New Instruments and Measurement Methods

METHODS FOR EXTRACTING A BEAM FROM A SYNCHROCYCLOTRON

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1. INTRODUCTION

THE principle of synchrocyclotron (FM cyclotron) acceleration has much in common with the principle of cyclotron acceleration. Therefore the same term, cyclotron, is used to designate both types of accelerators.* However, the FM acceleration mode has specific features which must be taken into account when the beam is extracted, for unlike the ordinary cyclotron, no electric deflector can be used to extract a beam of accelerated ions.

The increase Δr in the radius of the equilibrium orbit per revolution is determined by the well-known formula [1]

$$\Delta r = \frac{2eVr\sin\varphi}{(1-n)\beta^2 E}. \quad (1)$$

Here r is the radius of the equilibrium orbit, $2eV \times \sin\varphi$ the equilibrium potential difference, $\beta = v/c$, v the velocity of the equilibrium ion, c the velocity of light in vacuum, E the total energy of the equilibrium ion, $n = d\ln H/d\ln r$, and H the intensity of the magnetic field along the equilibrium orbit. Substituting in the right half of (1) values typical of large synchrocyclotrons, we can verify that Δr is on the order of 0.1 mm. Δr is small principally owing to the small equilibrium potential difference employed in present operating synchrocyclotrons. Since the thickness of the forward deflecting plate of an ordinary electrostatic deflector is about 1 mm, the ordinary electric deflector can in general not be employed in an FM cyclotron.

In such an analysis of the problem we neglect the precession effect. Owing to the combined action of radial betatron oscillations and precession of the center of the curvature of the ion trajectory, the spread of ions is larger than Δr. Quantitatively, the maximum spread is determined by the approximate formula [2]

$$h_{\max} \approx \pi n \sqrt{2A_r \Delta r_{\mathrm{pr}}}, \quad (2)$$

where

$$\Delta r_{\mathrm{pr}} = \frac{\Delta r}{1-\sqrt{1-n}} \quad (3)$$

and A_r is the amplitude of radial betatron oscillations. As can be seen from (1)—(3), this effect may turn out to be appreciable for small synchrocyclotrons, owing to the relatively small values of β and E at the end of the acceleration. Thus, for example, in the 37-inch cyclotron, the model of the well known California 184-inch synchrocyclotron, $h_{\max}$ amounts to 0.5 cm [3,4]. This is why an ordinary electric deflector could be used in the 37-inch synchrocyclotron to extract 10% of the beam circulating in the chamber.

In the general case, to extract a beam from an synchrocyclotron it is necessary to employ other methods. This raises difficulties when the ions deviate from the limiting working radius to the region of radial instability, where $n > 1$. The reason is that in the overwhelming majority of the operating synchrocyclotrons the limiting working radius is smaller than the radius for which $n = 0.2$, and consequently the deflecting system must act over a relatively larger distance than in the cyclotron. This necessitates the use of a so-called magnetic channel to deflect the ions to the region of radial instability. The purpose of the magnetic channel is to reduce the magnetic field intensity along the ion trajectory. The corresponding decrease in the curvature of the ion trajectory causes the ion to enter the radial instability region on leaving the magnetic channel. The magnetic channel leads unavoidably to changes in the main magnetic field of the synchrocyclotron, and can therefore not be located in the immediate vicinity of the limiting working radius, and special methods must be used to direct the ions behind the inner plate of the magnetic channel. At the present time the following methods are known: 1) pulsed electric deflector, 2) multiple scattering by a target, and 3) a regenerative deflector.

One method of extracting a beam from a synchrocyclotron without using a magnetic channel will be considered in Sec. 6.

2. PULSED ELECTRIC DEFLECTOR

The deflecting system of the pulsed deflector consists of four curved strips (Fig. 1). Naturally, the electric field produced with the aid of the curved strips must not interfere with the normal synchrocyclotron acceleration. It follows therefore that a pulsed voltage must be applied to the deflector. The duration of the voltage pulse must not exceed the

*As a rule, the Soviet scientific literature employs the term "phasotron" to denote a cyclotron with frequency modulation of the accelerating electric field.

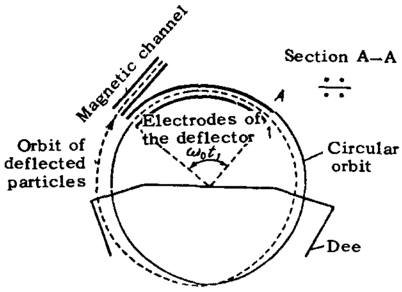

FIG. 1.

period of ion revolution at the start of the extraction process.

The pulsed electric field between the external and internal pairs of strips excites radial oscillations of the ions. These oscillations can be taken into account mathematically by adding to the right half of the usual equation of radial betatron oscillations [[5], Eq. (2.18)] a term proportional to the electric field intensity [6]:

$$\frac{d^2\varrho}{dt^2} + \omega_r^2\varrho = \frac{e\mathcal{E}}{m}. \qquad (4)$$

Here ρ is the deviation of the ion from the equilibrium orbit, t the time, $\omega_r = \omega_0\sqrt{1-n}$, ω_0 the circular frequency of revolution of the equilibrium orbit, e the ion charge, and m the ion mass.

Assuming the electric field between the pairs of strips to be homogeneous in the region occupied by the beam, we can readily integrate Eq. (4). For initial conditions $t = 0$, $\rho_0 = \delta_1 \cos \alpha_1$, and $(d\rho/dt)_0 = -\omega_r\delta_1 \sin \alpha_1$ we obtain for the region between the pairs of strips

$$\varrho = \delta_1 \cos(\omega_r t + \alpha_1) + \delta_2(1 - \cos \omega_r t), \qquad (5)$$

where

$$\delta_2 = \frac{e\mathcal{E}}{m\omega_r^2}. \qquad (6)$$

For an ion leaving the deflector we obtain [with account of (5)]

$$\varrho = \delta_1 \cos(\omega_r t + \alpha_1) + 2\delta_2 \sin\frac{\omega_r t_1}{2} \sin \omega_r\left(t - \frac{t_1}{2}\right), \qquad (7)$$

where t_1 is the instant of time of departure of the ion from the deflector.

The entrance to the magnetic channel should be preferably located at an azimuth for which the solution of (7) has a maximum. The determination of the maxima of (7) is difficult because δ_1 and α_1 are unknown, and we therefore confine ourselves here to an examination of the limiting case of infinitesimally small amplitudes of radial betatron oscillations ($\delta_1 = 0$). Then it follows directly from (7) that ρ assumes a maximum value at the instant of time

$$t_m = \frac{1}{2}\left(\frac{4k + \mathrm{sqn}\,\delta_2}{\omega_r}\pi + t_1\right), \qquad (8)$$

where k is an integer.*

*We note that owing to the simple trigonometric transformation used to derive (7), we can immediately write out an analytic expression for t_m, something not done in [6].

The pulsed electric deflector was first used in the California 184″ synchrocyclotron [6-8]. For this deflector $\delta_0 t_1 = 120°$. Since the beam current in a synchrocyclotron decreases sharply at the radius where $n = 0.2$, it is necessary to employ values $n < 0.2$ when the beam enters the deflector. The pulsed electric deflector of the California 184″ synchrocyclotron was designed for $n = 0.18$. Then when $\delta_2 < 0$ (the deflector displaces the ions inward)* the first maximum is at 357° after the entrance to the deflector, and when $\delta_2 > 0$ (the deflector shifts the ions outward) the first maximum is situated 159° after the enterance to the deflector. Owing to structural considerations (the magnetic channel is located outside the dee) the case $\delta_2 < 0$ was realized in the California synchrocyclotron, i.e., the entrance to the magnetic channel was located 357° past the entrance to the deflector. The maximum ion displacement was 7 cm. At 200 kV, the distance between pole pairs was 2.5 cm. The deflector was fed from a pulse transformer [7]. After passing through the magnetic channel, the beam goes to a rotation magnet (which also focuses the beam in the horizontal plane) and, if necessary, to a special focusing device [9] to increase the beam current density. The internal current of the beam, $J_i = 1.2 \times 10^{-6}$ A prior to entering the deflector, decreases to $J_d \approx 10^{-8}$ A at the entrance to the magnetic channel, and to $J_e = 5 \times 10^{-9}$ A in the test room (after passing through the magnetic channel). We shall use the symbols J_i, J_d, and J_e in what follows to determine the following new quantities, which characterize the extraction of the beam: $D = J_d/J_i$—the coefficient of admission of the beam into the magnetic channel, $A = J_e/J_d$—the coefficient of transmission of the magnetic channel, and $R = J_e/J_i$—the efficiency of beam extraction (in the example given above, $D = 0.84\%$, $A = 50\%$, and $R = 0.42\%$).

The duration of the current pulse of the extracted beam is $T\Delta\varphi/2\pi$, where $\Delta\varphi$ is the difference between the maximum and minimum phases of the ions in the beam and T is the period of ion revolution. The duration of the current pulse of the extracted beam has an order of magnitude 0.1 microsecond. The short duration of the extracted beam pulse makes it difficult to use electron counters in experiments with the extracted beam. This is the main shortcoming of the pulsed electric deflector method.

We note that the pulsed electric deflector is used not only in the California synchrocyclotron but also in the Carnegie Institute of Technology [10] and in the Harwell synchrocyclotron [11].

It was proposed to use a vertical pulsed electrical field to deflect the beam into the magnetic channel [11]. The magnetic channel would then be located either above or below the central plane of the synchrocyclotron magnet. Since the expected efficiency of beam extraction is of the same order in this method as in

*This is the case corresponding to Fig. 1.

the case of the radial field, this method has never been used. However, a pulsed vertical electric field is used in the Harwell synchrocyclotron to deflect the proton beam on to an internal target and obtain a short neutron pulse [12].

3. SCATTERING BY A TARGET

The simplest method of deflecting ions to the magnetic channel is to scatter them by a target.

The ions scattered by the target start to execute free radial oscillations. For certain ions, the displacement in the direction of deflection from the equilibrium orbit turns out to be precisely such that they enter the magnetic channel. It is advantageous to use targets made of a heavy substance, because 1) the energy lost by the incident particle (the energy transferred to the scattering nucleus) is smaller in scattering by heavy nuclei than by light ones, and 2) the Coulomb scattering cross section increases in proportion to the square of the atomic number of the scatterer.

The duration of the current pulse of a beam extracted by scattering can be estimated from the expression $(A_r)_{max} mT/\Delta r$, where $(A_r)_{max}$ is the maximum amplitude of radial oscillations of the ion and m is the average number of the passages of the ion through the target. By substituting into this expression typical experimental values of the quantities, we can verify that the duration of the current pulse of the extracted beam exceeds in the present case by $\sim 10^3$ times the corresponding value for the pulsed electric deflector. However, the extraction efficiency in the multiple scattering method turns out to be $\sim 10^2$ times lower than the efficiency of beam extraction with a pulsed electric deflector.

Data on the method of scattering by a target can be found in [13-18]. By way of illustration we give here the data for the Harwell synchrocyclotron [16]: $J_i = 1.3 \times 10^{-6}$ A, uranium target 0.32 cm thick, $J_e = 3 \times 10^{-10}$ A (at a distance of 12 meters from the synchrocyclotron), $R \approx 0.22\%$, $m = 15$, duration of extracted beam current 150 μsec.

It must be noted that fast protons scattered by nuclei become polarized because of the spin-orbit interaction between the protons and the nuclei. This circumstance must be taken into account when setting up experiments. It is curious to note that the first note on the use of scattering by a target to deflect ions into a magnetic channel dates back to 1949 [14], whereas proton polarization was first observed in 1952.[19] In the first case a thorium target was used, and in the latter a carbon target. It is known (see the review [20]) that the proton polarization decreases with increasing atomic number of the scattering nucleus (polarizer). Therefore proton polarization could be observed first by scattering with light nuclei. This law must likewise be taken into account in accelerator technology if a beam of polarized protons is required. Thus, for example, beryllium and carbon targets are used for this purpose [20]. As to the use of a magnetic channel in these experiments, it must be noted that a magnetic channel is used if the proton scattering angle in the horizontal plane is small (less than 10°), then, but not at large scattering angles [21], for in this case the protons enter rapidly the region of radial instability.

Since experiments with beams of polarized protons are of great interest at the present time, it has been proposed to accelerate previously polarized ions [22,24] and to extract these ions by more efficient methods (see Sec. 4). It is then necessary, of course, to investigate the possible depolarization of the beam during the acceleration process [24].

4. REGENERATIVE DEFLECTOR

In 1950 Tuck and Teng[25,26] proposed a new method of deflecting accelerated ions into a magnetic channel. They proposed to replace the normal magnetic field on the edge of the synchrocyclotron so as to cause the amplitude of the radial oscillations to increase while the stability of the vertical oscillations is maintained.

A diagram of the extraction device proposed by Tuck and Teng is shown in Fig. 2. The normal synchrocyclotron acceleration continues up to a radius r_s, at which beam extraction begins. Outside of r_s,

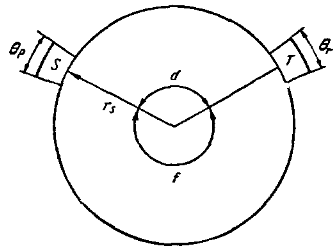

FIG. 2

the vertical component of the magnetic field H decreases along the radius $(n > 0)$ in a region of angular width θ_p, called the exciter (region S), and increases along the radius $(n < 0)$ in a region of angular width θ_r, called the regenerator (region D). Such an extraction device is called a regenerative deflector.

Physically the operating principle of the regenerative deflector is based on excitation of parametric resonance between the ion revolutions of circular frequency ω_0, and the radial betatron oscillations of angular frequency ω_r [27]. As is well known [5], a simple relationship $\omega_r = \omega_0 \sqrt{1-n}$ exists between ω_0 and ω_r. In synchrocyclotrons n is small. Therefore ω_r is close to ω_0 and by introducing local variations in the decrement of the magnetic field intensity it is possible to excite the above-mentioned resonance. Radial oscillations are induced in this case by an exiter with $n > 0$, while the precession of the centers of curvature of the ion orbits is compensated for with the aid of a regenerator with $n < 0$.*

Tuck and Teng have verified the feasibility of such a deflector by graphically plotting the trajectories of the extracted ions for different combinations of fields in the regenerator and in the exciter. The main shortcoming of such a method of investigation is the large amount of labor involved. Therefore, in order to find the optimal values of the parameters characterizing the action of the regenerative deflector, the problem was considered analytically by various authors.

a) Linear theory of the regenerative deflector. The basic results of the linear theory of the regenerative deflector are contained in the papers by LeCouteur[28,29], Dmitrievskiĭ[30,31], Barden[27], Crewe[32], and Cohen and Crewe[33,34]. The initial equations in these investigations are the ordinary betatron oscillation equations [[5], formula (2.18)], i.e., it is assumed that the magnetic fields of the generator and the exciter and the normal magnetic field of the synchrocyclotron depend on the radius linearly. As is well known, there is no connection between the radial and vertical oscillations in this case, and the solution is simpler.

Various methods have been employed to solve the equation of the betatron oscillations under the conditions where a regenerative deflector is in operation. Barden used Fourier analysis, Dmitrievskiĭ the operator method, and LeCouteur, Crewe, and Cohen used a matrix method to solve the differential equations.

Owing to the greater clarity of the matrix method, we consider it appropriate to present here LeCouteur's most important results. He starts from the following equations:

$$\left.\begin{aligned} &\frac{d^2\varrho}{d\theta^2}+\Omega_r^2\varrho=0, \quad \frac{d^2z}{d\theta^2}+\Omega_z^2 z=0,\\ &\Omega_r=\sqrt{1-n}, \quad \Omega_z=\sqrt{n}\,; \end{aligned}\right\} \tag{9}$$

$$\left.\begin{aligned} &\frac{d^2\varrho}{d\theta^2}-(p^2-1)\,\varrho=0, \quad \frac{d^2z}{d\theta^2}+p^2z=0,\\ &p^2=-\left(\frac{r}{H}\frac{\partial H}{\partial r}\right)_{\text{exc}}\,; \end{aligned}\right\} \tag{10}$$

$$\left.\begin{aligned} &\frac{d^2\varrho}{d\theta^2}+(q^2+1)\,\varrho=0, \quad \frac{\partial^2 z}{\partial\theta^2}-q^2z=0,\\ &q^2=\left(\frac{r}{H}\frac{\partial H}{\partial r}\right)_{\text{reg}}\,. \end{aligned}\right\} \tag{11}$$

Here $\rho(z)$ is the radial (vertical) deflection of the ion from the equilibrium orbit of radius r_s. Equations (9) hold true for the regions of the unperturbed field, while (10) and (11) apply to the exciter and regenerator regions, respectively. Because to the proposed shape of the deflector field (Fig. 2), Eqs. (10) and (11) are applicable only when the radial deflection of the ion in the exciter and the regenerator are directed outward.

In order to present the results in compact form, we introduce the following notation:

$$\boldsymbol{\varrho}=\begin{pmatrix}\varrho\\ \frac{d\varrho}{d\theta}\end{pmatrix}, \quad \mathbf{z}=\begin{pmatrix}z\\ \frac{dz}{d\theta}\end{pmatrix} \tag{12}$$

$$\left.\begin{aligned} &U(a,\ x)=\begin{pmatrix}\cos ax & \frac{\sin ax}{a}\\ -a\sin ax & \cos ax\end{pmatrix},\\ &V(a,\ x)=\begin{pmatrix}\operatorname{ch} ax & \frac{\operatorname{sh} ax}{a}\\ a\operatorname{sh} ax & \operatorname{ch} ax\end{pmatrix}. \end{aligned}\right\} \tag{13*}$$

By direct verification we can check the correctness of the following relations:

$$\left.\begin{aligned} &U(a,\ x)\,U(a,\ y)=U(a,\ x+y),\\ &V(a,\ x)\,V(a,\ y)=V(a,\ x+y). \end{aligned}\right\} \tag{14}$$

Let us trace now the radial motion of the ion during one revolution. We begin, for example, with the entry of the ion in the exciter. Then, according to (10), the matrix $V(\sqrt{p^2-1},\ \theta_p)$ transforms the vector ρ corresponding to the entry of the ion in the exciter into the vector ρ corresponding to extraction of the ion from the exciter:

$$\boldsymbol{\varrho}_{\text{out exc}}=V\left(\sqrt{p^2-1},\ \theta_p\right)\boldsymbol{\varrho}_{\text{in exc}}. \tag{15}$$

The vector $\rho_{\text{out exc}}$ specifies the initial conditions when the ion moves in an interval of angular width $f-(\theta_p+\theta_q)/2$. At the end of this section, according to Eq. (9), we have

$$\boldsymbol{\varrho}_{\text{out}\,f}=U\left(\Omega_r,\ f-\tfrac{1}{2}\theta_p-\tfrac{1}{2}\theta_q\right)\boldsymbol{\varrho}_{\text{out exc}} \tag{16}$$

The vector $\rho_{\text{out f}}$ specifies the initial conditions for the motion of the ion in the regenerator. Further, according to (11), the vector ρ for extraction of the ion from the regenerator is determined in the following manner:

$$\boldsymbol{\varrho}_{\text{out reg}}=U\left(\sqrt{q^2+1},\ \theta_q\right)\boldsymbol{\varrho}_{\text{out}\,f}. \tag{17}$$

The vector $\rho_{\text{out reg}}$ specifies the initial conditions for the motion of the ion in an interval with angular width $d-(\theta_p+\theta_q)/2$, and at the end of this interval we have in accordance with (9)

$$\boldsymbol{\varrho}_{\text{out}\,d}=U\left(\Omega_r,\ d-\tfrac{1}{2}\theta_p-\tfrac{1}{2}\theta_q\right)\boldsymbol{\varrho}_{\text{out reg}}. \tag{18}$$

Substituting in succession (15) in (16), (16) in (17), and (17) in (18), we obtain

$$\boldsymbol{\varrho}_{\text{out}\,d}=C_r\boldsymbol{\varrho}_{\text{in exc}}, \tag{19}$$

where

$$\begin{aligned} C_r=&\,U\left(\Omega_r,\ d-\tfrac{1}{2}\theta_p-\tfrac{1}{2}\theta_q\right)U\left(\sqrt{q^2+1},\ \theta_q\right)\\ &\times U\left(\Omega_r,\ f-\tfrac{1}{2}\theta_p-\tfrac{1}{2}\theta_q\right)V\left(\sqrt{p^2-1},\theta_p\right). \end{aligned} \tag{20}$$

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*It is known that the circular frequency of precession of the center of curvature of ion orbits for small n (as is the case in a synchrocyclotron) is given by the expression $\omega_{pr} \approx n\omega_0/2$. Consequently, to compensate for this precession it is necessary to introduce a region with negative n.

*sh = sinh, ch = cosh.

The vector $\rho_{\text{out d}}$ specifies the initial conditions for the motion of the ion in the exciter during the next revolution. Thus, a single passage of the ion through all the sectors of the deflector is described by the following relation:

$$\boldsymbol{\rho}_{m+1} = C_r \boldsymbol{\rho}_m, \tag{19'}$$

where ρ_m denotes the vector ρ at the entry of the ion in the exciter in the m-th revolution. m-fold passage through all the sectors of the deflector is described by the relation

$$\boldsymbol{\rho}_m = C_r^{m-1} \boldsymbol{\rho}_1, \tag{21}$$

where ρ_1 denotes the vector ρ at the entry of the ion in the exciter during the first revolution.

Analogously we have for the vertical oscillations

$$\mathbf{z}_m = C_z^{m-1} \mathbf{z}_1, \tag{22}$$

where $\mathbf{z}_m$ denotes the vector $\mathbf{z}$ at the entry of the ion in the excitor during the m-th revolution and

$$C_z = U\left(\Omega_z,\ d - \frac{1}{2}\theta_p - \frac{1}{2}\theta_q\right) V(q,\ \theta_q) \times U\left(\Omega_z,\ f - \frac{1}{2}\theta_p - \frac{1}{2}\theta_q\right) U(p,\ \theta_p). \tag{23}$$

In order to investigate (21) and (22) it is advantageous to introduce the eigenvalues and the corresponding eigenvectors of the matrix $C_{r,z}$. Since det $C_{r,z} = 1$ in accordance with (13), the product of the eigenvalues of the matrix $C_{r,z}$ is equal to unity. We therefore put

$$\left.\begin{aligned} C_{r,z}\mathbf{R}_{r,z} &= \sigma_{r,z}\mathbf{R}_{r,z}, \\ C_{r,z}\mathbf{R}'_{r,z} &= \frac{1}{\sigma_{r,z}}\mathbf{R}'_{r,z}, \end{aligned}\right\} \tag{24}$$

where

$$\mathbf{R}_{r,z} = \begin{pmatrix} R_{r,z1} \\ R_{r,z2} \end{pmatrix}, \quad \mathbf{R}'_{r,z} = \begin{pmatrix} R'_{r,z1} \\ R'_{r,z2} \end{pmatrix}.$$

The eigenvalues of the matrix $C_{r,z}$ are denoted here by $\sigma_{r,z}$ and $1/\sigma_{r,z}$ and the eigenvectors corresponding to them are $\mathbf{R}_{r,z}$ and $\mathbf{R}'_{r,z}$.

Let us express $\rho_1(\mathbf{z}_1)$ in terms of $\mathbf{R}_{r,z}$ and $\mathbf{R}'_{r,z}$:

$$\boldsymbol{\rho}_1(\mathbf{z}_1) = x_{r,z}\mathbf{R}_{r,z} + y_{r,z}\mathbf{R}'_{r,z}. \tag{25}$$

Here

$$\left.\begin{aligned} x_{r,z} &= \frac{\rho_1(z_1) R'_{r,z2} - \dot{\rho}_1(\dot{z}_1) R'_{r,z1}}{R_{r,z1}R'_{r,z2} - R_{r,z2}R'_{r,z1}}, \\ y_{r,z} &= \frac{\rho_1(z_1) R_{r,z2} - \dot{\rho}_1(\dot{z}_1) R_{r,z1}}{R_{r,z2}R'_{r,z1} - R'_{r,z2}R_{r,z1}}, \end{aligned}\right\} \tag{25'}$$

where the dot denotes differentiation with respect to θ.

According to (21), (22), (24), and (25) the effect of m revolutions is written in the form

$$\boldsymbol{\rho}_m(\mathbf{z}_m) = x_{r,z}\sigma_{r,z}^m\mathbf{R}_{r,z} + y_{r,z}\sigma_{r,z}^{-m}\mathbf{R}'_{r,z}. \tag{26}$$

The deflector should be made such that one of the eigenvalues, say

$$\sigma_r \equiv e^{\Lambda}, \tag{27}$$

exceeds unity. Then $1/\sigma_r \equiv r^{-\Lambda}$ must of necessity be less than unity. Therefore $e^{-\Lambda m}$ becomes negligibly small after several revolutions and we get from (26)

$$\boldsymbol{\rho}_m = x_r e^{+\Lambda m}\mathbf{R}_r. \tag{28}$$

Expression (28) shows that when condition (27) is satisfied the ion is captured in a motion in which its amplitude of radial oscillations increases by a factor e^{Λ} during each revolution, while the phase at the entrance to the exciter remains constant. The quantity e^{Λ} is called by LeCouteur the coefficient of amplification of the radial oscillations.

In order to avoid an infinite increase in the amplitude of the vertical oscillations of the ion, the moduli of the eigenvalues σ_z and $1/\sigma_z$ should be equal to unity, i.e.,

$$\sigma_z \equiv e^{i\lambda}. \tag{29}$$

Condition (24) in expanded form represents a system of homogeneous linear equations for the determination of $R_{r,z1}$ and $R_{r,z2}$. In order for this system to have a nontrivial solution its determinant must equal zero. Therefore, taking (27) and (29) into account, we get

$$\left.\begin{aligned} \operatorname{ch}\Lambda &= \frac{1}{2}\operatorname{Sp} C_r = \frac{1}{2}(C_{r11} + C_{r22}), \\ \cos\lambda &= \frac{1}{2}\operatorname{Sp} C_z = \frac{1}{2}(C_{z11} + C_{z22}). \end{aligned}\right\} \tag{30}$$

To obtain the real values of Λ and λ the following conditions should be satisfied*

$$\frac{1}{2}\operatorname{Sp} C_1 > 1, \tag{31}$$

$$\frac{1}{2}|\operatorname{Sp} C_z| < 1. \tag{32}$$

If the angles θ_p and θ_q are small, we can simplify the following factors

$$\left.\begin{aligned} V\left(\sqrt{p^2-1},\ \theta_p\right) &\approx B(S), \\ U\left(\sqrt{q^2+1},\ \theta_q\right) &\approx B(-T), \\ U(p,\ \theta_p) &\approx B(-S), \\ V(q,\ \theta_q) &\approx B(-T), \end{aligned}\right\} \tag{33}$$

which enter into the equation $C_{r,z}$; here

$$B(x) = \begin{pmatrix} 1 & 0 \\ x & 1 \end{pmatrix}, \tag{34}$$

$$S = p^2\theta_p, \quad T = q^2\theta_q. \tag{35}$$

According to (15), (17), and analogous expressions for the vertical oscillations, we obtain with allowance for (33)

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*We exclude from consideration here the limits of the stability boundaries, corresponding to $\Lambda = 0$ and $\lambda = s\pi$ (s integer).

$$\left.\begin{aligned} \varrho_{\text{out exc}} &= \varrho_{\text{in exc}} \equiv \varrho_{\text{exc}}, \\ (d\varrho/d\theta)_{\text{out exc}} &= (d\varrho/d\theta)_{\text{in exc}} + S\varrho_{\text{exc}}, \end{aligned}\right\} \quad (36)$$

$$\left.\begin{aligned} \varrho_{\text{out reg}} &= \varrho_{\text{in reg}} \equiv \varrho_{\text{reg}}, \\ (d\varrho/d\theta)_{\text{out reg}} &= (d\varrho/d\theta)_{\text{in reg}} - T\varrho_{\text{reg}}, \end{aligned}\right\} \quad (37)$$

$$\left.\begin{aligned} z_{\text{out exc}} &= z_{\text{in exc}} \equiv z_{\text{exc}}, \\ (dz/d\theta)_{\text{out exc}} &= (dz/d\theta)_{\text{in exc}} - Sz_{\text{exc}}, \end{aligned}\right\} \quad (38)$$

$$\left.\begin{aligned} z_{\text{out reg}} &= z_{\text{in reg}} \equiv z_{\text{reg}}, \\ (dz/d\theta)_{\text{out reg}} &= (dz/d\theta)_{\text{in reg}} + Tz_{\text{reg}}. \end{aligned}\right\} \quad (39)$$

It follows from (36)—(39) that in the case of small θ_p and θ_q, the exciter and the regenerator do not change the coordinates ρ and z of the ion, but change jumpwise the "velocities" $d\rho/d\theta$ and $dz/d\theta$.

Substituting (33) in (20) and (23), and multiplying the matrices, we obtain by means of (30)

$$\operatorname{ch}\Lambda = \operatorname{ch} 2\pi\Omega_r - \frac{T-S}{2\Omega_r}\sin 2\pi\Omega_r - \frac{TS}{\Omega_r^2}\sin\Omega_r f \sin\Omega_r d, \quad (40)$$

$$\cos\lambda = \cos 2\pi\Omega_z + \frac{T-S}{2\Omega_z}\sin 2\pi\Omega_z - \frac{TS}{\Omega_z^2}\sin\Omega_z f \sin\Omega_z d. \quad (41)$$

Although condition (32) ensures the absence of infinite amplification of the amplitude of the vertical oscillations, a correlation nevertheless arises between z and $dz/d\theta$, and this can lead to a broadening of the beam [28,35]. The existence of a correlation between z and $dz/d\theta$ can be verified by using (38) and (39). For this purpose we square (39), for example, and average ($\langle \ldots \rangle$) over many revolutions

$$\left\langle \left(\frac{dz}{d\theta}\right)^2_{\text{out reg}} \right\rangle = \left\langle \left(\frac{dz}{d\theta}\right)^2_{\text{in reg}} \right\rangle + 2T\left\langle z_{\text{reg}}\left(\frac{dz}{d\theta}\right)_{\text{in reg}} \right\rangle + T^2\langle z^2_{\text{reg}}\rangle. \quad (42)$$

Since the amplitude of the vertical oscillations cannot increase without limit the term in the left half of (42) is equal to the first term in the right half. It follows therefore immediately (see, for example, [36]) that the line of regression of $(dz/d)_{\text{in reg}}$ on z_{reg} has the form

$$\left(\frac{dz}{d\theta}\right)_{\text{in reg}} = -\frac{1}{2}Tz_{\text{reg}}. \quad (43)$$

Expression (43) shows that a correlation exists between $(dz/d\theta)_{\text{in reg}}$ and z_{reg}, but does not characterize the value of this correlation. To estimate this value it is necessary to calculate the correlation coefficient r. In view of the simple derivation of (43), it is not advantageous to calculate the regression line of z_{reg} on $(dz/d\theta)_{\text{in reg}}$ in this case. In the particular case of $S = 0$ and $d = 0$, LeCouteur obtained

$$z_{\text{reg}} = -\frac{1}{2}\,\frac{T}{\Omega_z^2 - \dfrac{T}{\Omega_z}\cos 2\pi\Omega_z}\left(\frac{dz}{d\theta}\right)_{\text{in reg}} \quad (44)$$

Taking (43) and (44) into account, it follows that

$$r = \frac{T}{2\sqrt{\Omega_z^2 - \dfrac{T}{\Omega_z}\cos 2\pi\Omega_z}}. \quad (45)$$

For example, if $\Omega_z = 1/4$ (corresponding to $n = 0.0625$), then $r = 2T$. Since the choice of T is limited by condition (32), we get $r < 1$.[35]

LeCouteur gives for the change in amplitude of the vertical oscillations under the influence of a regenerative deflector the following estimate [28]:

$$\overline{h^2} = \frac{h_0^2}{\sin\varepsilon}, \quad (46)$$

$$\sin\varepsilon = \frac{\sin\lambda\sin 2\pi\Omega_z}{1 - \cos\lambda\cos 2\pi\Omega_z + \dfrac{TS}{2\Omega_z^2}\sin^2\Omega_z d}. \quad (47)$$

In (46), $\overline{h^2}$ is the geometric mean of the square of the maximum and minimum amplitudes of the vertical oscillations under the action of the regenerative deflector, and h_0 is the amplitude of the vertical oscillations prior to the action of the regenerative deflector. We note that LeCouteur uses the formula for harmonic oscillations to calculate h. This, however, is a crude approximation, for once the regenerative deflector goes into action the vertical oscillations do not remain harmonic.

For final extraction from the synchrocyclotron, the beam must pass through a magnetic channel. It is obvious that in the last revolution prior to entering the channel the radial deviation should increase by an amount exceeding the thickness of the internal wall of the magnetic channel. From this requirement follows for the amplification coefficient of the radial oscillations the following inequality:

$$e^{\Lambda} \geqslant 1 + \frac{d_1}{d_2}, \quad (48)$$

where d_1 is the thickness of the internal wall of the magnetic channel and d_2 is the distance from the internal edge of the internal wall of the magnetic channel to the equilibrium orbit of radius r_s.*

b) Nonlinear theory of the regenerative deflector. The linear theory can be used to construct a regenerative deflector in the case when $\partial H/\partial r$ is constant at the exciter and the regenerator locations. In actual synchrocyclotrons, however, $\partial H/\partial r$ is constant for radii that are smaller than the radius for which $n = 0.2$ (for a typical form of the $H(r)$ curve see, for example, [37]). Therefore such a deflector makes it possible to extract ions of energy several percent below the maximum energy. This raises the important problem of using a regenerative deflector in the region where $\partial H/\partial r$ is not constant, in order to extract ions with maximum possible energy.

An account of the nonlinearity of $H(r)$ was first carried out in the theory of the regenerative deflector by LeCouteur [29,37] in 1953. Since now $\partial^2H/\partial r^2 \neq 0$, a connection arises between the vertical and radial oscillations. Principal attention was paid in [37] to the maintenance of stability of the vertical oscillations.

*The word "internal" is used here in the sense "closer to the center of the synchrocyclotron."

It was found necessary to employ a "weak" exciter (small values of S) and a "strong" regenerator (large values of T). Consequently the normal magnetic field of the synchrocyclotron can be used as the exciter.

Thus, the idea arises of creating a regenerative deflector without using an exciter. The deflector construction becomes simpler in this case, since it is necessary to produce only one region of local inhomogeneity in the magnetic field intensity. The theory of such a deflector is developed in the papers of LeCouteur and Lipton [38], Verster [39], Stubbins [40,41] and Matora [42]. Since the theory of LeCouteur and Lipton has been widely used until now, we shall stop to discuss this theory.

LeCouteur and Lipton start from the following equations

$$\frac{d^2\varrho}{d\theta^2}+\varrho+\frac{r_s}{H_s}(H-H_s)\left(1+\frac{2\varrho}{r_s}\right)+\frac{\varrho^2 H}{r_s H_s}=0,$$

$$\frac{d^2z}{d\theta^2}-z\frac{r_s}{H_s}\frac{\partial H}{\partial r}\left(1+\frac{\varrho}{r_s}\right)^2=0, \qquad (49)$$

where H_X is the vertical component of the magnetic field intensity along an equilibrium orbit of radius r_S, and H is the same component at a point with coordinates ($r = r_S + \rho$, z).

According to (49), a narrow regenerator changes the "velocities" in the following fashion:

$$\left.\begin{aligned}\left(\frac{d\varrho}{d\theta}\right)_{\text{out reg}}-\left(\frac{d\varrho}{d\theta}\right)_{\text{in reg}}&=-\theta_q\frac{r_sH_q}{H_s}\left(1+\frac{\varrho}{r_s}\right)^2\equiv-T_r\varrho,\\ \left(\frac{dz}{d\theta}\right)_{\text{out reg}}-\left(\frac{dz}{d\theta}\right)_{\text{in reg}}&=\theta_q\frac{r_s}{H_s}\frac{\partial H_q}{\partial r}\left(1+\frac{\varrho}{r_s}\right)^2z\equiv T_zz,\end{aligned}\right\} \qquad (50)$$

where H_q is the vertical component of the magnetic field in the regenerator. Inasmuch as $H_q(r)$ is a nonlinear function, we have $T_r \neq T_Z$. The discussion in [38] pertains to the Liverpool synchrocycloton, where r_S is chosen such that Eqs. (49) can be represented in the form

$$\frac{d^2\varrho}{d\theta^2}+\Omega_r^2\varrho=0, \quad \frac{d^2z}{d\theta^2}+\Omega_z^2z=0, \qquad (51)$$

$$\frac{d^2\varrho}{d\theta^2}+\Omega_r^2\varrho(1-0{,}152\varrho)+0.13z^2=0; \quad \frac{d^2z}{d\theta^2}+(\Omega_z^2+0.26\varrho)z=0, \qquad (52)$$

where (51) holds for $\rho < 0$ and (52) for $\rho > 0$. $\Omega_r^2 = 1 - n = 0.955$ and $\Omega_Z^2 = n = 0.045$, with ρ and z in inches.

According to [38] it is possible to describe in first approximation the oscillations when $\rho > 0$ by means of the frequencies Ω_r' and Ω_Z', which depend on ρ. The expressions for Ω_r' and Ω_Z' can be obtained from (52) with the aid of the method of nonlinear mechanics:

$$\left.\begin{aligned}\Omega_r'&=\frac{2\Omega_r}{1+(1-0{,}152\varrho)^{-\frac{1}{2}}},\\ \Omega_z'&=0.24\Omega_z+0.76(\Omega_z^2+0{,}26\varrho)^{\frac{1}{2}}.\end{aligned}\right\} \qquad (53)$$

After choosing Ω_r' and Ω_Z' in accordance with (53), we can use the results of the linear theory [28], where it is necessary to replace in the formulas the value of T by T_r for the radial oscillations and by T_Z for the vertical oscillations. The fact that $T_r \neq T_Z$ permits the radial deflection of the ion to be increased while the vertical oscillations of the ion are maintained constant.

The results of the approximate theory were verified by numerical integration of (51) and (52) with the aid of an electronic computer. The need for using an electronic computer in this problem was pointed out also in [41]. To carry out the numerical calculations it was assumed in [38] that

$$\theta_q\frac{r_s}{H_s}H_q=T\varrho+V\varrho^2+W\varrho^3, \qquad (54)$$

where T, V, and W are constant. A modified Runge-Kutta method was used in the solution. The values of ρ, z, $d\rho/d\theta$, and $dz/d\theta$ after each revolution are assumed, with account of (50) and (54), to be the initial conditions for the next revolution. The calculations were carried out for the following set of parameters:

Case	T, in^{-1}	V, in^{-2}	W, in^{-3}
A	0.2	0.2	0
B	0.2	0.25	0
C	0.2	0.25	0.01
D	0.2	0.25	0.02
E	0.2	0.3	0.04

The results of the calculations have shown that in cases A and B it is possible to obtain an increment in the radial deflection of the ion larger than 1 inch per revolution. In cases C, D, and E (when $W \neq 0$) in the case of large radial ion deflections the vertical oscillations become unstable.

Since a regenerator located where $\partial H/\partial r \neq$ const makes it possible to obtain larger radial deviations of the ion and larger increments per revolution, the magnetic channel can have in this case a larger aperture and can be relatively short. This circumstance enables us to increase the efficiency of beam extraction.

c) Practical application. The use of the regenerative deflector raises the efficiency of beam extraction to several percent. This is why the regenerative deflector is at present the most widely used means of beam extraction. The first regenerative deflectors built and used were those of the Liverpool synchrocyclotron [43-46] and of the synchrocyclotron of the Joint Institute for Nuclear Research [30,31,47,48]. These deflectors were designed on the basis of the linear theory, correspondingly developed in [28] and [30]. For the Liverpool synchrocyclotron D = 0.2, A = 0.15 and R = 0.03, while for the synchrocyclotron of the Joint Institute for Nuclear Research R = 0.05—0.06.

Since then, regenerative deflectors have been used extensively only in conjunction with regenerators (without an exciter), constructed on the basis of the

Table I. Methods of beam extraction, used in various synchrocyclotrons.

Location of accelerator	Main parameters of accelerator: Diameter of magnet pole	Kind and energy of accelerated particles, MeV	Current of internal beam, microamperes	Methods of beam extraction (literature): Electric pulsed deflector	Scattering by a target with large mass number	Regenerative deflector	Passage through region where n = 1
1. University of California	184″=4,6 m	720p 430d 880α	0.1	6-8	14, 15	50	72
2. Joint Institute for Nuclear Research (USSR)	m	680p 405d 811α	0.3		13	30, 31 47, 48	
3. CERN (Switzerland)	5 m	600p	0.1			51—53	
4. University of Chicago	170″=4.25 m	450p 260d 520α	1—2		17	54, 55	43
5. Carnegie Institute of Technology	140″=3.5 m	450p	1	10			70, 71
6. Liverpool University	156″=3.9 m	410p	1			43—46	
7. University of Rochester	130″=3.25 m	240p	0.1			56	
8. Institute of Nuclear Chemistry, Uppsala (Sweden)	2.3 m	192p	1			57, 58	
9. Atomic Energy Research Center, Harwell (England)	110″=2.75 m	175p	1—2		16		
10. Harvard University	95″=2.38 m	160p	0.3		18	59	
11. Paris University	2.8 m	160p 80d 160α	7			60, 61	
12. Tokyo University	1.6 m	65p	1				73, 74
13. National Commission on Atomic Energy, Buenos Aires	1.75 m	29d	30			62	

Remarks: 1. The indicated literature contains data on the use of the corresponding method of beam extraction for the specific synchrocyclotron.
2. The main parameters of the accelerator are taken essentially from [77,78]. There are no data on the extracted beam in the table, since the literature does not contain enough data for a systematic description.

nonlinear theory and yielding extracted ions with almost maximum energy. A summary of the corresponding literature is given in Table I.

Measurements of the energy of the extracted-beam ions have shown that the energy spread is small. Thus, for example, in the case of the synchrocyclotron of the Joint Institute for Nuclear Research, the energy spectrum $\Phi(E)$ of the extracted beam [49] is well described by the Gaussian curve

$$\Phi(E) = \exp\left\{-\frac{(E-\bar{E})^2}{2(\Delta E)^2}\right\} \qquad (55)$$

with mean energy $\bar{E}$ = 665 MeV and variance ΔE = 2.8 MeV.

The magnetic fields of the regenerator and exciter were produced by placing iron masses near the orbit where the beam extraction begins. The calculation of the magnetic fields is carried out in this case assuming axial magnetization of the magnetic masses to saturation [30 31]. We note that in [27] it is proposed to produce the regenerator and exciter fields with the aid of conductors located in the synchrocyclotron chamber and fed by current pulses.

To focus the beam after passage through the magnetic channel, magnetic quadrupole lenses are usually employed. However, it is also possible to focus with the non-working region of the synchrocyclotron magnet [63]* or with a deflecting magnet [65]. It is necessary then to add iron masses of definite shape to these magnets to produce the focusing fields.

5. MAGNETIC CHANNEL

The common element of the various extraction devices based on the principles developed in Sec. 2—4 is the magnetic channel. The purpose of the magnetic channel is to reduce appreciably the magnetic field intensity along the orbit of the extracted ions. The magnetic channel consists usually of several sections, each comprising two iron bars in the form of rectangular parallelepipeds. The magnetic channel is located near the edge of the pole of the synchrocyclotron magnet. It is obvious that in the space between the bars constituting the magnetic channel, the magnetic field of the synchrocyclotron is reduced in intensity.

The following circumstance is used to calculate the magnetic channel: if we place in the magnet gap an iron mass of arbitrary form, with the dimension in the direction of the magnetic field larger than at least one dimension in a direction perpendicular to the magnetic field, then such an iron mass is magnetized almost to saturation in a field stronger than 9,000 Oe [6,66]. The intensity of the magnetic field produced by the iron mass is then given by the formulas

$$\mathbf{H} = -\operatorname{grad}\Phi, \tag{56}$$

$$\Phi(x, y, z) = M \int \frac{(z-z')\,dx'\,dy'\,dz'}{[(x-x')^2+(y-y')^2+(z-z')^2]^{\frac{3}{2}}}. \tag{57}$$

It is assumed in (57) that the external magnetic field is directed along the z axis. The integration is over the volume of the iron mass. For the magnetization M for most grades of iron we can assume the value [6,66]

$$M = \frac{21000 \pm 500}{4\pi}\ \text{Oe}.$$

The dimensions of the iron bars used to produce the magnetic channel, and the magnetic field of the synchrocyclotron where these bars are located, satisfy the foregoing requirements. Therefore the design of the magnetic channel [66] is based on the use of the formulas (56) and (57)†. In [66] the results are represented in a convenient graphic form, which facilitates the design of the magnetic channel.

The magnetic channel unavoidably gives rise to a certain change in the magnetic field in the region of the equilibrium orbit, up to which the normal synchrocyclotron acceleration continues. In order to eliminate this perturbation, it is necessary to use suitably chosen shims. The theory of the shims is based on the use of formulas (56) and (57) and is developed in [6,66,68].

Experimental methods of adjusting and shimming the magnetic channel are described in [6,44,55,59].

The focusing of the beam can be regulated by varying the magnetic field intensity gradients in the sections of the channel. Usually the beam leaving the channel has appreciable dimensions in the horizontal plane. To eliminate this, the last sections (in the direction of motion of the beam) can be made to consist of only one internal plate [59], with the aid of which a horizontally focusing magnetic field is produced.

It has also been proposed to remove the internal plate in the first section of the channel in order to increase the effective extraction of the beam [30,31,42]. In the case of a regenerative deflector the remaining plate acts then like an exciter. Thus, the magnetic channel ceases to be a "passive" element in the method of the regenerative deflector, and participates in the buildup of the radial oscillations.

6. PASSAGE OF BEAM THROUGH A REGION WHERE n = 1.

In 1951 Hamilton and Lipkin [69] proposed a method of beam extraction based on the fact that the ions are accelerated to an energy corresponding to a radius on which $n = 1$, and are then extracted with the aid of an electric deflector. The use of the electric deflector is possible in this case (compare with Sec. 1), for when $n > 1$ there is instability of the radial oscillations [the radial deflection increases by a factor $\exp(2\pi\sqrt{n-1})$ per revolution]. Such a method of beam extraction has the advantage that the ions extracted are accelerated to the maximum possible energy in the given synchrocyclotron, since the magnetic stiffness Hr has a maximum at $n = 1$.

An important assumption in the Hamilton and Lipkin method is that a considerable part of the beam can be brought from a radius where $n = 0.2$ to a radius where $n = 1$, i.e., the resonances at $n = 0.2$, 0.25, 0.5, 0.75, and 1 should not be dangerous. This can be attained if the synchrocyclotron magnetic field is homogeneous to a high degree. In most existing synchrocyclotrons this condition is not satisfied. Consequently the Hamilton and Lipkin method did not find successful application until recently. Only a few attempts to accelerate the ions to a radius where $n = 1$ and to extract them by natural untwisting were made in the Chicago [43], Carnegie [70,71] and California [72] synchrocyclotrons, but the beam losses were appreciable.

In 1959 it became possible to apply successfully the Hamilton method in the 160 cm synchrocyclotron of the Tokyo University [73,74]. The high degree of homogeneity of the magnetic field of this synchrocyclotron made it possible to guide the beam successfully through the mentioned resonances. The reso-

*Abstracts of the papers by the Soviet physicists[31,47,48,63] on the extraction of a beam from synchrocyclotrons can be found in [64].

†We note that a similar method has been recently used successfully to design accelerators with spatial variation of the magnetic field.[67]

nance at $n = 0.25$ has been found to be the most dangerous. Recent calculations by Dmitrievskiĭ et al [75], also indicate that the most dangerous resonance in the coupling between the radial and vertical oscillations occurs at $n = 0.25$.

It is shown in [73] that even in the case of a sufficiently homogeneous synchrocyclotron magnetic field the following two factors exert an appreciable influence on the extraction of the beam by the Hamilton and Lipkin method:

1) Type of ion source. Measurements have shown that when an ion source with feelers (projections) on the dee is used, the beam current at the radius where $n = 1$ is more than double that if an ion source without feelers is used. Use of an ion source with feelers makes is possible to obtain at the radius where $n = 1$ some 80% of the ion current existing up to the radius where $n = 0.2$.

2) Bias voltage on the dee. As is well known [76], a constant bias voltage is applied to the dee system to prevent high frequency discharge in the synchrocylotron chamber. This voltage causes the centers of the orbits to shift along the edge of the dee. Measurements have shown that with increasing bias voltage the current decreases on the radius where $n = 1$. A partial increase in the bias voltage can be compensated for by changing the position of the ion source.*

When the beam reaches the radius where $n = 1$, an electric deflector installed inside the dee permits extraction of 80% of the ions that have reached this radius, and have an energy 57 MeV. The beam leaving the deflector has considerable horizontal dimensions. To obtain horizontal focusing, iron masses were used to produce a region where the magnetic field intensity increased with increasing radius.

Although the iron masses were arranged in this case in the same way as in the focusing devices of [63,65], the authors of [73] called their focusing unit a magnetic channel. It must be noted that such a definition does not agree with the conventional one which we used in Secs. 1–5.

The horizontal dimensions of the beam leaving the focusing unit are greatly reduced. The extraction efficiency reaches in this case 50% at a current of 1 μA inside the region where $n = 0.2$. This is the greatest beam-extraction efficiency attained so far with a synchrocyclotron. However, in view of the danger of resonance at $n = 0.25$ in large synchrocyclotrons [75], the use of such an effective method of beam extraction is limited only to synchrocyclotrons with pole-piece diameter not exceeding 2 meters.

7. ADDITIONAL REMARKS

a) New methods of beam extraction from a synchrocyclotron. Stubbins [80] proposes to use a radio-frequency electric field to extract a beam from a synchrocyclotron. The betatron oscillations are then described by a Hill equation. The frequency of the electric field must be chosen such as to make the radial oscillations unstable and the vertical ones stable. No detailed derivations are given in [80], and only the most important results are indicated: 1) sufficiently large increments of the radial deflection per revolution can be attained and 2) the maxima of the radial deflection in successive revolutions are observed at approximately the same azimuth. Stubbins proposes that this method is simpler than the regenerative-deflector method.

Veksler, Kolomenskiĭ, and Burshteĭn proposed a stochastic acceleration mode [81]. At the present time a stochastic attachment is being developed for the CERN synchrocyclotron [82,83]. In this connection, it is pointed out in [82] that a stochastic system of beam extraction, permitting an appreciable increase in the duration of the current pulse of the extracted beam, is feasible.

b) Use of regenerative deflector in other accelerators (other than the synchrocyclotron). The success of the regenerative deflector in the extraction of a beam from a synchrocyclotron has suggested the possible use of this method for other accelerators.

It is proposed in [33,34] to use a regenerative deflector to extract a beam from a weak-focusing betatron and synchrotron, while in [33,34,84–87] it is proposed to use a regenerative deflector in accelerators in which the particles move in magnetic periodic systems. The feasibility of using a regenerative deflector with an electronic model of a PPSF* accelerator with radial sectors [89] has been demonstrated experimentally.

A so-called distributed regenerative action is proposed in [90] for the weak-focusing synchrotron. Distributed regenerative action differs from the regenerative action considered so far, where local regions of variable magnetic field are introduced, in that the field forming the deflector is distributed over the entire equilibrium orbit, starting with a certain definite value of the radial deflection.

We note that the reaction of the regenerative deflector is proposed in [33,34,84–87,91] as a method for injecting particles into accelerators.

The procedure of the theory of the regenerative deflector has been described in greater detail in the paper by Cohen and Crewe [33,34], and we present only their main results.

Cohen and Crewe proposed to apply the action of the regenerative deflector every other revolution, i.e., if the particle enters the regenerator during the m-th revolution, then it bypasses the regenerator during the next $(m + 1)$st revolution and enters again into the regenerator during the $(m + 2)$nd revolution. Let

*We note that an error has crept into the translation of [73], namely that in Fig. 3 the designations of the curves have been interchanged.

*The abbreviation PPSF adopted in [88] stands for an accelerator with constant magnetic field and strong focusing.

us consider here in greater detail the application of this method to a weak-focusing accelerator without straightline sections. We denote the decrement of the magnetic field intensity by n in the unperturbed region and by n_r in a regenerator of angular width θ_r. Let us trace the motion of the particle during two revolutions. We start, for example, with the entrance of the particle into the regenerator. The matrix $U(\sqrt{1-n_r}, \theta_r)$ transforms the vector ρ at the entry of the particle in the regenerator into the vector ρ corresponding to the exit of the particle from the regenerator:

$$\varrho_{\text{out reg}} = U(\sqrt{1-n_r}, \theta_r)\,\varrho_{\text{in reg}}. \qquad (58)$$

The vector $\rho_{\text{out reg}}$ specifies the initial conditions when the particle moves in a region with angular width $2\pi - \theta_r$. The vector ρ at the particle exit from this region has the form

$$\varrho_{2\pi} = U(\sqrt{1-n}, 2\pi-\theta_r)\,U(\sqrt{1-n_r}, \theta_r)\,\varrho_{\text{in reg}}. \qquad (59)$$

According to the condition for the action of the deflector, the vector ρ corresponding to the reentrance of the particle into the regenerator has after one revolution the form

$$\varrho_{4\pi} = U(\sqrt{1-n}, 2\pi)\,\varrho_{2\pi} = U(\sqrt{1-n}, 2\pi)\,U(\sqrt{1-n}, 2\pi-\theta_r) \times U(\sqrt{1-n_r}, \theta_r)\,\varrho_{\text{in reg}},$$

or with allowance for (14)

$$\varrho_{4\pi} = U(\sqrt{1-n}, 4\pi-\theta_r)\,U(\sqrt{1-n_r}, \theta_r)\,\varrho_{\text{in reg}}. \qquad (60)$$

The product of the matrices in (60) can be represented in accordance with (14) in the form

$$U(\sqrt{1-n}, 4\pi-\theta_r)\,U(\sqrt{1-n_r}, \theta_r) = U(\sqrt{1-n}, 4\pi)\,U(\sqrt{1-n}, -\theta_r)\,U(\sqrt{1-n_r}, \theta_r). \qquad (61)$$

At small values of θ_r (narrow regenerator) we obtain

$$U(\sqrt{1-n}, -\theta_r)\,U(\sqrt{1-n_r}, \theta_r) = B(-T), \qquad (62)$$

where

$$T = (n-n_r)\,\theta_r, \qquad (63)$$

and the matrix B is determined by formula (34).

In the case of a narrow regenerator we thus obtain from (59) and (60)

$$\varrho_{2\pi} = U(\sqrt{1-n}, 2\pi)\,B(-T)\,\varrho_{\text{in reg}}, \qquad (59')$$

$$\varrho_{4\pi} = U(\sqrt{1-n}, 4\pi)\,B(-T)\,\varrho_{\text{in reg}}. \qquad (60')$$

In order for the particle to bypass the regenerator during the (m + 1)st revolution, the deflection of the vector $\rho_{2\pi}$ should have a sign opposite to the deflection of the vector $\rho_{\text{in reg}}$. According to [33,34], this takes place when

$$\tfrac{1}{2}\,\mathrm{Sp}\,[U(\sqrt{1-n}, 2\pi)\,B(-T)] < -1. \qquad (64)$$

In order for the particle to enter during the (m + 2)nd revolution in the regenerator, we should have

$$\tfrac{1}{2}\,\mathrm{Sp}\,[U(\sqrt{1-n}, 4\pi)\,B(-T)] > 1. \qquad (65)$$

After multiplying the matrices we can rewrite conditions (64) and (65) in the form

$$T_{r1}^{-}(T - T_{r1}^{-}) > 0, \qquad (66)$$

$$T_{r2}^{+}(T - T_{r2}^{+}) > 0, \qquad (67)$$

where

$$T_{r1}^{-} = 2\sqrt{1-n}\,\mathrm{ctg}\,\pi\sqrt{1-n}, \qquad (68)^*$$

$$T_{r2}^{+} = -2\sqrt{1-n}\,\mathrm{tg}\,2\pi\sqrt{1-n}. \qquad (69)$$

We can reason analogously in the case of vertical oscillations. In order for the vertical oscillations to remain stable, the condition to be satisfied is

$$\tfrac{1}{2}\left|\mathrm{Sp}\,[U(\sqrt{n}, 4\pi)\,B(T)]\right| < 1, \qquad (70)$$

or

$$T_{z2}^{\pm}(T - T_{z2}^{\pm}) < 0, \qquad (71)$$

where,

$$\left.\begin{aligned} T_{z2}^{-} &= -2\sqrt{n}\,\mathrm{ctg}\,2\pi\sqrt{n}, \\ T_{z2}^{+} &= 2\sqrt{n}\,\mathrm{tg}\,2\pi\sqrt{n}. \end{aligned}\right\} \qquad (72)$$

On the basis of formulas (56), (69), (71) and (72), it is possible to plot curves showing the regions of the parameter T for which the deflector is effective. Such an analysis will show that such regions of T exist for accelerators for which n is close to 0.75, i.e., for synchrotrons and betatrons. LeCouteur proposed in [28] the use of a regenerative deflector for synchrotrons and betatrons, but with buildup of vertical oscillations and with the radial oscillations maintained stable, and also with the deflector operating during each revolution.

Cohen and Crewe [33,34] considered also the use of a regenerative deflector for the extraction of a beam from weak-focusing accelerators with straight-line sections. Since the analysis is perfectly similar to that considered above, we present only the final results, for example in the case when the deflector acts during each revolution. We denote by N the number of elements in the periodicity of the magnetic system per revolution. The periodicity element consists in this case of a straight-line section of length L and a magnetic sector with radius of equilibrium-orbit curvature R and a decrement of magnetic field intensity n. Let the regenerator be made up by introducing a perturbing intensity gradient $\partial H/\partial r$ (which begins with the equilibrium orbit) and a straight-line section of length $l \ll L$. For successful operation of the regenerative deflector the following condition should be satisfied

$$\left.\begin{aligned} &\tfrac{1}{2}\,\mathrm{Sp}\,[A_r^N B(-T)] > 1, \\ &\tfrac{1}{2}\left|\mathrm{Sp}\,[A_z^N B(T)]\right| < 1, \end{aligned}\right\} \qquad (73)$$

where $A_{r,z}$ transforms the vector $\rho(z)$ from the

*ctg = cot; tg = tan.

start to the end of the unperturbed periodicity element

$$\left.\begin{aligned} &A_{r,z}=U\left(\Omega_{r,z},\ \frac{2\pi}{N}\right)U\left(0,\ \frac{L}{R}\right),\\ &\Omega_r=\sqrt{1-n},\ \Omega_z=\sqrt{n},\\ &T=-\frac{l}{HR}\frac{\partial H}{\partial r}. \end{aligned}\right\} \tag{74}$$

Conditions (73) can be rewritten in the form

$$\left.\begin{aligned} &T_{r1}^{+}(T-T_{r1}^{+})>0,\\ &T_{z1}^{\pm}(T-T_{z1}^{\pm})<0, \end{aligned}\right\} \tag{75}$$

where

$$\left.\begin{aligned} &T_{r1}^{+}=-\frac{2+2a_{N-1}^{r}-a_{N}^{r}\operatorname{Sp}A_{r}}{a_{N}^{r}A_{r12}},\\ &T_{z1}^{\pm}=\frac{\pm 2+2a_{N-1}^{z}-a_{N}^{z}\operatorname{Sp}A_{z}}{a_{N}^{z}A_{z12}}. \end{aligned}\right\} \tag{76}$$

The quantities $a_N^{r,z}$ must be calculated in accordance with the following recurrence formula

$$a_N^{r,z}=a_{N-1}^{r,z}\operatorname{Sp}A_{r,z}-a_{N-2}^{r,z}, \tag{77}$$

where

$$a_0^{r,z}=0,\quad a_1^{r,z}=1.$$

We were able, however, in [92-95], to greatly simplify and generalize the results of Cohen and Crewe. For brevity we introduce the following concept of a cycle: during the action of the deflector in each revolution the concept cycle coincides with the concept revolution, but if the deflector operates every other revolution, the cycle consists of two revolutions, and the particle enters into the regenerator during one of the revolutions and bypasses it during the other revolution, in a definite sequence.

We define a narrow regenerator as one for which the matrix $C_{r,z}$, which transforms the vector $\rho(z)$ from the start of one cycle to the start of the next cycle, has the form

$$C_{r,z}=A_{r,z}^{M}B(\pm T). \tag{78}$$

Here the matrix $A_{r,z}$ transforms the vector $\rho(z)$ from the start of a periodicity element to the start of the next periodicity element in the unperturbed part of the magnetic system of the accelerator. $M = N$ and $2N$ for deflector operation every revolution and every other revolution, respectively. T is a parameter that depends on the specific construction of the generator. The minus sign pertains to the index r and the plus to the index z.

For the mode wherein the deflector operates during each revolution, the following conditions should be satisfied

$$\left.\begin{aligned} &T/T_{r1}^{+}>1,\\ &T/T_{z1}^{\pm}<1, \end{aligned}\right\} \tag{79}$$

and for the deflector operation every other revolution the conditions to be satisfied are

$$\left.\begin{aligned} &T/T_{r2}^{+}>1,\\ &T/T_{z2}^{\pm}<1,\\ &\cos N\mu_r<0, \end{aligned}\right\} \tag{80}$$

where

$$\cos\mu_{r,z}=\frac{1}{2}\operatorname{Sp}A_{r,z}. \tag{81}$$

For the quantities $T_{r\frac{M}{N}}^{+}$ and $T_{z\frac{M}{N}}^{\pm}$ the following simple formulas are applicable:

$$\left.\begin{aligned} &T^{+}_{r\frac{M}{N}}=-\frac{2\sin\mu_r}{A_{r12}}\operatorname{tg}\frac{M\mu_r}{2},\\ &T^{+}_{z\frac{M}{N}}=\frac{2\sin\mu_z}{A_{z12}}\operatorname{tg}\frac{M\mu_z}{2},\\ &T^{-}_{z\frac{M}{N}}=-\frac{2\sin\mu_z}{A_{z12}}\operatorname{ctg}\frac{M\mu_z}{2}. \end{aligned}\right\} \tag{82*}$$

The value of formulas (82) lies in the fact that they show directly how $T_{r\frac{M}{N}}^{+}$ at $T_{z\frac{M}{N}}^{\pm}$ depend on the quantities that characterize the unperturbed motion.

To investigate the variation of the amplitude of the vertical oscillations when a beam is extracted with the aid of a regenerative deflector, we have employed the method of envelopes. A measure of the variation of the amplitude of the vertical oscillations was chosen to be the ratio v of the envelopes of the perturbed and unperturbed motions. By averaging over the periodicity elements the square of the maximum of v relative to the initial phase of the vertical oscillations we obtain, for the case of a narrow regenerator, the following simple estimate of the variation of the amplitude of the vertical oscillations:

$$\sqrt{\bar{v}^2_{\max}}=\sqrt{\frac{1-\cos\lambda\cos M\mu_z}{1-\cos\lambda\cos M\mu_z-|\cos\lambda-\cos M\mu_z|}}, \tag{83}$$

where

$$\cos\lambda=\cos M\mu_z+\frac{TA_{z12}}{2}\frac{\sin M\mu_z}{\sin\mu_z}. \tag{84}$$

On the basis of our formulas it is necessary to modify the results of Cohen and Crewe as follows:

1) In the case of a weak-focusing accelerator without straight-line sections, the condition (67) should be replaced, in accordance with the third formula (80), by

$$\cos 2\pi\sqrt{1-n}<0. \tag{85}$$

2) In the case of a weak-focusing accelerator with straight-line sections we obtain from (80) in place of of the recurrence formulas (76) and (77) the following simple formulas:

$$\left.\begin{aligned} &T^{+}_{r\frac{M}{N}}=-\frac{2\sqrt{1-n}\sin\mu_r\operatorname{tg}\frac{M\mu_r}{2}}{\left(1+\frac{L}{R}\sqrt{1-n}\operatorname{ctg}\frac{2\pi}{N}\sqrt{1-n}\right)R\sin\frac{2\pi}{N}\sqrt{1-n}},\\ &T^{+}_{z\frac{M}{N}}=\frac{2\sqrt{n}\sin\mu_z\operatorname{tg}\frac{M\mu_z}{2}}{\left(1+\frac{L}{R}\sqrt{n}\operatorname{ctg}\frac{2\pi}{N}\sqrt{n}\right)R\sin\frac{2\pi}{N}\sqrt{n}},\\ &T^{-}_{z\frac{M}{N}}=-\frac{2\sqrt{n}\sin\mu_z\operatorname{ctg}\frac{M\mu_z}{2}}{\left(1+\frac{L}{R}\sqrt{n}\operatorname{ctg}\frac{2\pi}{N}\sqrt{n}\right)R\sin\frac{2\pi}{N}\sqrt{n}}. \end{aligned}\right\} \tag{86}$$

3) Cohen and Crewe attempted to generalize formally the estimate of LeCouteur (46) for the varia-

*tg = tan.

tion of the amplitude of the vertical oscillation, to include the case of accelerators in which the particles move in magnetic periodic systems. However, in view of LeCouteur's incorrect procedure (see Sec. 4), we cannot regard the estimate of Cohen and Crewe as reliable. It is therefore necessary to employ our estimate (83).

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