EFFECTS PRODUCED BY AN ARTIFICIAL SATELLITE RAPIDLY MOVING IN THE IONOSPHERE OR IN AN INTERPLANETARY MEDIUM

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I. INTRODUCTION

1. Formulation of Problem, Initial Data

The use of artificial satellites and space rockets to investigate the structure and properties of the outer ionosphere and the interplanetary medium has intensified the interest in effects arising in the vicinity of bodies moving in a plasma. It is very important to take these effects into account in the planning and interpretation of various experiments made with satellite-borne instruments. The questions connected with the interaction between moving bodies and a plasma are in themselves unique and, from various points of view, in themselves of theoretical interest.

The main feature of the motion of bodies in the upper atmosphere and in interplanetary and cosmic space is that it takes place in a highly rarefied medium, where the particle mean free paths Λ are large compared with the characteristic dimensions of the body R_0. This can be seen from Table I, which lists the main physical parameters of the ionosphere and of the interplanetary gas. The usual methods of hydrodynamics or aerodynamics cannot be employed to describe the phenomena that occur in the vicinity of a body in such a rarefied medium. It becomes necessary here to use kinetic theory, which takes account of the fact that the plasma is not a continuous medium but an aggregate of individual molecules, atoms, electrons, and ions.

Neutral particles, that is, molecules or atoms, interact only with the surface of the body. Reflection of the incoming stream creates an excess of particles in front of the body, where a "condensation" region is produced (see Fig. 2). Behind the body, to the contrary, a "rarefaction" region is produced, since the moving body "sweeps" the particles, as it were, and they cannot fill this region behind the body immediately. The length of the rarefaction region obviously increases with the velocity v_0 of the body like v_0/v_n, where $v_n = (2kT/M)^{1/2}$ is the mean thermal velocity of the molecules.

The charged particles—electrons and ions—interact not with the surface of the body alone, for their motion is greatly influenced by electric and magnetic fields. The electric field is produced both by the charge of the body itself and by the space charge created in the plasma due to the difference between the electron and ion concentrations.

Inasmuch as under ionospheric conditions the velocity of the body v_0 is usually much larger than the thermal velocity of the ions and much smaller than the thermal velocity of the electrons (see Tables I and II), the character of their response to the electrical field is essentially different. The electron distribution is completely determined by the electric field under these conditions. On the other hand, the influence of the electric field on the ions is not decisive, since the energy of the particles incident on the body greatly exceeds their thermal energy, $Mv_0^2 \gg kT$, and exceeds consequently the potential energy of the ion in the electric field, since $e\varphi \sim kT$. More important is the influence of an external (the earth's) magnetic field, which holds back the ions and prevents the filling of the rarefied region. The character of the filling and the dimensions of the rarefaction region depend essentially on the angle between the velocity of the body $\mathbf{v}_0$ and the magnetic field $\mathbf{H}_0$.

The dimensions of the rarefaction region greatly exceed the dimensions of the body itself. In other words, the "trail" stretches a long distance behind the body. This can lead to a considerable scattering of electromagnetic waves by the electron-density disturbances in such a trail. It is important, of course, to know the character of the interaction of the plasma particles, and also the corpuscular radiation and light, with the surface of the body. These questions have been investigated very little up to now, and an all-out primarily experimental investigation is still needed. The interaction of the particles and of the radiation with the surface of the body greatly depends on the surface material. In the theoretical calculations it is assumed, for the sake of being definite, that the particles interact with the surface in very simple manner—they are either absorbed or reflected. Such important processes as damage to the surface by collision with the particles (see [26,27,38]) or by the corpuscular radiation (knocking out of the electrons from the surface by the light—photoeffect), etc, are neglected. The role of all these processes in the formation of the distrubed zone is not clear at the present time.

The particles reflected from the body can in principle heat and even ionize the gas in front of the body. It is easy to verify, however, that the heating of the gas can be neglected if the molecule mean free path is sufficiently large. In fact, the summary energy of the particles reflected within a time Δt from the surface of the body is obviously equal (in order of magnitude) to $\mathcal{E}_{\mathrm{ref}} \sim Mv_0^2 \cdot n_0 R^2 \cdot v_0 \Delta t$, where n_0 is the particle density, M the particle mass, and R the dimension of the body. These particles scatter without collision over a distance on the order of the mean free path Λ. Consequently, they are slowed down within a cylinder of volume $V \sim \Lambda^2 v_0 \Delta t$. The total number of particles in this volume is $n_0 V \sim n_0 \Lambda^2 v_0 \Delta t$. The mean particle energy in the gas ahead of the body is therefore increased by the collisions with the reflected particles by an amount $\Delta \mathcal{E} \sim Mv_n^2 R^2/\Lambda^2$. At sufficiently large mean free paths $[\Lambda \gg (v_0/v_n) R]$ this change in energy is small, $\Delta \mathcal{E} \ll kT$. For bodies whose dimensions are on the order of 1 meter, the condition $\Lambda \gg (v_0/v_n) R$ is well satisfied in the ionosphere at altitudes exceeding 200 km.

The ionization of the gas ahead of the body, due to collisions with the reflected particles, is also insignificant, inasmuch as the velocity of the reflected molecules and ions is only of the order of the velocity of the body, that is, $\sim 10^6$ cm/sec. It is much smaller than the velocity of the electrons in the atoms, so that the probability of ionization by collision with a reflected particle is negligibly small.* Analogously, the additional ionization ΔN_f due to fast electrons emitted from the surface of the body under the influence of the incident ultraviolet radiation from the sun is also small. Actually, $\Delta N_f \simeq \sigma_e n (S/h\nu) \alpha_\nu \Delta t$, where $\sigma_e \sim 10^{-17}\,\mathrm{cm}^2$ is the effective cross section for the ionization by slow electrons, $S/h\nu \sim 10^{11}-10^{12}$ is the flux of quanta in the incident radiation from the sun, $\alpha_\nu \sim 10^{-1}$ is the coefficient of emission of electrons from the surface of the body, and $\Delta t = R_0/v_0 \sim 10^{-4}$ sec. We therefore have everywhere in the ionosphere $\Delta N_f \lesssim 10^{-1}-1$ electron/cm^3.

A very important problem is that of excitation of waves by the body. Inasmuch as the body moves in the medium with supersonic velocity, it can in principle excite both sound waves and ionic plasma waves, that is, it can give rise to Cerenkov radiation. In an isothermal plasma, however, all these modes are very strongly attenuated if the wavelength is of the order of or shorter than the mean free path. They cannot therefore exert a noticeable influence on the processes occurring near the body. Sound waves with a wavelength much larger than the molecule mean free path can be radiated by the body, as recently calculated by Dokuchaev [36]. The condition $\lambda \gg \Lambda$, which limits the length of the sound wave, is satisfied in the ionosphere only for waves with frequency of several cycles and below. Electronic plasma waves are generally speaking not generated by the body, since its velocity is much lower than the thermal velocity of the electrons, and consequently the velocities of the corresponding waves are also much lower. To be sure, the scattering of the electric field of the body by the inhomogeneities of the medium could play an important role here and could lead to a certain weak excitation of plasma or even electromagnetic waves [22]. It must be borne in mind that the presence of the earth's magnetic field makes the question of the spectrum of the excited waves and their attenuation much more complicated; in particular, it may turn out that the motion of the body along the direction of the magnetic field exhibits special features. Therefore, until a consistent theory is developed, it is hardly possible to draw any qualitative conclusions regarding this matter.

The investigation of the stability of the perturbed zone near a body moving in a plasma, as in ordinary hydrodynamics, reduces to an investigation of the character of development of small deviations from equilib-

*The larger values obtained in [35] for the additional ionization are due to the erroneous assumption that the cross section for the ionization is equal to the gas-kinetic value $\sigma \sim 10^{-16}\,\mathrm{cm}^2$. In point, apparently, σ does not exceed $10^{-40}\,\mathrm{cm}^2$ (there are no exact data).

Table I. Plasma Parameters

Height z, km	n, cm^{-3}	N, cm^{-3}	T, °K	Λ_n, cm	$\Lambda_{e,i}$, cm	ν_n, sec^{-1}	$\nu_{e,n+i}$, sec^{-1}	$\nu_{ii'}$, sec^{-1}	$v_{n,i}$, cm/sec	v_e, cm/sec
Ionosphere										
200	$(2-5)\cdot10^{10}$	$(3-50)\cdot10^4$	450–800	$8\cdot10^3$	$9\cdot10^3$	$9\cdot10^0$	$1.6\cdot10^3$	8	$7\cdot10^4$	$1.3\cdot10^7$
300	$3\cdot10^9$	$(10-20)\cdot10^5$	1000	10^5	$7\cdot10^3$	$9\cdot10^{-1}$	$3\cdot10^3$	15	$9\cdot10^4$	$1.7\cdot10^7$
400	$5\cdot10^8$	$(5-15)\cdot10^5$	1500	$7\cdot10^5$	$1.6\cdot10^4$	$1.6\cdot10^{-1}$	$1.4\cdot10^3$	7	10^5	$2.0\cdot10^7$
700	$6\cdot10^6$	$(2-5)\cdot10^5$	2000	$5\cdot10^7$	$1.3\cdot10^5$	$3\cdot10^{-3}$	$2\cdot10^2$	1	$1.6\cdot10^5$	$2.6\cdot10^7$
1000	10^5	10^5	3000	$8\cdot10^8$	$8\cdot10^5$	$2\cdot10^{-4}$	40	0.2	$2\cdot10^5$	$3.0\cdot10^7$
3000	1 (?)	$7\cdot10^3$	4000	$2\cdot10^{14}$	$3\cdot10^6$	10^{-9}	14	0.1	$2.3\cdot10^5$	$3.6\cdot10^7$
Interplanetary gas										
$(3-4)\cdot R_0$	0 (?)	$3\cdot10^2$	$5\cdot10^3$ (?)	—	$3\cdot10^9$	—	10^{-2}	$5\cdot10^{-4}$	$2\cdot10^6$	$4\cdot10^7$
$100\cdot R_0$	0 (?)	10^2								
Cosmic space										
—	0 (?)	1	10^4 (?)	—	$7\cdot10^{11}$	—	$8\cdot10^{-5}$	$3\cdot10^{-6}$	$3\cdot10^6$	$5\cdot10^7$

Notation: n, cm^{-3} – concentration of neutral particles; N, cm^{-3} – concentration of electrons or ions; T – temperature; Λ_n – mean free path of neutral particles; $\Lambda_{e,i}$ – mean free path of electrons or ions; R_0 – radius of the earth; ν_n – number of collisions between neutral particles; $\nu_{e,n+i}$ – number of collisions between electrons and ions or neutral particles; $\nu_{ii'}$ – number of collisions between ions; $v_{n,i}$ – mean thermal velocity of neutral particles or ions; v_e – mean thermal velocity of electrons.

Table II. Plasma Parameters

Height z, km	v_0, cm/sec	H_0, Oe	ω_H, sec^{-1}	Ω_H, sec^{-1}	ϱ_H, cm	ω_0, sec^{-1}	D, cm	M_0	N/n	ω_H/ω_0	v_0/v_n
Ionosphere											
200	10^6	0.45	$7.9\cdot10^6$	$1.7\cdot10^2$	$4.1\cdot10^2$	$(1-4)\cdot10^7$	0.2–1	24	$10^{-5}-10^{-6}$	0.4	13
300		0.44	$7.7\cdot10^6$	$1.6\cdot10^2$	$5.6\cdot10^2$	$(1.8-8)\cdot10^7$	0.14–0.7		$(0.3-2)\cdot10^{-3}$	0.15	11
400		0.40	$7\cdot10^6$	$1.8\cdot10^2$	$5.5\cdot10^2$	$(4-7)\cdot10^7$	0.2–0.4	20	$(1-3)\cdot10^{-3}$	0.1	10
700		0.35	$6\cdot10^6$	$2\cdot10^2$	$8\cdot10^2$	$(2.5-4)\cdot10^7$	0.7–0.4	16 (?)	$(0.3-1)\cdot10^{-1}$	0.2	6
1000		0.33	$5.8\cdot10^6$		10^3	$1.8\cdot10^7$	1.2	14–16	~1	0.3	5
3000		0.16	$3\cdot10^6$	90	$2.5\cdot10^3$	$1.5\cdot10^6$	4	7	~10^4	1.8	4
Interplanetary gas											
$(3-4)\cdot R_3$	$6\cdot10^5$	$(0.5-1.5)\cdot10^{-2}$	$(0.9-3)\cdot10^5$	55	$3.6\cdot10^4$	10^6	30	1	∞ (?)	0.2	0.3
$100\cdot R_3$	$2\cdot10^5$	$(0.5-1)\cdot10^{-3}$	$2\cdot10^4$	5.5	$3.6\cdot10^5$	$5.6\cdot10^5$	50			0.03	0.1
Cosmic space											
—	10^5	10^{-5}	$2\cdot10^2$	$5.5\cdot10^{-2}$	$5.5\cdot10^7$	$5.6\cdot10^4$	700	1	∞ (?)	0.003	0.03

Notation: v_0 – velocity of satellite or rocket moving away from the earth, H_0 – earth's magnetic field; ω_H – Larmor frequency of the electrons (eH/mc); ω_0 – frequency of plasma oscillations $\sqrt{4\pi Ne^2/m}$; D – Debye radius $\sqrt{kT/4\pi Ne^2}$; M_0 – average molecular weight of neutral particles or ions; Ω_H – Larmor frequency of ions (eH/M_ic); ρ_H – Larmor radius of ions (v_i/Ω_H).

rium: the region under consideration is unstable if these deviations increase with time, and stable if arbitrary disturbances damp out. This question also calls for a special analysis. However, there are grounds for assuming that the perturbed region is stable.

It is important to emphasize that with changing altitude in the ionosphere, and particularly on going over to interplanetary or cosmic space, the characteristic parameters of the plasma in which the body moves change appreciably, as is seen from Tables I and II. The character of the phenomena occurring in the vicinity of the moving body changes accordingly; for example, in the part of the ionosphere near the earth, up to a height of several thousand kilometers, the satellite velocity is on the order of 10^6 cm/sec, while the ion and molecule velocities are on the order of 10^5 cm/sec. We deal here, consequently, with fast supersonic motion of the body. However, with increasing distance from the surface of the earth, owing to the increase in temperature and decrease in the mean mass of the particles, their velocity increases to an order of 10^6 cm/sec; the velocity of the body, to the contrary, decreases, so that v_0 and v_n become equal and even the inverse condition $v_0 \ll v_n$ may set in (see Tables I and II). The character of this phenomenon, naturally, depends greatly on the extent to which the earth "drags" its surrounding gas shell at large distances from the earth. Further, in the ionosphere the dimensions of the bodies are always large compared with the Debye radius. Under such conditions the factor most important to the distribution of the electric field and the charged particles in the vicinity of the body is the Debye screening. In interplanetary and interstaller gas the Debye radius is already comparable with the dimensions of the body (see Table II). Therefore the conditions in the vicinity of the body, and consequently also the structure of the disturbed zone in the lower layers of the ionosphere and in its remote regions, should differ greatly.

Thus, the problems that arise in the examination of effects in the vicinity of satellites and rockets in the ionosphere and interplanetary gas are greatly varied in their general formulation and encompass a large branch of plasma physics. We plan to cast light here on a much narrower circle of problems, pertaining essentially to the motion of bodies in regions of the ionosphere not too far from the earth. The body velocities are much larger under these conditions than the mean thermal velocity of the ions and the molecules, and the dimension is much larger than the Debye radius. In Chapter II we consider the perturbations produced by such a body in the medium, while in Chapter III we calculate the scattering of the radiowaves by the "trail" of the medium. Finally, in Chapter IV we discuss the particle fluxes in the vicinity of the body, a problem of interest in itself and also important to the analysis of the results of soundings of the ionosphere. This problem, as well as the problem of interaction between a plasma and a slowly moving body, will be considered in a separate article.

2. Brief Summary of the Literature

In the present article we report essentially the results obtained in [1-7]. It is therefore of interest to dwell here briefly on the contents of other published papers devoted to these or allied problems.

In one of the earliest papers devoted to a theoretical consideration of the effects due to motion of a body in a plasma (Jastrow and Pearse [9]), an attempt was made to obtain only a qualitative description of the expected phenomena. The distribution of the potential around a rapidly moving body was assumed to be spherically symmetrical with the electrons having a Boltzmann distribution around the body and the ion density equal to the undisturbed value. We show later on, however, that in fact the potential distribution around the body is far from spherical, and the ion concentration is highly disturbed.

A similar problem was solved more rigorously by Kraus and Watson [10]. They used the kinetic equation to calculate in first approximation of disturbance theory the ion density distribution and the electric potential around a small point-like charge moving in the ionosphere, that is, a weakly charged body with dimensions much shorter than the Debye radius. In the ionosphere, on the other hand, as was already noted above, the opposite case is of importance, where the Debye radius is much smaller than the dimensions of the body. In addition, as will be shown below, the second-order terms (with respect to the charge of the body), omitted by the authors, turn out to be more important at large distances than the first-order terms. Therefore the results of [10] are generally incorrect for large distances from the body. In particular, at large distances the disturbances of the density decrease as $1/r^2$, and not as $1/r^3$ as would follow from [10].

Several papers essentially analogous to [10] were published by Rand [11]. His calculations were carried out for a two-dimensional case, that is, for a thin weakly charged wire. The results of [11] can be significant in an analysis, say, of phenomena near moving satellite antennas. Interesting from the methodological point of view is the author's attempt to generalize his results to a large body by considering the disturbance produced by the sharp edge of the body. However, the character of the assumptions made in these calculations remains unclear. Chopra and Singer [12] calculated the deceleration force of the body, under the assumption that the main contribution is made by that region of space where the satellite has a pure Coulomb field. This may be true at the highest altitudes. In the ionosphere, the main contribution is made by the field of the "trail" of the body, where the distribution of the potential has nothing in common with a Coulomb distribution.* Greifinger [16] considered the motion of a point-like charge in a plasma in a magnetic field by the same method as Kraus and Watson and neglecting, furthermore, the thermal motion of the ions.

Davis and Harris [34] integrated numerically the equations of motion of the ions near the body simultaneously with the equation for the electric potential. They neglected the thermal motion of the ions and the magnetic field. Such an approximation is incorrect, at any rate for large distances from the body. In addition, at small distances behind an absorbing body the electron distribution may be far from equilibrium, unlike the assumptions in [34]. Nonetheless, this paper is of considerable interest, for only by using numerical methods is it possible to take a consistent account of the influence of the electric field on the motion of the ions near the body.

An interesting mechanism for the disturbance of the plasma around the body was considered recently by Getmantsev and Denisov [37]. The point is that near the antennas of a rocket or a satellite there is produced a sufficiently strong high-frequency electromagnetic field which influences greatly the concentration of the electrons and ions. The authors have calculated the plasma disturbances near an antenna mounted on a resting or slowly-moving body.

A considerable number of papers is devoted to an investigation of magnetic phenomena caused by a body moving in a conducting medium. However, all the papers published on this topic are based on a macroscopic magnetohydrodynamic analysis. The applicability of their results to the motion of rockets and satellites in the upper ionosphere and in interplanetary medium is therefore doubtful.

II. STRUCTURE OF DISTURBED REGION IN THE VICINITY OF A BODY MOVING RAPIDLY IN A PLASMA

3. Initial Equations

All the problems solved here call for a kinetic analysis, since the mean free path Λ of the particles is much larger than the linear dimension R_0 of the body. The distribution function of the neutral particles is determined by the kinetic equation

$$\frac{\partial f}{\partial t}+\mathbf{v}\frac{\partial f}{\partial \mathbf{r}}-\frac{1}{M}\frac{\partial U}{\partial \mathbf{r}}\frac{\partial f}{\partial \mathbf{v}}=0. \tag{2.1}$$

Here $f = f(\mathbf{r}, \mathbf{v}, t)$ is the distribution function of the neutral particles (molecules, atoms), M their mass, and $U = U(\mathbf{r}, t)$ the potential energy of interaction between the particles and the surface of the body. If the body moves uniformly with velocity $\mathbf{v}_0$, then $U = U(\mathbf{r} - \mathbf{v}_0 t)$. In this case it is convenient to consider the problem in a coordinate system fixed in the moving body. The particle distribution is then stationary and described by the equation

$$\mathbf{v}\frac{\partial f}{\partial \mathbf{r}}-\frac{1}{M}\frac{\partial U}{\partial \mathbf{r}}\frac{\partial f}{\partial \mathbf{u}}=0, \tag{2.2}$$

where $\mathbf{u} = \mathbf{v} + \mathbf{v}_0$. It is necessary to take into account the fact that in the coordinate system fixed in the body, the latter experiences a flux of particles with velocity $-\mathbf{v}_0$. This means that at large distances from the body (as $r \to \infty$), where the motion of the particles is not disturbed, we have a Maxwellian distribution function

$$f_0(u)=n_0\left(\frac{M}{2\pi kT}\right)^{3/2}\exp\left\{-\frac{Mu^2}{2kT}\right\}. \tag{2.3}$$

Here n_0 is the undisturbed density of the neutral particles and T their temperature. We note that the interaction between the particles and the surface of the body can be described either by stipulating a special form of the potential $U(\mathbf{r})$, or by introducing definite boundary conditions for the distribution function on the surface of the body. For example, if all the particles colliding with the body are absorbed, then the boundary condition for the distribution function on the surface of the body S has the form

$$f(\mathbf{r}_0, \mathbf{u})_S=0,$$

if $\mathbf{n}\cdot\mathbf{v} > 0$, where $\mathbf{n}$ is the outward normal to the surface.

The electrons and ions interact not only with the body but also with the electric and magnetic fields in the plasma. In the coordinate system fixed in the body, the equations for the distribution functions of these particles have the form

$$\mathbf{v}\frac{\partial f_e}{\partial \mathbf{r}}-\left\{\left(1\frac{e}{m}\frac{\partial \varphi}{\partial \mathbf{r}}+\frac{1}{m}\frac{\partial U}{\partial \mathbf{r}}\right)-\frac{e}{mc}[\mathbf{H}, \mathbf{u}]\right\}\frac{\partial f_e}{\partial \mathbf{u}}=0, \tag{2.4*}$$

$$\mathbf{v}\frac{\partial f_i}{\partial \mathbf{r}}-\left\{\left(\frac{e}{M_i}\frac{\partial \varphi}{\partial \mathbf{r}}+\frac{1}{M_i}\frac{\partial U}{\partial \mathbf{r}}\right)+\frac{e}{M_i c}[\mathbf{H}, \mathbf{u}]\right\}\frac{\partial f_i}{\partial \mathbf{u}}=0. \tag{2.5}$$

Here $f_e(\mathbf{r}, \mathbf{u})$ and $f_i(\mathbf{r}, \mathbf{u})$ are the electron and ion distribution functions, e is the charge of the ion (we assume for simplicity that the ions are singly charged), m and M_0 are the masses of the electron and the ion, $\varphi = \varphi(\mathbf{r})$ is the potential of the electric field, and $\mathbf{H}$ is the magnetic field. At infinity the functions f_i and f_e have the form (2.3).

The magnetic field in (2.4) and (2.5) can be regarded as specified (in the ionosphere, $\mathbf{H}$ is the earth's magnetic field). To the contrary, the electric field is itself due to the difference in the concentration of the electrons and the ions in the disturbed zone. It is defined by the Poisson equation

$$\Delta\varphi=4\pi e\left(\int f_e(\mathbf{r}, \mathbf{u})\,d^3u-\int f_i(\mathbf{r}, \mathbf{u})\,d^3u\right). \tag{2.6}$$

At infinity $\varphi \to 0$ and the boundary condition for the potential $\varphi(\mathbf{r})$ on the surface of the body depends on the type of the surface (dielectric, metal) and on the

*We do not have paper [12], and we make use of its brief summary in the review of Chopra.[13]

*$[\mathbf{H}, \mathbf{u}] = \mathbf{H}\times\mathbf{u}$.

charge of the body itself. By finding the solution of (2.2) for neutral particles or of the system (2.4)–(2.6) for charged particles, we solve completely the problem of the disturbances produced by the body in the plasma.

4. Disturbances of the Neutral-particle Concentration

The distribution function f of the neutral particles is conveniently represented in the form

$$f(\mathbf{r},\ \mathbf{u})=f_1(\mathbf{r},\ \mathbf{u})+f_2(\mathbf{r},\ \mathbf{u}), \tag{2.7}$$

where $f_1(\mathbf{r}, \mathbf{u})$ is the distribution function of the particles that experience no collisions with the body; $f_2(\mathbf{r}, \mathbf{u})$ is the distribution function of the particles reflected from the body and depends on the form of the surface and on the character of the particle reflection. We shall consider usually a rapidly moving sphere of radius R_0. In this case the most important is the rarefaction region, which extends over a long distance from the body.

a) Rarefaction region behind a rapidly moving body of arbitrary shape. By virtue of the fact that the velocity of the particle stream incident on the body is much larger than the thermal velocity, collisions between the particles and the surface of the body during the filling of the rarefied region have low probability. Therefore away from the body these collisions are of little significance, that is, $f \simeq f_1$. This means also that the specific shape of the body is insignificant at considerable distances from the body in the rarefied zone: what is important is only the maximum cross section of the body in the plane perpendicular to the incoming stream. Consequently, in approximate calculations, we can replace the body by its cross section; for example, the spherical surface considered above can be replaced by a round disc of radius R_0 located at the point $z = 0$.

We note first that the thermal motion of the particles in the direction of the z axis, parallel to the velocity v_0, is of little importance because $v_0 \gg \sqrt{kT/M}$. We can therefore assume that all the particles move in the z direction with identical velocity v_0. Then the problem of determining the particle density in the rarefied zone becomes actually a dimensional one: it is merely necessary to determine how the particles fill in the course of time an empty region equal to the cross section of the body in the plane (x, y), perpendicular to the direction z of the motion of the body. Account must then be taken of the fact that during this time t all the particles move as a unit a distance $v_0 t$ in the z direction, that is, one must change over by means of the simple substitution $t = z/v_0$ to the coordinate system that moves together with the body.

The distribution function of the particles in the (x, y) plane has thus at the initial instant of time the form

$$f(x,\ y;\ u_x,\ u_y;\ 0)=\begin{cases} n_0\dfrac{M}{2\pi kT}\exp\left[-\dfrac{M(u_x^2+u_y^2)}{2kT}\right], \\ \text{if the point}(x,\ y)\text{ lies outside } S \\ 0,\ \text{if }(x,\ y)\text{ lies inside } S, \end{cases} \tag{2.8}$$

where S is the cross section of the body. At any other instant of time the distribution function describing the free motion of the particles has the form

$$f(x,\ y;\ u_x,\ u_y;\ t)=f[x_0(x,\ u_x,\ t),\ y_0(y,\ u_y,\ t);\ u_x,\ u_y;\ 0],$$

where (x_0, y_0) is the point where the particle has been located at the initial instant of time if it is situated at the instant t in the point (x, y) and has a velocity (v_x, v_y); $f(\ldots, 0)$ is the distribution function at the initial instant of time, given by (2.8). It is easy to verify directly that this expression satisfies (2.2) outside the body.

Naturally,

$$\left.\begin{aligned} &x_0=x-u_xt,\quad y_0=y-u_yt, \\ &f\left(x,\ y;\ u_x,\ u_y;\ \frac{z}{v_0}\right)=f\left(x-\frac{u_x}{v_0}z,\ y-\frac{u_y}{v_0}z;\ u_x,\ u_y;\ 0\right). \end{aligned}\right\} \tag{2.9}$$

In the last expression the time t has already been replaced by z/v_0. Integrating now the distribution function (2.9) with respect to the velocity, we can obtain the particle density in the disturbed zone

$$\begin{aligned} n(x,\ y,\ z)&=\int f\left(x,\ y;\ u_x,\ u_y;\ \frac{z}{v_0}\right)du_x du_y \\ &=\left(\frac{v_0}{z}\right)^2\int f\left(x_0, y_0;\ \frac{x-x_0}{z}v_0,\ \frac{y-y_0}{z}v_0;\ 0\right)dx_0\,dy_0. \end{aligned} \tag{2.10}$$

In the integral (2.10), the variables u_x and u_y are replaced by $x_0 = x - u_x z/v_0$ and $y_0 = y - u_y z/v_0$, that is, the integration is carried out over the initial coordinates of the particles. By virtue of the properties of the initial function, the integration in (2.10) is actually carried out only over regions outside the cross section of the body. The same integral taken over the cross section of the body is obviously equal to the disturbance of the particle density $\Delta n = n_0 - n$, inasmuch as the integral over the entire region is equal to n_0. Thus

$$\begin{aligned} \Delta n(x,\ y,\ z)&=\left(\frac{v_0}{z}\right)^2\int_S dx_0\,dy_0 f\left(x_0,\ y_0;\ \frac{x-x_0}{z}v_0,\ \frac{y-y_0}{z}v_0;\ 0\right) \\ &=n_0\frac{Mv_0^2}{2\pi kTz^2}\int dx_0\,dy_0\exp\left[-\frac{Mv_0^2}{2kT}\frac{(x-x_0)^2+(y-y_0)^2}{z^2}\right] \\ &=n_0\frac{Mv_0^2}{2\pi kTz^2}\exp\left[-\frac{Mv_0^2}{2kT}\frac{x^2+y^2}{z^2}\right]\int_S dx_0\,dy_0 \\ &\times\exp\left[-\frac{Mv_0^2}{2kT}\frac{x_0^2+y_0^2-2x_0x-2y_0y}{z^2}\right]. \end{aligned} \tag{2.11}$$

This expression for the perturbed molecule density has a form which is very simple and convenient to integrate. This integration will be carried out below for cases when the cross section of the body in a plane perpendicular to the direction of motion is a circle or a rectangle. In addition, a simple expression will be obtained for the disturbances of the particle concentration in the case of an arbitrary cross section, an expression correct at sufficiently large distances from the body.

Body of circular cross section. Assume that the cross section of the body is a circle of radius R_0. Of course, this is also the cross section of the spherical body considered above.

In calculating Δn we change over in the integral (2.11) to polar coordinates φ and $\rho_0 = \sqrt{x_0^2 + y_0^2}$. If, furthermore, we measure the angle φ from the (x, y) direction, so that $x_0 x + y_0 y = \rho_0 \rho \cos\varphi$, we obtain in place of (2.11)

$$\Delta n(x, y, z) = \frac{n_0 M v_0^2}{2\pi k T z^2} \exp\left[-\left(\frac{\varrho}{z}\right)^2 \frac{M v_0^2}{2kT}\right] \int_0^{R_0}\int_0^{2\pi} \varrho_0\, d\varrho_0\, d\varphi$$
$$\times \exp\left\{\left[-\left(\frac{\varrho_0}{z}\right)^2 + \frac{2\varrho\varrho_0}{z^2}\cos\varphi\right]\frac{M v_0^2}{2kT}\right\}$$
$$= 2n_0 \exp\left[-\left(\frac{\varrho}{z}\right)^2 \frac{M v_0^2}{2kT}\right] \int_0^{\frac{R_0}{z}\sqrt{\frac{M v_0^2}{2kT}}} t\,dt\,e^{-t^2} I_0$$
$$\times \left(\frac{\varrho}{z}\sqrt{\frac{2Mv_0^2}{kT}}\,t\right),$$
$$(\varrho = \sqrt{x^2 + y^2}, \quad z > 0). \tag{2.12}$$

On the axis, that is when $\rho = 0$, the integral in (2.12) can be evaluated, and we obtain simply

$$n = n_0 \exp\left[-\frac{M v_0^2}{2kT}\left(\frac{R_0}{z}\right)^2\right]. \tag{2.13}$$

At large distances from the body, when

$$\left(\frac{M v_0^2}{2kT}\right)\left(\frac{R_0}{z}\right)^2 \ll 1$$

we get

$$n - n_0 = -\frac{M v_0^2}{2kT}\left(\frac{R_0}{z}\right)^2,$$

that is, the disturbances decreases in proportion to $1/z^2$.

The function $n(\rho, z)/n_0$ behind the body, given by formula (2.12), is shown in Fig. 1 for $\sqrt{Mv_0^2/2kT} = 8$. It is seen from the figure that a large "rarefaction region" is produced behind a rapidly moving body. Thus, in the case considered here $n(0, z) = 0.5 n_0$ even for $z \simeq 10R_0$.

Body of rectangular cross section. Let us consider now a body of rectangular cross section with dimensions $2R_x$ and $2R_y$. This case is less realistic, but in the presence of a magnetic field calculations of the ion perturbation for a rectangle lead to simpler ex-

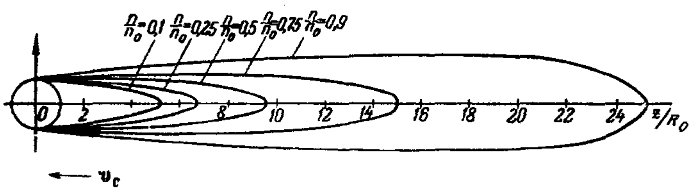

FIG. 1. Curves of constant ratio $n(\rho, z)/n_0$ behind a spherical body in the rarefaction region ($\sqrt{Mv_0^2/2kT} = 8$).

pressions. In this case the integration in expression (2.11) can be readily carried out, and we obtain

$$\Delta n(x, y, z)$$
$$= \frac{n_0 M v_0^2}{2\pi k T z^2} \int_{-R_x}^{R_x}\int_{-R_y}^{R_y} dx_0\, dy_0 \exp\left[-\frac{(x-x_0)^2 + (y-y_0)^2}{z^2}\frac{M v_0^2}{2kT}\right]$$
$$= \frac{n_0}{4}\left|\Phi\left(\frac{x - R_x}{z}\sqrt{\frac{Mv_0^2}{2kT}}\right) - \Phi\left(\frac{x + R_x}{z}\sqrt{\frac{Mv_0^2}{2kT}}\right)\right|$$
$$\times \left|\Phi\left(\frac{y - R_y}{z}\sqrt{\frac{Mv_0^2}{2kT}}\right) - \Phi\left(\frac{y + R_y}{z}\sqrt{\frac{Mv_0^2}{2kT}}\right)\right| \tag{2.14}$$

Here, as before, $z > 0$ and

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$$

is the probability integral.

Disturbances at large distances from a body of arbitrary cross section. Formula (2.11) enables us to obtain a simple expression for the perturbation of the particle concentration at large distances behind a body of arbitrary cross section. In fact, at large z the argument in the exponential term of the integral in (2.11) is always small and the term itself is consequently close to unity. We therefore have at large z

$$\Delta n(x, y, z) = \frac{n_0 S M v_0^2}{2\pi k T z^2} \exp\left[-\frac{M v_0^2}{2kT}\frac{x^2 + y^2}{z^2}\right]. \tag{2.15}$$

On the z axis, that is, at $x = y = 0$, the perturbation has the form

$$\Delta n = n_0 \frac{S M v_0^2}{2\pi k T z^2}. \tag{2.16}$$

The physical meaning of this expression is clear: the region S perturbed by the body melts away uniformly with velocity $\sim\sqrt{kT/M}$, that is, after a time $t = z/v_0$ it spreads out into a circle of radius $\sqrt{kT/M}(z/v_0)$. Thus, at large distances from the body the perturbation of the density decreases as $1/z^2$.

b) Region of condensation ahead of a rapidly moving body. We consider now the region of "condensation," that is, we determine the excess concentration n_2, due to the presence in the medium of additional particles reflected from the surface of the body. We assume for simplicity that the surface of the body is a sphere of radius R_0.

Specular reflection. We assume first that the particles are specularly reflected upon striking the surface of the body. Then the number of excess particles in a small volume dV_{ρ_z} near an arbitrary point (ρ, z) (Fig. 2a) is equal to the number of particles in a corresponding small volume dV_{ρ_0} in the incoming stream, and consequently $n_2(\rho, z) = n_0 dV_{\rho_0}/dV_{\rho_z}$. Taking it also into account that the velocity is not changed by elastic collision of the particle with the body, we find that the ratio of the volumes is equal to the ratio of their cross sections:

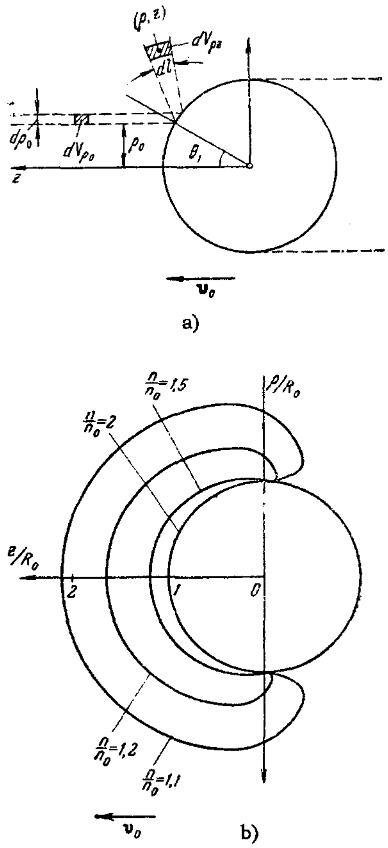

FIG. 2. a) Illustrating the derivation of the particle concentration in the "condensation" region. b) Curves of constant ratio $n(\rho, z)/n_0$ in the "condensation" region ahead of the body in the case of specular reflection from the surface of the body ($\sqrt{Mv_0^2/2kT} = 8$).

$$\frac{dV_{\varrho_0}}{dV_{\varrho z}} = \frac{\varrho_0 d\varrho_0}{\varrho\, dl},$$

where $d\rho_0$ and dl are shown in Fig. 2a. All these quantities are expressed by elementary geometry in terms of ρ, z, R_0, and the angle θ_1 (the angle between the normal to the sphere at the point of collision and the z axis; see Fig. 2a):

$$\varrho_0 = R_0 \sin\theta_1, \quad d\varrho_0 = R_0 \cos\theta_1\, d\theta_1,$$

$$dl = \frac{2\varrho\left(1 - \frac{R_0}{r}\sin^3\theta_1\right)}{\sin 2\theta_1}\, d\theta_1.$$

We then obtain

$$n_2 = n_0 \frac{R_0^2}{\varrho^2} \frac{\sin^2\theta_1 \cos^2\theta_1}{1 - \frac{R_0}{\varrho}\sin^3\theta_1}, \tag{2.17}$$

and the total concentration of the particles in the "condensation region" is

$$n(\varrho, z) = n_0 + n_2(\varrho, z).$$

The angle θ_1 can be expressed here in terms of ρ and z with the aid of the relation

$$2z\cos\theta_1 + 2\varrho\sin\theta_1 - \frac{\varrho}{\sin\theta_1} = R_0. \tag{2.18}$$

As $\rho \rightarrow 0$, that is, near the z axis, the expression for n_2 becomes quite simple

$$n_2(0, z) = n_0 \frac{R_0^2}{(2z - R_0)^2}, \quad n(0, z) = n_0\left(1 + \frac{R_0^2}{(2z - R_0)^2}\right). \tag{2.19}$$

It is clear therefore that the density of the particles increases noticeably ahead of the body. The variation of the particle concentration in the "condensation" region, calculated from formula (2.17) for different ρ and z, is shown in Fig. 2, where we see that $n/n_0 = 2$ near the surface of the body, $n/n_0 = 1.5$ at a distance $0.2R_0$, and $n/n_0 = 1.1$ at a distance R_0. With increasing distance from the surface of the body, the concentration disturbances decrease more rapidly in the "condensation" region than in the "rarefaction" region.

Diffuse reflection. Let us assume now that the molecules are diffusely scattered upon colliding with the surface of the body, that is, that the surface is very rough, so that the particles are reflected at any angle with equal probability.

Arguments analogous to those given above for the case of specular reflection lead to the following expression for the additional density n_2 in the case of diffuse reflection:

$$n_2(\varrho, z) = n_0 \frac{R_0^2}{2\pi} \int_0^{2\pi} d\varphi_1 \int_0^{\arccos \frac{R_0}{\sqrt{\varrho^2+z^2}}} \frac{\cos\theta' \sin\theta_1\, d\theta_1}{\varrho^2 + z^2 + R_0^2 - 2R_0\sqrt{\varrho^2+z^2}\cos\theta_1} D(\theta'), \tag{2.20}$$

where*

$$\cos\theta' = \cos\theta_1 \cos\theta + \sin\theta_1 \sin\theta \cos\varphi_1, \quad \theta = \operatorname{arctg}\frac{\varrho}{z},$$

$$D(\theta') = \begin{cases} 1 & 0 \leqslant \theta' \leqslant \frac{\pi}{2}, \\ 0 & \frac{\pi}{2} < \theta' < \pi. \end{cases}$$

This expression for n_2 in the case $\rho = 0$ (that is, on the z axis) greatly simplifies:

$$n_2(0, z) = \frac{n_0 R_0}{2z}\left\{\frac{R_0}{z} - 1 + \frac{1}{2}\left(\frac{R_0}{z} + \frac{z}{R_0}\right)\ln\frac{z + R_0}{z - R_0}\right\}. \tag{2.21}$$

It follows therefore that the additional concentration n_2 ahead of the body is always larger in diffuse than in specular reflection. Thus, at large z ($z \gg R_0$) we have $n_2 \approx n_0(R_0/z)^2/2$ in the case of diffuse reflection and $n_2 \approx n_0(R_0/z)^2/4$ in the case of specular reflection. However, the concentration of the particles changes most appreciably near the surface of the body (for $\Delta R \ll R_0$). Here

$$n_2 = \frac{n_0}{2}\ln\frac{2R_0}{\Delta R},$$

where $\Delta R = \sqrt{\rho^2 + z^2} - R_0$ is the distance from the surface of the sphere. As $z - R_0 \rightarrow 0$ the concentration increases logarithmically and can become considerably larger than n_0.

The concentration of reflected particles near the surface of the body increases even more strongly in the presence of accommodation. In this case the con-

*arctg = $\tan^{-1}$.

centration of the reflected particles increases in addition by a maximum factor v_0/v_n.

In the rarefaction region, as indicated above, the role of the reflected particles is not important. Therefore formulas (2.12) and (2.13) (see also Fig. 1) remain in force regardless of the character of reflection of the particles from the surface of the body.

c) Concentration of neutral particles around a sphere moving with arbitrary velocity. In the present section we present without proof formulas for the concentration of neutral particles around a sphere with arbitrary velocity. Although in this case we can no longer state that the rarefaction region contains only particles that do not collide with the body, and that on the forward side the blocking of the particles by the body is insignificant, it is nevertheless convenient to represent n, as before, in the form

$$n = n_1 + n_2.$$

The expression for n_1 can be obtained here directly from geometrical considerations, by recognizing that the body sweeps in velocity space a region corresponding to the angle subtended by the sphere at the given point. We ultimately obtain

$$n_1(\varrho, z) = n_0\left(\frac{M}{2\pi kT}\right)^{3/2} \int\limits_{\arcsin \frac{R_0}{\sqrt{\varrho^2+z^2}}}^{\pi} \sin\theta\, d\theta \int\limits_0^{\infty} v^2\, dv$$
$$\times \exp\left\{\frac{-M\left(v_0^2 + v^2 - 2vv_0\cos\theta \frac{z}{\sqrt{\varrho^2+z^2}}\right)}{2kT}\right\} I_0$$
$$\times \left(\frac{Mvv_0}{kT}\frac{\varrho}{\sqrt{\varrho^2+z^2}}\sin\theta\right). \tag{2.22}$$

If the body moves rapidly, that is, $v_0 \gg \sqrt{kT/M}$, then the previously obtained formula (2.12) follows from (2.22) at distances z such that ρ/z and $R_0/z \ll \sqrt[4]{kT/Mv_0^2}$.* The value of n_2 depends, as in the case of a rapidly moving body, on the character of reflection of the molecules from the surface of the body. The calculations reduce here, roughly speaking, to an averaging of formulas (2.17) and (2.20) over the directions of the particles incident on the body. We thus have in lieu of (2.20)

$$n_2(\varrho, z) = n_0\left(\frac{M}{2\pi kT}\right)^{3/2} \int\limits_0^{2\pi} d\varphi \int\limits_{\arcsin \frac{R_0}{\sqrt{\varrho^2+z^2}}}^{\pi} \sin\theta\, d\theta \int\limits_0^{\infty} v^2\, dv$$
$$\times \exp\left\{-\frac{M(v_0+v)^2}{2kT}\right\} \frac{\frac{R_0^2}{\varrho^2+z^2}\cos^2\theta' \sin^2\theta'}{\sin^2\theta - \sin\theta \sin^3\theta' \frac{R_0}{\sqrt{\varrho^2+z^2}}}. \tag{2.23}$$

The angle θ' in formula (2.23) is given by

$$\sin\theta' \frac{R_0}{\sqrt{\varrho^2+z^2}} + 2\cos(\theta+\theta')\sin\theta' + \sin\theta = 0. \tag{2.24}$$

*We note that a different criterion for the applicability of formulas (2.11)–(2.17) is given erroneously in [7], namely $R_0/z \ll \sqrt{kT/Mv_0^2}$.

5. Influence of Magnetic Field on the Ion-concentration Disturbance

The system (2.4)–(2.6), which describes simultaneously the distribution of the ions, electrons, and electric field in the plasma, is rather complicated. It can be solved only by allowing for specific conditions that make certain simplifications possible. In particular, as can be seen from (2.4)–(2.6), the equations for the electron and ion distribution function are coupled because the motion of the charged particles is influenced by the electric field, which in turn depends on the concentration distribution of these particles. At the same time, in the case of a rapidly moving body, $v_0 \gg \sqrt{kT/M}$ and the energy $Mv_0^2/2$ of the ions incident on the body is large compared with the thermal energy kT. The potential energy of the ion in an electric field, resulting from the disturbance in the plasma, is, as will be shown, only somewhat higher than the thermal energy. We must therefore expect the influence of the electric field on the ion motion to be negligible in first approximation. In this approximation Eq. (2.5), which describes the distribution of the ions, can be solved independently of (2.4)–(2.6). A solution with account of the influence of the electric field on the ion motion will be obtained below in the approximation linear in the field.

If we disregard the magnetic field, then the motion of the ions does not differ at all from the motion of the neutral particles, considered in the preceding section. In this case, consequently,

$$\frac{N_i(\mathbf{r})}{N_{i0}} = \frac{n(\mathbf{r})}{n_0}. \tag{2.25}$$

The expressions for $n(\mathbf{r})/n_0$ are given above. Thus, the problem consists only of taking account of the influence of the external constant magnetic field on the motion of the ions. We consider here only the case of a body moving rapidly in a plasma.

In this case the presence of a magnetic field **H** influences appreciably only the distribution of the ions in the shadow of the body. The magnetic field in the condensation zone is practically insignificant. Indeed, the velocity of the particles reflected in this zone is of the same order as the velocity v_0 of the body. Therefore their Larmor radius, $\sim v_0/\Omega_H$, is very large and if the radius of the body $R_0 \ll v_0/\Omega_H$, then the magnetic field begins to come into play only at distances where the particle density is practically equal to N_{i0}. Since the collisions between the particles and the surface of the body can be neglected in the shadow zone, only the maximum transverse cross section of the body in a plane perpendicular to the direction of motion is important in the corresponding calculations. Starting from this, the equation for the distribution function of the ions in the presence of a constant magnetic field can be written in the form

$$\mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + \frac{e}{cM_i}[\mathbf{v}+\mathbf{v}_0, \mathbf{H}]\frac{\partial f}{\partial \mathbf{v}} = 0 \tag{2.26}$$

with the usual boundary condition

$$f_{z=0} = \begin{cases} \left(\frac{M_i}{2\pi kT}\right)^{3/2} \exp\left\{\frac{-M_i(\mathbf{v}+\mathbf{v}_0)^2}{2kT}\right\}, & \text{if } (x, y) \text{ is outside S,} \\ 0, & \text{if } (x, y) \text{ is inside S,} \end{cases}$$

where S is the transverse cross section of the body.

The characteristic equations for (2.26) are, as is well known, the equations of motion of the ions; they have the form

$$\frac{d\mathbf{v}}{dt} = \frac{e}{M_i c}[\mathbf{v}+\mathbf{v}_0, \mathbf{H}], \quad \frac{d\mathbf{r}}{dt} = \mathbf{v}, \tag{2.27}$$

where the time t is a free parameter.

Let us choose the z axis, as usual, along the direction of motion of the body, the y axis perpendicular to the ($\mathbf{H}$, $\mathbf{v}_0$) plane, and the x axis in the ($\mathbf{H}$, $\mathbf{v}_0$) plane, perpendicular to the $\mathbf{v}_0$ direction.

The solution of the characteristic system (2.27) has in these coordinates the form

$$x = x_0 + [v_x^0 \sin\alpha - (v_z^0 + v_0)\cos\alpha]\, t\cdot\sin\alpha - \frac{u_\perp}{\Omega_H}\cos\alpha\,[\sin(\Omega_H t - \varphi) + \sin\varphi],$$

$$y = y_0 + \frac{u_\perp}{\Omega_H}[\cos(\Omega_H t - \varphi) - \cos\varphi],$$

$$z = z_0 - v_0 t + [v_x^0 \sin\alpha - (v_z^0 + v_0)\cos\alpha]\, t\cdot\cos\alpha + \frac{u_\perp}{\Omega_H}\sin\alpha \times [\sin(\Omega_H t - \varphi) + \sin\varphi],$$

$$v_x = v_x^0 \sin^2\alpha - (v_z^0 + v_0)\cos\alpha\sin\alpha - u_\perp\cos\alpha\cos(\Omega_H t - \varphi),$$

$$v_y = -u_\perp \sin(\Omega_H t - \varphi),$$

$$v_z = -v_0 + v_x^0 \sin\alpha\cos\alpha - (v_z^0 + v_0)\cos^2\alpha + u_\perp \sin\alpha\cos\alpha\,(\Omega_H t - \varphi). \tag{2.28}$$

Here

$$u_\perp = \sqrt{(v_y^0)^2 + [v_x^0\cos\alpha + (v_z^0 + v_0)\sin\alpha]^2}$$

is the projection of the velocity $\mathbf{u} = \mathbf{v} + \mathbf{v}_0$ on a plane perpendicular to $\mathbf{H}$; $\sin\varphi = v_y^0/u_\perp$; α is the angle between $\mathbf{v}_0$ and $\mathbf{H}_0$; x_0, y_0, and z_0 are the initial coordinates; v_x^0, v_y^0, and v_z^0 are the initial velocities, and $\Omega_H = eH/M_i c$ is the gyromagnetic frequency of the ions. It is important that the ion merely rotates freely in a plane orthogonal to $\mathbf{H}$. Because of this, the total energy of the particles and the modulus of the projection of the velocity on this plane remains constant in time:

$$(v_z + v_0)^2 + v_x^2 + v_y^2 = (v_z^0 + v_0)^2 + v_x^{0^2} + v_y^{0^2},$$

$$v_y^{0^2} + [v_x^0\cos\alpha + (v_z^0 + v_0)\sin\alpha]^2 = v_y^2 + [v_x\cos\alpha + (v_z + v_0)\sin\alpha]^2.$$

Using these relations and recognizing that the boundary function $f_{z=0}$ depends only on the coordinates and on $(\mathbf{v} + \mathbf{v}_0)^2$, we can readily write the solution of (2.26):

$$f[z, x, y; (\mathbf{v}+\mathbf{v}_0)^2] = f\Big[0,\ x - \{u_\perp\cos\alpha\cos(\Omega_H t - \varphi) + v_x\}t + \frac{u_\perp}{\Omega_H}\cos\alpha\,[\sin(\Omega_H t - \varphi) + \sin\varphi],\ y - \frac{u_\perp}{\Omega_H}\{\cos(\Omega_H t - \varphi) - \cos\varphi\};\ (v_z + v_0)^2 + v_y^2 + v_x^2\Big], \tag{2.29}$$

where the parameter t is defined by the relation

$$v_z t - u_\perp \sin\alpha\left[\cos(\Omega_H t - \varphi)t - \frac{1}{\Omega_H}\{\sin(\Omega_H t - \varphi) + \sin\varphi\}\right] = z. \tag{2.30}$$

In the absence of a magnetic field, the distribution function (2.29) coincides, of course, with (2.9). Integrating the obtained expression for the distribution function with respect to the velocities, we can determine the ion density in the shadow zone. The form of the latter depends essentially on the angle α between the direction of the magnetic field and the direction of motion of the body.

a) Motion of body along the magnetic field ($\mathbf{v}_0 \parallel \mathbf{H}$). We consider first a simple case when the body moves along the magnetic field $\mathbf{H}$, that is, when $\alpha = 0$. We assume here that the cross section of the body is a circle of radius R_0. Expression (2.29) for the distribution function then assumes the form

$$f[z, x, y; (\mathbf{v}+\mathbf{v}_0)^2] = \begin{cases} \left(\frac{M_i}{2\pi kT}\right)^{3/2}\exp\left[-\frac{M_i(\mathbf{v}+\mathbf{v}_0)^2}{2kT}\right] & \text{for } \left\{x + \frac{u_\perp}{\Omega_H}\left[\sin\left(\Omega_H\frac{z}{v_z} - \varphi\right) + \sin\varphi\right]\right\}^2 + \left\{y - \frac{u_\perp}{\Omega_H}\left[\cos\left(\Omega_H\frac{z}{v_z} - \varphi\right) - \cos\varphi\right]\right\}^2 \geqslant R_0; \\ 0 & \text{for } \left\{x + \frac{u_\perp}{\Omega_H}\left[\sin\left(\Omega_H\frac{z}{v} - \varphi\right) + \sin\varphi\right]\right\}^2 + \left\{y - \frac{u_\perp}{\Omega_{Hz}}\left[\cos\left(\Omega_H\frac{z}{v_z} - \varphi\right) - \cos\varphi\right]\right\}^2 < R_0. \end{cases} \tag{2.31}$$

Here

$$u_\perp = v_\perp = \sqrt{v_x^2 + v_y^2}. \tag{2.32}$$

We change over to new variables u and φ_1, which are determined by the relations

$$\begin{aligned} u_\perp \cos\left(\frac{\Omega_H z}{2v_z} - \varphi\right) &= \frac{u\cos\varphi_1 - \varrho\Omega_H}{2\sin\frac{\Omega_H z}{2v_z}}, \\ u_\perp \sin\left(\frac{\Omega_H z}{2v_z} - \varphi\right) &= \frac{u\sin\varphi_1}{2\sin\frac{\Omega_H z}{2v_z}}. \end{aligned} \tag{2.33}$$

Inasmuch as

$$dv_x\, dv_y = \frac{u\, du\, d\varphi_1}{4\sin^2\frac{\Omega_H z}{2v_z}},$$

We introduce the variable $\mathbf{v}' = \mathbf{v}\sqrt{M_i/2kT}$ and obtain from (2.31)

$$N_i(\varrho, z) = \frac{N_0}{\sqrt{\pi}} \int_{-\infty}^{\infty} dv_z' \exp\left[-(v_0' + v_z')\right]^2 \frac{1}{\pi}$$

$$\times \frac{1}{\pi}\int_0^{2\pi} \int_{\frac{R_0}{2\varrho_H\left|\sin\frac{\Omega_H z}{2v_z}\right|}}^{\infty} \frac{u\, du\, d\varphi_1}{4\sin\frac{\Omega_H z}{2v_z}}$$

$$\times \exp\left\{-\frac{(u\cos\varphi_1 - \varrho\Omega_H)^2 + u^2\sin^2\varphi_1}{\left(2\sin\frac{\Omega_H z}{2v_z}\right)^2}\right\},$$

or after integration with respect to φ_1

$$N_i(\varrho, z) = \frac{N_0}{\sqrt{\pi}} \int_{-\infty}^{\infty} dv_z' \exp\left[-(v_0' + v_z')^2\right]$$

$$\times \left\{2\exp\left[-\frac{\varrho^2}{\left(2\varrho_H \sin\frac{\Omega_H z}{2v_z}\right)^2}\right]\right.$$

$$\times \int_{\frac{R_0}{2\varrho_H\left|\sin\frac{\Omega_H z}{2v_z}\right|}}^{\infty} u e^{-u^2} I_0\left(\frac{\varrho u}{\varrho_H\left|\sin\frac{\Omega_H z}{2v_z}\right|}\right) du, \tag{2.34}$$

where $\rho_H = (c/eH)\sqrt{2kTM_i}$ has the meaning of the average Larmor radius of the ion, and I_0 is a Bessel function of zero order of imaginary argument. Recognizing furthermore that $v_0 \gg \sqrt{kT/M_i}$, we replace v_z everywhere in the curly bracket by $-v_0$ and integrate with respect to v_z. We ultimately obtain

$$N_i(\varrho, z) = 2N_0 \exp\left\{-\frac{\varrho^2}{\left(2\varrho_H \sin\frac{\Omega_H z}{2v_0}\right)^2}\right\}$$

$$\times \int_{\frac{R_0}{2\varrho_H\left|\sin\frac{\Omega_H z}{2v_0}\right|}}^{\infty} u e^{-u^2} I_0\left(\frac{\varrho u}{\varrho_H\left|\sin\frac{\Omega_H z}{2v_0}\right|}\right) du. \tag{2.35}$$

At small distances from the body we have

$$\sin\frac{\Omega_H z}{2v_0} \approx \frac{\Omega_H z}{2v_0} \quad \text{for} \quad z \ll \frac{v_0}{\Omega_H}, \tag{2.35a}$$

and formula (2.35) coincides with (2.12). This confirms the assumption made above, that the influence of the magnetic field is insignificant in the near zone (when $z \ll v_0/\Omega_H$). When $z \gtrsim v_0/\Omega_H$, the influence of the magnetic field, to the contrary, is very large. It is obvious from (2.35) that $N_i(\rho, z)$ is a periodic function of z with a period $T_z = 2\pi v_0/\Omega_H$.

The character of variation of the ion density in the shadow zone, of course, depends appreciably on the relation between R_0 and the Larmor radius ρ_H. If the dimensions of the body are very large ($R_0 \gg \rho_H$) then the perturbed zone represents actually a semi-infinite cylinder of radius R_0, inside of which the ion concentration is zero; the boundary of the cylinder is smeared over a distance on the order of ρ_H.

For small bodies with $R_0 \ll \rho_H$, to the contrary, the ion concentration in the shadow zone changes very little over the length of the period $2\pi v_0/\Omega_H$.

On the z axis (for $\rho = 0$) expression (2.35) assumes a particularly simple form. Here

$$N_i(0, z) = N_{i0} \exp\left[-\frac{R_0^2}{4\varrho_H^2 \sin^2\frac{\Omega_H z}{2v_0}}\right]. \tag{2.36}$$

For the case $\mathbf{v}_0 \parallel \mathbf{H}$ considered here, the disturbances of the ion concentration

$$\frac{\Delta N_i}{N_{i0}} = -\frac{N_i(\varrho, z) - N_0}{N_0},$$

along the z axis for $\rho = 0$, are shown in Fig. 3, while Fig. 4 shows curves of equal values of $N_i(\rho, z)/N_{i0}$. The corresponding curves are calculated for $R_0 = \rho_H$ and $\sqrt{Mv_0^2/2kT} = 8$. In this case the period of variation of $\Delta N_i/N_{i0}$ is equal to $50.24 R_0$.

It must be noted that under real conditions the structure of the rarefied region behind the moving body is neither strictly periodic nor semi-infinite. This is due to the fact that in the derivation of the previous formulas we took no account of the collisions of the ions and the electric field. It is understandable that when the collisions are taken into account the disturbances of the region behind the moving body can have a strictly periodic structure only up to distances of order Λ_i —the mean free path of the ions. Collisions lead to a change in the periodicity in z and to a smearing of the effect over distances on the order of Λ_i.

b) <u>Motion of body transverse to the magnetic field ($\mathbf{v}_0 \perp \mathbf{H}$)</u>. If the body moves in a direction perpendicu-

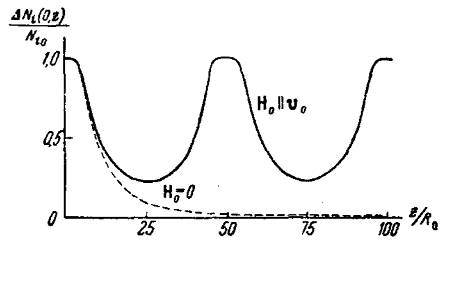

FIG. 3. Variation of $\Delta N_i/N_{i0}$ on the z axis in the "rarefaction" region of a spherical body moving parallel to the magnetic field. Dashed curve – the same for $H_0 = 0$.

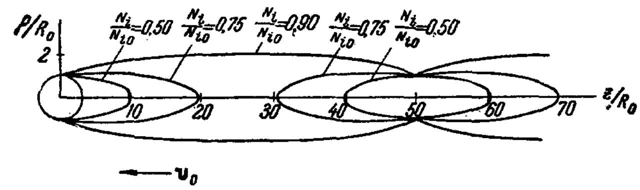

FIG. 4. Curves of constant ratio $N_i(\rho, z)/N_{i0}$ in the "rarefaction" region of a spherical body when $v_0 \parallel H_0$, $\sqrt{M_i v_0^2/2kT} = 8$, $R_0/\rho_H = 1$.

lar to $\mathbf{H}$, then the problem has no axial symmetry and the expression for the ion density for a body of circular cross section has a very complicated form. Simpler and clearer formulas are obtained if the cross section of the body in a plane perpendicular to the direction of motion is a rectangle with dimensions $2R_x$ and $2R_y$.

Expression (2.29) for the distribution function assumes in this case the form

$$f[z, x, y;\ (\mathbf{v}+\mathbf{v}_0)^2] = \begin{cases} \left(\frac{M_i}{2\pi kT}\right)^{3/2} N_0 \exp\left[-\frac{M_i(\mathbf{v}+\mathbf{v}_0)^2}{2kT}\right], & \text{if } R_x \gg |x - v_x t| \\ \text{and} R_y \gg \left| y - \frac{u_\perp}{\Omega_H}\left[\cos\left(\Omega_H \frac{z}{v_z} - \varphi\right) - \cos\varphi\right]\right|, & \\ 0, \quad \text{if } R_x < |x - v_x t| & \\ \text{and} R_y < \left| y - \frac{u_\perp}{\Omega_H}\left[\cos\left(\Omega_H \frac{z}{v_z} - \varphi\right) - \cos\varphi\right]\right|, & \end{cases} \quad (2.37)$$

where the parameter t is defined by the relation

$$v_z t - u_\perp\left[\cos(\Omega_H t - \varphi)\,t - \frac{\sin(\Omega_H t - \varphi) + \sin\varphi}{\Omega_H}\right] = z, \quad (2.38)$$

and

$$u_\perp = \sqrt{v_y^2 + (v_z + v_0)^2}, \quad \sin\varphi = \frac{v_y}{u_\perp}.$$

The sought ion density in the shadow zone is determined by integrating the distribution function (2.37) with respect to the velocities. Recognizing that $\sqrt{kT/M_i} \ll v_0$, we obtain ultimately

$$N_i(z, x, y) = N_{i0} - \Delta N_i(z, x, y) = N_{i0} - \frac{N_{i0}}{4}\left|\Phi\left(\frac{x - R_x}{z}\sqrt{\frac{M_i v_0^2}{2kT}}\right) - \Phi\left(\frac{x + R_x}{z}\sqrt{\frac{M_i v_0^2}{2kT}}\right)\right| \times \left|\Phi\left(\frac{y - R_y}{2\varrho_H \sin\frac{\Omega_H z}{2v_0}}\right) - \Phi\left(\frac{y + R_y}{2\varrho_H \sin\frac{\Omega_H z}{2v_0}}\right)\right| \quad (2.39)$$

Here, as before, Φ is the probability integral, $\rho_H = (c/eH)\sqrt{2M_i kT}$, and the z and x axes coincide with the directions of the motion of the body and the magnetic field, respectively. It is seen from (2.39) that in the case $\mathbf{v}_0 \perp \mathbf{H}$ considered here the dependence of N_i on x is similar to that when $\mathbf{H} = 0$ [compare with (2.14)], while the y-dependence changes radically because of the magnetic field. At small distances from the body $z \ll v_0/\Omega_H$ formula (2.39) coincides with formula (2.14), obtained in the absence of a magnetic field, as should be the case.

However, when $z \geq v_0/\Omega_H$, the influence of the magnetic field is large: the second factor in (2.39) is, as in the case when $\mathbf{v}_0 \parallel \mathbf{H}$, a periodic function of z with a period $2\pi v_0/\Omega_H$.

The distribution of the ion concentration in the shadow zone has, owing to the influence of the magnetic field, a rather complicated form. An idea of the character of the disturbance of the ion concentration $\Delta N_i/N_{i0}$ can be gained from Figs. 5–7.

The variation of $\Delta N_i/N_0$ along the z axis for a body with square cross section, with $R_x = R_y = \rho_H$ and $\sqrt{Mv_0^2/2kT} = 8$, is shown in Fig. 5. The disturbance does not remain constant, but decreases with distance as $1/z$. We recall here that $\Delta N_i/N_{i0}$ decreases with the distance like $1/z^2$ in the case when $H = 0$ (dashed curved in Fig. 5), whereas when $\mathbf{H} \parallel \mathbf{v}_0$ the average variation of $\Delta N_i/N_{i0}$ does not increase at all with increasing distance.

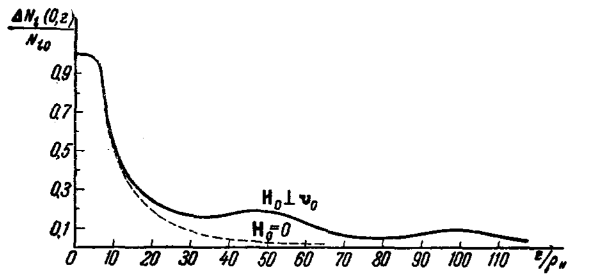

FIG. 5. Variation of $\Delta N_i/N_{i0}$ on the z axis in the "rarefaction" region of a body of square cross section, moving perpendicular to the magnetic field ($v_0 \perp H$, $\sqrt{M_i v_0^2/2kT} = 8$, $R_x = R_y = \rho_H$).

Figure 6 shows the traces in the (x, y) plane of the surface $N_i(x, y, z)/N_{i0} = 0.5$ for different values of z, that is, different distances behind the body. We see from the figure that this surface has a rather interesting structure; for example, in the case when $\rho_H = 0.3R_x = 0.3R_y$ the form of the cross section of the body first spreads out, and then again assumes its initial sharp boundaries. This is seen even more clearly in Fig. 7, which shows the general form of the surface $N_i(x, y, z)/N_{i0} = 0.8$. With increasing N_i/N_{i0} the surface $N_i/N_{i0} = \text{const}$ stretches over larger distances behind the body; the influence of the magnetic field naturally, increases then. The influence of the magnetic field becomes stronger also with decreasing Larmor radius of the ions, more accurately, with decreasing ratio ρ_H/R_x or ρ_H/R_y. This is seen, in particular, from Fig. 6, where the cross sections $N_i(x, y, z)/N_{0i} = 0.5$ are shown for different ratios of the Larmor radius to the dimension of the body. The influence of the magnetic field is completely insignificant when $\rho_H/R_x = \rho_H/R_y = 3$ (the dashed curves on Fig. 6 show the same sections in the absence of a magnetic field), it becomes noticeable when $\rho_H/R_x = \rho_H/R_y = 1$, and is quite large when $\rho_H/R_x = \rho_H/R_y = 0.3$.

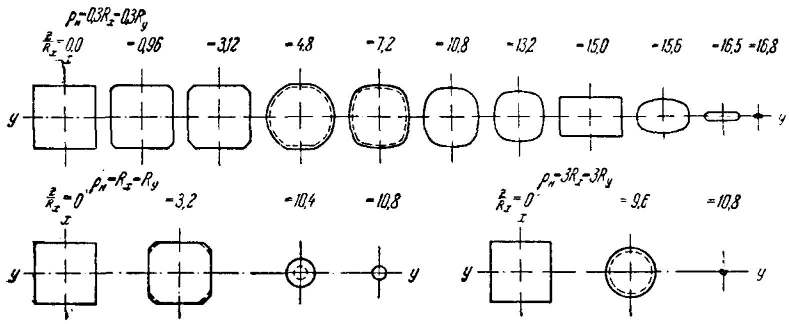

FIG. 6. Cross sections through the surfaces of constant ratio $N_i/N_{i0} = 0.5$ in the (x, y) plane in the "rarefaction" region for different values of z/R_x, as indicated in the figure. The body has a square cross section. Dashed curve – the same for $H_0 = 0$.

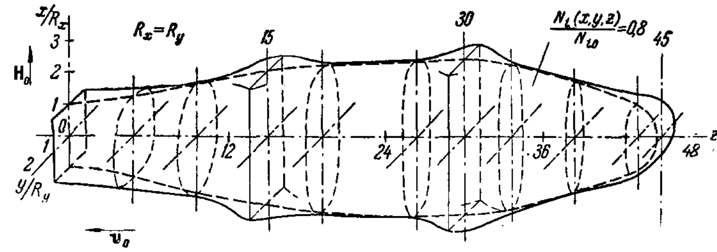

FIG. 7. Surface of constant ratio $N_i/N_{i0} = 0.8$ in the "rarefaction" region of a body of square cross section, $R_x = R_y$, for the case when $v_0 \perp H_0$, $H_0 \parallel x$, $\rho_H = 0.3R_x$.

If the linear dimension of the body in a direction perpendicular to $\mathbf{H}_0$ is large, that is, much larger than the Larmor radius ($R_y \gg \rho_H$), then the disturbed zone at large distances from the body represents a plate in the form of a strip lying in the (x, z) plane, that is, in the plane ($\mathbf{v}_0 \cdot \mathbf{H}$):

$$N_i(x, y, z) \approx N_0 \left[1 - \frac{1}{2} \left| \Phi\left(\frac{x - R_x}{z} \sqrt{\frac{M_i v_0^2}{2kT}} \right) - \Phi\left(\frac{x + R_x}{z} \sqrt{\frac{M_i v_0^2}{2kT}} \right) \right| \begin{Bmatrix} 0, & |y| > R_y \\ 1, & |y| < R_y \end{Bmatrix} \right]. \tag{2.40}$$

The thickness of the plate is somewhat smeared out (by an amount ρ_H) and oscillates with a period $2\pi v_0/\Omega_H$. It follows from formula (2.40), in particular, that owing to the influence of the magnetic field appreciable disturbances in the density are maintained at larger distances from the body.

c) Arbitrary direction of motion. To calculate the distribution function in this general case it is convenient to integrate (2.29) with respect to the velocity by changing over from the velocities v_x, v_y, and v_z to the velocity $u_\perp$, defined in accordance with (2.32), and the velocity $v_\parallel$, parallel to the magnetic field. In this case at large distances from the body ($z \gg v_0/\Omega_H$), where the influence of the magnetic field is most noticeable, the following expression holds for the ion concentration:

$$N_i(x, y, z) = N_0 \left\{ 1 - \frac{1}{4} \left| \Phi\left(\sqrt{\frac{M_i v_0^2}{2kT}} \frac{x - R_x}{z \sin\alpha + x \cos\alpha} \right) - \Phi\left(\sqrt{\frac{M_i v_0^2}{2kT}} \frac{x + R_x}{z \sin\alpha + x \cos\alpha} \right) \right| \cdot \left| \Phi\left(\frac{y - R_y}{2\rho_H \sin \frac{\Omega_H z}{2v_0}} \right) - \Phi\left(\frac{y + R_y}{2\rho_H \sin \frac{\Omega_H z}{2v_0}} \right) \right| \right\} \tag{2.41}$$

It is assumed here that the angle α is not very small ($\sin\alpha > 2R_x/z$). The form of the disturbed zone is in this case, naturally, rather complicated. On the whole, the length of the perturbed zone increases with decreasing angle α like $1/\sin\alpha$. The mean value of the disturbance of the concentration of the ions for any $\alpha \neq 0$ decreases in proportion to $1/z$. The motion of a body along the magnetic field is in this sense a special case.

6. Electric Field Around a Body

By disturbing the density of the electrons and ions, the moving body upsets the quasineutrality of the plasma in its vicinity. As a result, an electric field is produced here, which itself influences the distribution of the charged particles. Therefore, to find the electric field it is necessary, generally speaking, to solve Eq. (2.6) for the potential of the field simultaneously with Eqs. (2.4) and (2.5), which determine the distribution of the charged particles.

On the whole, this system of equations is quite complicated, and a complete solution can be obtained only in the region of a plasma which is weakly perturbed by the motion of the body. An approximate solution of these equations in the region of a strongly disturbed plasma is based on the fact that near the body the distribution of the heavy particles (ions) is disturbed essentially because of their interaction with the body itself, so that in first approximation the influence of the electric field on the motion of the ions can be neglected.

We note that the distribution of the charged particles and consequently also the electric field near the body are influenced by the character of interaction of the particles with the surface of the body; accordingly, the form of U(r) or the boundary conditions in Eqs. (2.4) and (2.5) also change. If, for example, all the particles are elastically reflected from the surface of the body, then the potential energy U(r) is as before equal to infinity on the surface of the body and to zero outside the body. On the other hand, if the particles are absorbed upon contact with the surface of the body, for example the ions become neutralized and the electrons absorbed, or if the collisions between the particles and the surface are inelastic, then the corresponding expressions change; furthermore, in this case there are added to Eqs. (2.4) and (2.5) terms that describe the creation and absorption of the particles. New terms must be added also when account is taken of other effects on the surface of the body (photoeffect, thermionic emission, etc.). One can show, however, that sufficiently far away from the body in the disturbed zone the concentrations of the ions and electrons, and consequently also the electric field, are actually independent of the character of interaction between the particles and the surface of the body. It is therefore advantageous to consider first the simplest case, when all the particles are reflected from the surface of the body. The influence of particle absorption will be taken into account later on.

a) Body reflects elastically particles incident on it. Electron density. In the conditions of interest to us, the velocity of the body can always be regarded as smaller than the thermal velocity of the electrons $v_e = \sqrt{2kT/m}$. Therefore the disturbances due to the motion of the body are small and the distribution of the electrons should be close to equilibrium. Solution of (2.4) can be sought in the form of a series of successive approximations, choosing as the zeroth approximation the Maxwell-Boltzmann equilibrium distribution

$$f_{e0}(\mathbf{u}, \mathbf{r}) = N_0\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} \exp\left\{-\frac{mu^2 - e\varphi(\mathbf{r})}{kT}\right\}. \quad (2.42)$$

The next term of the expansion, as can be readily seen, is $\sqrt{mv_0^2/kT}$ times smaller than f_{e0}, and is neglected throughout. *

For the electron density we obtain from formula (2.24), naturally,

$$N(\mathbf{r}) = N_0 \exp\left\{\frac{e\varphi(\mathbf{r})}{kT}\right\}. \quad (2.43)$$

Electric field. Equation (2.6) for the potential of the electric field is now written in the form

*It must be noted that inasmuch as the spreading out of the disturbance transversely to the magnetic field is hindered when the disturbed region is very strongly elongated along the magnetic field, no Maxwell-Boltzmann distribution is established for the electron distribution function, and the expression for $f_e(\mathbf{u}, \mathbf{r})$ and $N(\mathbf{r})$ has a more complicated form. Such a case is realized, however, only if the body motion is strictly longitudinal, when the angle α is very small.

$$\Delta\varphi(\mathbf{r}) = -4\pi e N_0\left(\frac{N_i(\mathbf{r})}{N_0} - \exp\left[\frac{e\varphi(\mathbf{r})}{kT}\right]\right), \quad (2.44)$$

where

$$N_i(\mathbf{r}) = \int f_i\, d^3u$$

is the ion density and N_0 is the undisturbed electron density. *

Let us rewrite (2.44) in terms of the dimensionless variables $y = e\varphi/kT$ and $x = r/R_0$:

$$\Delta_x y = -A\left\{\frac{N_i(\mathbf{r})}{N_0} - \exp y\right\}, \quad (2.45)$$

where A is a certain constant

$$A = \frac{4\pi e^2 N_0 R_0^2}{kT} = \left(\frac{R_0}{D}\right)^2 = 2.1\cdot 10^2 \frac{N_0}{T}\left(\frac{R_0}{1\mathrm{m}}\right)^2, \quad (2.46)$$

$D = \sqrt{kT/4\pi e^2 N_0}$ is the Debye radius. In the ionosphere $A \sim 10^3$–10^4 (for R_0 = 1 meter), that is, quite large. This means that the characteristic dimensions which arise in an examination of the disturbances due to moving bodies in the ionosphere are large compared with the Debye radius; the Debye screening plays therefore a most important role here.

The character of the solution of (2.45) in the presence of a large parameter A in the right half of the equation is of course determined by this parameter. In this case it is convenient to separate two regions: the region where the concentration of the ions is not very small, so that $AN_i(\mathbf{r})/N_0 > 1$, and the region near the body (we shall call it the "region of maximum rarefaction"), where, to the contrary, $AN_i(\mathbf{r})/N_0 < 1$. In the first region the principal role in (2.45) is played by the nonlinear term exp y, and the equation can therefore be rewritten in the form

$$-y = \ln\left\{\frac{N_0}{N_i(\mathbf{r}) + \frac{N_0}{A}\Delta_x y}\right\}. \quad (2.47)$$

A solution of this equation can be readily obtained by iteration: in the first approximation

$$-y_1 \approx \ln\frac{N_0}{N_i(\mathbf{r})}; \quad (2.48)$$

In the next approximation

$$-y_2 = \ln\left[\frac{N_0}{N_i(\mathbf{r}) - \frac{N_0}{A}\Delta_x \ln\frac{N_0}{N_i(\mathbf{r})}}\right]$$

$$= \ln\frac{N_0}{N_i(\mathbf{r})} - \ln\left[1 - \frac{N_0}{N_i(\mathbf{r})A}\Delta_x \ln\frac{N_0}{N_i(\mathbf{r})}\right], \quad (2.49)$$

etc. In the first region $[AN_i(\mathbf{r})/N_0 > 1]$ this method yields good convergence.

In the second region, the region of maximum rarefaction, $[AN_i(\mathbf{r})/N_0 < 1]$, the concentration of the ions is very small and accordingly the role of the ions is insignificant. In this region, however, the solution

*For simplicity we assume everywhere that the ions are singly charged, that is, $N_{i0} = N_0$.

of (2.47) depends strongly on the electric properties of the body itself. We assume here that the body is a dielectric with uncharged surface (since we are considering the case when all the particles striking the surface of the body are completely reflected by the body). Then the solution of (2.47) in the region under consideration, as can be readily seen, has the form

$$-y = \ln A + y_1(x), \qquad (2.50)$$

where y_1 satisfies the equation

$$-\Delta_x y_1 = \exp\{-y_1(x)\} \qquad (2.45a)$$

(inside the body, naturally, $\Delta y_1 = 0$). The value of y_1 on the boundary of the considered region is determined by the conditions of matching the solution of (2.50) to the solution in the first region. As can be seen from (2.48), the boundary value y_1 turns out to be only of the order of unity. Consequently, Eq. (2.45a) and its boundary condition do not contain any large parameters, so that it is clear that $y_1 \lesssim 1$ everywhere in the region under consideration. By virtue of this, in first approximation, we can neglect the function $y_1(x)$ in (2.50), that is, we can assume that in the entire region of maximum rarefaction $y = -\ln A$. In the same approximation it is necessary to take into account in the second region only the first iteration (2.48) for $y(x)$.

Thus, the potential of the electric field has the following approximate form (accurate to terms of order $1/\ln A$ near the body and with accuracy of order $1/A$ away from the body)

$$-\varphi(\mathbf{r}) = \begin{cases} \frac{kT}{e} \ln \frac{N_0}{N_i(\mathbf{r})} & \frac{N_i(\mathbf{r})}{N_0} A \gg 1, \\ \frac{kT}{e} \ln A & \frac{N_i(\mathbf{r})}{N_0} A \ll 1. \end{cases} \qquad (2.51)$$

By determining with the aid of this expression for the potential the density of the electrons (2.48), we verify that in the first region the concentrations of the ions and electrons coincide; in the second region (near the body) the electron concentration is on the order of N_0/A, while the ion concentration is much smaller. Thus, the difference in the electron and ion concentrations does not exceed N_0/A anywhere and is small compared with the undisturbed concentration N_0.

Further, if we disregard the influence of the electric field on the motion of the ions, formula (2.51) with account of the expressions for the disturbances of the ion concentrations, obtained in the preceding section, completely determine the potential of the electric field in the disturbed zone. The distribution of the potential in the vicinity of a spherical body and in the absence of a magnetic field is shown in Fig. 8. Since $\ln A \sim 10$ in the ionosphere, the potential φ in the vicinity of the maximum rarefaction is one order of magnitude larger than kT/e, that is, $\varphi \sim 1$ volt. Ahead of the body, to the contrary, φ is only of the order of kT/e, that is, $\varphi \sim 0.05-0.1$ volt.

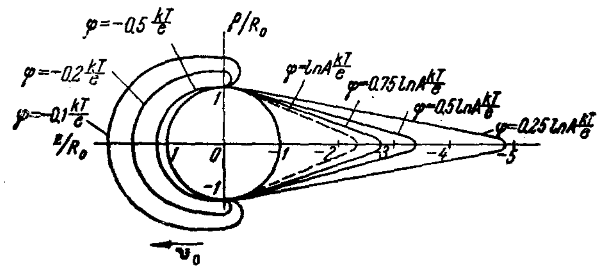

FIG. 8. Distribution of potential in the vicinity of a specularly reflecting spherical body in the absence of a magnetic field ($\sqrt{Mv_0^2/2kT} = 8$).

b) Body absorbs the incident particles. When all the particles incident on the surface of the body are completely absorbed or neutralized by it, the concentration of the ions in the region of condensation is equal to the concentration of the ions in the incoming stream, since there are no reflected ions. Behind the body, in the rarefaction region, the concentration of the ions is determined essentially only by the free incoming ions in this zone, and therefore does not depend on the character of interaction between the ions and the surface of the body.

Let us proceed now to a consideration of the potential of the electric field φ. In the analysis of Eq. (2.45), above, we separated for φ the first region and the second region—the region of maximum rarefaction. The boundary separating these regions was determined by the condition $AN_i/N_0 = 1$; it is shown dashed in Fig. 8. It is easy to verify that the character of the interaction of the particles with the surface of the body does not influence at all the expression for the potential of the electric field in the first region, where

$$-\varphi = \frac{kT}{e} \ln \frac{N_0}{N_i}. \qquad (2.52)$$

Consequently, the value of the potential φ in the first region changes only to the extent that the concentration of the ions changes; this concentration, as we have seen above, remains essentially in the same form, except that in front of the body we now have $N_i = N_0$ and $\varphi = 0$. The electron density in the first region is determined as before by expression (2.43) and, naturally, coincides with the ion density.*

In the second region, the region of maximum rarefaction, the potential of the electric field can, to the contrary, change appreciably. In fact, if the particles incident on the surface of the body are absorbed, then the potential of the surface itself changes. In this case, if the surface of the body is a dielectric, the potential

*It must be emphasized that when electrons are absorbed on the surface of the body their density, strictly speaking, no longer obeys the Boltzmann distribution (2.43). However, if there is a sufficiently large negative potential on the surface of the body, formula (2.43) holds true in any case in those points $\mathbf{r}$ where $\varphi_0 - \varphi(\mathbf{r}) < -kT/e$.

of the field at each point of the surface is determined from the condition that the current in this region be equal to zero (that is, the number of electrons absorbed from the plasma per unit time should equal the number of neutralizing ions at the same point of the surface). If the body surface is metallic, then the potential of the surface is determined from the condition that the total current on the body vanish. Furthermore, on the boundary of the considered region of maximum rarefaction, shown dashed in Fig. 8, the potential of the field is constant and equal to $-(kT/e)\ln A$. Thus, the potential φ in the region of maximum rarefaction is determined by (2.6) with the conditions indicated above on the boundaries of the region.

It is important that the density of the ions in the region under consideration is negligibly small. Likewise small is the electron density: it does not exceed N_0/A, since the presence of electron absorption on the surface of the body does not increase their concentration near the body in any case. Therefore the role of the free charges is small in this case compared with the influence of the conditions on the boundary of the region, and can be neglected; Eq. (2.6) is consequently rewritten in the form

$$\Delta\varphi = 0 \tag{2.53}$$

with boundary condition $\varphi = \varphi_0(S)$ on the surface of the body and $\varphi = (kT/e)\ln A$ on the remaining part of the surface of the region of maximum rarefaction. A solution of this equation can be obtained numerically only under specific conditions, using ordinary methods of electrostatics. The result of the corresponding calculation of the potential φ for a metallic surface under the conditions of the F layer of the ionosphere $\sqrt{Mv_0^2/2kT} = 8$ and $-\varphi_0 = 0.25(kT/e)\ln A$ is shown in Fig. 9.*

Under the conditions of the ionosphere $\varphi_0 \sim -(2\text{–}3)kT/e$. In the calculation of the curves of Fig. 9 it was assumed that $\varphi_0 \simeq -0.25(kT/e)\ln A$, since in the F layer $\ln A = 10$ for $R_0 \sim 1$ meter; consequently $0.25(kT/e)\ln A \sim 2.5kT/e$. It is seen from the figure that the variation of the potential φ near the surface (in the region of maximum rarefaction) has

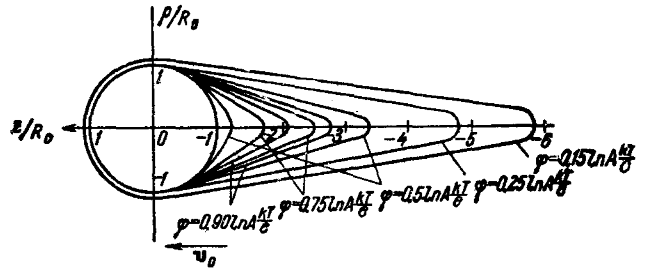

FIG. 9. Distribution of the potential in the vicinity of a metallic sphere $[\sqrt{Mv_0^2/2kT} = 8,\ \varphi_0 = 0.25(kT/e)\ln A]$.

changed appreciably (compared with the case of a reflecting sphere), as should be the case. The maximum value of φ, however, is as before equal to $(kT/e)\ln A$; it is attained not near the surface of the body, but at a distance on the order of R_0.

The intensity of the electric field on the surface of the body in the rarefied zone is $E \sim kT(\ln A)/eR_0$. The value of E is minimal in the $-z$ direction, opposite the direction of the motion of the body; here $E = 1.3(kT/eR_0)\ln A$. On approaching a direction orthogonal to $\mathbf{v}_0$, E increases appreciably to a value on the order of kT/eD, where D is the Debye radius. In the opposite side, in the double layer on the forward surface of the body, the field intensity has an opposite sign; as was shown by Gintsburg [19], its absolute value is also of the order of kT/eD.

The potential of the field φ remains of the same form also in the presence of other conditions in the surface of the body; for example, if only part of the ions or electrons is absorbed, or if other processes in the surface of the body are taken into account, such as photoemission, thermionic emission, etc. Only the form of the potential φ in the region of maximum rarefaction depends significantly on the conditions on the surface. Of course, this holds true only if the dimensions of the body are much larger than the Debye radius.

c) Influence of electric field on the motion of the ions. In the preceding section we have calculated the electric field due to the plasma perturbation caused by the moving body. It was assumed here that it is possible to neglect the reaction of the electric field on the perturbation, that is, on the motion of the ions. Actually, of course, this is true only in the first approximation. It goes without saying that the electric field influences the motion of the ions; however, as indicated above, owing to the high ion velocity relative to the body, this influence is not decisive for the problem considered here, since, as is clear from the results of the preceding section, $Mv_0^2 \gg e\varphi(r)$.

In the condensation region ahead of the body, an account of the influence of the electric field leads only to insignificant corrections of the order of kT/Mv_0^2. To the contrary, the ions filling the rarefaction region, are acted upon by the electric field for a long time, so that here the influence of the field is much more

*The potential of a metallic sphere φ_0 is determined, as noted above, from the condition that the total current on its surface vanish. The ion current is obviously $I_i = eN_0\pi R_0^2 v_0$, where v_0 is the velocity of the body. The electron current is $I_e = (1/2)eN_0\pi R_0^2 v_e \times \exp[e\varphi_0/kT]$. It is assumed here that the body is negatively charged with potential φ_0, and that the electrons in the plasma have a Maxwellian distribution. Account is also taken of the fact that the electrons are absorbed essentially only on one hemisphere (since the number of electrons in the "rarefied region" is very small), $v_e = \sqrt{8kT/\pi m}$ is the average electron velocity. If we neglect the photocurrent, thermionic emission, and other processes, then the potential φ_0 of the body is determined simply from the relation $I_i = I_e$; this yields

$$-\varphi_0 = \frac{kT}{e}\ln\frac{v_e}{2v_0} = \frac{kT}{e}\ln\sqrt{\frac{2kT}{\pi m v_0^2}}.$$

appreciable. The filling of the rarefaction region occurs, as can be readily understood, only as a result of thermal motion of the particles in a plane perpendicular to the direction of motion of the body. The potential energy of the ion in the field, on the other hand, is of the same order as the energy of thermal motion. Therefore an account of the influence of the electric field on the filling of the rarefaction region with the ions should turn out to be essential in the general case. A corresponding calculation, carried out in Sec. 8 of Chapter III for the far region, shows that the disturbances of the ion density in the rarefaction region decrease to approximately one-half as the result of the influence of the electric field. In this case the form of the disturbed "trail" of the body also changes somewhat. An account of the influence of the electric field does not change, however, the fundamental qualitative features of the behavior of the ion concentration in the rarefaction region, as noticed above.

In particular, we shall show in the present section that even in a rigorous account of the electric field in the absence of a magnetic field, the ion-density disturbances decrease far away from the body like $\sim 1/r^2$, which coincides with the results obtained above without account of the influence of the electric field. To prove this we write down the exact equation (2.5) for the ion distribution function, with account of the electric field (when $\mathbf{H}_0 = 0$) and the Poisson equation (2.44)

$$\mathbf{v}\frac{\partial f}{\partial \mathbf{r}} - \frac{1}{M}\frac{\partial f}{\partial \mathbf{u}}\frac{\partial}{\partial \mathbf{r}}(U + e\varphi) = 0, \tag{2.54}$$

$$\Delta\varphi = -4\pi e\left\{\int f d^3u - N_0 e^{\frac{e\varphi}{kT}}\right\} - \frac{Q}{R_0^2}\delta(r - R_0). \tag{2.55}$$

The last term in (2.55) describes the distribution of the charge Q over the surface of the sphere. We now put $f = f_0(\mathbf{u}) + f'(\mathbf{u}, \mathbf{r})$, where f_0 is the Maxwellian distribution function; we change over to Fourier components, that is, we multiply both equations by $\exp(-i\mathbf{q}\cdot\mathbf{r})$ (where $\mathbf{q}$ is the wave vector) and integrate with respect to d^3r. We obtain in place of (2.54)

$$i\mathbf{q}\mathbf{v}f'_{\mathbf{q}} - \frac{e}{M}\frac{\partial f_0}{\partial \mathbf{u}} i\mathbf{q}\varphi_{\mathbf{q}} - \frac{1}{(2\pi)^3}\int i\mathbf{q}_1(U_{\mathbf{q}_1} + e\varphi_{\mathbf{q}_1})\, d^3q_1$$
$$-\frac{i\mathbf{q}}{M}\frac{\partial f_0}{\partial \mathbf{u}} U_{\mathbf{q}} = 0, \tag{2.56}$$

where

$$\mathbf{u} = \mathbf{v} + \mathbf{v}_0$$

and

$$f'_{\mathbf{q}} = \int e^{-i\mathbf{q}\mathbf{r}} f'(\mathbf{u}, \mathbf{r})\, d^3r. \tag{2.57}$$

φ_q and U_q are determined accordingly.

The behavior of $f(r)$ and $\varphi(r)$ at large distances, that is, as $r \to \infty$, is determined by the behavior of f_q as $q \to 0$. We can therefore let q approach 0 in (2.56). Then the last term drops out. (The first two terms must be retained, since, as will be shown below, f'_q and φ_q tend to infinity as $q \to 0$.) In the third term we can also put $q = 0$. Thus

$$i\mathbf{q}\mathbf{v}f'_{\mathbf{q}} - \frac{e}{M}\frac{\partial f_0}{\partial \mathbf{u}} i\mathbf{q}\varphi_q = J(\mathbf{u}), \tag{2.58}$$

where

$$J(\mathbf{u}) = \frac{1}{M}\int i\mathbf{q}_1(U_{\mathbf{q}_1} + e\varphi_{\mathbf{q}_1})\frac{\partial f'_{q_1}}{\partial \mathbf{u}}\frac{d^3\mathbf{q}_1}{(2\pi)^3} \tag{2.59}$$

We now change over to the Fourier components in (2.55). Then

$$\frac{Q}{R^2}\int \delta(r - R_0)\, e^{-i\mathbf{q}\mathbf{r}}\, d^3r = 4\pi Q\frac{\sin qR_0}{qR_0}.$$

In addition, in the expansion of $[\exp(e\varphi/kT) - 1]$ in powers of φ

$$N_q = N_0\frac{e}{kT}\varphi_{\mathbf{q}} + \frac{e^2}{2(kT)^2}\int \varphi_{\mathbf{q}_1}\varphi_{\mathbf{q}-\mathbf{q}_1}\frac{d^3q_1}{(2\pi)^3} + \dots$$

the first term behaves like $1/q$, and the remainder tend to constant values, so that they can be neglected when $q \to 0$. Therefore, as $q \to 0$,

$$\frac{4\pi N_0 e^2}{kT}\varphi_{\mathbf{q}} = 4\pi\int f'_q\, d^3u + \frac{4\pi Q}{e}\frac{\sin qR_0}{qR_0} \approx 4\pi\int f'_{\mathbf{q}}\, d^3u + \frac{4\pi Q}{e}. \tag{2.60}$$

When $q \ll 1$ the term $4\pi Q/e$ can be neglected, inasmuch as $\int f_q d^3u \sim 1/q$. Therefore,

$$\frac{4\pi N_0 e^2}{kT}\varphi_{\mathbf{q}} = 4\pi\int f_{\mathbf{q}}\, d^3u. \tag{2.60a}$$

Solving (2.58) and (2.60a) simultaneously, we obtain

$$N_{\mathbf{q}} = \int f'_q\, d^3u = \frac{\frac{1}{i}\int\frac{J(\mathbf{u})}{\mathbf{q}\mathbf{v}}d^3v}{1 + \int\frac{q(\mathbf{v} + \mathbf{v}_0)}{\mathbf{q}\mathbf{v}}f_0\, d^3v}. \tag{2.61}$$

It is seen from (2.61) that as $q \to 0$ the Fourier components of the ion density are proportional to $1/q$. This means that when account is taken of the electric field (and consequently also the plasma waves), the ion-density disturbances decrease as $1/r^2$ with increasing distance from the body, so that

$$\delta N(r) \sim \int\frac{1}{q}e^{i\mathbf{q}\mathbf{r}}d^3q \sim \frac{1}{r^2}\int\frac{e^{i\mathbf{s}\mathbf{n}}}{s}d^3S,$$

where

$$\mathbf{s} = \mathbf{q}r, \quad \mathbf{n} = \frac{\mathbf{r}}{r}.$$

We see that the behavior of N_q at small q [or, what is the same, the behavior of $\delta N(r)$ for large $r(\to\infty)$] is determined by the value of $J(\mathbf{u})$, which in accord with (2.59) is proportional to the product of φ by f' and differs from zero only in the second perturbation-theory approximation in Q.

III. SCATTERING OF RADIOWAVES BY THE "TRAIL" OF A BODY MOVING RAPIDLY IN A PLASMA

7. Formulation of the Problem

In the preceding section it was shown that the homogeneity of the plasma is disturbed in the vicinity of a rapidly moving body. A "condensation" region appears in front of the body, and a rarefaction region behind. This causes an inhomogeneous "trail" of the body to move together with the body (say an artificial satellite) in the ionosphere, and possibly to scatter the radiowaves incident on it.

The greatest role in scattering is played by the "rarefaction" region, since its dimension along the direction of motion of the body is considerably larger than the dimensions of the body itself and may reach the mean free path of the particles.

It is clear from the very outset that scattering on the trail may exceed appreciably scattering on the body itself (for example, on a metal sphere) only if the linear dimension of the body R_0 is shorter than the wavelength λ, that is,

$$\lambda \gg R_0. \tag{3.1}$$

It is also clear that the closer the frequency of the scattered wave to the plasma frequency, the larger the scattering. If the frequency of the wave ω is sufficiently close to ω_0, special types of effects can occur, which, however, are not considered here. Inasmuch as at distances on the order of R_0 away from the body the perturbations of the dielectric constant ϵ are already small compared with unity, it is natural to use perturbation theory for the solution of this problem. Bearing in mind further that real interest attaches to cases when the frequency of the wave is much larger than the Larmor frequency of the electrons

$$\omega \gg \omega_H, \tag{3.2}$$

we can assume that

$$\varepsilon = 1 - \frac{4\pi N e^2}{m\omega^2} \tag{3.3}$$

and consequently

$$\delta\varepsilon = -\frac{4\pi e^2}{m\omega^2}\delta N. \tag{3.4}$$

The condition (3.2) will be needed below only to ensure the correctness of (3.4).

We can now write down directly from the known perturbation-theory formula an expression for the amplitude of the scattered wave at distances that are large compared with the wavelength [14]. Namely

$$\mathbf{E}' = \frac{e^2}{m\omega^2\varepsilon}\frac{e^{ikR}}{R}[\mathbf{k}'[\mathbf{k}'\mathbf{E}_0]]N_{\mathbf{q}}. \tag{3.5}$$

Here $\mathbf{E}_0$ is the amplitude of the incident wave, $\mathbf{k}'$ the wave vector of the scattered wave ($|\mathbf{k}'| = k = \sqrt{\epsilon}\ \omega/c$), ϵ the dielectric constant of the plasma, N_q the Fourier component of the electron-density perturbation

$$N_{\mathbf{q}} = \int \delta N(\mathbf{r}) \exp(-i\mathbf{q}\mathbf{r})\, d^3r,$$
$$\mathbf{q} = \mathbf{k}' - \mathbf{k}, \quad |\mathbf{q}| = 2k \sin\frac{\psi}{2}, \tag{3.6}$$

$\mathbf{k}$ the wave vector of the incident wave, and ψ the scattering angle, that is, the angle between $\mathbf{k}$ and $\mathbf{k}'$ (see Fig. 10). We note that inasmuch as the wave-

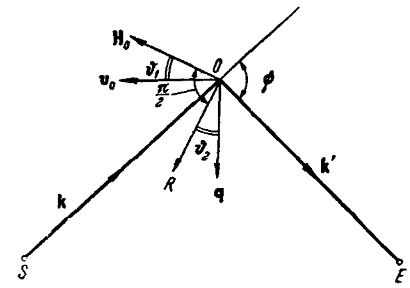

FIG. 10. Arrangement of the vectors in space.

length λ enters into (3.6) only through q, condition (3.1) is actually written more accurately in the form

$$qR_0 \ll 1. \tag{3.7}$$

At the end of this section we present a calculation of N_q without the limitation (3.7). The differential effective scattering cross section (within a solid-angle element do) is given by the formula

$$\sigma = \frac{1}{16\pi^2\varepsilon^2}\left(\frac{\omega_0}{\omega}\right)^4 \frac{|N_{\mathbf{q}}|^2}{N_0^2} k^4 \sin^2\psi_1\, do, \tag{3.8}$$

where ψ_1 is the angle between $\mathbf{k}'$ and $\mathbf{E}_0$, $\omega_0^2 = 4\pi N_0 e^2/m$, and N_0 is the undisturbed electron density.

We thus see from (3.5) that calculation of the effective scattering cross section for electromagnetic waves reduces to a calculation of the Fourier components of the variation of the electron density. In principle, to obtain N_q we can use the results of the calculation of $N(\mathbf{r})$ given in Chapter II. It turns out, however, that it is more convenient to determine N_q directly from the kinetic equation. It is possible then to solve the problem more rigorously and to take into account the influence of the electric field on the motion of the ions, something not done in the previous calculation of $N(\mathbf{r})$.

8. Calculation of the Fourier Components of the Electron-Concentration Disturbance

We represent the ion distribution function $f_i(\mathbf{u}, \mathbf{r})$ in the form

$$f_i(\mathbf{u}, \mathbf{r}) = f'(\mathbf{u}, \mathbf{r}) + f_{i0}(\mathbf{u}),$$
$$f_{i0}(\mathbf{u}) = N_{i0}\left(\frac{M_i}{2\pi kT}\right)^{3/2} \exp\left(-\frac{M_i u^2}{2kT}\right), \tag{3.9}$$

where N_{i0} is the unperturbed ion density and M_i the ion mass. It is clear from the very outset that at small values of q the large distances from the body (much larger than R_0) will be significant in the calculation of N_q. At such distances, however, the disturbances of the distribution function f' and the electric potential φ are small, so that the motions of the ions will at such distances be described by kinetic equations that are linearized in f' and φ. The body itself, and also the region around it where the electric field is strong, can be regarded as point-like and their presence can be taken into account by adding to the right half of (2.5) a term with the meaning of the "collision integral" of the ions with the body. Such a term should, obviously, be different from zero only at the point where the body is situated, that is, it should have the form

$$J(\mathbf{u})\,\delta(\mathbf{r}),$$

where $J(\mathbf{u})$ is some function of the ion velocity. We assume that the body is at the origin. Taking the foregoing into account, we find that the function f' satisfies (in the coordinate system where the body is at rest and f' does not depend on the time) the equation

$$\frac{\partial f'}{\partial \mathbf{r}}(\mathbf{u}-\mathbf{v}_0)+\frac{\partial f'}{\partial \mathbf{u}}\frac{e}{M_i c}[\mathbf{uH}]+\frac{e}{kT}f_0\mathbf{u}\frac{\partial \varphi}{\partial \mathbf{r}}=J(\mathbf{u})\,\delta(\mathbf{r}), \quad (3.10)$$

Account is taken in (3.10) of the fact that $\partial f_0/\partial \mathbf{u} = -M_i f_0 \mathbf{u}/kT$.

We note that the term $J(\mathbf{u})\delta(\mathbf{r})$ in (3.10), written out from simple considerations, coincides actually with the Fourier transform of the term (2.59) in (2.58), obtained by a rigorous transition to the limit as $q \to 0$. Of course, account must be taken of the fact that $H = 0$ in (2.58). From the very form of the initial equation (3.10) it is obvious that $J(\mathbf{u})$ plays the role of a "source," that is, it is equal to the number of ions which acquire per unit time, as a result of collision with the body, a velocity $\mathbf{u} = \mathbf{v} + \mathbf{v}_0$. Later on we also assume that the surface is metallic and completely neutralizes, that is, absorbs, all the ions incident on it. In addition, we introduce an essential approximation, neglecting the influence of the electric field on $J(\mathbf{u})$, that is, assuming that $J(\mathbf{u})$ is the number of ions freely impinging on the sphere per unit time, that is, that

$$J(\mathbf{u})=\pi R_0^2\,|\mathbf{u}-\mathbf{v}_0|\,f_0=\sigma_0 f_0 v_0, \quad (3.11)$$

where σ_0 is the transverse cross section of the body. This approximation is justified because $M_i v_0^2/2 \gg e\varphi$. We present also a formula for $J(\mathbf{u})$, valid when account is taken of the electric field around the body, but under the condition that the body does not absorb the particles incident on it, but merely scatters them elastically. Since $J(\mathbf{u})d^3u$ is the number of particles which acquire velocity in the interval d^3u near the body, it is clear that $\mathbf{u}$ is the collision integral of the particles with the body.

This enables us, repeating the arguments used to find the form of the ordinary collision integral, to obtain a formula for $J(\mathbf{u})$. Let the ion that passes near a body with impact parameter ρ and azimuth angle φ acquire after scattering (owing to interaction with the surface of the body and the electric field surrounding it) a velocity $\mathbf{v}$.* Then the initial ion velocity is $\mathbf{v}_1 = \mathbf{v}_1(\mathbf{v}, \rho, \varphi)$, where the functions $\mathbf{v}_1(\mathbf{v}, \rho, \varphi)$ are determined by the scattering law, and the number of ions which acquire during unit time a velocity $\mathbf{v}$ is simply the number of incident ions with velocity $\mathbf{v}_1(\mathbf{v}, \rho, \varphi)$, that is

$$\varrho\, d\varrho\, d\varphi\, v n_0 \left(\frac{M_i}{2\pi kT}\right)^{3/2}\exp\left\{\frac{-M_i[\mathbf{v}_1(\mathbf{v},\varrho,\varphi)+\mathbf{v}_0]^2}{2kT}\right\}.$$

Account was taken here of the fact that the scattering is elastic ($|\mathbf{v}_1| = |\mathbf{v}|$) and that the incident ions have at infinity a Maxwellian distribution in the immobile reference frame. To find $J(\mathbf{u})$ it is necessary also to subtract from this expression the number of ions with velocity $\mathbf{v}$, knocked out as a result of collision with the body:

$$\varrho\, d\varrho\, d\varphi\, v n_0 \left(\frac{M_i}{2\pi kT}\right)^{3/2}\exp\left\{\frac{-M_i(\mathbf{v}+\mathbf{v}_0)^2}{2kT}\right\}.$$

Ultimately

$$J(\mathbf{u})=J(\mathbf{v}+\mathbf{v}_0)=n_0\left(\frac{M_i}{2\pi kT}\right)^{3/2} v\int \varrho\, d\varrho\, d\varphi \times\left\{\exp\frac{-M[\mathbf{v}_1(\mathbf{v},\varrho,\varphi)+\mathbf{v}_0]^2}{2kT}-\exp\frac{-M_i(\mathbf{v}+\mathbf{v}_0)^2}{2kT}\right\}$$
$$=f_0(\mathbf{u})\,v\int \varrho\, d\varrho\, d\varphi\left\{\exp\frac{-M_i\mathbf{v}_0\Delta\mathbf{v}(\mathbf{v},\varrho,\varphi)}{kT}-1\right\}, \quad (3.12)$$

where $\nabla\mathbf{v} = \mathbf{v}_1 - \mathbf{v}$ is the change in the velocity of the ion upon scattering.

Thus, $J(\mathbf{u})$ can be simply calculated if we know the law governing the scattering of the ions by the body, with account of the electric field. Of course, formula (3.12) does not enable us to calculate $J(\mathbf{u})$ in the general case, if for no other reason than that the electric field itself around the body is unknown. Nonetheless, it turns out to be useful in the approximate calculations.

To obtain the complete system of equations it is necessary to add to (3.10) the Poisson equation for the potential (2.44)

$$\Delta\varphi=-4\pi e\left\{\int f' d^3u-\delta N\right\}, \quad (3.13)$$

where the disturbance of the electron density δN is connected in the linear approximation with φ, in accordance with (2.43), by the equation

$$\delta N = N_0\frac{e\varphi}{kT}. \quad (3.14)$$

Taking the Fourier transforms of (3.10), (3.13), and (3.14), that is, multiplying them by $\exp(-i\mathbf{q}\cdot\mathbf{r})$ and integrating with respect to d^3r, we obtain

$$i\mathbf{q}(\mathbf{u}-\mathbf{v}_0)f_\mathbf{q}+\frac{e}{M_i c}[\mathbf{uH}]\frac{\partial f_\mathbf{q}}{\partial\mathbf{u}}+\frac{e}{kT}f_0 i\mathbf{q}\mathbf{u}\varphi_\mathbf{q}=J(\mathbf{u}). \quad (3.15)$$

Here

$$q^2\varphi_\mathbf{q}=4\pi e\left\{\int f_\mathbf{q}\, d^3u-N_\mathbf{q}\right\}, \quad (3.16)$$

$$N_\mathbf{q}=\frac{N_0 e}{kT}\varphi_\mathbf{q}, \quad (3.17)$$

where

$$f_\mathbf{q}(\mathbf{qu}) = \int f' e^{-i\mathbf{qr}}\, d^3r, \quad \varphi_\mathbf{q}=\int \varphi e^{-i\mathbf{qr}}\, d^3r.$$

*We recall that $\mathbf{u}$ denotes the velocity in the immobile reference frame and $\mathbf{v}$ in a frame attached to the body.

For small values of q we can neglect in (3.16) the time proportional to q^2, and we obtain in place of (3.17) and (3.16)

$$N_{\mathbf{q}} = \int f_{\mathbf{q}}\, d^3u, \quad \varphi_{\mathbf{q}} = \frac{kT}{N_0 e} N_{\mathbf{q}} \tag{3.18}$$

Let us consider first the problem neglecting the magnetic field. This means that we should put $\mathbf{H} = 0$ in (3.15). Then the system (3.15) and (3.18) coincides with Eqs. (2.58) and (2.60a), and its solution is given by (2.61), in which $J(u)$ from (3.11) should be substituted. Then there appears in the numerator and denominator of (2.61) one and the same integral. The integral has a singular denominator, which, as indicated by Landau [15], must be taken by circuiting around the singularity. To take this into account, it is necessary to replace $\mathbf{q}\cdot\mathbf{v}$ by $\mathbf{q}\cdot\mathbf{v} - i\delta$, where $\delta \to +0$. Changing over then from integration with respect to d^3v to integration with respect to d^3u ($\mathbf{u} = \mathbf{v} + \mathbf{v}_0$) and using the formula

$$\int_{-\infty}^{\infty} \frac{e^{-y^2}}{y-a-i\delta}\, dy = 2\sqrt{\pi}\left(i\frac{\sqrt{\pi}}{2} - \int_0^a e^{y^2}\, dy \right) e^{-a^2}, \tag{3.19}$$

we obtain ultimately*

$$N_{\mathbf{q}} = \frac{-\pi R_0^2 n_0}{q} \frac{\left(\frac{M_i v_0^2}{2kT}\right)^{1/2}\left(\frac{\sqrt{\pi}}{2} + i\int_0^a e^{x^2}\, dx\right) e^{-a^2}}{1 - a e^{-a^2}\int_0^a e^{x^2}\, dx + ia\frac{\sqrt{\pi}}{2}e^{-a^2}}$$

$$\left(\begin{array}{l} a = \mathbf{n}\mathbf{v}_0\sqrt{\dfrac{M_i}{2kT}}, \\ \mathbf{n} = \dfrac{\mathbf{q}}{|\mathbf{q}|} \end{array} \right) \tag{3.20}$$

The denominator in (3.20) is connected with an account of the influence of the electric field on the motion of the ions. If we neglect this influence, that is, if we set the denominator equal to unity, and take the inverse Fourier transform then we obtain (2.15) as we should.

We turn now to the case of motion in a magnetic field. Transforming (3.15) to cylindrical coordinates in velocity space with an axis along the direction of the magnetic field, we reduce it to the form

$$\frac{\partial f'_{\mathbf{q}}}{\partial\beta} - i(\mu + \gamma\cos\beta) f'_{\mathbf{q}} = B(\beta), \tag{3.21}$$

where

$$\mu = \frac{q_z u_z - \mathbf{q}\mathbf{v}_0}{\Omega_H}, \quad \gamma = \frac{q_\perp u_\perp}{\Omega_H}, \quad \Omega_H = \frac{eH}{M_i c},$$

$$B = \frac{ie}{\Omega_H kT}(\mathbf{q}\mathbf{u}) f_0 \varphi_{\mathbf{q}} - \frac{J}{\Omega},$$

$\mathbf{q}_\perp$ and $\mathbf{u}_\perp$ are the projections of the vectors $\mathbf{q}$ and $\mathbf{u}$ on a plane perpendicular to the magnetic field, and β is the angle between $\mathbf{q}_\perp$ and $\mathbf{u}_\perp$.

Equation (3.17) has a solution of the form

$$f'_{\mathbf{q}} = e^{i(\mu\beta + \gamma\sin\beta)} \int_c^\beta B(t)\, e^{-i(\mu t + \gamma\sin t)}\, dt.$$

The constant c must be chosen such as to obtain a function that is periodic in β. Putting $c = \infty$ and $t = x + \beta$, we have

$$f'_{\mathbf{q}} = -\int_0^\infty \exp\left[-i\{\mu x + \gamma[\sin(\beta + x) - \sin\beta]\}\right] B(x+\beta)\, dx.$$

Eliminating f'_q and φ_q and shifting the origin of the angle β, we obtain

$$N_{\mathbf{q}} = \frac{\frac{1}{\Omega_H}\int J(u)\exp\left[-i\left\{\mu x + 2\gamma\cos\beta\sin\frac{x}{2}\right\}\right] dx\, d^3u}{2 + i\frac{\mathbf{q}\mathbf{v}_0}{\Omega_H}\int \frac{f_0}{N_0}\exp\left[-i\left\{\mu x + 2\gamma\cos\beta\sin\frac{x}{2}\right\}\right] dx\, d^3u}; \tag{3.22}$$

the integral with respect to d^3u in the denominator of (3.22) can be calculated in terms of elementary functions. Indeed, representing d^3u in the form $du_z u_\perp du_\perp d\beta$ we obtain

$$\left(\frac{M_i}{2\pi kT}\right)^{3/2}\int_0^\infty dx \int \exp\left\{-\frac{M_i}{2kT}(u_z^2 + u_\perp^2) - i\frac{q_z u_z - \mathbf{q}\mathbf{v}_0}{\Omega_H}x - \frac{i2q_\perp u_\perp}{\Omega_H}\sin\frac{x}{2}\right\} du_z u_\perp\, du_\perp\, d\beta = \left(\frac{M_i}{2\pi kT}\right)^{3/2}\int_0^\infty dx$$

$$\times \exp\left\{-\frac{2kT}{M_i\Omega_H^2}q_\perp^2\sin^2\frac{x}{2}\right\}\int \exp\left(-\frac{M_i w^2}{2kT}\right) d^2w$$

$$\times \int_{-\infty}^{\infty}\exp\left(-\frac{iq_z u_z}{\Omega_H} - \frac{M_i u_z^2}{2kT}\right) du_z$$

$$= \int_0^\infty \exp\left\{\frac{i\mathbf{q}\mathbf{v}_0}{\Omega_H}x - \frac{kT}{2M_i\Omega_H^2}\left(q_z^2 x^2 + 4q_\perp^2\sin^2\frac{x}{2}\right)\right\} dx$$

$$\left(\mathbf{w} = \mathbf{u}_\perp + 2\mathbf{q}_\perp\frac{\sin\frac{x}{2}}{M_i\Omega_H}kT\right)$$

As a result we get

$$N_{\mathbf{q}} = \frac{\frac{1}{\Omega_H}\int J(u)\exp\left[-i\left\{\mu x + 2\gamma\cos\beta\sin\frac{x}{2}\right\}\right] dx\, d^3u}{2 + \frac{i}{\Omega_H}\mathbf{q}\mathbf{v}_0\int_0^\infty \exp\left\{\frac{i\mathbf{q}\mathbf{v}_0}{\Omega_H}x - \frac{kT}{2M\Omega_H^2}\left(q_z^2x^2 + 4q_\perp^2\sin^2\frac{x}{2}\right)\right\} dx} \tag{3.23}$$

Substituting now $J(u)$ from (3.11) into (3.23) and calculating by the same method the integral in the numerator, we obtain ultimately

$$N_{\mathbf{q}} = -\pi R_0^2 v_0 N_0$$

$$\times \frac{\frac{1}{\Omega_H}\int_0^\infty \exp\left\{\frac{i\mathbf{q}\mathbf{v}_0}{\Omega_H}x - \frac{kT}{2M\Omega_H^2}\left(q_z^2x^2 + 4q_\perp^2\sin^2\frac{x}{2}\right)\right\} dx}{2 + i\frac{\mathbf{q}\mathbf{v}_0}{\Omega_H}\int_0^\infty \exp\left\{i\frac{\mathbf{q}\mathbf{v}_0}{\Omega_H}x - \frac{kT}{2M\Omega_H^2}\left(q_z^2x^2 + 4q_\perp^2\sin^2\frac{x}{2}\right)\right\} dx}. \tag{3.24}$$

*It can be shown that the denominator in (3.20), due to the account of the electric field, can be expressed in a simple manner in terms of the dielectric constant of the plasma with allowance for spatial dispersion. It is interesting that similar denominators connected with the dielectric constant appear when the magnetic field and even collision between particles are taken into account.

If

$$\frac{\mathbf{q}\mathbf{v}_0}{\Omega_H}=\frac{M_i c}{eH}\mathbf{q}\mathbf{v}_0 \gg 1,$$

then the integrand in (3.24) begins to oscillate rapidly when $x \gtrsim 1$. In this case small values of x become important in the integral. Putting $\sin(x/2) \approx x/2$ and calculating the integral with respect to dx (taking μ to mean $\mu - i\delta$, $\delta \to +0$) we arrive at formula (3.20). Thus, (3.25) is the condition under which the influence of the magnetic field can be neglected. We note that (3.25) is essentially equivalent to condition (2.35a) of Chapter II. This can be readily verified by putting $q \sim 1/z$ and $\mathbf{q}\cdot\mathbf{v}_0 \sim v_0/z$. Expression (3.24) has a sharp maximum when $q_z \to 0$ and $\mathbf{q}\cdot\mathbf{v}_0 \to 0$. We therefore present without derivation the formula obtained from (3.24) when

$$\frac{q_z}{\Omega_H}\sqrt{\frac{kT}{M_i}} \ll 1, \quad \frac{\mathbf{q}\mathbf{v}_0}{\Omega H} \ll 1.$$

Under these conditions

$$\begin{aligned} N_{\mathbf{q}} = &-\frac{\pi R_0^2 v_0 N_0}{q_z}\left[\sqrt{\frac{M_i}{2kT}}\left(\sqrt{\pi}+2i\int_0^{a_1} e^{x^2}\,dx\right)e^{-a_1^2}\right.\\ &\left.\times \exp\frac{-q_\perp^2 kT}{M_i\Omega^2 H}\, I_0\left(\frac{q_\perp^2 kT}{M_i\Omega^2 H}\right)\right]\\ &\times\left[2+ia_1\left(\sqrt{\pi}+2i\int_0^{a_1} e^{x^2}\,dx\right)e^{-a_1^2}\right.\\ &\left.\times \exp\frac{-q_\perp^2 kT}{M_i\Omega^2}\, I_0\left(\frac{q_\perp^2 kT}{M_i\Omega^2 H}\right)\right]^{-1}, \end{aligned} \tag{3.25}$$

where

$$a_1=\frac{\mathbf{q}\mathbf{v}_0}{q_z}\sqrt{\frac{M_i}{2kT}}$$

We see therefore that if $q_z \to 0$ and $|\mathbf{q}\cdot\mathbf{v}_0|/q_z < \infty$, then N_q becomes infinite in proportion to $1/q_z$, which is connected with the slowness of the decrease in the disturbances in the magnetic field, as noted in Chapter II. Formula (3.24) is obtained if condition (3.7) is satisfied, that is, when the body can be regarded as point-like. We can, however, obtain a more general formula, suitable for $qR_0 \sim 1$. For this purpose it is necessary to introduce into the right half of equation (3.10) in place of the ion and body "collision integral," which differs from zero only at the "point" where the body is situated, a term that describes the absorption of the ions by each surface element of the body. Recognizing that in unit time a unit surface absorbs the same number of ions with velocity $\mathbf{v}$ as are incident on it, namely,

$$-f_0(\mathbf{v}+\mathbf{v}_0)(\mathbf{v}\mathbf{s}) \approx f_0(\mathbf{u})(\mathbf{v}_0\mathbf{s})$$

(s is the normal to the surface, $\mathbf{v}\cdot\mathbf{s} < 0$) we find that the right half of (10) assumes the form

$$\left.\begin{matrix} -f_0\dfrac{\mathbf{v}_0\mathbf{r}}{r}\,\delta(r-R_0) & \mathbf{v}_0\mathbf{r}>0, \\ 0 & \mathbf{v}_0\mathbf{r}<0. \end{matrix}\right\} \tag{3.26}$$

Inasmuch as the velocity of the body is much larger than the velocity of the ions, the ions actually strike only the front half of the body, which is taken into account by the limitation $\mathbf{v}_0\cdot\mathbf{r} > 0$. After taking the Fourier transform we obtain by simple calculations from (3.26) that the right half of (3.15) is replaced by the function

$$\begin{aligned} -f_0\pi R_0^2\cdot 2\int_0^{\pi/2} &\sin\vartheta\cos\vartheta\exp\{iqR_0\cos\vartheta\cos\theta\}\\ &\times I_0(qR_0\sin\vartheta\sin\theta)\,d\vartheta, \end{aligned}$$

where I_0 is the Bessel function of real argument, and θ is the angle between $\mathbf{v}_0$ and $\mathbf{q}$. We see therefore that if the finite nature of the dimension of the body is taken into account, formula (3.24) is multiplied by the factor

$$\Phi(qR_0,\ \cos\theta)=2\int_0^{\pi/2}\sin\vartheta\cos\vartheta e^{iqR_0\cos\vartheta\cos\theta} I_0(qR_0\sin\vartheta\sin\theta)\,d\vartheta. \tag{3.27}$$

When $qR_0 \to 0$, as can be readily noted, $\Phi(qR_0, \cos\theta) \to 1$. If the angle θ is close to $\pi/2$, then we can put $\cos\theta = 0$ in (3.27), and

$$\Phi(qR_0,\ 0)=2\frac{I_1(qR_0)}{qR_0}. \tag{3.28}$$

Let us consider now the derivation of N_q with account of the collisions between particles. For this purpose it is necessary to introduce into the initial equations the integral of collisions between the ions themselves and between the ions and other particles. We confine ourselves for this purpose to introducing an effective number of collisions ν, expressed in terms of the collision integral Y in the form

$$\begin{aligned} Y &= -\nu\left(f-\frac{f_0}{N_0}\int f\,d^3u\right)\\ &= -\nu\left(f'-\frac{f_0}{N_0}\int f'd^3u\right) \end{aligned} \tag{3.29}$$

Such a form of the collision integral does not affect the law of particle conservation, since $\int Y d^3u = 0$. If we add (3.29) in (3.15), then we obtain in place of (3.24) as an end result a formula for N_q in the form

$$N_{\mathbf{q}}=\frac{-\dfrac{\pi R_0^2 N_0 v_0}{\Omega_H}\left[\displaystyle\int_0^\infty \exp\left\{\frac{i\mathbf{q}\mathbf{v}_0-\nu}{\Omega_H}x-\frac{kT}{2M_i\Omega^2 H}\left(q_z^2x^2+4q_\perp^2\sin^2\frac{x}{2}\right)\right\}dx\right]}{2+\dfrac{i\mathbf{q}\mathbf{v}_0-2\nu}{\Omega_H}\displaystyle\int_0^\infty \exp\left\{\frac{i\mathbf{q}\mathbf{v}_0-\nu}{\Omega_H}x-\frac{kT}{2M_i\Omega^2 H}\left(q_z^2x^2+4q_\perp^2\sin^2\frac{x}{2}\right)\right\}dx} \tag{3.30}$$

which is used as the basis for further calculations.

9. Effective Cross Section for the Scattering by the Trail of the Body

Substituting (3.30) into (3.8) with account of (3.27), we find that the differential effective scattering cross section (that is, the intensity of the wave scattered in a given direction) is

$$\sigma(\vartheta_1, \vartheta_2, \varphi) = \left\{\frac{1}{16}\left(\frac{\omega_0}{c}\right)^4 \frac{R_0^4 v_0^4}{\Omega_H^2} \sin^2\psi_1\right\}$$

$$\times F_3(\alpha, \beta, \gamma, \delta)\,|\Phi(qR_0, \cos\theta)|^2. \tag{3.31}$$

Here ψ_1 is the angle between the electric field E and the wave vector of the scattered wave $\mathbf{k}'$; ϑ_1, ϑ_2, and φ are angles which determine the direction of the incident and scattered waves relative to the body (see Fig. 10); naturally, the total effective scattering cross section is

$$\sigma = \int \sigma(\vartheta_1, \vartheta_2, \varphi)\, do, \tag{3.32}$$

where do is the solid-angle element in the scattering direction.

In formula (3.31) we have

$$F_3(\alpha, \beta, \gamma, \delta) = \frac{F_1^2 + F_2^2}{(2-2\beta F_1 - \alpha F_2)^2 + (\alpha F_1 - 2\beta F_2)^2}, \tag{3.33}$$

and

$$F_1 = e^{-\delta}\int_0^\infty \cos\alpha x \exp\{-\beta x - \gamma x^2 + \delta\cos x\}\, dx, \tag{3.34}$$

$$F_2 = e^{-\delta}\int_0^\infty \sin\alpha x \exp\{-\beta x - \gamma x^2 + \delta\cos x\}\, dx, \tag{3.35}$$

where

$$\alpha = \frac{(\mathbf{q}\mathbf{v}_0)}{\Omega_H} = b(\cos\vartheta_1 \sin\vartheta_2 + \sin\vartheta_1\cos\vartheta_2\cos\varphi) = b\cos\theta;$$

$b = qv_0/\Omega_H$, and the vector $\mathbf{q} = \mathbf{k}' - \mathbf{k}$ is directed along the bisector of the angle between the wave vectors $\mathbf{k}$ and $\mathbf{k}'$, or, what is the same, between the rays SO and OE, which join points of radiation and observation S and E with the point O where the body is located. The vector $\mathbf{q}$ lies in the plane of SO and OE, with

$$q^2 = 4\frac{\omega^2}{c^2}\left(1 - \frac{\omega_0^2}{\omega^2}\right)\sin^2\frac{\psi}{2}, \tag{3.36}$$

and the angle ψ between $\mathbf{k}$ and $\mathbf{k}'$ is determined as in Fig. 10. The angles ϑ_1 and ϑ_2 are determined in terms of the scalar products $(\mathbf{v}_0 \cdot \mathbf{H})$ and $(\mathbf{q} \cdot \mathbf{H})$ by the formulas

$$\cos\vartheta_1 = \frac{(\mathbf{v}_0\mathbf{H}_0)}{v_0 H}, \qquad \sin\vartheta_2 = \frac{\mathbf{q}\mathbf{H}}{qH}. \tag{3.37}$$

Thus, ϑ_1 is the angle between the direction of the magnetic field $\mathbf{H}$ and the velocity $\mathbf{v}_0$. The angle ϑ_2 between $\mathbf{q}$ and the normal OR to $\mathbf{H}_0$, lying in the plane $(\mathbf{q}\mathbf{H}_0)$, is positive or negative if the vector $\mathbf{q}$ is turned clockwise or counterclockwise, respectively, relative to OR; φ is the angle between the planes $(\mathbf{v}_0\mathbf{H}_0)$ and $(\mathbf{q}\mathbf{H})$. The remaining notation assumed in the formulas is:

$$\beta = \frac{\nu}{\Omega_H}, \quad \gamma = \frac{a}{2}\sin^2\vartheta_2, \quad \delta = a\cos^2\vartheta_2, \quad a = \frac{kT}{M_i\Omega_H^2}q. \tag{3.38}$$

Thus, the effective scattering cross section is described by a complicated function F_3, the analysis of which can be carried out only if numerical calculations are used. The tabulation of $\Phi(qR_0, \alpha)$ does not entail great difficulty.

We shall henceforth call F_3 the scattering function. The tabulated scattering functions (3.33) for different values of α, β, γ, and δ have made it possible to ascertain the character of the behavior of this function with altitude, wavelength, temperature, and direction of motion of the body.

The main feature of $F_3(\alpha, \beta, \gamma, \delta)$ is its oscillating character. The maxima and minima of F_3 correspond to definite values of α. For the values of the parameters a, b, and β used by us, F_3 displays in addition to the principal maximum another six or eight maxima and six or eight minima, located symmetrically about the principal maximum when $\vartheta_1 = 0$. The principal maximum of F_3 [which we also call the maximum of zeroth order (0)] corresponds to the value

$$\alpha_0 = 0. \tag{3.39}$$

The side maxima and minima [we denote them (± 1 max), (± 2 max), ... and (± 1 min), (± 2 min), ... respectively] occur (when $\vartheta_1 = 0$) for the values

$$\left.\begin{array}{l} \alpha_{\max} \simeq \pm 1.22, \ \pm 2.18, \ \pm 3.15, \ \pm 4.23 \\ \alpha_{\min} \simeq \pm 0.73, \ \pm 1.70, \ \pm 2.91, \ \pm 3.86. \end{array}\right\} \tag{3.40}$$

The principal maximum of $F_3(\alpha_0 = 0)$ has the largest value when $\vartheta_1 = 0$ or $\vartheta_2 = 0$. For other values of ϑ_1 or ϑ_2, $F_3(\alpha_3 = 0)$ can be smaller than the maximum of the first order of $F_3(\alpha_{\max} = \pm 1.22)$. In the case when $\vartheta_1 = 0$, that is, when the velocity of the body $\mathbf{v}_0$ and the magnetic field $\mathbf{H}_0$ are collinear, $F_3(\alpha = 0)$ is always much larger than the side maxima, which decrease as their number increases. The number of observed side maxima depends essentially on β and γ, which determine the convergence of the integrals (3.34). On Fig. 11 we show for illustration two $F_3(\alpha)$ curves for $\vartheta_1 = 0$ or $\vartheta_2 = 0$ respectively, calculated for a = 1, β = 0.06, and b = 14.

If $\vartheta_1 = 0$ ($\mathbf{v}_0 \parallel \mathbf{H}_0$), then $F_3(\alpha)$ does not depend on φ. Therefore the surface $F_3(\alpha, \varphi)$ is formed in this case as the result of rotation of the curves shown in Fig. 11 about the axis $\mathbf{v}_0$ (or $\mathbf{H}_0$). The corresponding three-dimensional representation of $F_3(\vartheta_2, \varphi)$ for $\vartheta_1 = 0$ ($\mathbf{H}_0 \parallel \mathbf{v}_0$) (Fig. 12) shows the intersections of the surface $F_3(\vartheta_2, \varphi)$ and the planes $\varphi = 0$ and $\varphi = \pi/2$. In Fig. 12 the angle ϑ_2 is laid off directly along the vertical axis v_0, for owing to the rapid variation of $F_3(\vartheta_2)$ the construction of the corresponding surface in polar coordinates is difficult. It is seen

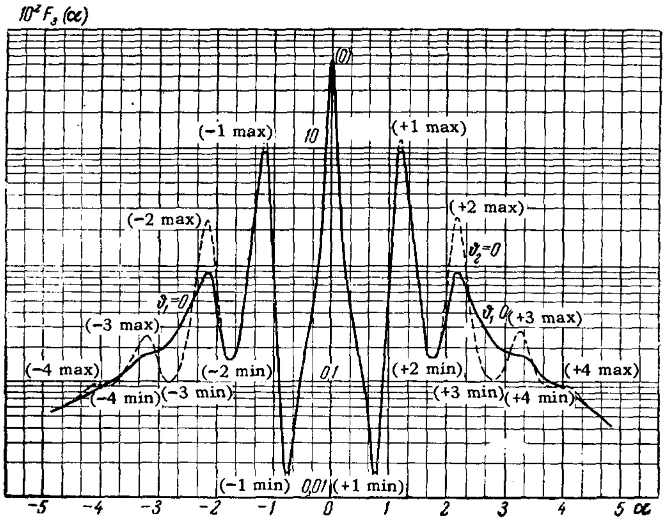

FIG. 11. Variation of the function $F_3(\alpha)$ for $a = 1$, $\beta = 0.06$, $b = 14$, $z = 300$ km, and $\lambda = 30$ meters, for $\varphi = 0$. The maxima and minima of different order are marked in the figure: (0), (±1 max), (±1 min), (±2 max), (±2 min), etc.

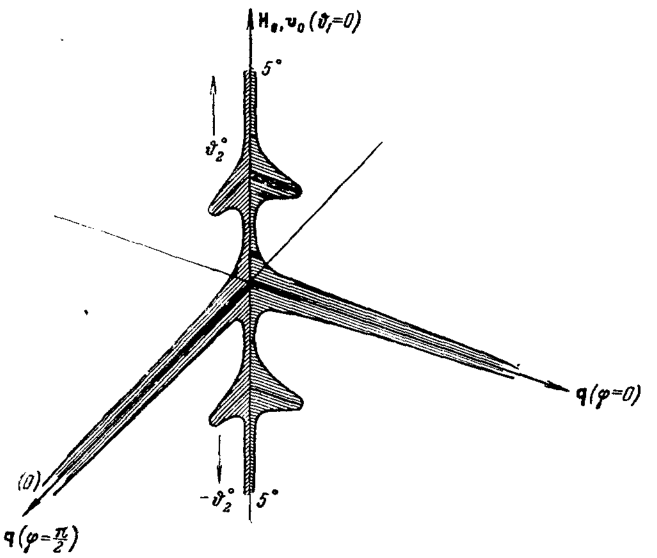

FIG. 12. Three-dimensional representation of the function $F_3(\vartheta_2, \varphi)$ in the case when $v_0 \parallel H_0$ ($\vartheta_1 = 0$) ($a = 1$, $b = 14$, $\beta = 0.06$). The sections are in the planes $\varphi = 0$ and $\varphi = \pi/2$.

from Fig. 11 and 12 that F_3 varies rapidly as a function of the angle ϑ_2 (or of the angle ϑ_1 for a fixed value of ϑ_2). In this case the width of the principal and side maxima is of the order of a fraction of a degree or of one or two degrees. With increasing intensity of the principal maximum, which corresponds to an increase in α and a decrease in β (an increase in the height of the ionosphere), its width decreases and simultaneously its ratio to the maximum of the ± first and other orders decreases. In Fig. 11, where the $F_3(\alpha)$ are plotted in a logarithmic scale, the rapid changes of the scattering function are strongly smoothed out.

The symmetry of the function $F_3(\alpha)$ relative to the angle ϑ_2 is disturbed when the direction of the velocity v_0 does not coincide with $\mathbf{H}_0$ ($\vartheta_1 \neq 0$). In this case α depends on the angle φ between the planes $(\mathbf{v}_0\mathbf{H})$ and $(\mathbf{qH})$, and since the condition $\alpha = 0$ is satisfied for a specified value $\varphi \neq \pi/2$ and a negative value of ϑ_2, the principal maximum drops below the plane $\vartheta_2 = 0$. In this case the line made up of the principal maxima on the $F_3(\vartheta_2, \varphi)$ surface is no longer a circle lying in the plane $\vartheta_2 = 0$, as in the case when $\vartheta_1 = 0$, but represents a non-plane curve of elliptic type, crossing the plane $\vartheta_2 = 0$ when $\varphi = \pi/2$ and $3\pi/2$. At these two points the zero-order maximum has the same values as in the case when $\vartheta_1 = 0$. The character of the variation of $F_3(\vartheta_2, \varphi = 0)$ when $\vartheta_1 = 0$ can be traced on Fig. 13, where the calculations have been carried out for $a = 1$, $b = 14$, and $\beta = 0.06$. The three-dimensional plot of $F_3(\vartheta_2, \varphi)$ in two mutually perpendicular planes is shown in Fig. 14.

For an analysis of the effective scattering cross section, the scattering function F_3 has been calculated for three heights in the ionosphere, $z = 300$, 400, and 700 km. For these values of z and for $v_0 = 8$ km/sec and known ionospheric data [8], the values obtained for the first factor of (3.31) in the curly brackets are listed in Table III. The third factor of (3.31) depends principally on the ratio R_0/λ and is calculated for $\vartheta_1 = \vartheta_2 = \varphi = 0$. In this case we have

$$\Phi_3 = \left\{ \frac{J_1\left(\frac{4\pi R_0}{\lambda}\right)}{4\pi R_0/\lambda} \right\}^2 .$$

An analysis of the formula for Φ_3 shows that the overall course of $\Phi_3(4\pi R_0/\lambda)$ changes little for other values of the angles.

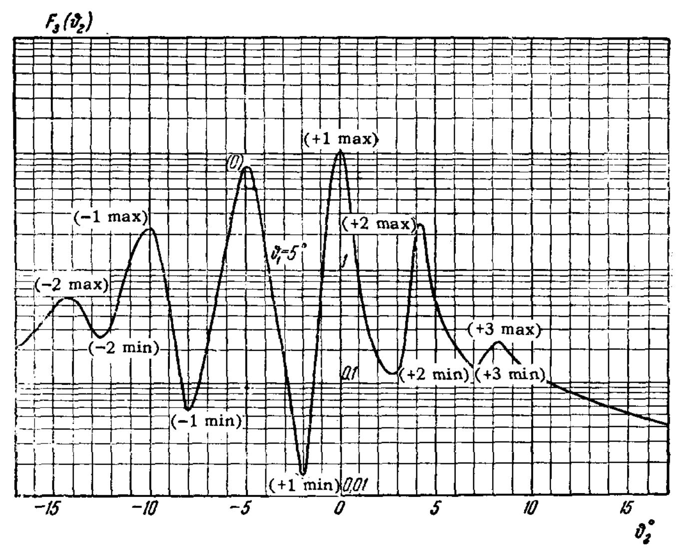

FIG. 13. Variation of the function $F_3(\vartheta_2)$ with $a = 1$, $b = 14$, $\beta = 0.06$ ($z = 300$ km, $\lambda = 30$ meters), $\varphi = 0$ and $\vartheta_1 = 1°$ and $5°$. The maxima and minima of different order (0), (±1 max), (±2 max), etc. are marked in the figure.

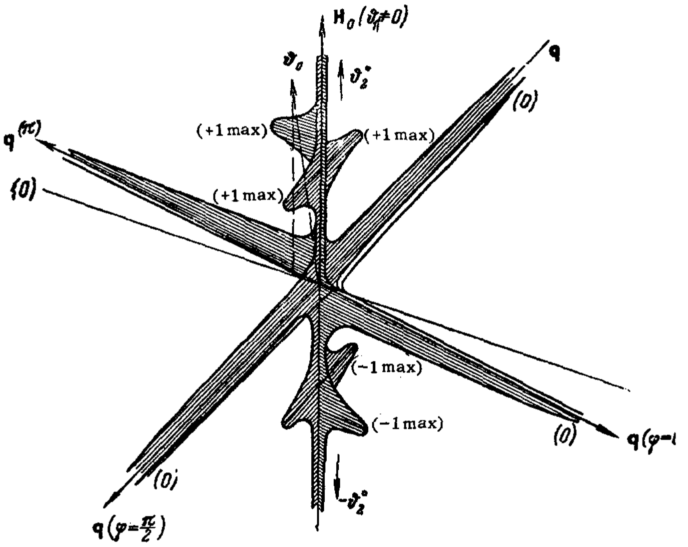

FIG. 14. Three-dimensional plot of the function $F_3(\vartheta_2, \varphi)$ in the case when $\vartheta_1 \neq 0$ ($a = 1$, $b = 14$, $\beta = 0.06$). Sections through the planes $\varphi = 0$ and $\varphi = \pi/2$.

Table III. Values of

$$\frac{1}{16}\left(\frac{\omega_0}{c}\right)^4 \frac{R_0^4 v_0^2}{\Omega_H^2} \sin^2\psi_1$$

R_0, m \ z, km	Day			Night		
	300	400	700	300	400	700
0.5	$1.8\cdot10^{-2}$	$1.2\cdot10^{-2}$	$1.2\cdot10^{-3}$	$6\cdot10^{-5}$	$1.2\cdot10^{-3}$	$1.8\cdot10^{-4}$
1	0.3	0.2	$2\cdot10^{-2}$	10^3	$2\cdot10^{-2}$	$3\cdot10^{-3}$
2	4.8	3.2	0.32	$1.6\cdot10^{-2}$	0.32	$4.8\cdot10^{-2}$
3	24	16	1.6	$8\cdot10^{-2}$	1.6	0.24

It is seen from the foregoing that when $\mathbf{v}_0 \parallel \mathbf{H}_0$ ($\vartheta_1 = 0$) the principal maximum of the effective scattering cross section lies in the direction of the "specular reflection" of the wave from the direction of the earth's magnetic field. In this case the bisector of the angle ($\mathbf{kk}'$), namely the vector $\mathbf{q}$, coincides with the normal to $\mathbf{H}_0$. On the other hand, if $\vartheta_1 = 0$, then the vector $\mathbf{q}$ is turned relative to the normal to $\mathbf{H}_0$ by an angle $\pm\vartheta_2$, determined from equation (3.31) with $\alpha = 0$ for specified values of ϑ_1 and φ. Thus, $\mathbf{q}$ makes in this case an angle ($\pi/2 \pm \vartheta_2$) with $\mathbf{H}_0$; for example, when $\vartheta_1 = 5°$ and $\varphi = 0$, the principal maximum will be directed along the vector $\mathbf{k}'$, chosen in such a way that the vector $\mathbf{q}$ makes an angle ($\pi/2 - 5°$) with $\mathbf{H}_0$. The position of the maxima of higher order, that is, the angles ϑ_2 through which they are turned, are determined from the corresponding values of α_{max}; for example, when $\varphi = 0$, $\vartheta_1 = 0$, and $b = 14$ as used in the calculations, the principal maximum and the maxima of orders $\pm$ 1, 2, 3 correspond to values of the angles

$$\vartheta_{2\,\max} \approx 0°,\ \pm 5°,\ \pm 9°,\ \pm 13°, \tag{3.41}$$

and when $\vartheta_1 = 1°$ ($\varphi = 0$, $b = 14$) the maxima of the same orders correspond to the angles

Table IV. $F_3(\vartheta_2, \vartheta_1)$ for $\lambda = 30$ m, $\varphi = 0$

$\vartheta_2, \vartheta_1^0$ \ z, km	$\vartheta_1 = 0$			$\vartheta_2 = 0$		
	300	400	700	300	400	700
0	53.46	134,4	1535	53.46	134.4	1535
0.02	—	—	613.9	—	—	—
0.03	—	—	355.1	—	—	—
0.05	49.20	103.2	150.7	—	—	—
0.1	39.72	60.86	40.8	—	—	—
0.2	22.44	23.10	10.31	—	23.07	10.0
0.3	12.98	11.30	4.56	—	—	4.42
0.5	5.44	4.20	1.58	5.39	4.14	1.53
1.0	1.32	0.94	0.32	1.30	0.92	0.31
2.0	0.17	0.10	0.019	0.17	0.099	0.018
3.0	0.015	0.011	0.037	0.013	$7.8\cdot10^{-3}$	0.023
4.0	0.85	0.67	0.56	0.98	0.69	0.68
5.0	9.39	13.18	3.18	10.26	12.61	3.41
6.0	0.88	1.44	0.28	0.77	0.61	0.19
7.0	0.18	0.10	0.025	0.18	0.11	0.010
8.0	0.22	0.18	0.14	0.26	0.053	0.10
9.0	0.89	0.98	0.45	2.48	5.31	28.9
9.5	0.83	1.15	0.62	0.78	1.05	1.57
10	0.53	0.70	0.53	0.45	0.52	0.47
11	0.25	0.25	0.21	0.23	0.22	—
12	0.19	0.19	0.18	0.10	0.046	—
12.5	0.187	0.20	—	0.32	0.89	—
13	0.18	0.21	0.24	0.28	0.47	—
14	0.16	0.19	0.25	0.16	0.19	—
15	0.12	0.14	0.20	0.12	0.13	—
16	0.10	0.11	—	0.088	0.086	—
17	—	0.091	—	0.091	0.16	—
17.5	0.077	—	—	0.080	0.020	—
18	0.071	0.078	—	0.072	0.16	—
19	0.062	0.067	—	0.062	0.070	—
20	0.054	0.058	—	0.054	0.057	—

$$\vartheta_{2,\max} \approx -1°;\quad (+4°,\ -6°);\quad (+8°,\ -10°);\quad (+12°,\ -14°). \tag{3.42}$$

An idea of the character of variations of $F_3(\lambda, \vartheta_1, \vartheta_2, z)$ as functions of the angles ϑ_1 and ϑ_2, the height z, and the wavelength λ can be obtained by examining Tables IV–V; for a wavelength $\lambda = 30$ meters and heights $z = 300$, 400, and 700 km respectively for $\vartheta_1 = 0$ ($\mathbf{H}_0 \parallel \mathbf{v}_0$) the behavior is illustrated in Figs. 15–18 (in Fig. 18 $z = 300$ km and $\vartheta_1 = 5°$).

A change in the temperature, T and accordingly in the number of collisions ν naturally brings a change in a and β (see[8]). Some of the data in Table VI give an idea of the changes in F_3 due to the change in T.

The results of the calculation of the effective cross section σ in the principal maximum for different wavelengths and spheres with radii $R_0 = 0.5$, 1, 2, and 3 meters are shown in Table VII and in Fig. 19. The same table gives the ratio of $\sigma(\vartheta_1 = 0, \vartheta_2 = 0)$ to the total effective cross section σ_0 of an ideally conducting sphere. In analyzing σ/σ_0 we must keep in mind that inasmuch as σ is the differential effective cross section, this ratio diminishes by a factor 4π the effect of increase in scattering by the "trail," as compared with σ_0. However, since σ is large only in a narrow region of angles, and the sphere scatters practically isotropically, the time of action of the scattering of a metallic sphere at the point of observation is appreciably larger.

It is seen from these data that the differential effective cross section of the principal maximum $\sigma(0, 0)$ of the "trail" of a satellite can greatly exceed in the

Table V. $F_3(\vartheta_2, \lambda)$ in the vicinity of the principal maximum for $\vartheta_1 = 0$ and $\varphi = 0$

z, km	300				400				700		
λ, m / ϑ_z^0	30	20	15	10	30	20	15	10	30	20	15
0	53.46	11.07	4.82	1.72	134.4	31.05	14.10	5.20	1535	479.3	241.3
0.02	—	—	—	—	—	—	—	—	613.9	—	—
0.03	—	—	—	0.080	—	—	—	0.063	355.1	—	—
0.05	49.20	9.75	3.96	—	103.2	20.54	7.71	—	150.7	25.3	8.67
0.1	39.72	7.16	2.58	—	60.86	10.2	3.27	—	40.8	6.58	2.20
0.2	22.44	3.16	—	—	23.10	3.36	—	—	10.3	1.63	—
0.3	12.98	1.83	0.51	—	11.30	1.55	0.42	—	4.56	0.70	—
0.5	5.44	0.68	0.61	0.016	4.20	0.52	0.12	0.11	1.58	0.22	0.056
1.0	1.32	0.11	0.01	0.020	0.94	0.074	0.005	0.017	0.34	0.027	0.001
1.5	0.47	—	0.03	—	0.31	—	0.027	—	—	$8\cdot10^{-5}$	0.023
2	0.17	0.031	0.48	0.016	0.10	$3.6\cdot10^{-3}$	—	0.010	0.019	0.027	—

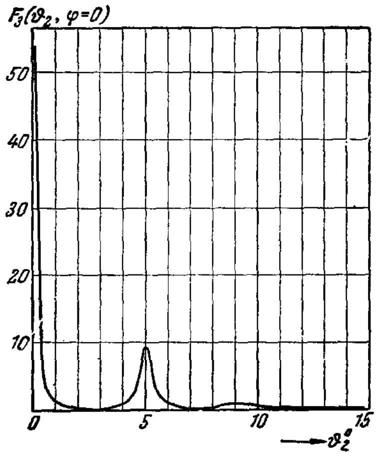

FIG. 15. Dependence of $F_3(\vartheta_2, \varphi = 0)$ on ϑ_2 for $\lambda = 30$ meters, $\vartheta_1 = 0$, and $z = 300$ km.

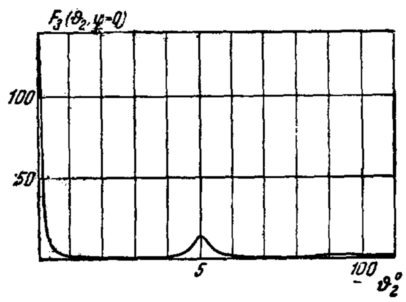

FIG. 16. The same as Fig. 15 but $z = 400$ km.

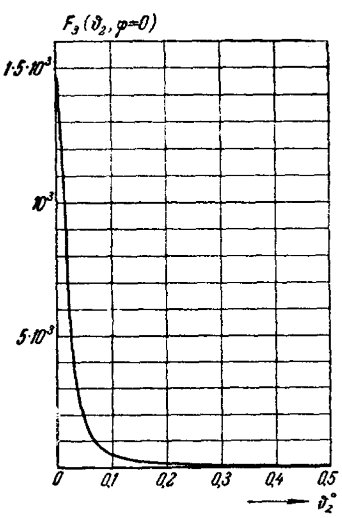

FIG. 17. The same as Fig. 15 but $z = 700$ km.

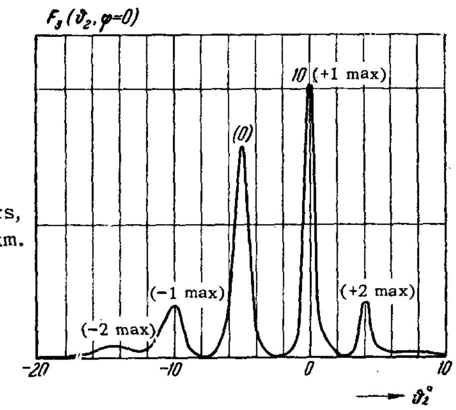

FIG. 18. Dependence of $F_3(\vartheta_2, \varphi = 0)$ on ϑ_2 for $\lambda = 30$ meters, $\vartheta_1 = 5°$, and $z = 300$ km.

Table VI. Values of the principal maximum of $F_3(\vartheta_1, \vartheta_2 = 0)$ for different values of the temperature T at $\lambda = 30$ meters and $z = 300$ and 400 km.

T, deg	800	1660	2000	1000	2000
	z=300 km			z=400 km	
$\vartheta_1 = 0°$	53.46	14.04	9.67	134.4	38.14
$\vartheta_1 = 5°$	10.26	4.12	—	12.61	3.16

daytime, and in many cases also at night, the effective cross section of the metallic sphere σ_0, and may reach many tens or even hundreds of square meters. However, inasmuch as these values correspond to only one wave direction, in estimating the true scattering effect it is necessary to take account also of the width of the corresponding lobe. The scattering increases with height in the region 400–700 km, varying with the wavelength approximately exponentially as $\exp(-1/\lambda)$; in practice σ is small when $\lambda < 15$. We note here that scattering from the "trail" depends generally speaking little on the properties and form of the body itself and on the character of its surface, being determined only by the velocity of the body and its linear dimension ρ_0. At the same time, scattering from the body itself depends appreciably on its

Table VII. Differential effective cross section $\sigma(\mathrm{m}^2)$ of the "trail" in the direction of the principal maximum; $\sigma_0(\pi)$ is the effective cross section of a metalic sphere*

	z, km		300			400			700		
	R_0, m	λ, m	30	20	15	30	20	15	30	20	15
Day	0.5	σ	0,97	0,20	0.08	1,60	0.36	0.2	1,8	0.6	0.3
		σ/σ_0	1380	47	7	2290	90	14	2400	160	22
	1	σ	15	3	1	26	5.6	2	30	8	4
		σ/σ_0	350	14	1.6	590	25	3	630	36	5
	2	σ	210	35	10	350	65	20	400	100	34
		σ/σ_0	72	3	0.3	120	5	0.6	134	79	1.2
		σ	810	98	18	1420	184	35	1620	280	60
		σ/σ_0	28	1	0.3	50	2	0.5	58	2.8	0.83
Night	0.5	σ	$3\cdot10^{-3}$	$7\cdot10^{-4}$	$2\cdot10^{-4}$	0.2	0,04	0,02	0.3	$8\cdot10^{-2}$	$4\cdot10^{-2}$
		σ/σ_0	4.3	0.2	0,02	229	9	1,4	373	22	3.61
	1	σ	0.05	10^{-2}	$3\cdot10^{-3}$	2.6	0.61	0.2	4.6	1.3	0.6
		σ/σ_0	1	0.05	0.014	60	2.5	0.3	96	57	0,8
	2	σ	0.7	0.1	0.03	35	6.5	2.0	61	15	5.3
		σ/σ_0	0,24	$3\cdot10^{-3}$	$2.4\cdot10^{-3}$	12	0.5	0.06	20	1.2	0.2
	3	σ	2.7	0.3	0.06	140	18	3.5	240	51	9
		σ/σ_0	0.1	$4\cdot10^{-8}$	$7\cdot10^{-4}$	5	0.2	0.05	8.5	0.6	0.8

*Because of an arithmetical error in [6], the original table had incorrect values of σ for z = 700 km. This error has been corrected here.

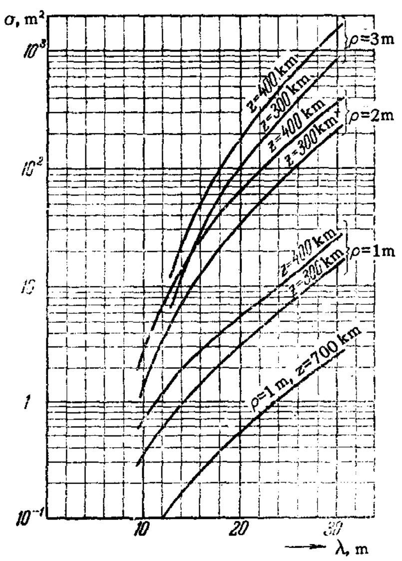

FIG. 19. Dependence of the effective cross section σ during the day on the wavelength for altitudes z = 300, 400 and 700 km and for sphere radii R_0 = 1, 2, and 3 meters.

properties. In this respect a smooth metallic sphere is optimal for scattering by a body in the investigated wavelength range. Other bodies, of analogous dimension but with different surface character, have considerably smaller values of σ_0. We note also that the relative effect of scattering by an inhomogeneous formation, that is, the ratio σ/σ_0, increase rapidly with decreasing radius of the sphere R_0. Indeed, when $\frac{2\pi R_0}{\lambda} \ll 1$

$$\sigma_0 \sim \frac{R_0^6}{\lambda^4}, \quad \sigma \sim R_0^4 \text{ and } \frac{\sigma}{\sigma_0} \sim \frac{1}{R_0^2} \to \infty,$$

and when $\varrho_0 \to \infty$

$$\sigma_0 \approx R_0^2, \quad \sigma \sim R \text{ and}$$

$$\frac{\sigma}{\sigma_0} \sim \frac{1}{R_0} \to 0.$$

10. Character of the Field of the Scattered Wave at the Point of Observation

In conclusion let us consider the overall pattern of the effect of scattering at the point of observation and let us summarize the calculation results given above.

Assume that the body moves longitudinally, that is, the velocity vector is close to the direction of the permanent magnetic field $\mathbf{H}_0$, and the vector $\mathbf{k}$ of the incident electromagnetic wave is normal to $\mathbf{H}_0$ (or $\mathbf{v}_0$). Then the surface of revolution formed by the field of the scattered wave around the vector $\mathbf{v}_0$ (or $\mathbf{H}_0$) has many lobes and its principal lobe is directed along the normal $\mathbf{v}_0$ (or $\mathbf{H}_0$), while the side lobes are symmetrical with respect to the principal lobe. The angular aperture filled with several lobes amounts to not more than 15 or 20° to the normal to $\mathbf{H}_0$. With further increase in the angle ϑ_2, the intensity of the field of the scattered wave decreases monotonically. Thus, in some point near the earth's surface, the following picture will be observed as the body approaches. First the scattering field increases monotonically, after which bursts of intensity occur, corresponding to the positive side maxima (+ 2 max, + 1 max), to the principal maximum (0), and to the negative maxima (− 1 max, − 2 max, ...), after which the field again decreases monotonically. Inasmuch as the effective scattering cross section is sufficiently large only in the principal and one of the two side maxima, practically the field of the scattered wave is sufficiently intense at the point of observation only for several instants. Let us estimate, for the sake of being definite, the corresponding effect when the body passes at an

Table VIII. Characteristics of intensity bursts produced at the point of observation by a wave scattered by the "trail" of a satellite in a plasma during the day ($\vartheta_1 = 0$, $\mathbf{v}_0 \parallel \mathbf{H}_0$).

z, km	λ, m	Bursts data					
		R, m	Number of lobe	$\frac{\sigma}{\sigma_0}$	δt, sec	δr, km	Δt, sec
300	30	1	(0)	175	0.6	5	—
			(± 1 max)	35	—	—	3.4
		2	(0)	36	1	8	—
			(± 1 max)	7	0.8	—	3,4
		3	(0)	14	1.2	9	—
			(± 1 max)	2.8	1.0	—	3.4
400	30	1	(0)	293	0,4	3.5	—
			(± 1 max)	29	—	—	4.5
		2	(0)	60	0.8	6.5	—
			(± 1 max)	6	0,4	3.5	4.5
700	30	1	(0)	29	—	—	—
		2	(0)	6	0.15	1.2	—
300	20	1	(0)	6.8	—	—	—
		2	(0)	1.4	0.25	2	—
400	20	1	(0)	12.7	—	—	—
		2	(0)	2,6	0.25	2	—

altitude $z \sim 400$ km. In this case $\sigma = 350\ \mathrm{m}^2$ in the principal maximum at a body radius $\rho_0 = 2$ meters and a wavelength $\lambda = 30$ meters, while the width of the principal lobe, defined as corresponding to a decrease in σ to a value σ_0, is $\delta\vartheta_2 \sim 0.6°$. Therefore during the time that the principal lobe covers the distance $\delta r \sim z\delta\vartheta_2 \sim 4$ km past the point of observation, that is, during a time $\delta t \sim \delta r/v_0 \sim 0.5$, the average intensity of the scattered wave at the point of observation is determined from the value $\sigma \sim 170\ \mathrm{m}^2$, since $\sigma(\vartheta_2)$ varies approximately linearly in this interval of ϑ_2. The first two side maxima ($\pm$ 1 max) are still sufficiently intense, since σ is commensurate with σ_0. The higher-order lobes are already difficult to observe against the general background of scattering by the body itself.

We see that the effect of scattering at the point of observation manifests itself in the form of bursts. The duration and the relative intensity of the effect as a whole will change for different body dimensions, character, and surface form, depending on the sensitivity of the indicators.

If the motion of the body is not along the magnetic field, but at not too large an angle to it, then the surface is formed by a scattered-wave field of more complicated form. The surface elements produced by each of the lobes become curved. However, the general character of the field structure near the earth's surface remains the same. The quantitative changes, however, can be appreciable. Thus, if the transmission and reception points—the rays OS and OE (see Fig. 10)—lie in the same plane as $\mathbf{v}_0$ and $\mathbf{H}_0$ ($\varphi = 0$ or π), a larger number of lower-intensity identical lobes is observed. On the other hand, if the rays OS and OE lie in a plane perpendicular to the ($\mathbf{v}_0\mathbf{H}_0$) plane ($\varphi = \pi/2$ or $3\pi/2$), then the field will be the same as for $\mathbf{v}_0 \parallel \mathbf{H}_0$, and the principal maximum will remain of the same magnitude. Consequently, it the transmission of the main beam and the reception of the scattered field occur in a plane normal to ($\mathbf{v}_0\mathbf{H}_0$), then the effect of scattering at the point of observation is not smaller than the effect in the case of longitudinal motion of the body.

In conclusion, Table VIII lists, for different heights of the ionosphere and for different wavelengths, data on the scattering bursts occurring on the earth's surface during the flight of the body, when the intensity of the field exceeds or is commensurate with the scattered field from a metallic sphere of suitable size. In the table δr and $\delta t = \delta r/v_0$ are the width of the illuminated area and the duration of the effect at the point of observation, while Δr and $\Delta t = \Delta r/v_0$ are the distance between the centers of the individual areas illuminated by the scattered field and the time intervals between the successive bursts at the point of observation.

It follows from table VIII, that one passage of the body produces at the point of observation scattered-wave bursts that repeat at intervals of several seconds, the duration of each burst being of the order of and less than one second. Naturally, such a situation occurs only when the body is beamed by a plane wave in one direction. If, however, the path of the body is beamed from several points (S_1, S_2, S_3, ...) at different angles, then several scattered waves will be observed (Fig. 20) at different angles in one point (E) near the earth's surface, and the "lifetime" of the scattering effect will increase appreciably.

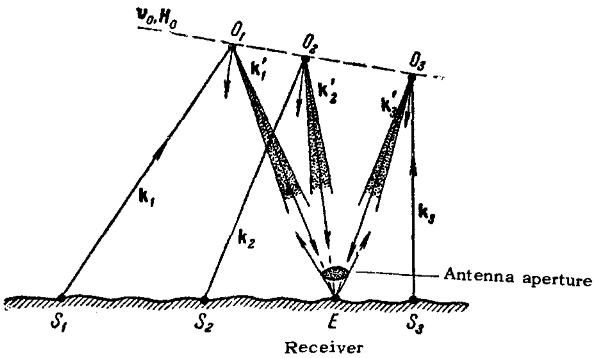

FIG. 20. Schematic diagram of the scattering field for a body irradiated from different points.

11. Effective Scattering Cross Section in the Absence of an External Magnetic Field

Let us consider, for the sake of completeness, the results of the calculation of the effective cross section when there is no external constant magnetic field in the plasma ($\mathbf{H}_0 = 0$). The corresponding formula is obtained in this case by taking the limit as $\mathbf{H}_0 \rightarrow 0$ in (3.31). We can then put $\nu = 0$ in (3.31), for in the absence of a magnetic field the account of the collisions influences little the effective scattering cross section. As a result $\sigma(\theta)$ has the form

$$\sigma(\theta) = \left\{\frac{1}{16}\left(\frac{\omega_0}{c}\right)^4 \frac{R_0^4 \sin^2 \psi_1}{q^2}\right\} F(b_1, \theta)\Phi(qR_0, \cos\theta), \quad (3.36')$$

where

$$\left.\begin{aligned} F(b_1, \theta) &= \frac{\frac{\pi}{4} b_1^2 \exp(-2b_1^2 \cos^2\theta) + [b_1 W(b_1 \cos\theta)]^2}{\frac{\pi}{4} b_1^2 \cos\theta \exp(-2b_1^2\cos^2\theta) + [1 - b_1\cos\theta W(b_1\cos\theta)]^2}, \\ W(S) &= e^{-S^2}\int_0^S e^{t^2}\,dt, \end{aligned}\right\} \quad (3.37')$$

$\cos\theta = \mathbf{q}\cdot\mathbf{v}_0/(qv_0)$, and, in addition to the notation used above,

$$b_1^2 = \frac{b^2}{2(\delta+2\gamma)} = \frac{Mv_0^2}{2kT}. \quad (3.38')$$

For the ionosphere heights used above, namely 300, 400 and 700 km, we obtain $b_1 = 9.8$, 8.6, and 6.2, respectively.

The dependence of the function $F(b_1, \theta)$ on the angle θ between the velocity $\overline{\mathbf{v}}_0$ and the vector $\mathbf{q}$, directed along the bisector of the angle between the wave vectors of the incident and scattered waves $\mathbf{k}$ and $\mathbf{k}'$, is listed for these values of b_1 in Table IX and in Fig. 21. The value of $F(b_1, \theta)$ corresponding to some angle θ characterizes here the intensity of the scattered wave in the direction $\mathbf{k}'$ for a specified direction of the incident wave $\mathbf{k}$. Thus, the field of the waves scattered by the moving body represents a smeared out "specular reflection" of the incident wave from the direction of the velocity $\mathbf{v}_0$, so that the three-dimensional scattering diagram is a double-hump surface of revolution.

The axis of revolution of this surface is the velocity $\mathbf{v}_0$. The function $F(b_1, \theta)$ has a minimum for strictly specular reflection, that is, for the vector $\mathbf{k}'$ ($\theta = \pi/2$) such that $\mathbf{q} \perp \mathbf{v}_0$. The half-width of the cross section of this surface is $\sim \Delta\theta/2$, if it is measured from the direction $\theta = \pi/2$ and determined from the value of θ for which $F(b_1, \theta) \approx 10^{-1} F(b_1, \theta)_{max}$, varies at different altitudes in the following fashion:

z, km	300	400	700
$\frac{\Delta\theta^\circ}{2}$	18	20	30

We see that the width of the lobe increases with increasing height. Variation of the height z is accompanied by variations of $F(b_1, \theta)_{max}$, $F(b_1, \pi/2)$, and $\Delta\theta_{max}/2$, that is, the angle between the direction of the maximum $F(b_1, \theta)$ and the direction of $F(b_1, \pi/2)$ vary as shown in Table X.

For the indicated values of $F(b_1, \theta)_{max}$ and $F(b_1, \pi/2)$ we obtain from (3.36) the values of the effective cross section in the direction of maximum scattering, showing that only for bodies of small dimension does the effective scattering cross section σ exceed the corresponding value σ_0 for the scattering by the sphere itself. For example, when $\lambda = 30$ m and $z = 300$ km, for a sphere of radius $R_0 = 0.5$ m, we have $\sigma_{max} \sim 5 \times 10^{-3}$ m^2 and $\sigma_{max}/\sigma_0 \sim 7$, that is, an inhomogeneous formation scatters more than a sphere, and even for $R_0 = 1$ m we get $\sigma_{max} \sim 2 \times 10^{-2}$ m^2 and $\sigma_{max}/\sigma_0 \sim 0.5$, so that the sphere scatters more than the "trail" of the body. We note in conclusion that with decreasing dielectric constant ϵ of the plasma we get, as follows from (3.36), $\sigma(\theta) \approx 1/\epsilon$. Therefore the scattering should increase noticeably if the body is situated in a region where ϵ

Table IX. Dependence of the function $F(b_1, \theta)$ on the angle θ at different heights of the ionosphere.

θ, deg	z=300 km	z=400 km	z=700 km	θ, deg	z=300 km	z=400 km	z=700 km
0	1,07	1,02	1,06	78	50,88	61,20	54,60
10	1,05	1,08	1,09	80	95,70	98,25	50,18
20	1,16	1,18	1,20	81	120,1	108,35	46,09
30	1,37	1,39	1,43	82	135,6	108,74	43,00
40	1,77	1,80	1,86	83	133,4	100,89	39,48
50	2,55	2,70	2,79	84	121,0	90,26	37,06
60	4,37	4,80	5,17	85	106,8	80,86	34,83
65	6,37	6,59	8,66	86	93,4	73,09	33,00
70	10,56	11,26	17,70	87	62,5	67,39	31,75
72	12,83	15,05	25,84	88	79,8	63,77	30,87
74	19,08	21,50	37,80	89	76,6	61,50	30,35
76	29,30	34,86	50,04	90	75,4	60,79	30,18

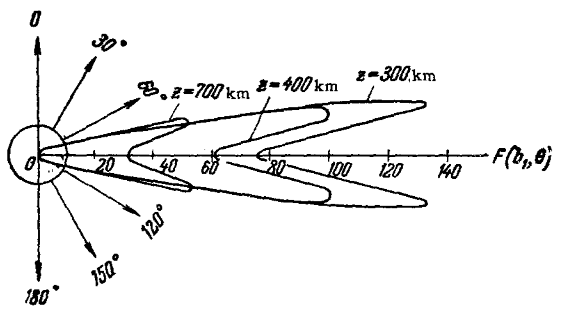

FIG. 21. Dependence of the scattering function $F(b_1, \theta)$ on the angle θ for different values of b_1 ($z = 300$, 400, 700 km) when $H_0 = 0$.

Table X.

z, km	300	400	700
$F(b_1, \theta)_{max}$	135.6	108.8	54.6
$F(b_1, \frac{\pi}{2})$	75.4	60.1	30.2
$\left(\frac{\Delta\theta_{max}}{2}\right)^2$	8	8	12

is close to zero. This case, however, calls for special consideration and it is hardly possibly to draw any quantitative conclusions about the behavior of σ in the region $\epsilon \to 0$ without further analysis.

12. Disturbances Brought About by a Point-like Body

Let us consider the disturbance produced by a body with dimensions much smaller than the Debye radius. Such a problem was considered by Kraus and Watson [10]. We have mentioned that their results are incorrect at large distances from the body, because they have carried out the calculation only in first approximation, that is, they left out the term $J(u)$ and retained the last term in (2.60).

We carry out the calculations with Fourier components. We first obtain, for small $q \to 0$, an expression for the Fourier components of the ion or electron concentration (at distances large compared with the Debye radius these quantities coincide).

Let us calculate $J(u)$ with the aid of (3.12). If the charge of the body is sufficiently small, the main contribution to the integral with respect to $d\rho$ is made by values of ρ much smaller than the Debye radius. At such distances we have a purely Coulomb field. If the charge is small, then the ion scattering angle ion ϑ, that is, the angle between $\mathbf{v}$ and $\mathbf{v}_1$, is also small and given by the formula

$$\vartheta = \frac{2Qe}{M_i v^2}\frac{1}{\varrho} \approx \frac{2Qe}{M_i v_0^2 \varrho}, \tag{3.39'}$$

where Q is the charge of the body.

Let us expand the right half of (3.12) in powers of ϑ, retaining terms $\sim \vartheta^2$. Such an expansion is possible if the terms containing ϑ in the exponent of (3.12) are small, which leads to the condition

$$\sqrt{\frac{M_i v_0^2}{kT}}\,\vartheta \sim \frac{Qe}{\sqrt{M_i v_0 kT}}\frac{1}{\varrho} \ll 1. \tag{3.40'}$$

This condition should be satisfied at any rate for $\rho \lesssim 1/D$, which imposes the following condition on the charge:

$$\frac{Qe}{\sqrt{M_i v_0^2 kT}}\frac{1}{\mathrm{v}} \ll 1. \tag{3.41'}$$

Expanding and integrating with respect to $d\varphi$, we obtain

$$J = 2\pi f_0 v_0 \left\{\frac{M_i v_0^2}{2kT} + \frac{M_i}{4(kT)^2}\left[v_0^2 v^2 - (\mathbf{v}_0\mathbf{v})^2\right]\right\} \int \vartheta^2(\varrho)\,\varrho\, d\varrho. \tag{3.42}$$

The integral in the right half of (3.42) diverges logarithmically. At large ρ the integral must be cut off at $\rho \sim D$, and at small ones it must be cut off at those values of ρ, for which condition (3.40) ceases to be satisfied, that is, when

$$\varrho \sim \varrho_1 = \frac{Qe}{\sqrt{M_i v_0^2 kT}}; \tag{3.43}$$

ultimately

$$J(\mathbf{u}) = -\frac{4\pi Q^2 e^2}{M_i kT} f_0 \ln\frac{R_D}{\varrho_1}\frac{1}{v_0^3}\left\{v_0^2 - \frac{M_i}{2kT}\left[v_0^2 u^2 - (\mathbf{v}_0\mathbf{u})^2\right]\right\}. \tag{3.43'}$$

We have assumed here a pure Coulomb field up to distances $\rho \sim \rho_1$, for which the dimension of the body should satisfy the condition

$$R_0 \lesssim \varrho_1 = \frac{Qe}{\sqrt{M_i v_0^2 kT}} . \tag{3.44}$$

Substituting (3.43) in (2.61) and (3.23) we obtain after transformations, without a magnetic field and in a field, respectively,

$$N_q = -\frac{1}{q} \frac{2\pi Q^2 e^2 n_0}{v_0^3 (2k^3T^3M_i)^{1/2}} \ln \frac{R_D}{\varrho_1} [v_0^2 - (\mathbf{v}_0\mathbf{u})^2] \times \left[(1-2a^2)\left(\sqrt{\pi} + 2i \int_0^a e^{x^2} dx\right) e^{-a^2} + 2ia \right] \times \left[2 - 2a \left(\int_0^a x^2 dx - i \frac{\sqrt{\pi}}{2} \right) e^{-a^2} \right]^{-1}, \tag{3.45}$$

$$N_q = \frac{2\pi Q^2 e^2 n_0}{M_i kT v_0^3} \ln \frac{R_D}{\varrho_1} \left[\frac{kT}{M_i \Omega^2} \int_0^\infty v_0^2 \left(q_z^2 x^2 + q_\perp^2 4 \sin^2 \frac{x}{2} \right) - \left(q_z v_{0z} x + q_\perp v_{1\perp} 2 \sin \frac{x^2}{2} \right) \exp\{\ldots\} dx \right] \times \left[2 + i \frac{\mathbf{q}\mathbf{v}_0}{\Omega} \int_0^\infty \exp\left\{ i \frac{\mathbf{q}\mathbf{v}_0}{\Omega} - \frac{kT}{2M_i\Omega^2} \left(q_z^2 x^2 + 4q_\perp^2 \sin^2 \frac{x}{2} \right) \right\} dx \right]^{-1} \tag{3.46}$$

Here $\mathbf{v}_{1\perp}$ is the vector obtained by rotating $\mathbf{v}_{0\perp}$ through an angle $-x/2$ (the expression in the curly brackets in the numerator of (3.46) is the same as in the denominator). Expressions (3.45) and (3.46) are proportional to the square of the charge of the body. They can therefore not be obtained in first perturbation-theory approximation in Q (which was used, for example, by Kraus and Watson). These expressions are contained, of course, in the second perturbation-theory approximation. Formulas (3.45) and (3.46) hold true only for the very smallest values of q. At large q it is necessary to add to them the terms given by the first perturbation-theory approximation, that is, the expression

$$\frac{Q/e}{2(Dq)^2 + 2 - 2a \left(\int_0^a e^{x^2} dx - \frac{i\sqrt{\pi}}{2} \right) e^{-a^2}} \tag{3.47}$$

in the absence of a magnetic field and

$$\frac{Q/e}{2(Dq)^2 + 2 + i \frac{\mathbf{q}\mathbf{v}_0}{\Omega} \int_0^\infty \exp\left\{ i \frac{\mathbf{q}\mathbf{v}_0}{\Omega} x - \frac{kT}{2M\Omega^2} \left(q_z^2 x^2 + 4q_\perp^2 \sin^2 \frac{x}{2} \right) \right\} dx} \tag{3.48}$$

in a magnetic field. If q is not small compared with 1/D, then the expressions for the electron and ion concentrations begin to differ somewhat. We have written out here the expressions corresponding to the electron concentration.

To obtain $\delta N(\mathbf{r})$ in coordinate space it is necessary to take the inverse Fourier transform. We shall not do so here. We note merely, that in the region where (3.45) is valid $\delta N(\mathbf{r})$ decreases like $1/r^2$, as it should. In the region of applicability of (3.47), as shown in [10], $\delta N \sim 1/r^3$ (for $r \gg D$). Expressions (3.46) and (3.48) lead to a very complicated dependence of δN on the coordinates.

IV. PARTICLE FLUX IN THE VICINITY OF THE BODY

13. General Remarks

The disturbances produced in a plasma by a moving body cause particle fluxes to differ from those in the unperturbed plasma. The determination of the particle flux through an arbitrarily oriented elementary area in the vicinity of the body is essential for the interpretation of the results of various sounding measurements.

The calculation of the neutral-particle flux $\overline{nv}$ entails no principal difficulties and is made difficult in many specific cases only by the complexity involved in calculating the corresponding integrals. Inasmuch as the trajectories of motion of the neutral particles remain straight lines, since there is no potential field to influence their motion, there is no need at all for solving the kinetic equation to calculate the particle flux, and it is enough to start from the geometrical picture of the motion of the particles, with account of the occultation of the body and the reflection of the particles from its surface. Such a method of calculation leads to the same result as the solution of the kinetic equation, and is more convenient.

To calculate the ion and electron flux it is necessary to solve the corresponding kinetic equation. This is particularly essential for the near region surrounding the body. The influence of the electric field is large here and the trajectories of the motion of the charged particles are strongly bent, while in many cases they can become in general finite. The scale of this zone is determined by the double layer produced around the body, that is, by distances from its surface on the order of the Debye radius D. On the other hand, the influence of the electric field outside the double layer, particularly in front of a rapidly moving body, is not so important, since, as we have seen above, the energy of the ion and the electric field $e\varphi(r)$ does not exceed in general the average thermal energy of the particles kT. Therefore the trajectories of motion of the ions do not change appreciably here. Ahead of the body the flux $\overline{N_i v}$ of the charged particles, in the case when $R_0 \gg D$, can apparently be determined with sufficiently high accuracy from the formulas derived for neutral particles. This statement, of course, calls for a more rigorous proof. Naturally, if for some reasons the body is strongly charged, so that $e\varphi \gtrsim M_i v_0^2$, it is necessary to carry out the calculations with the attraction potential of the body itself already taken into account. The repulsion potential always influences strongly the flux of the particles, independently of the ratio R_0/D. Thus, the problem of the calculation of the flux $\overline{N_i v}$ calls for special analysis, particularly under conditions when the dimensions of the body are small or commensurate with the Debye radius. In this case the ordinary approaches to the calculation of the particle flux, based on the formulas for the neutral particles or on theoretical calculations of the flux for the charged particles, where the corresponding problem has not yet been studied in sufficient detail (for example, using Langmuir's formulas [17 21]) may lead, as is known from the literature (see [28-33,23-25,39]) to incorrect results. Such a case is realized apparently, on going over to interplanetary media, where $D \gtrsim R_0$ and the velocity of the satellites and space rockets can be of the same order as or smaller than the thermal velocity of the particles.

In the next section we give the results of calculations of the flux of neutral particles for several cases of practical interest. Taking the foregoing into consideration, the formulas obtained can be used, by exercising certain caution, for the calculation of the flux $\overline{N_i v}$. These formulas, as well as those for the fluxes of the charged particles analyzed by Kagan and Perel' [28-31], show that the formulas employed in the literature for the analysis of the result of sounding measurements are suitable only in a limited number of cases. We shall expound on this in greater detail in a separate article.

14. Flux of Neutral Particles in the Vicinity of a Rapidly Moving Body

Naturally, the calculation of the flux of the particles must be carried out for each particular case, that is, for a specified location of the probe in the vicinity of the moving body. Depending on the position of the probe relative to the body and depending on the reflecting properties of the surface of the body itself, the magnitude of the particle flux will change. An idea of the general properties of the flux can be obtained by solving particular problems. We give below the results of the calculation of the flux on a spherical surface of a probe of radius r, located near the surface of a large specularly-reflecting sphere of radius $R_0 > r$. We shall assume for simplicity that the axis joining the centers of the sphere and the probe is parallel to the motion of the body. We calculate separately the fluxes per unit surface $\overline{nv}$ for cases when the spherical surface is located in front of and behind the sphere.

a) Flux on a probe in front of a sphere. It is seen from Fig. 22 that it is possible to consider two regions on the surface of the sphere r. In one region (arc ABC on Fig. 22) only particles which experienced no collision with the sphere, or the "direct" particles, are incident; in the second region (APC) there enter both direct particles and particles reflected from the sphere.

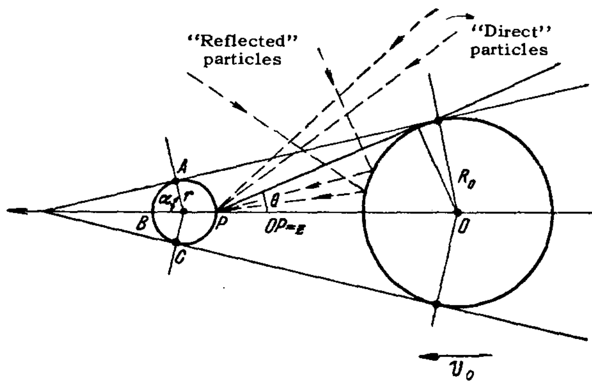

FIG. 22. Illustrating the determination of the particle flux.

The particle flux density in the first region ABC is calculated in elementary fashion. The particle flux density in the second region was calculated by us separately for the "direct" and "reflected" particles only at the symmetrically located point P of intersection of the rear surface of the probe with the OP axis. The calculation of the flux density on the entire surface of the probe in the region APC leads to complicated integrals, which call for numerical calculation.

The particle flux density in an arbitrary point of the first region ABC is obviously equal to

$$\overline{(n\mathbf{v})} = n_0 \left(\frac{M}{2\pi kT}\right)^{\frac{3}{2}} \int_0^{\infty} dv_z v_z \int_{-\infty}^{+\infty} dv_x \times \int_{-\infty}^{\infty} dv_y e^{-\frac{M}{2kT}[v_x^2+(v_y+v_{0y})^2+(v_z+v_{0z})^2]}, \tag{4.1}$$

where $f_0(\mathbf{v})$ is the Maxwellian distribution function (3), and $v_n = \sqrt{2kT/M}$ is the thermal velocity of the particles.

It follows from (4.1) that

$$\overline{(n\mathbf{v})} = \frac{n_0}{2}\left\{\frac{v_n}{\sqrt{\pi}} e^{-\left(\frac{v_0}{v_n}\right)^2 \cos^2\vartheta_0} + v_0 \cos\vartheta_0 \left[1 + \Phi\left(\frac{v_0 \cos\vartheta_0}{v_n}\right)\right]\right\}. \tag{4.2}$$

When $v_0 \cos\vartheta_0/v_n \gg 1$ this leads to the obvious formula

$$\overline{(n\mathbf{v})} \simeq n_0 v_0 \cos\vartheta_0. \tag{4.3}$$

In formula (4.2) $\Phi(x)$ is, as usual, the probability integral.

Point P is struck by the "direct" and "reflected" particles. Assuming that the mean free path of the particles is $\Lambda \gg R_0$, we neglect, as usual, the collisions between the particles and the disturbed region in the vicinity of the sphere. Then the particles striking P, whose velocities make with the OP axis an angle ϑ greater than θ, travel past the sphere—the "direct" particles— and the particles whose velocities make an angle $\vartheta < \theta$ with the OP axis strike P after being reflected from the surface of the sphere—the "reflected" particles (see Fig. 22). Accordingly, the particle distribution function f(v) has the following values

$$f(\mathbf{v}) = \begin{cases} f_0(\mathbf{v}+\mathbf{v}_0) = n_0\left(\frac{M}{2\pi kT}\right)^{\frac{3}{2}} \exp\left[-\frac{M(\mathbf{v}+\mathbf{v}_0)^2}{2kT}\right] & 0 < \vartheta < \frac{\pi}{2}, \\ f_2(\mathbf{v}) & \vartheta < \theta. \end{cases} \quad (4.4)$$

Then we have at the point P for the flux density of the "direct" particles

$$\overline{(\mathbf{nv})}_1 = \int_0^{2\pi} d\varphi \int_\theta^{\frac{\pi}{2}} \cos\vartheta \sin d\vartheta \int_0^\infty v^3\, dv f_0(\mathbf{v}+\mathbf{v}_0), \quad (4.5)$$

and for the reflected particles

$$\overline{(\mathbf{nv})}_2 = \int_0^{2\pi} d\varphi \int \cos\vartheta\, d\vartheta \int_0^\infty v f_2(\mathbf{v}+\mathbf{v}_0)\, dv, \quad (4.6)$$

where f_2 and the limits of the integral with respect to ϑ call for a special determination. The function f_2 depends on the character of the reflection of the particles from the surface of the sphere. If the collisions of the particles with the sphere are elastic, that is, the moduli of their velocities before and after reflection from the surface of the sphere are equal, then

$$f_2(\mathbf{vr})\, d^3v\, d^3r = f_0(\mathbf{v}+\mathbf{v}_0)\, d^3v\, d^3r_1, \quad (4.7)$$

inasmuch as the number of particles does not change upon reflection. Here d^3r is the volume which is occupied by the particles after reflection by those particles which occupied a volume d^3r_1 before reflection. It follows from (4.7) that

$$f_2(\mathbf{vr})\, d^3v = f_0(\mathbf{v}+\mathbf{v}_0)\, d^3v_1 \frac{d^3r_1}{d^3r}. \quad (4.8)$$

Calculations analogous to those carried out in Sec. 4b, show that

$$\frac{d^3r_1}{d^3r} = \frac{\sin^2\theta\cdot\cos^2\theta'\cdot\sin^2\theta'}{\sin^2\vartheta - \sin\vartheta\cdot\sin\theta\cdot\sin^3\theta'}, \quad (4.9)$$

where

$$\sin\vartheta = -2\sin\theta'\cdot\cos(\vartheta+\theta') - \sin\theta\cdot\sin\theta'. \quad (4.10)$$

Inasmuch as the sphere has a finite radius r, part of those particles, which for a point-like sphere would reach the surface of the sphere and would be reflected from it, now no longer reach the surface of the sphere, since they are obscured by the sphere. In order to take into account this occultation, it is necessary to consider the trajectories of those particles, which in the case of a point-like probe were reflected from the sphere and struck the point P.

As an end result we determined the limits of the integral (4.6) with respect to ϑ, and the expression for the flux density of the reflected particles at the point P becomes

$$\overline{(\mathbf{nv})}_2 = \int_0^{2\pi} d\varphi \int_0^{\pi-\alpha} \cos\vartheta \sin\vartheta \int_0^\infty v^3 \frac{d^3r_1}{d^3r} f_0(\mathbf{v}+\mathbf{v}_0)\, dv, \quad (4.11)$$

where

$$\sin\alpha = \frac{r+R_0\sin\theta'}{z+r}. \quad (4.12)$$

From (4.5) we obtain for the direct particles behind the sphere

$$\overline{(\mathbf{nv})}_1 = \frac{n_0}{2\sqrt{\pi}} v_n \cos^2\theta \exp\left[-\left(\frac{v_0}{v_n}\right)^2\right] - n_0 v_0 \cos^3\theta \exp\left[-\left(\frac{v_0}{v_n}\right)^2 \sin^2\theta\right] \frac{1}{2}\left[1-\Phi\left(\frac{v_0\cos\theta}{v_n}\right)\right] \quad (4.13)$$

or for $v_0(\cos\theta)/v_n \gg 1$

$$\overline{(\mathbf{nv})}_1 \approx \frac{n_0}{4\sqrt{\pi}} v_n \left(\frac{v_n}{v_0}\right)^2 \exp\left[-\left(\frac{v_0}{v_n}\right)^2\right]. \quad (4.14)$$

The integral (4.11), which determines the flux of the reflected particles can in general be evaluated only numerically. However, for cases of practical interest, when

$$\left.\begin{array}{l} \alpha \approx \frac{1}{2}\frac{r}{z}\frac{2-\sin\theta}{1-\sin\theta} \gg \frac{v_n}{v_0} \ll 1 \\ \text{or} \\ \alpha \ll \frac{v_n}{v_0} < \cos\theta, \end{array}\right\} \quad (4.15)$$

the corresponding calculations of (4.11) by the saddle-point method yield

$$\overline{(\mathbf{nv})}_2 = n_0 v_0 \frac{\sin^2\theta}{2-\sin\theta} e^{-\left(\frac{v_0}{v_n}\right)^2 \alpha^2}. \quad (4.16)$$

From formulas (4.13), (4.14), and (4.16) we see that the flux of the direct particles on the rear surface of the probe is much smaller than the flux of the reflected particles. The flux of the reflected particles is commensurate here and in many cases with the flux of the direct particles on the forward surface of the sphere and depends essentially on the occultation of the body by the probe.

b) Flux on a probe behind a sphere. The calculation of the particle flux on a spherical probe located behind a sphere leads to the following results: for the direct particles behind the probe, that is, in a region analogous to the region in front of the sphere (see Fig. 22)

$$\overline{(\mathbf{nv})}_1 = -\frac{n_0 v_0 \cos\theta}{2}\left[1-\Phi\left(\frac{v_0\cos\theta}{v_n}\right)\right] + \frac{n_0 v_n}{2\sqrt{\pi}} e^{-\frac{v_0^2\cos^2\theta}{v_n^2}}, \quad (4.17)$$

and for the direct particles in front of the sphere at the point P

$$\overline{(\mathbf{nv})}_1 = \frac{n_0}{2\sqrt{\pi}} v_n \cos^2\theta \exp\left[-\left(\frac{v_0}{v_n}\right)^2\right] + n_0 v_0 \cos^3\theta \frac{1}{2}\left[1+\Phi\left(\frac{v_0}{v_n}\cos\theta\right)\right] \exp\left[-\left(\frac{v_0}{v_n}\right)^2 \sin^2\theta\right] \quad (4.18)$$

In both cases when $v_0(\cos\theta)/v_n \gg 1$ the particle flux is small, since it is proportional to the factor $\exp[-(v_0/v_n)^2]$.

For the reflected particles at the point P we obtain an integral analogous to (4.11). In the limiting case when $v_0(\cos\theta)/v_n \gg 1$ and $v_0(\sin\theta)/v_n \gg 1$, that is, when the angle at which the sphere is seen from the point P is not small, we have

$$\overline{(nv)}_2 = \frac{\sqrt{\pi}\, n_0}{4\sqrt{2}}\, v_n \left(\frac{v_n}{v_0}\right) \cos\theta \exp\left[-\left(\frac{v_0}{v_n}\sin\theta\right)^2\right]. \quad (4.19)$$

Comparison of formulas (4.17)–(4.19) shows that the flux of reflected particles on a spherical probe placed behind a sphere is less than the flux of direct particles, and that the main contribution to the flux density is given in this case by the flux of the direct particles at the point P.

V. CONCLUSION

In the present article we considered the results of theoretical investigations of the interaction of moving bodies with a rarefied plasma. Principal attention was paid to the case when the velocity of motion of the body is much larger than the thermal velocity of the neutral particles and ions, while the dimensions of the body are sufficiently large compared with the Debye radius. Such conditions are realized in the motion of artificial satellites of space ships in the ionosphere or in the interplanetary medium closest to the earth. Although this case has been on the whole quite thoroughly investigated, many problems still call for further analysis. It is necessary to take into account primarily the influence of the electric field on the motion of the ions in the near zone behind the body. Another very important problem is that of the magnetic perturbations. In the problem involving the scattering of radiowaves by the "trail" of the body, it is of special interest to know how strongly the effective cross section increases in the resonant region, when $\epsilon \to 0$. Many other uninvestigated problems, occurring in the analysis of phenomena in the vicinity of a moving body, have been mentioned in the Introduction.

In the lower layers of the ionosphere it is necessary to take already into account the fact that the dimension of the body becomes comparable with the particle mean free path. Under these conditions an interesting problem is that of heating and additional ionization of the plasma, destruction of the surface of the body, and radiation of waves. At very large distances from the earth's surface, the dimensions of the body may become comparable with the Debye radius, and the velocity of the body in a definite region can be smaller than the thermal velocity of the particles. The character of different perturbations brought about by the body under such conditions also calls for a special analysis.

Thus, the interaction between a moving body and a plasma leads to unique and exceedingly varied effects. The perturbations caused by the body are very appreciable, so that the physical state of the region surrounding the body differs strongly from the state of the unperturbed medium.

The results obtained show that the phenomena in the vicinity of satellites or space ships in the ionosphere or in interplanetary medium must be taken into account when processing the results of experimental investigations whose purpose is to obtain data on the state of the unperturbed medium. This is particularly important in the analysis of the results of measurements with the aid of various types of probes. Failure to take these effects into account may lead to appreciable errors.

All-inclusive experimental and theoretical investigations of the structure of the perturbed region in the vicinity of moving bodies in a plasma is of great interest. These investigations make it possible, in particular, to develop the most effective methods for the investigation of the properties of media through which satellites and space ships travel.

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