THE MICROTRON

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INTRODUCTION

THE microtron is a cyclic resonance accelerator of electrons with a guiding magnetic field that is constant in time. The electrons are accelerated by a high-frequency electric field produced in a cavity resonator, in which one uses a special type of resonance acceleration—"resonance with variable harmonic number."

The idea of the microtron was proposed by V. I. Veksler[1] in 1944.* After that, during the course of several years, only a few papers appeared in which various aspects of the operation of such an accelerator were discussed.[2,3] The first description of an operating microtron, which was constructed by a group of Canadian physicists, was published in 1948.[4,5] Some features of the microtron, and in particular the possibility of obtaining very short bursts of electrons with good energy homogeneity of the particles, drew the attention of many laboratories to this accelerator; as a result, at present in various countries of the world one can count a total of fifteen operating microtrons. For the most part these instruments are intended for acceleration of electrons up to 2.5 or 5 Mev; in only one of the microtrons are the electrons accelerated up to 29 Mev.

Until recently the principal defect of most of the operating microtrons was the low electron current at the output of the accelerator—the time average of the electron current with an energy of ~ 5 Mev did not exceed one microamp. A second principal defect of the microtron was that the pole diameter was several times greater than the pole diameter of a betatron or synchrotron with the same final energy of the electrons.

Recently there have appeared some interesting papers in which a description is given of experimental investigations of the operation of a microtron.[6,7] Of especial interest are the new ideas described in the paper of S. P. Kapitza and his coworkers.[7] The results obtained by them show that the microtron, after introduction of a few relatively simple improvements, can give electron beams of extremely high intensity and can be much more compact than previous microtrons.

In the present summary we consider the basic experimental and theoretical data concerning the operation of the microtron, present the engineering parameters of all known microtrons, describe the position of the microtron among the various electron accelerators, and consider the various applications of microtrons.

*In various papers the statements are made that the idea of the microtron was independently proposed by Schwinger and also by Alvarez. However, these proposals were not published.

1. CONDITIONS FOR RESONANCE ACCELERATION OF ELECTRONS. DIFFERENT MODES OF OPERATION OF THE MICROTRON

All the microtrons so far constructed are of the same type: the magnetic field is produced in a gap between cylindrical poles of an electromagnet, and the accelerating resonator is placed near the edge of the gap. The ideal trajectory of an electron has the form of a plane spiral, all the turns of which are formed by circles which are tangent at the same point (Fig. 1). These turns of the trajectory are called orbits; after the first passage through the resonator, the electron moves along the first "orbit," after the second, it moves along the "second," etc.

The accelerating voltage which acts in the resonator is given by the formula

$$V(t) = V_a \cos \omega_a t = V_a \cos 2\pi \frac{t}{T_a}. \quad (1)$$

Let us find the conditions for resonance acceleration of an electron in the microtron. This question will be treated here more consistently (without

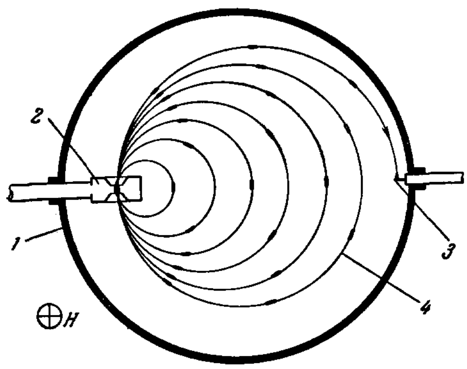


FIG. 1. Schematic of construction of a microtron. 1 – vacuum chamber; 2 – resonator; 3 – target; 4 – electron trajectory. We show the location of the electron bunches which are simultaneously present in the chamber. The force lines of the guiding magnetic field are perpendicular to the plane of the drawing.

unjustified assumptions) than has been done in the various papers concerning the microtron.

By resonance acceleration we mean, as usual, a process in which the electron, at each passage through the accelerating gap, has exactly the same phase.* However, let us make the assumption that this requirement is not necessary for the first crossing of the gap. Thus we require that

$$\varphi^{(2)} = \varphi^{(3)} = \ldots = \varphi_s = \text{const.} \tag{2}$$

The phase φ_s is called the equilibrium or resonant phase, and an electron satisfying condition (2) is called a resonant electron.

The change in energy of the electron in passing across the resonator is given by the well-known formula

$$\Delta W = eV_a \frac{\sin \theta/2}{\theta/2} \cos \varphi, \tag{3}$$

where θ is the so-called phase angle

$$\theta = \omega_a \tau, \tag{4}$$

τ is the transit time across the accelerating gap, φ is the phase angle. We note that formula (3) is valid to very high accuracy under the following conditions: 1) the electron moves in the homogeneous rf field parallel to the field lines and outside the region of the accelerating gap the electric field intensity is equal to zero; 2) the relative change in electron velocity which is associated with the passage of the electron through the accelerating gap is very small, i.e., $(v_2 - v_1)/v_1 \ll 1$. The second condition in particular is always satisfied for electrons, for which $v_1 \simeq c$.

In operating microtrons the value of ΔW is not less than ~ 250 kev (while in most cases, $\Delta W \simeq 500$ kev). Under these conditions the time τ does not change by more than 26% for all passages of the electron through the resonator beginning with the second, since the electron velocity is 0.74 c already at 250 kev. The function $(\sin \theta/2)/(\theta/2)$ changes very slowly in the range $0 < \theta < 40°$ so that with these small changes in the value of τ (and correspondingly of θ) the value of the factor $(\sin \theta/2)/(\theta/2)$ in Eq. (3) practically does not change. We may therefore assume that the quantity τ does not depend on the "turn number" ν, if $\nu > 1$, and is given by the formula

$$\tau = \frac{d}{c} = \text{const}, \tag{5}$$

where d is the length of the accelerating gap. Consequently for $\nu > 1$

$$\frac{\theta}{2} = \frac{1}{2}\omega_a \frac{d}{c} = \frac{\pi d}{\lambda} = \pi l = \text{const}, \tag{6}$$

where

$$l = \frac{d}{\lambda}; \tag{7}$$

λ is the wavelength of the accelerating voltage.

The change in energy of the resonant electron for $\nu > 1$ is independent of ν and is given by the formula

$$(\Delta W)_\nu = eV_a \frac{\sin l\pi}{l\pi} \cos \varphi_s = eV_s = \text{const}, \tag{8}$$

where

$$V_s = V_a \frac{\sin l\pi}{l\pi} \cos \varphi_s. \tag{9}$$

The quantity V_s is called the equilibrium or resonant accelerating voltage. A graph of the function $(\sin l\pi/l\pi)$ is shown in Fig. 2.

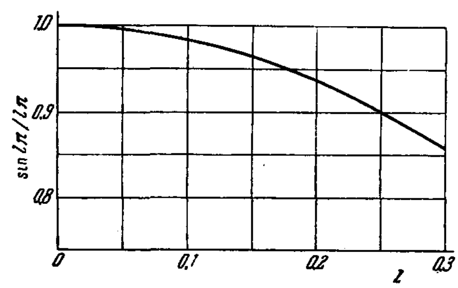


FIG. 2. Transit time factor as a function of dimensionless length of accelerating gap.

The period of revolution of the electron with total energy E in a magnetic field with field intensity H is

$$T = \frac{2\pi E}{ecH} \quad (E = W + E_0, \quad E_0 = m_0c^2). \tag{10}$$

Since the energy of the resonant electron after passage across the resonator increases each time by exactly the same value $\Delta W = eV_s$ (if $\nu > 1$), the period of revolution of the electron also increases each time by the same amount

$$\Delta T = T_{\nu+1} - T_\nu = \frac{2\pi V_s}{cH} \quad (\nu = 2, 3, \ldots). \tag{11}$$

*The phase φ of the electron is that value of the phase of the rf field which occurs at the moment of passage of the electron through the middle of the accelerating gap. The numerical value of the phase is always kept within the limits $(-\pi, +\pi)$ by using the relation $\varphi = \omega_a t - n\pi$ where n is an interger.

This fact is precisely the basic idea of the microtron. In fact, it follows from (11) that, although the period of rotation of the accelerating electron increases from turn to turn, while the period of the accelerating field does not change, nevertheless one can maintain resonant acceleration of the electron with constant phase in accordance with requirement (2). To do this we must satisfy the condition

$$\Delta T = bT_a, \tag{12}$$

where b is a constant integer not equal to zero. In this case we can produce conditions in which the period T_ν of rotation of the electron on any orbit with $\nu > 1$ will be a multiple of the period T_a of the accelerating rf field, as a result of which condition (2) will be satisfied.

From conditions (11) and (12) we obtain

$$\frac{2\pi V_s}{cH} = bT_a. \tag{13}$$

Condition (13) is the fundamental condition for resonant acceleration in the microtron and must be fulfilled in all the various possible modes of operation of this accelerator. However, it is only a necessary and not a sufficient condition. As a second condition, we choose the following: let

$$T_2 = mT_a, \tag{14}$$

where m is an integer (as we shall show later, m cannot be less than 2).*

If conditions (13) and (14) are simultaneously satisfied, the period of rotation of the electron in any orbit beginning with the second will be given by the expression

$$T_\nu = [m + (\nu - 2)\, b]\, T_a. \tag{15}$$

Consequently, the harmonic number ("multiplicity of the resonance"), i.e., the quantity $g_\nu = T_\nu / T_a$, changes from turn to turn:

$$g_\nu = m + (\nu - 2)\, b. \tag{16}$$

This is illustrated in Fig. 3 in which we show the special case m = 3, b = 1.

We denote the kinetic energy W_1 of the electron after the first passage through the resonator by $c_1 E_0$:

*Usually the second condition is written in the following form: $T_1 = aT_a$ where a is an integer. However, if by the period of rotation of the electron we mean the time interval from the moment of passing the center of the accelerating gap to the next crossing, then strictly speaking the usual formula $T_1 = 2\pi E_1/ecH$ is not exact, since over the interval AB from the middle of the gap to the exit from the rf field, the electron velocity changes significantly. Therefore, the mean velocity of the electron over the segment AB differs from the velocity of the electron in its circular orbit outside the region of the resonator. Of course, the true value of T_1 under actual conditions in a microtron differs very little from the value $2\pi E_1/ecH$ and consequently from aT_a. For the second orbit this sort of error already can be disregarded, since the electron velocity becomes practically constant ($v \simeq c$).

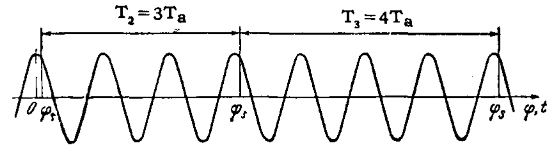


FIG. 3. Change in multiplicity of resonance in passage of the resonant electron from the second orbit to the third.

$$W_1 = W_{inj} + \Delta W_1 \equiv c_1 E_0, \tag{17}$$

where W_{inj} is the kinetic energy of the electrons injected into the resonator.

We also use the notation

$$\Delta W = eV_s \equiv c_2 E_0. \tag{18}$$

Then condition (14), when we take into account Eq. (10), is rewritten as follows:

$$T_2 = \frac{2\pi E_0}{ecH}(1 + c_1 + c_2) = mT_a. \tag{19}$$

Using the expression for T_a from (13) we obtain on the basis of (18) and (19)

$$\frac{m}{b} - 1 = \frac{1 + c_1}{c_2}. \tag{20}$$

This relation, like (13), is a condition for resonance acceleration in the microtron expressed in its most general form. Since the minimum value of b is 1, while c_1 and c_2 are positive numbers, it follows from (20) that $m_{min} = 2$.

Condition (2) can be satisfied in many different ways, i.e., very many different modes of operation of the microtron are possible;[8,9] for example, one can choose m = 2, b = 1; then $c_1 = c_2 - 1$, and consequently only values $c_2 > 1$ will be possible (i.e., $V_s > 511$ kv), and the value of W_1 expressed in kev must be 511 less than V_s in kv. If we choose m = 3, b = 1, then relation (20) takes the form: $1 + c_1 = 2c_2$. Consequently, for example, the following modes are possible: $c_1 = c_2 = 1$; $c_1 = 1.4$, $c_2 = 1.2$; $c_1 = 0.8$, $c_2 = 0.9$, etc.

In practice, in all operating microtrons b = 1, since in this case, as we see from (13), we get the largest value of H in the microtron for given values of V_s and T_a. This in turn means that it is possible to obtain maximum energy of the electron for a given pole diameter of the electromagnet.

Therefore in the following presentation we shall consider only the case of b = 1.

As one sees from the examples given above, the mode of operation in which b = 1 and m = 2 can be established only under special conditions (concerning which we shall speak in more detail later on); such a regime has not yet been applied in practice. There has been wide application of the operating conditions for which b = 1 and m = 3. Therefore this is usually regarded as the fundamental mode of operation of the microtron.

Condition (13) with b = 1 can be rewritten as the following working formula:

$$H\lambda = \frac{2\pi E_0}{e} c_2 = 10.697 c_2 \text{ koe-cm.} \qquad (21)$$

The second resonance condition for b = 1 has the form

$$m - 1 = \frac{1 + c_1}{c_2}. \qquad (22)$$

We denote by H_c that value of the magnetic field intensity ("cyclotron field") for which the period of rotation of the slow electron would be equal to the given value of T_a. Then

$$H_c = \frac{2\pi E_0}{ecT_a} = \frac{2\pi E_0}{e\lambda} = \frac{10.697}{\lambda} \text{ koe-cm.} \qquad (23)$$

Thus the coefficient c_2 is expressed in terms of H_c:

$$c_2 = \frac{H}{H_c}. \qquad (24)$$

From (10), (13), (8), and (15) we can obtain

$$E_\nu = \frac{T_\nu}{T_a} \Delta W = (m + \nu - 2) \Delta W = E_0 (m + \nu - 2) c_2 \; (\nu = 2, 3, \ldots) \qquad (25)$$

In reference 10 the possibility of using "soft acceleration modes" in a microtron is considered. It is shown that the "resonance part of the energy" eV_s, defined by condition (13), can be given to the electron not by a single passage through the accelerating gap, but by several successive passages forming a cycle. The advantage of such a mode of acceleration is the possibility of increasing the operating value of H by a factor of 1.4 to 1.7. However, the fraction of electrons captured into acceleration is decreased markedly as compared with the usual operating conditions of the microtron and consequently the beam current drops. In addition, if one operates the microtron in the "soft" mode," one would require a very high stability of the values of H, V_a, and λ.

Let us dwell briefly on the choice of parameters in designing a microtron. In constructing a microtron one will try to make the resonant magnetic field intensity as large as possible, since the greater H, the smaller will be the pole diameter of the electromagnet and consequently the more compact will be the whole accelerator. As we see from (21), to increase the required value of H one must choose a maximum possible value of the ratio c_2/λ, i.e., one must choose the biggest possible of V_a and the smallest value for λ. At the present time the lower limit for the wavelength of the field in a resonator is determined by the fact that as one decreases λ, the dimensions of the resonator decrease, and consequently one decreases the length of the accelerating gap and increases the amplitude E_a of the electric field intensity in the resonator for a given value of V_a ($E_a = V_a/d$). The maximum permissible value of E_a, for which there will still be no breakdown in the resonator, depends to a large extent on the state of the emitting surfaces at the edges of the orifices in the resonator. We may assume that $E_{a,max} \simeq 1$ Mv/cm. Then, for example, for $V_a = 560$ kv and $l = d/\lambda = 0.1$, the minimum wavelength is $\lambda_{min} \simeq 5.6$ cm.

We note that with decrease in wavelength we obtain the following additional advantage: reduction of the diameter of the resonator permits us correspondingly to reduce the height of the gap between the poles of the microtron electromagnet and thus to increase the diameter of the region inside of which H (r) = const to the required degree of accuracy.

It is understood that in choosing the magnitude of λ one takes account of the available data concerning the dependence on λ of the power and stability of the rf oscillator used to produce an electric field of the required intensity inside the resonator.

In most operating microtrons one uses an accelerating field with λ ~ 10 cm. If we take $c_2 = 1$ (i.e., $V_a \sim 560$ kv) and λ = 10 cm, then, according to (21), H ≃ 1.07 koe. This example shows that in microtrons one uses extremely low values of the field intensity of the guiding magnetic field, much lower than the values of H which can easily be obtained in the gap between the poles of an electromagnet with iron pole pieces, even for low magnetic quality of the iron. For comparison we point out that, for example, in cyclotrons one usually uses a magnetic field with H ~ 15—20 koe and more, while in betatrons $H_{orb} \sim 4$—9 koe.

Because of the small value of H, the pole of the microtron magnet has a diameter which is much greater than, for example, that of a betatron intended for acceleration of electrons to the same energy. With this smallness of H there is, however, associated a certain advantage: the construction of the electromagnet is very simple, and the weight of the magnet is relatively small.

Many authors have made all sorts of proposals intended to increase the operating value of H and thus to make the microtron more compact. These proposals will be discussed later.

A microtron with a given value of H can operate at various frequencies of the accelerating field. All the possible variants of mode of operation in this case, as one sees from (21), will be subjected to the condition c_2/λ = const. This means that when we decrease the wavelength, we must correspondingly decrease V_a, so that we require a much smaller power for the generator feeding the resonator. Under certain conditions this will be the decisive fact in choosing the mode of operation of the microtron. One should, however, keep in mind that it is desirable to use the maximum possible value of c_2 and consequently of V_a, since in this case a given final energy of the electrons will be reached after a smaller number of passages through the resonator and the loss of electrons during the acceleration process will be a minimum.

In addition to choosing the values of λ and c_2, for the construction of the microtron we must also choose the parameter m. Knowing m we can calculate c_1 according to (22). Further problems are the achievement of such a mode of injection of electrons that one obtains the required value of c_1. The methods for achieving these conditions depend on the type of injection.

Finally, we choose the value of the equilibrium phase φ_s (usually $\varphi_s \simeq 13-20°$, cf. below) and, knowing the length d of the accelerating gap, we calculate V_a according to formula (9). Experience shows that the value of V_a obtained from (9) should be increase by 5—7% in order to take account of the fringing of the rf field beyond the limits of the geometrical length of the accelerating gap and the effects of curvature of the electron path in the resonator.

2. INJECTION OF ELECTRONS INTO THE MICROTRON

Up to the present time five methods of injection have been tried out in practice. Although the oldest variant is still the most widely used, it is unquestionably outmoded for most problems by the more recent, more complete methods of injection.

Let us consider all these methods individually.

a) Injection using field emission from the metal of the resonator. This type of injection we shall refer to briefly as field emission injection.

The structure of the resonator usually employed in the microtron is shown schematically in Fig. 4. The maximum field intensity of the electric field during the operation of the resonator occurs at the surface of the ring regions denoted in Fig. 4 by the letters A and B. These ring metallic surfaces are also the sources of an intense flux of electrons, part of which can be used for further acceleration in the microtron.* Since injection into the microtron should be from one side, one must take measures to increase the intensity of the field emission from one of the ring regions (from ring A in Fig. 4) and to significantly reduce the emission from the second region. If we permit two-sided field emission, the result will be an increase in loading of the resonator with electrons that are not usable later on and consequently an increase in the required power supply of the resonator.

The one-side emission is achieved by appropriate treatment of the metal surface and also by selection of suitable metals. The emission is markedly increased, for example, by oxidation or roughening of the surface and is reduced significantly by polishing, by careful cleaning, or by gilding the surface. In some microtron resonators an aluminum liner, placed in one of the cones of the resonator, is used to increase the field emission.

It is well known that the electron current I from field emission depends very strongly on the field intensity at the cathode surface. If we use the Fowler-Nordheim formula and assume that the electric field intensity in the resonator changes as $E(t) = E_a \cos \omega_a t$, we can obtain the following relation:

$$I = I_0 \cos^2 \omega_a t \cdot \exp\left(-\frac{B}{\cos \omega_a t}\right), \tag{26}$$

where I_0 and B are constants, proportional to the value of E_a and depending on the structure of the emitting surfaces.

A graph of $I(\omega_a t)$ according to formula (26) is shown in Fig. 5. As we see from the graph, with field emission injection we have a pulsed source of electrons where the emission current is different from zero practically only for phases of emission (φ_{em}) lying within the range approximately from $-52°$ to $+52°$.

For this injection method, $W_{inj} = 0$, so that according to (17)

$$c_1 E_0 = \Delta W_1. \tag{27}$$

From (22) we get

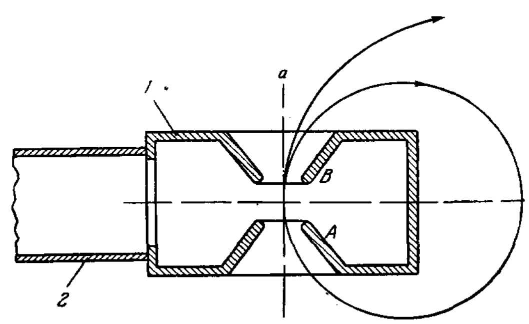


FIG. 4. Construction of a toroidal resonator. 1 – resonator (figure of rotation about axis a); 2 – waveguide.

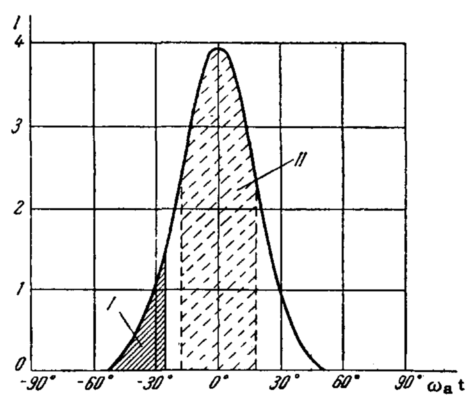


FIG. 5. Field emission current (in relative units) as a function of phase of the rf field.

*It should be mentioned that the assumption that the emission of electrons from the lining of the resonator occurs through field emission has not been demonstrated as yet by direct experiments. The opinion has been expressed that the observed electron emission probably is the result of a combination of several processes, field emission, secondary emission, and photoemission.[5]

$$\Delta W_1 = (m-1)V_s - 511 \text{ kev, kv.} \quad (28)$$

With field injection, c_1 cannot be greater than c_2, and usually $c_1 \simeq c_2$. If $c_1 = c_2$, the resonance conditions (21) and (22) simplify and take a form which has been very widely used in the literature:

$$eV_s = \frac{E_0}{m-2}, \quad (29)$$

$$H\lambda = \frac{10.697}{m-2}. \quad (30)$$

In accordance with these formulas m cannot be less than 3, and V_s cannot be greater than 511 kv, and can take on only the following values: 511; 255.5; 170.3 kv, etc.

If actually the equality $c_1 = c_2$ is satisfied only approximately, then formulas (29) and (30) will be approximate formulas. In addition, in the case of $c_1 \neq c_2$ the operating condition with m = 2 is not excluded.

For each specific microtron, if we know its parameters V_a, φ_3, and d, we can from (9) find V_3 and from (28) calculate the required value of ΔW_1. On the other hand, the change in energy of the electron in an rf field with given values of V_a and $l = d/\lambda$ depends on the phase of emission of this electron and can be calculated according to a well-known method.[11,5] In Fig. 6 we show a graph of $\Delta W_1(\varphi_{em})$ calculated for the following conditions[12]: V_a = 280 kv, l = 0.066. Using this graph, let us consider the following example. Let us assume that in a given microtron m = 4 and c_2 = 0.5 [this value of c_2 corresponds to V_s = 255.5 kv; consequently, according to Eq. (9), $\varphi_s \simeq 24°$]. Then condition (28) requires that ΔW_1 be equal to 255.5 kev. Such a change in energy is given to an electron with an emission phase $\varphi_{em} \simeq -60°$ (cf. Fig. 6). It may turn out that this electron will have a phase $\varphi^{(2)} = \varphi_s$; in this case it will be a resonant electron. However, fulfillment of the condition $\varphi^{(2)} = \varphi_s$ is not necessary since in the microtron we can also accelerate nonresonant electrons if their energy and initial phase $\varphi^{(2)}$ is sufficiently close to the energy and phase of the resonance electron (cf. Sec. 3). Therefore in such a microtron there will be stable acceleration not only of electrons with $\varphi_{em} = -60°$, but also of electrons which are emitted with other values of φ_{em}. The computation carried out for this microtron[12] showed that only electrons whose emission phases lie within the limits from −61 to −26° can pass eight times through the resonator and be accelerated to the required energy of 2.044 Mev.

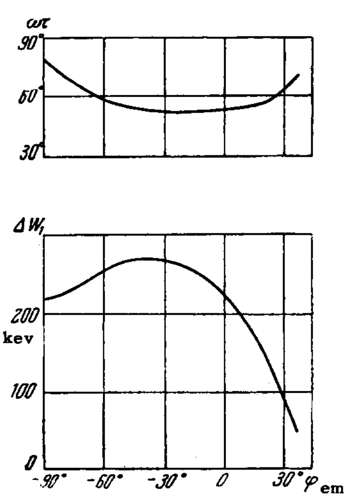


FIG. 6. Energy of electron after first passage through the resonator and transit phase angle as a function of phase of emission of the electron. V_a = 280 kv, l = 0.066, initial electron velocity v_0 = 0.

Using the graph of $I(\varphi_{em})$ (cf. Fig. 5), one can easily determine, for example by using a planimeter, the electron current corresponding to this favorable region of emission phases (a measure of this current is the area of region I), which amounts to about 7% of the total electron current passing through the resonator and entering the first orbit. There emerges from the resonator a diverging beam of electrons; since the homogeneous magnetic field of the microtron does not produce focusing forces in the axial direction, only ~ 10% of all the electrons injected into the first orbit enter the opening in the resonator for the second acceleration. Thus, only ~ 0.7% of those electrons which are in the first orbit enter the second orbit. This result is in agreement with the well-known fact observed with all microtrons using field emission; when one goes from the first orbit to the second, there is a very great loss of electrons (one frequently loses more than 90%). As a result, such microtrons are low-current accelerators, with a beam current at the output of less than one microamp (time average of the current).

An extremely effective method for increasing the "coefficient of utilization" of field emission could be to increase the amplitude of the electric field intensity in the resonator. For example, if one could double the value of E_a, first of all the emission current would be increased according to (26) by approximately a factor of 30, and secondly as a result of change in shape of the curve, shown in Fig. 6, the region of admissible emission phases would be shifted to the right in Fig. 5, which would lead to an increase in the relative size of the shaded area by a factor of 4.4,[12] and the beam current at the output of the microtron would be increased by a factor of 130.

However, the practical problem of increasing the value of E_a is very difficult. In order to increase E_a one must either raise V_a in the same resonator, or use a different resonator with a shorter accelerating gap. The value of V_a is already chosen to be the maximum possible from other considerations (cf. above). Consequently one must decrease the length d. This, however, involves an increase in danger of breakdown of the resonator and spoiling of its parameters; the Q value of the resonator and its shunt resistance are reduced, as a result of which greater power is required in order to obtain the same value of V_a. Usually with λ = 10 cm and V_a ~ 560 kv, the length d is not less than 8 mm.[13]

One can calculate the conditions under which the favorable emission phases will be located symmetrically around the peak of the field emission current (cf. Fig. 5). To do this one must compute the length d of the accelerating gap in such a way that the favorable emission phases are grouped around the phase $\varphi_{em} = 0$. Using the graph of Fig. 5 one can easily calculate that in this case (if we use the same width of the favorable region of emission phases, i.e., 35°) the relative magnitude of the shaded region would be equal not to 7, but to 70% (area of Fig. II).

Computations of the optimum length of the accelerating gap have been published recently.[14,15] Two somewhat different approaches were assumed in this computation. Let us consider each of them briefly.

1. The problem is formulated as follows.[14] An electron is emitted with phase $\varphi_{em} = 0$, when the field emission current is maximum, and in passing across the resonator should attain the maximum possible energy ΔW_1. This means that the phase φ_{out} with which the electron leaves the resonator should be equal to 90°. We require that such an electron, having made one revolution in its orbit, enters the resonator with phase $\varphi^{(2)} = \varphi_s$, and then with phase $\varphi^{(3)} = \varphi_s$, etc. The length of accelerating gap for which this requirement is satisfied will be assumed to be optimal. It turns out that for given values of φ_s, λ, and m this requirement can be achieved only for one definite value of V_a, and the same applies to the value of d_0. In Table I we present the results of the computation from reference 14; it was assumed that $\varphi_s = 18°$, b = 1.

Table I

m	V_a, Mv	λ~10 cm H, oe	d_0 mm	D_1, mm	λ~3 cm H, oe	d_0, mm	D_1, mm
2	2.369	4398	20.7	27.7	14660	6.2	8.3
3	0.427	828	12.8	42.8	2761	3.6	12.8
4	0.238	473	10.1	53.0	1573	3.0	15.9
5	0.165	325	8.6	62.6	1083	2.6	18.8

From these results one can draw the following conclusions. In microtrons operating at λ ~ 3 cm, the length of the accelerating gap considerably exceeds the optimum size d_0 given by the computation. (For example, for m = 4 in operating microtrons d ≥ 4.5 mm, whereas d_0 = 3 mm.) In this way one can explain the small utilization coefficient for field emission. For microtrons operating at λ ~ 10 cm, it is entirely possible to make a resonator with a length of accelerating gap equal to the optimum value, if m > 2. However, the resonance value of the magnetic field intensity is very low and the accelerator will not be compact.

A still more important point is the following. It turns out that with the parameters found in this computation, the quantity E_a is smaller than usual. (For example, for λ ~ 10 cm and m = 3, E_a = 334 kv/cm.) This results in a significant reduction of the electron current from field emission.

Thus one comes to the conclusion that one should not attempt to make the length of the accelerating gap optimal in the sense used above, but rather that it have its smallest possible length; the gain in value of emission current associated with the fact that one has chosen the maximum achievable field intensity for the electric field in the resonator will apparently be much greater than the loss associated with the spoiling of the location of the region of acceptable emission phases with respect to the phase $\varphi_{em} = 0$.

An interesting case is that of m = 2, to which there correspond unusually high values of H (~ 4.4 koe) and $E_a \simeq 1.14$ Mv/cm. We note that in this case, as one can easily compute, $c_1 \simeq 2.7$ and $c_2 \simeq 3.7$. For the usual shape of the electron trajectory this variant cannot be achieved since the length of accelerating gap is too large compared to the diameter D of the first orbit. The author proposes that one can achieve an acceleration at H ~ 4.4 koe by ejecting the electrons from the resonator through an appropriate hole (Fig. 7). However, no computations have been made.

2. In the second variant of the computation of the optimum length of the accelerating gap, the problem is stated as follows.[15] An electron is emitted at a phase $\varphi_{em} = 0$, and emerges at the resonator at the phase φ_{out}, on whose value no conditions are imposed. One is required to choose the quantities V_a and d_0 so that the electron, having made one revolution in its orbit, enters the resonator at phase $\varphi^{(2)} = \varphi_s$; in addition, one poses a second requirement: the period of rotation T_1 of this electron should be precisely equal to an integer number of periods of the accelerating field.* We note that the imposition of this second requirement is hardly useful. It is

FIG. 7. Shape of electron trajectory proposed for increasing the upper limit on the value of the magnetic guide field intensity.

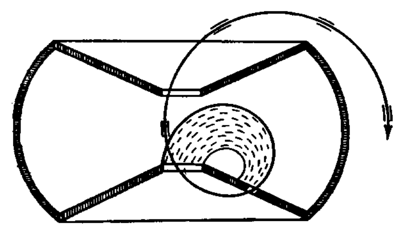


Table II

a	φ_s=18° V_a, kv	l_0	φ_{out}	φ_s=24° V_a, kv	l_0	φ_{out}
2	531	0.042	25.6°	578	0.0563	34.1°
3	271	0.0296	23.3°	283	0.0401	31.2°

*Cf. footnote on page 859.

sufficient that the first requirement be satisfied. In Table II we give the results of the computation. In this table $a = T_1/T_a$, $l_0 = d_0/\lambda$.

As we see from the data given in the table, the accelerating gap should have a shorter length than in actually operating microtrons. For example, when $a = 2$ and $\lambda = 10$ cm, $d_0 = 4.2$ mm. It is possible that one might succeed in achieving the elimination of breakdowns in a resonator with $d = 4.2$ mm and $V_a = 531$ kv; the Q value for it will be less than usual, and the required power will be greater than usual, but this would be balanced by an increase in beam current at the output of the microtron.

In the second variant of the calculation, just as in the first, the operating value of V_a (and the corresponding value of H, if we are given the equilibrium phase φ_s) is rigidly fixed, and a deviation from this required value should lead to a reduction in beam current. Unfortunately it has not yet been verified experimentally to what extent the computed parameters agree with the experimental values.

In the actual case of an operating microtron the resonator has a fixed value of the dimension d, and the choice of mode of operation corresponding to maximum beam current of accelerated electrons is made as follows: one varies V_a and chooses for each value of V_a the optimum value of H; as a result one finds opt V_a and opt H (cf. Sec. 3). With these values one could calculate what values of φ_s, c_1, and c_2 are optimal for the given microtron, and also one could establish the location of the optimum region of suitable phases of emission relative to the phase $\varphi_{em} = 0$.

Reference 16 describes the results of an experimental investigation of some processes occuring with field emission injection. To follow the trajectories of electrons emitted from a definite point on the surface of the conical liner of the resonator, they applied the following simple technique. On the well-polished edge of the resonator aperture there is produced an artificial center of emission in the form of a spot of Aquadag. Data concerning the shape of trajectories of electrons emerging from this point were obtained by using movable slits and a screen covered with a scintillator. Thus it was established that only two extremely small regions of the emitting surface produce those electrons which later on can pass unhindered through the resonator the required number of times. The dimensions of these little regions were the following: $x \simeq 0.3$ mm, $y \simeq 3$ mm (Fig. 8). The area a may be called the outer working region of emission and the area b the inner. The experiments showed clearly that the two beams of electrons emitted by the external and internal emission zones later on form two completely separate systems of orbits.

The practical conclusion from these observations is that one must take measures to reduce field emission from the whole surface of the edges of the aper-

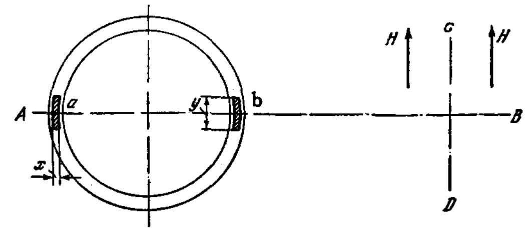


FIG. 8. Schematic of the location of the working regions of emission (a, b) on the edge of the aperture in the resonator. AB is the plane of the orbit, CD is the axis of symmetry of the magnetic field.

ture of the resonator which lie outside the limits of the areas a and b; thus, one will significantly reduce the power which is needed for the resonator. In reference 6 the brass conical liners of the resonator were well polished. Then a thin layer of Aquadag was deposited at the position of the operating zones of emission. During the period of breakdowns in the resonator, occurring as one slowly increased V_a, there occurs a "forming" of this deposit. After cessation of breakdown, such an emission zone gives a stable current of electrons from 100 to 200 microamps per pulse at the third orbit.*

In other microtrons with field emission injection the beam current at the last orbit usually turns out to be several times greater than that obtained in reference 6; it amounts to ~ 1 milliamp per pulse. Still larger currents (up to 7 milliamps per pulse) were obtained for this same type of injection in reference 7.

Thus a microtron with field emission injection, with an electron energy of 5–6 Mev, can give a beam current which is completely suitable for carrying out many physical investigations. Such a microtron is characterized by extremely simple construction.

b) Injection using a hot cathode located inside the resonator. Experiments with this type of injection are also described in reference 6. For the cathode they used a tantalum wire of diameter 0.4 mm, placed in the aperture of the resonator approximately in a vertical plane, close to zone b (cf. Fig. 8). The cathode operates for quite a long time if one does not heat it to too high temperatures. At the third orbit they obtained a stable current of 500 μa per pulse. Such a current is clearly close to the maximum obtainable by this method of injection (if one does not use an oxide cathode), since the area of the operating zone of emission is very small and the duration of the time interval in each cycle of the rf field during which there is capture of electrons into the acceleration regime, amounts to ~ $0.1\,T_a$.

The construction of the resonator is more complicated than for the case of field emission injection. There are some difficulties associated with the fact that the magnetic field of the cathode gives rise to a

*In reference 6 they also attempted to use a local radiator of electrons made in the form of a needle. It turned out that such a source of field emission is not suitable.

considerable vertical shift of the orbits; they were able to overcome these.

Since with injection of this type one eliminates the need to obtain the greatest possible value of E_a in the resonator, one can use a resonator with a longer accelerating gap. This results in an improvement of the Q value of the resonator.

Just as for field emission injection, we have the limitation $c_2 \leq 1$ for $m > 2$.

c) Injection using an electron beam. The following requirements must be imposed on an electron beam which is to be used as an injector into a microtron: it must have extremely narrow dimensions in order not to interfere with the motion of the electrons in the first orbit; the electrons must move in the beam through a quite large potential difference, in order that emerging from the beam they may enter the aperture of the resonator, despite the deflecting action of the magnetic field of the microtron; the electron beam must give a quite intense beam of electrons. The last two requirements contradict the first, and it is not easy to find a satisfactory compromise solution. If one could produce a small electron beam giving electrons with energy ~ 300 kev, the first requirement would be less stringent since the beam could be located at the "most open" position near the resonator (Fig. 9). However, as attempts at construction of injectors for betatrons and synchrotrons show, in miniature beams with pulsed supply one can obtain electrons with energies no greater than 80—100 kev. For such an energy the radius of curvature of the electron trajectory in a magnetic field $H \simeq 1.2$ koe will be about 8—9 mm. Therefore the injector must be placed near the aperture of the resonator. Upon entering the accelerating field of the resonator, the electron begins to pick up energy and the radius of curvature of its trajectory rapidly increases.

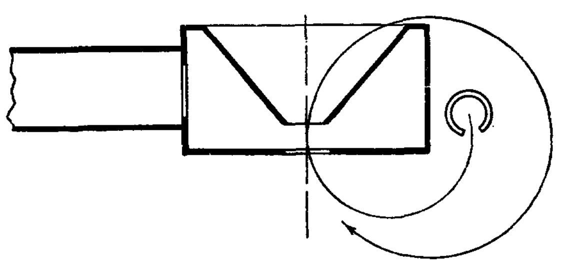


FIG. 9. Arrangement of injector suitable for high energy injection.

This variant of injector is applied in a microtron for 5.9 Mev which, in turn, will serve as the injector for a strong focusing synchrotron at 1.2 Gev which is being built at the University of Lund.[16,17] The arrangement of the electron beam with respect to the resonator is shown in Fig. 10.* At the injector cathode, made of a tungsten spiral, one applies negative pulses of amplitude 80 kv with respect to the grounded anode. The emission current amounts to ~ 500 ma per pulse; this beam enters the resonator. At the tenth orbit the current per pulse is usually equal to 20 ma. They have also used a dispenser cathode which made it possible to raise the current by a factor of 2—2.5; however, the length of service of such a cathode in a vacuum system which has not been aged is only a few hours.

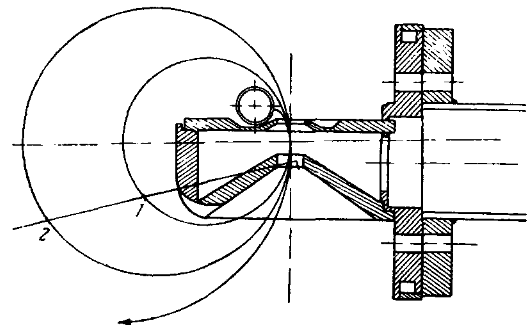


FIG. 10. Arrangement of injector in the Swedish microtron. A Faraday cylinder, with which one measures the beam current at the various orbits, moves along line 1–2.

Pulses applied to the cathode must have a duration of their flat part of ~ 2 microseconds, since the resonator is fed by pulses of length 2.7 microseconds and part of this time is expended in driving the oscillations in the resonator up to the stationary value V_a.

Computation showed,[16] all electrons whose phases lie within a region of width ~ 25° are stably accelerated even when W_{inj} ~ 60—70 kev.

From the parameters of this microtron ($H = 1.23$ koe, $\lambda \simeq 9.95$ cm, $b = 1$, $m = 3$) one can determine that $c_2 = 1.143$, i.e., $\Delta W = 584$ kev, $c_1 = 2c_2 - 1 = 1.286$ and $\Delta W_1 = 577$ kev. A larger value of c_2 could be obtained only with a higher injection energy. However, even for $W_{inj} = 300$ kev the value of c_2 would not exceed 1.576, so that with $\lambda = 9.95$ cm, the resonance value of the magnetic field intensity would be no greater than 1.7 koe.

d) Injection using a second resonator. As a high voltage electron beam, one can use a resonator operating at the same frequency as the main resonator of the microtron.[8] The source of electrons may be either a separate hot cathode (Fig. 11) or a suitably treated surface deposit in the resonator emitting electrons as a result of field emission. Contrary to the case of field emission, in the present case the coefficient of utilization of the field emission can be made extremely high. In fact, if in the supply for the injection resonator we introduce a phase-shifting system, then we can always select the phase shift between the oscillations in the injector and the principal resonators so that the maximum fraction of electrons of field emission will be captured into the acceleration mode. With such an optimum phasing of the resonators, the coefficient of utilization of field

*This drawing and the basic parameters of this microton were very kindly supplied to us by O. Wernholm.

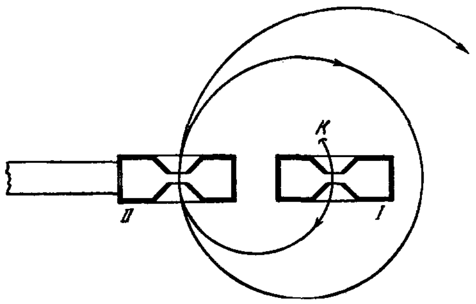


FIG. 11. Use of a separate resonator as injector. K – hot cathode; I – injection resonator; II – main resonator. The wave guide feeding resonator I must be outside the plane of the orbits.

emission will reach 70%, if one uses a phase angle of capture equal to 35° (cf. Sec. 2a).

The electron current which such an injection resonator can supply can be made sufficient for our purposes. According to the data of reference 18 one can obtain up to 0.9 amp per pulse at $\lambda \sim 10$ cm and up to 0.3 amp at $\lambda \sim 3$ cm just behind the resonator.

An attempt at practical application of an injection resonator has been made in only one case so far.[19,6] Both resonators were fed from the same magnetron. In addition to controlling the relative phase shift, one could independently control the amplitudes of the voltages on the resonators. The rated power of the magnetron was insufficient, and the experiments were not completed. Probably in future microtrons of this type one will feed the injection resonator with a portion of the power dissipated in the stabilizing load (cf. Sec. 5b). Instead of this load one should use a ferrite attenuator, which will tap off energy to the injection resonator.[12]

By using an injection resonator one can supply a large injection energy to the electron. However, it turns out that not every value of W_{inj} is achievable for a given value of H, because the injection resonator operates on the same wavelength as the main resonator and consequently has the same dimensions. Therefore the radius of curvature of the electron trajectories emerging from the first resonator and entering the aperture of the second must be approximately twice as large as the radius of the first orbit in an ordinary microtron (cf., for example, Figs. 4 and 11), while the radius of the first orbit in a two-resonator microtron must be approximately four times as great as in the usual case. If in the latter, for $\lambda = 10$ cm, electrons with energy 511 kev will traverse the resonator freely at H = 1.07 koe (since the diameter of the orbits $D_1 = 5.52$ cm and the radius of the toroidal resonator $R_{res} = (0.3-0.35)\lambda \approx 3-3.5$ cm), in a two-resonator accelerator an electron with $W_{inj} = 511$ kev can go from the first resonator to the second only when $H \leq 740$ oe. If we use the value H = 535 oe, then one can have the operating condition $c_2 = 0.5$, $m = 6$. On the first orbit the resonant electron will have an energy of 766.5 kev (with a corresponding magnetic rigidity of G = 3906 oe-cm), and the diameter of this orbit will be equal to 14.6 cm; this is hardly sufficient to miss the injection resonator. The following operating condition is also possible:[18] $W_{inj} = 255.5$ kev, $c_2 = 0.5$, $m = 5$. In both cases one should use a low value of the magnetic field intensity despite the relatively high value of W_{inj}. Only for very large W_{inj} is it possible to use operating conditions with a high value of H. For example, for very compact resonators ($R_{res} = 0.3\lambda$) the following operating condition is possible: $W_{inj} \simeq 1.53$ Mev, $m = 4$, $c_2 = 2$.

An injector in the form of a supplementary resonator has one further advantage over an injector in the form of an electron beam: it gives very short bursts of electrons with a repetition rate equal to the frequency of the field in the resonators; with correct relative phasing of the resonators, one will inject into the main resonator practically only "useful" electrons, whereas when one uses an electron beam operating continuously during its own whole operating interval, many "useless" electrons will enter into the main resonator (that is, electrons not capturable into an acceleration cycle), which leads to an increase in the required resonator power.

Still another variant of construction and arrangement of an injection resonator is possible.[20] The latter can be built in the form of a coaxial resonator of the quarter-wave type (Fig. 12), whose parameters are adjusted so that one obtains optimum bunching of electrons. For the arrangement of the bunching resonator shown in Fig. 12, it is necessary to change the sign of the curvature of the electron trajectories over the region between the two resonators. To do this it is proposed to use an electrostatic deflecting structure.

e) Injection using a hot cathode by the method of S. P. Kapitza, V. P. Bykov, and V. N. Melekhin. In a

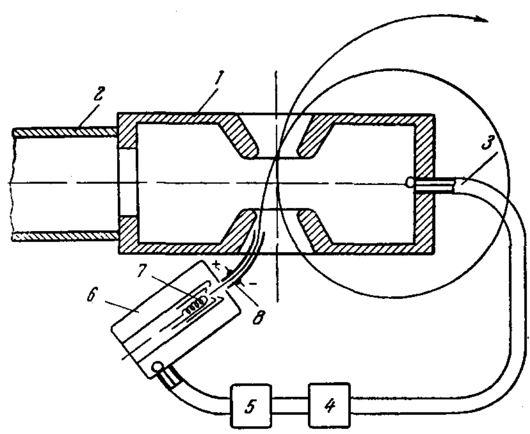


FIG. 12. Injector in the form of a coaxial resonator. 1 – main resonator; 2 – waveguide; 3 – coaxial feed for supplying the injection resonator; 4 – phase shifter; 5 – attenuator; 6 – injection resonator; 7 – electron beam; 8 – electrostatic deflecting structure (inflector).

recently published paper these authors[7] have proposed an interesting new arrangement; according to this proposal the conditions of injection are markedly changed, and it becomes possible to increase considerably the beam current of accelerated electrons. The resonator of traditional shape for a microtron is replaced by a cylindrical resonator in which one excites E_{010} oscillations. On the front wall of the resonator in the plane of the orbit, at a precisely computed distance from the axis of the resonator, one places a hot cathode. In calculating the trajectory of the thermionic electrons; one includes both the constant magnetic field and also the high frequency fields—electric and magnetic—in the interior of the resonator. In Fig. 13 we show one of the computed electron trajectories corresponding to emission phase 0°. For a fixed length of resonator there corresponds to the different locations of the cathode different values of c_2. For $l = 0.163$ the possible values of c_2 lie in the range 1—1.2.

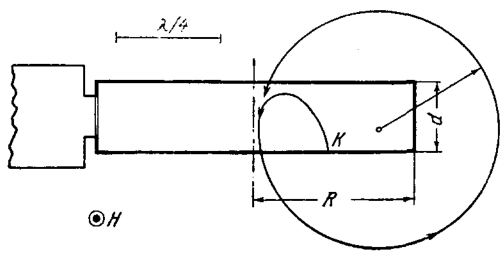


FIG. 13. Electron trajectory using a cylindrical resonator (first variant). K — hot cathode.

The experimental investigation of this type of injection was done with a microtron having a pole diameter 70 cm and a gap height between poles of 11 cm. The cylindrical resonator had a diameter 2R = 7.66 cm (this corresponds to $\lambda = 10$ cm) and a length d = 1.63 cm. The cathode made of lanthanum boride was heated to 1600° C and had an area of radiating surface ~ 1—2 mm². The center of this surface was at a distance of 1.75 cm from the axis of the resonator. Under the operating conditions with $c_2 = 1.1$ ($V_s = 563$ kv) the beam current at the 12-th orbit reached 15 ma per pulse for an electron energy of 6.8 Mev. With this value of the energy one can, on the basis of (25), establish that in this case m = 3. Consequently, according to (22), $c_1 = 1.2$. Thus, for motion inside the resonator the resonance electron should have an energy of 613 kev.

If $c_2 = 1.1$ and $\lambda = 10$ cm, then $H \simeq 1.18$ koe [cf. (21)]. Furthermore, using (15) and assuming that at an energy of ~ 7 Mev the electron velocity is equal to the velocity of light, one can compute that the diameter of the 12-th orbit is equal to $13\lambda/\pi = 414$ mm.

The detailed calculation carried out on a digital computing machine showed that one captures into the acceleration cycle electrons with an emission phase near $\varphi_{em} = 0$, where the current of these electrons amounts to ~ $1/30$ of the total emission current. The experimental data confirmed this result.

In reference 7 there is proposed a still more favorable variant using a hot cathode in a cylindrical resonator. If the hot cathode is located on the back wall of the resonator near to its axis, and an auxiliary hole in the plane of the orbit is made on this same wall, the electron trajectory shown in Fig. 14 is obtained. In this case one can operate with $c_2 = 2$—2.5, i.e., the compactness of the microtron is greatly increased. This injection type was checked in an experiment under the following conditions: the resonator had a diameter of 7.66 cm and length 2.32 cm, the hot cathode was located at a distance of 3.2 mm from the resonator axis. With H = 1.95 koe, i.e., $c_2 = 1.823$ and $V_s \simeq 932$ kv, they obtained a current of 5 ma per pulse at the 12-th orbit for an electron energy of 11.6 Mev. The diameter of the 12-th orbit was 414 mm, as in the first variant of operation of this kind of microtron; this is natural since the value of λ is the same in both cases.

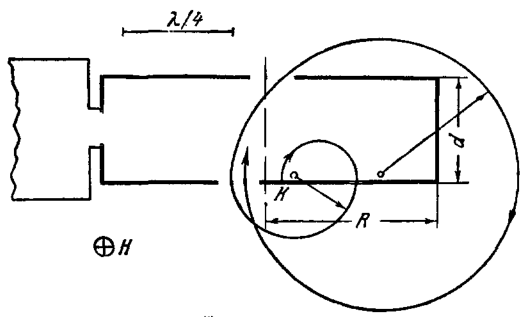


FIG. 14. Electron trajectory using a cylindrical resonator (second variant). K — hot cathode.

In the second mode of operation, $c_1 = 2.646$, i.e., $W_1 = 1.352$ Mev. This energy of the thermionic electron is reached in two steps; over the portion of the trajectory from the hot cathode to the output of the resonator (Fig. 14) and in the first transit through the resonator.

In this injection type one captures approximately $1/20$ of the total emission current into the acceleration cycle.

The resonators described here are characterized by a large length of the accelerating gap: in the first case $l = 0.163$, and in the second $l = 0.232$. However, detailed computations of the trajectories have shown that all the electrons captured into acceleration can proceed unhindered through the resonator for twelve and more times.

With respect to shunt resistance, for an accelerating gap of given length the cylindrical resonator is inferior to the toroidal resonator of special type which is usually used in microtrons. But because of the fact that cylindrical resonators with large values of l have been used, excitation of such a generator requires approximately the same power as in ordinary microtrons.

The computations show that it is worthwhile to replace the cylindrical resonator by a prismatic one. The use of such a resonator enables one to reduce the height of the gap between the poles of the electromagnet; in addition, the distribution of electromagnetic field in this resonator makes it possible to choose such a shape of the trajectory of the thermionic electrons that one increases the parameter c_2 even more, i.e., the microtron becomes even more compact.

Comparing this type of injection with that using an electron beam, one may note that in the second case one achieved somewhat larger beam currents at ~ 6 Mev. The microtron with a hot cathode and cylindrical resonator is characterized by many advantages over all the other types described earlier. It is the most compact, since it permits one to work with extremely large values of c_2. In construction it is simpler than the microtron with two resonators or with an injector in the form of an electron beam, and it permits one a significantly larger freedom in variation of parameters both with respect to design and construction. Another extremely important point is that all the parameters of this microtron are precisely calculable, whereas for other types of injection many of them must be arrived at empirically.

We note that two types of injection—those using a hot cathode and an electron beam—enable one to vary the beam current; this is accomplished by controlling the heater current.

To the five types of injection described above one should add still another, although it has not yet been tested in experiment. In reference 2 it was proposed to place a ring source of electrons (hot cathode or source of field emission) not at the edge of the accelerating gap, but in some plane located approximately in the middle of the gap. This proposal had for its purpose to establish the operating conditions with $m = 2$, $b = 1$, to which, as was pointed out in Sec. 1, there corresponds the condition $c_1 = c_2 - 1$. In this case one can select a large value of c_2 (the author proposed to use a cylindrical resonator and chose $c_2 \simeq 2$); correspondingly, H is doubled compared with the usual value of ~ 1 koe. The difficulty in achieving the operating conditions with $m = 2$, $b = 1$ is that the diameter of the first orbit is too small and the electrons cannot miss the resonator. It was therefore proposed to make cuts in both flat walls of the resonator in the plane of the orbit in order to allow the electrons to pass through. In a later paper[22] it is reported that, as shown by numerical computations of the motion of electrons in the first orbit, such cuts do not achieve the desired purpose. Therefore another method is developed for achieving the conditions with $m = 2$, $b = 1$; the first orbit is made elliptical in form, as a result of which the electrons circle the resonator unhindered. The required change in shape of the orbit is accomplished

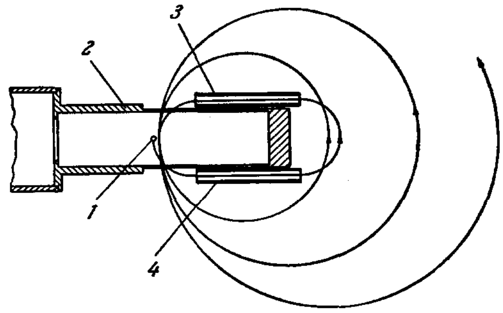


FIG. 15. Schematic of arrangement of magnetic shields which enable one to increase the upper limit of the intensity of the magnetic guide field. 1 – cathode; 2 – cylindrical resonator; 3, 4 – magnetic shields.

by allowing the electrons to pass through two iron pipes, inside of which $H \simeq 0$ (Fig. 15).

The length of the first orbit is increased so much that the period of rotation of the electron satisfies the condition $T_1 \simeq 2T_a$, instead of the condition $T_1 \simeq T_a$. On the second orbit $T_2 = 2T_a$.

Preliminary experiments have been made and have given encouraging results.

3. PHASE FOCUSING IN THE MICROTRON

The phenomenon of phase focusing in cyclic accelerators, the discovery of which greatly broadened the possibilities of accelerator technology, was first described by V. I. Veksler in the same paper in which he published the idea of the microtron.[1] It was shown that in a microtron one can accelerate without limit not only the resonance electrons, but also many other electrons, provided that in phase and energy they differ not too much from resonance. The automatic phase focusing results from the fact that the phase of a resonant electron, as it changes from turn to turn, oscillates around the equilibrium phase φ_s.

The theory of phase oscillations in the microtron was later developed in detail.[2,23,8,24,25]* This theory answers many practically important questions. What is the optimum value of the equilibrium phase, i.e., what is the best value of V_a for a given value of V_s? What fraction of the electrons injected into the resonator in a continuous beam can be captured into acceleration for different values of φ_s? What is the spread in energy of the accelerated electrons? Within what limits can one vary the electron energy for electrons moving in a given orbit of the microtron?

The first theoretical papers showed that the microtron is essentially different from all other accelerators based on the phase focusing principle with respect to the change in phase of the accelerating particles. In these other instruments the phase of the particle changes only in small jumps, i.e., the

*The references are given in chronological order.

phase difference $\Delta\varphi_\nu = \varphi_{\nu+1} - \varphi_\nu$ is always small. This makes it possible, in carrying out the mathematical analysis of the phase equation, to go over from difference equations to differential equations. In the case of the microtron the quantity $\Delta\varphi_\nu$ is not small and the replacement of the differences by differentials can lead to large errors.[26]

The phase equation of a microtron with b = 1 has the form[25]

$$\Delta^2\varphi_\nu - \frac{2\pi\cos\varphi_{\nu+1}}{\cos\varphi_s} = -2\pi. \qquad (31)$$

The values φ_s and $-\varphi_s$ are special stationary solutions of this equation.

In the case of small phase oscillations one can assume that

$$\varphi_\nu = \varphi_s + \eta_\nu, \quad |\eta_\nu| \ll \varphi_s. \qquad (32)$$

Then (31) is transformed into a linear difference equation whose solution has the form

$$\left.\begin{aligned} \eta_\nu &= a\cos(\varepsilon\nu + \delta), \\ \cos\varepsilon &= 1 - \pi\,\mathrm{tg}\,\varphi_s, \end{aligned}\right\} \qquad (33)^*$$

where a and δ are constants. From (33) it follows that the electron will be accelerated stably, i.e., we will have phase focusing, if the value of the tangent of the equilibrium phase does not go beyond the following limits:

$$0 < \mathrm{tg}\,\varphi_s < \mathrm{tg}(\varphi_s)_{\lim} = \frac{2}{\pi}, \qquad (34)$$

from which we obtain for φ_s the limits

$$0 < \varphi_s < 32.5°. \qquad (35)$$

Here again we see an important difference of a microtron with respect to the phase motion as compared to all other accelerators using phase focusing; for these others the upper limit of $|\varphi_s|$ is 90°. The condition (35) means that in the microtron the value of V_a can exceed V_s only by little more than 16% (since cos 32.5° = 0.843).

If the initial conditions, i.e., the phases and energies of the electrons entering the accelerator have a uniform distribution, then on the basis of (33) one can show that the optimum value of φ_s is ~ 17.7°; for such a value of φ_s the fraction of electrons captured into acceleration cycles will be a maximum.

In Fig. 16 we show the limiting curve $\eta_0(\varphi_s)$ obtained in reference 2 by numerical computation. All electrons for which, with a given value of φ_s, the values of $\eta_0 = \varphi_0 - \varphi_s$ correspond to points lying within the limiting curve, will be stably accelerated.

When $\varphi_s \simeq 17.7°$ the period ν_p of the phase oscillations, expressed in terms of the number of passages through the accelerating gap, is equal to 4. As φ_s changes from 0 to 32.5°, the period ν_p changes from ∞ to 2.

*tg = tan.

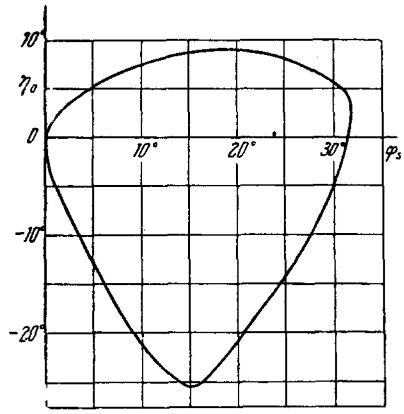


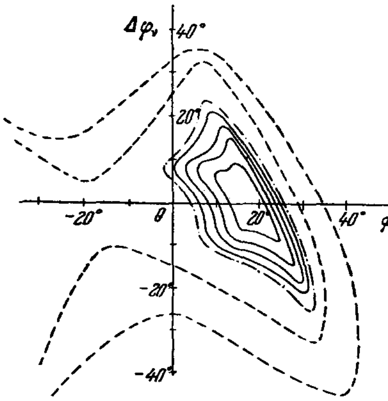
FIG. 16. Limiting curve $\eta_0(\varphi_s)$ for the case of small phase oscillations.

As already mentioned above, the quantity a in (33) is constant, i.e., the small phase oscillations have a constant amplitude. In other accelerators using phase focusing the small phase oscillations are damped.

In treating large phase oscillations one can use equation (31) as a recursion formula and carry out the computations numerically. There have also been developed several ingenious graphical methods,[8,24,27] which enable one, for given values of the phase φ_ν and energy W_ν of the electron, to determine $\varphi_{\nu+1}$ and $W_{\nu+1}$, after which one can find $\varphi_{\nu+2}$ and $W_{\nu+2}$, etc. Especially interesting is the graphical method described in references 28 and 27, since it does not exclude even those cases where one of the series of values of W, for example, W_q, is smaller than the preceding value W_{q-1} (i.e., the case where the electron was slowed down in the resonator).

By using one or another of these computational or graphical methods, one can obtain, in particular, a picture of the motion of the electron's phase in the coordinates φ_ν, $\Delta\varphi_\nu$, the so-called phase orbits (Fig. 17). If the initial conditions are such that the phase point is close to equilibrium (φ_s, 0), then later on the phase point will move around in jumps on the corresponding phase orbit, and its motion will be close to periodic. Open phase orbits mean that the phase point does not move periodically and

FIG. 17. Phase orbits of an electron for φ_s = 17.7°.



that the corresponding electron is not captured into a cycle of unlimited acceleration.

The computation shows that if one requires the electron to pass through the resonator a very large number of times, then for $\varphi_s = 17.7°$ the phase angle of capture is equal to $\sim 29°$. However, one should keep in mind that in microtrons at 5—6 Mev the electron passes through the resonator altogether 10—12 times; therefore some of those electrons will also reach the target for which, from the point of view of the criterion of an unlimited number of accelerations, one does not satisfy the conditions of phase focusing.[27] On the other hand, data concerning the beam current of electrons on different orbits obtained in several operating microtrons[5,13,29] show that in the transition from one orbit to another the loss of electrons is insignificant beginning with the second or third orbit (Fig. 18). This means that the phase angles of capture calculated, for example, for the third and for the tenth orbit will differ very little from one another.

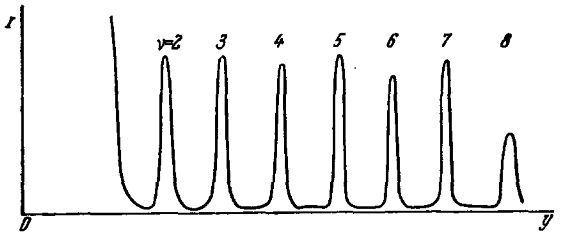


FIG. 18. Change in current at target as it is moved along the common diameter of the orbits (according to the data of reference 15). The point y = 0 corresponds to the center of the accelerating gap.

Since in a microtron electrons which have different input parameters are captured into acceleration cycles, one can find electrons with several different energies on each orbit, including the last. For a precise computation of the width of the energy spectrum of the electrons at the last orbit, one should use the same "step by step" method as one uses in calculating the change in the phase of the electrons. The computations show that the energy spectrum of electrons at the output of the microtron is extremely narrow. For example, with W = 4.09 Mev, H = 1.07 koe, $V_s = 511$ kv, and $\varphi_s = 11.5°$, the total energy spread according to reference 27 is 1.2%, i.e., $W \simeq (4.09 \pm 0.025)$ Mev.*

In such a computation one determines at the same time the duration θ_p of the electron pulses at the output of the microtron. The electron bunches have a short duration, and for $\lambda = 10$ cm ($f \simeq 3 \times 10^9$ cps, $T_a \simeq 3.3 \times 10^{-10}$ sec) the value of θ_p will be approximately 3×10^{-11} sec. Such bunches follow one another with a frequency of 3×10^9 cps over a period of 1.5—2 μsec, after which the resonator is disconnected, for example, for 2 millisec.

In some applications of the microtron, in particular if it is used for generating submillimeter electromagnetic waves,[30,18] it is desirable to obtain the shortest possible electron bunches. It turns out[31] that theoretically under appropriate conditions one can obtain a marked squeezing down of the bunch on one of the orbits; for example, instead of the initial angular spread of the bunch over 8°, one can obtain 30′ at the 8-th orbit.

Successful application of a small microtron for generating oscillations with a wavelength ~ 8 mm at a power level of ~ 0.5 milliwatts is described in reference 32.

The presence of even a relatively narrow, but finite, phase stable region enables one to vary smoothly (within definite limits) the energy of the electrons at each orbit, which significantly increases the value of the microtron as an instrument for physical investigations. This possibility is achieved in the following way. Let us assume that the microtron operates at certain values of H and V_a, to which there corresponds a definite value of φ_s. If, by altering the power which is supplied to the resonator, we change V_a and at the same time vary H by the same amount, then according to (21), (18), and (9) the microtron will operate with a new value of c_2, but with the same value of φ_s. If the value of H is changed in a way which is not proportional to V_a, then we will obtain both a new value of c_2 and another value of φ_s.

According to (22), a change of c_2 means that we must necessarily also change c_1. The method for accomplishing this depends on the type of injection used in the particular microtron. In particular, for field emission injection a change in c_1 is accomplished automatically by shifting the region of permissible phases of emission of the electrons. This in turn affects the beam current (cf. Sec. 2a). We can therefore expect that, with this type of injection, for each given microtron there will be certain optimal values of V_a and H, for which the beam current will be a maximum. This has also been observed experimentally[16] (Fig. 19).

Thus, by changing H we vary the value of c_2 and consequently the energy of the electrons at each orbit, in accordance with (25); the diameter of each orbit, except for the first, remains unchanged, and the beam current will change as shown in Fig. 19. If we demand that the beam current should not go lower than, say, $\frac{1}{4}$ of the maximum current for all possible changes in operating conditions, then under the conditions shown in the figure, i.e., for changes of V_a from 516 to 590 kv, the admissible values of c_2 turn out to lie between the limits 0.95—1.12; consequently, the energy of the electrons at the 8-th orbit can be

*In a microtron which accelerates electrons to higher energies the width of the spectrum is not increased, so that the quantity $\Delta W/W$ will be even smaller.

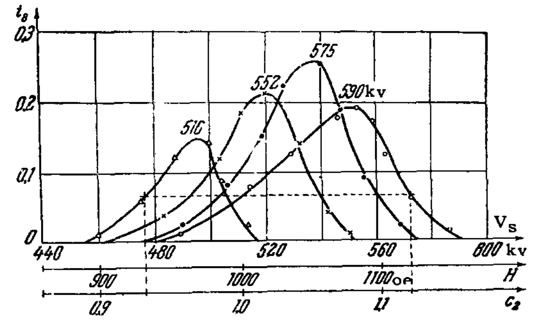


FIG. 19. Beam current as a function of H for different values of V_a, under the operating conditions m = 3, b = 1. The ordinates give the ratio of the current at the 8-th orbit to the current at the first orbit for the same conditions of acceleration. The dashed horizontal line is drawn at a level which is ¼ of the maximum ordinate of the curve for V_a = 575 kv.

varied smoothly within the limits from 3.86 to 4.64 Mev. If we require electrons with lower energy than 3.86 Mev, we should use electrons from the 7-th orbit (W_7 = 3.36—4.06 Mev), etc. Thus we can obtain from a given microtron electrons with an energy varying smoothly within the limits from 1.9 Mev (= $W_{4,min}$) to 4.64 Mev. Electrons with W < 1.9 Mev cannot have any arbitrary energy since W_3 = 1.43—1.78 Mev. Consequently, for example, the energy 1.85 Mev cannot be obtained either at the fourth or at the third orbit.* For the resonator of this microtron $l \simeq 0.094$ and $(\sin l\pi/l\pi) \simeq 0.986$ (cf. Fig. 2). From the data in Fig. 19 and formula (9), one can calculate that the optimum values of φ_s which are attained on the orbit are: approximately 13° for V_a = 516 kv, approximately 19° for V_a = 575 kv, etc.†

Because the region of phase stability in the microtron is quite narrow, there are very rigid requirements on the stability of the values of V_a and H. For example, if the microtron operates with $\varphi_s = 13°$ ($\cos\varphi_s = 0.974$), then a reduction of V_a by 2.6% shifts the equilibrium phase to zero and the beam current drops to zero. The admissible limits of instability of the value of H depend on the required stability of the beam current and can be determined from experimental curves of the same type as Fig. 19.

*As we see from Fig. 19, when V_a is reduced below 516 kv we will reach a value of V_a for which one can no longer obtain a non-zero current for any value of H. However, with further reduction of V_a (and corresponding reduction of H) the beam appears once more – the microtron will operate in the mode with m = 4. In this case $c_1 = 3c_2 - 1$, and the maximum value of c_2 will be 0.5, since with field emission injection we have the restriction $c_1 \leqslant c_2$. In this operating condition we can again obtain a family of curves of the same type as shown in Fig. 19. However, the current at the last orbit (in this case the 7-th) will be significantly lower in absolute value, since the field emission current falls markedly because of the reduction of E_a. One can also work under operating conditions with m = 5, $c_{2,max} = ⅓$, etc.

†In these computations the corrections referred to at the end of section 1 are not included.

4. FOCUSING OF ELECTRONS

In the preceding sections it has been assumed that there is a uniform magnetic field in the microtron. Such a field gives no axial focusing of the accelerated electrons and therefore it would appear that repeated passage across the aperture of the resonator is possible only for those electrons for which there is a sufficiently small axial (z) component of the initial velocity. In actuality, however, there is a small axial focusing in the microtron. It arises as a result of the operation of two factors: 1) the resonator is placed near the edge of the pole, and the center of the accelerating gap is located at a point where $H(r) \neq$ const, where the relative fall-off δH of the magnetic field intensity compared to H at the central region of the pole gap is 1—2%; 2) the rf electric field of the resonator acts like a weak focusing lens on the electrons passing through it. Initially it was assumed that the second effect is practically absent, since the velocity of the electrons is very close to the velocity of light. However, later on it was shown[33] that this conclusion is incorrect. The resonator focusing has the consequence that the electrons carry out small oscillations around the plane of the orbits, where the amplitude of these oscillations slowly increases. A very abbreviated summary of information concerning methods and results of theoretical computations of the combined action of these two focusing factors is given in reference 22. The problem of possible methods for improving the focusing is especially important in the case of a microtron which is intended for acceleration of electrons to high energy, since an electron in such an accelerator must pass through the resonator several tens of times. In a 29-Mev microtron[22] the first focusing effect was increased by a special shimming of the magnetic field in the region of location of the resonator, since the computation showed that without these measures a very small inclination of the axis of the resonator to the plane of the orbit, which is practically unavoidable, would lead to a complete disappearance of the beam at the succeeding orbits.

In reference 2 it was proposed to produce magnetic focusing in a microtron by using an inhomogeneous magnetic field for which H(y) = const, while H(x) is a slowly decreasing function with H(x) = H(−x); the y axis is directed along the common diameter of all the orbits. No detailed computations were published.

In the few experimental investigations of focusing of electrons in a microtron, the following results were obtained. By using photographic plates placed at different azimuths on the first orbit of a microtron for 2 Mev (m = 4), it was established that after one turn only $^1/_{10}$-th of the electrons passed through the resonator which left the resonator on the first orbit.[12] In a 4.5-Mev microtron,[6] using the technique of artificial centers of field emission (cf. Sec. 2a) and using a probe with a fluorescent screen, the parameters of the axial oscillations of electrons were measured separately for electrons emitted by the outer and inner emission zones. The results of these measurements are shown in Fig. 20. During the time of ten revolutions in their trajectory, the electrons make approximately only one full oscillation around the plane z = 0. Note that the amplitude of oscillation is increasing.

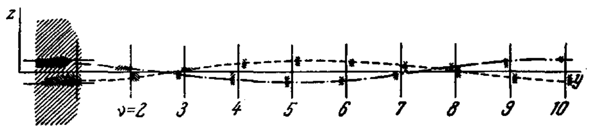


FIG. 20. Results of experimental investigation of axial oscillations of electrons. The dashed areas show the location and size of the luminous spot on the screen as the latter moves along the y axis. The left spot on each orbit is associated with electrons from the outer emission zone, the right spot with those from the inner. The y axis coincides with the common diameter of the orbits, the z axis is perpendicular to the plane of the orbits. The vertical lines show the theoretical positions of the points of intersection of the corresponding orbits with the y axis.

One can get some notion of the focusing of a beam in a microtron by considering the experimental data concerning the distribution of beam current in the orbits. If the losses of electrons in the acceleration process are small beginning with the second orbit (for example, in Fig. 18), this shows not only that the loss which is associated with the moving of electrons out of the phase stable region is small, but it also shows that there is satisfactory focusing. We note that in certain cases[13,34] the losses of electrons were *significantly* greater than those shown in Fig. 18. In the 29-Mev microtron[22] the beam current from the fifth orbit to the 32nd orbit remained at approximately the same level and then steadily fell off to $\frac{1}{3}$ of this value for $\nu = 56$. It is assumed that the main reason for these losses is the small inhomogeneity of the magnetic field. Part of the loss is explained by the fact that the duration of the acceleration of the electron up to the 56-th orbit is not small compared with the duration of the pulse feeding of the resonator (~ 0.5 μsec and 3 μsec respectively); after switching off the resonator, the value of V_a rapidly decreases, and further resonance acceleration of electrons which arrive at this moment, for example, at the 30-th or 40-th orbit, becomes impossible.[35*]

*Such electrons will bombard the resonator, giving rise to hard bremstrahlung. This undesirable phenomenon can be eliminated by placing a structure in the region of the first few orbits for bending the electrons on to one of its lids. In particular, one can use a pair of electrodes placed above and below the plane of the orbits, to which one applies a pulsed voltage shortly before the moment when the resonator is disconnected.[36]

5. BASIC INFORMATION CONCERNING MICROTRON CONSTRUCTION

a) Electromagnet. A special characteristic of microtron electromagnets is the fact that the magnetic field intensity in the gap is small compared with the induction which one can achieve in iron (1—2 koe and 10—15 kilogauss respectively). This means that the cross section of the yoke must be very much smaller than the pole cross section. The double-yoke which is characteristic for cyclotron magnets is used in microtrons only as an exceptional case.[37] The most widely used shape of core of a microtron electromagnet is shown in Fig. 21. To increase the ratio of the diameter of the region of uniform magnetic field to the pole diameter one can use ring shims. The power dissipated in the windings of the electromagnet usually does not exceed 500 watts. Therefore air or water cooling of the windings is rarely used. The constant current feeding the windings of the electromagnet is stabilized by one or another electronic circuit; a current *stabilization to* $\sim 0.1\%$ is sufficient.

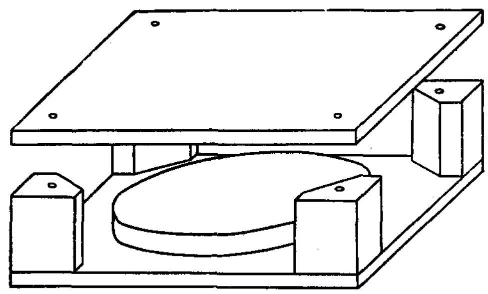


FIG. 21. Construction of magnetic circuit of a microtron electromagnet. The upper cylindrical pole, which is similar to the lower one, is not shown in the figure.

Very stringent requirements on the magnet system arose in designing the microtron for 29 Mev (with 56 orbits). In this case one had to use special measures in order to reduce to a minimum the precession of the orbits around their common diameter; otherwise the electrons would not pass through the resonator the required number of times. To do this they either had to produce a very homogeneous field in the working region of the chamber, or at least get the maximum possible symmetry of the distribution of field with respect to the line of the common diameter of the orbits.[38]

A tank construction was chosen for the magnet (Fig. 22). The mechanical treatment of the pole surfaces was carried out very carefully. The magnet was supplied with a system of correcting windings which were attached to the surfaces of the poles which face one another. Each of these windings had a separate power supply.

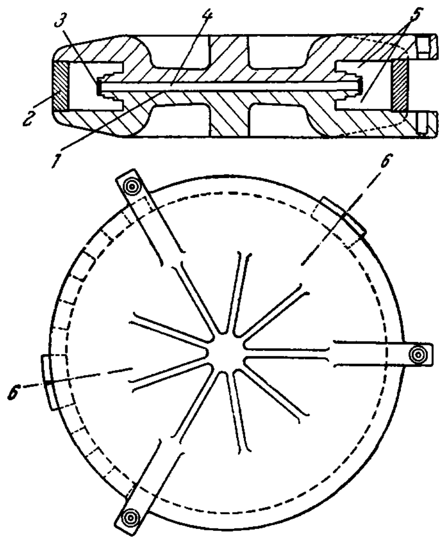


FIG. 22. Construction of magnetic circuit of a microtron for 29 Mev. 1 – pole; 2 – iron posts of height ~40 cm; 3 – aluminum ring forming a vacuum chamber with an internal diameter of 203.2 cm; 4 – pole gap of height 127 mm; 5 – location of electromagnet windings; 6 – axes of tubes for vacuum pumps.

The summary of the main parameters of microtron magnets which have been constructed will be given later on.

b) High-frequency system. Existing microtrons differ greatly from one another in the construction of the system for feeding rf power to the resonator. As a rule, the rf oscillator is a magnetron, and only in one case[39] has a triode generator with a klystron amplifer been used. The typical arrangement of the rf equipment of a microtron is shown in Fig. 23. In working in the 10-cm range one requires magnetrons with a power per pulse not greater than 1 megawatt; only the 29-Mev microtron uses a magnetron power of 2 megawatts.

As an illustration of the balance of power consumption, we give data for systems which have quite efficient resonators:[7] of the 600 kw power supplied to the resonator, 400 kw are lost in the walls of the resonator, 100 kw are used for acceleration of all the electrons inside the resonator, and another 100 kw for the further acceleration of the resonance electrons (the beam current is 15 μa, the electron energy ~ 7 Mev).

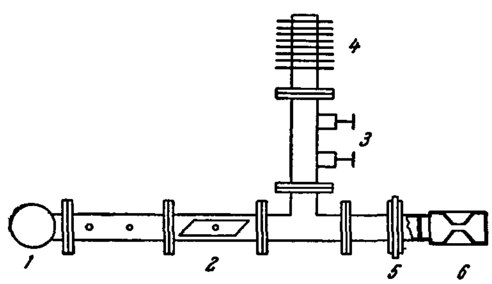


FIG. 23. Arrangement of rf system of a microtron. 1 – magnetron; 2 – phase shifter; 3 – regulator of the fraction of power which is tapped off to the stabilizing load 4; 5 – vacuum baffle; 6 – resonator.

Unlike other cyclic accelerators which use phase focusing, the microtron in principle is an accelerator that operates continuously: the process of capture of electrons into the acceleration cycle is repeated completely in each cycle of the rf field. However, the large amount of power required for exciting the resonator requires one to use pulsed operation with high duty cycle (usually approximately 1000), in order that the time average of the power supplied to the resonator not exceed admissible limits. Possibly in the future one will succeed in essentially reducing the duty cycle and consequently will raise the beam current, by using magnetrons with high average power and resonators with low losses and with very intense cooling.

In microtrons, as a rule, one uses non-tunable magnetrons, so that the resonator must be supplied with some sort of arrangement for remote tuning of the natural frequency. Usually this is accomplished by means of a mechanical or thermal system which produces controlled deformations of one of the walls of the resonator.

The stabilizing load (cf. Fig. 23) is usually regulated so that one dissipates in it approximately one-half the power supplied by the magnetron. In order to obtain very high stability of the amplitude of the accelerating voltage, it is necessary to introduce a stabilization of the magnetron supply.

In using field emission injection, because of the fact that the emission current depends on E_a and because of the influence of the current in the beam of accelerated electrons on the value of the field in the resonator, there is an automatic mechanism for stabilization of the value of V_a. Possibly a similar mechanism also acts in hot cathode injection.

Questions of construction of a resonator for a microtron have been described in a long paper by Kaiser.[40] Recently it has been pointed out[6] that one should re-examine some of the requirements imposed on this part of the microtron. In particular, in the construction of the resonator it is unreasonable to attempt to achieve maximum shunt resistance, since in such a resonator the value of V_a will depend too strongly on the beam current.

Some special electronics questions associated with the design and construction of rf systems for a microtron are discussed in references 41–43.

c) Vacuum system. The vacuum chamber of a microtron usually has a cylindrical shape. The chamber is made of non-magnetic metal, and the lids of iron. In some equipments, the lids serve as the poles of the magnet. During the operation of the microtron, the chamber is attached to a pumping system which usually consists of an oil diffusion pump and a rotating pump. The vacuum requirements of the microtron are extremely reasonable. The electrons quickly reach high energy, so that even with poor vacuum losses of electrons because of scattering by molecules of the residual gas practically do not occur. Therefore the upper limit of admissible pressure in the chamber is determined by the conditions of operation of the resonator, or the wave guide which feeds it. In some equipments it is sufficient to reduce the pressure to $p \leq 10^{-4}$ mm Hg in order to make possible normal operation of the accelerator. However, with such a high pressure one quickly spoils the polished surfaces of the interior of the resonator, and one gets more and more frequent breakdowns in the resonator. Therefore it is usually desirable to operate with pressures $p \leq 10^{-5}$ mm Hg.

With field emission injection, according to the data of reference 18, one observes a dependence of the emission current on the pressure in the chamber, and the optimum pressure lies in the range 10^{-4}–10^{-5} mm Hg. A higher vacuum may be required when one uses certain types of hot cathode, because of the possibility of "poisoning" by oxygen at high temperatures.

d) Auxiliary equipment for observation of the acceleration of the electrons. To measure the beam current at different orbits one uses a Faraday cup whose container is brought out from the chamber through a sliding vacuum lock. For visual observation of the beam one uses a coated luminescent screen which is attached to a holder with the same type of gasket. The illumination of the screen when bombarded by electrons is observed through an appropriate window in the vacuum chamber, where one can use a system of mirrors and television equipment.[6] A special system for visual observation of the beam is used in the Swedish microtron.[39] On the bottom lid of the chamber, approximately at its outer edge, there is a long arm on which there are attached about 15 horizontal tungsten wires of diameter 0.1 mm placed one above the other at distances of 1.25 mm. During the operation of the microtron these wires are heated at those points where the electron orbits pass through, and thus one can observe all the orbits simultaneously and obtain information concerning the dimensions of the cross section of the electron bunches at each of them.

e) Extraction of electrons from the chamber. A special feature of the microtron is the large change in energy of the accelerated particle after each passage through the resonator, and, as a consequence, the large separation between neighboring orbits. If the "pitch" γ of the helical trajectory of the electron is measured along the common diameter of the orbits, then $\gamma_\nu = D_{\nu+1} - D_\nu$ where D_ν is the diameter of the ν-th orbit. From the familiar relation between the magnetic rigidity G of the electron and its total energy E, $eG = \sqrt{E^2 - E_0^2}$, using formulas (21) and (25) one easily finds that

$$D_\nu = \frac{\lambda}{\pi}\sqrt{(m-2+\nu)^2 - \frac{1}{c_2^2}} \qquad (\nu = 2, 3, \ldots). \qquad (36)$$

For $\nu \geq 5$ and $c_2 \geq 0.5$, the approximate formula

$$D_\nu \simeq \frac{\lambda}{\pi}(m-2+\nu) \qquad (37)$$

gives a result with an error of less than 6%. Thus for large ν,

$$\gamma \simeq \frac{\lambda}{\pi}, \qquad (38)$$

which for $\lambda = 10$ cm reaches ~ 3.18 cm. With such a large pitch in the helix, the problem of extraction of the beam of accelerated particles outside the magnetic field presents no difficulties.

Up to the present time a large number of methods have been developed for the extraction. In each of these one uses an "antimagnetic channel"—an iron tube inside of which the magnetic field intensity is very small because of the screening action of the walls of the pipe. Upon entering this channel the electron beam moves approximately in a straight line and is easily extracted from the chamber. The most universal extraction set-up satisfies the following requirements: 1) it should be possible to extract electrons from any orbit starting with the second; 2) the extracted beam should enter the same fixed aperture of the experimental equipment in which it is used, independent of the orbit from which it is extracted; 3) at this fixed aperture the beam should always emerge in the same direction. In Fig. 24 we show schematically all the proposed variants of the extraction equipment, except for the most recent,[13] where it was desired to extract the beam only from the last orbit. All four variants satisfy the first requirement, but only variants c and d satisfy the second and third. With regard to variant c, we should note that as yet there is no information concerning its application in practice.

In one case (in the Swedish microtron[39]) a com-

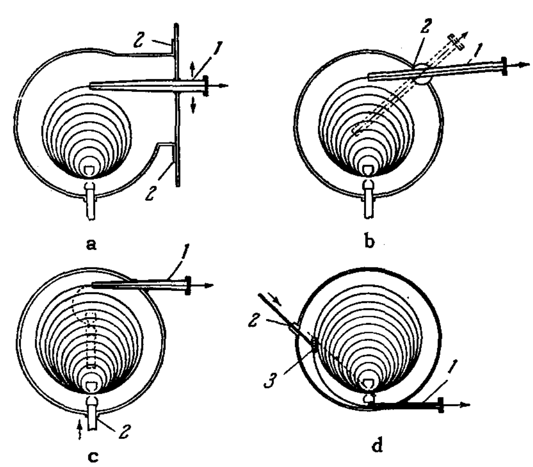


FIG. 24. Various types of structure for extraction of the beam from the vacuum chamber. 1 – iron tube; 2 – sliding vacuum gasket (miscellaneous constructions); 3 – short iron tube. Variants a – d are described respectively in references 37, 44, 45, and 46.

Table III

Microtron parameters	Location of Microtron: Moscow[7]	Tomsk[47]	London (Canada)[5,44]	London (Great Britain)[13]	Washington (U.S.A.)[34]		Washington (U.S.A.)[37]	Braunschweig (West Germany)[6]	Naples (Italy)[29]	Budapest (Hungary)[48,12]	London (Great Britain)[22,38]	Mainz (West Germany)[49]	Stockholm (Sweden)[17,39]
Maximum kinetic energy of electrons, in Mev	6.8 (11.6)	5.1	5	4.5	0.8	3.3	3.06	4.6	2.5	2	29	10	5.9
Electromagnet													
Pole diameter, cm	70	55	41	43.2			~20	56	60	46	203.2	50	47
Height of gap between poles (or inner height of the chamber), cm	11	12.5		9.5				10		11.7	12.7		
Nominal magnetic field intensity, koe	1.18 (1.95)	1.07	~1	~1	up to 1.68	up to 1.68	up to 1.68	1	0.52	0.5	1.07		1.23
Power input, kw				0.45						<0.3			
Weight in tons											20		0.7
Radiofrequency system													
Wave length, cm	10	10	10.7	10	3.2	3.2	3.2	10.7	10.23	10.61	10		9.95
Equilibrium voltage, kv	562 (932)	511	511	490	up to 255	up to 255	up to 255	~500	255	250	511		585
Length of accelerating gap, mm	16.3 (23.2)		10	7.8				10	7.95	7	7.8		10
Diameter of aperture in resonator, mm		10	9.5					11	7.2		8.5		9
Q of the resonator (unloaded)			10^4	9500	4100			7000 (4500)	8000	5300	9500		6000
Shunt resistance of the resonator, megohms			2		1.3			0.52	1		1—2		0.70
Power supplied to the resonator (pulsed), kw	600 (?)		300	250			<50	300 (without beam)	200	125	~10^3		
Pulse length of resonator supply, μsec			2	2	0.25		0.12—1	1—3	1.6	2.2	3		2.7
Repetition rate, pulses per second		345	435	200	до 10^3		200—2000	13—1000	200	300	100		12.5
Characteristics of the beam													
Number of orbits	12	9		8		14	12	8	10	8	56		10
Diameter of the last orbit, cm. .	41.4	34.9		30		15.22	14.1	~34	38		184		34.8
Current in the beam at the last orbit (per pulse), milliamp....	15 (5)		1	0.5		0.04	0.1	up to 0.1	1	0.01	0.05		20
Maximum obtainable efficiency in extraction of beam, %	80			70				70		60	30		

bined system of beam extraction is used: first it is deflected by an electrostatic deflector with E = 45 kv/cm, and then it enters an "antimagnetic channel."

The current in the extracted beam in microtrons usually is 50—70% of the circulating current at the corresponding orbit.

In conclusion we give a summary of the main parameters of operating microtrons and of one under construction[47] (Table III). Some of the microtrons were not described in detail in the literature, so that we can only give incomplete data concerning them.

f) Miscellaneous proposed modifications and improvements in microtron construction. In 1946 there appeared a short communication[3] concerning a modification of the microtron proposed by Schwinger. He proposed to use a sector magnet consisting of two sectors which are formed if a magnet with cylindrical poles is cut along its diameter and the halves which are thus obtained are displaced by a certain distance. The accelerating system may consist of one or several resonators; in the latter case the resonators should be excited from a single generator and have an appropriate phase shift relative to one another. The resonators are placed in the gap between the magnet sectors. This makes it possible to reduce considerably the height of the gap between the magnet poles by making it only a little greater than the diameter of the aperture in the resonator.

A powerful accelerating system can give the electrons greater energy changes at each turn. One can get $c_2 \geq 2$, so that the magnet system will be extremely compact.

In this variant of the microtron, as in the usual microtron, there is only a weak focusing of the beam during its transit through the resonator. However, in addition to the resonator, one can introduce a stronger lens, for example, a set of quadrupole magnets. In addition the electron beam can be well focused just before it enters the magnetic field, since in this type of construction there is sufficient room for a high-quality electron gun; one can also obtain a high injection energy.

In the microtron with a sector magnet, the resonance conditions are somewhat different from the usual case.[18,50,12] Questions of the theory, in particular the problem of the width of the region of phase stability as a function of the distance between the magnet sectors, were considered in reference 50. Many other questions, for example, the effect of the fringing magnetic field on the electron motion, have as yet not been worked out.

Up to now there has been no announcement of the building of a microtron of this type.

In 1953 in the Soviet Union[51] and Japan,[52] and independently in 1955 in the U.S.A.[52] there were proposed cyclic strong-focusing accelerators with a magnetic field constant in time. (In English these systems were called "FFAG accelerators.")

Information concerning possible application of the principle of these accelerators to the microtron has as yet not appeared in the literature, except for the most preliminary considerations.[18]

Still another new type of accelerator with a constant magnetic field and strong focusing, the so-called accelerator with crossed orbits, was proposed and worked out in detail theoretically in a series of papers by E. M. Moroz.[53-55] The magnetic system of this accelerator consists of several sectors of specially calculated shape with a uniform magnetic field in the pole gap of each sector.

As a result of the action of the edge magnetic field, at the points of entrance of the accelerated particles into the magnet sectors and at the points where they emerge from the sectors, there is a strong focusing of the particles. Thus a microtron with a magnetic system of this type will have the same advantages as the microtron with a sector magnet described above, but it will be free from its fundamental defect—the absence of magnetic focusing of the particles. Calculations showed that one can construct a sector magnet system with an extremely broad region of stability of motion of the accelerated electrons.

Unfortunately, so far there has been no work in which all these theoretical conclusions were verified in experiment. A recent communication[32] states that a microtron with a four-sector magnet system for 4—12 Mev is near completion. Interesting information concerning the construction and operation of this microtron is given in reference 62.

6. CONCLUSION

At the present time it is difficult to state definitely what is the maximum energy of electrons which future microtrons will reach. One of the main advantages of a 5—10 Mev microtron is its simplicity. Such a microtron is the only accelerator which can be made in any physics laboratory having a master mechanic. In going to higher energies, the advantage of simplicity is lost. Thus, for the 29-Mev microtron, the parameters of which are given above, one required a very carefully prepared magnet with a very complex system of correcting windings, and the beam current obtained at the last orbits after long adjustment of the accelerator was very small. Apparently the upper limit in energy of electrons accelerated in a microtron lies in the range 50—100 Mev.[7,56]

The powerful magnetrons available at present can operate with pulse lengths of rf voltage not exceeding a very definite value (~ 3—$8\ \mu$sec). This imposes a limit on the energy of electrons obtainable in a microtron, since with the large number of orbits necessary the total length of the acceleration process may become comparable with the duration of the operating interval of the resonator. However, it appears that the values of the limiting energies of electrons obtainable on the basis of such considerations are very high,[35] so that the maximum achievable electron energy in a microtron practically will be determined not by the magnetron pulse length, but by other causes, in particular the difficulty in obtaining a magnetic field with the required high degree of uniformity. (We are talking about a microtron without sector focusing.)

In principle there exists a possibility of producing an accelerator of the microtron type in which not only electrons, but also ions can be accelerated to arbitrarily high energies, beginning from an energy ~ 1 Gev.[57] For this purpose one should apply the sector magnet system of the same type as proposed by E. M. Moroz,[54] and an injector in which the particles are accelerated to an energy exceeding 0.5 Gev. As an accelerating system it is proposed to use a specially designed linear accelerator. Of course, such an instrument is no longer a microtron. The author proposed to call this accelerator an oxinotron.

One important question is the maximum current in a beam of accelerated electrons attainable in a microtron (per pulse). This quantity is determined by the maximum injection current, the degree of focusing of the electrons during the acceleration process, and the available magnetron power. If the latter is, for example, 800 kw at $\lambda \sim 10$ cm, then taking into account the losses in the resonator and its supply, to accelerate the electrons one may use a power ~ 300 kw. This means that for an electron energy equal to 5 Mev the beam current may reach 60 milliamp per pulse, and for an energy of 20 Mev, 15 milliamp.* For higher power of the magnetron the limiting beam current will be correspondingly greater.

There is, however, another point which may limit the attainable beam current. The electrons in a microtron move along curved trajectories and therefore lose energy through electromagnetic radiation. The radiative energy loss for the individual electron in the usual microtron is extremely small (for example, for $W = 29$ Mev and $H = 1.07$ koe, $\Delta W \simeq 0.07$ ev/turn) and does not have any effect whatsoever on the process of acceleration of the electrons. However, in view of the fact that the electrons in a microtron form extremely compact bunches, the effect of coherent radiation is greatly increased.[3] As we know, in this case each electron loses by radiation per unit length of its path an energy proportional to the number of electrons in the bunch. Thus the magnitude of the energy loss depends strongly on beam current. If the beam current should exceed some definite value, the energy losses to radiation will be so great that they will give rise to a removal of electrons from the region of phase stability.

In computing the value of the time average of the beam current, it is necessary to remember the effective duty cycle of the resonator, i.e., the duty cycle calculated taking account only of that part of each pulse during which the amplitude of the accelerating voltage already has its stationary value and is maintained at this level. In addition, it is necessary to use the relation between the effective pulse length from the resonator and the duration of the process of acceleration of the electrons to a given energy.[35]

Other well developed and widely used electron accelerators aside from the microtron are the accelerator with an electrostatic generator (ESG), the betatron, the synchrotron, and the linear wave guide accelerator (linac).

We give a comparison of the microtron with these four accelerators with regard to the most important parameters.

1. Upper limit of attainable electron energy. With respect to this parameter, only the electrostatic generator is inferior to the microtron, while the other three accelerators enable one to reach tens and hundreds of times greater electron energies than with the microtron.

*We recall that the beam currents attained in a microtron are 20 milliamp at 6 Mev[39] and 5 milliamp at 12 Mev.[7]

2. Upper limit of obtainable beam current. In betatrons and synchrotrons the beam current per pulse (even if one makes the pulse length very, very short, ~ 2 μsec) is by factors of ten less than that obtainable in a microtron. The electrostatic generator gives a continuous beam, where the beam current is very much greater than the time average of the beam current in a microtron, but smaller than the current per pulse. The linear accelerator at medium energies (up to 100 Mev) can give up to 0.8 ampere per pulse.

3. Energy homogeneity of the accelerated electrons. With respect to this factor only the electrostatic generator is superior to the microtron. As already pointed out, as a result of the small width of the phase-stable region the spread in energy of electrons in the microtron does not exceed ± 50 kev. Thus in a microtron at $W = 20$ Mev, $\delta W = \pm 2.5 \times 10^{-3}$. For the linear accelerator one can get the value of δW to approach that for the microtron only by using special techniques, which result in a loss in beam intensity.

4. Constancy of average value of the energy of accelerated electrons over a long time interval. With respect to this parameter the microtron also has an advantage over the other accelerators, except for the electrostatic generator. Constancy of the energy of the accelerated electrons is achieved by stabilizing the intensity H of the magnetic guide field; since in a microtron one uses a constant magnetic field, one can achieve a very high coefficient of stabilization of H.

5. Duration of the pulse of electron current impinging on a target. Here we have in mind not a macropulse whose width is determined by the length of the pulse fed to the magnetron and is usually 1.5–2 μsec, but the "micropulse" whose duration is determined by the length of the individual electron bunch and is about 3×10^{-11} sec for $\lambda = 10$ cm (and under certain conditions still smaller[31]). The microtron gives the shortest micropulse of all operating pulsed accelerators, which is a decisive advantage for certain applications.

If we restrict ourselves only to the first four parameters mentioned above, one can conclude that only the linear accelerator can replace the microtron in almost all cases. In a more detailed comparison of the different electron accelerators, carried out taking into account the required type of operation of the given accelerators, there may also be other parameters which are important; for example, stability of the equipment, its mobility, its total volume or specific volume (i.e., the volume per kilowatt of beam power), the energy efficiency of the accelerator, the possibility of varying the energy of accelerated electrons continuously and over wide intervals, the characteristics of the extracted beam—the extraction coefficient, the beam diameter, the angular divergence.

The parameters of a microtron make it a very efficient injector for a large synchrotron,[15,16,17] a suitable accelerator for generating submillimeter waves[31,32] and for nuclear-physics investigations by the time of flight method.[18]

We mention an application of a microtron for studying the scattering of electrons (it is proposed to make a determination of the radiative correction in elastic scattering of electrons[58]) and for studying the effect of 2-Mev electrons on the angle of rotation of the plane of polarization in a liquid.[59]

The short duration of the electron micropulses in the microtron makes it difficult to set up experiments in which one uses particle or quantum counters which have a relatively long dead time.[6] However, this defect can be avoided by using a rational construction of the apparatus.

Up to now papers published by laboratory workers constructing microtrons were devoted mainly to questions of investigating the accelerator itself. Undoubtedly in the future there will appear more and more publications concerning physics investigations carried out using electron beams accelerated in microtrons.

The present-day 10—20 Mev microtron, constructed with account of the advances made in this field, will be a compact accelerator with quite high beam current. Such a microtron probably will eliminate the betatron applied at present for industrial defectoscopy and for medical purposes.

[1] V. I. Veksler, DAN SSSR **43**, 346 (1944); J. Phys. U.S.S.R. **9**, 153 (1945).

[2] J. Itoh and D. Kobayashi, Kagaku **17**, 34 (1947); Sci. papers from the Osaka University, No. 12, 1 (1949).

[3] L. I. Schiff, Rev. Sci. Instr. **17**, 6 (1946).

[4] Henderson, LeCaine, and Montalbetti, Nature **162**, 699 (1948).

[5] Redhead, LeCaine, and Henderson, Can. J. Res. **A28**, 73 (1950).

[6] H. Reich, Z. angew. Phys. **12**, 481 (1960).

[7] Kapitza, Bykov, and Melekhin, JETP **39**, 997 (1960), Soviet Phys. JETP **12**, 693 (1961).

[8] C. Schmelzer, Z. Naturforsch. **7A**, 808 (1952).

[9] H. F. Kaiser, Rev. Sci. Instr. **25**, 1025 (1954).

[10] E. M. Moroz, DAN SSSR **106**, 986 (1956); Soviet Phys. Doklady **1**, 137 (1956).

[11] H. L. Knudsen, Trans. of the Danish Acad. of Techn. Sci., No. 3, 3 (1949).

[12] É. Kisdi-Koszó and L. Turi, Nucl. Instr. and Methods **9**, 137 (1960).

[13] Henderson, Heymann and Jennings, Proc. Phys. Soc. (London) **B66**, 654 (1953).

[14] A. Paulin, Nucl. Instr. and Methods **5**, 107 (1959).

[15] A. Paulin, Nucl. Instr. and Methods **9**, 113 (1960).

[16] M. Sedlaček, Ark. fys. **11**, 129 (1957).

[17] O. Wernholm, Ark. fys. **16**, 525 (1960).

[18] H. Kaiser, IRE Trans. NS-3, No. 2, 17 (1956).

[19] H. Reich and J. Trier, Phys. Verhandl. **8**, 163 (1957).

[20] B. Z. Kanter, Izv. Vuzov (Fizika), No. 3, 138 (1960).

[21] D. K. Aitken, Proc. Phys. Soc. (London) **A70**, 550 (1957).

[22] Aitken, Heymann, Jennings, and Kalmus, Proc. Phys. Soc. (London) **A77**, 769 (1961).

[23] A. A. Kolomenskii, Trudy FIAN **13**, 3 (1960) (cf. ref. 16 in this paper).

[24] Henderson, Heymann, and Jennings, Proc. Phys. Soc. (London) **B66**, 41 (1953).

[25] A. A. Kolomenskii, ZhTF **30**, 1347 (1960), Soviet Phys. Tech. Phys. **5**, 1278 (1961).

[26] M. Rabinovich, J. Phys. U.S.S.R. **10**, 530 (1946).

[27] É. Kisdi-Koszó, Acta Physica Acad. Sci. Hungar. **4**, 377 (1955).

[28] É. Kisdi-Koszó, Repts. of Central Inst. for Physics of the Hungarian Acad. of Sci. **4**, 499 (1954).

[29] A. Carrelli and F. Porreca, Nuovo cimento **6**, 729 (1957).

[30] V. L. Ginzburg, Izv. AN SSSR., ser. fiz. **11**, 165 (1947).

[31] I. Kaufman and P. D. Coleman, J. Appl. Phys. **27**, 1250 (1956).

[32] Brannen, Froelich, and Stewart, J. Appl. Phys. **31**, 1829 (1960).

[33] J. S. Bell, Proc. Phys. Soc. (London) **B66**, 802 (1953).

[34] H. F. Kaiser, J. Franklin Inst. **257**, 89 (1954).

[35] F. Porreca, Nuovo cimento **11**, 283 (1959).

[36] R. E. Jennings, Elec. Rev. (London) **161**, 309 (1957).

[37] H. F. Kaiser and W. T. Mayes, Rev. Sci. Instr. **26**, 565 (1955).

[38] Davies, Jennings, Porreca, and Rand, Nuovo cimento **17**, suppl. No. 2, 202 (1960).

[39] O. Wernholm, private communication.

[40] H. F. Kaiser, J. Franklin Inst. **259**, 25 (1955).

[41] J. Lehöcz and J. Pálinkás, Repts. of the Central Inst. for Physics of the Hungarian Acad. of Sci. **5**, 43 (1957) (cf. also p. 483).

[42] H. Reich, Nucl. Instr. and Methods **3**, 97 (1958).

[43] M. Innas Ali, Pakistan J. Sci. and Industr. Res. **1**, 26 (1958).

[44] E. Brannen and H. I. S. Ferguson, Rev. Sci. Instr. **27**, 833 (1956).

[45] Hsieh Chia-lin, Sci. Record, New ser. **1**, 227 (1957).

[46] H. Reich, Z. Naturforsch. **13a**, 1003 (1958).

[47] A. A. Vorob'ev, in Вопросы истории естествознания и техники (Problems of the History of Science and Technology), issue 8, Moscow, AN SSSR 1959, p. 33.

[48] É. Kisdi-Koszó and L. Turi, Acta Physica Acad. Sci. Hungar. **12**, 273 (1960).

[49] G. Lehr, Atomwirtschaft **4**, 277 (1959).

[50] L. Turi, Repts. of the Central Inst. for Physics of the Hungar. Acad. of Sci. **7**, 111 (1959).

[51] Kolomenskii, Petukhov, and Rabinovich, Некоторые вопросы теории циклических ускорителей (Some Problems of the Theory of Cyclic Accelerators), Moscow, AN SSSR 1955, p. 7.

[52] Symon, Kerst, Jones, Laslett, and Terwilliger, Phys. Rev. **103**, 1837 (1956).

[53] E. M. Moroz, DAN SSSR **108**, 436 (1956), Soviet Phys. Doklady **1**, 326 (1956).

[54] E. M. Moroz, DAN SSSR **115**, 78 (1957), Soviet Phys. Doklady **2**, 311 (1958).

[55] E. M. Moroz, Atomnaya Energiya **4**, 238 (1958).

[56] R. E. Jennings, Contemp. Phys. **2**, 277 (1961).

[57] A. Roberts, Ann. Phys. **4**, 115 (1958).

[58] Bhiday, Jennings, and Kalmus, Proc. Phys. Soc. (London) **72**, 973 (1958).

[59] A. Carrelli and F. Porreca, Nuovo cimento **19**, 844 (1961).

[60] V. P. Bykov, JETP **40**, 1658 (1961), Soviet Phys. JETP **13**, 1169 (1961).

[61] A. Paulin, Nucl. Instr. and Methods **12**, 155 (1961).

[62] E. Brannen and H. Froelich, J. Appl. Phys. **32**, 1179 (1961).

[63] Kapitza, Bykov, and Melekhin, JETP **41**, 368 (1961), Soviet Phys. JETP **14**, 266 (1962).

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