\nabla)\mathbf{u}_\alpha$ on the left side of (1.1) can also be neglected. This procedure is valid if the directed velocity of the particles is much smaller than the random (thermal) velocity:

$$\mathbf{u}_\alpha \ll \sqrt{\frac{T_\alpha}{m_\alpha}}. \quad (1.6)$$

This inequality is obviously the criterion for the diffusional nature of the transport process.

Making use of the simplifications given above and (1.2) and (1.3), we write (1.1) in the form of an equation expressing equilibrium between the electric force, the Lorentz force, the pressure gradient (per particle), and the effective friction force:

$$Z_\alpha e\mathbf{E}+Z_\alpha e\frac{[\mathbf{u}_\alpha\mathbf{H}]}{c}-T_\alpha\frac{\nabla n_\alpha}{n_\alpha}-m_\alpha\sum_{(\beta)}\nu_{\alpha\beta}(\mathbf{u}_\alpha-\mathbf{u}_\beta)=0. \quad (1.7)$$

In (1.7) the temperature is assumed to be constant and is taken outside the gradient operator since thermal diffusion effects are not treated.

The system of equations (1.7) written for all the plasma particles allows us to determine the directed velocities, that is to say, this system is used to solve the problem of directed particle motion.

2. Directed motion of charged particles in a neutral gas. We first consider the motion of charged particles in a weakly ionized gas, in which case collisions of charged particles with each other are not important; that is to say, the electron-neutral and ion-neutral collision frequencies are much higher than the electron-ion collision frequency:

$$\nu_{en} \gg \nu_{ei}, \quad \nu_{in} \gg \nu_{ie}. \quad (1.8)$$

Under these conditions the transport equations (1.7) for the different charged particles are independent of each other and assume the form

$$Z_\alpha e\mathbf{E}+Z_\alpha e\frac{[\mathbf{u}_\alpha\mathbf{H}]}{c}-T_\alpha\frac{\nabla n_\alpha}{n_\alpha}-m_\alpha\nu_{\alpha n}\mathbf{u}_\alpha=0. \quad (1.9)$$

Here, $\nu_{\alpha n}$ is the frequency of collisions between particles of type α and neutrals; it is evident that at low ionization the directed velocity of the neutrals is much smaller than the directed velocity of the charged particles.

Taking the projection of the vector equation (1.9) in the direction of the magnetic field we find an expression for the longitudinal directed velocity

$$\left.\begin{aligned} u_{\alpha\|} &= -D_{\alpha\|}\frac{\nabla_\| n_\alpha}{n_\alpha}+\mu_{\alpha\|}E_\|,\\ D_{\alpha\|} &= \frac{T_\alpha}{m_\alpha\nu_{\alpha n}}, \quad \mu_{\alpha\|}=\frac{Z_\alpha e}{m_\alpha\nu_{\alpha n}} \end{aligned}\right\} \quad (1.10)$$

($u_{\alpha\|}$, $E_\|$, $\nabla_\| n_\alpha$ are the projections of the corresponding vectors in the direction of the magnetic field).

The projection of (1.9) on the plane perpendicular to the magnetic field can be used to determine the transverse components of the directed velocity—the transverse component of the velocity in the direction of the density gradient and the electric field $\mathbf{u}_\perp$, and the velocity perpendicular to the magnetic field and the density gradient $\mathbf{u}_T$:

$$\left.\begin{aligned} \mathbf{u}_{\alpha\perp} &= -D_{\alpha\perp}\frac{\nabla_\perp n_\alpha}{n_\alpha}+\mu_{\alpha\perp}\mathbf{E}_\perp,\\ D_{\alpha\perp} &= \frac{T_\alpha}{m_\alpha\nu_{\alpha n}\left(1+\frac{\omega_\alpha^2}{\nu_{\alpha n}^2}\right)}, \quad \mu_{\alpha\perp}=\frac{Z_\alpha e}{m_\alpha\nu_{\alpha n}\left(1+\frac{\omega_\alpha^2}{\nu_{\alpha n}^2}\right)}; \end{aligned}\right\} \quad (1.11)$$

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*$[\mathbf{u}_\alpha\mathbf{H}] = \mathbf{u}_\alpha \times \mathbf{H}$.

†Quantities referring to the electrons are denoted by the subscript $\alpha = e$, quantities referring to ions by the subscript $\alpha = i$, and neutral particles by $\alpha = n$.

$$\mathbf{u}_{\alpha T}=\frac{cT_\alpha[\mathbf{h}\nabla n_\alpha]}{Z_\alpha eHn_\alpha\left(1+\frac{\nu_{\alpha n}^2}{\omega_\alpha^2}\right)}-\frac{c[\mathbf{hE}]}{H\left(1+\frac{\nu_{\alpha n}^2}{\omega_\alpha^2}\right)}. \tag{1.12}$$

Here, $\mathbf{E}_\perp$ and $\nabla_\perp n$ are the projections of the vectors $\mathbf{E}$ and ∇n on the plane perpendicular to the magnetic field; $\mathbf{h}$ is a unit vector in the direction of the magnetic field; ω_α is the Larmor frequency

$$\omega_\alpha=\frac{|Z_\alpha|eH}{m_\alpha c}. \tag{1.13}$$

The motion of the particles in the direction of the density gradient and the electric field is described by the longitudinal and transverse diffusion coefficients $D_{\alpha\|}$ and $D_{\alpha\perp}$, and the corresponding mobilities $\mu_{\alpha\|}$ and $\mu_{\alpha\perp}$. These quantities are related by the Einstein relation

$$\frac{D_{\alpha\|}}{\mu_{\alpha\|}}=\frac{D_{\alpha\perp}}{\mu_{\alpha\perp}}=\frac{T_\alpha}{Z_\alpha e}. \tag{1.14}$$

The difference between the magnitudes of $D_{\alpha\|}$ and $D_{\alpha\perp}$ (or $\mu_{\alpha\|}$ and $\mu_{\alpha\perp}$) is responsible for the anisotropic transport effects in the presence of the magnetic field.

We note that in strong magnetic fields ($\omega_\alpha \gg \nu_{\alpha n}$) the motion in the direction perpendicular to the density gradient and electric field is determined by the drift velocity of the charged particles.

Using the equations of motion we have obtained approximate expressions for the flux of charged particles in a neutral gas. We now compare these expressions with more exact expressions obtained for certain cases by means of the kinetic equation.

The kinetic equation can be integrated to describe the motion of electrons in a gas only when elastic collisions between electrons play the most important role. The analysis leads to the following expression for the diffusion coefficient:[20]

$$D_{e\|}=\frac{1}{3}\int_{(\mathbf{v})}\frac{v^2}{\nu_{en}}f_e(v)\,d\mathbf{v},\quad D_{e\perp}=\frac{1}{3}\int_{(\mathbf{v})}\frac{v^2\nu_{en}}{\omega_e^2+\nu_{en}^2}f_e(v)\,d\mathbf{v}, \tag{1.15}$$

where $\mathbf{v}$ is the electron velocity, $f_e(v)$ is the velocity distribution function

$$\nu_{en}^*=n_n v s^*_{en}(v),\quad s_{en}^*=\int_{(\Omega)}\sigma_{en}(v,\vartheta)(1-\cos\vartheta)\,d\Omega, \tag{1.16}$$

ν_{en}^* is the "diffusion" collision frequency, s_{en}^* is the momentum transfer coefficient, $\sigma_{en}(v,\vartheta)$ is the differential cross section for scattering of electrons by neutrals, and the integration in (1.15) is carried out over the entire electron velocity space.

The expressions in (1.15) and (1.10)—(1.11) coincide when the electron-ion collision frequency ν_{en}^* is independent of velocity. If ν_{en}^* depends on velocity, a correspondence can be obtained between the formulas by introducing certain averaged collision frequencies in (1.10) and (1.11). In this case, the quantity ν_{en} that appears in $D_\perp$ is found to depend on magnetic field. Nonetheless, the general dependence of the diffusion coefficients on magnetic field given by (1.11) is approximately correct if the dependence of ν_{en}^* on v is not too strong.

The kinetic equation that describes the motion of ions in a neutral gas can be integrated easily if the effective cross section for ion-neutral collisions varies inversely with their relative velocity, $\sigma_{in} \sim 1/v$ (it is assumed that only elastic collisions are important).[18] In this case, integration of the kinetic equation yields expressions for the flux which coincide with (1.10)—(1.12), where the effective collision frequency is given by

$$\nu_{in}=\frac{m_n}{m_i+m_n}n_n\int_{(\Omega)}v\sigma_{in}(v,\vartheta)(1-\cos\vartheta)\,d\Omega. \tag{1.17}$$

3. Ambipolar diffusion of charged particles in a weakly ionized gas. The neutrality of the plasma is conserved in the diffusion of charged particles (this statement holds if the dimensions of the inhomogeneous region are much larger than the Debye radius). The neutrality condition for a plasma consisting of electrons and ions of charge Ze is

$$n_e=Zn_i. \tag{1.18}$$

The expression in (1.18) relates the fluxes flowing into each volume element

$$\nabla(n_e\mathbf{u}_e)=Z\nabla(n_i\mathbf{u}_i). \tag{1.19}$$

In many cases (1.19) is responsible for the absence of any electric current in the direction of the density gradient, i.e., it is responsible for the balance between the corresponding components of the directed velocities of the electrons and ions

$$u_{e\|}=u_{i\|},\quad \mathbf{u}_{e\perp}=\mathbf{u}_{i\perp}. \tag{1.20}$$

This relation holds, for example, for diffusion in a dielectric chamber (discussed in greater detail in Sec. 3 of Part I).

We note that (1.18) and (1.19) do not impose any limitations on the particle flux in the direction perpendicular to the density gradient [it is evident that $\nabla(n\mathbf{u}_T)=0$]. This follows because this flux causes no change in the particle density. The diffusion regime for which (1.20) holds is called ambipolar diffusion.

We now consider ambipolar diffusion in a weakly ionized gas, where the basic mechanisms are electron-neutral and ion-neutral collisions ($\nu_{en} \gg \nu_{ei}$, $\nu_{in} \gg \nu_{ie}$). In this case the directed electron and ion velocities are given by (1.10)—(1.12). Substituting these relations in (1.20) and taking account of (1.18) we determine the space-charge electric field required to maintain ambipolar diffusion:*

*The irrotational electric field given by (1.21) and (1.22) can exist if the density distribution is a product of functions that depend on the longitudinal and transverse coordinates $n = n_\|(\mathbf{r}_\|) \times n_\perp(\mathbf{r}_\perp)$.

$$E_{\parallel} = \frac{(D_{i\parallel}-D_{e\parallel})\nabla_{\parallel} n_e}{(\mu_{i\parallel}-\mu_{e\parallel})\, n_e} \approx -\frac{T_e \nabla_{\parallel} n_e}{e n_e}, \tag{1.21}$$

$$\mathbf{E}_{\perp} = \frac{(D_{i\perp}-D_{e\perp})\nabla_{\perp} n_e}{(\mu_{i\perp}-\mu_{e\perp})\, n_e} \approx -\frac{T_e\left[1-\frac{\omega_e\omega_i T_i}{Z\nu_{en}\nu_{in}T_e}\right]\nabla_{\perp} n_e}{e\left[1+\frac{\omega_e\omega_i}{\nu_{en}\nu_{in}}\right] n_e}. \tag{1.22}$$

The approximate relations in (1.21) and (1.22) as well as the following relations, (1.23)—(1.25), are obtained under the assumption that $m_i\nu_{in} \gg Zm_e\nu_{en}$, $m_i\nu_{in} \gg T_e/T_i m_e\nu_{en}$; these conditions are satisfied in almost all cases since $m_i \gg m_e$.

It is evident from (1.21) that the longitudinal electric field associated with ambipolar diffusion tends to accelerate the diffusional motion of the ions and to retard the motion of the electrons. In weak magnetic fields the transverse electric field also accelerates the ion diffusion; in strong fields, in which the ion diffusion coefficient is larger than the electron coefficient, the electric field changes sign.

Using (1.10), (1.11), (1.21), (1.22) we find the ambipolar diffusion rates for electrons and ions

$$\left.\begin{aligned} u_{en} &= u_{in} = -D_{a\parallel}\frac{\nabla_{\parallel} n_e}{n_e}, \\ D_{a\parallel} &= \frac{T_i+ZT_e}{m_i\nu_{in}+Zm_e\nu_{en}} \approx \frac{T_i+ZT_e}{m_i\nu_{in}}; \end{aligned}\right\} \tag{1.23}$$

$$\left.\begin{aligned} \mathbf{u}_{e\perp} &= \mathbf{u}_{i\perp} = -D_{a\perp}\frac{\nabla_{\perp} n_e}{n_e}, \\ D_{a\perp} &= \frac{T_i+ZT_e}{m_i\nu_{in}+Zm_e\nu_{en}+\frac{Zm_e\omega_e^2}{\nu_{en}}+\frac{m_i\omega_i^2}{\nu_{in}}} \approx \frac{T_i+ZT_e}{m_i\nu_{in}\left(1+\frac{\omega_e\omega_i}{\nu_{en}\nu_{in}}\right)}. \end{aligned}\right\} \tag{1.24}$$

The quantities $D_{\alpha\parallel}$ and $D_{\alpha\perp}$ introduced here are called the longitudinal and transverse coefficients of ambipolar diffusion.

Using (1.12) and (1.22) it is not difficult to find the particle velocities in the direction perpendicular to the density gradient in ambipolar diffusion:

$$\left.\begin{aligned} \mathbf{u}_{e\top} &\approx -\frac{c(T_i+ZT_e)[\mathbf{h}\nabla n_e]}{ZH\left(1+\frac{\nu_{en}\nu_{in}}{\omega_e\omega_i}\right)\left(1+\frac{\nu_{en}^2}{\omega_e^2}\right)n_e}, \\ \mathbf{u}_{i\top} &\approx \frac{cm_e\nu_{en}(T_i+ZT_e)\,[\mathbf{h}\nabla n_i]}{Z^2eHm_i\nu_{in}\left(1+\frac{\nu_{en}\nu_{in}}{\omega_e\omega_i}+\frac{\omega_i^2}{\nu_{in}^2}\right)n_i}. \end{aligned}\right\} \tag{1.25}$$

These velocities determine the density of the diamagnetic current which accompanies ambipolar diffusion

$$\mathbf{j} = Zen_i\mathbf{u}_i - en_e\mathbf{u}_e \approx \frac{c\,(T_i+ZT_e)}{ZH\left(1+\frac{\nu_{en}\nu_{in}}{\omega_e\omega_i}\right)}[\mathbf{h}\nabla n_e] \ (\text{for } \nu_{en} \ll \omega_e). \tag{1.26}$$

4. Diffusion in a fully ionized gas. We now treat diffusion in a fully ionized gas consisting of electrons and ions of charge Ze.

We note that each volume element in a fully ionized gas (if $\nabla H \perp H$) is subject to only two forces: the pressure gradient $\nabla(p_i + p_e)$ and the magnetic pressure gradient $\nabla(H^2/8\pi)$, which are perpendicular to the magnetic field. Diffusion, i.e., uniform motion of the gas, can obtain when these forces are in equilibrium. For this reason the notion of longitudinal diffusion in a fully ionized gas is, in general, not meaningful: if a pressure gradient exists the longitudinal motion of the plasma is an accelerated motion. The transverse diffusion of a fully ionized gas can be treated meaningfully in terms of equilibrium of the gas as a whole:

$$-\nabla_{\perp}(p_i+p_e) = \nabla_{\perp}\left(\frac{H^2}{8\pi}\right). \tag{1.27}$$

The magnetic field in the diffusion region can be taken as uniform only if the kinetic pressure is much smaller than the magnetic pressure:

$$\beta = \frac{8\pi(p_i+p_e)}{H^2} \ll 1. \tag{1.28}$$

The transverse directed motion of the electrons and ions is given by (1.7)

$$\left.\begin{aligned} e\mathbf{E} + \frac{e[\mathbf{u}_e\mathbf{H}]}{c} + T_e\frac{\nabla n_e}{n_e} + m_e\nu_{ei}(\mathbf{u}_e-\mathbf{u}_i) &= 0, \\ Ze\mathbf{E} + \frac{Ze[\mathbf{u}_i\mathbf{H}]}{c} - T_i\frac{\nabla n_i}{n_i} - m_i\nu_{ie}(\mathbf{u}_i-\mathbf{u}_e) &= 0. \end{aligned}\right\} \tag{1.29}$$

In finding the common solution of these equations we must keep in mind the neutrality condition (1.18) and the relation between the collision frequencies (1.4). Solving the system in (1.29) it is an easy matter to find the directed particle velocities. The electron and ion velocities are the same in the direction of the density gradient:

$$\mathbf{u}_{e\perp} = \mathbf{u}_{i\perp} = -D_{\perp}\frac{\nabla_{\perp} n_e}{n_e}, \quad D_{\perp} = \frac{\left(T_e+\frac{1}{Z}T_i\right)\nu_{ei}}{m_e\omega_e^2}, \tag{1.30}$$

that is to say, the diffusion of a fully ionized gas across a magnetic field is ambipolar regardless of the magnitude of the electric field in the plasma. There is no particle flux in the direction of the electric field.

The electron and ion velocities in the direction perpendicular to the density gradient and the field are given by

$$\left.\begin{aligned} \mathbf{u}_{e\top} &= -\frac{c}{H}[\mathbf{h}\mathbf{E}] - \frac{cT_e}{eH}\frac{[\mathbf{h}\nabla n_e]}{n_e}, \\ \mathbf{u}_{i\top} &= -\frac{c}{H}[\mathbf{h}\mathbf{E}] + \frac{cT_i}{ZeH}\frac{[\mathbf{h}\nabla n_i]}{n_i}. \end{aligned}\right\} \tag{1.31}$$

These velocities determine the electron and ion drift that exists regardless of electron-ion collisions.

The density of the diamagnetic current in a fully ionized gas is

$$\mathbf{j} = n_e e(\mathbf{u}_{i\top} - \mathbf{u}_{e\top}) = \frac{c}{ZH}(T_i+ZT_e)[\mathbf{h}\nabla n_e]. \tag{1.32}$$

The relations in (1.30)—(1.32) coincide with the corresponding expressions obtained from the kinetic analysis when $\omega_e \gg \nu_{ei}$ [20-30] if the effective electron-ion collision frequency is taken to be

$$\nu_{ei} = \frac{4\sqrt{2\pi}}{3}n_i L\left(\frac{Ze^2}{T_e}\right)^2\left(\frac{T_e}{m_e}\right)^{1/2} \tag{1.33}$$

(L is the Coulomb logarithm [6]).

Table I

	Longitudinal diffusion coefficient $D_{\parallel}$	Longitudinal electric field in ambipolar diffusion $E_{\parallel}$	Transverse diffusion coefficient $D_{\perp}$	Diamagnetic current density $\mathbf{j}$	Transverse electric field in ambipolar diffusion $E_{\perp}$	Conditions under which formula applies
Diffusion of electrons in neutral gas	$\frac{T_e}{m_e\nu_{en}}$	—	$\frac{T_e}{m_e\nu_{en}\left(1+\frac{\omega_e^2}{\nu_{en}^2}\right)}$	$\frac{cT_e\,[\mathbf{h}\nabla n_e]}{H\left(1+\frac{\nu_{en}^2}{\omega_e^2}\right)}$	—	$l_{\parallel}\gg\lambda_e$, $l_{\perp}\gg\lambda_e$ for $\nu_{en}>\omega_e$, $l_{\perp}\gg\bar{\varrho}_e$ for $\omega_e>\nu_{en}$
Diffusion of ions in neutral gas	$\frac{T_i}{m_i\nu_{in}}$	—	$\frac{T_i}{m_i\nu_{in}\left(1+\frac{\omega_i^2}{\nu_{in}^2}\right)}$	$\frac{cT_i\,[\mathbf{h}\nabla n_i]}{H\left(1+\frac{\nu_{in}^2}{\omega_i^2}\right)}$	—	$l_{\parallel}\gg\lambda_i$, $l_{\perp}\gg\lambda_i$ for $\nu_{in}>\omega_i$, $l_{\perp}\gg\bar{\varrho}_i$ for $\omega_i>\nu_{in}$
Ambipolar diffusion in a weakly ionized gas ($\nu_{en}\gg\nu_{ei}$)	$\frac{T_i+ZT_e}{m_i\nu_{in}}$	$-\frac{T_e}{e}\frac{\nabla_{\parallel}n}{n}$	$\frac{T_i+ZT_e}{m_i\nu_{in}\left(1+\frac{\omega_e\omega_i}{\nu_{en}\nu_{in}}\right)}$	$\frac{c\,(T_i+ZT_e)\,[\mathbf{h}\nabla n_e]}{ZH\left(1+\frac{\nu_{en}\nu_{in}}{\omega_e\omega_i}\right)}$	$-\frac{T_e\nabla_{\perp}n_e\left(1-\frac{T_i\omega_e\omega_i}{ZT_e\nu_{en}\nu_{in}}\right)}{en_e\left(1+\frac{\omega_e\omega_i}{\nu_{en}\nu_{in}}\right)}$	$l_{\parallel}\gg\lambda_i$, $l_{\perp}\gg\lambda_i$ for $\nu_{en}\nu_{in}>\omega_e\omega_i$, $l_{\perp}\gg\bar{\varrho}_e$ for $\omega_e\omega_i>\nu_{en}\nu_{in}$
Diffusion in a fully ionized gas	—	—	$\frac{(T_i+ZT_e)\,\nu_{ei}}{Zm_e\omega_e^2}$	$\frac{c}{ZH}(T_i+ZT_e)\,[\mathbf{h}\nabla n_e]$	—	$l_{\perp}\gg\bar{\varrho}_i\frac{\nu_{ei}}{\omega_e}$ for $\nu_{ei}>\omega_e$, $l_{\perp}\gg\bar{\varrho}_i$ for $\omega_e>\nu_{ei}$
Ambipolar diffusion in a highly ionized gas	$\frac{T_i+ZT_e}{m_i\nu_{in}}$	$-\frac{T_e}{e}\frac{\nabla_{\parallel}n}{n}$	$\frac{T_i+ZT_e}{m_i\nu_{in}\left[1+\frac{\omega_e\omega_i}{(\nu_{en}+\nu_{ei})\,\nu_{in}}\right]}$	$\frac{c\,(T_i+ZT_e)\,[\mathbf{h}\nabla n_e]}{ZH\left[1+\frac{(\nu_{en}+\nu_{ei})\,\nu_{in}}{\omega_e\omega_i}\right]}$	$-\frac{T_e\nabla_{\perp}n_e}{en_e}\times\frac{1-\frac{T_i}{ZT_e}\frac{\omega_e\omega_i}{(\nu_{en}+\nu_{ei})\,\nu_{in}}}{1+\frac{\omega_e\omega_i}{(\nu_{en}+\nu_{ei})\,\nu_{in}}}$	Combination of the preceding conditions

5. Ambipolar diffusion in a highly ionized gas. We now consider diffusion in a gas containing electrons, positive ions of one kind, and neutral particles, when the collision frequencies for collisions between the charged particles and between the charged and neutral particles are comparable (highly ionized gas). In this case the equations in (1.7) become

$$\left.\begin{aligned} &e\mathbf{E}+\frac{e\,[\mathbf{u}_e\mathbf{H}]}{c}+T_e\frac{\nabla n_e}{n_e}+m_e\nu_{ei}(\mathbf{u}_e-\mathbf{u}_i)+m_e\nu_{en}\mathbf{u}_e=0,\\ &Ze\mathbf{E}+\frac{Ze\,[\mathbf{u}_i\mathbf{H}]}{c}-T_i\frac{\nabla n_i}{n_i}-m_i\nu_{ie}(\mathbf{u}_i-\mathbf{u}_e)-m_i\nu_{in}\mathbf{u}_i=0.\end{aligned}\right\} \quad (1.34)$$

Here we have taken account of plasma neutrality and the relation between the collision frequencies (1.4). As before, it is assumed that the directed velocity of the neutrals is much smaller than the directed velocity of the charged particles.

The common solution of (1.34) determines the velocities $\mathbf{u}_e$ and $\mathbf{u}_i$. We do not write the complicated expressions that result here. Equating the velocity components of the electrons and ions in the direction of the density gradient it is easy to find the ambipolar velocity of the particles and the electric field responsible for the ambipolar diffusion. The longitudinal and transverse components of these quantities are

$$\mathbf{u}_{\parallel}=-D_{a\parallel}\frac{\nabla_{\parallel}n_e}{n_e},\quad D_{a\parallel}=\frac{T_i+ZT_e}{m_i\nu_{in}}, \quad (1.35)$$

$$\mathbf{E}_{\parallel}=-\frac{T_e}{e}\frac{\nabla_{\parallel}n_e}{n_e}, \quad (1.36)$$

$$\mathbf{u}_{\perp}=-D_{a\perp}\frac{\nabla_{\perp}n_e}{n_e},\quad D_{a\perp}=\frac{T_i+ZT_e}{m_i\nu_{in}\left[1+\frac{\omega_e\omega_i}{(\nu_{ei}+\nu_{en})\nu_{in}}\right]}, \quad (1.37)$$

$$\mathbf{E}_{\perp}=-\frac{T_e\left[1-\frac{T_i}{ZT_e}\frac{\omega_e\omega_i}{(\nu_{ei}+\nu_{en})\,\nu_{in}}\right]}{e\left[1+\frac{\omega_e\omega_i}{(\nu_{ei}+\nu_{en})\,\nu_{in}}\right]}\frac{\nabla_{\perp}n_e}{n_e}. \quad (1.38)$$

We also give the formula for the current density in ambipolar diffusion ($\nu_{en}\ll\omega_e$)

$$\mathbf{j}=n_e e\,(\mathbf{u}_{i\top}-\mathbf{u}_{e\top})=-\frac{c\,(T_i+ZT_e)\,\nabla n_e}{ZH\left[1+\frac{\nu_{in}(\nu_{ei}+\nu_{en})}{\omega_e\omega_i}\right]}. \quad (1.39)$$

The relations in (1.35)—(1.39) are obtained under the assumption that $m_i\nu_{in}\gg m_e\nu_{en}$. A kinetic analysis of diffusion in a plasma consisting of electrons, ions, and neutrals is given in [43] for the case in which the "diffusion" electron-neutral and ion-neutral collision frequencies are velocity independent and the inequality $m_e\nu_{ei}\ll m_i\nu_{in}$ is satisfied. The expressions for the ambipolar diffusion rate and electric field given in [43] are approximately the same as the expressions given above if ν_{en}, ν_{in}, and ν_{ei} are taken from (1.16), (1.17), and (1.33).

6. Summary of results. In Table I we summarize the formulas for the longitudinal and transverse diffusion coefficients, the electric field responsible for ambipolar diffusion, and the diamagnetic current of charged particles. The conditions for which the formulas apply are also given. These conditions follow from the inequality in (1.6), according to which the directed velocity of the particles must be much smaller than the thermal velocity if diffusion motion is to occur. In order to introduce (1.6) explicitly we have used

the formulas given above for the components of the directed velocity $u_{\parallel}$, $u_{\perp}$, and u_T (the electron and ion temperatures are assumed to be of the same order of magnitude). In the table, the quantities $l_{\parallel} \approx n/|\nabla_{\parallel} n|$ and $l_{\perp} = n/|\nabla_{\perp} n|$ are the characteristic scale sizes of the inhomogeneous region of the plasma; $\lambda_i = (1/\nu_{in}) \times \sqrt{2T_i/m_i}$ and $\lambda_e = (1/\nu_{en})\sqrt{2T_e/m_e}$ are the mean free paths; $\bar{\rho}_i = (1/\omega_i)\sqrt{2T_i/m_i}$ and $\bar{\rho}_e = (1/\omega_e) \times \sqrt{2T_e/m_e}$ are the Larmor radii for the ions and electrons respectively.

2. Transverse Diffusion in a Strong Magnetic Field

1. Fundamental relations. In this section we treat the transverse diffusion of charged particles in strong magnetic fields; in strong magnetic fields the ions and electrons execute many revolutions about the magnetic force lines in the time interval between collisions and the Larmor radii of the particles are much smaller than the characteristic scale sizes, i.e.,

$$\omega_e \gg \nu_e, \ \omega_i \gg \nu_i, \ \bar{\varrho}_e \ll l_{\perp}, \ \bar{\varrho}_i \ll l_{\perp}. \tag{2.1}$$

Under these conditions the effect of particle collisions on the motion can be treated as a small perturbation.

In the absence of collisions the motion of charged particles in a magnetic field can be conveniently regarded in terms of rotation at the Larmor frequency about the guiding centers. The coordinates of the guiding center and the particle coordinates are related by

$$\mathbf{R}_\alpha = \mathbf{r}_\alpha + \boldsymbol{\varrho}_\alpha, \ \boldsymbol{\varrho}_\alpha = \frac{cm_\alpha}{Z_\alpha eH}[\mathbf{w}_\alpha \mathbf{h}]. \tag{2.2}$$

Here, $\mathbf{R}_\alpha$ is the radius vector of the guiding center, $\mathbf{r}_\alpha$ is the radius vector of the particle, ρ_α is the Larmor radius and $\mathbf{w}_\alpha$ is the rotation rate.

The guiding centers themselves can move with arbitrary velocity along the magnetic field ($\mathbf{w}_{\alpha\parallel}$). Furthermore, in a transverse electric field the guiding centers drift with velocity

$$\mathbf{u}_E = \frac{c}{H}[\mathbf{E}\mathbf{h}]. \tag{2.3}$$

Thus, the vector velocity of the particle is given by the sum

$$\mathbf{v}_\alpha = \mathbf{w}_{\alpha\perp} + \mathbf{w}_{\alpha\parallel} + \mathbf{u}_E. \tag{2.4}$$

The rotational velocity of particles, the longitudinal velocity, the rotational phases φ and the transverse coordinates of the guiding center $\mathbf{R}_\perp$ (in the presence of a transverse electric field, the coordinate along this field) are integrals of the motion. In the absence of collisions the particle distribution function can be an arbitrary function of the integrals of motion. We assume that the function describing the distribution in velocity $\mathbf{w}$ is Maxwellian with a density depending on the transverse coordinates of the guiding center:*

*It is easily shown that $dw_\perp dw_\parallel d\varphi = \frac{1}{w} dw_x dw_y dw_z = \frac{1}{w} d\mathbf{w}$.

$$\left.\begin{aligned} F_\alpha(\mathbf{R}_{\alpha\perp}, w_{\alpha\perp}, w_{\alpha\parallel}, \varphi)\, dw_{\alpha\perp}\, dw_{\alpha\parallel}\, d\varphi &= n_\alpha(\mathbf{R}_{\alpha\perp}) f_\alpha(\mathbf{v}_\alpha)\, d\mathbf{w}_\alpha, \\ f_\alpha(\mathbf{v}_\alpha) &= \left(\frac{m_\alpha}{2\pi T_\alpha}\right)^{3/2} e^{-\frac{m_\alpha(\mathbf{v}_\alpha - \mathbf{u}_E)^2}{2T_\alpha}}. \end{aligned}\right\} \tag{2.5}$$

The distribution in (2.5) leads to the familiar expression for the mean particle drift velocity under the effect of a density gradient

$$\mathbf{u}_{\alpha g} = \int \mathbf{w} n_\alpha(\mathbf{R}_{\alpha\perp}) f_\alpha(\mathbf{w}_\alpha)\, d\mathbf{w}_\alpha = \frac{cT_\alpha}{Z_\alpha eHn_\alpha}[\mathbf{h}\nabla n_\alpha]. \tag{2.6}$$

The displacement of the guiding centers (correspondingly, the displacement of the particles themselves) across the magnetic field in the direction of a density gradient or an electric field is a result of collisions. Under the assumptions made above the particle flux can be found from the theory of random fluctuating motion developed by Chandrasekhar.[12,46]

We take the OZ axis along the magnetic field and the OX axis in the direction of the density gradient. The flux of guiding centers through the plane X = const is given by the obvious relation

$$\Gamma_\alpha = \frac{1}{\Delta t}\left\{\int_0^\infty d(\Delta X) \int_0^{\Delta X} dX' [n_\alpha(X - X') W_\alpha(X - X', \Delta X) - n_\alpha(X + X') W_\alpha(X + X', -\Delta X)]\right\}, \tag{2.7}$$

where $W_\alpha(X, \Delta X)$ is the probability that in a time Δt a guiding center located at point X will be displaced by a distance ΔX. The quantity Δt extends over many collisions but is such that the mean displacement is appreciably smaller than the characteristic scale size $(1/\nu_\alpha \ll \Delta t \ll l^2/\bar{\rho}_\alpha^2 \nu_\alpha)$. In this case we can write $n_\alpha W_\alpha$ as an expansion

$$n_\alpha(X - X') W_\alpha(X - X', \Delta X) = n_\alpha(X) W_\alpha(X, \Delta X) - X' \frac{\partial}{\partial X}(n_\alpha W_\alpha). \tag{2.8}$$

Substituting (2.8) in (2.7), integrating over X′, and averaging over the velocities of particles of type α we find the following expression for the flux:

$$\Gamma_\alpha = n_\alpha \langle \Delta X_\alpha \rangle - \frac{1}{2}\frac{\partial}{\partial X}[n_\alpha \langle (\Delta X_\alpha)^2 \rangle]. \tag{2.9}$$

Here, the symbol $\langle \ldots \rangle$ denotes summation over collisions per unit time and averaging over velocity:

$$\langle \Delta X_\alpha \rangle = \frac{1}{\Delta t}\int_{-\infty}^{\infty} \overline{(\Delta X) W_\alpha(X_\alpha, \Delta X)}\, d(\Delta X). \tag{2.10}$$

The averaging process is carried out for a fixed value of X_α.

If the gas contains many particle types, the transverse flux of particles of each type in the direction of the density gradient can obviously be written in the form of a sum of fluxes associated with collisions of particles of the given type with particles of all other types:

$$\Gamma_\alpha = \sum_{(\beta)} \Gamma_{\alpha\beta}, \quad \Gamma_{\alpha\beta} = n_\alpha \langle \Delta X_{\alpha\beta} \rangle - \frac{1}{2}\frac{\partial}{\partial X}[n_\alpha \langle (\Delta X_{\alpha\beta})^2 \rangle] \tag{2.11}$$

($\Delta X_{\alpha\beta}$ is the displacement of a particle of type α as a result of collisions with particles of type β). Hence, in what follows we treat separately diffusion across the magnetic field associated with different kinds of collisions.

When the particle Larmor radius is much greater than the interaction range the magnetic field has essentially no effect on the collision. In these cases the collision changes the velocity of the charged particle by an amount $\Delta \mathbf{v}_\alpha$; in accordance with (2.2) and (2.4) the displacement of the guiding center is then given by

$$\Delta\mathbf{R}_\alpha = \frac{m_\alpha c}{Z_\alpha eH}[\Delta\mathbf{v}_\alpha\mathbf{h}],$$

$$\Delta R_\alpha = \frac{|\Delta v_{\alpha\perp}|}{\omega_\alpha}, \quad \Delta X_\alpha = \frac{m_\alpha c\Delta v_{\alpha y}}{Z_\alpha eH}. \tag{2.12}$$

In collisions in which the relative change of transverse velocity is appreciable the guiding center is displaced by a distance of the order of the Larmor radius (Fig. 1).

Summation of the displacements due to collisions of particles of type α with particles of type β and averaging over velocity yield

$$\langle\Delta X_{\alpha\beta}\rangle = \frac{cm_\alpha}{Z_\alpha eH}\int\limits_{(\mathbf{v}_\alpha)} f_\alpha(\mathbf{v}_\alpha)\,d\mathbf{v}_\alpha\int\limits_{(\mathbf{v}_\beta)} n_\beta(X_\beta)f_\beta(\mathbf{v}_\beta)\,d\mathbf{v}_\beta \times\int\limits_{(\Omega)}(\Delta v_{\alpha y})v\sigma_{\alpha\beta}(v,\vartheta)\,d\Omega, \tag{2.13}$$

where $v = |\mathbf{v}_\alpha - \mathbf{v}_\beta|$ is the relative velocity.

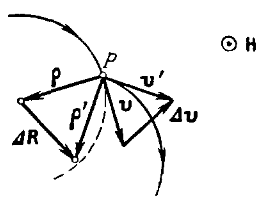

FIG. 1

The collision integral is computed in the usual way:[19]

$$\int\limits_{(\Omega)}(\Delta v_{\alpha y})v\sigma_{\alpha\beta}(v,\vartheta)\,d\Omega = -\frac{m_\beta}{m_\alpha+m_\beta}v_y v s^*_{\alpha\beta}, \tag{2.14}$$

$$s^*_{\alpha\beta} = \int\limits_{(\Omega)}\sigma_{\alpha\beta}(v,\vartheta)(1-\cos\vartheta)\,d\Omega. \tag{2.15}$$

In the integrals in (2.13) it is convenient to change from the velocity variables $\mathbf{v}_\alpha$ and $\mathbf{v}_\beta$ to the velocities

$$\mathbf{v} = \mathbf{v}_\alpha - \mathbf{v}_\beta, \quad \mathbf{v}_0 = \frac{m_\alpha T_\beta\mathbf{v}_\alpha + m_\beta T_\alpha\mathbf{v}_\beta}{m_\alpha T_\beta + m_\beta T_\alpha}. \tag{2.16}$$

Substituting (2.14) in (2.13) and taking account of (2.16) we find

$$\langle\Delta X_{\alpha\beta}\rangle = -\frac{\mu_{\alpha\beta}c}{Z_\alpha eH}\int\limits_{(\mathbf{v}_0)} d\mathbf{v}_0\int\limits_{(\mathbf{v})} d\mathbf{v}f_\alpha(\mathbf{v}_\alpha)f_\beta(\mathbf{v}_\beta)n_\beta(X_\beta)vv_y s^*_{\alpha\beta}(v). \tag{2.17}$$

Here, $\mu_{\alpha\beta}$ is the reduced mass

$$\mu_{\alpha\beta} = \frac{m_\alpha m_\beta}{m_\alpha + m_\beta}. \tag{2.18}$$

In (2.17) we substitute expressions for the distribution functions (2.5) in which collisions are neglected inasmuch as collisions have very little effect on the motion of the charged particles for the conditions being considered [cf. (2.1)].

The expression for $\langle(\Delta X_{\alpha\beta})^2\rangle$ is obtained in similar fashion:

$$\langle(\Delta X_{\alpha\beta})^2\rangle = \frac{2}{3}\frac{\mu^2_{\alpha\beta}c^2}{Z^2_\alpha e^2H^2}n_\beta\int\limits_{(\mathbf{v}_0)} d\mathbf{v}_0\int\limits_{(\mathbf{v})} d\mathbf{v}f_\alpha(\mathbf{v}_\alpha)f_\beta(\mathbf{v}_\beta)v^3 s_{\alpha\beta}(v). \tag{2.19}$$

In this expression we have neglected the variation in n_β under the integral sign, i.e., in a change of coordinate by an amount of the order of the Larmor radius. Substituting the Maxwellian distribution function in the integral in (2.19) it is an easy matter to reduce it to the form*

$$\langle(\Delta X_{\alpha\beta})^2\rangle = \frac{8\pi}{3}\frac{\mu^2_{\alpha\beta}c^2}{Z^2_\alpha e^2H^2}n_\beta\left(\frac{\mu_{\alpha\beta}}{2\pi T_{\alpha\beta}}\right)^{3/2}\int\limits_0^\infty v^5 s^*_{\alpha\beta}(v)e^{-\frac{\mu_{\alpha\beta}v^2}{2T_{\alpha\beta}}}dv, \tag{2.20}$$

$$T_{\alpha\beta} = \frac{m_\alpha T_\beta + m_\beta T_\alpha}{m_\alpha + m_\beta}. \tag{2.21}$$

Using (2.11), (2.17), and (2.20) we can determine the fluxes associated with collisions between particles of different types.

2. Diffusion caused by collisions between charged particles and neutrals. As before, we assume that the density of neutral particles is independent of coordinates and that the velocity distribution is Maxwellian.

Under these conditions, (2.17), which describes the displacement of charged particles of type α caused by collisions with neutrals, is reduced to the following form after substitution of (2.5):

$$\Delta X_{\alpha n} = \frac{4\pi\mu^2_{\alpha n}c^2 n_n E_x}{3Z_\alpha eH^2 T_{\alpha n}}\left(\frac{\mu_{\alpha n}}{2\pi T_{\alpha n}}\right)^{3/2}\int\limits_0^\infty v^5 s^*_{\alpha n}(\vartheta)e^{-\frac{\mu_{\alpha n}v^2}{2T_{\alpha n}}}dv. \tag{2.22}$$

We shall be interested here only in terms proportional to the first power of E (the drift velocity u_E is assumed to be small compared with the thermal velocity $\sqrt{T_\alpha/m_\alpha}$).

The relations in (2.11), (2.20), and (2.22) yield an expression for the transverse flux of charged particles caused by collisions with neutrals:

$$\Gamma_{\alpha n} = \frac{Z_\alpha\nu_{\alpha n}}{m_\alpha\omega^2_\alpha}n_\alpha eE_x - \frac{T_{\alpha n}\nu_{\alpha n}}{m_\alpha\omega^2_\alpha}\frac{\partial n_\alpha}{\partial X}, \tag{2.23}$$

in which the effective collision frequency $\nu_{\alpha n}$ is given by

$$\nu_{\alpha n} = \frac{8\mu_{\alpha n}}{3\sqrt{\pi}m_\alpha}\left(\frac{\mu_{\alpha n}}{2T_{\alpha n}}\right)^{5/2}\int\limits_0^\infty v^5 s_{\alpha\beta}(v)e^{-\frac{\mu_{\alpha\beta}v^2}{2T_{\alpha\beta}}}dv. \tag{2.24}$$

*In obtaining (2.20) we have assumed $\mathbf{v} = \mathbf{w}$ in (2.5), taking account of the fact that the drift velocity u_E is much smaller than the thermal velocities $\sqrt{T_\alpha/m_\alpha}$ and $\sqrt{T_\beta/m_\beta}$.

It should be noted that (2.23) and (2.24) can be simplified for electrons since $m_e \ll m_n$ and [cf. (2.21)]

$$T_{en} \approx T_e, \quad \mu_{en} \approx m_e.$$

The ion temperature can be taken equal to the temperature of the neutral gas:

$$T_i \approx T_n, \quad T_{in} \approx T_i.$$

A complete equilibration of energy between ions and atoms of comparable mass occurs within a few collisions. In this time period the ion is displaced by several Larmor radii, that is to say, by a distance appreciably smaller than the characteristic scale size [cf. (2.1)]. Hence, we may assume that thermal equilibrium obtains between the ions and atoms during the diffusion time.

The relation in (2.23) gives the corresponding values for the diffusion coefficients and the directed velocities of the particles:

$$\left.\begin{aligned} D_{\alpha n} &= \frac{T_\alpha \nu_{\alpha n}}{m_\alpha \omega_\alpha^2} = \nu_{\alpha n} \bar{\varrho}_\alpha^2, \\ \bar{v}_E &= \frac{Z_\alpha \nu_{\alpha n}}{m_\alpha \omega_\alpha^2} eE = -\nu_{\alpha n} \left(\frac{m_\alpha c}{Z_\alpha eH}\right) u_E. \end{aligned}\right\} \quad (2.25)$$

The physical meaning of these expressions is clear. The diffusion coefficient is equal to half the mean square displacement of the particle per unit time. Inasmuch as the particle is displaced by a distance of the order of the Larmor radius ρ_α in each collision (cf. Fig. 1), the diffusion coefficient must be of order $\nu_{\alpha n}\bar{\rho}_\alpha^2$ (2.25).

In accordance with (2.12) (cf. Fig. 1) the displacement of the particle in the direction of the electric field is determined by the change of velocity in the perpendicular direction $\Delta X_\alpha = (m_\alpha c/Z_\alpha eH)\Delta v_{\alpha y}$. Correspondingly, the mean velocity in the direction of the field must be proportional to the number of collisions and the mean velocity in the direction perpendicular to the field $\langle \Delta X_\alpha \rangle = -\nu_{\alpha n}(m_\alpha c/Z_\alpha eH)\bar{v}_{\alpha y}$ as given by (2.25).

The relation in (2.23) can also be used to determine the rate of ambipolar diffusion caused by collisions of charged particles with neutrals; this has been done in an earlier section. In the case of a gas consisting of electrons, one kind of positive ion, and neutrals, the relation that determines the ambipolar diffusion rate is obtained by equating Γ_{en} and $(1/Z)\Gamma_{in}$:

$$u_x = \frac{\Gamma_{\alpha n}}{n_\alpha} = -D_{\alpha\perp}\frac{\partial n}{\partial X}, \quad D_{\alpha\perp} = \frac{\left(T_e + \frac{1}{Z}T_i\right)\nu_{en}}{m_e \omega_e^2}. \quad (2.26)$$

In this case the electric field is given approximately by

$$E_x \approx \frac{T_i}{eZ_i n_e}\frac{\partial n_e}{\partial X}. \quad (2.27)$$

3. Diffusion caused by collisions of charged particles with each other. [35] When the effect of collisions of charged particles with each other is important in diffusion, the time for exchanging energy between charged particles of different types is found to be much smaller than the diffusion time. For example, equilibration of energy between electrons and ions requires m_i/m_e collisions.* This same number of collisions is required for the electrons or ions to diffuse a distance of the order of the ion Larmor radius $\bar{\rho}_i$ as a result of collisions. The relation in (2.1) indicates that the characteristic scale sizes of the plasma are appreciably greater than this so that thermal equilibrium must be established between the charged particles during the diffusion time. Using this result, we assume below that the temperatures of charged particles of different kinds are the same:

$$T_\alpha = T_\beta = T_{\alpha\beta} = T. \quad (2.28)$$

We now use (2.17) to determine the mean displacement in collisions of charged particles $\Delta X_{\alpha\beta}$. In the calculation we must take account of the coordinate dependence of the density of particles of type β

$$n_\beta(X_\beta) = n_\beta(X_\alpha) + (X_\beta - X_\alpha)\frac{\partial n_\beta}{\partial X}. \quad (2.29)$$

The difference in coordinates $X_\beta - X_\alpha$ in a collision (for a fixed value of X_α) is, in accordance with (2.2), (2.4), and (2.16),

$$\begin{aligned} X_\beta - X_\alpha &= \frac{c}{eH}\left(\frac{m_\beta w_{y\beta}}{Z_\beta} - \frac{m_\alpha w_{y\alpha}}{Z_\alpha}\right) \\ &= \frac{c}{eH}(v_{0y} - u_{Ey})\left(\frac{m_\beta}{Z_\beta} - \frac{m_\alpha}{Z_\alpha}\right) - \frac{c\mu_{\alpha\beta}}{eH}v_y\left(\frac{1}{Z_\alpha} + \frac{1}{Z_\beta}\right). \quad (2.30) \end{aligned}$$

The quantity $\Delta X_{\alpha\beta}$ can be computed easily by substituting (2.29) and (2.30) in (2.17). In (2.29) we omit terms proportional to the second or higher derivatives of the density since the change of density within a Larmor radius is small; thus $\langle \Delta X_{\alpha\beta} \rangle$ is

$$\begin{aligned} \langle \Delta X_{\alpha\beta} \rangle &= \frac{4\pi\mu_{\alpha\beta}^2 c^2}{3Z_\alpha e^2 H^2}\left(\frac{1}{Z_\alpha} + \frac{1}{Z_\beta}\right) \\ &\times \frac{\partial n_\beta}{\partial X}\left(\frac{\mu_{\alpha\beta}}{2\pi T}\right)^{3/2}\int_0^\infty v^5 s_{\alpha\beta}^*(v)\, e^{-\frac{\mu_{\alpha\beta}v^2}{2T}}\, dv. \quad (2.31) \end{aligned}$$

The relations in (2.11), (2.20), and (2.31) give an expression for the diffusion flux caused by the collisions between charged particles:

$$\Gamma_{\alpha\beta} = -\frac{\nu_{\alpha\beta}T}{m_\alpha \omega_\alpha^2}\left(\frac{\partial n_\alpha}{\partial X} - \frac{Z_\alpha}{Z_\beta}\frac{n_\alpha}{n_\beta}\frac{\partial n_\beta}{\partial X}\right), \quad (2.32)$$

in which $\nu_{\alpha\beta}$ is given by (2.24). Substituting the Rutherford cross section in (2.24) and (2.15) we obtain $\nu_{\alpha\beta}$ in explicit form:

$$\nu_{\alpha\beta} = \frac{4Z_\alpha^2 Z_\beta^2 e^4 (2\pi\mu_{\alpha\beta})^{1/2}}{3m_\alpha T^{3/2}} L n_\beta. \quad (2.33)$$

Here, L is the Coulomb logarithm

*Here we consider near collisions, which cause the electron velocity to turn through an angle $\approx \pi/2$, or an aggregate of remote interactions equivalent to a near collision.

$$L = \ln \frac{p_{\max}}{p_{\min}},$$

$$p_{\max} = r_d = \left(\frac{T}{4\pi n e^2} \right)^{1/2},$$

$$p_{\min} = r_0 = \begin{cases} r_h = \dfrac{h}{(\mu_{\alpha\beta} T)^{1/2}} & \text{for} \quad r_h \gg r_c, \\ r_c = \dfrac{e^2}{T} \, | Z_\alpha Z_\beta | & \text{for} \quad r_c \gg r_h. \end{cases} \tag{2.34}$$

If (2.33) is taken into account the diffusion flux (2.23) can be written in the form

$$\Gamma_{\alpha\beta} = -\frac{4}{3} \left(\frac{2\pi \mu_{\alpha\beta}}{T} \right)^{1/2} \frac{c^2 e^2}{H^2} L n_\alpha n_\beta Z_\beta \left(Z_\beta \frac{1}{n_\alpha} \frac{\partial n_\alpha}{\partial X} - Z_\alpha \frac{1}{n_\beta} \frac{\partial n_\beta}{\partial X} \right). \tag{2.35}$$

It should be noted that collisions of charged particles lead to an ambipolar flux in the direction of the density gradient regardless of the electric field. As follows from (2.35):

$$Z_\alpha \Gamma_{\alpha\beta} = -Z_\beta \Gamma_{\beta\alpha}. \tag{2.36}$$

4. Effect of collisions of like charged particles on plasma diffusion across a magnetic field.[35,39] It is evident from (2.35) that collisions of like particles cannot lead to a diffusion flux proportional to the density gradient, i.e.,

$$\Gamma_{\alpha\alpha} = 0. \tag{2.37}$$

In some work (for example, in [6]) this effect is attributed to the fact that the "center of gravity of the guiding centers" is not displaced in collisions of like particles. This explanation is unsatisfactory, however, since diffusion can occur even if the position of the center of gravity of the particles remains fixed.*

It has been pointed out in [35], that in collisions of like particles the diffusion flux, defined as the mean square displacement, is balanced by a flux in the opposite direction proportional to the mean displacement. It is not difficult to understand the origin of this flux. As is evident from Fig. 2, in the collision of particle 1 with the same particle 2 the guiding center of particle 1 is displaced in the direction of particle 2 (along the curve I). In the presence of a density gradient the collisions are more frequent in the direction in which the density is greater. Hence, collisions of like particles lead to a mean displacement and, correspondingly, to a particle flux in the direction of increasing density. This flux balances the usual diffusion flux in the direction of lower density. Complete compensation of the fluxes is obtained from (2.35) if the calculation is carried out with an accuracy up to the first spatial derivative of the density.

Calculations which take account of higher derivatives of the density allow us to determine the diffusion rate caused by collision of like particles.[35] The expression for the diffusion flux is

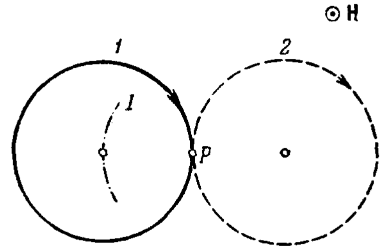

FIG. 2

$$\Gamma_{\alpha\alpha} = \frac{8}{15} \pi^{1/2} \frac{c^4 m_\alpha^2}{H^4} \left(\frac{T}{m_\alpha} \right)^{1/2} L n^2 \frac{\partial}{\partial X} \left(\frac{1}{n_\alpha} \frac{\partial^2 n_\alpha}{\partial X^2} \right)$$
$$\div \frac{c^4 m_\alpha^4}{H^4} \left(\frac{T}{m_\alpha} \right)^{1/2} L \frac{n}{l_\perp^3}. \tag{2.38}$$

In accordance with (2.35) and (2.38) the ratio of the ion flux due to ion-ion collisions to that caused by electron-ion collisions is of order (if $n_i \approx n_e$)

$$\frac{\Gamma_{ii}}{\Gamma_{ie}} \div \left(\frac{m_i}{m_e} \right)^{1/2} \frac{\overline{\varrho_i^2}}{l_\perp^2}. \tag{2.39}$$

It has been noted in reference 35 that under certain conditions the ion flux due to ion-ion collisions (2.39) can become important. However, as is evident from (2.38), collisions of like particles result in different diffusion rates for the electrons and ions (the ions diffuse more rapidly). For this reason a charge separation arises in the plasma and a transverse electric field is produced; in the general case this field is not uniform.[39] The drift velocities of the colliding particles are different in the inhomogeneous electric field (since their guiding centers become spatially separated). As a result of the collisions there arises a "friction force" which is proportional to the mean relative drift velocity and is directed along this velocity (i.e., perpendicular to both the electric and magnetic fields). The friction force produces a particle drift parallel to the electric field. This drift is in opposite directions for particles of opposite charge. Thus, the inhomogeneous electric field changes the particle flux associated with collisions and can lead to ambipolar diffusion mode, i.e., the absence of an electric current in the direction of the gradients. It is shown in [39] that the electric field responsible for ambipolar diffusion in a gas consisting of electrons and singly charged ions is of order

$$E \div \frac{T}{e l_\perp} \tag{2.40}$$

(the field distribution is determined by the condition that the transverse current vanishes). The ratio of the ambipolar diffusion flux, which is proportional to the higher density derivative (Γ^{II}) to the flux proportional to the density gradient (Γ^{I}) is of order

$$\left| \frac{\Gamma^{\mathrm{II}}}{\Gamma^{\mathrm{I}}} \right| \div \frac{\varrho_i^2}{l_\perp^2}. \tag{2.41}$$

Thus, the ambipolar flux, which is proportional to the higher density derivative, that is to say, the flux

*We note from (2.35) that the diffusion flux caused by collisions of particles of different mass, in which the center of gravity of the guiding centers is displaced, can also vanish (for example, if $Z_\alpha = -Z_\beta$, $n_\alpha = n_\beta$).

caused by like-particle collisions is negligibly small and need not be considered under the present conditions [cf. (2.1)].

5. Effect of collisions of ions with different charges on diffusion. If a plasma contains ions of different charge, collisions between these ions can have an important effect on transverse plasma diffusion.[2,37] Each collision of this kind leads to an appreciable change in ion momentum, that is to say, to a displacement of the guiding center of the ion by a distance of the order of the Larmor radius; on the other hand, the collision of an ion with an electron leads to a much smaller displacement of the ion, since the relative change of ion momentum in this case is of the order of the ratio of electron mass to ion mass.

The effect of collisions between ions with different mass can be estimated from (2.35). Given the same relative values of the particle density gradients $\left(\frac{1}{n_\alpha}\frac{\partial n_\alpha}{\partial X} = \frac{1}{n_\beta}\frac{\partial n_\beta}{\partial X}\right)$ the ratio of the diffusion flux of ions with charge $Z_\alpha e$ caused by collisions with ions of charge $Z_\beta e$ to the flux caused by collisions with electrons is

$$\frac{\Gamma_{\alpha\beta}}{\Gamma_{\alpha e}} = \left[\frac{m_\alpha m_\beta}{(m_\alpha + m_\beta)\, m_e}\right]^{1/2} \frac{Z_\beta (Z_\beta - Z_\alpha)\, n_\beta}{(Z_\alpha + 1) n_e}. \tag{2.42}$$

This ratio can be large even if $n_\beta \ll n_e$, i.e., even if there is only a small number of impurity ions with charge different from that of the primary ions; this result follows because the ion masses m_α and m_β are much larger than the electron mass m_e.

In a plasma containing two kinds of positive ions collisions result in diffusion fluxes in opposite directions for ions of different charge [cf. (2.36)]. For the same relative density gradients it follows from (2.35) that the ions with lower charge diffuse in the direction of lower density (the usual diffusion direction) whereas the ions with the higher charge diffuse in the direction of increasing density.

When the ions are formed in the central part of the plasma volume and the ion densities diminish toward the periphery, the ion diffusion for both kinds must also be directed toward the periphery of the volume in the stationary state. This result means that a stationary density distribution cannot have the same relative gradients for both ion species. It is evident that the ions with higher charge will be concentrated in the central region of the volume to a greater extent than those with the smaller charge.

As an example we consider the diffusion of multiply charged impurity ions of type α in a plasma consisting of singly charged ions and electrons (it is assumed that $n_\alpha \ll n_i \approx n_e = n$). The diffusion flux of the impurity ions is given by (2.35)

$$\Gamma_\alpha = \Gamma_{\alpha i} + \Gamma_{\alpha e} = -\frac{4c^2e^2}{3H^2}\left(\frac{2\pi\mu_{\alpha i}}{T}\right)^{1/2} n_\alpha n \left\{\left[\frac{\partial \ln n_\alpha}{\partial X} - Z_\alpha \frac{\partial \ln n}{\partial X}\right] + \left(\frac{m_e}{\mu_{\alpha i}}\right)^{1/2}\left[\frac{\partial \ln n_\alpha}{\partial X} + Z_\alpha \frac{\partial \ln n}{\partial X}\right]\right\}. \tag{2.43}$$

The second term in the curly brackets is much smaller than the first since $m_e \ll \mu_{\alpha i}$. Hence, the following inequality must be satisfied if the flux of impurity ions is to be directed in the direction of diminishing density:

$$\frac{\partial \ln n_\alpha}{\partial X} > Z_\alpha \frac{\partial \ln n}{\partial X}; \tag{2.44}$$

this result means that the variation of impurity density with coordinate must be faster than n^{Z_α}. This inequality is responsible for the sharp drop in the density of multiply charged impurity ions toward the periphery of the plasma volume.

6. Diffusion in a strong magnetic field that affects particle collisions. Up to this point we have considered diffusion for cases in which the magnetic field had no direct effect on the collisions, that is to say, the Larmor radii of the particles were much greater than the interaction range.

In strong magnetic fields, however, the interaction range of the charged particles (Debye radius) is comparable with the Larmor radius of the electrons or greater and we must take account of the effect of the field on particle collisions. An analysis of the diffusion of charged particles with the effect of the magnetic field on collisions taken into account is given in [31,38].*

As before, the diffusion flux caused by collisions can be determined from (2.11). However, the quantities $\langle \Delta X_{\alpha\beta}\rangle$ and $\langle (\Delta X_{\alpha\beta})^2\rangle$ in this expression must now be computed taking account of the effect of the magnetic field on particle motion. In carrying out the calculations we assume that the interaction of the charged particles is described by a Coulomb potential with cutoff at the Debye radius.

It is convenient to introduce the notion of an impact parameter p to characterize collisions in a strong magnetic field.

Relating the particle displacement in a collision $\Delta X_{\alpha\beta}$ and the impact parameter, we can obtain the average value $\langle \Delta X_{\alpha\beta}\rangle$ from the obvious relation

$$\langle \Delta X_{\alpha\beta}\rangle = \iint\limits_{(p)} dp_1\, dp_2 \int\limits_{(v_\alpha)} f(v_\alpha)\, dv_\alpha \int\limits_{(v_\beta)} f(v_\beta)\, dv_\beta n_\beta (X_\beta)\, v(\Delta X_{\alpha\beta}), \tag{2.45}$$

where p_1 and p_2 are the projections of the impact parameter on mutually perpendicular axes in the plane perpendicular to the relative velocity v.

The quantity $(\Delta X_{\alpha\beta})^2$ is defined in similar fashion. The calculation of $\langle \Delta X_{\alpha\beta}\rangle$ and $\langle (\Delta X_{\alpha\beta})^2\rangle$ and the corresponding diffusion flux for an arbitrary effect of magnetic field on collisions is difficult. Hence, we shall treat two limiting cases. In the first case the motion of one of the particles participating in the col-

*In [36] an attempt is also made to treat diffusion in a magnetic field that affects collisions. However this work contains an error: the authors start from the unjustified assumption that the total particle pressure is constant, neglecting the magnetic pressure (this work is discussed in greater detail in [38]).

lision is strongly "magnetized" while the motion of the second is essentially unaffected by the magnetic field. This case is realized in electron-ion collisions when $\rho_e \ll p \ll \rho_i$. In the second case the motion of both particles participating in the interaction is "magnetized" (electron-electron collisions for $\rho_e \ll p$ and electron-ion and ion-ion collisions for $\rho_i \ll p$).

7. Electron diffusion caused by electron-ion collisions for $\rho_e \ll p \ll \rho_i$. In a collision in which the impact parameter is much greater than the Larmor radius the electron drifts in the electric field of the ion (Fig. 3). The transverse displacement of the guiding center of the electron due to the collision is

$$\Delta \mathbf{R}_{e\perp} = -\frac{c}{eH}\int_{-\infty}^{\infty} [\mathbf{F}_{ei}\mathbf{h}]\, dt = \frac{Z_i ec}{H}\int_{-\infty}^{\infty} V\,[\mathbf{rh}]\, dt, \quad (2.46)$$

where $\mathbf{F}_{ei} = -Z_i e^2 \mathbf{r} V(r)$ is the force exerted on the electron by the ion, $\mathbf{r}$ is the radius vector from the ion to the electron,

$$V(r) = \frac{1}{r^3} \quad \text{for} \quad r < r_d, \qquad V(r) = 0 \quad \text{for} \quad r > r_d. \quad (2.47)$$

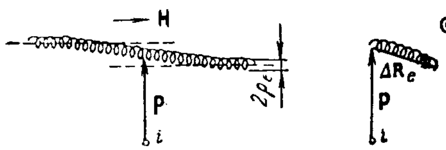

FIG. 3

Since the mean thermal velocity of the ions is much smaller than the mean electron velocity we first consider the electron drift, taking the ions to be fixed. Furthermore, inasmuch as the mean electron energy in the present case is much greater than the interaction energy in the collision (since the Larmor radius $\bar{\rho}_e$ is much greater than the radius of the "strong interaction" r_c) we assume that the longitudinal electron velocity is unchanged. It follows from (2.46) that the projection of the trajectory of the electron guiding center on the plane XY is the arc of a circle whose center lies at the ion site (cf. Fig. 3). The arc length traversed by the guiding center during the collision is determined [in accordance with (2.46)] from the relation

$$s_e = \frac{cp}{eHv_{ez}}\int_{-\infty}^{\infty} V(r)\, dz \left(r = \sqrt{p^2+z^2}\right). \quad (2.48)$$

It is evident from Fig. 3 that the vector representing the displacement of the guiding center is

$$\Delta \mathbf{R}_{e\perp} = \frac{s_e\,[\mathbf{ph}]}{p} - \frac{s_e^2 \mathbf{p}}{2p^2}, \quad (2.49)$$

where we have taken account of the fact that

$$\frac{s_e}{p} \approx \frac{\varrho_e}{p}\frac{r_c}{p} \ll 1; \quad (2.50)$$

the direction of the vector $\mathbf{p}$ is from the ion to the electron.

The relations in (2.49) and (2.50) give the electron displacement in the direction of the gradients

$$\Delta X_e = \frac{cp_y}{eHv_{ez}}\int_{-\infty}^{\infty} V(r)\, dz - \frac{1}{2}\frac{c^2 p_x}{e^2H^2v_{ez}^2}\left[\int_{-\infty}^{\infty} (V(r)\, dz)\right]^2. \quad (2.51)$$

In computing $\langle \Delta X_e \rangle$ from (2.45) we must take account of the variation of ion density within the interaction region (this variation is assumed to be small)

$$n_i(X_i) = n_i(X_e) - p_x \frac{\partial n_i}{\partial X}. \quad (2.52)$$

The relative velocity in the integrals in (2.45) is obviously the longitudinal velocity of the electron $v = |v_{ez}|$. Calculations making use of (2.45) and (2.51), taking account of (2.52) and (2.5), lead to the following result:

$$\langle \Delta X_{eI} \rangle = \left(\frac{2\pi m_e}{T}\right)^{1/2} \frac{c^2e^2}{H^2} L_p L_v Z_i^2 \frac{\partial n_i}{\partial X}, \quad (2.53)$$

$$L_p = \ln \frac{p_{\max}}{p_{\min}}, \quad (2.54)$$

$$L_v = 2\int_{(v)} e^{-\frac{m_e v^2}{2T}} \frac{dv}{v}. \quad (2.55)$$

In similar fashion we find the quantity

$$\langle (\Delta X_e)^2 \rangle = 2\left(\frac{2\pi m_e}{T}\right)^{1/2} \frac{c^2e^2}{H^2} L_p L_v Z_i^2 n_i. \quad (2.56)$$

The calculation is carried out up to terms proportional to $1/H^2$ (terms proportional to higher powers of $1/H$ are omitted). The resulting error is of order ρ_e/p.

The integral over velocity L_v diverges at the lower (zero) limit. This divergence arises because in the analysis given here the time of the electron-ion interaction increases without limit as the longitudinal velocity of the electron diminishes. Actually, the interaction time is bounded. It cannot be greater than $p/v_{i\perp}$, where $v_{i\perp}$ is the transverse component of ion velocity. When $v_{ez} < v_{i\perp}$ the ion leaves the interaction region before the electron. Another limitation is due to the longitudinal acceleration of the electron in the collision process. Near the ion the longitudinal velocity of the electron cannot be smaller than a quantity of order

$$v_c = \left(\frac{e^2}{pm}\right)^{1/2} = \bar{v}_e\left(\frac{r_c}{p}\right)^{1/2} \quad (2.57)$$

However, since the integral L_v diverges logarithmically we can replace the exact expression by a "cutoff" integral with limiting velocity v_0 equal to v_c or $\bar{v}_i$. Thus

$$L_v = \ln\frac{\bar{v}_e^2}{v_0^2} = \begin{cases} \ln\dfrac{m_i}{m_e} & \text{for} \quad \dfrac{m_i}{m_e} < \dfrac{\bar{p}}{r_c}, \quad (2.58\text{a}) \\ \ln\dfrac{\bar{p}}{r_c} & \text{for} \quad \dfrac{\bar{p}}{r_c} < \dfrac{m_i}{m_e}, \quad (2.58\text{b}) \end{cases}$$

$$\bar{p} = (p_{\max}p_{\min})^{1/2}.$$

The expressions for $\langle \Delta X_{eI} \rangle$ and $\langle (\Delta X_e)^2 \rangle$ have been obtained under the assumption that the ion remains fixed. The existence of a directed ion velocity caused by a pressure gradient leads to an additional directed electron displacement ΔX_{eII}. The magnitude of this displacement can be determined from the Einstein relation between the diffusion and mobility constants of the particle. In order to apply the Einstein relation we must convert to a reference system in which the ions are at rest. With respect to the laboratory system this system moves with a velocity equal to the directed velocity of the ions u_{ig} [cf. (2.6)]. An electric field arises in the moving system; this field is given by

$$E=\frac{1}{c}[\mathbf{u}_{ig}\mathbf{H}]=\frac{T\nabla n_i}{Z_i e n_i},\quad E_x=\frac{T}{Z_i e n_i}\frac{\partial n_i}{\partial X},\quad E_y=E_z=0. \tag{2.59}$$

Under the effect of the weak electric field, in collisions the colliding electrons are displaced along the electric field and the mean displacement per unit time is proportional to the field strength:

$$\langle \Delta X_{eII}\rangle=\mu E. \tag{2.60}$$

The electron mobility μ can be determined from the Einstein relation

$$\mu=-\frac{e}{T}D. \tag{2.61}$$

As is well known, the diffusion coefficient in this relation is determined by the mean-square displacement:

$$D=\frac{1}{2}\langle(\Delta X_e)^2\rangle. \tag{2.62}$$

Using (2.59)—(2.62) we obtain an expression for the mean electron displacement due to the directed ion motion:

$$\langle \Delta X_{eII}\rangle=-\frac{1}{2Z_i}\frac{1}{n_i}\frac{\partial n_i}{\partial X}\langle(\Delta X_e)^2\rangle. \tag{2.63}$$

A more detailed analysis of the electron displacement caused by the relative transverse motion of the electron and ion is given in [38], where it is shown that the mean displacement $\langle \Delta X_{eII}\rangle$ is determined primarily by electrons with low longitudinal velocities. Thus, any deviation from the Maxwellian distribution in the region of low longitudinal velocities can lead to a significant change in $\langle \Delta X_{eII}\rangle$ and to a corresponding change in the particle diffusion flux.

The relations in (2.53), (2.56), and (2.63) determine the quantities $\langle \Delta X_e\rangle$ and $\langle(\Delta X_e)^2\rangle$ ($\langle \Delta X_e\rangle$ is obtained by summing $\langle \Delta X_{eI}\rangle$ and $\langle \Delta X_{eII}\rangle$). Using these expressions and (2.11) we obtain the following expressions for the electron flux in the direction of the density gradient caused by electron-ion collisions for $\rho_e \ll p$:

$$\Gamma_{ei}=-\left(\frac{2\pi m_e}{T}\right)^{1/2}\frac{c^2e^2}{H^2}L_pL_v n_e n_i Z_i\left(Z_i\frac{1}{n_e}\frac{\partial n_e}{\partial X}+\frac{1}{n_i}\frac{\partial n_i}{\partial X}\right). \tag{2.64}$$

8. Ion diffusion caused by electron-ion collisions for $\rho_e \ll p \ll \rho_i$. When $\rho_i \gg p$ the effect of the magnetic field on the ion motion can be neglected. An important result of the collision for an ion in this case is the change of ion velocity. In accordance with (2.12) the transverse displacement of the ion guiding center is

$$\Delta\mathbf{R}_{ie\perp}=\frac{m_i c}{Z_i e H}[\Delta\mathbf{v}_i\mathbf{h}]=\frac{c}{Z_i e H}\int_{-\infty}^{\infty}[\mathbf{F}_{ie}\mathbf{h}]\,dt. \tag{2.65}$$

The integral on the right side of (2.65) determines the moment of the force acting on the ion in the collision process. A comparison of (2.65) and (2.46) shows that the ion and electron displacements are related by

$$\Delta\mathbf{R}_{ie\perp}=\frac{\Delta\mathbf{R}_{ei\perp}}{Z_i}. \tag{2.66}$$

The relation between the averaged and summed (over collisions) displacements $\langle \Delta X_{ie}\rangle$ and $\langle \Delta X_{ei}\rangle$ can be determined from (2.45):

$$\langle \Delta X_{ie}\rangle=-\frac{1}{Z_i}\langle \Delta X_{ei}\rangle\qquad (n_i\longrightarrow n_e) \tag{2.67}$$

(the density n_i in the expression for $\langle \Delta X_{ei}\rangle$ must be replaced by n_e).

In accordance with (2.45) and (2.66) the relation between $\langle(\Delta X_{ie})^2\rangle$ and $\langle(\Delta X_{ei})^2\rangle$ is

$$\langle(\Delta X_{ie})^2\rangle=\frac{1}{Z_i^2}\langle(\Delta X_{ei})^2\rangle\quad (n_i\rightarrow n_e). \tag{2.68}$$

Using (2.11), (2.67), (2.68), (2.53), (2.62), and (2.56) it is an easy matter to determine the ion diffusion flux in the direction of the density gradient caused by ion collisions with electrons:

$$\Gamma_{ie}=-\left(\frac{2\pi m_e}{T}\right)^{1/2}\frac{c^2e^2}{H^2}L_pL_v n_e n_i\left(\frac{1}{n_i}\frac{\partial n_i}{\partial X}+Z_i\frac{1}{n_e}\frac{\partial n_e}{\partial X}\right)=\frac{1}{Z_i}\Gamma_{ei}. \tag{2.69}$$

It is evident from (2.64) and (2.69) that collisions of electrons with ions lead to an ambipolar diffusion flux regardless of the strength of the electric field when $\rho_i \gg p \gg \rho_e$ just as in the case $\rho_e \gg p$.

9. Diffusion caused by collisions in which $\rho \ll p$ for both colliding particles. In collisions in which $\rho_k \ll p$ for both particles the particles drift in the direction perpendicular to the magnetic field and the line that connects them. The drift trajectories of the guiding centers are shown in Fig. 4. In collisions of particles with like sign the guiding centers drift in a circle around each other (Fig. 4a). This motion results in a mean displacement in the direction of the density gradient. Particles with equal and opposite charge drift along lines that are perpendicular to the

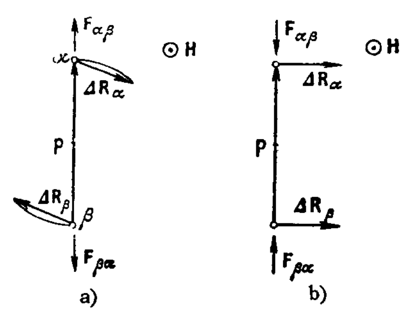

FIG. 4

impact parameter (Fig. 4b). In these collisions there is no mean displacement parallel to the density gradient.

The transverse displacement of the guiding centers of particles of type α that collide with particles of type β is determined by the drift velocity:

$$s_\alpha(t) = \frac{c}{Z_\alpha eH}\int_{-\infty}^{t} [\mathbf{F}_{\alpha\beta}\mathbf{h}]\,dt = \frac{Z_\beta ec}{H}\int_{-\infty}^{t} V(r_{\alpha\beta})\,[\mathbf{r}_{\alpha\beta}\mathbf{h}]\,dt. \quad (2.70)$$

Since the particle interaction is assumed to be weak (cf. page 172) the components of the vector $\mathbf{r}_{\alpha\beta}$ directed from the guiding center of particle β to the guiding center of particle α are

$$x_{\alpha\beta} = p_\alpha + s_{\alpha x} - s_{\beta x}, \quad y_{\alpha\beta} = p_y + s_{\alpha y} - s_{\beta y}, \quad z_{\alpha\beta} = v_z t. \quad (2.71)$$

Since the drift displacement is much smaller than the impact parameter $s \ll p$ [cf. (2.50)], in computing the particle displacement by means of (2.70) and (2.71) we use successive approximations. In the second approximation [i.e., carrying out the calculation to terms proportional to $(s/p)^2$] we obtain the following expression for the particle displacement:

$$\Delta X_{\alpha\beta} = s_\alpha(\infty) = \frac{Z_\beta ec}{Hv_z}\,p_y\int_{-\infty}^{\infty} V(r_{\alpha\beta})\,dz - \frac{1}{2}\,\frac{Z_\beta(Z_\alpha+Z_\beta)e^2c^2}{H^2v_z^2}\,p_x\left[\int_{-\infty}^{\infty} V(r_{\alpha\beta})\,dz\right]^2. \quad (2.72)$$

This expression can also be used, as in the other cases, to compute the quantities $\langle \Delta X_{\alpha\beta}\rangle$ and $\langle(\Delta X_{\alpha\beta})^2\rangle$ and the diffusion flux:

$$\Gamma_{\alpha\beta} = -\left(\frac{2\pi\mu_{\alpha\beta}}{T}\right)^{1/2}\frac{c^2e^2}{H^2}\,L_pL_v n_\alpha n_\beta Z_\beta\left(Z_\beta\frac{1}{n_\alpha}\frac{\partial n_\alpha}{\partial X} - Z_\alpha\frac{1}{n_\beta}\frac{\partial n_\beta}{\partial X}\right). \quad (2.73)$$

The coefficient L_v in (2.73) is determined from (2.58b).

It is evident from (2.73) that the diffusion flux caused by collisions of like particles vanishes when $\rho \ll p$ just as when $\rho \gg p$; the flux due to the mean displacement in the direction of the density gradient balances the diffusion flux due to the mean-square displacement in the direction opposite to the gradient. As in the case $\bar\rho \gg p$, the flux vanishes only when the calculation is carried out for the first density derivative. Taking account of the higher derivatives and the inhomogeneous electric field (cf. page 170) results in a nonvanishing flux associated with like-particle collisions.

In a plasma consisting of electrons and ions of one kind the ratio of the ambipolar diffusion flux (proportional to the higher derivatives in density) to the flux proportional to the density gradient (caused by electron-ion collisions) is of order [39]

$$\frac{\Gamma^{II}}{\Gamma^{I}} - \frac{\bar\varrho_i^2}{l_\perp^2} \quad \text{for} \quad \varrho_e \ll p \ll \varrho_i, \quad (2.74)$$

$$\frac{\Gamma^{II}}{\Gamma^{I}} - \frac{p_{\max}p_{\min}}{l_\perp^2} \quad \text{for} \quad \varrho_i \ll p. \quad (2.75)$$

Thus, the contribution due to like-particle collisions is unimportant.

10. Summary of results. In Table II we summarize the expressions for the transverse flux of charged particles (in the direction of the density gradient) caused by various kinds of collisions (the formulas are written in vector form).

The expressions for the flux due to collisions between charged particles are defined for the three cases characterized by the different ratios of Debye radius (r_d) to mean Larmor radii of the electrons and ions $\bar\rho_e$ and $\bar\rho_i$.

When $\bar\rho_e \gg r_d$ the diffusion flux is determined by (2.35).

When $\bar\rho_i \gg r_d \gg \bar\rho_e$ the diffusion flux caused by electron-ion collisions is the sum of the flux due to collisions for which $p < \bar\rho_e$ [this flux is determined from (2.35)] and the flux due to collisions with $p > \bar\rho_e$ [this flux is determined by (2.64) and (2.69)].

When $r_d \gg \bar\rho_i$ the diffusion flux of electrons and ions is obtained by summing the fluxes for the three ranges of p ($p < \bar\rho_e$, $\bar\rho_e < p < \bar\rho_i$, $p > \bar\rho_i$) determined from (2.35), (2.64), (2.69), and (2.73).

Thus the formulas in the table can be used to determine the diffusion flux of charged particles across a magnetic field in a plasma with various components. Using these formulas it is an easy matter to obtain [cf. (2.26)] the ambipolar flux of charged particles in a plasma consisting of electrons, ions of one kind, and neutrals ($n_e = Zn_i$):

$$\Gamma_{e\perp} = \Gamma_{en} + \Gamma_{ei} = -D_{a\perp}\nabla_\perp n_e, \quad (2.76)$$

$$\Gamma_{i\perp} = \Gamma_{ie} + \Gamma_{in} = -D_{a\perp}\nabla_\perp n_i, \quad (2.77)$$

$$D_{a\perp} = \frac{Z_i+1}{Z_i}\,\frac{T}{m_e\omega_e^2}(\nu_{en}+\nu_{ei}), \quad (2.78)$$

$$\nu_{ei} = \frac{4}{3}(2\pi)^{1/2}\frac{Z_i^2e^4n_i}{(m_e)^{1/2}T^{3/2}}\,\Lambda, \quad (2.79)$$

$$\Lambda = \begin{cases} \ln\dfrac{r_d}{r_0} & \text{for } r_d \ll \bar\varrho_e, \quad (2.80a) \\ \ln\dfrac{\bar\varrho_e}{r_0} + \dfrac{3}{4}\ln\dfrac{r_d}{\bar\varrho_e}\ln\dfrac{\bar v_e^2}{v_0^2} & \text{for } \bar\varrho_e \ll r_d \ll \bar\varrho_i, \quad (2.80b) \\ \ln\dfrac{\bar\varrho_e}{r_0} + \dfrac{3}{8}\ln\dfrac{m_i}{m_e}\ln\dfrac{\bar v_e^2}{v_0^2} + \dfrac{3}{4}\ln\dfrac{r_d}{\bar\varrho_i}\ln\dfrac{(r_d\bar\varrho_i)^{1/2}}{r_c} & \text{for } r_d \gg \bar\varrho_i; \quad (2.80c) \end{cases}$$

$\dfrac{\bar v_e^2}{v_0^2}$ is the smaller of the quantities $\dfrac{m_i}{m_e}$ and $\dfrac{\bar\varrho_e r_d}{r_c}$.

We have introduced here the ambipolar diffusion coefficient $D_{a\perp}$ determined by the effective electron-neutral collision frequency (2.24) and electron-ion collision frequency (2.79). In this calculation it is assumed that the electron and ion temperatures are the same [cf. Eq. (2.28)].

3. Solution of Certain Diffusion Problems

We present below the solutions of a number of boundary-value problems involving the diffusion of

Table II

Type of flux	
Flux of charged particles caused by collisions with neutrals	$\Gamma_{an}=-\frac{c^2 m_\alpha T_\alpha \nu_{an}}{Z_\alpha^2 e^2 H^2}\nabla_\perp n_a+\frac{c^2 m_\alpha \nu_{an}}{Z_\alpha e H^2} n_\alpha \mathbf{E}_\perp$
Fluxes caused by collisions of charged particles with $\bar{\varrho}\gg r_d$	$\Gamma_{\alpha\beta}=-\frac{4}{3}\left(\frac{2\pi\mu_{\alpha\beta}}{T}\right)^{1/2}\frac{c^2e^2}{H^2}\left(\ln\frac{r_d}{r_0}\right)n_\alpha n_\beta Z_\beta$ $\times\left(Z_\beta\frac{1}{n_\alpha}\nabla_\perp n_\alpha-Z_\alpha\frac{1}{n_\beta}\nabla_\perp n_\beta\right),$ $\Gamma_{ei}=Z_i\Gamma_{ie}=-\frac{4}{3}\left(\frac{2\pi m_e}{T}\right)^{1/2}\frac{c^2e^2}{H^2}\left(\ln\frac{r_d}{r_0}\right)n_e n_i Z_i$ $\times\left(Z_i\frac{1}{n_e}\nabla_\perp n_e+\frac{1}{n_i}\nabla_\perp n_i\right)$
Fluxes caused by collisions of electrons and ions with $\bar{\varrho}_e\ll r_d\ll\bar{\varrho}_i$	$\Gamma_{ei}=Z_i\Gamma_{ie}=-\left(\frac{2\pi m_e}{T}\right)^{1/2}\frac{c^2e^2}{H^2}$ $\times\left(\frac{4}{3}\ln\frac{\bar{\varrho}_e}{r_0}+\ln\frac{r_d}{\bar{\varrho}_e}\ln\frac{\bar{v}_e^2}{v_0^2}\right)n_e n_i Z_i$ $\times\left(Z_i\frac{1}{n_e}\nabla_\perp n_e+\frac{1}{n_i}\nabla_\perp n_i\right),\quad \frac{\bar{v}_e^2}{v_0^2}=\frac{m_i}{m_e}$ or $\frac{(\bar{\varrho}_e r_d)^{1/2}}{r_c}$ (equal to the smaller of the quantities)
Fluxes caused by collisions of charged particles with $\bar{\varrho}\ll r_d$	$\Gamma_{ee}=0,$ $\Gamma_{ei}=Z_i\Gamma_{ie}=-\left(\frac{2\pi m_e}{T}\right)^{1/2}\frac{c^2e^2}{H^2}\left(\frac{4}{3}\ln\frac{\bar{\varrho}_e}{r_0}+\frac{1}{2}\ln\frac{m_i}{m_e}\right.$ $\left.\times\ln\frac{\bar{v}_e^2}{v_0^2}+\ln\frac{r_d}{\bar{\varrho}_i}\ln\frac{(r_d\bar{\varrho}_i)^{1/2}}{r_c}\right)$ $\times n_e n_i Z_i\left(Z_i\frac{1}{n_e}\nabla_\perp n_e+\frac{1}{n_i}\nabla_\perp n_i\right),$ $\Gamma_{\alpha\beta}=-\left(\frac{2\pi\mu_{\alpha\beta}}{T}\right)^{1/2}\frac{c^2e^2}{H^2}\left(\frac{4}{3}\ln\frac{\bar{\varrho}_i}{r_d}+\ln\frac{r_d}{\bar{\varrho}_i}\right.$ $\left.\times\ln\frac{(r_d\bar{\varrho}_i)^{1/2}}{r_c}\right)n_\alpha n_\beta Z_\beta\left(Z_\beta\frac{1}{n_\alpha}\nabla_\perp n_\alpha-Z_\alpha\frac{1}{n_\beta}\nabla_\perp n_\beta\right)$ (particles α and β are ions)

plasma in a magnetic field. These solutions are required for the analysis of the experimental results.

1. Diffusion equations and boundary conditions. In this section we consider a plasma consisting of electrons, singly charged ions, and neutral particles. It is assumed that the electron and ion temperatures remain unchanged in the diffusion process (constant in space and time).

The change in plasma density ($n = n_i = n_e$) is determined by the diffusion flux of charged particles ($\Gamma = nu$), and by volume ionization and recombination processes:

$$\frac{\partial n}{\partial t}+\nabla\Gamma=\frac{\delta n}{\delta t}. \tag{3.1}$$

Because of the plasma neutrality condition it is evident that the electron and ion fluxes are related by the expression

$$\nabla\Gamma_e=\nabla\Gamma_i. \tag{3.2}$$

The change in the density of charged particles due to volume processes $\delta n/\delta t$ can be written as a sum

$$\frac{\delta n}{\delta t}=z_i n-\alpha n^2, \tag{3.3}$$

in which the first term gives the ionization rate and the second the electron-ion recombination rate. The mean ionization rate z and the recombination coefficient α are assumed to be independent of density. In (3.2) we have neglected the change in plasma density due to electron capture by neutral particles and the subsequent ionic recombination. For this reason the results obtained with (3.1) and (3.3) apply for the analysis of the diffusion in gases with small effective cross sections for the formation of negative ions (for example, in electropositive inert gases).*

We now apply the diffusion equation (3.1) to three cases.

a) Diffusion of charged particles in a weakly ionized gas in a chamber with dielectric walls. When the plasma is contained by dielectric walls the electron and ion fluxes must be the same at all parts of the wall. For this reason we seek a solution of (3.1) and (3.2) for which the electron and ion fluxes in the direction of the density gradient are the same over the entire volume:

$$\Gamma_{e\parallel}=\Gamma_{i\parallel}, \qquad \Gamma_{e\perp}=\Gamma_{i\perp}, \tag{3.4}$$

that is to say, the diffusion is ambipolar (cf. page 164). In (3.1) we substitute (1.23) and (1.24) which give the ambipolar diffusion flux, thereby obtaining the ambipolar diffusion equation

$$\frac{\partial n}{\partial t}-D_{a\parallel}\frac{\partial^2 n}{\partial z^2}-D_{a\perp}\Delta_{\perp}n=z_i n. \tag{3.5}$$

Here, $\Delta_{\perp}n=\frac{\partial^2 n}{\partial x^2}+\frac{\partial^2 n}{\partial y^2}$; it is assumed that recombination is unimportant in the weakly ionized gas.

The ambipolar diffusion equation (3.5) also applies if a current flows through the plasma provided the electron and ion fluxes can be written as a sum of "current" components (vanishing divergence) and "diffusion" components related by (3.4).

The electric field distribution in ambipolar diffusion is given by (1.21) and (1.22)

$$E_z=\frac{D_{i\parallel}-D_{e\parallel}}{\mu_{i\parallel}-\mu_{e\parallel}}\frac{1}{n}\frac{\partial n}{\partial z}, \qquad \mathbf{E}_{\perp}=\frac{D_{i\perp}-D_{e\perp}}{\mu_{i\perp}-\mu_{e\perp}}\frac{1}{n}\nabla_{\perp}n. \tag{3.6}$$

The condition for the existence of a potential electric field ($\nabla\times\mathbf{E}=0$) that follows from these conditions is

$$\frac{\partial^2 \ln n}{\partial x\,\partial z}=\frac{\partial^2 \ln n}{\partial y\,\partial z}=0; \tag{3.7}$$

it follows that ambipolar diffusion can occur in a magnetic field only if the spatial distribution of the density can be written as a product of longitudinal and transverse functions:

$$n(x, y, z)=n_{\parallel}(z)\,n_{\perp}(x, y). \tag{3.8}$$

In what follows we shall consider only distributions of this kind.

According to (3.6) the potential of the dielectric walls of the chamber containing a plasma in a magnetic field is nonuniform. In a strong magnetic field the walls perpendicular to the magnetic field are charged negatively (since the electron diffusion coefficient along the magnetic field is much greater than the ion diffusion coefficient) whereas the walls parallel to the field are charged positively (the transverse electron diffusion coefficient is much smaller than the ion coefficient).

b) Diffusion of charged particles in a weakly ionized gas in a chamber with conducting walls. If the chamber walls are highly conducting the wall potentials are equalized; the electric field distribution in the volume is changed and the ambipolar diffusion mechanism no longer operates. This effect was first noted by Simon,[47,48] who called it the short circuit effect. The analysis of diffusion of a weakly ionized gas in a chamber with conducting walls is carried out by means of equations obtained by substituting the flux expressions (1.10) and (1.11) in (3.1) and (3.2):

$$\frac{\partial n}{\partial t}-D_{e\parallel}\left[\frac{\partial^2 n}{\partial z^2}+\frac{e}{T_e}\frac{\partial}{\partial z}(nE_z)\right]-D_{e\perp}\left[\Delta_{\perp}n+\frac{e}{T_e}\nabla_{\perp}(n\mathbf{E})\right]=z_i n, \tag{3.9}$$

$$D_{e\parallel}\left[\frac{\partial^2 n}{\partial z^2}+\frac{e}{T_e}\frac{\partial}{\partial z}(nE_z)\right]+D_{e\perp}\left[\Delta_{\perp}n+\frac{e}{T_e}\nabla_{\perp}(n\mathbf{E})\right]=D_{i\parallel}\left[\frac{\partial^2 n}{\partial z^2}-\frac{e}{T_i}\frac{\partial}{\partial z}(nE_z)\right]+D_{i\parallel}\left[\Delta_{\perp}n-\frac{e}{T_i}\nabla_{\perp}(n\mathbf{E})\right]. \tag{3.10}$$

These equations must be supplemented by the specification of the equipotential condition at the walls (i.e., the plasma boundaries).

In what follows we seek solutions of (3.9) and (3.10) assuming, as in the case of ambipolar diffusion, that the components of electric field are proportional to the components of the density gradient:

$$E_z=\xi\frac{1}{n}\frac{\partial n}{\partial z}, \qquad \mathbf{E}_{\perp}=\xi\frac{1}{n}\nabla_{\perp}n. \tag{3.11}$$

If we assume that the boundary density of the plasma is constant the equipotential condition leads (in contrast with the case of ambipolar diffusion) to the result that the coefficient ξ is the same in the expressions for E_z and $E_{\perp}$.*

Substituting (3.11) in (3.9) and (3.10) we have

$$\frac{\partial n}{\partial t}-D_{e\parallel}\left(1+\frac{e}{T_e}\xi\right)\frac{\partial^2 n}{\partial z^2}-D_{e\perp}\left(1+\frac{e}{T_e}\xi\right)\Delta_{\perp}n=z_i n, \tag{3.12}$$

$$\left(1+\xi\frac{e}{T_e}\right)\left(D_{e\parallel}\frac{\partial^2 n}{\partial z^2}+D_{e\perp}\Delta_{\perp}n\right)=\left(1-\xi\frac{e}{T_i}\right)\left(D_{i\parallel}\frac{\partial^2 n}{\partial z^2}+D_{i\perp}\Delta_{\perp}n\right). \tag{3.13}$$

*We note that the results obtained in this section can be generalized to the case in which electron capture is important. We do not give the appropriate relations here in order to avoid complicating the paper.

*Within reasonable limits a variation in the boundary density is not important since the wall potential is a logarithmic function of boundary density (3.11).

These equations will be used to analyze diffusion in chambers with metal walls.

c) Transverse diffusion in a strong magnetic field. In analyzing transverse diffusion we assume that the plasma density is independent of the z coordinate, which is along the magnetic field. In this case the diffusion can be ambipolar regardless of the conductivity of the walls containing the plasma. In accordance with (1.22) the potential at any point in transverse ambipolar diffusion is determined uniquely by the plasma density. For this reason the wall potentials are not affected by the conductivity. Substituting the expression for the ambipolar diffusion coefficient (2.78) in (3.1) we obtain an equation that describes transverse diffusion in a strong magnetic field:

$$\frac{\partial n}{\partial t} - D^0_{a\perp}\nabla_\perp[(1+\gamma n)\nabla_\perp n] = z_i n - \alpha n^2. \qquad (3.14)$$

In (3.14) the quantity $D^0_{a\perp}$ is the ambipolar diffusion coefficient associated with collisions of charged particles and neutrals (the "linear" part of the diffusion coefficient)

$$\gamma n = \frac{\nu_{ei}}{\nu_{en}}, \qquad (3.15)$$

while $D_{ei} = \gamma n D^0_{a\perp}$ gives the diffusion coefficient associated with electron-ion collisions. In accordance with (2.79), the factor γ is a logarithmic function of density. This weak dependence of γ on n will be disregarded below.

In order to obtain a unique solution for the above equation we must establish the boundary values of the plasma density. The density close to absorbing walls in the absence of a magnetic field has been treated by a number of authors (cf. reference 49). In [50] the effect of a transverse magnetic field on the boundary conditions has been analyzed for certain particular cases. We shall not present the results of this work. We indicate only that in general, if the criteria for the application of the diffusion analysis (cf. Table I) are satisfied the plasma density close to an absorbing wall is appreciably smaller than the density in the central regions. Thus, in solving the diffusion equations (3.5), (3.12), (3.13), and (3.14) we take the density at the boundaries (close to the chamber walls) to be zero:

$$n_{\text{bound}} = 0. \qquad (3.16)$$

2. Stationary plasma diffusion. It is well-known that stationary transverse diffusion in a weakly ionized gas can be realized in a "long" positive column in a discharge in which the magnetic field is directed along the axis. In this case (3.5) becomes

$$\Delta_\perp n + \frac{z_i}{D_{a\perp}} n = 0. \qquad (3.17)$$

The density distribution is determined by the nonnegative solution of (3.17) that vanishes at the boundaries. (Such a distribution is called a diffusion distribution.) We must observe the characteristic relation

$$z_i = \frac{D_{a\perp}}{\Lambda_0^2}, \qquad (3.18)$$

which represents a balance condition, i.e., the balance between the rates of production and removal of charged particles. The quantity Λ_0 is of the order of the transverse dimensions of the plasma container and is called the diffusion length. For a plasma container of cylindrical shape the diffusion distribution and the diffusion length are given by the expressions

$$n = n_0 J_0\left(\frac{r}{\Lambda_0}\right), \qquad \Lambda_0 = \frac{a}{2.405}, \qquad (3.19)$$

where J_0 is the Bessel function and a is the chamber radius.

The quantities z_i and $D_{a\perp}$ are uniquely related to the electron temperature. For this reason (3.18) can be used to calculate the temperature. Since the electron temperature in the positive column of a discharge is determined by the longitudinal electric field, we can also compute this field as well as the stationary plasma density, which is inversely proportional to the electric field.

As the magnetic field increases the necessary electron temperature and longitudinal electric field both diminish because of the reduction in the diffusion coefficient (cf. Fig. 14). Correspondingly the plasma density for a fixed discharge current increases.

Thus, by determining experimentally the magnetic field dependence of the electron temperature, the electric field, and/or the plasma density in the positive column of a discharge one can obtain an idea of the effect of the magnetic field on transverse diffusion. It should be noted that the quantities T_e, E_z, and n_0 vary relatively slowly as $D_{a\perp}$ changes. When $D_{a\perp}$ varies by a factor of 2 or 3 the quantities E_z and n_0 vary by 10–30%.

3. Diffusion from the plasma of a stationary discharge. If the active discharge region only occupies part of the volume of the discharge chamber charged particles diffuse out of this region. At some distance from the active region of the discharge the electron temperature will drop to a value at which gas ionization ceases. Below we consider certain particular cases of the diffusion of charged particles from a stationary discharge into a region in which there is no ionization.

a) Weakly ionized gas; dielectric chamber; boundary of the active region of the discharge parallel to the magnetic field (Fig. 5). For this case ($\partial/\partial t = 0$, $z_i = 0$) the diffusion equation (3.5) assumes the form

$$D_{a\parallel}\frac{\partial^2 n}{\partial z^2} + D_{a\perp}\Delta_\perp n = 0. \qquad (3.20)$$

We present a solution of this equation of the class in (3.8) which vanishes at the boundaries and increases monotonically from the boundary to the center of the volume. If the boundary of the diffusion region and the chamber walls are plane this solution is

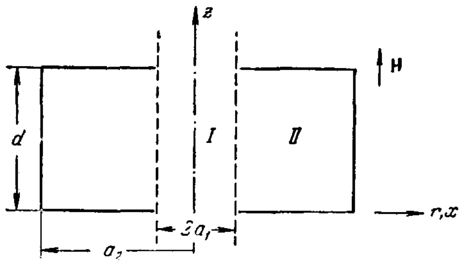

FIG. 5. Diagram of the discharge volume. I) Active region of the discharge; II) diffusion region.

$$n(x,z)=A\sin\frac{\pi z}{d}\sinh\left[\frac{\pi}{d}\sqrt{\frac{D_{a\parallel}}{D_{a\perp}}}(a_2-x)\right]. \quad (3.21)$$

If $\sqrt{D_{a\parallel}/D_{a\perp}}\,\dfrac{\pi(a_2-x)}{d}\gg 1$, the hyperbolic sine can be replaced by the exponential:

$$n(x,z)\approx B\sin\frac{\pi z}{d}e^{-\frac{x}{s_\perp}}, \quad (3.22)$$

$$s_\perp=\frac{d}{\pi}\sqrt{\frac{D_{a\perp}}{D_{a\parallel}}}. \quad (3.23)$$

This solution gives the transverse reduction in density if the longitudinal distribution at the boundaries of the active region of the discharge is sinusoidal ($\sin \pi z/d$). For other boundary distributions, at a sufficiently large distance from the boundaries of the active region ($\dfrac{x-a_1}{s_\perp}\gg 1$) one expects that a density distribution similar to (3.22) will be obtained.

A similar technique is used to find the solution of (3.20) for the case in which the boundaries of the active discharge and the chamber are both cylindrical. If $(a_2-r)\gg s_\perp$, $(r-a_1)\gg s_\perp$ this solution is of the form

$$n(r,z)=B\sin\frac{\pi z}{d}\frac{e^{-\frac{r}{s_\perp}}}{\sqrt{r}}. \quad (3.24)$$

Thus, the plasma density falls off exponentially at a sufficiently large distance from the boundary of the active region of the discharge and the walls of the chamber. The characteristic density decay length $s_\perp$ (the length in which the density is reduced by a factor of e) is of the order of the distance through which the charged particles can diffuse across the magnetic field in one lifetime. The relation (3.23) for $s_\perp$ can be written as follows.

$$s_\perp=\sqrt{D_{a\perp}\tau_\parallel},\quad \tau_\parallel=\frac{d^2}{\pi^2 D_{a\parallel}}. \quad (3.25)$$

The charged-particle lifetime in this case is equal to the time of longitudinal diffusion.

b) Weakly ionized gas; metal chamber; plasma boundary parallel to the magnetic field. As we have indicated, the ambipolar diffusion mechanism (in a magnetic field) does not operate in a chamber with conducting walls. Diffusion from a plasma in a stationary discharge for this case was considered by Simon[47,48] by means of (3.9) and (3.10). In the analysis given by Simon the terms containing the transverse electric field $E_\perp$ are neglected in these equations. It has been pointed out by Zharinov[51] and Tonks[52] that this procedure is not justified. It is evident that the terms containing $E_\perp$ (3.9) and (3.10) are of the same order as the terms describing the transverse diffusion. A correct analysis has been given in [52] for the case of diffusion from the plasma of a stationary discharge in a magnetic field (plane geometry) for boundary conditions corresponding to dielectric walls and metal walls. However, the results obtained in this work apply only if the ratio of longitudinal to transverse chamber dimensions lies within certain limits.

Diffusion in a chamber with metal walls can be analyzed easily by means of (3.12) and (3.13). To describe diffusion from the plasma in a stationary discharge ($\partial/\partial t=0$, $z_i=0$) these equations are written in the form

$$\left.\begin{aligned}\left(1+\frac{e}{T_e}\xi\right)\left(D_{e\parallel}\frac{\partial^2 n}{\partial z^2}+D_{e\perp}\Delta_\perp n\right)=0,\\ \left(1-\frac{e}{T_i}\xi\right)\left(D_{i\parallel}\frac{\partial^2 n}{\partial z^2}+D_{i\perp}\Delta_\perp n\right)=0.\end{aligned}\right\} \quad (3.26)$$

There are two possible classes of solutions for (3.26); these correspond to positive and negative potentials at the metal walls. In most cases the walls must be charged negatively since the electrons reach the walls more rapidly than the ions. In these cases (3.26) reduces to the following:

$$D_{i\parallel}\frac{\partial^2 n}{\partial z^2}+D_{i\perp}\Delta_\perp n=0, \quad (3.27)$$

$$\xi=-\frac{T_e}{e}. \quad (3.28)$$

In accordance with (3.11) the electric field is given by:

$$\mathbf{E}=-\frac{T_e\nabla n}{en}. \quad (3.29)$$

It is evident [cf. (1.10) and (1.11)] that the field distribution given by these relations indicates zero electron flux over the entire diffusion region. The electrons are "trapped" in this region because of the negative potential at the walls.* The distribution of charged particle density is given by (3.27), which describes ion diffusion. This equation is the same as (3.20). Hence, the expressions obtained above (3.21)—(3.25) can be applied to the case at hand if the ambipolar coefficients are replaced by the coefficients for ion diffusion. In particular, the characteristic density

*We note that the total number of electrons and ions reaching an insulated metal wall must be the same in a stationary mode. However, in this case the electrons must move to a wall outside the region being considered – in the region of the active discharge or to the boundary of the diffusion region.

decay length for diffusion in a metal chamber is given by [cf. (3.23) and (3.25)]

$$s_\perp = \sqrt{D_{i\perp}\tau_{i\parallel}} = \frac{d}{\pi}\sqrt{\frac{D_{i\perp}}{D_{i\parallel}}}. \tag{3.30}$$

c) Weakly ionized gas; dielectric chamber; boundary of the active region of the discharge perpendicular to the magnetic field (Fig. 6). For the case of cylindrically symmetric boundaries the solution of (3.20) of interest here is

$$n(r, z) = AJ_0\left(\frac{r}{\Lambda_0}\right)\sinh\left[\sqrt{\frac{D_{a\perp}}{D_{a\parallel}}}\frac{(d-z)}{\Lambda_0}\right], \tag{3.31}$$

or, when $(d-z) \gg \Lambda_0\sqrt{D_{a\parallel}/D_{a\perp}}$

$$n(r, z) = BJ_0\left(\frac{r}{\Lambda_0}\right)e^{-\frac{z}{s_\parallel}}, \tag{3.32}$$

$$s_\parallel = \sqrt{\frac{D_{a\parallel}}{D_{a\perp}}}\Lambda_0 = \sqrt{D_{a\parallel}\tau_\perp} \quad \left(\tau_\perp = \frac{\Lambda_0^2}{D_{a\perp}}\right). \tag{3.33}$$

In this case the characteristic length $s_\parallel$ is determined by the distance through which the charged particles can diffuse along the magnetic field in one lifetime (the time for transverse diffusion).

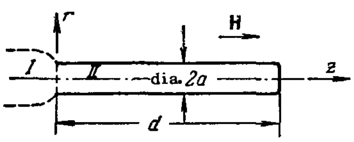

FIG. 6. Diagram of the discharge volume. I) Active region of the discharge; II) diffusion region.

By measuring the quantity $s_\parallel$ one can obviously determine the ratio $D_{a\perp}/D_{a\parallel}$. It should be noted that in a strong magnetic field the length $s_\parallel$ can be much greater than the transverse dimensions of the chamber.

In [53] the diffusion equation (3.5) has been analyzed for the more complicated case in which the plasma density at the boundary of the active region varies periodically in time. The reduction of the mean density in time as well as the phase variation in the density oscillation along the length of the chamber are again determined by the ratio of the diffusion coefficients $D_{a\perp}/D_{a\parallel}$.*

d) Transverse diffusion in a highly ionized gas in a strong magnetic field. When diffusion is due primarily to collisions between charged particles, transverse diffusion from a stationary plasma is described by (3.14), where we take $\partial n/\partial t = 0$, $z_i = 0$, $\gamma n \gg 1$:

$$\frac{1}{2}\gamma D^0_{a\perp}\Delta_\perp(n^2) - \alpha n^2 = 0. \tag{3.34}$$

The solution of this equation for a cylindrical boundary (vanishing at $r = a_2$) can be written in the form

$$n^2(r) = A\left[I_0\left(\frac{2a_2}{s_\perp}\right)K_0\left(\frac{2r}{s_\perp}\right) - K_0\left(\frac{2a_2}{s_\perp}\right)I_0\left(\frac{2r}{s_\perp}\right)\right], \tag{3.35}$$

$$s_\perp = \sqrt{\frac{\gamma D^0_{e\perp}}{\alpha}} = \sqrt{\frac{D_{ei}}{\alpha n}}. \tag{3.36}$$

If $r \gg s_\perp$, $a_2 - r \gg s_\perp$ (i.e., at large distances from the axes and from the walls of the chamber), the relation in (3.35) can be simplified by making use of the asymptotic form of the Bessel functions

$$n(r) \approx B\frac{e^{-\frac{r}{s_\perp}}}{(r)^{1/4}}. \tag{3.37}$$

The characteristic length $s_\perp$ is the effective length for transverse diffusion in one recombination lifetime of the particles.

4. Diffusion in the afterglow. In this section we consider plasma decay which occurs after the ionization source is turned off (for example, at the end of a pulsed discharge) in a cylindrical chamber with axis parallel to the magnetic field. The analysis is carried out for the later stage of decay, where the equilibrium temperature of the charged particles is approximately that of the surrounding medium and no gas ionization takes place ($z_i = 0$).

a) Weakly ionized gas; dielectric chamber. The equation describing the density variation in the afterglow is obtained from (3.5):

$$\frac{\partial n}{\partial t} - D_{a\parallel}\frac{\partial^2 n}{\partial z^2} - D_{a\perp}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial n}{\partial r}\right) = 0. \tag{3.38}$$

The solution of this equation of class (3.8) that vanishes at the walls of the cylindrical chamber ($z = 0, d$ and $r = a$) can be written in the form

$$n(r, z, t) = \sum_k A_k e^{-\left(\frac{D_{a\perp}}{\Lambda_k^2} + \frac{D_{a\parallel}\pi^2}{d^2}\right)t} J_0\left(\frac{r}{\Lambda_k}\right)\sin\frac{\pi z}{d},$$

$$\Lambda_k = \frac{a}{\varkappa_k}, \quad \varkappa_0 = 2.405, \quad \varkappa_1 = 5.520, \quad \varkappa_2 = 8.653, \ \ldots \tag{3.39}$$

Here, κ_k is the root of the Bessel function $J_0(\kappa_k) = 0$ and k increases with increasing root number.

The coefficients A_k are determined by the initial transverse density distribution (the longitudinal distribution is assumed to be sinusoidal). If the initial distribution is a diffusion distribution (3.19) only one term appears in the sum in (3.39):

$$n(r, z, t) = n_0 e^{-\frac{t}{\tau}} J_0\left(\frac{r}{\Lambda_0}\right)\sin\frac{\pi z}{d}, \tag{3.40}$$

$$\frac{1}{\tau} = \frac{1}{\tau_\perp} + \frac{1}{\tau_\parallel}, \quad \frac{1}{\tau_\perp} = \frac{D_{a\perp}}{\Lambda_0^2}, \quad \frac{1}{\tau_\parallel} = \frac{D_{a\parallel}\pi^2}{d^2}. \tag{3.41}$$

In this case the spatial distribution does not change in time while the plasma decays exponentially. The decay time constant τ is determined by the diffusion coefficients. For an arbitrary initial transverse density distribution the diffusion distribution (3.40) is established

*After the present review was written a paper by Golubev and Granovskiĭ [118] was published describing an experimental investigation of diffusion in a cylindrical chamber with fixed and periodically varying density at the boundary of the diffusion region. In this work a probe was used to determine the density distribution along the length of the chamber. The results of the investigation include data on the diffusion of charged particles in helium and argon plasmas at magnetic fields up to 1500 Oe. These results verify the possibility of using the method of investigating diffusion proposed by the authors in [53].

in a time of several $\tau_\perp$ after plasma decay starts; this follows because the terms in the series (3.39) corresponding to higher-order distributions (terms with $k > 0$) decay more rapidly than the main term.

b) Weakly ionized gas; metal chamber. Plasma decay in a metal chamber is described by equations obtained from (3.12) and (3.13):

$$\left.\begin{aligned}\frac{\partial n}{\partial t} &= -D_{e\parallel}\left(1+\frac{e}{T_e}\xi\right)\frac{\partial^2 n}{\partial z^2} - D_{e\perp}\left(1+\frac{e}{T_e}\xi\right)\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial n}{\partial r}\right),\\ \frac{\partial n}{\partial t} &= -D_{i\parallel}\left(1-\frac{e}{T_i}\xi\right)\frac{\partial^2 n}{\partial z^2} - D_{i\perp}\left(1-\frac{e}{T_i}\xi\right)\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial n}{\partial r}\right).\end{aligned}\right\} \tag{3.42}$$

It is not difficult to find the solution of this set in the form of a diffusion distribution (3.40).[54] This distribution is established a rather long time after the start of plasma decay. Substituting (3.40) in (3.42) and solving the systems of equations for τ and ξ [in accordance with (3.11) the quantity ξ determines the potential of the metal walls] we find

$$\tau = \frac{\tau_i + \frac{T_e}{T_i}\tau_e}{1+\frac{T_e}{T_i}}, \tag{3.43}$$

$$\frac{e\xi}{T_e} = \frac{\tau_e - \tau_i}{\tau_i + \frac{T_e}{T_i}\tau_e}. \tag{3.44}$$

The quantities τ_e and τ_i denote the effective time for "free" (i.e., not affected by space charge) diffusion of electrons and ions:

$$\frac{1}{\tau_e} = \frac{D_{e\perp}}{\Lambda_0^2} + \frac{D_{e\parallel}\pi^2}{d^2}, \quad \frac{1}{\tau_i} = \frac{D_{i\perp}}{\Lambda_0^2} + \frac{D_{i\parallel}\pi^2}{d^2}. \tag{3.45}$$

The rate of free diffusion for particles of each type is, by (3.45), equal to the sum of the rates for longitudinal and transverse diffusion. Correspondingly, the effective diffusion time (τ_e, τ_i) is determined for each particle by the faster process. In turn, the plasma decay constant is determined by the larger of the quantities τ_e or τ_i (when $T_e \approx T_i$).

It should be noted that the flux distributions for the electrons and ions at the chamber walls are not generally the same. Thus, when $D_{\parallel e}(\pi^2/d^2) \gg D_{\perp e}/\Lambda_0^2$, $D_{\parallel i}(\pi^2/d^2) \ll D_{\perp i}/\Lambda_0^2$ (this is the case in a strong magnetic field with $d \gg a$) the electrons diffuse along the magnetic field to the end walls while the ions diffuse across the fields to the side surface of the chamber. The currents arising in this process are short-circuited through the metal walls of the chamber.

We have treated diffusion decay in a plasma bounded by metal on all sides. When only part of the surface bounding the plasma is metal (the ends or side surface, for example) the solution of the decay equation is difficult.

c) Transverse diffusion in a highly ionized gas. Plasma decay occurring as a result of diffusion across a strong magnetic field and recombination is described by an equation derived from (3.14):

$$\frac{\partial n}{\partial t} = D^0_{a\perp}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial n}{\partial r}\right) + \gamma D^0_{a\perp}\frac{1}{r}\frac{\partial}{\partial r}\left(rn\frac{\partial n}{\partial r}\right) - \alpha n^2. \tag{3.46}$$

If there is appreciable ionization, in which case both diffusion due to electron-ion collisions and recombination are important, the decay equation (3.46) is nonlinear. In this case it is convenient to characterize the plasma decay by a time constant defined in terms of the mean density over the cross-section:

$$\tau = \frac{1}{d\ln\bar{n}/dt}, \tag{3.47}$$

$$\bar{n} = \frac{2}{a^2}\int_0^a n(r)\,r\,dr. \tag{3.48}$$

We note that because of the nonlinearity the quantity τ depends on density, that is to say, the plasma decay is not exponential.

Equation (3.36) cannot be solved in general form. In [54], through the use of a series of limiting cases, it has been possible to find an approximate solution of the equation which evidently describes the last stage in plasma decay. In solving (3.46), as a first approximation it is assumed that the right side of the equation is independent of coordinates. A method of successive approximations is then used to obtain a more accurate solution.

In Fig. 7 we show the radial distribution of density obtained by the approximate solution in various cases.

The curve marked 1 holds when nonlinear processes are unimportant ($\gamma n \ll 1$, $\alpha n \ll D^0_{a\perp}/a^2$) and gives an idea of the convergence of the method. The density distribution obtained in the second approximation does not differ greatly from the diffusion distribution (3.19), which is an exact solution of the equation (dashed curve).

The curve marked 2 gives the density distribution for the case in which the basic electron-removal process is diffusion due to electron-ion collisions ($\gamma n \gg 1$, $\alpha \ll \gamma D^0_a/a^2$). In this case the plasma decay constant is given by

$$\frac{1}{\tau_{(2)}} = 3.8\,\frac{\gamma D^0_{a\perp}\bar{n}}{a^2} = 3.8\,\frac{\bar{D}_{ei}}{a^2}. \tag{3.49}$$

When diffusion due to electron-ion and electron-neutral

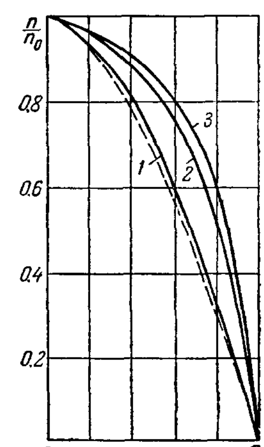

FIG. 7. Radial distribution of charged particles in the plasma.

collisions is important but recombination is not, the density distribution in plasma decay is given by the curve marked 1 rather than curve 2. The reciprocal time constant in this case can be approximated by the sum of the values corresponding to the limiting cases:

$$\frac{1}{\tau_{(2)}}=\frac{5.8}{a^2}D^0_{a\perp}+\frac{3.8}{a^2}\overline{D}_{ei} \tag{3.50}$$

(the accuracy in this approximation is better than 20%).

In a highly ionized gas, in which diffusion caused by electron-neutral colisions is unimportant ($\gamma n \gg 1$), and the basic processes for the removal of electrons are nonlinear diffusion and recombination, one finds that a time independent radial distribution is established. As is evident from the figure (curve 3), even with an appreciable recombination effect ($\alpha a^2/\gamma D^0_{a\perp} = 4.5$) the distribution does not differ greatly from the nonlinear diffusion distribution (curve 2). To an accuracy of 10% the plasma decay constant for $\alpha a^2/\gamma D^0_{a\perp} < 5$ can be approximated by

$$\frac{1}{\tau_{(2)}}=\frac{3.8}{a^2}\overline{D}_{ei}+1.2\alpha\overline{n}. \tag{3.51}$$

When both linear diffusion and recombination are important the described method cannot be used to find the solution of (3.46). In [55] an approximate analysis is given for this case which takes account of longitudinal diffusion; this analysis is based on an average of the decay equation taken over the volume. In taking an average of the equation it is assumed that the spatial distribution of density is a diffusion distribution [Eq. (3.40), curve 1 in Fig. 7]. Hence the solution applies if recombination is not important. The plasma decay constant obtained as a result of this solution in the case in which longitudinal diffusion is unimportant is

$$\frac{1}{\tau}=\frac{5.8}{a^2}D^0_{a\perp}+1.4\alpha\overline{n}. \tag{3.52}$$

In the general case in which all three of the electron-removal processes considered here are important (linear diffusion, nonlinear diffusion, recombination) the radial density distribution must obviously be given by a curve lying between the curves marked 1 and 3 in Fig. 7 (if $\alpha a^2/\gamma D^0_{a\perp} < 5$). Correspondingly, an approximate expression for the reciprocal of the plasma decay constant can be obtained by combining (3.49)–(3.51):

$$\frac{1}{\tau}=\frac{1}{\tau_0}+\frac{1}{\tau_{ei}}+\frac{1}{\tau_r}\quad\left(\text{for } \frac{1}{\tau_r}<1.5\frac{1}{\tau_{ei}}\right), \tag{3.53}$$

$$\frac{1}{\tau_0}=\frac{5.8D^0_{a\perp}}{a^2},\quad \frac{1}{\tau_{ei}}=\frac{3.8\overline{D}_{ei}}{a^2},\quad \frac{1}{\tau_r}=1.2\alpha\overline{n}. \tag{3.54}$$

II. EXPERIMENTAL INVESTIGATION OF THE DIFFUSION OF CHARGED PARTICLES IN A WEAKLY IONIZED GAS IN A MAGNETIC FIELD

4. Diffusion of Electrons in a Neutral Gas

Before discussing experimental investigations of the diffusion of a plasma across a magnetic field, we summarize the experimental data on transverse diffusion of electrons in a neutral gas.

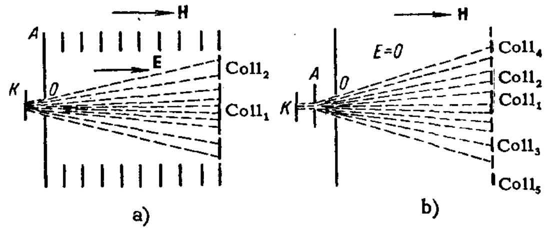

FIG. 8. Diagram of the experimental devices.

The diffusion spreading of an electron beam passing through a gas parallel to a magnetic field was investigated in early work by Bailey.[56] It was established in this work that the transverse diffusion of electrons in hydrogen at pressures of 2–16 mm Hg and magnetic fields below 800 Oe is described accurately by (1.11).

A more detailed investigation was carried out by Bickerton.[57] A diagram of the apparatus is shown in Fig. 8. In this device the electrons emitted by the thermionic cathode K move in a uniform electric field through a small aperture O, entering a cylindrical diffusion chamber. The transverse diffusion of the electrons takes place in the chamber, whose axis coincides with the direction of the magnetic field. The increase in the transverse dimension of the electron beam in the time the electrons move from the input aperture to the collector is given by the approximate relation

$$\Delta r\approx\sqrt{4D_{e\perp}\tau}=\sqrt{4D_{e\perp}\frac{d}{u_{e\parallel}}} \tag{4.1}$$

(d is the chamber length, $u_{e\parallel}$ is the longitudinal velocity of the electrons in the electric field).

By measuring the ratio of current to an annular collector $Coll_2$ and to the central collector $Coll_1$, which gives the beam spreading, it is possible to determine $u_{e\parallel}/D_{e\perp}$. In Fig. 9 we show the results of the determination of this quantity in helium at low currents; at low currents the space charge has no effect on electron motion. These data were obtained under conditions for

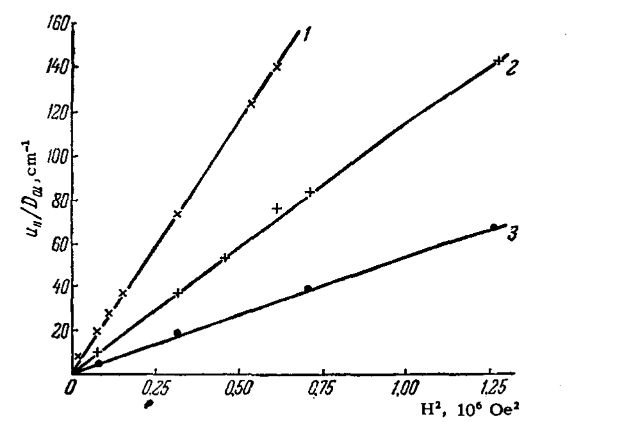

FIG. 9. Dependence of $u_{e\parallel}/D_{e\perp}$ on H. E/p = 10 V/cm·mm Hg; 1) p = 0.5 mm Hg; 2) 1 mm Hg; 3) 2 mm Hg.

which there was no electron emission from the collector, which could distort the measured result. The experimentally determined dependence of $u_{e\parallel}/D_{e\perp}$ (in helium and hydrogen) on magnetic field ($u_{e\parallel}/D_{e\perp} \sim H^2$) and pressure ($u_{e\parallel}/D_{e\perp} \sim 1/p$) is in good agreement with the theoretical relation given in (1.10) and (1.11):

$$\frac{u_{e\parallel}}{D_{e\perp}} = \frac{3}{2}\frac{eE}{\bar{K}_e}\left(\frac{\omega_e}{\nu_{en}}\right)^2 = \frac{3}{2}\frac{\left(\frac{eE}{p}\right)}{\bar{K}_e}\left(\frac{p}{\nu_{en}}\right)^2\frac{e^2H^2}{m_e^2c^2p} \quad (\text{for } \omega_e \gg \nu_{en}) \tag{4.2}$$

($\bar{K}_e$ is the mean electron energy).

An investigation of transverse diffusion of electrons at lower pressures and higher magnetic fields has been carried out by Zhilinskiĭ, Terent'ev, and the author. A diagram of the apparatus is shown in Fig. 8b. In this scheme the electrons, which are first accelerated, pass through an aperture and then reach the diffusion chamber, in which there is no electric field. In the chamber the electrons reach the collector by diffusing along the magnetic field. During the longitudinal diffusion the electron beam spreads to dimensions of order

$$r = r_0 + \Delta r, \quad \Delta r \approx \sqrt{4D_{e\perp}\tau} \approx \sqrt{2d^2\frac{D_{e\perp}}{D_{e\parallel}}}. \tag{4.3}$$

Measurement of the transverse current distribution by means of a sectionalized collector makes it possible to determine the ratio of longitudinal to transverse diffusion coefficients. The measurements were carried out in helium. The electron energy was kept below 10 eV in order to eliminate inelastic electron collisions. The lower limit on the pressure is given by the condition $\lambda_e \ll d$, which must be satisfied if the electron motion is to be diffusional. The highest pressure is determined by the condition that the electrons must suffer small energy losses during the diffusion time $\left(\frac{m_e}{m_i}\frac{d^2}{D_{e\parallel}} = \nu_{en} < 1\right)$.

The results of measurements of the ratio of the diffusion coefficients for helium pressures ranging from 0.01 to 0.12 mm Hg in magnetic fields up to 2000 Oe are shown in Fig. 10. In this same figure we show the theoretical curve computed from (1.10) and (1.11):

$$\frac{D_{e\parallel}}{D_{e\perp}} = \frac{\omega_e^2}{\nu_{en}^2} = \frac{e^2H^2}{m^2c^2\nu_{en}^2}. \tag{4.4}$$

In these calculations it is assumed that $\nu_{en}/p = 2.5 \times 10^9\ \text{sec}^{-1}$ mm Hg (it is well known that the collision frequency for electrons and helium atoms is essentially constant for energies ranging from 2 to 15 eV). It is evident from the curve that the experimental results are in good agreement with the theoretical predictions for both the dependence of the ratio $D_{e\parallel}/D_{e\perp}$ on H and p as well as the absolute values.

Thus, the experimental data indicate that the diffusion of electrons across a magnetic field in a neutral gas (in the absence of a transverse electric field and with low space charge) can be described accurately by the familiar theoretical formulas. We note, however, that when there is a transverse electric field or when space charge is important the transverse electron motion becomes more complicated. For instance, in static magnetrons at low gas pressures one observes unstable electron motion, strong oscillations, and noise (cf. [58]).

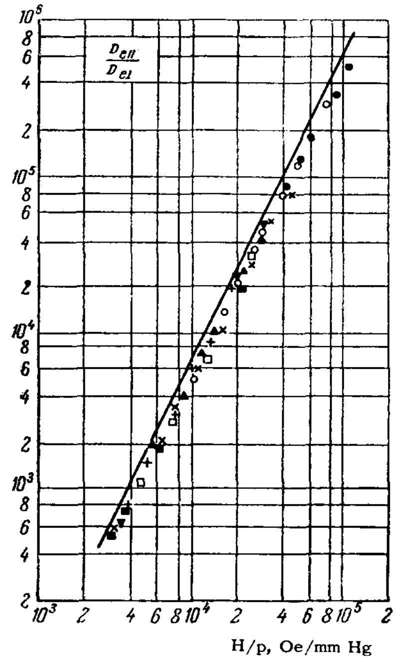

FIG. 10. The dependence of $D_{e\parallel}/D_{e\perp}$ on H. Experimental points: ● – p = 0.01 mm Hg; ○ – 0.02 mm Hg; ▲ – 0.035 mm Hg; × – 0.06 mm Hg; + – 0.075 mm Hg; □ – 0.085 mm Hg; ■ – 0.12 mm Hg; — theoretical curve.

5. Diffusion of Charged Particles from a Plasma in a Discharge with a Thermionic Cathode

The first experimental data on diffusion of charged particles in a magnetic field were obtained in studies of ion sources published in 1949.[59] A diagram of the experiments described in [59] is given in Fig. 11. Between the thermionic cathode K and the anode A there is a stationary discharge at low gas pressure (10^{-4}–10^{-2} mm Hg). The plasma moves along the lines of force of the magnetic field and passes through an aperture into the cavity of a graphite anode block. The size and shape of the primary plasma region in the cavity I are determined by the size and shape of the anode aperture (in the experiments described here the aperture is a narrow slit while the cavity in the anode is a rectangular parallelepiped). The charged particles of the primary plasma diffuse to the periphery of the anode cavity. By measuring the density distribution of charged particles in a plane perpendicular

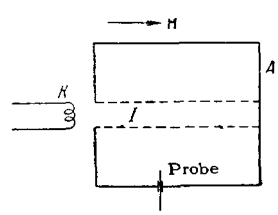

FIG. 11. Diagram of the experimental device.

to the magnetic field one can, as shown in Sec. 3 of Part I, obtain information on the transverse diffusion coefficient. The density distribution is measured by means of a movable probe P. In order to reduce the effect of the magnetic field on the probe measurements the plasma density is determined from the ion current to the probe (the probe is maintained at a negative potential with respect to the plasma).

The results of the measurements were used to determine the characteristic density decay length $s_\perp$ for several modes of operation in argon and hydrogen at a pressure of approximately 10^{-3} mm Hg and a magnetic field of 3000–4000 Oe. Bohm compared the experimental values of $s_\perp$ with the results of calculations based on the assumption of ambipolar particle motion and concluded that the transverse diffusion rate was approximately two orders of magnitude greater than that corresponding to ambipolar diffusion across the magnetic field.[59] The discrepancy was explained by Simon.[7,47,48] Simon showed that equalization of the potential at the boundaries of the plasma means that diffusion in the volume bounded by the conducting walls is not ambipolar. As we have shown in Sec. 3 of Part I, the conducting walls must be charged to a negative potential of order $(kT_e/e)\ln(n/n_{\text{bound.}})$ in order for the electron flux in the diffusion region to vanish (i.e., the conducting cavity must be a potential well for the electrons).

This description is verified by measurements of the distribution of anode current described in a paper by Zharinov.[51] The measurements were carried out with a probe that was moved close to the end surface of the anode (a cylindrical device similar to that shown in Fig. 11 was used). It was established, in accordance with the considerations given above that the electron flow to the anode is much smaller than the ion flow outside the region of primary plasma.

If the electron flux vanishes the density distribution of the plasma is determined by ion diffusion. The characteristic length $s_\perp$ is determined, in accordance with (3.30), by the distance over which ions can diffuse during one lifetime:*

$$s_\perp = \sqrt{D_{i\perp}\tau_{i\parallel}}, \quad \tau_{i\parallel} = \frac{d^2}{\pi^2 D_{i\parallel}}. \tag{5.1}$$

Substituting (1.10) and (1.11) for the ion diffusion coefficients and assuming that $\omega \gg \nu_{in}$, we have

$$s_\perp = \frac{d}{\pi}\frac{\nu_{in}}{\omega_i}. \tag{5.2}$$

This relation applies if the longitudinal ion motion is a diffusion motion, i.e., if $\lambda_i \ll d$. When $\lambda_i > d$ the ion lifetime is determined by the ion thermal velocity

$$\tau_{i\parallel} \approx \frac{d\sqrt{2m_i}}{\sqrt{T}} \quad (\text{for } T_i \approx T_e), \tag{5.3}$$

and $s_\perp$ is given approximately by

$$s_\perp \approx \left(\frac{2T}{m_i}\right)^{1/4}\frac{(\nu_{in}d)^{1/2}}{\omega_i}. \tag{5.4}$$

Measurements of the dependence of $s_\perp$ on magnetic field[7,48] have been carried out in order to verify the interpretation given by Simon. The measurements were carried out in a cylindrical anode block made of metal using the arrangement shown above (Fig. 11). Most of the results were obtained in a chamber filled with nitrogen at a pressure of 10^{-3}–5×10^{-3} mm Hg in magnetic fields of 2000–14000 Oe. The ionization in the primary plasma was less than 1%. A typical dependence of $s_\perp$ on H is shown in Fig. 12. It is evident from (5.1) that this dependence is in agreement with the theory; it corresponds to an inverse quadratic dependence of the transverse diffusion coefficient on magnetic field ($D_\perp \sim 1/H^2$). Measurements were also carried out to determine the dependence of $s_\perp$ on nitrogen pressure. When $\lambda_i \ll d$ ($\lambda_i < 6$ cm, $d = 26$ cm) a linear dependence of $s_\perp$ on pressure is observed, in accordance with (5.2); when $\lambda_i \approx d$ ($\lambda_i \approx d = 6$ cm) it is found that $s_\perp \sim \sqrt{p}$ [cf. (5.4)].

The absolute values of $s_\perp$ obtained in the experiments described in [48] and in the first experiments[59] are of the same order of magnitude as the theoretical values. One does not expect a better correspondence since the effective collision cross section and the ion temperature are only known approximately.

Investigations of the density distribution of charged particles diffusing from a stationary plasma have also been described in [60]. The difference in this work is the fact that diffusion was studied in a long chamber (1.2–1.5 m) and that an appreciable part of the lateral surface of the chamber was made of glass. A rigorous analysis of diffusion in such a "mixed" chamber surface is difficult. It is probable, however, that the boundary conditions at the side walls have little effect

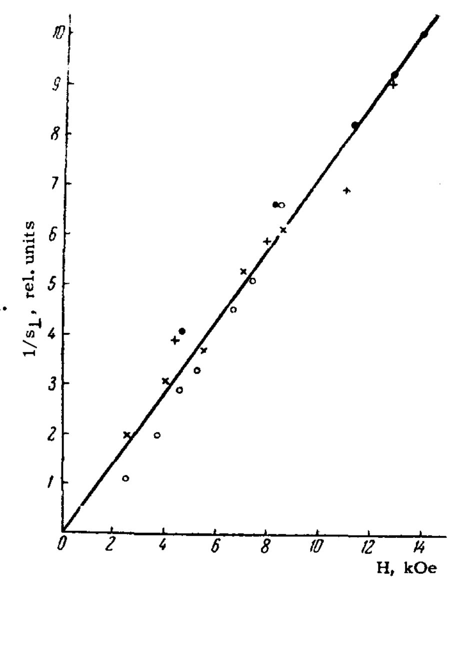

FIG. 12. The dependence of $1/s_\perp$ on H.

*We note that the formulas used in [47] and [48] differ from (3.30) by a factor $\sqrt{2}$. This results from the fact that Simon has neglected the transverse electric field in his calculations (cf. page 178).

when $s_\perp \ll a$ and that the results of the measurements of the diffusion length can be compared with (5.2) and (5.4).

Measurements carried out in nitrogen and hydrogen at a pressure of 10^{-3} mm Hg showed that the length $s_\perp$ is inversely proportional to the magnetic field H (up to values of 3500 Oe). The order of magnitude of $s_\perp$ is in agreement with that given by (5.2). The quantity $s_\perp$ was also determined in H_2, He, N_2, Ne, Ar, and Kr at $p = 10^{-5}$ mm Hg. However, it is difficult to interpret the low pressure measurements since $s_\perp$ is of the order of ion Larmor radius and the conventional diffusion theory does not apply.

Thus, the experimental data available at the present time concerning the density distribution in a plasma diffusing across a magnetic field from a discharge with a thermionic cathode are in satisfactory agreement with collisional diffusion theory.

It should be noted that these data do not yield information on the transverse motion of the electrons because under the conditions of these experiments (chamber with conducting walls) the density distribution is determined completely by an ion diffusion.[51]

A number of experiments have revealed anomalous effects in the plasma in a discharge with a thermionic cathode; these can be attributed to changes in the nature of the transverse motion of the charged particles in the magnetic field.

As far back as the first experiments described by Bohm[59] it was established that the ratio of electron saturation current (to the probe) to ion saturation current in a strong magnetic field is much greater than the expected value (in these experiments the collecting surface of the probe was perpendicular to the magnetic field). This led to the belief that the transverse motion of the electrons is more rapid than a diffusion motion. In fact, however, this assumption was not justified completely since the analysis of the electron branch of the probe curves in the magnetic field was very approximate.

A further investigation of probe characteristics in a plasma in a discharge with a thermionic cathode has been carried out by Zharinov.[61,62] In these experiments apertures were cut into the end of the anode and the volt-ampere characteristics were measured by probes located beyond these apertures. With increasing magnetic field (up to some critical value) the ratio of electron saturation current to ion current was observed to diminish monotonically. At the critical magnetic field this ratio increased suddenly; simultaneously, strong oscillations were observed in the probe current. Systematic measurements of the critical magnetic field were carried out in hydrogen, nitrogen, and helium in the pressure range 10^{-2}—10^{-3} mm Hg. It was found that the critical field increases approximately linearly with pressure; at a pressure of approximately 5×10^{-2} mm Hg this field is approximately 500 Oe for H_2 and approximately 1500 Oe for N_2 and He.

Comparison of the currents to different probes (of the phases of the current oscillations) shows that fields greater than some critical field cause the formation of one or two plasma "tongues" and that these tongues rotate about the axis of the discharge. As the magnetic field increases the period of rotation is reduced. Similar effects are described in[63]. The authors of this work observed rotation of an arc in different gases, H_2, He, N_2, Ar, Kr, and Xe at pressures of 10^{-4}—10^{-5} mm Hg in the anode cavity (the pressure in the space between the cathode and anode was maintained at approximately 10^{-3} mm Hg). The authors of [63] report no observation of arc rotation at anode cavity pressures of 10^{-3} mm Hg. A special method has been developed by Elizarov and Zharinov to make it possible to study in detail the formation and rotation of plasma formations.[64] In this work the end surface of the anode is a fine metal grid. The flux of electrons or ions passing through the grid is transformed into a light flux. The electrons are accelerated by short voltage pulses and strike a luminescent screen, causing it to emit. In the "ion conversion" mode the ions are accelerated and eject secondary electrons from the grid; these electrons are then accelerated toward the luminescent screen. In these measurements the discharge was formed by pulses with lengths up to 1 millisec. By photographing the emission from the luminescent screen at different "probe pulse" delay with respect to the initiation of the discharge it was possible to determine the distribution of plasma density over the anode cross section during the time in which the plasma formations were in the rotating state. Preliminary results published at the present time indicate that the shapes of the plasma formations and their motion are extremely complicated.

It should be noted that the results of investigations of "anomalous" behavior of a discharge with a thermionic cathode described in [61–64] are quite contradictory. The mechanism responsible for the rotation of the plasma formation is not known. No relation has yet been established between the observed effects and various plasma transport effects.

In a recently published paper Guest and Simon[65] have tried to relate the observed anomalous behavior to a helical plasma instability similar to the positive column instability considered earlier by Nedospasov and Kadomtsev (page 186). The helical instability can be caused by a longitudinal current in the diffusion region (as noted above, in diffusion in a metal chamber the ion current to the periphery is not compensated by the electron current).

It is not yet clear whether this mechanism can serve as an explanation for the anomalous diffusion of plasma from a discharge with a thermionic cathode, since no detailed comparisons are available of the conditions necessary for the formation of the instability and the experimental data.

6. Diffusion in a Low-Pressure Arc

In many modes of operation of a low-voltage arc (gas pressures of 0.1—10 mm Hg) ionization occurs primarily at the cathode (cf. [66,67]). The plasma density distribution in the space between the cathode and anode is determined by the diffusion of charged particles from the region contiguous to the cathode. If it is assumed that the temperature of the charged particles is constant over the entire volume (outside the ionization region) the results of the analysis in Sec. 3, Part I can be applied. It follows from this analysis that at some distance from the cathode the plasma density must fall off exponentially toward the anode. The characteristic density decay length in a cylindrical discharge chamber (dielectric) with magnetic field along the axis is given by (3.33):

$$s_{\parallel} = \frac{a}{2.405}\sqrt{\frac{D_{a\parallel}}{D_{a\perp}}}. \tag{6.1}$$

Nedospasov[68] has reported experimental data on the plasma density distribution between the cathode and anode of a low-voltage argon arc in a longitudinal magnetic field. These data have been obtained by measuring the ion current to a wall probe that could be moved along the side wall of the discharge chamber. These results were used to determine the characteristic length $s_{\parallel}$ and the ratio of the longitudinal and transverse diffusion coefficients $D_{a\parallel}/D_{a\perp}$ [cf. (6.1)]. The ratio $D_{a\parallel}/D_{a\perp}$ as a function of magnetic field is given in Fig. 13. It is evident from the curve that the experimental dependence obtained with magnetic fields below 1000 Oe at argon pressures of 0.25—1 mm Hg is in good agreement with the theoretical function obtained from (1.23) and (1.24):

$$\frac{D_{a\parallel}}{D_{a\perp}} = 1 + \frac{\omega_e\omega_i}{\nu_{en}\nu_{in}} = 1 + \frac{e^2H^2}{c^2 m_e m_i \nu_{en}\nu_{in}}. \tag{6.2}$$

7. Diffusion in the Positive Column

We have indicated above (cf. page 177) that the basic characteristics of the positive column—electron temperature, density distribution, maximum density, and longitudinal electric field, are determined by the diffusion of charged particles to the side wall of the discharge chamber. Hence, by studying these characteristics in the presence of a magnetic field along the discharge axis one can obtain information concerning the transverse diffusion of charged particles.

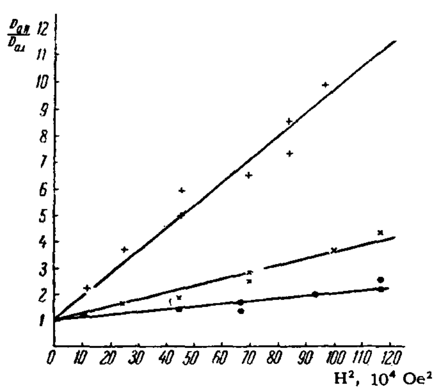

FIG. 13. The dependence of $D_{a\parallel}/D_{a\perp}$ on H. + – p = 0.25 mm Hg; × – p = 0.7 mm Hg; • – p = 1.0 mm Hg.

Probe measurements of the characteristics of the positive column of a discharge in helium at low magnetic fields (up to 400 Oe) have been reported by Bickerton and Engel.[69] The measured results were found to be in good agreement with the diffusional theory for the positive column. As the magnetic field increases the electron temperature and longitudinal electric field are reduced while the plasma density increases. These results have been verified by other workers. Vasil'eva and Granovskiĭ have carried out a direct measurement of the diffusion coefficient in the positive column.[70] Using probes (ion characteristics) these workers determined the plasma flux to the side wall and the density gradient close to the wall. The ratio of these quantities is equal to the diffusion coefficient. The variation in the transverse diffusion coefficient in helium at magnetic fields of 500—1000 Oe (pressures of 0.07—1 mm Hg) corresponds to that given by (1.24).

However, the behavior of a positive column cannot be described in terms of diffusion associated with particle collisions if the field becomes too high. As the magnetic field increases a critical value is reached at which the particle loss rate increases. Simultaneously one observes strong oscillations and noise. This effect was first reported by Lenhert[71] and investigated in detail by Lenhert and Hoh,[72-74] Granovskiĭ and his colleagues,[75] Allen et al[76,77] and a number of other authors.[78,80] We present here a brief review of the experimental results.

In [71-74] and [76-77] the subject of investigation was the positive column in a long discharge (2.4—4.1 m) located in a uniform magnetic field. The cathode and anode were outside the magnetic field. The potential difference between identical probes located at different points in the plasma was measured and used to deduce the longitudinal electric field. The results of the measurements in helium for a chamber radius of 1 cm are shown in Fig. 14 by the solid curves. In this same figure the dashed curves represent the theoretical curves calculated under the assumption that diffusion is due to collisions.*

At low magnetic fields the theoretical and experimental curves are found to be in agreement. There are certain discrepancies that can be attributed to the uncertainties in the values of the collision cross sec-

*The theoretical curves are plotted for the case in which the plasma contains only the atomic ion He^+. The presence of molecular ions does not affect the calculations greatly.

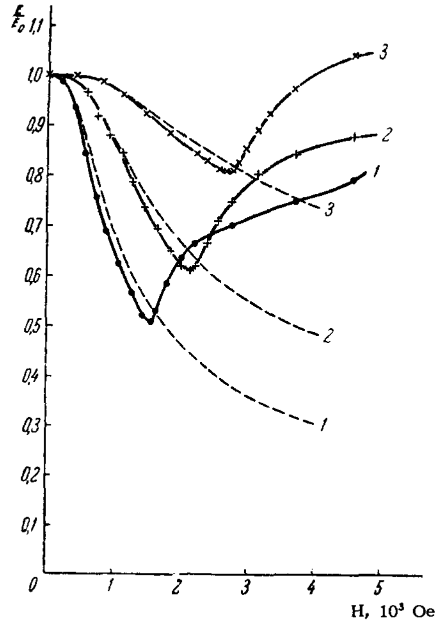

FIG. 14. The longitudinal electric field as a function of magnetic field. 1) p = 0.45 mm Hg; 2) 0.9 mm Hg; 3) 1.8 mm Hg.

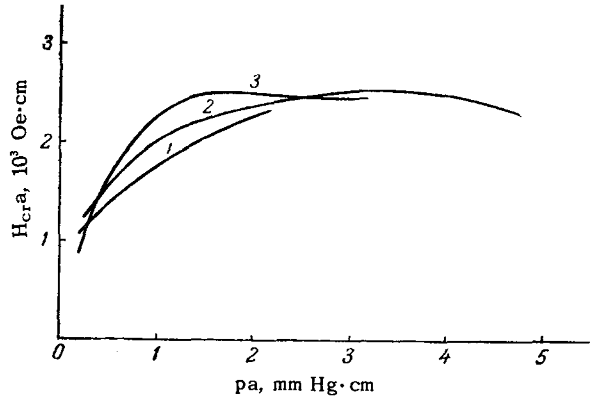

FIG. 15. The dependence of $H_{cr}a$ on pa. 1) Calculated from the formula given in [81]; 2) experimental curve using the data of [76]. a = 0.9–1.3 cm; 3) experimental curve from the data of [73]; a = 0.75 cm.

tions used in the calculations. At some critical magnetic field H_{cr}, however, the longitudinal electric field in the positive column starts to increase. This behavior indicates the enhanced loss of charged particles from the plasma. The enhanced rate of loss of charged particles at magnetic fields beyond the critical value is also verified by measurements of the ion current to wall probes. [74,75]

At high magnetic fields (4,000–6,000 Oe) the longitudinal electric field (and consequently the rate of loss of particles from the plasma) is found to be of the same order as that with no magnetic field and remains fairly constant.

The increased longitudinal electric field at $H > H_{cr}$ is accompanied by a sharp rise in the noise intensity. The values of the critical magnetic fields given by the minimum in the E(H) curves and the rise in noise intensity are found to coincide.

In Fig. 15, using the data of [73] and [76] we show the dependence of H_{cr} on helium pressure for several values of chamber diameter. It is established that H_{cr} increases slightly as the discharge current is increased but that it changes relatively little as the length of the discharge region in the magnetic field is varied from 0.5 to 4 m. We have only given the results for helium. However, similar data have been obtained in investigations of the positive column in discharges in hydrogen, nitrogen, neon, argon, and krypton. [73,74,76,77]

The anomalously rapid loss of particles from the plasma in the positive column has been explained by Kadomtsev and Nedospasov, [81] who have shown that this loss can be associated with the presence of instabilities of the positive column in the magnetic field.

This work considers a helical distortion of the pinch.* The growth of the perturbation is inhibited by diffusion since the diffusion flux to the wall is found to be largest where the pinch is closest to the wall. At the same time the helical distortion in the magnetic field subjects the pinch to a force parallel to the wall. This force is associated with the azimuthal component of current produced in the distortion. As the magnetic field increases, this force, which tends to increase the distortion, is also increased, while diffusion, which inhibits the perturbation, is reduced. Hence an instability arises at some critical value of the magnetic field. Calculations based on formulas given in [81] for H_{cr} for this instability (cf. Fig. 15) are in good agreement with the results of an experimental investigation of this quantity in the positive column.†

The helical perturbation of the positive column predicted by Kadomtsev and Nedospasov has been observed directly in magnetic fields greater than the critical value in [76] and [77]. The authors of these papers have successfully photographed a time-resolved pattern of the emission from a pinch in two projections. These photographs indicate clearly a right-handed screw distortion of the pinch. An additional verification has been given by measurements of the positive column in the presence of a high-frequency field. [80]

*The observed anomalies have been attributed by Hoh[82] to an instability in the boundary layer between the plasma and the wall (not associated with the discharge current). This interpretation can not explain all the observed effects. Judging by later papers,[83,84] the author himself has abandoned it and supports the picture that a helical instability arises in the column, as suggested by Nedospasov and Kadomtsev.

†The theoretical curve in Fig. 15 is plotted from data kindly furnished to the author by A. V. Nedospasov. These same data have been used in a paper by Vdovin and Nedospasov,[119] which was published after the present review was written, and which contains calculations of H_{cr} for helium, hydrogen, neon, argon, and mercury and contains an analysis of the appropriate experimental data.

A field at a frequency of 4 Mc was applied to a special solenoid inside of which the discharge chamber was located. In the presence of additional ionization caused by the high-frequency electric field it was found that the fixed electric field required for maintaining particle balance in the positive column was reduced. Correspondingly, the electron density was higher for a given discharge current. Hence, in distortion of the pinch the stabilizing effect of the diffusion is enhanced and H_{cr} is increased. The experiments verified the predicted increase of H_{cr}.

The authors of [81] have not only obtained criteria for the development of the helical instability, but have also considered the growth of the instability at values of H close to H_{cr}. The analysis yields the oscillation frequency of the pinch and the rate of anomalous diffusion associated with these oscillations. The results are found to be in agreement with experiment.

At fields much higher than H_{cr} the pinch is capable of supporting many kinds of oscillations simultaneously; in this case the oscillations become irregular. A theory for the development of this kind of instability has been given by Kadomtsev[85] by analogy with the theory of turbulent convection of a liquid. This theory predicts that the turbulent diffusion rate of a plasma should be independent of magnetic field when $H \gg H_{cr}$. This and other predictions of the theory are in agreement with the experimental results.

Thus, the anomalous rate of loss of charged particles from a positive column in a strong magnetic field and the observed oscillations and noise can be attributed to a helical instability of the pinch.

8. Diffusion in an Oscillating-Electron (Penning) Discharge

A diagram of the oscillating-electron (Penning) discharge is shown in Fig. 16. In this discharge the electrons oscillate between the two electrodes K or between the cathode and a reflector (connected electrically) as long as displacements across the magnetic field do not cause them to strike the anode A. The ratio of plasma density at the periphery of the discharge (close to the lines of force of the magnetic field that intersect the anode) to the density at the center n_a/n_0 serves as a measure of the rate of diffusion of plasma across the magnetic field. This ratio has been investigated by a number of French workers under various experimental conditions.[86-88] The plasma density was measured with probes (ion current to the probe).

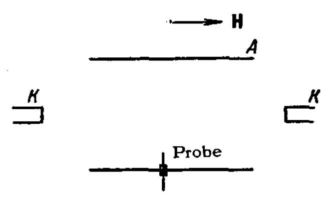

FIG. 16. Diagram of the oscillating-electron discharge (Penning discharge).

The measurements of n_a/n_0 for a cold-cathode Penning discharge in hydrogen are shown in Fig. 16. At low magnetic fields the ratio n_a/n_0 diminishes monotonically with increasing field and increases with pressure. This variation correlates with the change in diffusion rate caused by collisions. However, at some critical field H_{cr} the ratio n_a/n_0 starts to grow; in the "supercritical" mode n_a/n_0 diminishes as the pressure is increased. The value of H_{cr} increases as the pressure is increased and as the radius is reduced and remains essentially constant as the discharge current is varied by a factor of 10. Intense noise is observed at fields somewhat greater than the critical field. Measurements with an external receiver indicate that the low-frequency oscillations are accompanied by noise at frequencies of approximately 1000 Mc. The variation of noise amplitude is approximately the same as the variation of n_a/n_0. Similar effects have been observed in a Penning discharge with a thermionic cathode in a fairly large device (length approximately 1 m). The anomalous variation of n_a/n_0 and the appearance of intense noise at magnetic fields greater than some critical value evidently indicates the formation of an unstable state of the Penning discharge; this state is characterized by enhanced transverse transport of particles. The nature of this anomalous behavior is not clear.

It is reported in [87] and [88] that a Penning discharge is subject to instabilities associated with an anisotropic electron velocity distribution and with electric fields in the plasma. However, an analysis for the mechanism responsible for these instabilities is still lacking.

9. Diffusion in the Afterglow

We have presented above the results of a number of experiments in which anomalously rapid motion of charged particles across the magnetic field was observed. In most cases the observed anomalous behavior is associated with the presence of a current in the plasma or with a highly directed motion of the charged particles. For this reason, special interest, as far as diffusion theory is concerned, attaches to investigations of diffusion in the afterglow; these measurements can

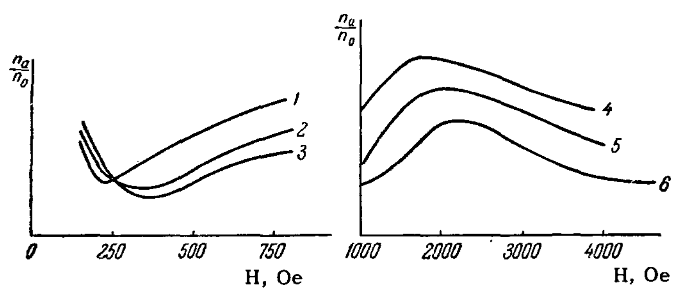

FIG. 17. The dependence of n_a/n_0 on H. 1) p = 0.016 mm Hg; 2) 0.024 mm Hg; 3) 0.03 mm Hg; 4) 0.021 mm Hg; 5) 0.025 mm Hg; 6) 0.03 mm Hg.

be carried out under conditions in which the plasma is essentially in equilibrium.

The first experiments on plasma decay in a magnetic field were carried out by Bostick and Levine in a toroidal chamber.[89] However, the magnetic field is not uniform in this geometry and plasma diffusion is, to a considerable degree, masked by the toroidal drift. Nonetheless, we shall consider [89] since these results have been frequently used in the analysis of transverse plasma diffusion.

The investigation was carried out in a metal toroidal vacuum chamber of rectangular cross section. This chamber served simultaneously as a resonator excited at two separate frequencies in the 10-centimeter region. A pulsed discharge was produced in the gas in the chamber by introducing high-frequency power at one of the resonance frequencies. In the time interval between pulses a measurement oscillator was used to measure the shift of the resonant frequency of the second resonance. Under proper conditions the frequency shift should be proportional to the mean plasma density (cf. [90]). This technique was used to determine the density variation in the decaying plasma and the time constant for plasma decay τ. The value of τ in helium increased approximately linearly as the pressure was increased from 0.05 to 0.35 mm Hg. As the magnetic field was varied from 160 to 1400 Oe the measured time constant went through a peak. The presence of this peak has been interpreted by many workers as evidence for "anomalous" transverse diffusion.* This interpretation is unconvincing because it does not take account of the effect on plasma decay due to the toroidal drift associated with the inhomogeneity of the magnetic field. Indeed, the relatively weak dependence of the plasma decay rate on magnetic field (as H is increased by a factor of 8–10, τ does not vary by more than 1.5–2 times) as well as the reduction in decay rate at increased pressure, reported in [89], are characteristic of toroidal drift of charged particles in a neutral gas. It thus appears that the results reported in [89] can be attributed to toroidal plasma drift. In any case, the data reported in this work can hardly give information on transverse diffusion in magnetic fields greater than 200–300 Oe because at these fields the diffusion rate is smaller than the toroidal drift rate.

Investigations of plasma decay in a uniform magnetic field have been carried out by both probe and microwave methods. Granovskiĭ and his colleagues [91–93] have investigated plasma decay by means of probes. The plasma was produced in cylindrical glass chambers 2–3 cm in diameter by a pulsed high-frequency discharge or by a thermionic-cathode discharge. The variation in ion current to probes (I_p) introduced into the discharge was measured between pulses. Using the slope of the $I_p = f(t)$ curves these workers determined the plasma decay time constant in the initial stage τ_0 (immediately after the discharge pulse) and in the later stages τ_1 (several hundred microseconds after the pulse). The measurements were carried out at plasma densities greater than 10^9–10^{10} cm^{-3}.

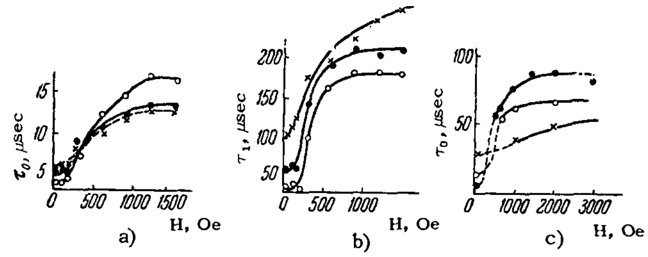

FIG. 18. The dependence of τ on H. a) He, τ_0; o – p = 0.06 mm Hg; × – 0.33 mm Hg; • – 0.11 mm Hg. b) He, τ_1; o – p = 0.06 mm Hg; × – 0.33 mm Hg; • – 0.11 mm Hg. c) Ar, τ_0; • – p = 0.03 mm Hg; o – 0.1 mm Hg; × – 0.3 mm Hg.

The dependence of τ on magnetic field in helium and argon for several cases obtained in [91] and [93] is shown in Fig. 18. The change in time constant is small at magnetic fields greater than 500–1000 Oe. The saturation of these curves can be attributed to volume processes for the removal of charged particles such as electron-ion recombination and electron capture by electronegative impurity molecules with subsequent ion recombination. To evaluate the effect of volume processes the authors of [92] determined the ratio between the total number of particles arriving at the chamber surface (as measured by the ion current to all probes) and the initial number of charged particles (estimated by the ion current to the probes in the chamber volume). The results of these measurements indicate a marked effect due to volume processes for the removal of charged particles in the decay of a helium plasma under the conditions of these experiments. Thus, the $\tau(H)$ curves reflect plasma diffusion only at low magnetic fields (500–1000 Oe). The qualitative behavior of these curves is in agreement with diffusion theory in this region. However, a quantitative comparison is difficult since the plasma parameters are not completely determined. In a magnetic field the plasma density can only be determined to within an order of magnitude by probe measurements. At the initial stages of plasma decay the electron and ion temperatures are both unknown. Appropriate choice of parameters can provide agreement between experimental values of τ and the calculated values.

The above considerations indicate that the experimental results reported in [91] and [93] do not contradict diffusion theory; on the other hand, these results are inadequate for quantitative verification of the theory.

*Other arguments for anomalous effects are sometimes based on the oscillations in plasma decay observed in [89]. However, it is not stated clearly in [89] under what conditions the oscillations are observed. For example, it is not clear whether or not there is a current through the plasma in the oscillation period. Moreover, a correlation is not established between the observed oscillations and the plasma decay rate.

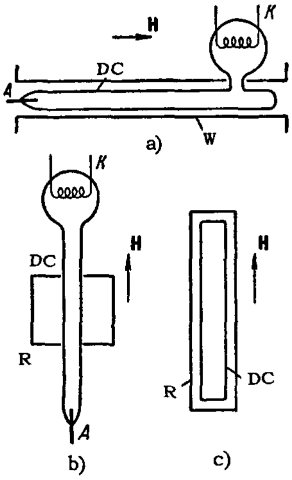

FIG. 19. Diagram showing the plasma in measurement waveguides and resonators. K (cathode), A (anode), DC (discharge chamber), W (waveguide), R (resonator).

The application of microwave methods [90] makes it possible to study plasma decay at densities from 10^7–10^8 to 10^{12}–10^{13} cm^{-3} (plasma decay in helium was studied). In this section we consider only the experimental results that have been obtained at low densities, in which case collisions between charged particles are unimportant.

Zhilinskiĭ and the author [94–95] used a waveguide technique to investigate plasma decay in a magnetic field. A cylindrical glass chamber (inner diameter 1.6–2.0 cm, length 110 cm) was placed in a cylindrical waveguide (Fig. 19a). The helium plasma was produced by current pulses 1–2 μsec in length. The time variation of the phase shift of 3 cm waves propagating through the plasma was determined in the time interval between pulses. The magnitude of this shift is determined by the mean electron density. Characteristic curves showing the variation of density on a semilogarithmic scale are given in Fig. 20. It is evident from the curve that plasma decay is exponential at densities below 10^9–10^{10} cm^{-3}. This means that nonlinear processes, such as diffusion caused by electron-ion collisions and recombination, are unimportant. Simple estimates indicate the establishment of thermal equilibrium between the electrons, ions, and neutrals and the establishment of a diffusion radial distribution in the early stages of plasma decay. This result is verified by the uniform behavior of decay curves at different initial densities. Thus, the slope of the $\ln \bar{n}(t)$ curves at low densities can, in accordance with (3.41), be used to determine the "linear" coefficient for transverse diffusion:

$$D_\perp^0 = \frac{\Lambda_0^2}{\tau_\perp}, \quad \frac{1}{\tau_\perp} = \frac{1}{n}\frac{d\bar{n}}{dt}, \quad \Lambda_0 = \frac{a}{2.405}. \tag{9.1}$$

In Fig. 21 are shown the results of the determination of the diffusion coefficient reported in [94] and [95] at magnetic fields up to 2500 Oe and helium pressures of 0.02–1 mm Hg.

Plasma decay has been investigated by a resonator method by Guzhova and Syrgiĭ. [96] A chamber containing the plasma was extended through the aperture in the end walls of a cylindrical 3-centimeter cavity (cf. Fig. 19b). The plasma density was deduced from the shift in the resonant frequency during decay. The decay constant was determined for magnetic fields up to 1200 Oe and helium pressures of 0.08–0.32 mm Hg.

Ganichev, Zhilinskiĭ and the author, using a resonator method (Fig. 19b), have determined the plasma decay constant in helium at pressures of 0.08–1.5 mm Hg, magnetic fields up to 6000 Oe, and chamber diameters of 0.4–1 cm.

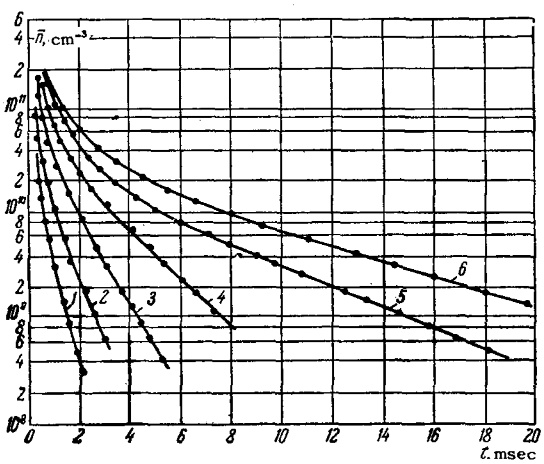

FIG. 20. Variation of charged particle density in the plasma afterglow. p = 0.08 mm Hg; 1) H = 300 Oe; 2) 450 Oe; 3) 650 Oe; 4) 1000 Oe; 5) 1500 Oe; 6) 2000 Oe.

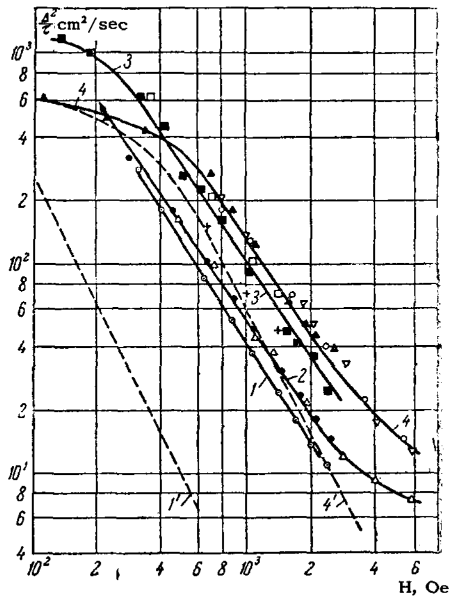

FIG. 21. The dependence of $D_\perp$ on H. Experimental points – curve 1: ⊙ – p = 0.025 mm Hg, a = 0.8 cm[95]; curve 2: • – p = 0.08 mm Hg, a = 0.8 cm[95]; + – p = 0.08 mm Hg, a = 0.7 cm[96]; △ – p = 0.08 mm Hg, a = 0.05 cm; curve 3: ■ – p = 0.35 mm Hg, a = 0.8 cm[95]; □ – p = 0.32 mm Hg, a = 0.7 cm[96]; curve 4: ▲ – p = 0.08 mm Hg, a = 0.8 cm[95]; ▽ – p = 1.5 mm Hg, a = 0.5 cm; ○ – p = 1.5 mm Hg, a = 0.2 cm. Theoretical curves: 1′ – p = 0.025 mm Hg; 4′ – 0.8 mm Hg.

It is evident from Fig. 21 that the results of various investigations of plasma decay in helium for discharge chamber diameters of 0.4–2 cm are all in good agreement. To an accuracy of 30–40%, the diffusion coefficient obtained in these experiments can be approximated by the empirical formula

$$D^0_{\perp} = 6 + \frac{(0.4+p)\,10^8}{H^2} \qquad (9.2)$$

where D is given in $cm^2 sec^{-1}$, H in Oe and p in mm Hg.

On the other hand, it follows from (1.24) that the coefficient for ambipolar diffusion due to collisions is

$$D^0_{a\perp} = \frac{(T_e+T_i)}{m_e\omega_e^2}\nu_{en} = \frac{7{,}5\cdot 10^7 p}{H^2}\left[\frac{cm^2}{sec}\right]. \qquad (9.3)$$

In computing $D^0_{a\perp}$ it is assumed that $T_e = T_i = 300°K$, $\nu_{en} = 2.5 \times 10^8 p\ sec^{-1}$, $\omega_e \gg \nu_{en}$. A comparison shows that only one of the terms in the empirical formula corresponds to the theoretical predictions. The experimental plasma decay rate is appreciably greater than the theoretical value, especially at low pressures. The reason for this discrepancy has not been established.* The component of decay rate that is independent of magnetic field may possibly be due to volume removal processes. However, there is an important factor in the diffusion coefficient that falls off rapidly with magnetic field and is only weakly dependent on pressure. It would appear that we are dealing here with a diffusion mechanism that is independent of electron-neutral collisions. The effective "collision frequency" for this mechanism is about $10^8\ sec^{-1}$.

Alikhanov, Demirkhanov, and their coworkers have investigated plasma decay in a glass chamber of large diameter (2a = 7 cm, d = 70 cm).[97] The entire chamber was located inside a cavity resonator (cf. Fig. 19c). The gas discharge was excited at one of the resonance frequencies. The density of the decaying plasma was deduced from the frequency shift of the other resonance. The results of measurements of the plasma decay constant in helium at pressures of 0.025–0.2 mm Hg and magnetic fields up to 6000 Oe are in agreement with the theoretical values. Because of the relatively small value of the length-diameter ratio of the discharge chamber (d/2a = 10) transverse plasma diffusion at magnetic fields greater than 100–500 Oe is masked by the longitudinal diffusion. The data of [97] indicate that the transverse plasma diffusion rate in a large-diameter chamber is in agreement with the theoretical value at magnetic fields smaller than several hundred oersteds.

All of the results given above have been obtained in dielectric discharge chambers. The effect of metal surfaces on plasma decay has been studied in [54]. A copper tube 110 cm long and approximately 2.0 cm in diameter was located inside a glass discharge chamber. This tube served as a waveguide, by means of which the plasma density was measured during decay. The measured values of the plasma decay constant at low densities are shown in Fig. 22. In a uniform magnetic field the decay rate in the presence of a metal tube is found to be approximately the same as in the chamber without metal. In order to simulate the decay of a plasma bounded on all sides by conducting surfaces these workers introduced a nonuniform magnetic field. The length of the solenoid producing the magnetic field was reduced so that the ends of the chamber (with the metal tube) were outside the solenoid; in this way the lines of force of the magnetic field were made to intersect the metal surface. As is evident from Fig. 22, in this case there is a marked reduction in the decay constant at high magnetic fields. Replacing the solid metal tube in the chamber by a tube consisting of insulated metal rings (each 10 cm in length) resulted in the disappearance of these effects. This result indicates that the change in plasma decay constant in a metal tube is due to the equalization of the potentials at the plasma boundaries, i.e., the short circuit effect. It is difficult to carry out a quantitative comparison of the experimental results with the theory of diffusion in a metal chamber given in Sec. 3 of Part I because the mechanisms responsible for the accelerated diffusion in the dielectric tube are not clear. The diffusion coefficient in the metal chamber can be computed if one assumes that the effective electron collision frequency increases in the magnetic field and then determines this frequency

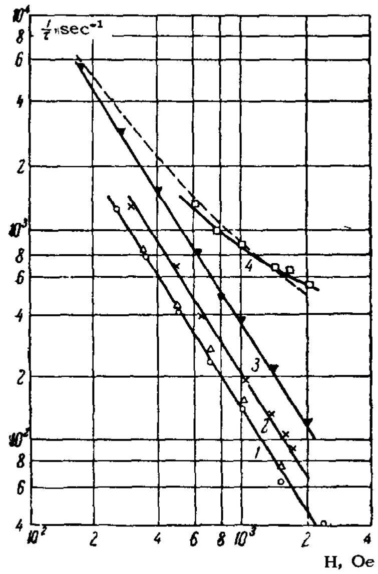

FIG. 22. The dependence of $1/\tau$ on H. The experimental points: 1) in a tubular chamber consisting of insulated metal rings, p = 0.25 mm Hg; o – uniform magnetic field; Δ – tube ends outside the field; 2) glass chamber without metal tubes, p = 0.035 mm Hg; 3) chamber with a solid metal tube, uniform magnetic field, p = 0.08 mm Hg; 4) chamber with a solid metal tube with ends outside the field, p = 0.08 mm Hg. The dashed line indicates the calculated curve for decay in a metal chamber with p = 0.08 mm Hg.

*It is possible that part of the discrepancy is due to differences between the electron temperature and the temperature of the gas.

from experiments on diffusion in the dielectric chamber, using (3.43), (3.45), (1.10), and (1.11). The results of these calculations are in agreement with the experimental data.

III. EXPERIMENTAL INVESTIGATIONS OF THE DIFFUSION OF CHARGED PARTICLES IN A HIGHLY IONIZED GAS IN A MAGNETIC FIELD

10. Decay of a Dense Plasma in a Magnetic Field

Nonlinear processes, such as diffusion due to electron-ion collisions, and recombination, play an important role at high plasma densities. Hence, by measuring plasma decay rates under these conditions it is possible to obtain information concerning diffusion due to electron-ion collisions.

We consider first the data obtained in [94] and [54] from measurements of the plasma decay rate in helium by a waveguide method. A diagram showing the method of measurement has been given above (cf. Fig. 19a) together with the results of investigations at low plasma densities (in which case the predominant mechanism is linear), which indicate that the plasma decay is exponential. At plasma densities above 10^9—10^{10} cm^{-3} the reciprocal decay time constant

$$\frac{1}{\tau}=\frac{1}{\bar{n}}\frac{d\bar{n}}{dt} \tag{10.1}$$

increases noticeably with density (cf. Fig. 20). The effectiveness of the nonlinear processes can be characterized by the quantity $\Delta(1/\tau) = 1/\tau - 1/\tau_0$ (τ_0 is the decay time constant at low densities). Analysis of the plasma decay curves shows that this quantity varies approximately in direct proportion to the plasma density at magnetic fields of 300—2000 Oe. In Fig. 23 we show the dependence of $\Delta(1/\tau)$ on magnetic field at $\bar{n} = 2 \times 10^{10}$ cm^{-3} for helium pressures of 0.02—0.8 mm Hg. It is evident that the nonlinear correction to the decay rate for a fixed plasma density is essentially independent of the pressure of the neutral gas. The

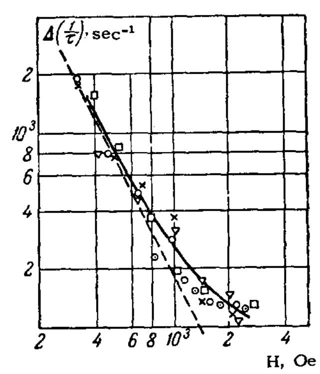

FIG. 23. The dependence of $\Delta(1/\tau)$ on H. Experimental points: ∇ – p = 0.025 mm Hg; × – 0.05 mm Hg; ○ – 0.08 mm Hg; □ – 0.35 mm Hg; ⊙ – 0.8 mm Hg. Theoretical curves: —— – $(1/\tau_{ei} + 1/\tau_r)$; – – – – $1/\tau_{ei}$ (we assume $\alpha = 3 \times 10^{-9}$ cm^3/sec, T = 300° K, Λ = 8, a = 0.8 cm).

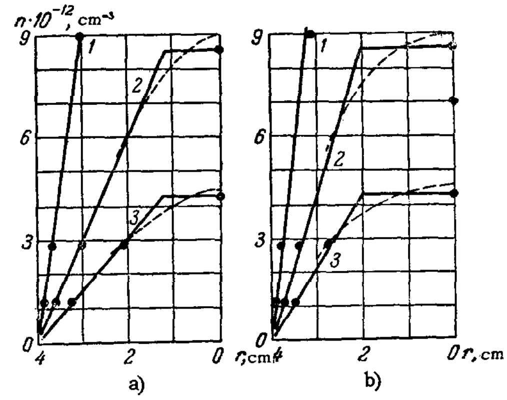

FIG. 24. Radial distribution of charged particles. a) H = 0; 1 – t = 0.3 msec, 2 – 0.5 msec, 3 – 0.65 msec; b) H ≈ 2000 Oe; 1 – t = 0.4 msec, 2 – 0.65 msec, 3 – 0.9 msec (t is taken from the termination of the discharge current pulse). The calculated curves are shown by the dashed lines.

curve in Fig. 23 can be analyzed by means of (3.53):

$$\Delta\left(\frac{1}{\tau}\right)=\frac{1}{\tau}-\frac{1}{\tau_0}=\frac{1}{\tau_{ei}}+\frac{1}{\tau_r},\quad \frac{1}{\tau_{ei}}\approx\frac{3.8\bar{D}_{ei}}{a^2},\quad \frac{1}{\tau_r}\approx 1.2\alpha\bar{n}. \tag{10.2}$$

The dependence of $\Delta(1/\tau)$ on magnetic field is found to be weak at magnetic fields greater than 1500 Oe. This evidently means that diffusion is masked by recombination. By determining the recombination coefficient ($\alpha = 3 \times 10^{-9}$ cm^3 sec^{-1}) from the curve in Fig. 23, it is an easy matter to determine $\bar{D}_{ei}$ by means of the formula given above. Values of $\bar{D}_{ei}$ for magnetic fields of 500—1000 Oe are given in Fig. 25. In accordance with the theory, $\bar{D}_{ei}$ varies in proportion to the mean density (in the range 5×10^9—10^{11} cm^{-3}) and as the inverse square of the magnetic field.

A "free space" microwave method has been used by Anisimov, Vinogradov, Konstantinov and the author [98] to investigate plasma decay at high densities. The

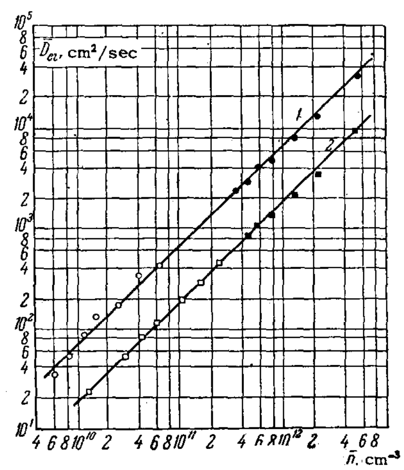

FIG. 25. The dependence of $\bar{D}_{ei}$ on $\bar{n}$. 1 – H = 500 Oe, 2 – H = 1000 Oe, ○, □ – data obtained from [54], ●, ■ – data obtained from [98], the solid curves are obtained using Eq. (2.78) assuming T = 300° K, Λ = 8.

plasma in these experiments was produced by a pulsed induction or electrode discharge in a cylindrical glass chamber 8.0 cm in diameter filled with helium. The density of the decaying plasma was measured by simultaneous microwave "probing" at three frequencies. By determining the phases of the reflected waves at several frequencies below the cutoff frequency it is possible to determine the positions of the reflection regions corresponding to these frequencies (i.e., the regions of critical density for each frequency. [99] The phase of the transmitted wave was used to find the mean density. Thus, these data could be used to deduce the spatial distribution of charged particles* (Fig. 24). These data were verified by measurements of the distribution of emission from the plasma. In the absence of a magnetic field the density distribution is essentially a diffusion distribution, described by the Bessel function. In a magnetic field of 1000–2000 Oe, the density distribution is flattened because of nonlinear effects. The distribution corresponds to the results of the approximate calculation given in Sec. 3 of Part I.

The plasma decay constant is determined from the time variation of the mean density. The plasma decay rate becomes independent of magnetic field at values greater than 1000–1500 Oe. The decay curves in this region are in good agreement with the curves obtained in investigations of plasma decay in strong magnetic fields (H = 30,000 Oe) on the American B-1 Stellarator. [100] As shown in [100] and [101], the curves reflect recombination caused by collisions of an ion with two electrons.

The diffusion time constant at magnetic fields below 1000 Oe can be determined by subtracting the recombination rate from the total decay rate $1/\tau_\alpha = 1/\tau - 1/\tau_r$. The dependence of this constant on magnetic field and density is in agreement with theory.

The results of measurements of the nonlinear diffusion coefficient are shown in Fig. 25. In the same figure we show the calculated values of $\bar{D}_{ei}$ obtained from (2.78). In the calculations the temperature and Coulomb logarithm are taken to be T = 300°K and Λ = 8 respectively. At plasma densities of 5×10^9–5×10^{12} cm^{-3} (2.80) yields Coulomb logarithm values Λ = 2.5–7. It should be noted, however, that Λ is small and the Debye radius and the Larmor radius of the electrons are approximately the same for the experimental conditions described here; hence (2.80) can only give Λ with an accuracy of several units. Thus, the agreement between the experimental values of $\bar{D}_{ei}$ and the theoretical values may be regarded as satisfactory.

It follows from the results of [54] and [98] that at plasma densities of 5×10^9–5×10^{12} cm^{-3} and magnetic fields up to 1500 Oe the nonlinear part of the diffusion coefficient is in good agreement with the theory of diffusion based on electron-ion collisions.

*A trapezoidal distribution was assumed in the analysis of the experimental data.

11. Diffusion in a Quiescent Cesium (Potassium) Plasma

Surface ionization can be used to produce a plasma in gases that possess a low ionization potential. This technique of accumulating ions makes it possible to obtain a quiescent plasma with a high degree of ionization in a magnetic field. A description of apparatus designed to produce such a plasma is given in [102]; the results of measurements of diffusion across a magnetic field made with this apparatus are given in [103] and [104].

A diagram of the device is shown in Fig. 26. The source of charged particles consists of two tungsten plates W heated to 2300°K. An atomic beam of cesium or potassium from the source S is aimed at the central part of the plates. The ions produced as a result of surface ionization are contained by the magnetic field for a long time period in the central volume (between the plates). The heated tungsten plates also serve as electron sources. The number of emitted electrons is automatically maintained at a fixed value because the plasma remains neutral over the entire volume, with the exception of sheaths with widths of the order of the Debye radius at the hot plates.

The walls of the chamber are maintained at a low temperature (approximately –10°C) to reduce the number of neutrals in the volume. The plasma density in the central volume and the degree of ionization are determined by the intensity of the neutral flux. At densities of 10^{10} cm^{-3} the ionization is approximately 40%; at densities of 10^{12} cm^{-3} the ionization is greater than 99%.

The apparatus is useful for studying diffusion in a magnetic field since the plasma it produces is essentially in equilibrium. Because of the high degree of ionization the diffusion due to charged-particle collisions is the only important diffusion mechanism ($\nu_{en} \ll \nu_{ei}$). Data on transverse plasma diffusion is obtained by means of a small movable probe (diameter 0.025 cm) which measures the radial density distribution. The difficulty in these measurements is the fact that the variation of density must be measured within the limits of the small transverse dimensions of the tungsten plates (approximately 1.4 cm).

It should be noted that the longitudinal density distribution is uniform since the charged particles strik-

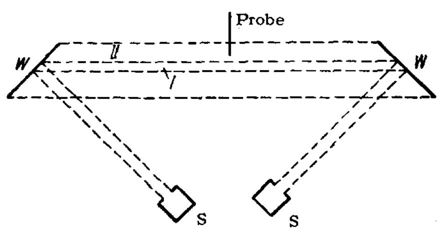

FIG. 26. Diagram showing the device used for obtaining a quiescent plasma. I) Region of plasma formation; II) region of transverse diffusion.

ing the hot tungsten surface are almost completely reflected. Consequently there is no longitudinal diffusion. Under these conditions, as shown in Sec. 3 of Part I, the characteristic density decay length $s_\perp$ depends on the ratio of the transverse diffusion coefficient to the recombination coefficient (3.36)

$$s_\perp = \sqrt{\frac{D_{ei}}{\alpha n}}. \tag{11.1}$$

Substituting (1.30) for D_{ei} in (11.1) we find

$$s_\perp = \frac{c}{eH}\sqrt{\frac{2\nu_{ei} m_e T}{\alpha n}}. \tag{11.2}$$

Under the experimental conditions described here the electron Larmor radius is smaller than the Debye radius ($\bar{\rho}_e < r_d$); hence, in computing the collision frequency ν_{ei} the Coulomb logarithm is determined from (2.80b).

The results of measurements of the characteristic length $1/s_\perp$ in cesium are given in Fig. 27. Analogous results were obtained for potassium. It is evident from the curve that $1/s_\perp$ is approximately proportional to H. Consequently, the transverse diffusion coefficient D_{ei} is inversely proportional to the square of the magnetic field, as it should be.

The absolute values of the diffusion coefficient cannot be found from measurements of $s_\perp$ [cf. (11.1)] since the value of the recombination coefficient is not known precisely. If one uses (11.2), obtained under the assumption that the plasma diffusion is due to electron-ion collisions, one can then determine the value of the recombination coefficient α from the experimental data. It is found that α increases with density ($\sim \bar{n}^{1/2}$) and is approximately 3×10^{-10} cm^3/sec at $\bar{n} \approx 5 \times 10^{11}$ cm^{-3}. The order of magnitude of this value is in agreement with a direct estimate of α and with the results of measurements carried out under other conditions. Some error may be due to the inaccuracy in the density measurements and the plasma temperature determinations. The dependence of α on n is also in agreement with the theoretical predictions concerning recombination in ion collisions with two electrons (three-body collisions). The observed correspondence of the data indicates that the order of magnitude of the transverse diffusion coefficient D_{ei} is given correctly by the theoretical formula (2.78).

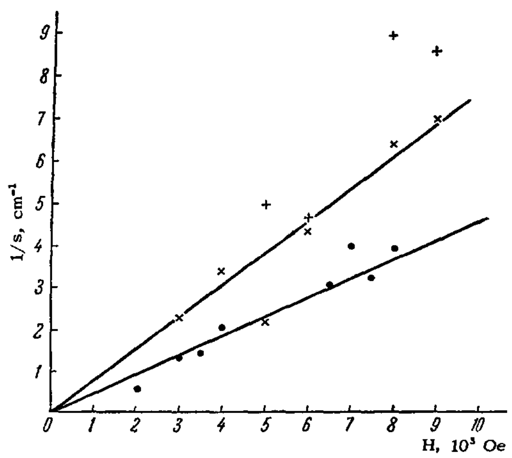

FIG. 27. Dependence of $1/s_\perp$ on H. •— n = 2.5×10^{10} cm^{-3}, ×— 3×10^{11} cm^{-3}; +— 5×10^{11} cm^{-3}.

12. Diffusion with Current Flowing Through a Plasma

In recent years the behavior of a highly ionized plasma in a magnetic field has been investigated in many experiments connected with the problem of controlled thermonuclear fusion. However, in almost all of these experiments the lifetime of the charged particles was much less than the diffusion lifetime. In various cases the lifetime was limited by hydromagnetic plasma instabilities, the loss of particles along the lines of force of the magnetic field, volume processes (cf. [1]). We limit ourselves here to a brief presentation of certain experimental results obtained with toroidal plasma devices with strong longitudinal magnetic fields under conditions such that the dangerous hydromagnetic instabilities caused by the plasma current are inhibited.

The most detailed investigations of particle lifetimes have been carried out in the American B-1 and B-3 Stellarators.[105-107] In these devices the discharge chamber is a torus that is "folded" into a "figure eight" in order to compensate for the toroidal particle drift. The plasma is produced in an external magnetic field (up to 40,000 Oe) parallel to the chamber axis. In these experiments the gas is ionized and the plasma is heated by means of a longitudinal electric field (0.05–0.3 V/cm). The maximum longitudinal current through the plasma is chosen so as to avoid the instability associated with the helical deformation of the plasma pinch characterized by low values of m (m characterizes the azimuthal symmetry of the perturbation); that is to say, the maximum current is limited by the Kruskal-Shafranov condition (cf. [1]).

The plasma density is measured during ionization and ohmic heating. The measurements are carried out by observing the phase shift of microwaves transmitted through the plasma; these measurements are carried out with 8-millimeter and 4-millimeter interferometers. Typical curves showing the variation of the plasma parameters in hydrogen are shown in Fig. 28. In the first stage (up to the point at which the density reaches a maximum) the gas is being ionized. At the time corresponding to maximum current (I) the gas in the chamber is almost completely ionized. This follows from the appreciable weakening of the emission of the hydrogen H_β line. The diminution in plasma density after the peak is due to the loss of charged particles from the plasma (a large part of these particles are evidently absorbed by the walls). Analyzing the curve showing the variation in density on the basis of reasonable assumptions concerning the ionization and dissociation of hydrogen molecules and the nature

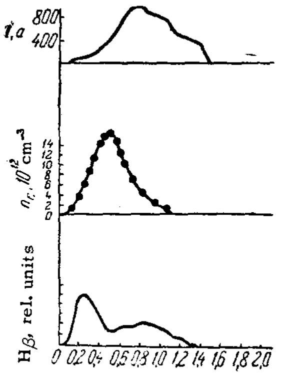

FIG. 28. Curves showing the time variation of the plasma parameters in the B-1 stellerator.

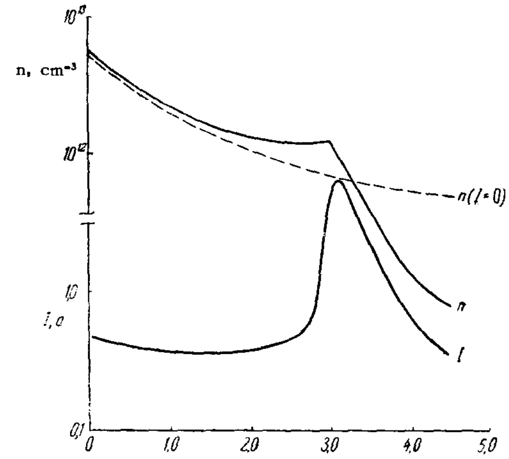

FIG. 29. Curves showing the variation of plasma current and density.

of the recycling of gas from the walls, the authors of [106] and [107] have determined the time constant for particle loss from the plasma in the ionization stage, and in the stage in which the density diminishes τ. The quantity τ is found to be 3 or 4 orders of magnitude smaller than the diffusion time across the magnetic field calculated on the basis of collisions between charged particles. In order to ascertain whether this effect was due to instabilities caused by higher-order helical instabilities of the pinch (high m) experiments were carried out with auxiliary stabilizing windings ($m = 3$). The stabilizing windings had very little effect on τ. The measurements have shown that τ increases with increasing magnetic field ($\tau \sim \sqrt{H}$ according to [106] and $\tau \sim H$ according to [107]) and that it diminishes as the effective radius of the chamber is reduced (according to [107] $\tau \sim a^2$). The dependence of τ on electron temperature was determined by making measurements in the "constant" electron temperature regime. The temperature was held constant (actually, the conductivity) by programming the voltage source in a suitable way. The quantity τ is found to be proportional to T_e. On the basis of measurements of τ in hydrogen at an initial pressure of 10^{-4}—10^{-3} mm Hg, plasma density 10^{12}—10^{13} cm^{-3}, electron temperature 2—20 eV, magnetic fields of 5000—40,000 Oe and a pinch diameter of 1—4 cm, the authors of [107] have obtained the following empirical formula for the anomalous diffusion coefficient:

$$D_{\perp} = \frac{a^2}{5.8\tau} \approx 2 \cdot 10^4 \frac{T_e}{H} \left(\frac{\text{cm}^2}{\text{sec}}\right) \tag{12.1}$$

(where T_e is in eV and H is in kOe).

We note that the results of different measurements are rather contradictory so that the empirical formula for $D_{\perp}$ is a very approximate one.

Anomalous diffusion has also been observed by Yavlinskiĭ and his coworkers in experiments on a toroidal device called Tokomak-2.[108] At an initial hydrogen pressure of 5×10^{-4}—5×10^{-3} mm Hg, a plasma density of approximately 10^{13} cm^{-3}, a longitudinal magnetic field of about 6000 Oe, a longitudinal electric field of 0.1—0.25 V/cm and a pinch diameter of 20 cm, the charged particle lifetime was found to be approximately 600 μsec. This value is of the same order of magnitude as that given by the empirical formula above.

Experiments have been carried out on the B-1 Stellarator in order to ascertain the minimum plasma current at which anomalous diffusion arises.[109] The investigations were carried out after the discharge current was turned off (afterglow) in a magnetic field of 18 kOe. As shown in [100], the plasma decay under these conditions is determined by volume recombination of electrons and ions (cf. the preceding section). A longitudinal electric field (frequency 20 kc and amplitude 0.01—0.03 V/cm) was induced in the decaying plasma. Characteristic curves showing the variation of plasma current and plasma density in helium at a pressure of 6×10^{-4} mm Hg are given in Fig. 29. At first the plasma is heated (as shown in [100] in the plasma afterglow $T_e \sim 0.1$ eV). Hence the recombination rate is reduced and the plasma conductivity, and correspondingly the current through the plasma, are increased. There is a critical current at which the decay rate increases sharply.* In [109] this increased decay rate is associated with the onset of anomalous diffusion. It has been established that the critical current is approximately proportional to the plasma density in the range 3×10^{11}—3×10^{12} cm^{-3}. The critical current density for a plasma density of approximately 10^{12} cm^{-3} and a helium pressure of 2×10^{-4}—4×10^{-3} mm Hg is approximately 0.2—0.3 A/cm^2. Similar results were obtained in argon and hydrogen.

Thus, in a highly ionized plasma a longitudinal current gives rise to an anomalously high loss of particles across the magnetic field, that is to say, anomalous diffusion.

*The reduction in plasma current at the end of decay is probably due to a reduction of plasma conductivity which, in this period, is due to collisions of electrons with neutrals.

A number of authors propose that the anomalous diffusion is due to various plasma instabilities—the excitation of ion waves [110,111] and the formation of a current-convective instability associated with the temperature gradient. [112] At the present time the available data are not adequate for choosing between these explanations. The origin of the anomalous diffusion is as yet unexplained.

CONCLUSION

An analysis of the experimental results has shown that in many cases the diffusion of charged particles across a strong magnetic field is much faster than that predicted by collisional diffusion theory.

The results of experiments on diffusion in the presence of a plasma current or a highly directed particle motion (in the positive column, in the Penning discharge, in a highly ionized plasma carrying current) are found to be in accord with theory only at low magnetic field. As the magnetic field is increased there is a critical value at which the diffusion rate increases. Simultaneously the plasma exhibits intense oscillations indicating the onset of an instability.

In experiments in which the plasma is essentially in equilibrium (plasma afterglow in a uniform magnetic field, quiescent current-free plasma in cesium or potassium), increasing the magnetic field leads to a monotonic reduction in the diffusion rate that is in approximate agreement with the collisional theory ($D_\perp \sim 1/H^2$). However, in a number of experiments on plasma decay at low collision frequencies the measured transverse diffusion coefficient has been found to be significantly larger than the expected value, as though there were some effective collision frequency larger than the calculated value by about 10^8 sec^{-1}. It should be noted that in no experiment has diffusion been observed to be smaller than that which would be calculated on the basis of a collision frequency of 10^7-10^8 sec^{-1}.

The anomalously rapid transport of plasma particles across a magnetic field (anomalous diffusion) observed in the various experiments has been associated with various instabilities, and with oscillations and plasma noise.

As far back as 1948, in order to explain the results of investigations on diffusion of charged particles from a stationary low-pressure plasma, Bohm proposed the existence of an anomalous mechanism for the transport of particles across a magnetic field due to plasma oscillations.* [59] The plasma oscillations produce alternating electric fields. The drift of charged plasma particles in these fields (in the plane perpendicular to the magnetic field) results in "collisionless" particle diffusion. Without derivation, Bohm gave the following expression for the coefficient of anomalous diffusion across a magnetic field due to oscillations:

$$D_\perp = \frac{cT}{16eH}.$$

Although this formula has not been justified, it is frequently used in the analysis of experimental results.

In various experiments the anomalous effects have been explained by making certain assumptions as to the nature of the instabilities, some of which we have touched upon in the appropriate sections of this review.

We have noted that in one case at least there has been success in identifying uniquely the instability causing the anomalous diffusion. The screw "current convective" instability described by Nedospasov and Kadomtsev [81] arises in the positive column. The development of this instability at high magnetic fields leads to the onset of a turbulent plasma state characterized by a broad oscillation spectrum. The difficulty in analyzing diffusion associated with a developed instability of this kind is that it is necessary to solve a system of nonlinear equations. Kadomtsev has formulated a theory of turbulent diffusion in the positive column on the basis of an analogy with turbulent motion in fluids. [85] The results of this theory are in agreement with the experimental data.

We may mention two other papers in which the coefficient of anomalous diffusion associated with the development of instabilities has been estimated. Each paper treats a fully ionized gas in the presence of a discharge current. Spitzer [111] has estimated the coefficient for anomalous diffusion associated with the excitation of ion acoustic waves. The expression for the diffusion coefficient is found to be approximately the same as the expression given by Bohm. Kadomtsev has obtained a relation for the diffusion coefficient in a developed current-convective instability associated with the temperature gradient in a fully ionized plasma. [112] As yet, the results of these papers have not been compared uniquely with the experimental data.

Several authors have tried to formulate a description of anomalous diffusion that does not depend on the actual form of the instability.

Ecker [113] has attempted to calculate anomalous diffusion by introducing a higher electron-ion collision frequency into the diffusion equation. Taylor has carried out an investigation with the purpose of establishing the limits on the anomalous diffusion coefficient. [114,115] This author concluded that the coefficient for transverse diffusion of ions in a fully ionized gas cannot exceed the Bohm coefficient by more than a factor of 4. Yoshikawa and Rose [116] tried to establish a relation between the diffusion coefficient in a turbulent plasma, the intensity of the oscillations, and the mean-square density fluctuations on the basis of rather general assumptions as to the nature of the turbulent motion. We shall not stop here to consider these papers

*As we have indicated (page 183) the results of measurements of the density distribution for diffusion in a stationary plasma described by Bohm[59] can be explained without recourse to anomalous diffusion.

in detail. We note only that these investigations cannot predict the conditions for which anomalous diffusion should arise.

In connection with the problem of the conditions required for the onset of anomalous diffusion special interest attaches to the so-called universal instabilities, i.e., instabilities that are not associated with the flow of current through the plasma, but which can arise by virtue of small gradients in density or temperature. One such instability has been reported in a theoretical paper by Rudakov and Sagdeev.[117] These investigators have shown that for certain definite relations between the density and temperature gradients $[\nabla(\ln T)/\nabla(\ln n) > 2$ or $\nabla(\ln T)/\nabla(\ln n) < 0]$ oblique ion acoustic waves are excited in a plasma. An analysis of effects associated with the finite Larmor radius of the ions has been given by Kadomtsev and Timofeev,[120] Galeev, Oraevskiĭ, and Sagdeev,[121] and Mikhaĭlovskiĭ and Rudakov,[122] who have shown that this instability can arise with a more realistic relation between the temperature and density gradients $[\nabla(\ln n) > \nabla(\ln T)]$.

The investigation of the criteria for the development of the universal plasma instability and the determination of the diffusion coefficient associated with this instability is, at the present time, one of the most important problems in the experimental and theoretical work on transport processes in plasma.

Note added in proof. We present a list of papers containing information on the diffusion of charged particles in a plasma in a magnetic field which appeared after the present paper was submitted for publication:

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