THE PRESENT STATE AND LINES OF DEVELOPMENT OF OPTICAL FLAME PYROMETRY*

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Usp. Fiz. Nauk 76, 683-710 (April, 1962)

INTRODUCTION

IN solving many scientific and technical problems, one has to measure the temperatures of flames of various sorts and of hot gases, e.g., in studying combustion processes in various furnace installations, metallurgical furnaces, internal-combustion engines, jet engines, etc.

The thermoelectric instruments widely applied to measure the temperatures of solids (thermocouples, resistance thermometers, etc.) are in a number of cases practically useless for the measurement of flame temperatures. Most of the commonly-used thermocouples (not to mention resistance thermometers) do not withstand temperatures above 1500°C, and hence cannot be applied to measure flame temperatures.

However, there are thermocouples made of refractory alloys which withstand higher temperatures (up to $2800-3000^{\circ}$ K). These thermocouples are used in measurement of flame temperatures. However, in measuring flame temperatures with thermocouples, additional measurement errors arise (as compared with the measurement of the temperature of a solid object). First of all, there are errors connected with the quite appreciable radiation losses at high temperatures, amounting to several hundreds of degrees at flame temperatures of $2000-3000^{\circ}$ K.

The use of protective shields, sometimes even specially heated, and other means of reducing the radiation loss from the thermocouple considerably complicate the construction and increase the clearance dimensions required by the thermocouple, without completely eliminating the errors.

At high velocities of hot gas streams (> 100 m/sec), thermocouples actually measure not the true thermodynamic temperature of the flame, but some temperature lying between the true temperature and the so-called impact temperature. In order to determine the true temperature, we must know the velocity of the current of hot gas and the coefficient of restitution. These quantities are usually unknown. The value of the coefficient of restitution depends on the shape of the thermocouple and the velocity of the gas flow. Additional errors arise in measuring the temperature of supersonic gas streams.

Errors also arise from the diffusion of one component of the thermocouple into the other at high temperatures.

Another important fact is that introducing a thermocouple into a flame may affect the combustion processes. In particular, the thermocouple may exert a catalytic effect on the combustion process, leading to an increase in temperature in the zones in contact with the thermocouple, and a corresponding distortion in the response of the thermocouple.

The thermoelectric methods of temperature measurement exhibit lag, and hence cannot be applied to measure rapidly changing temperatures.

The other methods for measuring flame temperatures involving the placing of a sensitive element in the flame (e.g., measurement with double-junction thermocouples, resistance thermometers, incandescent filaments, etc.) also have similar defects. Hence, although thermocouples are rather widely applied at present to measure flame temperatures, the region of applicability of thermoelectric methods of temperature measurement is essentially highly limited. Also, the errors of these sort of measurements at high tempera tures are large and difficult to correct.

The optical methods of temperature measurement, which make it possible to determine the temperature of a flame from its radiation without introducing a sensitive element into the flame, are in principle free from the defects mentioned above.

A large number of methods and instruments are mentioned in the literature, making possible the measurement of the temperature of either a transparent or a sooty (luminous) flame. These are the methods of luminance and color pyrometry, of comparison of luminances of emission and of absorption of sooty flames, spectral-line reversal methods, methods of relative intensities, methods of isolating the center of a spectral line for non-luminous flames, and several other methods. [1-32]

However, in applying these methods to measure the temperature of an actual flame, the errors of measurement are often considerably greater than expected, amounting to several hundred degrees.

As we shall see below, all of the methods of optical pyrometry cited above permit temperature measurement with satisfactory accuracy on a sort of ideal flame having the following characteristics:

1. The flame has a homogeneous temperature field.

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^{*}A revised draft of papers presented by the author at the First All-Union Conference on the Measurement of High Temperatures (September, 1960, L'vov) and at the Conference on Methods of Measuring the Temperatures of Flames, Gas Streams, and Plasmas (September, 1961, Leningrad).

The flame exhibits neither a monotonic radial temperature gradient, nor an accidental non-monotonic variation due to inhomogeneities of the flame in its fueloxidant ratio.

2. The temperature of the flame is not timedependent. Nor are there periodic oscillations in the temperature of any region of the flame.

3. The chemical processes in the flame may be considered to be at equilibrium, and the radiation from the flame is thermal radiation.

The problem of how well the emission from a flame obeys the laws of thermal emission has been subjected to many theoretical and experimental studies, [5, 33, 39]and we shall confine ourselves here to a few remarks.

In measuring the flame temperature in the combustion zone from the radiation of the combustion products, especially in low-pressure flames, chemiluminescence is often observed, leading to large errors in temperature measurement. A non-equilibrium energy distribution over the degrees of freedom is observed in rapid processes (e.g., in explosions). In such a case, we commonly speak of translational, rotational, vibrational, and electronic temperatures. The existence of uncompensated losses of energy by emission leads to a decrease in the number of excited molecules, and the measured temperature will be lower than the equilibrium temperature. However, this error becomes appreciable only in low-pressure flames, as it does not exceed 30° in flames burning at a pressure of 0.1 atm. [5,36] In any case, for an ordinary flame burning at atmospheric pressure, with a suitable choice of the working range of the spectrum and the method of measurement, as a rule we may exclude the effect of the factors cited above, and consider the flame emission to be thermal.

On the other hand, the requirements of homogeneity of the flame in space and time, as formulated in points 1 and 2, are never fulfilled in real flames.

Our problem consists in the theoretical and experimental analysis of the effect of inhomogeneities of the flame in space and time on the accuracy of measurement of the temperature by optical-pyrometry methods using various spectral regions. This makes it possible to evaluate the possibilities of optical flame pyrometry (in its present state) and to point out pathways of future development.

Why is inhomogeneity of the flame in space and time a source of error in the measurement of its temperature?

The problem is that when we measure by opticalpyrometric methods the temperature of a flame or a region of a flame having a temperature field which is inhomogeneous or varies in time, we are measuring a certain average optical temperature. In general, owing to the nonlinear dependence of the emission intensity on the temperature, this temperature will agree neither with the mass-average nor the arithmetic-mean temperature (the definitions of these temperatures will be given below) characterizing the physical processes occurring in the flame. This temperature will also differ from the maximum temperature of the flame, which is also an important characteristic of the flame.

Thus, our problem is reduced to analyzing the influence of the various properties of a real flame, the method of measurement chosen, and the spectral region used on the type of averaging process which occurs in measuring the flame temperature. In this analysis, we shall rely largely on the results of our own studies.

We have performed an experimental test of the theoretical concepts given in this article on two types of flames: a free high-velocity gasoline-air flame, of the type occurring in a ramjet engine, and a confined (air-illuminating gas) flame, of the type used in a metallurgical furnace. In the latter case, we provided the possibility of continuously varying the degree of mixing of the fuel and oxidant. This permitted us to test directly the effect of the degree of mixing on temperature measurements using the visible and infrared radiation.

The chief participants in the experiments on the free flame were O. N. Dubrovskaya and Ya. I. Merson. The following methods and instruments were used: calculation of the temperature field, developed by O. N. Dubrovskaya; ^[8,40] the IKP-3 infrared pyrometer, developed by Ya. I. Merson; ^[8] the OFO-3 apparatus for objective measurement of flame temperatures by the spectral-line reversal method, developed by M. R. Shamilev; ^[8] and local coloring of the flame, developed by M. R. Shamilev and N. I. Pomogin. ^[8] The measurements of temperature pulsations in the jet and the control determination of temperature from gasanalysis data were carried out by K. P. Vlasov.

The measurements on the confined flame were conducted on a test stand developed by V. A. Krivandin of the Furnace Department of the Steel Institute. The chief participant in the experiments was V. A. Dokuchaeva. The infrared pyrometer used was the standard instrument FÉP-3, in which the detector had been modified.^[46] A lead sulfide photoresistor (FSA-1) was used in the studies in the spectral region near 2.7μ , and also in the IKP-3. In the studies in the spectral region near 4.3 μ , a refrigerated (down to liquid-nitrogen temperature) photoresistor was used. The spectralline reversal method was applied in its visual variant. Temperature pulsations were measured with an ion gauge. The degree of mixing was varied by varying the path length through which the mixture of gaseous fuel and oxidant passed up to the point of ignition.

I. THEORETICAL SECTION

I. 1. Definition of the mean temperature of a flame having an inhomogeneous temperature field. Let us consider an axially symmetrical flame whose temperature is maximum in the center and decreases monotonically toward the edges. We shall assume the flame to be stationary. The effect of unsteadiness, i.e., timedependence of the flame temperature, as well as that of accidental non-monotonic temperature variations along the radius of the flame (due to poor mixing of the fuel and oxidizing agent), on the value of the measured average optical temperature will be discussed below.

Since the pyrometer will collect radiation from regions of varying temperatures through the thickness the flame (Fig. 1), measurement of the total radiation by any method will provide us with some sort of average of the temperature over the thickness. We shall call this temperature the mean optical temperature, with the symbol \overline{T} . The measured mean optical temperature will in general agree with neither the massaverage nor the arithmetic mean temperature. Naturally, it will also not agree with the maximum temperature at the axis of the flame being studied.



FIG. 1. The flame zone whose radiation is received by the pyrometer.

By definition, the arithmetic mean temperature is

$$\overline{T}_{arith} = \frac{\int_{0}^{d_{0}} T(x) dx}{d_{0}}, \qquad (1)$$

where T(x) is the true temperature at the point x, and d_0 is the diameter of the cross section of the flame being studied. T(x) characterizes the temperature profile, or variation of the temperature through the thickness of the flame. If the density of the hot gases in the flame also varies at different points through the thickness of the flame, then we can introduce the mass-average temperature

$$\overline{T}_{mass} = \frac{\int_{0}^{d_{0}} m(x) T(x) dx}{\int_{0}^{d_{0}} m(x) dx} , \qquad (2)$$

where m(x) gives the variation in the density of the hot gases through the thickness of the flame.

If we neglect the variation in density over the thickness of the flame, then $\overline{T}_{mass} = \overline{T}_{arith}$; for example, for the temperature profile shown in Fig. 2,

 $T_{\text{max}} = 3000^{\circ} \text{ K}, \quad \overline{T}_{\text{mass}} = \overline{T}_{\text{arith}} = C \cdot 3000^{\circ} \text{ K},$

where C = 0.52.



FIG. 2. The radial temperature distribution d = transverse coordinate of flame; $d_0 =$ flame diameter.

Since from the standpoint of studying combustion processes we are always interested in the arithmeticmean, the mass-average, or the maximum flame temperature, we must find the relation between \overline{T} and (e.g., for an axially-symmetric temperature profile) the maximum temperature at the axis of the flame. That is, we must find $\gamma' = \overline{T}/T_{max}$ as a function of the flame parameters for the various methods of measurement. Otherwise, we would be practically unable to utilize the results obtained by measurement of the mean optical temperature. If, however, we take \overline{T} to be the maximum or the mass-average temperature, we shall introduce errors of the order of several hundreds of degrees, as we shall see below.

If we measure the luminance or color temperature of the flame, rather than the true temperature, then in a flame having an inhomogeneous temperature field we shall measure the mean optical luminance temperature \overline{T}_l or the mean optical color temperature \overline{T}_c . The relation between these mean temperatures and the mean true temperature is given by the well-known formulas:^[1]

$$\frac{1}{\overline{T}_{l}} = \frac{1}{\overline{T}} - \frac{\lambda}{c_{2}} \ln \varepsilon_{\lambda T}, \qquad \frac{1}{\overline{T}_{c}} = \frac{1}{\overline{T}} - \frac{\ln \frac{c_{2}}{\varepsilon_{1}}}{c_{2} \left(\frac{1}{\lambda} - \frac{1}{\lambda}\right)}, \qquad (3)$$

where ε_i is the emissivity for the wavelength $\lambda_i.$

The averaging effect determines the quantity $\gamma' = \overline{T}/T_{max}$; γ' depends on the temperature profile and the value of the emissivity ϵ of the flame.

In order to determine $\gamma = \overline{T}_l / T_{max}$ or \overline{T}_c / T_{max} , we must transform from \overline{T} to \overline{T}_l or \overline{T}_c , respectively, by Eqs. (3).

To find the value of γ' , we may use the following method.^[4,8]

Let us consider some standard flame having the same optical parameters as the flame being studied. That is, the optical thickness, the distribution of emitting impurities, and hence also the value of the emissivity of the standard flame must agree with the corresponding values for the flame being studied. However, in distinction from the flame being studied, the standard flame must have a homogeneous radial temperature distribution. We shall denote as the "mean true temperature" of the studied flame that temperature of the standard flame at which the monochromatic luminance or the ratio of monochromatic luminances of this flame are equal to the corresponding values of the flame being studied.

Thus, we may find the pure averaging effect, or the value of γ' , which depends on the method of measurement used, as we shall see below.

We can easily see that this mean optical temperature can be greater than the arithmetic-mean or the mass-average temperature, since the emission intensity rapidly increases with temperature (increasingly as λ decreases). Thus, the specific contribution of the hotter layers to the total emission is much greater that that of the colder layers (and not in proportion to the temperature). However, on the other hand, if we consider flames with an axially-symmetric temperature distribution and a temperature maximum on the axis of the flame (as most often occurs in flames and hot-gas streams in practice), the radiation from the hotter but deeper layers will be absorbed more than that from the cooler layers nearer the observer. Thus, the two mentioned factors act in opposite directions, and the final result, i.e., the value of the mean optical temperature and its deviation from the arithmetic-mean temperature for a given temperature profile and absorption coefficient of the flame, depends on the degree of influence of each of these factors.

Let us consider the relation of the flame emission to the absorption coefficient $A_{\lambda T}$, i.e., to the concentration of emitting substances in the flame. The emission intensity of each volume element of the flame, and hence of the flame as a whole, must initially increase with $A_{\lambda T}$ according to Kirchoff's radiation law. However, absorption of the radiation from the deeper and hence hotter layers will play a greater role as $A_{\lambda T}$ increases. Thus, as the absorption increases, the hotter layers will play a smaller and smaller role. Hence, in the case being discussed, in distinction from that when the temperature is constant throughout the cross-section, the emission intensity of the flame (and hence, the luminance temperature) will initially increase with $A_{\lambda T}$, and then reach a maximum and begin to decline.

In order to determine the averaging process occurring in the measurement of the color temperature, we must compare the ratios of the monochromatic luminances for two wavelengths for the standard and the actual flame. In both cases, as has been stated above, we shall obtain thus the value of γ' , from which we can, if necessary, transform to γ , using the common expressions [Eq. (3)].

The luminance of the monochromatic emission of the standard flame $I_{\lambda T}$ is

$$I_{\lambda T} = I_{\lambda T}^{0} (1 - e^{-A_{\lambda T} d_{0}}), \qquad (4)$$

where $I_{\lambda T}^{0}$ is the luminance of a black body at a temperature equal to that of the standard flame; $A_{\lambda T}$ is the monochromatic absorption coefficient; and d_{0} is the diameter of the flame.

The monochromatic intensity of emission of a real flame having an inhomogeneous temperature field is defined by the expression

$$I_{\lambda} = -\int_{0}^{d_{0}} F(l) I_{\lambda T}^{I} e^{-A_{\lambda}T^{l}} dl,$$
 (5)

where F(l) gives the concentration distribution of the impurities through the cross-section, and T and $A_{\lambda T}$ are functions of l (the transverse coordinate of the flame).

If we write an expression for Kirchhoff's radiation law for a unit volume element, and assign values to F(l), T(l), and $A_{\lambda T}(l)$ [the value of I_{λ} is defined especially as a function of T(l), we can calculate I_{λ} by approximate-integration methods. Then, by comparison of the monochromatic luminances of the real and the standard flame, as was stated above, we can find the desired mean optical temperature, and then the values of \overline{T}_l/T_{max} , \overline{T}_c/T_{max} , and \overline{T}/T_{max} for various temperature profiles and values of $A_{\lambda}Td_0$.^[4,8]

As an illustration, Figs. 3, 4, and 5 give the results of calculations for the temperature profile shown in Fig. 2.



FIG. 3. Ratio of the mean luminance temperature to the maximum true temperature as a function of the optical density $A_{\lambda T}d_0$ for $\lambda = 0.4 \times 10^{-4}$ cm and $T_{max} = 3000^{\circ}$ K. $A_{\lambda T}$ = absorption coefficient; d_0 = flame diameter.

FIG. 4. Graph of the ratio of the mean color temperature to the maximum true temperature as a function of the diameter d_0 and the optical density $A_{\lambda}Td_0$. $A_{\lambda}T$ = absorption coefficient for $\lambda_1 = 0.4\mu$, $\lambda_2 = 0.6\mu$; $A_1/A_2 = 0.8/0.5$; T_{max} = 3000°K.





FIG. 5. Graph of the ratio of the mean true temperature to the maximum true temperature as a function of $A_{\lambda T}d_0$ for $\lambda = 0.6 \mu$, $T_{max} = 3000^{\circ}$ K. $A_{\lambda T}$ = absorption coefficient.

Thus, as is often the case, if the temperature profile and the value of $A_{\lambda}Td_0$ are unknown or poorly known, the value of the maximum or the arithmetic mean temperature may be determined only very inaccurately from the measured mean optical temperature.

The calculations indicate [4,8] that the value of the mean color temperature depends relatively little on the temperature profile, as the latter varies within rather wide limits. On the other hand, the mean true temperature, as measured, e.g., by the method of spectral-line reversal, depends relatively weakly on

the concentration of emitting impurities. All of this must be taken into account in choosing the method of measurement.

The nature of the averaging process depends considerably on the method of measurement applied. Hence, if we measure on one and the same flame, e.g., the mean luminance and color temperatures, and then transform by the usual Eqs. (3) from the mean luminance or color temperature to the mean true temperature, then, owing to the differing natures of the averaging process, we shall obtain mean true temperatures which differ from each other and from the mean true temperature as measured by the spectral-line reversal method by several hundreds of degrees.

For the same temperature profile and the same values of $A_{\lambda}Td_0 = 0.5$, we obtain for various wavelengths [4,8,45]

$$\begin{aligned} & \text{for} \quad \lambda = 0.6 \ \mu & \gamma' = 0.81, \\ & \lambda = 2.7 \ \mu & \gamma' = 0.64, \\ & \lambda = 4.3 \ \mu & \gamma' = 0.56, \\ & \lambda = 10 \ \mu & \gamma' = 0.545, \\ & \gamma'_{\text{arith}} = 0.52. \end{aligned}$$

The mean optical temperature is as a rule higher than the arithmetic mean temperature, but approaches it as the working wavelength increases.

This is explained by the fact that $I_{\lambda T}$ always increases with increasing temperature at a rate greater than linear, but more slowly as λT increases. Only for very large λT ($kT \gg h\nu$, where ν is the frequency of the radiation, and $h\nu$ is a quantum of the radiation energy) does Planck's formula go over into the Raleigh-Jeans formula $I_{\lambda T} = (8\pi/c^3) \nu^2 kT$. In this case, $I_{\lambda T}$ is proportional to T, and $\overline{T} = \overline{T}_{arith}$.

This is true when the absorption coefficient is not too large. When the values of the absorption coefficient in the external (cold) parts of the flame are large, the radiation from the hotter parts of the flame may not reach the pyrometer at all. This leads to a corresponding decrease in the value of \overline{T} measured. Thus for very large values of the absorption coefficient, \overline{T} may even be lower than the arithmetic mean temperature. In such a case, the optical temperature measured from the visible radiation may be even closer to the arithmetic mean than that measured from the infrared radiation. However, we encounter such flames rather seldom, and the methods of optical pyrometry are not very suitable for them, since the interior part of the flame is practically inaccessible (the radiation of the interior is absorbed en route and does not reach the pyrometer). In this case one actually measures the temperature of the periphery of the flame.

If we operate in the infrared region of the spectrum in the CO_2 bands at 2.7 μ and 4.3 μ , the difference \overline{T} $-\overline{T}_{arith}$ for various temperature profiles and specific absorption coefficients is no greater than $\pm (50-100^{\circ})$. Hence, if the temperature profile and the values of the emissivity of the flame are known only approximately, as is usual, the error in determination of \overline{T}_{arith} and T_{max} from the measured mean optical temperature will be considerably less if we work in the infrared than in the visible, since in this case the difference between \overline{T} and \overline{T}_{arith} is considerably less.

This conclusion may not hold, as was said above, when the absorption coefficient is very large, or also in those special cases in which the specific absorption coefficient of the cold layers of the flame is large in the infrared and small in the visible region of the spectrum (e.g., when the amount of soot is small, but the concentration of CO_2 is very high).

We note that the calculation methods in which the temperature profile is computed from the measured temperature values averaged over chords through the flame $[^{8,40,4}]$ make it possible to a considerable degree to reduce the error in the temperature determination, by virtue of the monotonic radial variation of the temperature.

I. 2. Determination of the temperature of a flame showing time pulsations of temperature. Besides spatially-inhomogeneous temperature fields in flames, we may observe periodic variations of temperature in time, which appear even in cases in which the fuel and the oxidant are supplied continuously. The combustion process itself involves temperature pulsations.

There are statements in the literature $[^{42-44}]$ of the existence of such pulsations in an actual turbulent flame. We have performed measurements on a free high-velocity flame of the type occurring in a ramjet engine $[^{4,41,45}]$ as well as on a confined flame of a type used in a metallurgical furnace $[^{4,46,47}]$ using an ion gauge and an infrared pyrometer. These measurements also revealed pulsations in the flame. The existence of pulsations in a flame has also been confirmed by temperature measurements made with a thermoanemometer. $[^{42-44}]$

The experiments with the ion gauge, which was shot through the flame, showed that in a free flame, at relatively high flow rates (of the order of 100-150 m/sec) the temperature is constant in the middle of the flame, but pulsates along the edges. The measurements of the temperature pulsations, which were made with the thermoanemometer, showed that the amplitude of the pulsations varies from zero in the central portion of the flame to $1000-1500^{\circ}$ K at the edges of the flame.

Finally, the direct measurements which we have performed on pulsations in the infrared emission of the flame (at about 2.7μ) for the same flame showed that the temperature averaged through the cross section of the flame pulsated with an amplitude of 200- 300° K. This is the mean pulsation. An estimate made by the method given in Sec. I. 1 for an ordinary temperature profile, e.g., that of Fig. 2 with $A_{\lambda T} d_0 = 0.1$, indicates that such a mean pulsation corresponds to pulsations varying from zero in the center of the jet to amplitudes of more than 1000°K at the edges of the flame.

The measurements which we have performed with the ion gauge on a confined flame having a low flow rate (several tens of meters per second) showed that in this case temperature pulsations take place throughout the thickness of the flame. The amplitude of the pulsations does not depend much on the fuel-oxident ratio, and amounts to several hundred degrees with a maximum temperature of about 2000°K.

Let us examine how these pulsations are reflected in the results of flame-temperature measurements by optical methods having some degree of lag and averaging the temperature over a time much greater than the period of the temperature pulsations of the flame. We might expect a priori that, owing to the nonlinear type of averaging discussed in the previous sections, as is characteristic of the optical methods (regardless of whether the temperature is averaged over space or time), the mean optical temperature \overline{T} will not equal the true mean flame temperature T_0 about which the temperature is pulsating. This difference will increase at shorter wavelengths.

We can show that the result of the measurements, i.e., the mean optical temperature \overline{T} , will be independent of the frequency of the pulsations, but will depend on the amplitude of the pulsations and the working wavelength.

For the case of temperature pulsations, \overline{T} can be found from a relation analogous to that given in Sec. I. 1, but with averaging in time rather than space:

$$\frac{1}{e^{\frac{c_2}{\lambda T}}-1} = -\frac{1}{\tau} \int_{0}^{\tau} \frac{1}{e^{\frac{c_2}{\lambda T}(t)}-1} dt,$$
 (6)

where τ is the period of the pulsations, and T(t) gives the temperature as a function of time. That is, essentially, we find the temperature of a stationary flame having a monochromatic luminance equal to the timeaveraged monochromatic lumiance of the pulsating flame being studied. By assigning graphically several curves for T(t) for $\overline{T}_0 = 1500^\circ$ and 2000° K, and integrating numerically for $\lambda = 0.6 \mu$, $\lambda = 2.7 \mu$, and λ = 4.3 μ , we obtain the following result. ^[4,41,45,46]

At $\overline{T}_0 = 1500^{\circ}$ K with $\lambda = 0.6 \mu$, for pulsation amplitudes of 500° and 1000°K, we find that \overline{T} exceeds T_0 by 300° and 600°K, respectively. At $T_0 = 2000^{\circ}$ K, this increase amounts to 250° and 500°K, respectively. At $\lambda = 2.7 \mu$, the increase is considerably less. In this case, the differences between \overline{T} and T_0 at T_0 = 1500°K are 50° and 170°K, respectively. At T_0 = 2000°K, they are 35° and 130°K. For $\lambda = 4.3 \mu$ at $T_0 = 1500^{\circ}$ K, we obtain 20° and 70°K, respectively, and at $T_0 = 2000^{\circ}$ K, we have 15° and 60°K.

It would be advantageous to choose an even longer working wavelength from the standpoint of the averaging process. However, this is not convenient in practice, since it is fundamentally carbon dioxide which radiates in the flame, and the bands of CO_2 at longer wavelengths at temperatures of 2000-3000° are relatively weak, and a number of difficulties are involved in recording them.

Incidentally, it may be possible to choose a longer working wavelength for non-carbonaceous flames.

Since the calculation was qualitative in nature, it was carried out for sinusoidal and near-sinusoidal temperature fluctuations. However, this does not limit the generality of the conclusions, since any periodic function can be expanded in a Fourier series, and the calculation may be carried out separately for each sinusoidal term. The result of the averaging for each sinusoidal term depends on the amplitude, but not on the frequency. The total result is determined by the amplitudes of all the sinusoidal terms. If the oscillation is not very far from harmonic, the result of the averaging can be determined mainly by the amplitude of the first harmonic. Taking the other harmonics into account gives a correction term. However, in the general case, we must add the mean radiation for all the harmonics and find the temperature corresponding to the total mean radiation.

If the oscillation is nearly rectangular (experiment shows that in a number of cases rectangular pulsations are observed), ^[44] we can perform the calculation algebraically and find $\gamma = \overline{T}/T_0$ as a function of the wavelength.

If the time intervals during which the temperature is T_1 or T_2 are unequal, but are equal to τ_1 and τ_2 , respectively, and $T_2 \ll T_1$, then we must write instead of Eq. (6) the following:

$$\frac{1}{\tau_1 + \tau_2} \int_0^{\tau_1} \frac{1}{e^{\frac{c_2}{\lambda T_0}} - 1} dt = \frac{1}{e^{\frac{c_2}{\lambda \tilde{T}}} - 1}$$
(6a)

and

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$$\gamma = \frac{c_2}{\lambda T_0 \ln \left[\frac{1}{A} e^{\frac{c_2}{\lambda T_0}} - \left(\frac{1}{A} - 1\right)\right]},$$
(7)

where $A = \tau_1/(\tau_1 + \tau_2)$. In the limiting cases, as $\lambda \rightarrow \infty$, $\gamma \rightarrow A$, and as $\lambda \rightarrow 0$, $\gamma \rightarrow 1$. Figure 6 shows the calculated relation of γ to λ for the case of rectangular temperature pulsations having $\tau_1 = \tau_2$.

I.3. Determination of the temperature of a flame inhomogeneous in Z (fuel-oxidant ratio). We shall discuss now a flame or region of a flame in which the temperature does not vary monotonically. For example, if the combustion process is already completed, the temperature in the core of a flame must be regarded as constant.

However, this is the case only with ideal mixing of the fuel and the oxidant, where Z (the fuel-oxidant ratio) is the same at different points of the flame or region of a flame being studied. This is not so in a real flame, and the degree of homogeneity in composition of the flame depends on a number of factors, as we shall see below.

Let us consider the effect of these inhomogeneities on the measurement of the flame temperature from the radiation, both in the visible and in the infrared. We shall be interested in the effect on the results of flame-temperature measurements produced by fluctuations in the value of Z at various points (volume elements) of the flame at a given instant of time; Z for the flame as a whole (or the region of the flame being studied) is fixed at the value Z_0 . Also, we shall be interested in the effect of time fluctuations of Z at a given point (volume element) of the flame, with the time-average value of Z fixed at Z_0 . Z_0 is defined as the ratio of the mean supplies of oxidant and fuel for the flame as a whole or some portion of it.

In order to make quantitative estimates, we shall make use of the fact that mixing processes are random processes, and can therefore be described by the Gauss formula. Thus, at a given instant of time in a given volume element of the flame, the probability that the oxidant-fuel ratio will have some value Z other than the mean value Z_0 is proportional to the quantity

$e^{-h^2(Z-Z_0)^2}\,dZ,$

where, as usual, $h = 0.7/\sigma$, where σ is the rms deviation of Z from its mean value Z_0 . More exactly, for a given concentration of fuel introduced into a homogeneous air current, this formula gives the probability of a certain deviation of this concentration from its mean value at a given instant of time or in a given volume element of the flame. This probability decreases as the difference between Z and Z_0 increases.

The relation of the temperature T to Z may be calculated from theory, ^[5] and as is known, follows a curve having a maximum near the stoichiometric value of Z.

Owing to the inhomogeneity in Z, the temperature in some regions of the flame will be higher, and in other regions lower than the value of the temperature corresponding to $Z = Z_0$. The temperature will be higher for the Z values nearer to stoichiometric, and lower for the Z values further from the stoichiometric value.

Since the function T = f(Z) is generally nonlinear, even the arithmetic mean (or mass-average) temperature (as determined, e.g., from gas-analysis data on a sample taken from a sufficiently large region of the flame containing regions of varying Z values) will also not be equal to the temperature T_0 corresponding to the given mean value $Z = Z_0$.

 \overline{T}_{arith} will coincide with T_0 only in case that the appreciable variations in Z are confined within the limits of a region of the T(Z) curve which may be considered linear.

However, if Z_0 is near the stoichiometric value (i.e., near the Z_0 value of the maximum in the T(Z) curve), the existence of any fluctuations in Z will

FIG. 6. $\gamma = \overline{T}/T_0$ as a function of the wavelength λ for the case of rectangular pulsations. Amplitude of pulsations: $I - 500^{\circ}$ K; II - 1000^oK; and III - 2000^oK. A = 0.5.



lead to a decrease in the arithmetic mean temperature \overline{T}_{arith} . Indeed, the presence of Z values either greater or smaller than the stoichiometric value will result in temperatures below that corresponding to the stoichiometric Z. In the last analysis, \overline{T}_{arith} will always be lower than the theoretical temperature value, i.e., the value for $Z = Z_0$ in the absence of fluctuations in Z. We shall denote this latter temperature as T_0 .

Hence, the deviation of \overline{T}_{arith} from the theoretical temperature value T_0 corresponding to the given value of Z_0 depends both on the absolute value of Z_0 and on the fluctuations in Z or degree of mixing, as characterized in this case by the value of h.^[4,45,46]

The arithmetic mean temperature \overline{T}_{arith} may be found, according to the definition of the mean value, from the relation

$$\bar{T}_{arith} = \frac{\int_{0}^{\infty} e^{-h^2 \left(\frac{Z-Z_0}{Z_0}\right)^2} T(Z) \, dZ}{\int_{0}^{\infty} e^{-h^2 \left(\frac{Z-Z_0}{Z_0}\right)^2} \, dZ} \,.$$
(8)

The integrand in the numerator contains the product of the temperature at a given Z value and the probability of that Z value. The integration is performed over all Z values. The denominator is a normalizing factor.

Thus we see that in general, $\overline{T}_{arith} = f(Z_0, h) \neq T(Z_0)$. By calculating the integral in Eq. (8) by a numerical or approximate method, we can find \overline{T}_{arith} as a function of Z_0 and h. The calculation indicates that the curve $\overline{T}_{arith} = f(Z_0)$ will follow the theoretical curve $T_0 = f(Z_0)$ only when the mixing is very good. However, in general, as is very important, this curve will have a flatter maximum, and the more so, the poorer the mixing.

Let us consider the mean optical temperature \overline{T} determined from the total monochromatic radiation from some region of the flame. \overline{T} will be higher than \overline{T}_{arith} in this case, as in the case discussed above, owing to the nonlinearity of the luminosity-temperature relation. Thus, the curve $\overline{T} = f(Z_0)$ will have an even flatter maximum than the curve $\overline{T}_{arith} = f(Z_0)$.

The mean optical temperature \overline{T} may be determined from the relation

$$\frac{1}{e^{\frac{c_2}{\lambda T}} - 1} = \frac{\int_{0}^{\infty} e^{-h^2 \left(\frac{Z-Z_0}{Z_0}\right)^2} \frac{1}{e^{\frac{c_2}{\lambda T(Z)}} - 1}}{\int_{0}^{\infty} e^{-h^2 \left(\frac{Z-Z_0}{Z_0}\right)^2} dZ}$$
(9)



FIG. 7. The theoretical dependence of the arithmetic mean and the mean optical temperatures on h (for $\lambda = 0.6\mu$ and $\lambda = 2.7 \mu$): dashed curves $-\lambda = 0.6\mu$; solid curves $-\lambda$ $\lambda = 2.7\mu$; dash-dot curves – arithmetic mean; 1: $Z_0 = 1; 2: Z_0 = 0.6.$

This relation differs from Eq. (8) in that the numerator of the right-hand side and the left-hand side of this equation contains the emission for the given temperature, instead of the temperature itself. Hence, \overline{T} is defined by a method analogous to that applied in Secs. I.1 and I.2, by comparison of the luminances of our real flame and a standard flame free from fluctuations in Z. The temperature of this standard flame when the emitted energies are equal is the sought mean optical temperature.

There is no need in these calculations to take into account the absorption of the radiation upon passing through the thickness of the flame. The absorption may be ignored, since the fluctuations are random, and the regions having higher and lower temperatures are distributed at random throughout the thickness of the flame. Hence, absorption has the same effect on the emission of the real flame and on the standard flame of uniform temperature being compared with it.

The absorption will also be the same for the real and the standard flames if the value of Z fluctuates in time.

This is the distinction between the averaging process when Z fluctuates and the process discussed in Sec. I.1, in which the temperature is averaged through the thickness of an axially-symmetric flame having a monotonically-varying temperature field. In the latter case, the mean optical temperature depends on the absorption.

The mean optical temperature \overline{T} depends on Z_0 , h, and $\boldsymbol{\lambda},$ and is not equal to the theoretical temperature, nor to \overline{T}_{arith} , but is generally larger than either.

 $\overline{\mathbf{T}}$ has been calculated by numerical integration for various values of h and λ .^[4,45,46]

The basic results of the calculations are given in Figs. 7 and 8.

On the basis of the calculations made, we may draw the following conclusions on the effect of fluctuations in Z on the results of measurements of flame temperatures by optical methods.

1. When $Z_0 = 1$, \overline{T} decreases as the fluctuations increase (i.e., as h decreases). This effect becomes more evident at longer wavelengths. Tarith decreases with increasing fluctuations even more sharply than \overline{T} . In the latter case, the difference between the temperatures measured under conditions of large and small fluctuations (with amplitudes varying over a fivefold range) can be as much as 400°.

FIG. 8. The theoretical dependence of the mean optical temperature on the wavelength λ for $Z_0 = 1$, h = 1.75; 3.5; 7. 1: h = 7; 2: h = 3.5; 3: h = 1.75.



2. When $Z_0 \neq 1$, \overline{T} increases as the fluctuations increase. However, this effect (essentially a difference effect) is small, and depends on the value of Z_0 and the trend of the Z(T) curve. This effect is more appreciable for $\lambda = 0.6 \mu$, and less so for longer λ values. It is also not very appreciable for \overline{T}_{arith} . Besides, the increase of \overline{T} as the fluctuations increase is not monotonic. The latter may be explained by the fact that the temperature of a given region of the flame increases the closer its Z value is to stoichiometric. Hence, when $Z_0 \neq 1$ (e.g., in the case $Z_0 < 1$), the number of regions having a greater temperature than that corresponding to the value Z_0 will increase with increasing fluctuations only up to the point at which an appreciable number of regions having Z > 1 appear. As the fluctuations increase further, the relative number of regions of temperature above that corresponding to Z_0 will decrease, and \overline{T} will also begin to decrease.

The same is true when $Z_0 > 1$. The value of h for which \overline{T} is maximum (for Z_0 \neq 1) depends essentially on the shape of the curve $T(Z_0)$, as well as on the values of Z_0 and λ .

3. Let us compare the calculated curves for $T(Z_0)$ for a flame having no fluctuations in Z_0 with those of a flame containing such fluctuations. That is, we shall compare the original curve for $T(Z_0)$ on which our calculations are based with that of \overline{T} calculated by Eq. (9) and T_{arith} calculated by Eq. (8).

When the fluctuations are very small (h is large), the curves of \overline{T} and \overline{T}_{arith} as functions of Z_0 follow the original curve $T(Z_0)$. As the fluctuations increase, the slopes of the $\overline{T}(Z_0)$ and $\overline{T}_{arith}(Z_0)$ curves become flatter. Here, for a given value of the fluctuation, the curve $\overline{T}(Z_0)$ becomes steeper at longer λ values. The curve of $\overline{T}_{arith}(Z_0)$ or $T_{mass}(Z_0)$ is even steeper.

Thus, when there are fluctuations, the curves of the relation of \overline{T} to Z_0 are flatter than the theoretical curve of $T_0(Z_0)$ or that of the arithmetic-mean temperature $\overline{T}_{arith}(Z_0)$. This difference in the trend of the curves increases with the fluctuations and with decrease in the wavelength.

4. It is an essential point that, even at $\lambda = 2.7 \mu$, but even more so at longer wavelengths, the trend of the $\overline{T}(Z_0)$ curve is close to that of $\overline{T}_{arith}(Z_0)$. However, depending on the size of the fluctuations, both curves may deviate considerably from the theoretical $T(Z_0)$ curve calculated with fluctuations in Z ignored. For $\lambda = 0.6 \mu$, the curve for $\overline{T}(Z_0)$ is much gentler in slope than that for \overline{T}_{arith} . This difference depends on the

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magnitude of the fluctuations, which are characterized in the present case by the value of h.

Hence, if we use the visible region of the spectrum to measure the temperature of a flame which is inhomogeneous in Z, i.e., has appreciable fluctuations in Z, we shall obtain for $Z_0 \neq 1$ temperature values much higher than the arithmetic-mean or mass-average temperatures; we shall also observe a much weaker dependence of the measured temperature value on Z_0 than that for the arithmetic-mean temperature, and even more so, if we compare it with the theoretical temperature value calculated neglecting the fluctuations in Z.

The temperature measured from the infrared radiation will be nearer to the arithmetic-mean temperature.

Since we usually do not know the degree of inhomogeneity of the flame in Z, we cannot in practice make corrections to the measured \overline{T} values to transform them to arithmetic-mean temperatures. Hence, in working in the infrared region of the spectrum, the errors in measurement of the temperature of a flame which is inhomogeneous in Z will be smaller than in the visible.

We can derive the same results without numerical integration by approximating the theoretical T(Z) curve with a step curve.

The results of the calculation will be cruder, but nevertheless we can obtain an analytical expression for \overline{T} , which is a function of h and λ .^[4]

II. EXPERIMENTAL SECTION

We have made an experimental test of the concepts developed above, as was stated in the introduction, on two types of flames: a free high-velocity flame of the type occurring in a ramjet engine, and a confined flame (air-illuminating gas) of a type used in a metallurgical furnace.

Using the methods and instruments mentioned in the introduction, we have determined the mean temperature and the temperature field from the emission, both in the visible (~ 0.6μ) and in the infrared (2.7μ and 4.3μ), for both types of flames. The temperature field in the visible was measured by the spectral-line reversal method using local coloring.^[8] The temperature field in the infrared was determined by a calculation method ^[8,40] from the mean luminance temperatures measured along chords in the flame and from the emissivity in the infrared.

In all cases, when there was an appreciable steady temperature gradient along the diameter, the mean temperatures measured by the visible radiation were higher than those measured in the infrared, in agreement with the calculations cited above.

II.1. The experimental test of the effect of pulsations on the measurement of flame temperatures in various spectral regions was performed on a free highvelocity flame, in which wide temperature fluctuations



FIG. 9. Radial temperature distribution of the flame. 1: measured by the spectral-line reversal method (o - lithium lines, Δ - sodium lines); 2: from the infrared emission; 3: from gasanalysis data. a) $Z_0 = 0.9$; b) $Z_0 = 1.0$. Distance from nozzle opening = 120 mm. Z_0 = fuel-oxidant ratio.

take place at the edges of the flame, and become wider as the portion of the flame being studied approaches the edge of the flame.

We determined the temperature field of the flame by the above-mentioned methods from the visible and infrared radiation, as well as by calculation from gasanalysis data. The latter method, as has been stated above, gives in principle the mass-average temperature.

The measurements were conducted for oxidant-fuel ratios not far from stoichiometric.

The measurements made at varying Z values, i.e., under varying combustion conditions, showed [4,41,45](Fig. 9) that all of these methods give concordant temperature values in the central part of the flame (within the accuracy of the measurements as defined by a total error of $\pm 50^{\circ}$). At the edge of the flame, the spectralline reversal method gives higher temperature values than the two other methods. This increase at the edges of the jet amounts to as much as 1000° K, i.e., greatly in excess of the limits of error of the measurements.

On the other hand, the infrared-pyrometry methods give temperature values, within the limits of accuracy of these measurements, which agree both in the center and at the edges of the flame with those obtained by gas analysis.

Thus, the theory presented above explains qualitatively, and to some extent even quantitatively, the observed difference in temperature profiles of a flame obtained by using the visible and the infrared radiation. It also explains the observed increase in the temperature values at the edges of the flame obtained when we use the visible region of the spectrum. The experimentally-observed difference between these two temperature profiles is still somewhat greater than that calculated above (especially under the assumption of rectangular temperature pulsations). We could not expect quantitative agreement here, since the shape of the pulsations was assumed rather arbitrarily in performing the calculations, the amplitude of the pulsations was known only crudely, and finally, the experimentally-observed effect in this case is an averaged effect.



FIG. 10. The experimental dependence of the maximum temperature T of a high-velocity flame on the fuel-oxidant ratio Z_0 , as measured by: ∇ the spectral-line reversal method; x - the infrared emission; o - gas-analysis data.

We shall return to this problem below.

II.2. In order to make a preliminary estimate of the effect of inhomogeneity of the flame in Z on the temperature measurement, we made measurements of the temperature of the central portion of a high-velocity gasoline-air flame of the type occurring in a ramjet engine in the visible and infrared radiation (for $\lambda = 0.6 \mu$ and $\lambda = 2.7 \mu$). The measurements were carried out for various Z values both above and below the stoichiometric ratio.

According to the measurements made, temperature pulsations do not occur in the central part of the flame, and the temperature gradient is small.

As a check, the temperature was also determined from gas-analysis data, which in principle give the mass-average temperature. Also, in several cases when the temperature was not too high, it was measured with thermocouples.

The experimental results are given in [4,41,45] and in Fig. 10. We see that at Z = 1, the spectral-line reversal method and the infrared pyrometer give results agreeing with each other as well as with those obtained by other methods, within the accuracy of the measurements performed.

When $Z \neq 1$, the spectral-line reversal method gives higher temperature values, while the infrared pyrometer gives temperature values closer to the mass-average temperature obtained by gas analysis.

Thus, the curve for $\overline{T}(Z_0)$ for $\lambda = 2.7 \mu$ is close to the curve for $\overline{T}_{mass}(Z_0)$ obtained by gas analysis, within the accuracy of our measurements and calculations. However, the curve of $\overline{T}(Z_0)$ for $\lambda = 0.6 \mu$ is much flatter, and gives higher temperature values for $Z_0 \neq 1$ than the gas analysis does.

The fact that the spectral-line reversal method often gives higher values of the temperatures being measured and a much weaker dependence of T on Z_0 than the theoretical (sometimes the measured temperature is practically independent of Z_0) has also been noted by other authors.^[11,19,48]

This result may be explained by the fact that actual turbulent flames contain fluctuations in Z. In line with our discussion in Sec. I.3, these lead to differing trends in the $T(Z_0)$ curves measured from the emission in the visible and in the infrared. In agreement with theory, the temperature measured by the spectral-line reversal method is considerably higher for $Z_0 \neq 1$ than that measured from the infrared emission. The $T(Z_0)$ curve is much flatter for the visible region of the spectrum than for the infrared, in agreement with theory. The qualitative agreement with theory is complete, but it is difficult to make a quantitative comparison, since we do not know the value of h characterizing the inhomogeneity of the flame in Z. Besides, the error in the temperature measurements in the infrared is at least $\pm 50^{\circ}$, owing mainly to error in the determination of the mean value of the emissivity in the infrared (see Sec. III). The errors in temperature determination from gas-analysis data are of the same order of magnitude.

As it seems, this explains the agreement of results to be noted in Fig. 10 between the temperature measurements from the infrared emission and from gasanalysis data. Obviously, the discrepancy between the temperatures cited above and the theoretical temperatures is within our limits of error. We might also note that the difference between the temperature values measured by the visible and the infrared emission is greater for $Z_0 \neq 1$ than we might expect from the results of the calculations given in Sec. I.3. We shall return to this problem below.

In order to test directly the theoretical ideas presented above concerning the effect of inhomogeneities in Z in the flame on the results of flame temperature measurements in the visible and the infrared, we must make flame temperature measurements from the visible and the infrared emission with varying degrees of mixing of the oxidant and the fuel, and thus, with varying degrees of inhomogeneity of the flame in Z.

Such measurements were performed on the vertical confined flame (air-illuminating gas) described above, a type of flame used in a metallurgical furnace. The length of the flame was about 3-5 meters, the width 40-50 cm, and the rates of efflux did not exceed 10 m/sec. The measurements were made through a series of windows arranged along the flame on all four sides of the apparatus. The instruments were moved along the flame by means of a worm drive. The presence of four sets of windows permitted us when necessary to measure simultaneously the true temperature of the flame by the spectral-line reversal method in the visible emission, and the luminance temperature and the emissivity of the flame in the infrared. We determined $\overline{\epsilon}$ for various lamp temperatures, and then used in the calculations the value of $\bar{\epsilon}$ determined at the lamp temperature nearest to the temperature of the flame at the given value of $Z_0^{[46,47]}$ (see also Sec. III). This permitted us to decrease considerably the errors in the temperature measurement from the infrared emission.

The degree of mixing was varied by varying the length of the tube Δl through which the cold gas-air mixture passed up to the point of ignition. The greater this length was, the better the degree of mixing. In our experiments, we varied this length from zero, corresponding to the poorest mixing, to 390 mm.



FIG. 11. The experimental dependence of the flame temperature on the fuel-oxidant ratio Z_0 for various degrees of mixing: a) For the visible region of the spectrum; b) for the infrared ($\lambda = 4.3 \mu$); d) for the visible ($\lambda = 0.6\mu$) and infrared ($\lambda = 2.7\mu$) with the maxima of all curves superimposed; e) for the infrared ($\lambda = 4.3\mu$), with the maxima of all curves superimposed. $1 - \Delta l = 0$; $2 - \Delta l$ = 140; $3 - \Delta l = 185$; $4 - \Delta l = 260$; $5 - \Delta l = 395$.

We determined the true flame temperatures and the function $\overline{T}(Z_0)$ from the visible and the infrared emission for Δl values of zero, 140 mm, 185 mm, 260 mm, and 395 mm. In measuring the temperature from the visible emission, we used the spectral-line reversal method, while in the temperature measurements from the infrared emission we measured the luminance temperature and the emissivity of the flame in the infrared (the spectral bands near 2.7μ and near 4.3μ were iso-

lated with interference filters). The measurements were made in the region along the length of the flame where the temperature was maximum.

The results of the measurements in the visible and the infrared $(2.7 \,\mu$ and $4.3 \,\mu$) are given in ^[4,46] and in Fig. 11.

We see that, in agreement with the theoretical ideas developed above, the slope of the $\overline{T}(Z_0)$ curves actually increases with better mixing, and that (in all cases) the $\overline{T}(Z_0)$ curves for the visible region of the spectrum are flatter than those for the infrared. The value of \overline{T} for $Z_0 = 1$ increases with better mixing, in agreement with theory. Here \overline{T} increases more rapidly in the infrared, while for $Z_0 = 1$ with poor mixing, \overline{T} is higher for the visible region of the spectrum.

Thus, at least qualitatively, experiment completely confirms the theoretical ideas given above. As for quantitative agreement, the $\overline{T}(Z_0)$ curve for the visible is flatter than we might expect from theoretical considerations for large values of Δl , i.e., good mixing (with corresponding large values of h). Also, the difference between the temperatures measured in the visible and in the infrared for $Z_0 = 1$ is greater than would be expected from theory.

This discrepancy, which exceeds the limits of error of the measurements, may be explained by the fact that experimentally we observe a composite effect involving: the effect of inhomogeneities of the flame in Z, temperature pulsations, and the monotonic temperature gradient along the radius of the flame. As we have seen, each of these factors has the result that the \overline{T} measured in the visible is higher than the \overline{T} measured in the infrared emission. Each of these factors enhances the difference between the temperatures measured from the visible and the infrared emission, and some sort of composite effect is observed experimentally.

The fact that the mean optical temperature measured in the visible radiation by the spectral-line reversal method with poor mixing of the fuel and oxidant is almost independent of the value of Z_0 , whereas



FIG. 12. The temperature of a free flame measured by the spectral-line reversal method for various fuel-oxidant ratios with poor mixing (I) and with good mixing (II).

with good mixing it is highly dependent, has also been observed with a free high-velocity flame of the ramjet type [49] (Fig. 12).

II.3. In order to compare the results of the calculattions with those of the experiments, we must calculate the composite averaging effect, and then compare the theoretical and experimental curves for $\overline{T}(Z_0)$ for various λ values. In order to carry out these calculations, we must know the temperature profile along the radius of the flame, the amplitude of the temperature pulsations, and the degree of inhomogeneity of the flame in Z.

The temperature gradient along the radius was estimated by the method indicated above. The amplitude of the pulsations was estimated from the pulsations in the ion current.

As for the homogeneity of the flame in Z, we assumed that, with the best mixing we attained, when $\Delta l = 395$, h = 7; and for the poorest mixing at $\Delta l = 0$, h = 2.21.

If we consider these effects to be independent, the temperature averaged over r, τ , and Z (radius, time, and inhomogeneities of composition), which we denote by \overline{T} , can be found by calculations analogous to those described above. That is, it may be found from an equation having $[\exp(c_2/\lambda \overline{T}) - 1]^{-1}$ on the left-hand side, and the product of the right-hand sides of Eqs. (5), (6), and (9) on the right-hand side.

The results of the calculations made and a comparison with the experimental results for $\lambda = 0.6\mu$, 2.7 μ , and 4.3 μ are given in ^[4,46] and in Figs. 13a, 13b, and 14. We must take into account, on the one hand, a certain arbitrariness in the choice of values of h, as well as the errors in determination of the temperature profile and the shape and amplitude of the pulsations. On the other hand, we must consider the errors in temperature measurement (especially in the infrared). Under these conditions, we may consider the agreement to be quite satisfactory and to corroborate the explanation proposed for the results of the measurements. We could not expect an agreement with a precision of better than 50–100° in this comparison.

III. CERTAIN PECULIARITIES IN INFRARED FLAME PYROMETRY

It follows from what has been said above that the methods of optical pyrometry making use of infrared radiation have an advantage in measuring the temperature of an actual flame over the methods using the visible.

However, the use of the infrared radiation to measure the flame temperature involves a number of specific difficulties at the present state of infrared technique. We shall discuss here certain of the most essential difficulties.^[4,8]

The trouble is that the transmission region of the existing infrared interference filters applicable in py-



FIG. 13. A comparison of the mean optical temperature calculated by taking into account the averaging along the diameter, over time, and over the imhomogeneities in the fuel-oxidant ratio with the experimental values for various values of the fuel-oxidant ratio. Dotted curve = theoretical; solid curve = experimental; curves 1 - infrared, $\lambda = 2.7\mu$; curves 2 - visible, $\lambda = 0.6\mu$; a) Δl = 395, h = 7; b) $\Delta l = 0$, h = 2.1.

rometers is very wide (with a half-width of about 0.3μ). The mean value of the emissivity $\bar{\epsilon}$ of a flame in this spectral range (it is precisely this value which we measure, and it is determined, in particular, by the relation between the luminance temperature and the true temperature) is usually very small (about 0.05 at $\lambda = 2.7 \mu$). This fact makes the measurements much more difficult, and increases the errors of measurement.

This conclusion is independent of the method of measurement applied. In the infrared, the most convenient method is to measure the luminance temperature and the emissivity.^[4]

Besides, with filters having wide transmission bands, $\bar{\epsilon}$ seems to depend on the temperature of the comparison source.^[4,8]

We shall discuss this problem in further detail. In general, the emissivity of a flame in any spectral re-



FIG. 14. The mean optical temperature \overline{T} calculated by taking into account the averaging along the diameter, over time, and over the inhomogeneities in the fuel-oxidant ratio for various fuel-oxidant ratios, for $\lambda = 4.3\mu$. Dotted curve – theoretical; solid curve – experimental; curve I: $\Delta l = 395$, h = 7; curve II: $\Delta l = 0$, h = 2.21.

gion may be measured by the usual method by determining the decrease in the radiation from a comparison source when the radiation passes through the flame, where it is partially absorbed.

Let I_1 be the luminance of the radiation from the comparison source in some spectral region $\Delta \lambda = \lambda_1$ $-\lambda_2$; I_2 be the luminance of the flame; and I_3 the total luminance of the radiation in the same spectral region. Then the measured value of the emissivity is

$$\bar{\epsilon}' = \bar{a}' = \frac{I_1 + I_2 - I_3}{I_1},$$
 (10)

where α' is the absorption coefficient.

However, we must note that the method described above for determining $\overline{\epsilon'}$ is not unequivocal, since the value of $\overline{\epsilon'}$ obtained thus will be determined not only by the properties of the flame and the width of the chosen spectral region, but will depend considerably on the energy distribution of the comparison source, and hence, on its temperature. Indeed, in this case the value of $\overline{\epsilon'}$ (from the general definition of the mean value of any quantity) for the spectral interval $\lambda_1 - \lambda_2$ is

$$\bar{\varepsilon}' = \frac{\int_{\lambda_1}^{\lambda_2} I(T_1, \lambda) \varphi(\lambda) \varepsilon(\lambda) d\lambda}{\int_{\lambda_1}^{\lambda_2} I(T_1, \lambda) \varphi(\lambda) d\lambda}, \qquad (11)$$

where $I(T_i, \lambda)$ is the monochromatic luminance of the radiation from the comparison source at temperature T_i , and $\epsilon(\lambda)$ is the actual value of the emissivity for a given value of λ .

$$\varphi(\lambda) = \tau(\lambda) \varphi_0(\lambda),$$

where $\varphi(\lambda)$ is the overall spectral characteristic of the instrument, $\tau(\lambda)$ is the transmission coefficient of the filter, and $\varphi_0(\lambda)$ is the spectral sensitivity of the radiation detector. It follows from Eq. (11) that, in general, the value of $\overline{\epsilon}'$ thus determined depends not only on the value of $\epsilon(\lambda)$, but also on the value of $I(T_1, \lambda)$, and hence, on the temperature T_1 of the source.

Only when we may consider the quantity $I(T_1, \lambda)$ to be constant within the interval from λ_1 to λ_2 , independently of λ (i.e., we may neglect the spectral energy distribution within this interval), may we bring this factor out of the integrand in both the numerator and the denominator. Then $I(T_1, \lambda)$ in the numerator and the denominator cancels out, and

$$\varepsilon' = \frac{\int_{\lambda_2}^{\lambda_2} \varepsilon\varphi(\lambda) d\lambda}{\int_{\lambda_2}^{\lambda_2} \varphi(\lambda) d\lambda}, \qquad (11a)$$

that is, $\overline{\epsilon}'$ would then have the meaning of the mean effective value of the emissivity of the flame over the interval from λ_1 to λ_2 , and would be an unequivocal characteristic of the flame.

In the visible region of the spectrum, the spectral regions which we can isolate are usually narrow enough, and the condition that $I(T_1, \lambda)$ be constant within the spectral range is generally satisfied. Hence, the problem of the possible dependence on the comparison-source temperature of the results of measurement of the emissivity of a luminous flame in the visible does not arise.

However, in measuring the emissivity of a flame in the infrared, when the transmission band width of the filter amounts to several tenths of a micron, we cannot consider the function $I(T_1, \lambda)$ to remain constant within such an interval. The value of $\bar{\epsilon}'$ will depend on the form of the function $I(T_1, \lambda)$, which will depend in turn on the temperature T_1 of the comparison source. If the interval from λ_1 to λ_2 isolated by the filter is wider than the absorption region (the region in which $\epsilon(\lambda)$ $\neq 0$), we can see from Eq. (11) that $\bar{\epsilon}'$ will have a maximum at a T_1 value such that the maximum of the $\epsilon(\lambda)$ curve and that of the $I(T_1, \lambda)$ curve within the interval from λ_1 to λ_2 coincide in position.

How, then, can we find the mean or effective value of the emissivity $\bar{\epsilon}$ of the flame, which we must know to transform the luminance temperatures which we have measured to true temperatures? This quantity $\bar{\epsilon}$, by definition, is equal to the ratio of the emission of the flame to that of a black body at the same temperature, i.e.,

$$\bar{\varepsilon} = \frac{\frac{\lambda_2}{\int_0^1 I_0(T, \lambda) \varphi(\lambda) \varepsilon(\lambda) d\lambda}}{\frac{\lambda_2}{\int_0^1 I_0(T, \lambda) \varphi(\lambda) d\lambda}}, \qquad (12)$$

where $I_0(T, \lambda)$ is the black-body radiation at the temperature of the flame being studied. Thus we see that $\overline{\epsilon'}$ will be equal to $\overline{\epsilon}$ only in case that

(13)

$$I(T_1, \lambda) = MI_0(T, \lambda),$$

where M is a constant.

Hence, the comparison source must be a black or gray body over the interval from λ_1 to λ_2 , with a temperature equal to that of the flame.

However, when the comparison source is a real body, condition (13) (the similarity of the energy-distribution curves as a function of wavelength for the comparison source and for a black body) will be satisfied if the color temperature of the comparison source is equal to the true temperature of the flame.

Condition (13) is not easy to satisfy in practice. However, it becomes easier to satisfy as the interval $\lambda_1 - \lambda_2$ becomes narrower. This is because the difference between the trends of the functions I and I_0 within this interval decreases as the interval becomes narrower. This condition is satisfied with sufficient accuracy when we use an interference filter and a value of T_1 not too far from T. It is satisfied much more poorly if an interference filter is not used.

We note that if $\bar{\epsilon}$ is determined from independently measured values of the luminance temperature and the true temperature, then in this case, other conditions remaining unchanged, the value of $\bar{\epsilon}$ determined will depend on the true temperature of the flame.^[4]

A calculation based on Eqs. (11) and (12) permits us to estimate numerically the difference between $\bar{\epsilon}'$ and $\bar{\epsilon}$ for a real flame as a function of the temperature difference $T - T_1$. If we do not use an interference filter, the spectral region $\lambda_1 - \lambda_2$ will be determined by the characteristics of the FS-A1 photoresistor, whose sensitivity falls off sharply at wavelengths longer than 3μ . Besides, the wavelengths shorter than 1μ may be eliminated with ebonite.

We have taken the function $\epsilon(\lambda)$ from the literature data (see, e.g., ^[5]). It is determined fundamentally by the absorption of CO₂ and partly by CO and water vapor. The function $\epsilon(\lambda)$ has a maximum at $\lambda = 2.7 \mu$ (provided that the radiation detector is not sensitive to $\lambda = 4.3 \mu$). The calculation gives: ^[4,8,47]

at	$T_{1} = 500^{\circ} \mathrm{K}$	$\bar{\epsilon}' = 0.10$,
	$T_1 = 1000^{\circ} \mathrm{K}$	$\bar{\epsilon}' = 0.075$
	$T_1 = 2000^{\circ} \mathrm{K}$	$\bar{\epsilon}' = 0.055$

Thus, we obtain correct values of $\tilde{\epsilon}$ if $T_1 = T$. The error in determination of $\tilde{\epsilon}$ at $T = 2000^{\circ}$ K amounts to 36% if $T_1 = 1000^{\circ}$, and almost 100% if $T_1 = 500^{\circ}$ K. This corresponds to temperature errors of 60° and 100°, respectively, when $\tilde{\epsilon} = 0.05$. Hence, when T_1 is greatly different from T, the error in measurement of $\tilde{\epsilon}$ due to this difference must not be ignored.

The error in determining $\tilde{\epsilon}$ will be much smaller if, in determining it by the method given above, we use an interference filter having a transmission maximum near 2.7 μ [near the maximum of the $\epsilon(\lambda)$ function], and a narrower transmission band (about 0.3 μ). Within the transmission band of the filter, $I_0(T, \lambda)$ will not vary much, and hence the error in determining $\overline{\epsilon}$ as a function of the value of T_1 will be much less. Then

at $T_1 = 500^{\circ} \text{K}$	$\epsilon' = 0.11,$
$T_1 = 1000^\circ \mathrm{K}$	$\bar{\epsilon}' = 0.092,$
$T_1 = 2000^{\circ} \mathrm{K}$	$\bar{\epsilon}' = 0.08.$

In this case, with a flame temperature of 2000°K, the error in determination of $\bar{\epsilon}$ is 15% at $T_1 = 1000$ °K, and 38% at $T_1 = 500$ °K. Hence, for $T_1 = 1000$ °K, we may neglect the error in the determination of $\bar{\epsilon}$. For lower values of T_1 , even in this case, we must make the appropriate correction to the measured value of $\bar{\epsilon}$.

However, all these calculations only serve for orientation, and the relation between the measured value of $\bar{\epsilon}$ and T_1 must be found experimentally in each separate case. The measurements which we have made of the relation of $\bar{\epsilon}$ to T_1 for various flames have confirmed the existence of such a relation, although the experimental dependence of $\bar{\epsilon}$ on T_1 is much stronger than the theoretical. ^[47] This difference may also be explained by the fact that the effective wavelength λ_{eff} also varies as T_1 varies (quite markedly with such wide-band filters). This is not taken into account in calculating the relation of $\bar{\epsilon}$ to T_1 .

If we use interference filters for $\lambda = 4.3 \mu$ with the same transmission bandwidth, the measured value of $\bar{\epsilon}'$ will be considerably less dependent on the temperature of the lamp, since the transmission maximum of the filter and the maximum of the emission are now considerably further apart than they were in working at $\lambda = 2.7 \mu$. Calculation indicates that the temperature of the comparison source may vary over a wide range, but the variation in $\bar{\epsilon}'$ is negligibly small. Even in this case, experiment indicates a stronger dependence of $\bar{\epsilon}'$ on the temperature of the comparison source than theory does, although the dependence is considerably weaker here than in the studies in the 2.7 μ region.

For broadband filters, λ_{eff} depends essentially on the flame temperature, the characteristics of the light filter, the radiation detector, and the absorption curve of the flame. If these quantities are not known exactly, there will be additional errors in the temperature measurement, which may amount to some tens of degrees, depending on the values of T and $\bar{\epsilon}$.^[47] In order to reduce the errors in the determination of $\bar{\epsilon}$ and λ_{eff} , we may apply the method of successive approximations.

The conclusions made above are also verified by the fact that the measured value of $\bar{\epsilon}$ is not affected by the placing of gray filters, which do not alter the spectral energy distribution, in front of the comparison source, as should be expected (in the infrared, screens of varying densities may be used as gray filters).

We have spoken of the advantages of infrared pyrometry. However, on the other hand, when we go into the infrared, the sensitivity of the methods of optical pyrometry is lessened, and the error of temperature determination is increased, for a fixed given accuracy of determination of the monochromatic luminance and the emissivity of the flame.

The relation of the error of measurement to the wavelength is given by the relation $\ensuremath{[4]}$

$$\frac{dT}{T} = \frac{\lambda T}{c_2} \left(\frac{1 - e^{\frac{c_2}{\lambda T}}}{e^{\frac{c_2}{\lambda T}}} \right) \left\{ \left| \frac{dI_{\lambda T}}{I_{\lambda T}} \right| + \left| \frac{d\varepsilon_{\lambda T}}{\varepsilon_{\lambda T}} \right| \right\}$$

where |dT/T|, $|dI_{\lambda T}/I_{\lambda T}|$, and $|d\epsilon_{\lambda T}/\epsilon_{\lambda T}|$ are the absolute values of the relative errors in the determination of the true temperature, the luminance and the emissivity of the flame. Within the limits in which Wien's formula may be applied, dT/T is directly proportional to the wavelength.

CONCLUSIONS

We have seen that, owing to the specific nature of the averaging process characteristic of optical pyrometry, large errors in measurement arise from the application of this method to real flames in technology, which are characterized by monotonic radial variation of the temperature, inhomogeneities in the fuel-oxidant ratio, and temperature pulsations.

Hence, before we proceed to measure the temperature of some type of flame, we must make a preliminary investigation of its degree of homogeneity and constancy. We must also determine the nature of the spectrum and estimate the emissivity of the flame. All of this will lead to a correct choice of method of measurement and working region of the spectrum and permit the making of corrections to the measured temperature values so as to diminish the error of measurement.

We have seen that these errors are especially large in the visible region of the spectrum (as much as hundreds of degrees). Hence, the methods using the visible, and in particular the widely applied spectral-line reversal method, have a limited region of applicability.

These methods may be conveniently applied at fueloxidant ratios near stoichiometric, and in cases in which good mixing of the fuel and oxidant is ensured by special measures. However, even here we can only measure the temperature of those regions of the flame where there are no appreciable gradients or pulsations of temperature. (These regions may be selected by means of local coloring). Naturally, it is far from often that these conditions are satisfied.

We may try to take another course, and make "instantaneous" temperature measurements at a "point." In this case, there will be no averaging effect, and the results of the measurements will not depend on the spectral region used.

We may regard a measurement as "instantaneous" if the time of measurement is much shorter than the shortest period of temperature fluctuation in the flame. On the other hand, we may consider that we are measuring the temperature at a point, provided that we may neglect both monotonic temperature variations and inhomogeneities of the flame in the fuel-oxidant ratio within the limits of the volume which we have selected for measurement. For various flames, these conditions are satisfied at various values of the selected volume and the time of measurement.

The existing apparatus does not permit the measurement on real flames of "instantaneous" temperature values at a "point." However, it would seem expedient to continue work on the design of such an apparatus, even if it were not universal but applicable only to certain types of flames.

Besides, since the mean optical temperature approaches the mass-average temperature as λT increases, for considerably higher temperatures (of the order of several tens of thousands of degrees), the errors of temperature measurement under discussion here will be considerably smaller for the visible region of the spectrum than at $T = 2000-3000^{\circ}K$.

Since these errors are smaller for longer wavelengths, they will be considerably diminished by going over to the infrared region of the spectrum.

Hence, the region of applicability of infrared pyrometry is much broader than that of the visible. At the present time, as we see it, the center of gravity of the studies in the field of optical pyrometry of flames must be shifted over into the region of infrared pyrometry. Of course, there may be conditions under which it would be inexpedient to use the methods of infrared pyrometry. For example, the flame may be surrounded by a cold layer of CO_2 or another gas which strongly absorbes the infrared but is transparent in the visible, or as was stated above, the absorption coefficients may be very high, such that $\overline{T}_{opt} < \overline{T}_{arith}$. In the latter case, application of optical-pyrometry methods is generally not very promising.

The fact that going over into the infrared involves an increase in the instrumental errors and a decrease in the sensitivity plays no decisive role with the present state of instrumentation.

In general, the errors associated with the specific nature of the averaging process in the optical pyrometry of a real flame are considerably greater, as a rule, than the instrumental errors. Hence a further improvement of the accuracy of the pyrometers used is not one of the major problems, as we see it. More important problems are the stability and reliability of the apparatus used, the design of apparatus permitting us to make the process of measurement objective and automatic, and the design of apparatus for reliable calibration of pyrometers up to high temperatures.

We have seen that the main difficulties in applying the methods of infrared pyrometry involve the fact that, with the existing radiation detectors and filters, the value of the effective emissivity $\bar{\epsilon}$ of the flame is small, at least for the 2.7 μ band, and the smaller $\bar{\epsilon}$ is, the greater the errors of measurement. Besides, with filters having wide transmission bands, the value of $\bar{\epsilon}$ itself, and also the effective wavelength λ_{eff} are subject to a high degree of error. This is mainly because the measured value of $\bar{\epsilon}$ depends on the temperature of the comparison source.

In order to increase the value of the apparent emissivity of the flame and to diminish the dependence of the apparent emissivity of the flame on the temperature of the comparison source, work is needed on the design of filters having considerably narrower bands in the 2.7 and $4.3\,\mu$ regions than those existing at present, as well as on radiation detectors sensitive enough in the $4.3\,\mu$ region. Going over into the $4.3\,\mu$ region, where there is an absorption band much more intense than at 2.7 μ , will result in an increase in the apparent value of $\overline{\epsilon}$. Experiment shows that with the existing filters and radiation detectors, the value of $\overline{\epsilon}$ is increased by 3-4 times hereby, while the dependence of the apparent value of $\bar{\epsilon}$ on the temperature of the comparison source is here considerably weaker than in studies in the 2.7 μ region. This facilitates and increases the accuracy of measurement of $\bar{\epsilon}$ and of the temperature. The use in the infrared pyrometer of an opticoacoustic detector developed by M. L. Veingrov, [50] combining a radiation detector and a narrow-band filter, is especially promising.

Further work is also necessary along the line of developing direct and calculation methods for determining the temperature field of a flame from the infrared radiation.

The temperature "spread" (the difference between the mean optical temperatures measured in the visible and the infrared) depends in particular, as we have seen, on the degree of mixing and the temperature pulsations in the flame. Thus, we may determine from this difference, at least qualitatively, some of the gasdynamic parameters of the flame.^[4]

We note that with a given mean ratio of fuel to oxidant (Z_0) , the radiation from the flame depends on the temperature pulsations and the degree of mixing of the fuel and the oxidant, since these quantities determine the mean optical temperature \overline{T}_{opt} . Hence, the ideas presented here may be applied in solving the technically-important problem of determining the conditions for optimum emission from a flame. When the fuel-oxidant ratio differs from the stoichiometric, the emission will be increased by poor mixing and temperature pulsations.

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Translated by M. V. King