METHODS OF MEASURING DIELECTRIC CONSTANTS OF SUBSTANCES AT MICROWAVE FREQUENCIES

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INTRODUCTION

KNOWLEDGE of the dielectric constant of a substance is of great importance in physics, chemistry, and engineering. Recent advances in science have expanded the use of dielectrics and have raised new problems involving the study of their properties. The latest progress in microwave physics has necessitated a study of the behavior of matter at these new wavelengths, and the accomplishments in microwave technology have given rise to new methods for the measurements of dielectric constants of substances.

There are many published methods of measuring dielectric constants of substances at microwave frequencies. We review in the present article the most frequently employed methods, indicate the progress made toward improving old methods and developing many new ones. Principal attention is paid to the physical aspects of the measurement of ϵ and $\tan\delta$. With this as a starting point, we classify the methods of measuring ϵ and $\tan\delta$, and also describe in greater detail some new methods for the measurement of these quantities, particularly the helical-waveguide method. Less attention is being paid to measurement techniques, and we do not consider at all questions connected with a detailed analysis of the measurement errors.

I. CLASSIFICATION OF METHODS USED IN THE MEASUREMENT OF THE DIELECTRIC CONSTANT

In principle, any measurement of the effect of a substance on an electromagnetic field can be used to determine ϵ and $\tan\delta$ of the substance. At low frequencies, the simplest is the interaction between the electric field of a capacitor and the dielectric used in it. Consequently all methods for the measurement of dielectric constants at low frequencies reduce to an evaluation of the change in capacitance brought about by introducing the investigated substance into the capacitor. The various methods of accounting for the change in capacitance predetermine the methods used for measurement of the dielectric constants. The most frequently encountered are bridge methods, resonance methods, and beat methods. The use of these methods results in sufficiently accurate measurements of the dielectric constants of the substances. Thus, in the case of nonconducting dielectrics, the resonance method yields an accuracy on the order of 0.01 percent in the determination of ϵ, while the beat method permits a determination of ϵ accurate to 5×10^{-6} dielectric-constant units.[1] The error increases sharply with increasing conductivity of the investigated dielectric.

In the microwave range, systems with lumped constants are replaced by systems with distributed constants. Accordingly, the methods used to measure the dielectric constant change. Some resonance procedures are retained. In addition, methods come into play in which interaction between guided waves and matter is used. There are several types of guides available for microwave such as the two-conductor line, the coaxial line, hollow waveguides, dielectric transmission lines, etc. The use of different transmission lines leads to different measurement methods. Finally, guided waves in free space can also be used, and this in turn necessitates a new technique.

The existence of a large number of methods of measurement of ϵ and $\tan\delta$ is due to the presence of various transmission lines, to the possibility of choosing different parameters suitable for the measurement, to the use of specimens of different shape, and to the choice of locations of these specimens in the system.

A common feature of all these methods is that they all involve a determination, in one manner or another, of the change in the phase constant of the propagation whenever the tested dielectric is introduced into the system, and the determination of the connection between this change and the value of the dielectric constant. These relations can be quite different in each individual case, and consequently the number of methods for measuring ϵ and $\tan\delta$ is large.

The methods used for microwave measurements of the dielectric constant are usually classified in the literature as follows:[2-4]

1) methods using waves in free space;
2) methods using guided waves;
3) resonant methods.

The most extensive group of methods, based on the use of guided waves, can be subdivided by the type of transmission line (two-conductor, waveguide, or coaxial line).

The two-conductor line served as the basis for the development of: 1) the first Drude method,[5] 2) the second Drude method (or the capacitor in two-conductor line method),[6] 3) Rozhanskii's plate method,[7] and 4) the Tatarinov method.[8] These methods were extensively used and developed in the Thirties. They were later replaced by better methods in which coaxial and

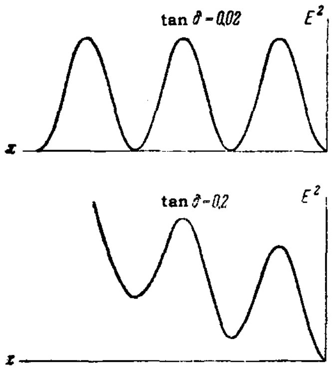


FIG. 1. Distribution of field intensity for low and medium losses.

waveguide lines were used. It must be noted, however, that the second Drude method is still used for decimeter waves, while the remaining methods are interesting because they were the first in which ideas were proposed for the derivation of the formulas and for the choice of parameters suitable for measurement; these methods were then transferred to the waveguide and cavity resonator methods.

The most widely used among the waveguide methods are those based on investigation of the waves transmitted through the specimen or reflected from it. The most popular variants are those in which a calibrated line is used,[9] although there are also bridge variants, based on a comparison of the waves reflected from the investigated specimen and from standard loads (see, for example, reference 10).

Methods employing waves in free space can also be subdivided into two subgroups, corresponding to the observation of the reflected and transmitted waves, respectively.

A similar subdivision can also be made when a coaxial line is used. In addition, a method in which a line segment with lumped capacitance is introduced is also worthy of attention. In the latter case equivalent-circuit calculations lead to simple formulas for ϵ and $\tan\delta$ of a substance placed in the capacitive part of the line.

Resonance methods have found extensive use. They differ in that the resonant systems can be made up of different transmission lines, in the type of the oscillations excited in these systems, in the placement of the specimen in the resonator, and in the shape of the specimen itself.

If we start from the common nature of the physical principles of the interaction between field and matter, all the foregoing methods can be subdivided into the following groups:

1) methods based on the study of the standing-wave field in the investigated dielectric,

2) methods based on an analysis of waves reflected from the investigated specimen,

3) methods based on a study of the waves transmitted through the dielectric,

4) resonance methods.

Worthy of particular attention are the investigations of N. A. Divil'kovskii and M. I. Filippov,[11] in which the dielectric constant is determined from the change in the temperature of a small dielectric sphere in a high-frequency field.

The existing methods can also be classified by the character of the waves used to interact with the substance. In most methods the phase velocity of the wave is either greater than the velocity of light (waveguide methods) or equal to the velocity of light (two-conductor lines, coaxial lines, free space). Methods exist, however, in which the phase velocity of the waves is less than the velocity of light (isolated dielectric rod, helical waveguide). In this review we shall call the first group fast-wave methods and the second group slow-wave methods.

II. FAST-WAVE METHODS OF MEASURING DIELECTRIC CONSTANTS

1. Methods Based on the Investigation of the Standing-Wave Field in the Dielectric

The simplest relations between ϵ and the measured parameters can be obtained by considering the propagation of waves in an unbounded dielectric medium or in a system completely filled with dielectric.

It is well known[12] that the propagation constant k_m of a wave in an unbounded dielectric and the propagation constant k_0 of a wave in free space are related by the equation

$$k_m = \sqrt{\varepsilon\mu}\, k_0, \tag{2.1}$$

where ϵ and μ, the dielectric constant and the permeability, are in general complex quantities.

If k_m can be measured when $\mu = 1$, it becomes possible to determine ϵ at a given frequency ($\omega_0 = k_0 c$). The simple connection between the dielectric constant and the propagation constant holds also for waves propagating in systems where the field structure is close to that of a plane wave, that is, in two-conductor and coaxial lines. If such lines are completely imbedded in the investigated medium, with $\epsilon \neq 1$ and $\mu = 1$, the dielectric constant of the substance is determined from the formula

$$\varepsilon = \left(\frac{k_m}{k_0}\right)^2 = \left(\frac{\lambda_0}{\lambda_d}\right)^2 \tag{2.2}$$

and the entire process of measuring ϵ reduces to a determination of the wavelength in the system without the dielectric (λ_0) and with the dielectric (λ_d). A measurement can be effected by placing an ideally reflecting plane perpendicular to the propagation direction of a plane wave and observing the standing-wave pattern in front of this plane. Figure 1 shows the distribution of the intensity of the electric fields for the case of low and medium losses. At low or medium losses, the distance between neighboring minima, l, is equal to half the wavelength, $\lambda_d = 2l$.

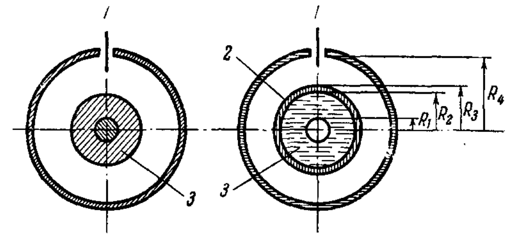


FIG. 2. Cross section of coaxial line with investigated dielectric. 1) Probe. 2) Cylindrical cuvette. 3) Dielectric.

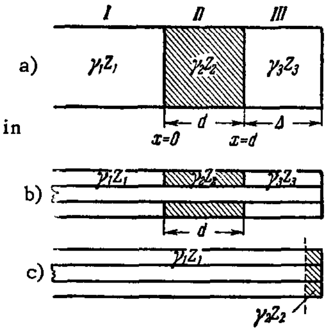


FIG. 3. Position of specimen in the method of the short circuited line.

The tangent of the loss signal can be determined here from

$$\operatorname{tg}\delta = \frac{2}{\pi}\frac{E_{\min}}{E_{\max}}, \tag{2.3*}$$

where E_{min} and E_{max} are the field intensities at the minimum and maximum of the standing wave.

The earliest attempt to use a two-conductor line for the measurement of ϵ with the line completely imbedded in the dielectric is the work by V. I. Kalinin[13] on the determination of the dielectric constant of water. The two-conductor line was drawn through a vessel filled with liquid. A probe was used to plot the field in the system with and without the liquid. The value of ϵ was calculated using (2.2). At 16.8 cm, ϵ of water was found to be 81.7, in good agreement with the results obtained by other methods. A determination of ϵ of liquids in a completely filled rectangular waveguide has been reported.[4] The dielectric constant was determined from two values of the wavelength in the guide, with and without the dielectric. In this system the connection between ϵ and the measured wavelengths is somewhat more complicated than for a two-conductor line, but all the measurements reduce in this case only to a determination of the standing wave in a medium completely filled with dielectric.

The determination of ϵ from the standing wave pattern in a dielectric was reported also in references 14 and 15, where ϵ and tan δ were measured in specimens of sufficient length, filling part of a short-circuited coaxial line. The specimen did not occupy the complete cross section of the line (Fig. 2). The probe was moved in the remaining free space. An equivalent-circuit analysis of the system yielded the following simple formulas for the determination of ϵ from the standing-wave pattern in the system:

$$\varepsilon = A\left[\left(\frac{\lambda_\varepsilon}{\lambda_0}\right)^2 + A - \eta^2\right]^{-1}, \tag{2.4}$$

where

$$A = \ln\frac{R_2}{R_1}\left(\ln\frac{R_4}{R_1}\right)^{-1},$$

$$\eta^2 = \left(\frac{\lambda_T}{\lambda_0}\right)^2 = \left(\ln\frac{R_2}{R_1} + \frac{1}{\varepsilon_T}\ln\frac{R_3}{R_2} + \ln\frac{R_4}{R_3}\right)\left(\ln\frac{R_4}{R_1}\right)^{-1},$$

λ_0 is the wavelength in the line without dielectric, λ_ϵ is the wavelength in the presence of dielectric, λ_T is the wavelength in the line in the presence of a tube, ϵ_T is the dielectric constant of the tube, while R_1, R_2, R_3, and R_4 are the radii of the internal wire, the tube (inside and outside diameters), and external wire of the coaxial line, respectively. The losses were determined from (2.3). It is noted in references 14 and 15 that a coaxial line can be used to measure dielectric constants $\epsilon \leq 20$ and medium losses in a broad band of frequencies.

A feature of the described methods is the simplicity of the mathematics. They are most suitable for the measurement of ϵ of liquids in different parts of a frequency band. Among their shortcomings is the need for a large quantity of the investigated substance, difficulties arising in the investigation of solid substances, and the fact that these methods cannot be used for substances with high losses.

2. Methods Based on Waves Reflected from the Specimen

The methods based on a study of the standing-wave field in the investigated dielectric are not used extensively because of several shortcomings, the principal among which is the need for a large quantity of the investigated substance. If small specimens are on hand, then methods based on a study of the waves transmitted through a limited portion of the investigated substances, or reflected from the substance, are more suitable. The most widely used are methods wherein the standing waves in front of the specimen are investigated with a known load behind the specimen. Many papers[9,16-18] have been devoted to such methods.

Let us examine the short circuited line method.[9] Assume that some portion of a transmission line (say a waveguide) is filled with the investigated dielectric (region II of Fig. 3). Each of the three regions will then be characterized by propagation constants γ_1, γ_2, and γ_3 and by wave impedances Z_1, Z_2, and Z_3. It is possible to determine the propagation constant in the investigated medium from the measured input impedance of the line. The short-circuiting plate can be located in different places.

*tg = tan.

1. Second medium short circuited ($\Delta = 0$). The input impedance of a line segment of length d, shorted at the end, is known to be[2]

$$Z_{\text{in}}^{(0)} = Z_2 \,\text{th}\gamma_2 d, \quad (2.5)^*$$

where $\gamma_2 = \alpha + i\beta_2$, α is the attenuation factor, β_2 is the phase constant; γ_2 is the propagation constant, and the normalized input impedance is

$$\frac{Z_{\text{in}}}{Z_1} = \frac{Z_2}{Z_1} \,\text{th}\gamma_2 d. \quad (2.6)$$

On the other hand, it is known that the characteristic impedance for a TE wave is connected with the propagation constant by the relation

$$Z_{TE} = \frac{i\omega\mu}{\gamma_2},$$

and consequently

$$\frac{Z_2}{Z_1} = \frac{\gamma_1}{\gamma_2} = \frac{i\beta_1}{\gamma_2}. \quad (2.7)$$

From (2.5) and (2.6) we get an equation for γ_2:

$$\frac{1}{i\beta_1 d}\frac{Z_{\text{in}}}{Z_1} = \frac{\text{th}\,\gamma_2 d}{\gamma_2 d}. \quad (2.8)$$

2. If the short-circuiting plate is located a quarter wavelength away from the rear wall of the specimen ($\Delta = \lambda/4$) and the attenuation in the third medium is equal to zero, we can write the following expression for the input impedance of the open line:

$$Z_{\text{in}}(0) = Z_2 \,\text{cth}\,\gamma_2 d. \quad (2.9)^\dagger$$

In this case we can also obtain an equation relating the unknown propagation constant in the second medium with the measured quantities (input impedance of the line and propagation constant in the first medium):

$$\frac{i}{\beta_1 d}\frac{Z_{\text{in}}}{Z_1} = \frac{\text{cth}\,\gamma_2 d}{\gamma_2 d}. \quad (2.10)$$

Relations (2.8) and (2.10) are transcendental equations with respect to the unknown propagation constant. They must be solved graphically. Many papers on the measurement of dielectric constants by similar methods list tables of the functions $\tanh\theta/\theta$.[1,9] In addition, methods wherein the system is only partially filled with dielectric lead to ambiguities in the determination of ϵ, and to eliminate these ambiguities one must resort to repeated measurements of ϵ of specimens of different thickness or determine beforehand the approximate value of the measured values of ϵ.

Several particular cases lead to simpler expressions for the propagation constant in the investigated medium.

a) Case of lossless dielectric. The propagation constant has in this case the form

$$\gamma_2 = i\beta_2.$$

The equation for γ_2 becomes

$$\frac{1}{i\beta_2 d}\frac{\chi(0)}{Z_1} = \frac{\text{tg}\,\beta_2 d}{\beta_2 d}, \quad (2.11)$$

where $\chi(0)$ is the input reactance of the line. Account of the equation

$$i\frac{\chi(0)}{Z_1} = -i\,\text{tg}\,\beta_1 d_{\min}, \quad (2.12)$$

where $d_{\min}$ is the distance from the specimen to the first minimum of the standing wave, leads to the following equation for γ_2:

$$\frac{\text{tg}\,\beta_1 d_{\min}}{\beta_1 d} = -\frac{\text{tg}\,\beta_2 d}{\beta_2 d}. \quad (2.13)$$

Equation (2.13) is simpler to solve than (2.8), since the function $\tan\theta/\theta = f(\theta)$ has been tabulated. In this case, however, it is necessary to take into account the ambiguity in the determination of ϵ.

b) In the case of high losses, when the reflected wave does not reach the air-dielectric interface, the determination of ϵ becomes much easier. Indeed, the absence of a reflected wave in the line leads to the equation

$$Z_{\text{in}}(0) = Z_2.$$

Since $Z_2/Z_1 = i\beta_1/\gamma_1$, we have

$$\gamma_2 = \frac{Z_1}{Z_{\text{in}}} i\beta_1. \quad (2.14)$$

In this case there is no need for solving a transcendental equation.

c) It is also possible to get rid of the transcendental equation by modifying the method to the so-called two-position method.[17] In this method the input impedances of the line are measured in the presence of a specimen, at two positions of the short-circuiting piston. In one of these positions the specimen is at the short circuit, and in the other the distance from the specimen to the short-circuiting plate is equal to one quarter wavelength. For the two measurements we have

$$\frac{\text{th}\,\gamma_2 d}{\gamma_2 d} = \frac{(Z_{\text{in}})_1}{Z_1}\frac{1}{i\beta_1 d}, \quad \frac{\text{cth}\,\gamma_2 d}{\gamma_2 d} = \frac{(Z_{\text{in}})_2}{Z_1}\frac{1}{i\beta_1 d}.$$

From these equations we obtain an expression for γ_2 in terms of the measured parameters:

$$\gamma_2^2 = \frac{\beta_1^2 Z_1^2}{(Z_{\text{in}})_1 (Z_{\text{in}})_2}. \quad (2.15)$$

All the equations derived for γ_2 hold also for measurements in a coaxial line (Fig. 3b).

Knowledge of the propagation constant enables us to determine the complex dielectric constant of the medium. For a two-conductor or coaxial line, the ϵ of the medium can be determined from the relation

$$\frac{\varepsilon_2^*}{\varepsilon_1} = -\left(\frac{\gamma_2}{\gamma_1}\right)^2, \quad (2.16)$$

where ϵ_1 and γ_1 are the dielectric constant and the propagation constant in the first medium, while ϵ_2^* and γ_2 refer to the second medium.

For a waveguide the following relation holds true:

*th = tanh.

†cth = coth.

$$\gamma_2^2 = k^2 - k_{cr}^2,$$

where $\gamma_2 = \alpha_2 + i\beta_2$ is the propagation constant in a waveguide with $\epsilon \neq 1$, $k = i\beta_0\sqrt{\epsilon\,(1 - i\tan\delta)}$ is the propagation constant in an unbounded medium with $\epsilon \neq 1$, $\beta_0 = 2\pi/\lambda_0$ is the propagation constant in free space with $\epsilon = 1$, and $k_{cr} = 2\pi/\lambda_{cr}$. The expressions for ϵ and $\tan\delta$ are obtained in the following form:

$$\varepsilon = \frac{k_{cr}^2 + \beta_2^2 - \alpha^2}{\beta_0^2}, \quad \mathrm{tg}\,\delta = \frac{2\alpha\beta_0}{k_{cr}^2 + \beta_2^2 - \alpha^2}. \tag{2.17}$$

The foregoing relations, which give the connection between γ_2 and the measured input impedance, hold for any transmission line (two-conductor, coaxial, or waveguide), when the specimen is located in some part of the line. The difference lies in the connection between γ_2 and the measured value of ϵ.

Notice should be taken of still another position of the specimen and somewhat different equations for γ_2 in the case of a coaxial line.[1,18] A specimen in the form of a disc (and not a ring) is located in the capacitive part, formed by the central conductor and the end half of the line (Fig. 3c). In this case the input impedance of the line can be determined from the formula

$$\frac{Z_{in}}{Z_1} = \frac{1}{Z_1}\frac{1}{i\omega C^*}, \tag{2.18}$$

where $C^* = \epsilon^* C_0$. From the input impedance of the line, the lumped capacitance C_0, and the geometry of the line we can determine the dielectric constant of the substance in explicit form:

$$\varepsilon^* C_0 = \left(i\omega Z_1 \frac{Z_{in}}{Z_1}\right)^{-1} = \left(867 i f \frac{Z_{in}}{Z_1} \lg\frac{D_2}{D_1}\right)^{-1}, \tag{2.19}$$

where D_1 and D_2 are the diameters of the conductors and f is the frequency in cycles.

For low-loss specimens we have

$$\varepsilon C_0 = \left(867 f \lg\frac{D_2}{D_1}\,\mathrm{tg}\,\frac{2\pi d_{min}}{\lambda_1}\right)^{-1}, \tag{2.20}$$

$$\mathrm{tg}\,\delta = \frac{\pi\Delta x}{\lambda_1}\left(\mathrm{tg}\,\frac{2\pi d_{min}}{\lambda_1} + \mathrm{ctg}\,\frac{2\pi d_{min}}{\lambda_1}\right). \tag{2.21}$$

We see that the formulas for ϵ and $\tan\delta$ are quite simple and a small quantity of the investigated substance is needed for the research.

Thus, the measurement of ϵ and $\tan\delta$ by the short-circuited line method reduces to a determination of the input impedance of the line, that is, to a measurement of the standing-wave coefficient and of the shift of the first minimum (as measured from the specimen). The errors of the method are determined by the errors of these measurements, and depend also on the gap between the specimen and the line. For specimens with medium values of ϵ and $\tan\delta$ ($\epsilon \leq 20$, $\tan\delta = 10^{-3} - 10^{-2}$), methods in which a measuring line is used yield an error of one percent in the determination of ϵ, while the accuracy of $\tan\delta$ is limited by the accuracy of the SWR measurement, which is approximately 2 percent.[4] Among the shortcomings of the method is that the specimen must be accurately adjusted to these dimensions of the system, something particularly important if large values of ϵ are measured. If the specimen is placed in the capacitive portion of a coaxial line, an additional difficulty is involved in the preparation of a special cell—the measuring capacitor. The short circuited line method is most convenient in the centimeter band. In the decimeter band the method is less suitable because of the need for a cumbersome measurement line. On the short-wave side, the use of the method is limited by the difficulty of constructing a short-circuited line with moving probe at wavelengths below 1 cm. The bandwidth of the method is determined by the transmission line employed. The width in the waveguide variant is less than when a coaxial line is used. Among the shortcomings of the method, in the general case, is the need for solving transcendental equations and the ambiguity of determination of ϵ. The method is suitable for the measurement of solid and liquid dielectrics.

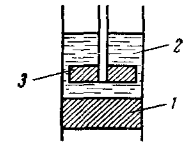


FIG. 4. Cell for measurement of ε and tan δ liquids.

Dielectric constants of liquids are usually measured in special cells such as shown schematically in Fig. 4. Here 1—a liner made of solid dielectric with very low losses and impervious to the liquid, 2—measured liquid dielectric, and 3—opening in the piston for the passage of the liquid. By connecting the cell to a vertically mounted measuring line and by plotting the readings of the indicator as the piston is displaced in the cell, a plot of the attenuation of the electromagnetic wave in the liquid dielectric is found as a function of the thickness of the liquid layer. From this curve it is possible to calculate the wavelength in the dielectric λ_d and the attentuation coefficient α. To determine the real and imaginary components (ϵ' and ϵ'') of the dielectric constant, the following relations are derived from waveguide theory:

$$\varepsilon' = \left(\frac{\lambda}{\lambda_d}\right)^2 + \left(\frac{\lambda}{\lambda_{cr}}\right)^2 - \left(\frac{\alpha\lambda_d}{2\pi}\right)^2\left(\frac{\lambda}{\lambda_d}\right)^2, \tag{2.22}$$

$$\varepsilon'' = \frac{1}{\pi}\left(\frac{\lambda}{\lambda_d}\right)^2 \alpha\lambda_d. \tag{2.23}$$

The results reported in reference 26 indicate that attenuation is determined by this method with high accuracy. A similar method can be used to measure the dielectric constants of liquids in the millimeter range. Heinken and Bruin[27] report measured values of ϵ for several alcohols and halogen-substitution benzenes in the 3 — 7.5 mm range. Burdun and Kantor[28] give measured values of the dielectric constant of

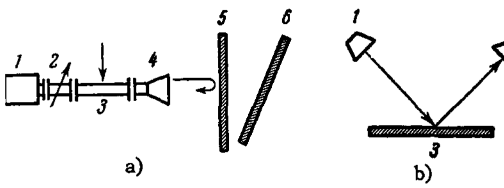


FIG. 5. Measurement of ε and tan δ by the reflection method.

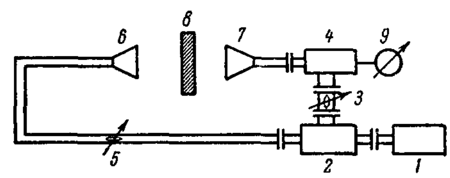


FIG. 6. Measurements of ε in free space by the transmission method.

water and formamide in the centimeter and millimeter band.

Recently published papers[19-21] report the use of the short circuited line method for the measurement of dielectric constants of substances with $\epsilon > 200$. To reduce the large amount of reflection occurring when large values of ϵ are measured, a plate made of a substance with low loss angle and with a dielectric constant equal to the square root of the dielectric constant of the investigated substance was placed in front of the tested specimen. Lipaeva and Skanavi[21] report dielectric constants, measured in this manner, for several special ceramics (barium titanate with $\epsilon = 656$, $\tan\delta = 0.41$; strontium-bismuth-titanate SVT-1 with $\epsilon = 385$, $\tan\delta = 0.34$, and others).

The short circuited line method used with waveguides has proved useful in the measurement of the temperature dependence of ϵ and $\tan\delta$. It is necessary to construct special thermostatic sections in this case. References 22 — 25 describe such sections, note the pecularities of the measurement method, and give the temperature dependence of ϵ and $\tan\delta$ for many substances. It is possible, as shown in reference 24, to measure ϵ with accuracy on the order of 1.5 — 3 percent, and $\tan\delta$ with accuracy on the order of 10 — 20 percent.

The method of two thicknesses is expedient for the measurement of dielectric constants of liquids. The equations for ϵ and $\tan\delta$ are obtained in explicit form, and no great difficulties are involved in producing dielectric layers of different thickness in the case of liquid dielectrics. The results of research carried out by such a method are reported in references 16 and 29.

The interest that is attached to the study of dispersion properties of matter make it frequently necessary to use coaxial lines for the measurement of ϵ and $\tan\delta$. Several published papers[18,30,31] describe the results of measurements of ϵ and $\tan\delta$ in the 5 — 40, 14 — 66, and 8 — 80 cm ranges. Particular interest attaches to the case when the specimen is placed in the capacitive portion of the line.[18]

Surface reflection of waves from the specimen can also be used to measure ϵ. Methods based on the use of surface reflection can be employed with some guided waves and with waves propagating in free space. In the study of waves reflected from the front wall of the specimen in a waveguide line, measures must be taken to eliminate reflection from the rear wall of the specimen. It becomes necessary to bevel the second wall of the specimen or to match the load in some fashion. The need for using a matched load behind the specimen makes this method less convenient than the short circuited line method. Data are reported in the literature on the measurement of the dielectric constant of water in a waveguide by the surface reflection method.[4] This method is extensively used with waves propagating in free space. The simplest formulas for the determination of ϵ and $\tan\delta$ are obtained (as corollaries of the Fresnel formulas) in the case of normal incidence of the waves on the specimen:

$$\varepsilon \sec\delta = \frac{1-2r_i\cos r_i'+r_i^2}{1+2r_i\cos r_i'+r_i^2}, \tag{2.24}$$

$$\sin\frac{\delta}{2} = \frac{1}{2}\left(\sqrt{\varepsilon\sec\delta} - \frac{1}{\sqrt{\varepsilon\sec\delta}}\right)\operatorname{tg} r_i', \tag{2.25}$$

where $r_i \exp[-ir_i']$ is the surface reflection coefficient. At arbitrary angle of incidence, the formulas become somewhat more complicated. A change to free space simplifies theory very little and necessitates the use of an entirely different experimental technique. Figures 5a and b show setups used to measure reflection in free space. Figure 5a shows the case of normal incidence. Here 1—generator, 2—attenuator, 3—line with probe, 4—antenna, 5—tested dielectric, 6—absorbing screen. The line and probe are used to measure the reflection from the specimen and from a metallic sheet placed in front of the antenna. If the antenna is matched with the space, the ratio of the two measured reflections yields the unknown reflection coefficient. When the incidence is far from normal (Fig. 5b, where 1 and 2 are the transmitter and receiver while 3 is the specimen), the reflection is measured by comparing the reflected power with that directly received. An example of the use of this method is reported in references 32 and 33.

3. Methods Based on Waves Transmitted Through the Dielectric

These methods can be used to study the passage of waves through a dielectric situated either in some waveguide system or in free space. What is measured is the complex transmission coefficient. The expression for the transmission coefficient of a plane wave normally incident on the interface between two dielectrics with constants ϵ_1 and ϵ_2 has the following form[4]

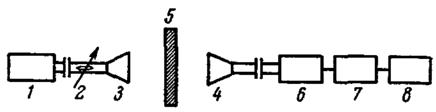


FIG. 7. Measurements of tan δ by the transmission method. 1 – generator, 2 – attenuator, 3 and 4 – horns, 5 – specimen, 6 – detector, 7 – amplifier, 8 – indicator.

$$t_i e^{-it_i'} = \frac{2\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1}+\sqrt{\varepsilon_2}}. \qquad (2.26)$$

It is assumed here that the wave propagates from the medium with dielectric constant ϵ_1 to the medium with ϵ_2. Measurement of the amplitude and phase of the transmission coefficient yields the complex dielectric constant of the investigated medium. By separating the real and imaginary parts of the transmission coefficient we obtain two equations for the real and imaginary parts of the dielectric constant. In general these are transcendental interdependent equations. If the losses are low, the equations are no longer transcendental. For H modes in a waveguide with cutoff wavelength λ_{cr} the expressions for ϵ and tan δ assume the form[1]

$$\frac{\varepsilon_2}{\varepsilon_1} = \frac{\lambda_{cr}^{-2} + \frac{\beta^2-\alpha^2}{4\pi^2}}{\lambda_{cr}^{-2}+\lambda_d^{-1}}, \quad \mathrm{tg}\,\delta = \frac{2\alpha\beta}{\beta^2-\alpha^2+\left(\frac{2\pi}{\lambda_{cr}}\right)^2}, \qquad (2.27)$$

where

$$\left.\begin{aligned} \alpha &= \frac{\Delta\alpha}{868\Delta l} \\ \beta &= \frac{\Delta\varphi}{57.3\Delta l} + \frac{2\pi}{\lambda_d \Delta l} \end{aligned}\right\} \qquad (2.28)$$

(radians per unit length), $\Delta\alpha$—attenuation, $\Delta\varphi$—phase shift, Δl—thickness of the specimen, and λ_d—wavelength in the system.

For the case of normal incidence of waves on a specimen (for two-conductor and coaxial lines), the formulas for the determination of ϵ and tan δ assume the following form:

$$\frac{\varepsilon_2}{\varepsilon_1} = \left(\frac{\lambda}{2\pi}\right)^2 (\beta^2 - \alpha^2), \qquad (2.29)$$

$$\mathrm{tg}\,\delta = \frac{2\alpha\beta}{\beta^2-\alpha^2}. \qquad (2.30)$$

The errors in this method are analogous to those in the measuring-line method. Inaccuracies in the trimming of the specimen to the dimensions of the measuring waveguide exert a similar influence. Another inconvenience is the need of using a matched load behind the specimen or for using specimens with beveled walls to prevent reflection.

This method finds greatest application when waves in free space are used. The dielectric constant is calculated from the phase difference of oscillations propagating in free space and in the investigated dielectric. The tangent of the loss angle is determined by measuring the power attenuation in the dielectric and in free space. A block diagram for the measurement of ϵ and tan δ is shown in Fig. 6. The signal flows from generator 1 to tee 2. One arm of the tee leads to attenuator 5 and to transmitting horn antenna 6. The other arm connects through attenuator 3 to detector 4. Horn 6 is first moved until the signal incident on detector 4 is of the opposite phase (the meter 9 reads minimum deflection then). Insertion of the specimen (8) disturbs the out of phase relationships and makes it therefore possible to determine the phase shift. The formula for ϵ is

$$\varepsilon = \left(1+\frac{\Delta}{d}\right)^2, \qquad (2.31)$$

where Δ is the horn displacement necessary to restore the phase and d is the thickness of the specimen. The tangent of the loss angle is determined with the aid of the circuit shown in Fig. 7. The signal is determined first without the specimen. The specimen 5 is then inserted and attenuator 2 adjusted until the signal returns to its value in the absence of the specimen. The difference in the attenuator readings determines the attenuation in the specimen. The attenuation, the known thickness of the specimen, and the magnitude of the dielectric constant yield the tangent of the loss angle. The phase shift can be read with sufficient accuracy, and consequently the measurements of ϵ by this method are highly accurate.[34]

The method is applicable in the short-wave part of the centimeter band and to the millimeter band. Among the shortcomings is the need for using large specimens.

4. Resonance Methods

Any transmission line, whether it be two-conductor, coaxial, or waveguide, can be made into a resonant system by shortcircuiting both ends of the line. Such a system has then a natural resonant frequency and internal losses. Introduction of a dielectric into the resonator changes the natural frequency and the losses of the resonant circuit. By determining the changes in the characteristics of the resonant circuit we can measure the electric parameters of the investigated dielectrics.

1. System completely filled with dielectric. Mathematically it is simplest to describe the processes when the resonator is completely filled with the substance. Indeed, let the resonator without dielectric have a resonant frequency $f_0 = c/\lambda_0$, where c is the velocity of propagation of the electromagnetic oscillations in free space, and λ_0 is the free-space wavelength at which the system is resonant. When the resonator is completely filled with dielectric and its length is unchanged, resonance takes place at a frequency

$$f_d = \frac{c}{\sqrt{\varepsilon}\,\lambda_0}.$$

The two resonant frequencies are related as

$$f_d = \frac{f_0}{\sqrt{\varepsilon}}.$$

Therefore the measurement of the resonant frequencies of a hollow resonator and one completely filled with matter enables us to determine the dielectric constant of the matter, using the formula

$$\varepsilon = \left(\frac{f_0}{f_d}\right)^2. \tag{2.32}$$

The dielectric constant of a substance can be measured also at a fixed generator frequency. In this case the resonance disturbed by introducing the dielectric is restored by changing the dimensions of the system.

Assume that the resonator is made from a waveguide. If resonance without dielectric occurs at a system wavelength

$$\lambda_d^0 = \frac{\lambda_0}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_{cr}}\right)^2}}, \tag{2.33}$$

then filling the resonator with dielectric causes the resonance (for the same free-space wavelength λ_0) in the system to occur at a wavelength

$$\lambda_d = \frac{\lambda_0}{\sqrt{\varepsilon - \left(\frac{\lambda_0}{\lambda_{cr}}\right)^2}}, \tag{2.34}$$

and this yields an expression for ϵ in the form

$$\varepsilon = \left(\frac{\lambda_d^0}{\lambda_d}\right)^2 \left[1 - \left(\frac{\lambda_0}{\lambda_{cr}}\right)^2\right] + \left(\frac{\lambda_0}{\lambda_{cr}}\right)^2. \tag{2.35}$$

The expression obtained is valid for substances with low losses.

The simplest relations are obtained for two-conductor and coaxial lines, for which $\lambda_{cr} = \infty$. For these lines

$$\varepsilon = \left(\frac{\lambda_d^0}{\lambda_d}\right)^2. \tag{2.36}$$

An example of the use of this method with a two-conductor line is the first method of Drude.[5] The gist of the method is to determine the position of two short-circuited bridges in a two-conductor line, for which an indicator placed between the bridges reads minimum intensity. This distance is equal to half a wavelength or an integral number of half wavelengths. The measurements are made for two cases, with and without dielectric. The dielectric constant is determined from (2.36). The absorption coefficient is determined from the distribution of the field intensity along the line. If I_0 and I_x are the amplitudes of the current when the indicator is located on the boundary of the liquid and at a depth x, the following relation holds true

$$I_x = I_0 e^{-4\pi n\varkappa \frac{x}{\lambda_0}}. \tag{2.37}$$

The coefficient of absorption $n\kappa$ is determined in this case from the current ratio at different bridge positions

$$\frac{I_1}{I_2} = e^{-4\pi n\varkappa \frac{x_1 - x_2}{\lambda_0}}. \tag{2.38}$$

In cavity resonators, complete filling is used to determine the dielectric constants of gases and non-polar liquids. The dielectric constant is then determined by formula (2.32), and the losses by the formula

$$\operatorname{tg} \delta = \frac{1}{Q} - \frac{1}{Q_0}, \tag{2.39}$$

where Q_0 and Q are the figures of merit of the resonator without and with dielectric.

The shortcomings inherent in this method are as follows: the difficulty of measuring solid specimens, the need for using a large amount of investigated substance, and large absorption when high-loss substances are investigated.

2. Partial filling of the system with dielectric. If the losses in the substance are large or if only a small specimen is available, or else if the investigated substance is difficult to machine, methods with a completely filled resonator become unsuitable and one must resort to partial filling of the system with dielectric. Although an instrument based on this method overcomes satisfactorily the difficulty of obtaining a Q high enough to perform the measurements with sufficient accuracy, the mathematical analysis becomes more complicated and as a rule involves numerical or graphical solution of transcendental equations. Let us consider several examples of partial filling of resonators, based on different transmission lines: two-conductor, coaxial, and waveguide.

The well known methods of measuring ϵ with the aid of a two-conductor line—the Rozhanskii-plate method and the capacitor method (or the second Drude method) are essentially the first versions of the method with a resonant system partially filled with dielectric. In the Rozhanskii method a thin plane-parallel layer of dielectric is placed perpendicular to the two-conductor line, in the voltage antinode. The shift in the resonant bridge after introducing the dielectric layer determines the dielectric constant ϵ, while measurement of the width of the resonance curve determines in this case tan δ. The formulas for ϵ' and ϵ'' are

$$\begin{aligned}(\varepsilon' - 1)\, d &= x_1 - x_0,\\ \varepsilon'' d &= x_2 - x_1,\end{aligned} \tag{2.40}$$

where x_0 is the distance from the moving bridge to the location of the dielectric (that is, to the voltage antinode in the absence of dielectric), x_1 is the same with the dielectric, x_2 is the distance from the boundary of the dielectric to the bridge position where the energy has half the maximum value, and d is the thickness of the specimen. These formulas have been derived assuming low losses and negligible thickness of the specimens, so as to obviate the use of transcendental equations for the determination of ϵ' and ϵ''.

In the capacitor method (the second Drude method), a capacitor filled with the investigated liquid is placed in the resonant circuit formed by the two short-circuiting bridges. The capacitor with liquid shifts the resonant point, and the value of ϵ is determined from this shift, while the losses are determined by the broaden-

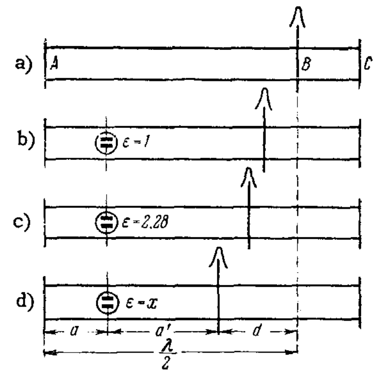


FIG. 8. Illustrating the measurement of ε and tan δ by the second Drude method.

ing of the resonant curve. Figure 8 shows the consecutive variation of the resonant wavelength and of the resonance width when a capacitor (Fig. 8b) is introduced in the system, when the capacitor is filled with a known control dielectric with $\epsilon = 2.28$ (Fig. 8c) and with an unknown dielectric with $\epsilon = x$ (Fig. 8d).

We used the following formulas to determine the dielectric constant:

a) Morton's formula,[35] derived under the assumption of low losses in the specimen and absence of losses in the line, and disregarding the effect of the suspensions:

$$\varrho_0 + \varepsilon\varrho = \frac{1}{2}\,\frac{\sin\beta d}{\sin\beta a\cdot\sin\beta\,(a+d)}\,, \tag{2.41}$$

where $\rho = 4\pi k \ln[D/R]$ and $\rho_0 = 4\pi k_0 \ln[D/R]$ are the capacitor constants, k_0 is the ballast capacitance of the capacitor (the capacitance of the leads inside and outside the glass), k the working capacitance, D the distance between the leads, R the radius of the leads, ϵ the unknown dielectric constant, and $\beta = 2\pi/\lambda$.

b) The Coolidge formula[36] for the absorption coefficient κ:

$$\frac{\varkappa}{1-\varkappa} = \frac{\gamma}{4\pi}\left[1 + \frac{2\pi}{\lambda}\,\frac{\dfrac{a\sin\beta\,(a+d)}{\sin\beta d} + \dfrac{\left(\dfrac{\lambda}{2} - a - d\right)\sin\beta a}{\sin\beta\,(a+d)}}{\sin\beta d}\right], \tag{2.42}$$

where $\gamma = \alpha\lambda$ and α is the attenuation of the line per unit length. These formulas are subject to errors which are particularly noticeable in highly-conducting liquids. In the thirties, a large number of investigations were made to obtain more accurate formulas, and also to develop and improve the method. The most important of these researches were made by V. N. Kessenikh and K. A. Vodop'yanov,[37] B. I. Romanov,[38] N. V. Malov,[39] S. L. Sosinskii and V. A. Dmitriev,[40] B. K. Maĭbaum,[41] and I. A. El'tsin.[42] Study of the peculiarities of these methods led to the following recommendations for the measurement of the dielectric constant of a substance by the capacitor and two-conductor line methods:

1. The capacitor employed should have low capacitance.

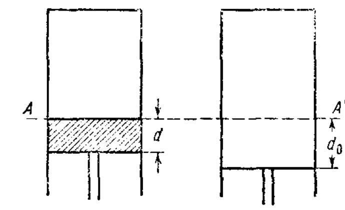


FIG. 9. Illustrating the measurement of ε with a resonator in the H_{011} mode.

2. The influence of the suspension must be taken into account, and also the influence of the conductivity of the dielectric and the attenuation of the measuring line itself.

3. The fluctuations in generator power must be minimized to the utmost by effecting optimal coupling with the generator.

Measurements of ϵ and tan δ of substances with the aid of a two conductor line have several important shortcomings, due essentially to radiation of electromagnetic energy and the influence of extraneous fields. Work on formulating the theory of this method is still going on, however,[43-45] and many papers have been published on the measurements of ϵ and tan δ in the meter and decimeter bands by the capacitor and two-conductor line method.[46-50]

Particularly interesting is reference 51, in which the two-conductor line capacitor method was modified to measure ϵ and tan δ of substances with high losses. This improvement is attained by connecting the measuring capacitor to the line not directly but through a quarter-wave loop perpendicular to the main line. Such a connection reduces the load on the main line, thereby permitting measurement of ϵ and tan δ of liquids with high losses.

At frequencies exceeding 3×10^9 cps, cavity resonators are used to measure the dielectric constant. The H_{011} and E_{010} modes are the most frequently used.

a) Measurements with the H_{011} mode. In these measurements the specimen is usually a cylindrical disc placed on the end of the cavity (Fig. 9). If resonance at a given λ_0 exists in the presence of a specimen of thickness d when the specimen surface coincides with the line AA', and if resonance in the absence of a specimen is produced when the piston is a distance d_0 away from the line AA', this means that the reactances of the two segments between the piston and the plane AA' are equal. Using the expression for the impedances of the H wave in a resonator with and without dielectric, we obtain the following expression:

$$\frac{\operatorname{tg}\beta d}{\beta d} = \frac{\operatorname{tg}\beta_0 d_0}{\beta_0 d_0}\,, \tag{2.43}$$

where β_0 is the propagation constant in the empty resonator

$$\beta_0 = \frac{2\pi}{\lambda_{d0}} = \frac{2\pi}{\lambda_0}\sqrt{1 - \left(\frac{\lambda_0}{\lambda_{cr}}\right)^2}\,;$$

$\beta = 2\pi/\lambda_d$ is the propagation constant in a waveguide completely filled with dielectric. The problem reduces to determining the propagation constant β in the given medium. This can be done by solving the transcendental equation (2.43). If we know the propagation constant $\beta = 2\pi/\lambda_d$ in a waveguide filled with the investigated substance, then the dielectric constant can be determined from the relation

$$\lambda_d = \frac{\lambda_0}{\sqrt{\varepsilon - \left(\frac{\lambda_0}{\lambda_{cr}}\right)^2}}. \tag{2.44}$$

Equation (2.43) neglects the losses in the substance. In addition, this is a transcendental equation, so that to determine ϵ it is necessary to know the approximate range of ϵ or to carry out the measurements with two specimens of different thickness.

To determine $\tan\delta$ of a substance partially filling the resonator it becomes necessary to calculate the Q of the resonator in the H_{011} mode with and without the dielectric. In reference 52 the following expression was obtained for Q:

$$Q = \frac{pD + \frac{1}{\varepsilon}L}{\frac{\Delta}{a}\left(\frac{1}{\beta^2 + k^2}\right)[k^2(pD+L) + \beta^2 p + \beta_0^2] + pD\,\mathrm{tg}\,\delta}, \tag{2.45}$$

where

$$D = 2d - \frac{1}{\beta}\sin 2\beta d, \qquad L = 2l - \frac{1}{\beta_0}\sin 2\beta_0 l,$$
$$p = \left(\frac{\sin\beta_0 l}{\sin\beta d}\right)^2, \qquad k = \frac{2\pi}{\lambda_{cr}},$$

a is the radius of the resonator, d is the thickness of the dielectric, l is the length of the part of the resonator filled with air, Δ is the depth of penetration of the field, β is the phase constant in the part of the resonator filled with the dielectric, β_0 is the phase constant in the part of the dielectric filled with air, and ϵ is the dielectric constant. When the losses in the dielectric are negligibly small we have $\tan\delta \approx 0$ and $Q = Q_m'$ (Q_m' *is the value of* Q *for a resonator filled with a perfect lossless dielectric having the same relative dielectric constant as the real dielectric*). From (2.45) we obtain

$$Q_m' = \frac{pD + \frac{1}{\varepsilon}L}{\frac{\Delta}{a}\frac{1}{\beta^2 + k^2}[k^2(pD+L) + 2a(\beta^2 p + \beta_0^2)]}, \tag{2.46}$$

From (2.45) and (2.46) we have

$$\mathrm{tg}\,\delta = \left(1 + \frac{1}{pl}\frac{L}{D}\right)\left(\frac{1}{Q} - \frac{1}{Q_m'}\right). \tag{2.47}$$

The depth of penetration Δ can be determined in the following manner. For an air-filled resonator we obtain theoretically

$$Q_m = \frac{a(k^2 + \beta_0^2)}{\Delta\left(k^2 + \frac{2a\beta_0^2}{l+d}\right)}. \tag{2.48}$$

We can determine Q_m experimentally. We then determine Δ from (2.48) and the known Q_m. This value of Δ is substituted in the expression for Q_m'; the calculation of $\tan\delta$ thus reduces to measuring the Q of the resonator and to calculating Q_m'. To determine Δ it is necessary to measure the Q of the empty resonator. If the losses are low, we can assume $Q_m' \approx Q_m$.

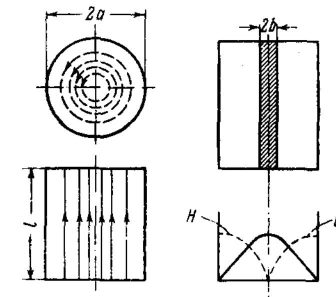


FIG. 10. Distribution of fields and position of specimen in the resonator method of measuring ε and tan δ in the E_{010} mode.

b) Measurements with E_{010} mode. The field distribution in such a resonator is shown in Fig. 10. When the specimen is on the axis of the system we can write for the field inside the resonator (by solving Maxwell's equation in cylindrical coordinates)

$$H_\varphi = AJ_1(kr)\,e^{i\omega t}, \qquad E_z = \frac{k}{\sigma + i\omega\varepsilon} AJ_0(kr)\,e^{i\omega t}, \tag{2.49}$$

where $k^2 = -i\omega\mu(\sigma + i\omega\epsilon')$, while J_0 and J_1 are Bessel functions of the first kind. In the region between the dielectric and the wall we have $r > 0$ and the fields are therefore given by the sum of two Bessel functions:

$$H_\varphi = [BJ_1(k_0 r) + CN_1(k_0 r)]\,e^{i\omega t},$$
$$E_z = \frac{k_0}{i\omega\varepsilon_0}[BJ_0(k_0 r) + CN_0(k_0 r)]\,e^{i\omega t}, \tag{2.50}$$

where $k_0 = \beta_0 = 2\pi/\lambda_0$, and N_0 and N_1 are Bessel functions of the second kind. The boundary conditions on the air-metal interface and on the air-dielectric interface yield the following equation for ϵ:

$$\varepsilon = \frac{\beta_1 J_0(\beta_0 b) J_1(\beta_0 b)}{\beta_0 J^0(\beta_0 b) J_1(\beta_1 b)} \frac{\frac{N_0(\beta_0 a)}{J_0(\beta_0 a)} - \frac{N_1(\beta_0 b)}{J_1(\beta_0 b)}}{\frac{N_0(\beta_0 a)}{J_0(\beta_0 a)} - \frac{N_0(\beta_0 b)}{J_0(\beta_0 b)}}. \tag{2.51}$$

where λ_0 is the resonant wavelength in free space and $\beta_1 = \beta_0\sqrt{\epsilon}$. For low losses this equation is exact but transcendental with respect to ϵ. It can be simplified by recognizing that

$$J_1(\beta_0 b) N_0(\beta_0 b) - J_0(\beta_0 b) N_1(\beta_0 b) = \frac{2}{\pi\beta_0 l},$$

and introducing the notation

$$F = \frac{\pi\beta_0 a}{2}[J_0(\beta_0 b) N_0(\beta_0 a) - J_0(\beta_0 a) N_0(\beta_0 b)].$$

We then obtain

$$\varepsilon = 1 + \frac{a}{b}\frac{J_0(\beta_0 a)}{J_1(\beta_0 a)}\left\{F\left[1 + \frac{1}{8}(\beta_0 b)^2\right] + \frac{1}{8}(\beta_0 b)^2 \frac{a}{b}\frac{J_0(\beta_0 a)}{J_1(\beta_0 b)}\right\}^{-1} \tag{2.52}$$

Plots of F vs. β_0a have been prepared for different values of b/a down to 0.3 (ϵ to 6). For very thin specimens, we can write the following expression for ϵ:

$$\varepsilon = 1 + 0.539\frac{V}{v}\frac{\Delta f}{f}. \tag{2.53}$$

The expression for the tangent of the loss angle can be written in the following form:

$$\operatorname{tg}\delta = \left[\left(\frac{a}{b}\right)^2 + f^2(\varepsilon - 1)\right]\left\{\varepsilon f^2\left[1 + \frac{J_1^2(\beta_1 b)}{J_0^2(\beta_1 b)}\right]\right\}^{-1}\left(\frac{1}{Q} - \frac{1}{Q'}\right) \tag{2.54}$$

where Q refers to the resonator with the specimen and Q′ refers to the resonator with fictitious lossless specimen. The theoretical value of Q′ is

$$Q' = \frac{1}{\alpha}\left[\left(\frac{a}{b}\right)^2 + f^2(\varepsilon - 1)\right]\left[\frac{a(a+b)}{b^2} + f^2(\varepsilon - 1)\right]^{-1}. \tag{2.55}$$

In practice it is necessary to obtain Q′ by comparing the experimental and the theoretical values of this quantity for a resonator without a specimen at the same frequency.

For very thin specimens we have

$$\operatorname{tg}\delta = \frac{0.263}{\varepsilon}\left(\frac{a}{b}\right)^2\left(\frac{1}{Q} - \frac{1}{Q'}\right). \tag{2.56}$$

Resonator methods permit highly accurate measurements of ϵ and $\tan\delta$. Thus, when operating in the H_{011} mode, the error in the measurement of ϵ in the three-centimeter band is ± 1.5 percent, while the accuracy of $\tan\delta$ is ± 5 percent.[3] Resonators in the E_{010} mode at 10 cm provide an accuracy of 5 percent in ϵ and 10 percent in $\tan\delta$, while ϵ measured with coaxial resonators in the 3-cm band is accurate to 1.5 – 5 percent. The relative simplicity and convenience, the manageable dimensions, and the feasible constructions make resonant methods with cavity resonators quite simple to operate. These methods have found wide use in microwave measurement practice. Among their shortcomings are the need for precisely made specimens of definite shape, an accurate determination of the wavelength, and generators of high stability. In addition, ordinary resonant methods are unsuitable for the measurement of ϵ and $\tan\delta$ of substances with high losses.

Recent work on the improvement of resonator methods has been aimed at broadening the range of measured values of ϵ and $\tan\delta$ and at the development of designs suitable for study of the temperature characteristics of the substances.

Tunable resonators, the use of which is limited to the centimeter band, permit measurement of ϵ up to about 200.[54] To measure large values of ϵ in the decimeter band, methods using semi-coaxial cavities have proved very convenient.[55,56] The equivalent circuits of such a cavity, which can be regarded as a resonant circuit with distributed inductance and lumped capacitance, yield rather simple formulas for ϵ in terms of the change in the resonant frequency and for $\tan\delta$ in terms of the change in Q brought about by introducing the specimen into the resonator. Similar cavities permit measurement of ϵ up to 1000. The method yields quite accurate value of ϵ. A shortcoming is the narrow bandwidth for specified usable cavity dimensions and the need for graduation (i.e., the need for control specimens).

As shown by G. V. Zakhvatkin,[57] resonators of the semi-coaxial type can be used to measure ϵ and $\tan\delta$ of substances with large losses. For this purpose the specimen should be placed not directly in the gap between the end of the cavity and the central rod, but in an additional capacitor such that the resultant air gap is in series with the measured specimen and decreases the drop in Q due to the introduction of the specimen in the cavity. This permits measurement of ϵ and $\tan\delta$ of substances with high losses.[55,56] Such measurements are possible even at shorter wavelengths (less than 10 cm) with ordinary resonators. In reference 58, the ϵ of substances with high losses (dipole liquids) were measured with a resonator in the H_{011} mode in a cylindrical specimen placed on the resonator axis, i.e., in the region where the field intensity is low. This choice of specimen location reduces the absorption of waves in the substance, meaning that substances with high losses can be investigated.

To determine the temperature variations of ϵ and $\tan\delta$ by the resonator method, it becomes necessary to place the measuring resonators in thermostatic ovens. References 24 and 56 contain descriptions of semi-coaxial resonators made of ceramics with silvered internal surfaces. The measurements, made on 3 and 10 cm, permitted the behavior of many high polymers and ferroelectrics to be traced over the investigated temperature range.

Various modifications of the resonator method have found application recently. Thus, ϵ and $\tan\delta$ of solid dielectrics are measured with a pi-shaped resonator.[59] Along with using H_{011} and E_{010} modes in cylindrical cavities, higher order modes are used. For example, the H_{111} is used in reference 60 and the H_{014} mode in reference 61. In reference 60 the expressions for ϵ and $\tan\delta$ are obtained in analytic form and can be measured accurate to 2 and 15 percent, respectively. Transcendental equations must be solved to find ϵ and $\tan\delta$ in accordance with the method of reference 61.

III. SLOW-WAVE METHODS OF MEASURING THE DIELECTRIC CONSTANT

In the methods considered up to now the dielectric constant was determined with systems in which the electromagnetic waves propagate with a velocity equal to the velocity of light, or with a phase velocity greater than the velocity of light. The dielectric constants can also be measured with slow waves, that is, waves in which the phase velocity is less than the velocity of light.

A large number of investigations have been devoted to the study of slow surface waves. From among the various slow-wave systems, particular interest is attached to systems of the helical type, the properties of which have been investigated by many.[62-66] A study of the dispersion characteristics of slow-wave systems containing dielectrics enable us to establish relationships between the dielectric constant and the phase velocity of the wave in the system. The use of helical and other slow-wave systems for the measurement of ϵ is characterized by various properties, principal among which are the following:

1. Helical type systems are of the broadband type and call for a broadband method of determining ϵ.

2. A helical-dielectric slow-wave system reduces appreciably the phase velocity of the wave, so that the dimensions of the measuring apparatus can be decreased.

3. A great degree of slowing down enables us to introduce certain simplifications in the derivation of the formulas and thereby get rid of transcendental terms in the expressions for ϵ.

4. In a helix, as in any other surface-wave system, the field on the axis is lower than on the surface itself. By placing a thin specimen with large losses in the region where the field intensity is reduced the field energy absorption is decreased, so that the values of ϵ of substances with large losses can be measured.

The classification of slow-wave methods for the measurement of ϵ and $\tan\delta$ can be the same as that of fast-wave methods.

1. Measurement of ϵ of Solid Dielectrics.[67,68]

1. Determination of ϵ of a substance completely filling the helix. If the internal region of a helical waveguide (helix radius a, winding angle θ) is filled with a perfect dielectric with constant ϵ, then the dispersion equation of such a system has the form

$$k^2 \operatorname{ctg}^2\theta = \frac{k_1' \dfrac{I_0(k_1'a)}{I_1(k_1'a)} + k_2' \dfrac{K_0(k_2'a)}{K_1(k_2'a)}}{\dfrac{\varepsilon}{k_1'} \dfrac{I_1(k_1'a)}{I_0(k_1'a)} + \dfrac{1}{k_2'} \dfrac{K_1(k_2'a)}{K_0(k_2'a)}}, \tag{3.1*}$$

where

$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda_0}, \quad k_1' = \sqrt{k_3'^2 - \varepsilon k^2}, \quad k_3' = \frac{\omega}{v'_{\text{ph}}}, \quad k_2' = \sqrt{k_3'^2 - k^2} \tag{3.2}$$

(λ_0 is the wavelength in free space, v'_{ph} is the phase velocity of the fundamental wave in the system). In the case of a closely-wound helix ($\cot\theta \approx 15-20$), we can put in (3.1)

$$k_1' \simeq k_2' \simeq k_3' = \frac{2\pi}{\lambda'_{\text{d}}}, \tag{3.3}$$

where λ'_{d} is the wavelength in the helix-dielectric system. This substitution enables us to simplify the dispersion equation (3.1) greatly and determine ϵ in explicit form

$$\varepsilon = \left(\frac{\lambda_0}{\lambda_{\text{d}}} \operatorname{tg}\theta\right)^2 \frac{I_0K_0}{I_1K_1}\left(1 + \frac{I_0K_1}{I_1K_0}\right) - \frac{I_0K_1}{I_1K_0}. \tag{3.4}$$

(The arguments of the modified Bessel functions I_0, I_1, K_0, and K_1 are $2\pi a/\lambda'_{\text{d}}$ and are omitted.)

*ctg = cot.

Thus, as in methods using fast waves, an explicit expression for ϵ can be obtained in the case of slow waves if the system is completely filled with the investigated substance.

In the case of Bessel functions of large arguments, when $2\pi a/\lambda'_{\text{d}} \geq 3$ (i.e., at high frequencies for a specified system geometry, or at large helix radii for a fixed frequency), the expression (3.4) for ϵ simplifies to

$$\varepsilon = 2\left(\frac{\lambda_0}{\lambda'_{\text{d}}} \operatorname{tg}\theta\right)^2 - 1. \tag{3.5}$$

2. Determination of ϵ in the presence of a gap between the cylindrical specimen and the helix. Such a system comprises a helix of radius a with winding angle θ, in which is placed coaxially a dielectric cylinder of radius b (Fig. 11a). For a closely-wound helix, the formula for ϵ is in this case

$$\varepsilon = \frac{\left(\dfrac{\lambda'_{\text{d}}}{\lambda_{\text{d}}}\right)^2 [\Delta_{01} l_{01} - I_1(b)\, q] + Q^a l_{01} [q_{00} I_1(b) - \Delta_{01} I_0(a)}{I_1(b)\left[Q^a l_{01} q_{00} - \left(\dfrac{\lambda'_{\text{d}}}{\lambda_{\text{d}}}\right)^2 q\right]}, \tag{3.6}$$

where

$$\Delta_{01} = I_0(b)K_1(b) + I_1(b)K_0(b), \quad l_{01} = I_0(a)K_1(a) + I_1(a)K_0(a),$$
$$q_{01} = I_0(b)K_1(a) + I_1(a)K_0(b), \quad q_{00} = I_0(a)K_0(b) - I_0(b)K_0(a),$$
$$q = q_{00}K_1(a) + q_{01}K_0(a) \tag{3.7}$$

(the factor $2\pi/\lambda'_{\text{d}}$ has been left out from the arguments of the Bessel functions), and

$$Q^a = \frac{K_0\left(\dfrac{2\pi a}{\lambda'_{\text{d}}}\right) I_1\left(\dfrac{2\pi a}{\lambda'_{\text{d}}}\right) K_1\left(\dfrac{2\pi a}{\lambda'_{\text{d}}}\right)}{I_0\left(\dfrac{2\pi a}{\lambda_{\text{d}}}\right) K_0\left(\dfrac{2\pi a}{\lambda_{\text{d}}}\right) I_1\left(\dfrac{2\pi a}{\lambda'_{\text{d}}}\right) K_1\left(\dfrac{2\pi a}{\lambda'_{\text{d}}}\right)} \tag{3.8}$$

(λ_{d} is the wavelength in the free helix). At high frequencies, (3.6) assumes the form

$$\varepsilon = \frac{1 - \left(\dfrac{\lambda'_{\text{d}}}{\lambda_{\text{d}}}\right)^2 + \exp\left\{\dfrac{4\pi}{\lambda'_{\text{d}}}(b-a)\right\}}{\left(\dfrac{\lambda'_{\text{d}}}{\lambda_{\text{d}}}\right)^2 - 1 + \exp\left\{\dfrac{4\pi}{\lambda'_{\text{d}}}(b-a)\right\}}. \tag{3.9}$$

If we put $a = b$ in (3.9), that is, change to the case when there is no gap between the dielectric and the helix, then (3.9) becomes

$$\varepsilon = 2\left(\frac{\lambda_{\text{d}}}{\lambda'_{\text{d}}}\right)^2 - 1. \tag{3.10}$$

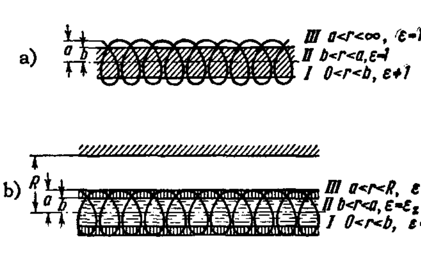


FIG. 11. "Helix plus dielectric" system.

As $a \to \infty$, the "helix plus dielectric" system changes into a "cylindrical dielectric rod" system in free space, and formula (3.6) becomes

$$\varepsilon = \frac{I_0\left(\frac{2\pi b}{\lambda'_d}\right) K_1\left(\frac{2\pi b}{\lambda'_d}\right)}{I_1\left(\frac{2\pi b}{\lambda'_d}\right) K_0\left(\frac{2\pi b}{\lambda'_d}\right)}. \qquad (3.11)$$

It is known that for an isolated dielectric rod the dispersion equation of an axially-symmetrical wave is written in the form[12]

$$\varepsilon = -\frac{gb}{pb}\,\frac{J_0(gb)\,K_1(pb)}{J_1(gb)\,K_0(pb)}, \qquad (3.12)$$

where $p = \sqrt{k_3'^2 - k^2}$, $g = \sqrt{\epsilon k^2 - k_3'^2}$; $J_0(gb)$ and $J_1(gb)$ are Bessel functions of the first kind of zero and first order, respectively.

Formula (3.11) coincides with (3.12) if the following substitution is possible (in the case of large slowing-down ratio)

$$p = k'_3, \quad g = ik'_3. \qquad (3.13)$$

Let us compare two methods of measuring ϵ: 1) the method of the isolated dielectric cylinder, and 2) the method of the dielectric cylinder on which a helix is wound. In the case of the isolated rod, the substitution (3.13) introduces very large errors in the determination of ϵ, since the large slowing-down ratio (that is, greater values of k'_3) are obtained only for large values of ϵ, but then $g = \sqrt{\epsilon k^2 - k_3'^2} \neq ik'_3$, and ϵk^2 can no longer be neglected compared with $k_3'^2$ without introducing a large error. Consequently ϵ should be determined from (3.12) and not (3.11). But (3.12) is a transcendental equation with respect to ϵ. Thus, to find ϵ it is necessary to use graphic calculations, and the isolated cylinder method becomes unavoidably cumbersome. In addition, the presence of a Bessel function of first order in (3.12) leads to ambiguity in the determination of ϵ. The isolated dielectric rod method is described in reference 69. In this method the determination of ϵ is reduced to a study of the standing-wave pattern along a cylindrical specimen made of the investigated dielectric. A comparison of the length of the wave slowed down by the rod with the length of the wave in free space yields the value of ϵ. The method can be used to measure ϵ on short waves. On longer wavelengths its use is more difficult, owing to the need for specimens with large diameters. Another shortcoming of this method is the need for matching the specimen with the line.

If the dielectric rod is placed inside a helix, the wave is slowed down both by the helix and by the dielectric. The surface character of the slowed-down electromagnetic waves is much more strongly pronounced in the "helix with dielectric" system than in the isolated "cylindrical rod." This makes the substitution $k'_1 \cong k'_2 \cong k'_3$ in the derivation of formulas (3.4) and (3.6) for ϵ quite legitimate, and this yields explicit expressions for ϵ and eliminates the ambiguity.

2. Measurement of ϵ of Liquid Dielectrics[70-72]

A helical line can also be used to measure ϵ of liquid dielectrics. In this case the helix can be either completely immersed in the dielectric, similar to the complete immersion of the two-conductor line in the liquid,[13] or wound on an insulating tube into which the investigated liquid is poured. The reduced dimensions of such systems, compared with two-conductor or coaxial lines, are a useful property of this method of measuring ϵ if employed on decimeter wavelengths.

1. Complete immersion of the helix in the dielectric. If the helix is completely immersed in an ideal unbounded dielectric medium with $\epsilon \neq 1$ and $\mu = 1$, then the expression for ϵ is

$$\varepsilon = \left(\frac{\lambda_0}{\lambda'_d}\,\mathrm{tg}\,\theta\right)^2 \frac{I_0\left(\frac{2\pi a}{\lambda'_d}\right) K_0\left(\frac{2\pi a}{\lambda'_d}\right)}{I_1\left(\frac{2\pi a}{\lambda'_d}\right) K_1\left(\frac{2\pi a}{\lambda'_d}\right)}. \qquad (3.14)$$

At high frequencies, (3.14) simplifies to

$$\varepsilon = \left(\frac{\lambda_0}{\lambda'_d}\,\mathrm{tg}\,\theta\right)^2 = \left(\frac{\lambda_d}{\lambda'_d}\right)^2. \qquad (3.15)$$

It is seen from (3.14) and (3.15) that the value of ϵ of the investigated liquid can be determined in the following fashion. Choosing a helix with sufficiently close winding ($\cot\theta \geq 10$), we measure the length of the standing wave λ_d in the helix in free space at the specified frequency f_0. The helix is then immersed in the investigated liquid and the length of the slow wave of the helix, λ'_d, is measured. These two measurements are sufficient to determine the dielectric constant of the liquid.

2. Liquid in tube. To measure the dielectric constants of liquids, it is more convenient in practice to use a system consisting of a dielectric tube (ϵ_T) of inside radius b, on which is wound a helix of radius a and which is filled with the investigated liquid (ϵ_l). In the case of close winding ($\cot\theta \geq 10$), for a known value of ϵ_T, the expression for ϵ_l is

$$\varepsilon_l = \frac{\varepsilon_T\{\Omega[\Delta'_{01}I_0(a) - q'_{00}I_1(b)] - \varepsilon_T[\Delta'_{01}I_1(a) - q'_{01}I_1(b)]\}}{I_1(b)[\varepsilon_T q'_{01} - \Omega q'_{00}]}, \qquad (3.16)$$

where

$$\left.\begin{aligned} \Omega &= \left(\frac{\lambda_0\,\mathrm{tg}\,\theta}{\lambda'_d}\right)^2 \left[\frac{I_0(a)}{I_1(a)} + \frac{K_0(a)}{K_1(a)}\right] - \frac{K_1(a)}{K_0(a)}, \quad \Delta'_{01} = \frac{\lambda'_d}{2\pi b}, \\ q'_{00} &= I_0(a)K_0(b) - K_0(a)I_0(b), \quad q'_{01} = I_0(b)K_1(a) + I_1(a)K_0(b). \end{aligned}\right\} \qquad (3.17)$$

We have left out the factor $2\pi/\lambda'_d$ in the arguments of the Bessel functions. As $b \to 0$ (the thickness of the tube tends to zero and $\epsilon_T \to \epsilon_l = \epsilon$), formula (3.16) becomes identical with (3.4). If $b \neq a$ but $\epsilon_T = 1$, we obtain from (3.16) formula (3.6) for the dielectric constant ϵ of a solid dielectric in the presence of a gap between the specimen and the helix.

In the case of high frequencies, when $2\pi a/\lambda'_d \geq 3$ and $2\pi b/\lambda'_d \geq 3$, formula (3.16) simplifies to

$$\varepsilon_l = \frac{\varepsilon_T\left(2E' - 1 - \frac{\varepsilon_T}{D'}\right)}{\varepsilon_T - \frac{2E' - 1}{D'}}. \qquad (3.18)$$

Equations (3.16) and (3.18) can be solved with respect to ϵ_T. When $\epsilon_l = 1$, we obtain expressions for the dielectric constant of a specimen in the form of a hollow dielectric cylinder. From (3.18) we get

$$\varepsilon_T = D'(E'-1) + \sqrt{(E'-1)D'^2 + 2E' - 1}, \qquad (3.19)$$

where

$$E' = \left(\frac{\lambda_0 \operatorname{tg}\theta}{\lambda'_d}\right)^2 = \left(\frac{\lambda d}{\lambda'_d}\right)^2, \quad D' = \frac{1 + \exp\left\{\frac{4\pi}{\lambda'_d}(b-a)\right\}}{1 - \exp\left\{\frac{4\pi}{\lambda'_d}(b-a)\right\}}. \qquad (3.20)$$

3. Measurement of ϵ with a Helical Waveguide and a Metal Shield

A helix with dielectric, placed in a metal shield, can be a useful instrument for the measurement of ϵ. This construction eliminates the influence of extraneous fields and therefore increases the measurement accuracy. The presence of the shield affects the form of the dispersion equation, and consequently the expression for ϵ. Figure 11b shows the most general system, consisting of a layered dielectric placed in a metallic waveguide of radius R. In the case of a dielectric placed in a tube of known value ϵ_2, the expression for ϵ_1 (when $\cot\theta \geq 10$) is given by

$$\varepsilon_1 = \frac{\varepsilon_2\left\{[A]\left[\frac{2\pi b}{\lambda'_d} I_1(b)\, q_{00}^{ab} - I_0(a)\right] + \varepsilon_2\left[I_1(a) - \frac{2\pi b}{\lambda'_d} I_1(b)\, q_{01}^{ab}\right]\right\}}{\frac{2\pi b}{\lambda'_d} I_1(b)\{q_{01}^{ab}[A] - \varepsilon_2 q_{01}^{aR}\}}, \qquad (3.21)$$

where

$$q_{01}^{ab} = I_0(b) K_1(a) + I_1(a) K_0(b), \quad q_{00}^{ab} = I_0(a) K_0(b) - I_0(b) K_0(a),$$
$$q_{11}^{aR} = I_1(R) K_1(a) - I_1(a) K_1(R), \quad q_{00}^{aR} = I_0(R) K_0(a) - I_0(a) K_0(R),$$
$$q_{01}^{aR} = I_0(R) K_1(a) - I_1(a) K_0(R), \quad q_{10}^{aR} = I_1(R) K_0(a) + I_0(R) K_1(R),$$
$$[A] = \left(\frac{\lambda_d}{\lambda'_d}\right)^2 \left[\frac{I_0(a)}{I_1(a)} + \frac{q_{10}^{aR}}{q_{11}^{aR}}\right] - \frac{q_{01}^{aR}}{q_{00}^{aR}} \qquad (3.22)$$

In the simpler case when the helix is completely filled with dielectric (and a shield is used), the following expression is obtained

$$\varepsilon = \left(\frac{\lambda_d}{\lambda'_d}\right)^2 \frac{I_0^2(a)}{I_1^2(a)} \frac{1 + \frac{I_1(a) K_0(a)}{I_0(a) K_1(a)}}{1 - \frac{I_1(a) K_1(R)}{I_1(R) K_1(a)}} - \frac{\frac{K_0(R)}{I_0(R)} - \frac{K_1(a)}{I_1(a)}}{\frac{K_0(a)}{I_0(a)} - \frac{K_0(R)}{I_0(R)}}. \qquad (3.23)$$

In the presence of a gap between the dielectric and the helix, the dielectric constant of the specimen is given by

$$\varepsilon = \frac{I_1(a) - I_0(a)\left\{\left(\frac{\lambda_d}{\lambda'_d}\right)^2\left[\frac{I_0(a)}{I_1(a)} + \frac{q_{10}^{aR}}{q_{11}^{aR}}\right] - \frac{q_{01}^{aR}}{q_{00}^{aR}}\right\}}{\frac{2\pi b}{\lambda'_d} I_1(b)\left\{q_{00}^{ab}\left[\left(\frac{\lambda_d}{\lambda'_d}\right)^2\left(\frac{I_0(a)}{I_1(a)} + \frac{q_{10}^{aR}}{q_{11}^{aR}}\right) - \frac{q_{10}^{aR}}{q_{00}^{aR}}\right] - q_{01}^{ab}\right\}} + 1. \qquad (3.24)$$

As $R \to \infty$, formulas (3.21), (3.23), and (3.24) go into the corresponding formulas for the "helix plus dielectric" system without a shield. The expressions for ϵ at high frequencies are the same as for an open helix, for then the waves are crowded closely against the helix, and therefore the shield exerts no influence on the slowing down factor.

4. Determination of tan δ by the Helical Waveguide Method[73]

The problem can be solved in two ways. The first is to obtain a dispersion equation for the investigated system from Maxwell's equations taking into account the fact that the dielectric constant is complex, and to separate from the dispersion equation the imaginary part, which characterizes the attenuation. The second reduces to solving the problem by the energy method, that is, to a determination of the attenuation coefficient in the system in terms of the ratio of the power lost to the total power flowing in the system.

1. Case of low and medium losses for cylindrical rods on which helices are wound. The dispersion equation of a helix closely wound on a dielectric rod with $\epsilon_r = \epsilon' - i\epsilon''$ has the form (when $\cot\theta \geq 10$)

$$\left(\frac{k}{k'_3}\operatorname{ctg}\theta\right)^2 = \frac{I_0(a) K_0(a)}{I_1(a) K_1(a)} \frac{k'_3 a}{\varepsilon_r I_1(a) K_0(a) + I_0(a) K_1(a)}. \qquad (3.25)$$

Since the dielectric is lossy, the propagation constant contains a term characterizing the attenuation, i.e., k'_3 can be written in the form

$$k'_3 = \gamma - i\alpha, \qquad (3.26)$$

where $\gamma = 2\pi/\lambda'_d$, and α takes the attenuation into account. In the case of low and medium losses, when $\epsilon'' < \epsilon'$ and $\alpha < \gamma$, we obtain from (3.25) the dispersion equation (3.1) for ϵ, and also an expression for tan $\delta = \epsilon''/\epsilon'$:

$$\operatorname{tg}\delta = \frac{\alpha a}{\varepsilon' I_0 I_1}\left[\left(\varepsilon' + \frac{\gamma^2 \operatorname{tg}^2\theta}{k^2}\frac{I_0^2}{I_1^2}\right)(I_1^2 - I_0 I_2) + \left(\frac{I_0^2}{K_0^2} + \frac{\gamma^2 \operatorname{tg}^2\theta}{k^2}\frac{I_0^2}{K_1^2}\right)(K_0 K_2 - K_1^2)\right]. \qquad (3.27)$$

The same formula is obtained for tan δ by the energy method.

At high frequencies, (3.27) simplifies considerably:

$$\operatorname{tg}\delta = \frac{\alpha\lambda'_d}{\pi}(\varepsilon' + 1). \qquad (3.28)$$

It is seen from (3.28) that at high frequencies the attenuation in the system, for a given specimen and for a given helix geometry, is proportional to the frequency. This becomes understandable from an analysis of the expressions for the power fluxes in the two regions of the helix. With increasing frequency, the power flux increases inside the helix and decreases outside. Because of the large interaction between the field and the substance, the attenuation of the system also increases. With increasing cotangent of the helix winding angle, the attenuation of the system increases, because the field becomes highly concentrated inside the rod and this increases the attenuation. Thus, to measure the attenuation of specimens with low losses it is necessary to modify the system so as to concentrate the greater part of the power flux inside the dielectric. This can be done by increasing cot θ or by changing over to a shielded system, for the shield essentially redistributes the power (increases the power flux inside the helix).

2. Determination of tan δ of a dielectric located inside a dielectric tube on which a helix is wound. Here, as in Sec. 3 of Ch. III, it is necessary to consider a helix with a layered dielectric. By subdividing the system into three regions:

$$\mathrm{I})\ 0 \leqslant r \leqslant b,\ \varepsilon_1 = \varepsilon_1' - i\varepsilon_1'';\quad \mathrm{II})\ b \leqslant r \leqslant a,\ \varepsilon_2 = \varepsilon_2' - i\varepsilon_2'';$$
$$\mathrm{III})\ a \leqslant r \leqslant \infty,\ \varepsilon = 1$$

and assuming that $\epsilon_1'' < \epsilon_1'$ and $\epsilon_2'' < \epsilon_2'$, we can determine ϵ by the energy method and obtain the following expression for the tangent of the loss angle of the dielectric situated inside the helix (ϵ_1', ϵ_2', and ϵ_2'' are known):

$$\operatorname{tg}\delta_1 = \frac{1}{\varepsilon_1' b I_0(b) I_1(b)} \{\alpha [b^2 M + \varepsilon_2' (A(a) a^2 - A(b) b^2) + A_2^2 (\Phi(b) b^2 - \Phi(a) a^2) + a^2 N] - \varepsilon_2'' (B(a) a - B(b) b)\}, \tag{3.29}$$

where

$$\left.\begin{aligned}
M &= \left\{\varepsilon_1' + \left(\frac{\lambda_0}{\lambda_{\mathrm{d}}'} \operatorname{tg}\theta\right)^2 \frac{1}{I_1^2(a)} \left[I_0(a) - k_3' b I_1(b) \left(1 - \frac{\varepsilon_1'}{\varepsilon_2'}\right) q_{00}\right]^2\right\} \\
&\quad \times [I_1^2(b) - I_0(b) I_2(b)], \\
N &= \left[I_0(a) - k_3' b \left(1 - \frac{\varepsilon_1'}{\varepsilon_2'}\right) I_1(b) q_{00}'\right]^2 \left[\frac{1}{K_0^2(a)} + \left(\frac{\lambda_0}{\lambda_{\mathrm{d}}'} \operatorname{tg}\theta\right)^2 \frac{1}{K_1^2(a)}\right] \\
&\quad \times [K_0(a) K_2(a) - K_1^2(a)], \\
A(r) &= [A_2 I_1(r) - B_2 K_1(r)]^2 + \frac{2}{k_3' r} [A_2 I_1(r) - B_2 K_1(r)] \\
&\quad \times [A_2 I_0(r) + B_2 K_0(r)] - [A_2 I_0(r) + B_2 K_0(r)]^2, \\
B(r) &= [A_2 I_1(r) - B_2 K_1(r)] [A_2 I_0(r) + B_2 K_0(r)]; \\
A_2 &= 1 - \frac{2\pi b}{\lambda_{\mathrm{d}}'} \left(1 - \frac{\varepsilon_1'}{\varepsilon_2'}\right) I_1(b) K_0(b), \quad B_2 = \frac{2\pi b}{\lambda_{\mathrm{d}}'} I_0(b) I_1(b) \left(1 - \frac{\varepsilon_1'}{\varepsilon_2'}\right), \\
\Phi(r) &= I_1^2(r) - I_0(r) I_2(r).
\end{aligned}\right\} \tag{3.30}$$

5. Measurements of ϵ with the Aid of a Moving Probe

To measure the dielectric constant by the helical waveguide method it is necessary to know the geometry of the specimen and of the helix, and also to determine the length of the slow wave λ_{d}' in the investigated system for a specified wavelength of the supply generator. The slow wavelength λ_{d}' can be measured with the system shown in Fig. 12. Probe diagrams plotted at constant generator power output can be used also to determine the attenuation coefficient α, which must be known to determine tan δ.

The dielectric constant of several cylindrical-rod samples was measured to verify formulas (3.4), (3.5), and (3.10). The substances investigated were vinyl plastic, organic glass, ebonite, porcelain, glass, and a special ceramic. Systematic investigations were made of the dispersion properties of "helix plus dielectric" type systems of different diameters and with different winding angles. Figures 13 and 14 show the dispersion relations for vinyl plastic and porcelain,

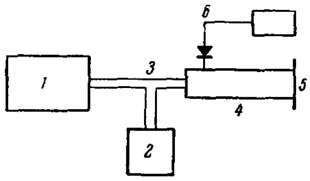


FIG. 12. Block diagram of probe methods for the measurement of ε: 1 – generator; 2 – wavemeter; 3 – coaxial cable; 4 – helix with dielectric; 5 – short circuit; 6 – probe, detector, and indicator.

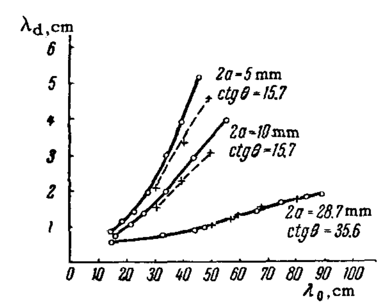


FIG. 13. Dispersion curves of the "helix plus dielectric" system (vinyl plastic). —— theory, —×— experiment.

calculated in accordance with (3.1) and plotted experimentally. A study of the dispersion curves and the frequency dependence of ϵ, made for specimens of the same dielectric but with different values of cot θ and different diameters D = 2a, shows that identical values of ϵ are obtained for identical D/λ_{d}'. Figures 15 and 16 show the dependence of ϵ on D/λ_{d}' for vinyl

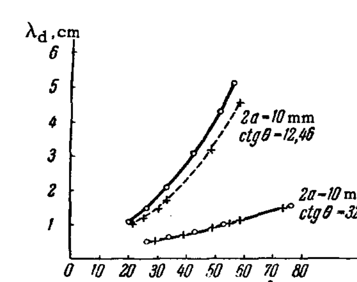


FIG. 14. Dispersion curves of the "helix plus dielectric" system (porcelain). —— theory, —×— experiment.

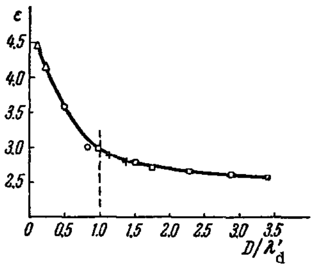


FIG. 15. Dependence of measured values of ε on D/λ'_d for vinyl plastic. Δ – 2a = 5 mm, cot θ = 15.7; □ – 2a = 28.7 mm, cot θ = 35.6; × – 2a = 28.7 mm, cot θ = 15; o – 2a = 10 mm, cot θ = 15.7.

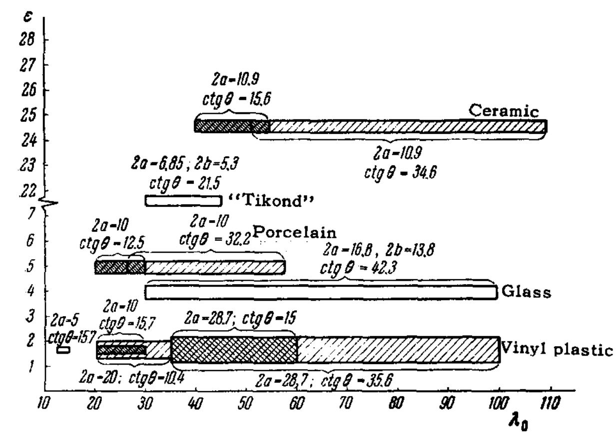


FIG. 17. Frequency range for specimens of different diameters, within which ε can be measured by the helical waveguide method with less than 10 percent error.

plastic and porcelain. From the plots of Figs. 15 and 16, and on the basis of the data obtained by investigating other substances (organic glass, ebonite, glass), we can draw the following conclusions:

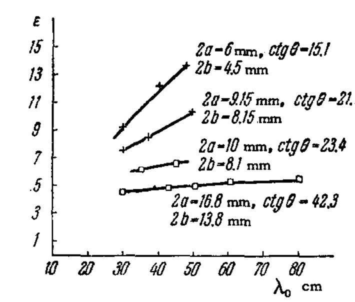
1. As the ratio D/λ'_d is decreased, ϵ first increases slowly and then (after passing through the value $D/\lambda'_d \sim 1$) the increase is faster. The guiding rule for measurements of ϵ must therefore be that the specimen dimensions and the frequency range be such that the condition $D/\lambda'_d > 1$ be satisfied.

2. Figures 15 and 16 show that the error in the measurement of ϵ varies with the interval of D/λ'_d. Thus, for substances with $\epsilon \sim 2-3$, the measurement error reaches 15 percent when $1 \leq D/\lambda'_d \leq 1.5$, dropping to 7 percent when $1.5 \leq D/\lambda'_d \leq 2$ and to 3 percent when $2 \leq D/\lambda'_d \leq 3.5$. In addition, as ϵ increases the region with the lower measurement error shifts toward the lower values of D/λ'_d. Thus, in the case of porcelain, the region of values of D/λ'_d for which the measurement error is less than 10 percent lies between 1 and 1.5.

3. A frequency range in which ϵ can be measured can be established for each specimen. On the short-wave side this range is limited by the frequency at which the slow wave becomes commensurate with the pitch of the helix, and on the long wave side the limit is the ratio D/λ'_d, which must not exceed 1.

4. To cover a wide range of frequencies, several specimens of different diameters must be used. Figure 17 shows the range of frequencies within which the error in the measurement must not exceed 10 percent for specimens of different diameters. Tubes made of glass, porcelain, and a special ceramic were measured to verify formula (3.20). Figure 18 shows the frequency dependence of the calculated values of ϵ while Fig. 19 illustrates the dependence of ϵ on D/λ'_d, where D is the outside diameter of the tube. From the curves of Fig. 19, and also from an analysis of the data obtained by investigating other substances, it can be concluded that to obtain more accurate values of ϵ it is necessary to satisfy the condition $D/\lambda'_d > 1$.

FIG. 18. Dependence of measured values of ε of glass tubes on the wavelength. □ – molybdenum glass; × – ordinary glass.



To verify formulas (3.16) and (3.18) for the dielectric constant of liquids, we measured the values of ϵ of gasoline, benzene, ether, acetone, and distilled water, in glass and ceramic tubes. Several theoretically calculated and experimentally plotted dispersion relations were compared for systems of the "helix plus layered dielectric" type. Figure 20 shows the theoretical and experimental curves for a glass tube filled with benzene or with distilled water, while Fig. 21 shows the dispersion curves for the case of acetone in a tikond tube ($\epsilon_T = 21$) and for acetone in a glass tube ($\epsilon_T = 6$). It is seen from these curves that a disparity exists between the theoretical and experimental curves in the short and long wave regions. The width of the region where the two coincide depends on

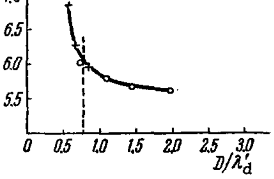
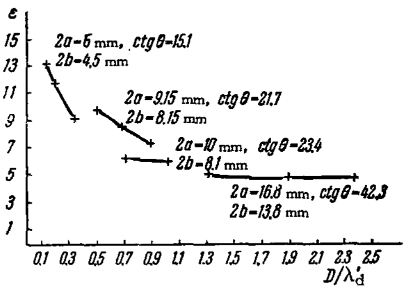


FIG. 16. Dependence of measured values of ε on D/λ'_d for porcelain. o – 2a = 10 mm, cot θ = 32.2; × – 2a = 10 mm, cot θ = 12.46.

FIG. 19. Dependence of measured values of ε on D/λ'_d, for tubes of different brands and diameters.



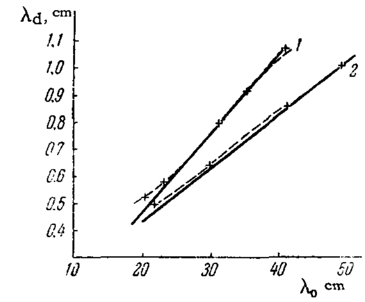


FIG. 20. Dispersion curves for "helix plus layered dielectric" systems. 1 – Benzene in glass tubes; 2 – distilled water in glass tubes; — theory, —×— experiment.

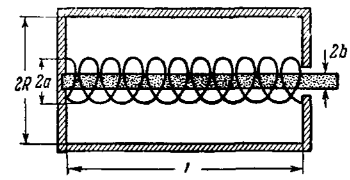


FIG. 22. Helical cavity with dielectric.

the geometry of the tubes and on the ratio of the dielectric constants of the tube and the liquid in the tube. It is seen from Fig. 19 that the region of agreement is broader for acetone in a "tikond" tube than for a glass tube. Measurements of substances in tubes with different ϵ_T lead to the conclusion that when $\epsilon_T < \epsilon_l$ it is necessary to work in the range $0.9 \leq D/\lambda_d' \leq 1.1$. If $\epsilon_T > \epsilon_l$ the range $0.7 \leq D/\lambda_d' \leq 1.5$ can be used. In this case the measurement errors do not exceed 10 percent.

The determination of tan δ of a substance has been reduced to a measurement of λ_d' and of the attenuation coefficient α and to calculations with formulas. Formulas (3.27) and (3.28) were checked for specimens made of vinyl plastic, ebonite, and getinaks (micarta). The values of tan δ of getinaks were found to be close to those tabulated, while the values for ebonite and vinyl plastic were somewhat higher than those in the table. The reason is that the attenuation produced in a low-loss dielectric is commensurate with the attenuation caused by the helix, and consequently a rigorous account of the losses in the helix becomes necessary.

6. Determination of ϵ and tan δ by the Helical and Loaded Cavity Methods[74]

Measurements of the dielectric constants of substances with high losses by the helical waveguide method are quite difficult, because the wave attenuates rapidly in such systems. The attenuation can be reduced by using thin specimens located on the axis of the system. But the slowing down effected by the introduction of a thin specimen is greatly reduced. Measurement of small changes of the length of the slow wave can lead to large errors. The above-described method can therefore be modified by changing over to the resonator method of measurement. A resonator is produced when a shielded helix is shorted on both sides by conducting planes. If the helix is then energized at high frequency, resonance will set in the system whenever the length of the helix is equal to a whole number of half waves. The presence of a helix in the cylinder produces slow waves of the helical type. Since the helix is a broadband system, a cavity of the helical type of fixed length will resonate in a broad band of frequencies. The resonance frequency of such a cavity is changed by placing on the axis of the system a thin specimen of the same length as the system. The shift in the resonant frequency will be proportional to the dielectric constant, and the change in the Q of the system yields the value of tan δ of the investigated substance. Since it is possible to measure relatively small frequency shifts, specimens with high losses can be investigated, and these specimens can be made sufficiently thin so as not to reduce greatly the value of Q, and to produce on the other hand a noticeable frequency deviation.

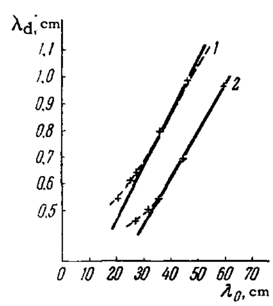


FIG. 21. Dispersion curves for "helix plus layered dielectric" systems. 1 – Acetone in glass tube, 2a = 10 mm, 2b = 8.1 mm, cot θ = 23.4, ε_2 = 6; 2 – acetone in tikond tube, 2a = 6.85 mm, 2b = 5.3 mm, cot θ = 21.5 ε_2 = 21.

Resonators can also be made with other slow-wave systems, such as a segment of a loaded waveguide. It should be noted that the pass band of the latter is much smaller than that of a helical cavity. The character of the dispersion determines the variation of the field intensity on the symmetry axis on going from one resonance to another. The intensity of the field changes slowly from one resonance to another in a helical resonator but abruptly in a loaded waveguide. This property makes the helical resonator suitable for the measurement of substances in a broad band of frequencies at an approximately uniform sensitivity. On the other hand, the sharp dependence of the field intensity on the frequency in a loaded cavity makes the latter suitable for measurement of ϵ of substances within a wide range of loss angle.

1. Cavity made of a segment of a coaxial helix. In measurements by the resonance method it is necessary to place the specimen, made in the form of a cylindrical rod, inside the cavity. It is more convenient in practice to use specimens with diameter less than that of the helix. To measure ϵ and tan δ of a high-loss substance the specimen diameter must be small. If the specimen diameter 2b (Fig. 22) is small enough to make a $\geq$ 5b (a is the radius of the helix), and if in addition the radius of the shield is R $\geq$ 5a, the expression for ϵ is

$$\varepsilon = \frac{2\Delta f}{f} \frac{\mu_1^b + \mu_0^b}{\mu_0^a - \mu_0^R - 2\frac{\Delta f}{f}\mu_0^b}. \tag{3.31}$$

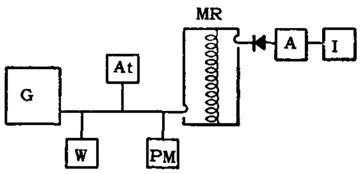


FIG. 23. Block diagram for the measurement of the frequency shift by the transmission method. G – generator, W – wavemeter, At – attenuator, PM – power meter, MR – measuring resonator, A – amplifier, I – indicator.

where

$$\mu_n^x = \frac{K_n\left(\frac{2\pi x}{\lambda'_d}\right)}{I_n\left(\frac{2\pi x}{\lambda'_d}\right)},$$

f_0 is the resonant frequency corresponding to the propagation constant $k_0 = 2\pi/\lambda_0$ and Δf is the frequency shift due to insertion of the dielectric in the cavity. The procedure for measuring ϵ can be as follows. We determine beforehand the slow wavelengths λ'_d at specified resonant frequencies of the investigated system (these are measured by the perturbation method). The specimen is then inserted at any of these frequencies and the frequency deviation measured.

The tangent of the loss angle is determined from the change in the Q brought about by inserting the investigated specimen into the cavity. If $a \geq 5b$ and $R \geq 5a$, then

$$\operatorname{tg}\delta = \frac{1}{\varepsilon}\left(\frac{1}{Q} - \frac{1}{Q_0}\right)\frac{2a}{k_3'^2 b^2}\left[G + \Gamma^2 \frac{q_{01}^{aR}}{q_{00}^{aR}}\right], \tag{3.32}$$

where

$$\left.\begin{aligned} &\Gamma = I_0(a) - \frac{1}{2}(k_3'b)^2(1-\varepsilon)\,q_{00}, \quad q_{00} = I_0(a)K_0(b) - I_0(b)K_0(a),\\ &q_{00}^{aR} = I_0(R)K_0(a) - I_0(a)K_0(R), \quad q_{01}^{aR} = I_0(R)K_1(a) + I_1(a)K_0(R),\\ &G = \left\{I_0(a) + \frac{1}{2}(k_3'b)^2(1-\varepsilon)\left[I_0(a)\ln\frac{k_3'b}{2} + K_0(a)\right]\right\}\\ &\qquad \times\left\{I_1(a) + \frac{1}{2}(k_3'b)^2(1-\varepsilon)\left[I_1(a)\ln\frac{k_3'b}{2} - K_1(a)\right]\right\}.\end{aligned}\right\} \tag{3.33}$$

Here Q_0 pertains to the air-filled cavity and Q to the system with the specimen.

2. Cavity made of segment of loaded waveguide. For specimens with small transverse dimensions we can use the following simplified formula for ϵ. It is shown in references 75 and 76 that the relative shift of the cavity frequency (whether the cavity be of simple or complex form), caused by insertion of a specimen of length equal to the length of the system, is

$$\frac{\Delta f}{f} = \frac{\int_V \mathbf{PE}\,dv}{W}, \tag{3.34}$$

where V is the volume of the perturbing body, **E** is the intensity of the field outside the perturbing body, $\mathbf{P} = (\epsilon - 1)\mathbf{E}_1$ is the vector of electric polarization of the dielectric body ($\mathbf{E}_1$ is the field intensity inside the perturbing body), ϵ is the dielectric constant of the perturbing body, and W is the total energy stored in the cavity. Using suitable simplifications, we obtain from (3.34)

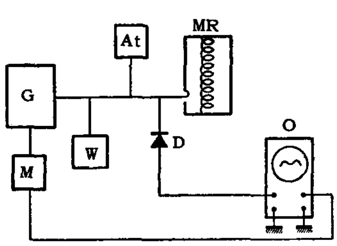


FIG. 24. Block diagram for the measurement of frequency shift by the leakage method. G – generator, M – modulator, W – wavemeter, At – attenuator, D – detector, MR – measuring resonator, O – oscilloscope.

$$\varepsilon = \frac{8\left(\frac{\Delta f}{f}\right)}{S v_{gr}\frac{|E_{z0}|^2}{P}}, \tag{3.35}$$

where $P = Wv_{gr}/l$ is the energy flux density in the system, S is the cross section area of the specimen, and v_{gr} is the group velocity. The quantity $|E_{z0}|^2/W$ is proportional to the coupling resistance and is fully determined by the geometry of the system and by the frequency. Therefore, by calculating or measuring beforehand the values of $|E_{z0}|^2/W$ at different resonant frequencies, we can use (3.35) to determine the dielectric constant. In the presence of a control specimen with known value of ϵ_1, we can determine the frequency shift Δf_1 due to introducing the control specimen. Then ϵ_x of an unknown specimen with the same transverse dimensions can be determined from the formula

$$\varepsilon_x = \frac{\Delta f_x}{\Delta f_1}(\varepsilon_1 - 1) + 1. \tag{3.36}$$

Measurement of ϵ by the resonator method reduces therefore to a measurement of the resonant-frequency shift due to introducing the investigated specimen into the cavity. Two methods were used to measure the frequency shifts. The first is illustrated in Fig. 23 and the second in Fig. 24. The Q of the resonator, needed to determine tan δ, was measured from the width of the resonant curve at the half-power level.

To verify formulas (3.31) and (3.32), we measured ϵ of solid-rod specimens and of liquids in capillary tubes. The measurements of ϵ of a polystyrol wire 1 mm in diameter yielded values $\epsilon = 2.3 - 2.6$ on wavelengths $10 - 50$ cm. The values of ϵ of a glass

Table I

Phase shift per cell, ψ		π		$\frac{2}{3}\pi$		$\frac{1}{3}\pi$		0	
Resonance of hollow cavity, Mc/sec		$f_1=2783$		$f_2=2750$		$f_3=2676$		$f_4=2638$	
Substance	Specimen diameter	Δf, Mc/sec	ε	Δf, Mc/sec	ε	Δf, Mc/sec	ε	Δf, Mc/sec	ε
Ebonite	5 mm	3.4	—	6.8	—	15	—	21.2	—
Vinyl plastic	5 mm	3.4	2.63	6.8	2.63	15	2.63	21	2.6
Organic glass	5 mm	3.4	2.63	6.8	2.63	15	2.63	21.2	2.63
Ordinary glass	5 mm	11.8	6.7	23.2	6.54	54.5	6.9	84.3	6.8
Bakelite	5 mm	6.4	4.06	13	4.2	29.5	4.2	42.3	4.25
Getinaks	5 mm	6.4	4.06	11.6	3.94	26.8	3.9	38.8	4.06

Table II

No.	1	2	3	4	5
ε_{calc}	23.5	48.3	50.22	51.5	63
ε_{exp}	22.6	46.6	50.0	50.4	65

rod 2 mm in diameter, in the same band, were 4.2 — 4.8, while tan δ ranged from 0.002 to 0.003. The measurements of ϵ and tan δ of liquids were carried out in capillaries with inside diameter 0.64 mm and wall thickness 0.075 mm. In the range 10 — 50 cm, ϵ of distilled water ranged from 60 — 70, while tan δ ranged from 0.15 to 0.2. Measurements were made of ϵ and tan δ of methyl and ethyl alcohol in the 51.7 — 15 cm range. The values obtained for both alcohols agreed well with the dispersion curves obtained for alcohols by various authors.[77,50]

To verify (3.35), we prepared specimens of ebonite, vinyl plastic, organic glass, glass, bakelite, and getinaks. The specimens were made into rods 5 mm in diameter and the same length as the system. The control specimen was ebonite with $\epsilon = 2.63$. The results of the measurements made at different resonances are listed in Table I. The dielectric constant of liquids was measured in glass capillaries with inside diameter 1 mm and outside diameter 1.1 mm. The control specimen was distilled water ($\epsilon_1 = 77$). The investigated substances were specially prepared mixtures of dioxane and distilled water. The results of measurements made at 2676 Mc/sec are listed in Table II.

Thus, the helical waveguide method can be used along with other well known methods for the measurement of ϵ and tan δ.

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