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THE CLASSIFICATION OF THE ELEMENTARY PARTICLES

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1. INTRODUCTION

THE purpose of the present article is to examine the existing attempts at a classification of the elementary particles. All contemporary studies in this direction start from the phenomenological scheme of Gell-Mann—Nishijima (hereafter called G-M—N), which is based on the introduction of a new property of particles—the strangeness. As is well known, besides the properties and conservation laws that are directly connected with the behavior of particles in the ordinary pseudo-euclidean space-time, particles have intrinsic or isotopic properties which characterize, in particular, their assignment to a definite charge multiplet or family. These properties include the isospin T, the baryon number N, the strangeness S, and also the often used combination called the hypercharge, Y = N + S. We have as the fundamental relation for the electric charge

$$Q = T_3 + \frac{S}{2} + \frac{N}{2} = T_3 + \frac{Y}{2},$$

which holds for all baryons: nucleons (p, n), hyperons (Λ, Σ, Ξ), and also for the mesons: π and K. On the other hand, rather paradoxically, the leptons: e, μ, ν, and the photons, not to speak of the hypothetical gravitons, occupy a secondary position in the development of classifications of the particles, and the problem of characterizing their isotopic properties is still far from solved.

It is already some time since proposals began to be made for considering the isotopic properties of particles in a space of three, four, or more dimensions of one type or another (Euclidean or pseudo-euclidean). An extremely successful theoretical interpretation of the G-M—N phenomenological scheme was given by d'Espagnat and Prentki (hereafter E — P), who took as the basis a three-dimensional space in which reflections as well as rotations are considered. At the same time Salam and Polkinghorne (hereafter S — P) developed a scheme with a four-dimensional space, which leads to very similar, though not identical, results. These papers are quite well known, and the methods have been expounded in a number of books[1,2] and review articles.[3,4,120] Later, in Sec. 2, we shall give only a very brief summary of the results of these papers.

But both the phenomenological scheme of G-M—N and the theories of E — P and S — P are by no means conclusive, even apart from certain differences between their results. For one thing, this can be seen from the following two important facts. The theory of E — P introduces eight independent constants for the interaction between baryons and π and K mesons, whereas the experimental data indicate that there are certain uniformities in these interactions. Secondly, in the schemes that have been mentioned the leptons are entirely ignored, and the question is even left open as to whether it is desirable to characterize them in terms of isotopic spin and strangeness.

During the last three or four years a large number of papers have appeared which attempt to get further with the problem of the systematics of particles. Many of these authors start from a natural desire to sketch a dynamical picture of the interactions of the particles. Since, however, the present level reached by experimental and theoretical studies of the properties of elementary particles is inadequate for a unique classification, these attempts make essential uses of different and often competing schemes of the intrinsic symmetries of the interactions of particles ("global," "fundamental," and "general" symmetries). These schemes correspond to one or another kind of equalizing treatment of groups of particles, for example, the treatment of all baryons as states of a single baryon field B and of all π and K mesons as states of a meson Π (see Sec. 3). In present attempts at classification of the particles the point of view of intrinsic symmetry is also used in treating such questions as the relative intensities of various interactions, the structure of particles, the explanation of the mass spectra of particles, parity conservation or nonconservation in various interactions, and so on. With such a treatment the answers to such questions can of course be only qualitative, but this does not decrease their importance. In Sec. 3 we shall consider papers that use various schemes with dynamical pictures of the interactions, and the classifications of particles that are obtained in this way.

In a number of papers (by Yang and Tiomno, by Salam and Taylor, and by D. Ivanenko together with M. Mirianashvili, A. M. Brodskiĭ, G. A. Sokolin, and others) attempts have been made to describe the intrinsic properties of particles in the framework of ordinary four-space, by using various representations of the Lorentz group that have usually not been taken into account, in particular the so-called anomalous spinors (cf. Sec. 4). Besides this, there have been attempts at a unified description of all matter on the basis of a nonlinear spinor field theory and new quantization rules (mainly in papers by Heisenberg and by a number of Soviet authors, see Sec. 5), and also there have been some preliminary discussions on a unified theory of matter, including gravitation, in the spirit of a topological geometrization of a single theory (papers by Wheeler and his co-workers). Also a number of authors, in particular Sakata, are now engaged in the development of rather similar models of compound particles constructed from a small number of fundamental fields — see Sec. 6. Questions connected with the classification of leptons will be touched on in Sec. 7.

Our task is the exposition of the main ideas bearing on the systematics of particles that have been raised in recent years. It must be agreed at once that so far none of these attempts has led to any final result. Moreover, in the last few years no result of an importance comparable with that of the introduction of strangeness has been obtained in an altogether convincing way. In spite of this we think that an analysis of the existing attempts at the classification of particles, both baryons and mesons, and also leptons, is extremely useful and can give indications of the most promising lines of study and stimulate further experiments both in the field of cosmic rays and in that of work with the powerful modern electron and proton accelerators.

2. THE THEORY OF STRANGENESS

a. Three-Dimensional Isospace

To overcome difficulties that arise in the study of processes of production of strange particles (hyperons and K mesons), Gell-Mann[5] and Nishijima[6] proposed a classification of particles into charge multiplets. This is based on the extension of the concept of isotopic spin (isospin) to strange particles and on the introduction in a phenomenonological way of a new quantum number S — the strangeness — whose physical meaning is a shift of the center of charge of the multiplet. The particles are grouped into the following multiplets: the isosinglet Λ^0, the isodoublets

$$N = \begin{pmatrix} p \\ n \end{pmatrix}; \; \Xi = \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix}; \; K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}; \; \overline{K} = i\tau_2 K^*, \tag{2.1}$$

and the isotriplets

$$\Sigma = \begin{pmatrix} \Sigma_1 \\ \Sigma_2 \\ \Sigma_3 \end{pmatrix}; \quad \pi = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix}. \tag{2.2}$$

The strangeness is connected with the electric charge of the particle in the following way:

$$Q = T_3 + \frac{N}{2} + \frac{S}{2}. \tag{2.3}$$

The strangeness is conserved in strong and electromagnetic interactions ($\Delta S = 0$); in weak interactions it is not conserved, and in nonleptonic decays of strange particles the selection rule that holds is $\Delta S = \pm 1$.

The G-M—N scheme made it possible to predict certain particles, Σ^0, Ξ^0, $\widetilde{K}^0$, and it is a remarkable fact that these were later found experimentally.

d'Espagnat and Prentki[7] used the full group of orthogonal transformations in three-dimensional isospace and assumed the existence of isospinors of the first and second kinds,[8] which transform differently under reflections in the isospace, to give a mathematical interpretation of strangeness and thus provide a theoretical foundation of the scheme of G-M—N. They introduced an isoparity operator U (the number of isofermions minus the number of antiisofermions) with the eigenvalues +1 for isospinors of the first kind and −1 for isospinors of the second kind, and, by assuming that the doublets N and K are isospinors of the first kind and the doublets Ξ and $\widetilde{K}$ are of the second kind, showed that

$$S = U - N \tag{2.4}$$

(on this basis the triplets Σ and π are isopseudovectors and Λ is an isoscalar). Then the expression (2.3) for the charge takes the form

$$Q = T_3 + \frac{U}{2}. \tag{2.5}$$

It must be particularly emphasized that here we have the first successful attempt to use a difference of the properties of spinors under reflections.

Thus the interaction Hamiltonian is invariant with respect to the full orthogonal group in three dimensional isospace, i.e., there is conservation of the isospin T and the isoparity U. The conservation of strangeness then follows from the conservation of the baryon number N. It must be noted that in a number of papers[9,10,11] there are suggestions about nonconservation of N (cf. Sec. 4).

The E — P formalism restricts the number of elementary particles. Whereas with the natural restriction $|Q| \le 1$ the Gell-Mann scheme allows the fermions Ω^- ($S = -3$, $T = 0$) and Z^+ ($S = +1$, $T = 0$) and the bosons ω^+ and ω^-, which have not been found experimentally, the existence of these particles is forbidden in the E — P scheme by the conservation of the isoparity U (cf. also reference 12). We note that there are indications, as yet only preliminary, of the existence of charged bosons of strangeness $S = \pm 2$, observed by Wang Kang-Ch'ang in Dubna in 1959.[13]

Still, despite the absence of contradictions with the existing experimental data and despite a certain elegance of the mathematical formulation, the G-M—N scheme has its weak sides. First, it is unable to give a dynamical picture of the observed mass spectrum of baryons; and second, it includes too many interactions and allows a great deal of arbitrariness in the choice of the coupling constants. By requiring invariance of the Lagrangian of the strong interactions with respect to charge conjugation one can show[14] that the theory will contain eight real coupling constants:

Table I. Table of the elementary particles. The main empirical characteristics and the classification of mesons and baryons into charge multiplets.*

Class of particles	Gravitons	Photons	Leptons			Mesons							Baryons							
						π mesons			K mesons				nucleons		hyperons					
Particle	g	γ	$\nu\tilde{\nu}$	e^-e^+	$\mu^+\mu^-$	π^+	π^-	π^0	K^+	K^-	K^0	$\tilde{K}^0$	p	n	Λ^0	Σ^+	Σ^0	Σ^-	Ξ^0	Ξ^-
Mass (m_e)	0	0	0	1	206.9	273.30		264,3	966.92		974,55		1836.12	1838.65	2183.30	2328.34	2333,02	2341.73	2566.6	2581.9
Lifetime (sec)	∞	∞	∞	∞	$2{,}26\cdot10^{-6}$	$2{,}56\cdot10^{-8}$		$<4\cdot10^{-16}$	$1{,}22\times10^{-8}$		K_1**: $1{,}00\times10^{-10}$; K_2: $6.1\cdot10^{-8}$		∞	$1.04\cdot10^{+3}$	$2{,}50\cdot10^{-10}$	$0.8\cdot10^{-10}$	$<0{,}1\cdot10^{-10}$	$1{,}59\cdot10^{-10}$	$1{,}5\cdot10^{-10}$	$1{,}9\cdot10^{-10}$
Spin	2?	1	1/2	1/2	1/2	0			0				1/2		1/2					
Isospin T						1			1/2				1/2		0	1			1/2	
Isospin T_3						+1	−1	0	+1/2	−1/2	−1/2	+1/2	+1/2	−1/2	0	+1	0	−1	+1/2	−1/2
Strangeness S						0					+1	−1	0		−1	−1			−2	
Baryon number N						0			0				+1		+1					
Isoparity U						0			+1	−1	+1	−1	+1		0	0			−1	

*At the present time the following antibaryons have been discovered: $\tilde{p}$, $\tilde{n}$, $\tilde{\Lambda}^0$, $\tilde{\Sigma}^+$, $\tilde{\Sigma}^0$, $\tilde{\Sigma}^-$. The values of the mass, lifetime, and spin for the antibaryons are the same as the corresponding values for the baryons; the isotopic characteristics (T_3, S, N, U) of antibaryons differ in sign from the corresponding values for the baryons.

** $K_1=\dfrac{K^0+\tilde{K}^0}{\sqrt{2}}$, $K_2=\dfrac{K^0-\tilde{K}^0}{\sqrt{2}}$.

$$L = g_1\bar{N}(i\gamma_5)\boldsymbol{\tau\pi}N + g_2 i\bar{\Sigma}(i\gamma_5)\times\boldsymbol{\pi\Sigma} + g_3[\bar{\Lambda}(i\gamma_5)\boldsymbol{\pi\Sigma} + \bar{\boldsymbol{\Sigma}}\boldsymbol{\pi}(i\gamma_5)\Lambda] + g_4\bar{\Xi}(i\gamma_5)\boldsymbol{\tau\pi}\Xi + f_1[\bar{N}\boldsymbol{\tau\Sigma}K + K^*\bar{\boldsymbol{\Sigma}}\boldsymbol{\tau}N] + f_2[\bar{N}K\Lambda + \bar{\Lambda}K^*N] + f_3[\bar{\Xi}\tau_2\boldsymbol{\tau\Sigma}K^* + K\boldsymbol{\tau}\bar{\boldsymbol{\Sigma}}\tau_2\Xi] + f_4[\bar{\Xi}\tau_2K^*\Lambda + \bar{\Lambda}K\tau_2\Xi], \quad (2.6)$$

which is unsatisfactory. In order to put further restrictions on the coupling constants, it is necessary to extend the isospace, increasing the number of dimensions and introducing more general invariance properties, i.e., a higher intrinsic symmetry of the elementary particles.

For the convenience of readers we present a table of the elementary particles, which gives in addition to the basic empirical data the isotopic characteristics of the baryons and mesons as they follow from the E — P scheme (Table I).

b. Four-Dimensional Isospace

The idea of extending the three-dimensional isospace to a four-dimensional space was first proposed by Pais[15] as a continuation of his work[16,17] in which he had attempted to set up a correspondence between the baryons and the various spinor representations of the full three-dimensional rotation group. The framework of such a treatment was too narrow, however, to include in the scheme the Ξ hyperon, which was discovered soon after the publication of the paper.

To find a way out of the resulting difficulty, Pais proposed the introduction of a four-dimensional intrinsic space.

As is known from the general theory of representations of rotation groups, the four-dimensional rotations defined by the six infinitesimal-rotation operators $T_{\alpha\beta} = -T_{\beta\alpha}$ ($\alpha, \beta = 1, 2, 3, 4$) can be represented as direct products of operators T_i and Z_i ($i = 1, 2, 3$) of independent three-dimensional rotations, which are defined by

$$T_i = \frac{1}{2}(T_{4i} + T_{jk}), \quad Z_i = \frac{1}{2}(T_{4i} - T_{jk}) \quad (2.7)$$

and satisfy the following commutation rules:

$$[T_i, T_j] = iT_k; \quad [Z_i, Z_j] = iZ_k, \quad [T_i, Z_j] = 0. \quad (2.8)$$

Thus the representations of the four-dimensional rotation group are specified by two numbers (T', Z'), and the sum $T' + Z'$ fixes the irreducible representations of this group: to half-integral $T' + Z'$ there correspond the double-valued spinor representations, and to integral $T' + Z'$ there correspond the tensor representations.

Regarding the baryons as belonging to the spinor representations and the mesons to the tensor representations and assuming that the electric-charge operator has integer values, i.e., that

$$Q = T_3 + Z_3 + \frac{1}{2} \text{ for baryons}$$

and

$$Q = T_3 + Z_3 \text{ for mesons,}$$

Pais arrived at a classification of the particles in terms of the representations of the four-dimensional isotopic-rotation group, in which, for example, the nucleon doublet is successfully described by the representation ($\frac{1}{2}$, 0), but which predicts doubly charged particles, which have so far not been observed. In spite of its importance as a stimulus, the Pais scheme was abandoned because of the defect just mentioned and certain other difficulties.

By using the tensor representations of the four-dimensional isotopic-rotation group for both mesons and baryons, Salam and Polkinghorne[18] avoided the difficulties of the Pais scheme and arrived at a systematics that is similar in its main features to the phenomenological classification of G-M—N. The operator for the electric charge is given by the formula

$$Q = T_3 + Z_3 \quad (2.9)$$

for all particles (in the notations of the paper in question $T_3 \equiv \tau_3$, $Z_3 \equiv \mu_3$). The connection with the Gell-Mann scheme becomes clear if we note the fact that T_3 has the meaning of the third component of the isospin, and $Z_3 = \frac{1}{2}S + \frac{1}{2}N = \frac{1}{2}U$. One of the interesting features of the S — P scheme is the complete symmetry between the baryon and meson families, which can be seen from Table II.

Table II. The systematics of Salam and Polkinghorne

	Representation	Particles	T_3	Z_3
Baryons	(1/2, 1/2)	p, n	+1/2, −1/2	+1/2
		Ξ^0, Ξ^-	+1/2, −1/2	−1/2
	(1, 0)	Σ^+, Σ^0, Σ^-	+1, 0, −1	0
	(0,0)	Λ^0	0	0
	(0, 1)	?	0	+1, 0, −1
Mesons	(1/2, 1/2)	K^+, K^0	+1/2, −1/2	+1/2
		$\tilde{K}^0$, K^-	+1/2, −1/2	−1/2
	(1, 0)	π^+, π^0, π^-	+1, 0, −1	0
	(0, 0)	?	0	0
	(0, 1)	?	0	+1, 0, −1

In the original form of this classification[18] the representation ($\frac{1}{2}$, $\frac{1}{2}$) in the meson family was associated with the θ mesons, and the representation (0, 1) with the τ mesons. In a later paper, however, which took account of the identity of θ and τ, the K mesons were put in correspondence with the representation $(\frac{1}{2}, \frac{1}{2})$. Like the Gell-Mann systematics, this scheme predicts some particles not yet discovered; the "free" representations in Table II correspond to these particles. In the boson class there are two such representations, (0, 0) and (0, 1). The first of these can be associated with the so-called ρ^0 meson, assumed on various grounds by a number of authors, and the second can be assigned to the D meson.[20]

A treatment of the particles that is closely similar to the S — P classification has recently been suggested by G. Sokolik. All known baryons and mesons can be divided into groups: two four-isovectors $\begin{pmatrix} N \\ \Xi \end{pmatrix}$ and $\begin{pmatrix} K \\ \tilde{K} \end{pmatrix}$; an antisymmetric isotensor of the second rank, which breaks up into two irreducible representations (0, 1) and (1, 0), which correspond to the triplets $(\Sigma^+, \Sigma^0, \Sigma^-)$ and (π^+, π^0, π^-); two singlets Λ^0 and ρ^0. It is easy to see that the representation that breaks up into these irreducible representations is given by matrices which satisfy the Duffin-Kemmer algebra

$$\beta_\mu\beta_\nu\beta_\lambda + \beta_\lambda\beta_\nu\beta_\mu = \delta_{\mu\nu}\beta_\lambda + \delta_{\lambda\nu}\beta_\mu$$

and the Ψ function of all 16 elementary particles transforms according to this representation.

In connection with the extension of the isospace to four dimensions the question arises as to its Euclidean or pseudoeuclidean character. Although the majority of authors incline toward the Euclidean four-isospace, this question is not yet finally settled.

There are two main objections against the pseudoeuclidean isospace:[21,22] the difficulties that arise in the definition of the probability amplitude and in the setting up of the commutation relations. Both of these objections are based on the absence of an analog of the Lorentz condition in the isospace, which is due to the absence of the concept of translation in the isospace.*

3. THE DYNAMICAL TREATMENT OF THE CLASSIFICATION OF PARTICLES

As we have seen, the starting point of the present theory of the "strong" particles is the systematics of Gell-Mann and Nishijima as interpreted mathematically by d'Espagnat and Prentki. Later there has been a tendency toward an equalizing treatment of the interaction of all the baryons, primarily the interaction with pions — the Gell-Mann[23] scheme of "global" symmetry, in which a universal interaction of the four baryon doublets with pions was introduced. In this the K coupling played only a subordinate role. On the other hand, Tiomno,[24] developing some ideas expressed in a preliminary way by Schwinger,[25] suggested the idea that there is a "fundamental" symmetry that manifests itself in a universal coupling of the baryon—K-meson interaction; here it is the interaction with pions that plays a secondary role. A natural generalization of these two schemes, which are in a certain sense alternatives, has been the recent papers of Feinberg and Gürsey[26] and also of Souriau[27] and of Umezawa and Visconti,[29] which use a "general" symmetry as the basis for introducing a universal interaction with a Π field which unites the π and K meson fields. Such a scheme is attractive in that it contains only one interaction constant and admits of a rather elegant mathematical treatment, for example, in the framework of a seven-dimensional intrinsic space. Pais and other authors[29,30,26] have shown, however, that such ultra-equalizing schemes, which possess a very high degree of symmetry, are in contradiction with experiment at a number of points.

A very important problem in the theory of baryons and mesons is that of constructing a dynamical interaction scheme and a corresponding classification of the elementary particles in such a way as to obtain the observed mass spectrum. The models of global, fundamental ("cosmic" in the terminology of Sakurai[31]), and general symmetry determine different approaches to the attempt to solve this problem.

All three of these models assume a hypothesis which is widely accepted at present, that nucleons and hyperons are different states of the same particle — the baryon (B) — in analogy with the concept of the proton and neutron as two states of the nucleon. Thus it is assumed, just as in the case of p and n, that in the absence of interactions that remove a degeneracy the masses of the various baryons are equal. According to all appearances the mass difference of p and n is due to electromagnetic interactions, namely to the interference of the electric and magnetic terms of the interaction energy,[33-35] and so also are the mass differences between charged and neutral Σ hyperons,[36] Ξ hyperons,[37] pions,[35] and K mesons.[38,39] It must be noted, however, that up to now there have been no completely convincing calculations of the mass differences.

The question arises as to what causes the mass differences between the various baryon multiplets. From the point of view of global symmetry the interaction of all the baryons with the pions is a universal one, which is the so-called strong coupling with the constant $g_\pi^2/4\pi \sim 15$ for the case of the ps-ps interaction. Only the inclusion of the interaction with the K mesons, which is assumed to be moderately strong, $g_K^2/4\pi \sim 0.1\, g_\pi^2/4\pi$, removes the degeneracy and leads to the mass differences between the various multiplets.[21] On the other hand, according to the hypothesis of fundamental symmetry the universal interaction is that between baryons and K mesons, and only the in-

*One can, however, as A. M. Brodskiĭ has shown, introduce a distinguished direction in the isospace, a certain vector k_μ, and require that in the momentum representation a condition like the Lorentz condition shall hold $[k_\mu A^\mu(k) = 0]$.

clusion of the additional interaction with the pions removes the degeneracy and leads to the mass differences.[31,32] To explain the mass differences in the framework of the model of general symmetry it is necessary to introduce interactions of a special type.[30,40,41]

Let us now turn to the main conclusions regarding the systematics of elementary particles that follow from the theory of global symmetry.

a. Global Symmetry

The theory of global symmetry is based on the introduction of baryon doublets[23]

$$N_1 = \begin{pmatrix} p \\ n \end{pmatrix}, \quad N_2 \equiv \begin{pmatrix} \Sigma^+ \\ Y^0 \end{pmatrix}, \quad N_3 \equiv \begin{pmatrix} Z^0 \\ \Sigma^- \end{pmatrix}, \quad N_4 \equiv \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix}, \tag{3.1}$$

where

$$Y^0 \equiv \frac{\Lambda^0 - \Sigma^0}{\sqrt{2}}; \qquad Z^0 \equiv \frac{\Lambda^0 + \Sigma^0}{\sqrt{2}}. \tag{3.2}$$

The Lagrangian for the interaction of baryons with pions then has the form

$$L_\pi = g_{N\pi} P_{N\pi} + g_{\Xi\pi} P_{\Xi\pi} + g\,(P_{\Sigma\pi} + P_{\Lambda\pi}), \tag{3.3}$$

where

$$\left.\begin{aligned} P_{N\pi} &= i\,[(\bar{p}\gamma_5 p - \bar{n}\gamma_5 n)\,\pi^0 + \sqrt{2}\,(\bar{p}\gamma_5\, n\pi^+ + \bar{n}\gamma_5\, p\pi^-)], \\ P_{\Sigma\pi} + P_{\Lambda\pi} &= i\,[(\bar{\Sigma}^+\gamma_5\Sigma^+ - \bar{Y}^0\gamma_5 Y^0)\,\pi^0 + \sqrt{2}\,(\bar{\Sigma}^+\gamma_5 Y^0\pi^+ \\ &\quad + \bar{Y}^0\gamma_5\Sigma^+\pi^-)] + i\,[(\bar{Z}^0\gamma_5 Z^0 - \bar{\Sigma}^-\gamma_5\Sigma^-)\,\pi^0 \\ &\quad + \sqrt{2}\,(\bar{Z}^0\gamma_5\Sigma^-\pi^+ + \bar{\Sigma}^-\gamma_5 Z^0\pi^-)]. \end{aligned}\right\} \tag{3.4}$$

Furthermore, because of the universality of the interaction,

$$g_{N\pi}^2 = g_{\Xi\pi}^2 = g^2. \tag{3.5}$$

Preliminary calculations made by Gell-Mann in lowest order in the coupling with the K mesons led to the relation

$$\frac{m_N + m_\Xi}{2} = \frac{3m_\Sigma + m_\Lambda}{4},$$

which is in fairly good agreement with the experimental data.

The further development of the idea of global symmetry, leading to a classification of the elementary particles, was carried out in a series of papers by Schwinger. Although in the first paper,[25] which was of a preliminary nature, the author assumes that the interaction of the baryons with K mesons is symmetrical and that the interaction with the pions introduces the asymmetry, in his subsequent papers Schwinger[21] uses the idea of global symmetry and constructs a dynamical theory of the particles, including leptons, and systematically applies the concept of successive decreases of symmetry with the inclusion of weaker and weaker interactions. Let us consider the main relations of Schwinger's theory.

We shall describe all particles by a many-component Hermitian field χ, which breaks up into a Fermi field ψ and a Bose field φ. It is at once evident that the spins of particles now known are confined to the values $\frac{1}{2}$ for fermions and 0 and 1 for bosons, and that the strong interactions involve particles with the minimum spins, $\frac{1}{2}$ and 0. Schwinger assumes that the Bose field with spin 1 represents an essentially different family of particles, including in particular the photon.

The existence of intrinsic degrees of freedom is expressed by an additional increase of the numbers of components of the fields ψ and φ. It is shown that in spite of the difference in the three-dimensional interpretations of the baryons (N and Ξ belong to the representation of the three-dimensional group of isotopic rotations with $T = \frac{1}{2}$, and Λ and Σ to the representations with $T = 0, 1$, respectively), there is a possibility of giving them a unified description in a four-dimensional intrinsic space. In fact, the same set of matrices T_α ($\alpha = 1, 2, 3, 4$) of four-dimensional rotations can be regarded as belonging both to the representations $T = 0, 1$ and also to the representation $T = \frac{1}{2}$; in this sense it is said that the four-dimensional description realizes a unified symmetry for the representations $T = \frac{1}{2}$ and $T = 0, 1$ of the three-dimensional rotation group.

Using the idea of the global symmetry of the pion-baryon interaction, Schwinger next prescribes that the nucleonic charge N is a common property of the baryons, which does not depend on the value of the isospin, and that the pion field is the dynamical agent that determines the nucleonic charge. The Lagrangian of the pion-baryon interaction then has the form

$$L_\pi = g_\pi \varphi_{(1)} \cdot \frac{1}{2} \left\{ \psi_{\left(\frac{1}{2}\right)} \beta\gamma_5 \nu\tau \psi_{\left(\frac{1}{2}\right)} + \psi_{(0)}\beta\gamma_5 i\nu\psi_{(1)} - \psi_{(1)}\beta\gamma_5 i\nu\psi_{(0)} + \psi_{(1)}\beta\gamma_5\nu\frac{1}{i}\cdot\psi_{(1)} \right\}, \tag{3.6}$$

where $\varphi_{(1)}$ is the pion field and $\psi_{(i)}$ are the baryon fields, with the index (i) indicating the representation of the three-dimensional isotopic rotation group; β and γ_5 are Dirac matrices referring to ordinary space; τ is the isotopic-spin matrix; and ν is the matrix of the nucleonic charge N (we recall that the spinors ψ are real):

$$\nu = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \tag{3.7}$$

In Eq. (3.6) the pion field $\varphi_{(1)}$ is described by a self-dual antisymmetric tensor (a possibility first pointed out by Salam and Matthews[42]):

$$\varphi_{(1)} = \begin{pmatrix} 0 & -i\pi_3 & i\pi_2 & 0 \\ i\pi_3 & 0 & -i\pi_1 & 0 \\ -i\pi_2 & i\pi_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{3.8}$$

The difference between baryons with integral and half-integral isospins is introduced by the interaction of baryons with K mesons, which also, according to Schwinger, destroys the four-dimensional symmetry in the representations $\psi_{(1/2)}$ and $\psi_{(0,1)}$ and thus leads to mass differences between the charge multiplets.

The dynamical effect of the electromagnetic field is to reduce the three-dimensional symmetry to a two-dimensional symmetry, in which there is the generally valid relation

$$Q = T_3 + \frac{1}{2} Y, \tag{3.9}$$

where Y is the hypercharge, numerically equal to the isoparity U. It is not hard to show that the electromagnetic interactions are invariant with respect to reversal of the charge,

$$R_Q^{-1} Q R_Q = -Q, \tag{3.10}$$

where R_Q is the unitary operator of charge reversal, given by

$$R_Q = R_N e^{i\pi\left(T_3 + \frac{1}{2} Y\right)}. \tag{3.11}$$

Here R_N is the unitary operator for reversal of the nucleonic charge ($R_N^2 = +1$). Furthermore the signs of N and Y change along with the sign of Q.

The interactions that involve leptons possess lower symmetry than those considered above. Therefore for the description of the leptons Schwinger assumes the representation T = 1 of the three-dimensional rotation group, and combines the leptons into a charge triplet. Here

$$Q = T_3 \tag{3.12}$$

with the eigenvalues 1, 0, −1.

In analogy with the nucleonic charge N, a leptonic charge L is introduced, which is represented by the matrix

$$\lambda = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \tag{3.13}$$

The leptonic charge (lepton number) was first introduced by Konopinski and Mahmoud,[43] who showed that if we regard μ^+, ν, and e^- as leptons (L = +1) and μ^-, $\tilde{\nu}$, and e^+ as antileptons (L = −1), then by postulating conservation of the lepton number one forbids all unobserved reactions involving leptons (for example, $\mu^+ \rightarrow e^+ + e^+ + e^-$, etc.), whereas all the observed reactions are allowed. We note that the theory of the two-component neutrino is incompatible with this definition of leptons and antileptons (in fact, according to this theory $\mu^+ \rightarrow e^+ + \nu + \tilde{\nu}$).

The existing symmetry in isotopic properties between heavy bosons and baryons leads to the idea of the existence of a family of bosons that realizes the representation T = 1 of the three-dimensional rotation group. The exceptional position of the photon in this classification and the formal possibility of identifying it with the third component of a three-dimensional isovector provide a possible answer. Thus we arrive at the concept of a family of bosons with spin 1 that consists of the photon (m = 0, Q = 0) and two charged vector particles $Z_\mu^\pm$, which have a nonvanishing rest mass.

Since the charged Z particles play the role of partners of the electromagnetic field Schwinger, and also Salam and Ward,[44] assume that the interaction constant of Z particles with fermions is a universal electromagnetic coupling $e^2/\hbar c$. If this is true, then the coupling with the charged Z field leads to a further contraction of the intrinsic symmetry, which will probably permit a description of the general mechanism of the weak interactions in terms of a hypothetical intermediate charged Z particle. This interaction must have even less symmetry than the electromagnetic interaction; it will destroy the invariance under the electric-charge reversal R_Q, Eq. (3.11), although it will be invariant under two-dimensional isotopic rotations (conservation of Q).

Studying the interactions of leptons with $Z^\pm$ particles, which destroy invariance under R_Q, Schwinger showed that such interactions must automatically destroy invariance under space reflection R_S:

$$L_{Zl} = g_Z Z^\mu \psi_l \beta \gamma_\mu (t - i\gamma_5 \{t_3, t\}_+) \psi_l. \tag{3.14}$$

Here $t = t_1$ or t_2; $Z^\mu = Z_1^\mu$ or Z_2^μ [$Z_{1,2}^\mu = (Z^+ \pm iZ^-)/2^{1/2}$; t_i (i = 1, 2, 3) are 3 × 3 isotopic matrices; and ψ_l is the lepton wave function].

It is easy to see that L_{Zl} is invariant under the product

$$R = R_s R_Q, \tag{3.15}$$

i.e., it conserves the combined parity.[45]

An analogous relation had also been established somewhat earlier in a paper by one of the writers and G. A. Sokolik,[46] which started from the idea of a combined description of the ordinary and isotopic spaces, and even reached conclusions about possible transitions from one space to the other. Very similar ideas on the connection of intrinsic (isotopic) properties with ordinary "external" properties have also been developed by Yukawa,[47] Pais,[48] Vigier,[49] and Raiskij.[50]

To assure that the neutrino mass is zero, one must require invariance under the transformation

$$\psi_l \rightarrow [1 + i\delta\varphi((1 - t_3^2) i\gamma_5 - t_3)] \psi_l, \tag{3.16}$$

which is an extension of the Salam-Touschek transformation[51,52,53] to the entire family of leptons. This invariance leads to conservation of the so-called neutrino charge n, for which the corresponding current is

$$j_n^\mu = \frac{1}{2} \psi_l \beta \gamma^\mu ((1 - t_3^2) i\gamma_5 - t_3) \psi_l. \tag{3.17}$$

The neutrino charge of the μ meson and the electron then has the sign opposite to the sign of the electric charge, and the neutrino charge of the neutrino is represented by the matrix γ_5, whose eigenvalues have the meaning of the spin projection along the direction of motion of the neutrino. Thus a neutrino with n = +1

(−1) can be regarded as a right-circularly (left-circularly) polarized neutrino. In lepton-pair production processes that involve the charged Z field the neutrino charge is conserved. Therefore a positively charged lepton is created with a right-circularly polarized neutrino, and a negatively charged lepton with a left-circularly polarized neutrino.

The law of conservation of the neutrino charge is quite independent of the conservation of leptonic charge (cf. also Pauli[54]); for example, a neutrino (L = +1) can accompany either a positron or a μ^- meson. Thus

$$Z^+ \longleftrightarrow \mu^+ + \tilde{\nu}_R \quad \text{or} \quad e^+ + \nu_R, \tag{3.18a}$$

$$Z^- \longleftrightarrow \mu^- + \nu_L \quad \text{or} \quad e^- + \tilde{\nu}_L, \tag{3.18b}$$

where the indices L and R denote left-handed and right-handed polarization. A similar theory of the neutrino has also been developed by Nishijima.[55] This theory does not coincide with the two-component theory. From the conservation of the leptonic and neutrino charges it follows that

$$\mu^+ \longrightarrow e^+ + \nu_R + \nu_L, \tag{3.19a}$$

$$\mu^- \longrightarrow e^- + \tilde{\nu}_R + \tilde{\nu}_L. \tag{3.19b}$$

In an analogous way we can construct the interaction of charged Z particles with baryons:

$$L_{ZN} = \frac{1}{\sqrt{2}} g_Z Z^\mu \cdot \frac{1}{2} \psi\beta\gamma_\mu \left(\tau - i\gamma_5 \left\{ \frac{\tau_3 - \zeta_3}{2}, \tau \right\}_+ \right) \psi. \tag{3.20}$$

Charged Z particles thus provide a coupling between lepton pairs and baryon pairs, playing the role of intermediate bosons, which have been considered in a number of other papers (cf. Sec. 7).

From the preceding discussion it follows that the classification of baryons and mesons according to Schwinger does not differ from the classification of d'Espagnat and Prentki, although it must be noted that pions can be introduced into this scheme in two alternative ways: either as a self-dual antisymmetric tensor (3.8) or as a four-vector (a charge triplet π and a charge singlet σ). Thus the scheme contains the prediction of a new neutral boson with T = 0 and S = 0. The classification of the leptons and the family of Z particles can be put in the form of Table III.

Thus we have before us an interesting attempt at a dynamical description of all known elementary particles, and the scheme predicts three new particles: a σ meson and Z^+ and Z^- particles. Moreover, Schwinger regards it as possible to include in the scheme also the interaction with the gravitational field, which, in his opinion, should lead to a degree of symmetry still lower than that of the weak interactions.

b. Fundamental Symmetry

As has already been stated, in his first paper,[25] devoted to the dynamical theory of elementary particles, Schwinger introduced the idea of a symmetrical interaction of baryons with K mesons within the framework of four-dimensional isospace. The interaction of baryons with pions introduces a preferred direction in one of the three-dimensional subspaces, and thus destroys the four-dimensional isotopic symmetry of the baryons; in analogy with the introduction of the electric charge Q, the corresponding invariant property associated with rotation around the preferred axis enables us to introduce the nucleonic charge N, equal to +1 for baryons, −1 for antibaryons, and 0 for mesons. Developing this theory, Schwinger shows that the K mesons have an analogous property of a type of charge that is realized dynamically by the coupling with the pion field. Thus there is introduced the hypercharge Y, with Y = +1 for K^+, K^0 and Y = −1 for K^-, $\tilde{K}^0$. The baryons that have isospin $T = \frac{1}{2}$ also possess hypercharge. Although this classification is not essentially different from that of d'Espagnat and Prentki, still it must be emphasized that Schwinger treats the hypercharge from the dynamical point of view (as a consequence of the interaction of pions with K mesons), and not from the geometrical point of view [the isoparity in Eq. (2.4) was defined as a reflection operator in three dimensional isospace].

Table III. Classification of the leptons according to Schwinger. The Z^0 boson is identified with the photon

Class of particles		Leptons				Antileptons				Z bosons		
Particle		μ^+	ν (ν_R)	ν (ν_L)	e^-	μ^-	$\tilde{\nu}$ ($\tilde{\nu}_R$)	$\tilde{\nu}$ ($\tilde{\nu}_L$)	e^+	Z^+	Z^0	Z^-
Lepton number L		+1				−1				0		
Isospin	T	1				1				1		
	T_3	+1	0		−1	−1	0		+1	+1	0	−1
Neutrino charge n		−1	+1	−1	+1	+1	+1	−1	−1	0		

The idea of the fundamental baryon—K-meson interaction received further development in a paper by Tiomno,[24] who in addition used an extremely general seven-dimensional auxiliary space (see also reference 57) for describing the intrinsic properties of particles. The empirical basis for this is the observation that the combination of a half-integral isospin for K mesons on one hand with an integral isospin for Λ and Σ hyperons on the other, which is the basis of the G-M—N classification, is not the only possible one, but follows from the relation

$$Q = T_3 + \frac{Y}{2}. \qquad (3.21)$$

It turns out that an alternative scheme is possible if one starts from the relations

$$Q = I_3 + J_3, \qquad (3.22)$$

$$Y = J_3 + J_3', \qquad (3.23)$$

where I_3, J_3, and J_3' are new quantum numbers. It follows from the fact that Q and Y are integers, that I_3, J_3, and J_3' must be simultaneously integers or half-integers. This leads to the classification of Table IV.

Tiomno then goes on to put a mathematical foundation under his proposed empirical scheme, as the investigation of d'Espagnat and Prentki did for the G-M—N phenomenological scheme. Since each of the numbers I_3, J_3, and J_3' takes the two values $\pm\frac{1}{2}$ (for baryons), their irreducible representation will be given by 8×8 matrices. Therefore the wave function of the baryons must have 8 components, which agrees with the number of known baryons: N, Ξ, Λ, Σ. As is well known, eight-component spinors correspond to a seven-dimensional space,[8] which is constructed as the direct sum of a three-dimensional isospace and a four-dimensional hypercharge space. Denoting the hyperspin vector by

$$\mathbf{J} = (M_{23},\ M_{31},\ M_{12}), \qquad (3.24)$$

we have

$$J_3 = \frac{1}{i} M_{12}, \quad J' = \frac{1}{i} M_{31}. \qquad (3.25)$$

Generally speaking, the requirement of invariance of all rotations in the seven-dimensional space is not needed as yet, since it leads to the prediction of three new particles, partners of the K mesons, which have not been observed so far. Therefore Tiomno requires invariance with respect to independent rotations in the three-dimensional isospace (conservation of I_3) and in the four-dimensional hyperspace (conservation of J_3 and J_3'). The mesons are then described by a tensor

$$B^{r_1 \ldots r_m}_{\lambda_1 \ldots \lambda_n} \quad (r_i = 1, 2, 3; \ \ \lambda_i = 1, 2, 3, 4) \qquad (3.26)$$

of rank m in the isospace and rank n in the hyperspace. As is known from the theory of representations of rotation groups, the maximum eigenvalues of I_3 and J_3 (or J_3') will be m and n, respectively. From the condition that the maximum electric charge is 1, we have

$$m + n = 1.$$

Table IV. Classification of particles in the doublet approximation

Field	π			K				B							
Charge states	π^+	π^0	π^-	K^+	K^0	$\widetilde{K}^0$	K^-	p	n	Σ^+	Y^0	Z^0	Σ^-	Ξ^0	Ξ^-
I_3	1	0	—1	0	0	0	0	1/2	—1/2	1/2	—1/2	1/2	—1/2	1/2	—1/2
J_3	0	0	0	1	0	0	—1	1/2	1/2	1/2	1/2	—1/2	—1/2	—1/2	—1/2
J_3	0	0	0	0	1	—1	0	1/2	1/2	—1/2	—1/2	1/2	1/2	—1/2	—1/2
$Y_3 = \frac{1}{2}(J_3 + J_3')$	0	0	0	1/2	1/2	—1/2	—1/2	1/2	1/2	0	0	0	0	—1/2	—1/2
$Z_3 = \frac{1}{2}(J_3 - J_3')$	0	0	0	1/2	—1/2	1/2	—1/2	0	0	1/2	1/2	—1/2	—1/2	0	0
$T_3 = L_3 + Z_3$	1	0	—1	1/2	—1/2	1/2	—1/2	1/2	—1/2	1	0	0	—1	1/2	—1/2

Thus there are two possibilities for mesons:

a) m = 1, n = 0 (isospin 1),

b) m = 0, n = 1 (hyperspin 1).

The first case corresponds to the pions, the second to the K mesons. In fact, it is well known that the pions form three charge states, and it is now proposed to describe the four charge states of the K mesons by a Hermitian hypervector

$$\mathscr{K}_\mu = \begin{pmatrix} \mathscr{K}_1 \\ \mathscr{K}_2 \\ \mathscr{K}_3 \\ \mathscr{K}_4 \end{pmatrix}.$$

The Hamiltonian for the interaction of baryons and K mesons is written in the form

$$H_K = g\mathscr{K}^\mu \bar{\psi}\Gamma_\mu\psi, \tag{3.27}$$

where Γ_μ ($\mu = 1, \ldots, 4$) are matrices which act on the hypercharge components of the baryon field ψ and commute with the isospin matrices. In addition, they satisfy the relations

$$\Gamma_\mu\Gamma_\nu + \Gamma_\nu\Gamma_\mu = 2\delta_{\mu\nu}. \tag{3.28}$$

As Tiomno points out, the arguments given above are based on a wider group of transformations than that used by d'Espagnat and Prentki. In fact, only by choosing a certain definite representation of the Γ_μ and then writing $\mathscr{K}^\mu\Gamma_\mu$ in the form of isospinors can one get a Hamiltonian

$$H_K = g\,[N\,(\Lambda + i\Sigma\tau)\,K + \Xi\,(\Lambda + i\Sigma\tau)\,K'] + \text{Herm. adj.} \tag{3.29}$$

where

$$K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}; \quad K' = \begin{pmatrix} -K^{0*} \\ K^- \end{pmatrix}, \tag{3.30}$$

which agrees formally with the Hamiltonian (2.6) of E — P[7] and Salam[14] in the parts that refer to the interaction with the K mesons.

The scheme of Tiomno is symmetrical with respect to all baryons as long as the electromagnetic and pion interactions are not introduced: the former introduces a preferred direction in the isospace (a mass difference between the charged and neutral components of a given charge multiplet), and the latter leads to a preferred direction in the hypercharge space (a mass difference between the different charge multiplets).

A rather elegant mathematical interpretation of the Tiomno scheme has been developed in a paper by Feinberg and Gürsey.[26] A rotation in the hypercharge four-space can be represented as the product of two commuting three-dimensional rotations:

$$\text{(I)}\quad \psi \longrightarrow \exp\left\{\frac{1}{2}\,i\,(1+\Gamma_5)\,\Sigma\mathbf{u}\right\}\psi, \tag{3.31}$$

$$\text{(II)}\quad \psi \longrightarrow \exp\left\{\frac{1}{2}\,i\,(1-\Gamma_5)\,\Sigma\mathbf{u}\right\}\psi, \tag{3.32}$$

where

$$\Sigma_{\mu\nu} = \frac{1}{2}\,(\Gamma_\mu\Gamma_\nu - \Gamma_\nu\Gamma_\mu).$$

It is easy to see that the transformation

$$\text{(III)}\quad \psi \longrightarrow a\psi - ib\Sigma_2\psi^*, \quad |a|^2 + |b|^2 = 1, \tag{3.33}$$

commutes with (I) and (II). It is the analog of the Pauli-Gürsey transformation[54,56] for the case of a four-dimensional Euclidean space, which is isomorphous to three-dimensional rotations. Thus the Tiomno scheme can be described in the framework of a four-dimensional intrinsic space.

In a number of papers of Dallaporta and his coworkers, which are characterized by starting from the idea of an equalizing "general" symmetry of the baryon-pion-kaon interaction, an attempt is made to give a qualitative explanation of the observed mass spectrum of the baryons by using the seven-dimensional intrinsic space of Tiomno. As has been shown in references 58 and 59, in the four-dimensional hyperspace one can introduce two groups of three-dimensional rotation generators: **Y**, the hypercharge spin, and **Z**, the hyperspin number, so that there is the following connection between Y_3 and Z_3 and Tiomno's quantum numbers J_3 and J_3':

$$J_3 = Y_3 + Z_3, \quad J_3' = Y_3 - Z_3 \tag{3.34}$$

(cf. Table IV).

The baryon—K-meson interaction is hypercharge-independent, that is, both **Y** and **Z** are conserved; the pion-baryon interaction is charge-independent (in the sense of the doublet approximation, in which all baryons have the isospin $\frac{1}{2}$), that is, **I** is conserved. Thus:

$$L = L_K + L_\pi = iF\sum_{k=1}^{4} \bar{X}\Omega_k G_5^{k+1} X\Phi_k + ig\sum_{j=1}^{3} \bar{X}T_j G_5^{j+1} X\pi_j, \tag{3.35}$$

where Ω_k and T_j are operators that act respectively on the hypercharge and isospin coordinates of a 32-component spinor X that describes the baryons; G_5 is a 32×32 matrix that is the extension to this case of the ordinary Dirac matrix γ_5. Introducing instead of Ω_k matrices Ω_k'

$$\Omega_k' = \begin{cases} -i\Omega_{k+1}\Omega_5 & \text{for } k = 1, 3, \\ i\Omega_{k-1}\Omega_5 & \text{for } k = 2, 4, \end{cases}$$

we get a new Lagrangian for the baryon—K-meson interaction, which in the combination L_K destroys the four-dimensional symmetry of the hypercharge space and can lead to a difference between the masses of N and Ξ:[49,59]

$$\bar{L}_k = \sum_{k=1}^{4} \bar{X}\,(iF\Omega_k + iF'\Omega_k' G_5)\,G_5^{k+1} X\Phi_k. \tag{3.36}$$

In fact, the constant of the K coupling will now be F − F′ for N and F + F′ for Ξ; then **Y** is no longer conserved, and the constants of the motion are Y_3 and **Z**.

In a recent paper by Dallaporta and Pandit[60] an attempt is made to explain the difference of the masses of Λ and Σ, owing to which we have instead of the two doublets of Gell-Mann a singlet Λ and a triplet Σ. This is accomplished by the introduction of an interaction Lagrangian which differs from the preceding formalism in that the fictitious neutral particles Y^0 and Z^0 enter in a symmetrical way. In other words, it is proposed to take a linear combination of the old Langrangian L_π and a new Langrangian L'_π in which $Y^0 \longleftrightarrow Z^0$:

$$\bar{L}_\pi = L_\pi + bL'_\pi,$$

where b is a real constant. It then turns out that the resulting Lagrangian is of exactly the form (2.6). Instead of eight coupling constants, however, one now introduces just four parameters, g, F, F′, and b. The connection between the constants of the two theories is given in the following equations:

$$\left.\begin{aligned} &g_1 = g_2 = g_3 = g, \\ &\frac{g_3}{g} = \frac{f_2}{f_1} = \frac{f_4}{f_3} = \frac{b-1}{b+1}, \\ &\frac{f_3}{f_1} = \frac{f_4}{f_2} = \frac{F'+F}{F'-F}. \end{aligned}\right\} \tag{3.37}$$

In the Lagrangian $\bar{L}_\pi$ the coupling of the Σ hyperon with the π field is proportional to (1 + b), whereas the coupling of the Λ hyperon with the π field is (1 − b). In the opinion of these authors this fact must lead to different masses for Σ and Λ.

The complete Lagrangian for the interaction of baryons with the K and π fields is invariant under rotations in a three-dimensional "effective" isospace; that is, there is conservation only of the "effective" isospin T:

$$\mathrm{T} = \mathrm{Z} + \mathrm{I}, \tag{3.38}$$

which coincides with the ordinary isospin of d'Espagnat and Prentki. In addition to this, Y_3 will be conserved. The series of papers by Dallaporta and his collaborators is of particular interest because it reflects the evolution of views that were first concerned with attempting to have fundamental and even general symmetry, and then departed from it to a certain extent.

Let us now consider the papers of Sakurai, in which the author has developed in detail ideas[25] about a symmetrical baryon—K-meson interaction (fundamental, or, in the author's notation, "cosmic" symmetry). Sakurai[31] draws a parallel between the electromagnetic and pion couplings, on one hand, and between the pion and K-meson couplings, on the other. The electromagnetic coupling does not allow processes with change of the electric charge of the particles involved, whereas the pion coupling can lead to such a change; similarly, the pion coupling cannot change the hypercharge of the interacting particles, whereas such a change occurs with the K coupling. The π-B interaction is charge-independent, and consequently possesses the corresponding intrinsic symmetry, which is destroyed by the electromagnetic interaction. By analogy we can expect that the K coupling is more symmetrical than the π coupling, and that the latter destroys this higher degree of symmetry. From this point of view the K coupling distinguishes neither baryons with different values of the hypercharge (Y = +1, 0, −1), nor K and anti-K mesons (Y = +1, −1, respectively); just as the π coupling is charge-independent, we can say that the K coupling is hypercharge-independent.

The corresponding Hamiltonian for the K coupling is of the form

$$H_k = \sqrt{2}\, G_k [\bar{N}YK^0 + \bar{N}ZK^+ \pm (\bar{\Xi}Y\tilde{K}^+ - \bar{\Xi}Z\tilde{K}^0)] + \text{Herm. adj.} \tag{3.39}$$

where $Y = \begin{pmatrix}\Sigma^+\\ Y^0\end{pmatrix}$, $Z = \begin{pmatrix}Z^0\\ \Sigma^-\end{pmatrix}$ are the so-called doublets of the "first kind" [cf. Eq. (3.1)].

On examining in detail the theoretical and experimental arguments that are the basis for the model of global symmetry, Sakurai concludes that so far these arguments do not allow us to say anything definite in its favor. Moreover, a calculation of the mass difference $\Xi^- - \Xi^0$ based on the model of global symmetry[37] gives the wrong sign, whereas the corresponding calculation with the model of fundamental symmetry gives the correct sign of the mass difference, with the value

$$m_{\Xi^-} - m_{\Xi^0} \simeq 3m_e.$$

The Hamiltonian for the interaction of pions with baryons is constructed in such a way that the π-B interaction will destroy the four-dimensional symmetry of the baryons. There are two possible types of N-π and Ξ-π interaction that are symmetrical in N and Ξ:

$$(\bar{N}\tau N + \bar{\Xi}\tau\Xi)\,\pi \text{ and } (\bar{N}\tau N - \bar{\Xi}\tau\Xi)\,\pi.$$

The symmetry between N and Ξ is destroyed if we assume that these two interactions occur together. In this case it turns out that the Ξ field is not directly coupled with the pion field ($g^2_{\Xi\pi} = 0$).

This fact led Sakurai[37] to the interesting prediction that the anomalous magnetic moment of the Ξ hyperons is zero:

$$\mu(\Xi^0) = \mu(\Xi^-) = 0.$$

The situation is different as to the destruction of the symmetry between the doublets Y and Z. In this case the interaction with the field must lead to a triplet Σ and a singlet Λ. In view of the small mass difference Λ − Σ, Sakurai supposes that $g_{\Lambda\Sigma\pi}$ and $g_{\Sigma\Sigma\pi}$ are equal in magnitude. In the case $g_{\Lambda\Sigma\pi} = g_{\Sigma\Sigma\pi}$ there would be four-dimensional symmetry; therefore one chooses $g_{\Lambda\Sigma\pi} = -g_{\Sigma\Sigma\pi}$. On this assumption Λ and Σ must be grouped into doublets of the "second kind":

$$V = \begin{pmatrix}\Sigma^+\\ -Z^0\end{pmatrix}, \quad W = \begin{pmatrix}-Y^0\\ \Sigma^-\end{pmatrix}. \tag{3.40}$$

The fact that the K coupling and the π coupling give rise to different four-dimensional symmetries for the baryons with Y = 0 was first noted by Schwinger.[21]

Thus the π-B interaction is characterized by two constants $g_{N\pi}$ and $g_{\Lambda\Sigma\pi} = -g_{\Sigma\Sigma\pi}$. In this sense the model of fundamental symmetry corresponds to the requirement of a minimum number of constants.[61]

Later Sakurai[31,32] makes an assumption about a possible connection between all the constants of the π-B interaction, by introducing a sort of "quantization of the coupling constants": for Ξ (S = −2) the constant is equal to zero, for Λ and Σ (S = −1) it is of moderate size, and finally, it is largest for N (S = 0). Similar considerations lead to an idea of Schwinger,[25] that the magnitude of the effective "charge" of the π-B interaction is given by the value of Y + B = 2B + S. If we accept such an assumption, then the Lagrangian for the π coupling will be of the form

$$L_\pi = g_\pi\left[\overline{N}\boldsymbol{\tau}N\boldsymbol{\pi} + \frac{1}{2}\overline{\boldsymbol{\Sigma}}\times\boldsymbol{\Sigma}\boldsymbol{\pi} - \frac{1}{2}(\overline{\Lambda}\boldsymbol{\Sigma}\boldsymbol{\pi} + \overline{\boldsymbol{\Sigma}}\Lambda\boldsymbol{\pi})\right]. \quad (3.41)$$

c. General Symmetry

We now consider the papers in which general symmetry is used as the basis for introducing a universal interaction between the baryon field and the combined π and K meson field.

In the paper of Feinberg and Gürsey a general 32-component B field

$$B = B(p,\ n,\ \Sigma^+,\ Y^0,\ -Z^0,\ -\Sigma^-,\ -\Xi^0,\ -\Xi^-),$$

is introduced, and its intrinsic rotations are written in the form

$$B = B\exp[i(\boldsymbol{\sigma}\mathbf{I} + \boldsymbol{\varrho}\mathbf{J}' + \boldsymbol{\xi}\mathbf{J})],$$

where $\boldsymbol{\sigma}$, $\boldsymbol{\rho}$, $\boldsymbol{\xi}$ are 8 × 8 rotation matrices that commute with each other. Thus each baryon can be associated with definite values of the I-spin, the J-spin, and the J′-spin introduced by Tiomno.[24] We can introduce three reflection operators, each changing the sign of one of these spins:

$$S_Q: (I_3,\ J_3,\ J_3') \to (-I_3,\ J_3,\ J_3'), \quad (3.42a)$$

$$S^{s_1}: (I_3,\ J_3,\ J_3') \to (I_3,\ -J_3,\ J_3'), \quad (3.42b)$$

$$S^{s_2}: (I_3,\ J_3,\ J_3') \to (I_3,\ J_3,\ -J_3'). \quad (3.42c)$$

These reflections are a natural generalization of the operation of charge symmetry for nucleons, for which

$$S_Q: \begin{pmatrix} p \\ n \end{pmatrix} \to \begin{pmatrix} p \\ n \end{pmatrix}\tau_1.$$

In fact, S_Q corresponds to the symmetrical interaction of baryons with pions, and S^{s_1} and S^{s_2} to the interaction with K mesons.

The interaction Lagrangian for the fields B, π, K, which is invariant under the transformations (3.42), will have the form

$$L = L_\pi + L_k = Tr\{G_\pi\gamma_5 B\Pi B^+\gamma_4 + G_k\gamma_5 BKB^+\gamma_4 + \text{Herm. adj.} \quad (3.43)$$

where π, K are 8 × 8 matrices for pions and K mesons, respectively (cf. also reference 62), and the pions and K mesons are the components of a seven-vector.

In this connection we call attention to the recent studies of Souriau[27] and of Umezawa and Visconti,[28] who by developing arguments that are essentially the same as the preceding principles of "general" symmetry arrive finally at a Lagrangian of the form

$$L = g\overline{\Psi}\gamma_5\Gamma_j\Phi_j\Psi \quad (j = 1,\ \ldots,\ 7), \quad (3.44)$$

where the matrices Γ_j, which are the generalization of the Dirac matrices for this case, generate a corresponding Clifford group.

Since the Lagrangian (3.43) describes a completely symmetrical interaction of baryons with π and K mesons, to obtain the mass differences of the baryons one must introduce additional interactions of a special type, which destroy this general symmetry. We may mention a suggestion made by Pais,[61] that the K^+ and K^0 mesons have different relative parities, so that an interaction between them of the type

$$\overline{K}^+K^0\pi^+ + \text{Herm. adj.} \quad (3.45)$$

will introduce an asymmetry into the Lagrangian (3.43); but such an assumption leads to a larger mass difference between K^+ and K^0.

For their part, Feinberg and Gürsey have made a detailed study of various forms of interaction of the type $K^2\pi^2$ and B^2K^2, which had been suggested earlier by various authors,[40,41] and have found that only an interaction of the type

$$L_A = \sum_{i=1}^{4}\overline{N}_i\boldsymbol{\tau}(a + b\gamma_5)N_i\overline{K}\boldsymbol{\tau}K \quad (3.46)$$

[N_i are the well known Gell-Mann doublets, Eq. (3.1), and a and b are arbitrary real numbers] leads to splitting of the masses of the baryon supermultiplet.

In concluding our brief exposition of the papers devoted to the classification of particles on the basis of the global, fundamental, and general symmetries, we must admit that, as is justly remarked in articles by various authors right up to the beginning of 1960, at present there are still no conclusive arguments in favor of a particular type of symmetry. It appears, however, that the more general and, one may perhaps say, more elegant general symmetry has too much equalizing effect and does not agree with a number of experimental facts.

4. ANOMALOUS SPINORS AND BOSONS

We shall now discuss attempts to describe the intrinsic (isotopic) properties of particles while remaining in the framework of ordinary space, by resorting to previously unused possibilities for different behaviors of spinors under reflections. It is in fact well known that if we define the square of the reflection operator as the identical transformation we get for the reflection operator the values $I = \pm 1$, but that if we define the square of reflection as rotation through 2π, then from the formulas for the transformation of spinors under rotations we find that this multiplies a spinor by -1, i.e., a single reflection will correspond to multiplication by $I = \pm i$. Thus in general we have for inversion of all three space axes

$$\psi' = \varrho_s \gamma_4 \psi \tag{4.1}$$

and for time reversal

$$\psi' = \varrho_t \gamma_1 \gamma_2 \gamma_3 \psi. \tag{4.2}$$

where ρ_s and ρ_t can independently take the values ± 1 and $\pm i$. Such differences in the behavior of spinors were first considered by Yang and Tiomno[63] (cf. also reference 64), who suggested that there could be four classes of spinors, A, B, C, D (ψ^A, ψ^B, ψ^C, ψ^D), corresponding to the values $\rho_s = +1, -1, +i, -i$. The extension to the case of time reversal was made in reference 65. It is convenient to introduce the obvious ψ^{AA}, ψ^{AB}, and so on. It is important to note that the difference between spinors of different classes is only a relative one and exists only in the presence of both fields; it must also be kept in mind that the presence of the coefficients ± 1, $\pm i$ corresponds to representations of the Lorentz group that are supplemented in different ways.

It was suggested[63] that by using the classes A, B, C, D one could assign individual spinors to the various particles: D spinors to the electron and the μ meson, a C spinor to the neutrino, and spinors A and B, respectively, to the proton and the neutron.

These ideas were developed by Marianashvili[65] and later by Gürsey,[66] who pointed out that bilinear combinations of spinors, and thus, in the spirit of the fusion method, boson functions, can be formed by taking spinors of quite different classes, for example $\psi^A\psi^C$; we then arrive at bosons that are multiplied by $\pm i$ on reflections of the coordinates, and not by ± 1, as in the case of scalars and pseudoscalars. In a natural way it is proposed to put the K mesons in correspondence with such bosons with imaginary spatial parity. In the Gürsey classification the various baryons, including hyperons, are characterized by the classes A, B, C, D, and thus strangeness is connected with the spatial parity P. It must be noted that for the classification of particles Yang and Tiomno[63] and Gürsey[66] use only spinors differing in spatial parity.

Later, mainly in papers by Soviet authors (the papers by D. Ivanenko with A. M. Brodskiĭ and with G. A. Sokolik and the mathematical studies of I. M. Gel'fand's group), attention was called to the possibility of introducing new, so-called "anomalous" types of spinors, which differ even more deeply in behavior under inversions from ordinary ("normal") spinors, owing to the use of the permissible additional factor of γ_5.[67,68]

To obtain the anomalous spinor representations of the Lorentz group we must turn our attention to the following theorem.[69] Let I_{4j} be the rotation operators in ordinary space, where j denotes one of the space axes, and let P be the operator for inversion of all three coordinates and T the operator for reversal of the time axis. Then, as can easily be seen,

$$\{I_{4j}, P\} = \{I_{4j}, T\} = 0$$

and in addition

$$[P, I_{ij}] = [T, I_{ij}] = 0.$$

These relations can at the same time be regarded as the conditions for the determination of the representations of P and T. The theorem in question asserts that these relations can be satisfied either by commuting (anomalous) representations of P, T,

$$P'T' = T'P', \tag{4.3}$$

or by anticommuting (ordinary or normal) representations,

$$PT = TP. \tag{4.4}$$

What has been said can be illustrated intuitively by examining the various reflections in the case of a vector, whose components are formed in the usual way from bilinear combinations of spinors. It then turns out that in the case of the vector one can obtain a reflection not only with the ordinary normal transformations of the spinors,

$$(x'_{1,2,3} \rightarrow -x_{1,2,3}) : \psi' = P\psi, \text{ where } P = \gamma_4,$$
$$(t' \rightarrow -t) : \psi' = T\psi, \text{ where } T = \gamma_1\gamma_2\gamma_3,$$

but also with anomalous transformations P′ and T′, where either P′ = P and T′ = iP, or else P′ = iT and T′ = T. Consequently, the reflection of all four coordinates, which is equal to γ_5 in the normal case, is equal to i in the anomalous case. It can be seen from this that in the normal case reflection of all four axes reduces to a product of two rotations, for example in the planes (t, z) and (x, y); on the other hand, in the anomalous case this operation cannot be reduced to four-dimensional rotations. It must be noted that a deep difference between spinors in spaces of even and odd numbers of dimensions shows itself in the fact that anomalous spinors can be introduced only in an even-dimensional space. It is curious to recall that in founding the theory of spinors Cartan[8] briefly indicated the existence of two possibilities for the reflection transformations of spinors, corresponding to the occurrence of an additional factor γ_5 in the way that has been discussed. At the same time, Cartan did not make the distinction between normal and anomalous spinors, which is based on including the factor γ_5 either for space reflections or for time reflection (A. M. Brodskiĭ).

It must be remembered that the anomalous spinors are realizations of representations of the product of the proper Lorentz group and the group of reflections, whereas the full Lorentz group ordinarily means the proper Lorentz group supplemented by the reflection of one or three axes.

We shall denote ordinary (normal) spinors by ψ^{11} and ψ^{22} (spinors of types I and II), and anomalous ("mixed") spinors by ψ^{12} and ψ^{21} (spinors of types III and IV). Here the first upper index refers to space reflection and the second to time reflection. We then have:

$$\psi^{11} \to \gamma_5 a_\mu \gamma_\mu \psi^{11} \text{ under } P \text{ and } T, \tag{4.5a}$$

$$\psi^{22} \to a_\mu \gamma_\mu \psi^{22} \text{ under } P \text{ and } T, \tag{4.5b}$$

$$\psi^{12} \to \begin{cases} \gamma_5 a_\mu \gamma_\mu \psi^{12} \text{ under } P, \\ a_\mu \gamma_\mu \psi^{12} \text{ under } T, \end{cases} \tag{4.5c}$$

$$\psi^{21} \to \begin{cases} a_\mu \gamma_\mu \psi^{21} \text{ under } P, \\ \gamma_5 a_\mu \gamma_\mu \psi^{21} \text{ under } T \end{cases} \tag{4.5d}$$

(a_μ is a unit vector normal to the hyperplane in which the reflection occurs).

Furthermore, by using the possibility of a further division of spinors into classes A, B, C, D, we arrive at 64 distinct types of spinors,[66] which it is convenient to designate, for example, by ψ^{1A2D}, etc. In a number of cases the transformation matrices of these spinors are equivalent by unitary transformation, and we can speak only of a relative difference. When, however, we take into account the antilinear transformations associated with antiparticle conjugation, the equivalence is destroyed.

From the point of view of our present argument the most important point is the relative difference in the behavior of the spinors under space and time reflections. We have here characterized spinors by two pairs of indices a, b and α, β. The index a takes the value 1 (or 2) for the presence (or absence) of a factor γ_5 for space reflections. The index b gives an analogous characterization of the behavior under time reflection. The indices α, β take four values (-1, 0, $+1$, $+2$) corresponding to the appearance of a factor $(i)^\alpha$ for space reflections and a factor $(i)^\beta$ for time reflection (classes A, B, C, D). The essential difference between spinors is characterized by the absolute values of the differences $(a-b)$ and $(\alpha-\beta)$. Since for "mixed" spinors the Dirac equation with a mass will be invariant only with respect to the strong inversion (combined parity) $P^S = PC$, we introduce the self-adjoint spinors[9]

$$\Psi(1) = \frac{1}{2}[(1+\gamma_5)\psi + (1-i\gamma_5)\psi^C], \tag{4.6a}$$

$$\Psi(2) = \frac{1}{2}[(1-\gamma_5)\psi + (1+i\gamma_5)\psi^C]. \tag{4.6b}$$

To characterize the behavior of the spinors Ψ under the strong inversions P^S and T^S ($T^S = TC$, where T is the Wigner time reversal), we need only the pair of indices $J = a + \alpha$, $K = b + \beta$. Furthermore the quantity

$$N = J - K = (a-b) + (\alpha - \beta) \tag{4.7}$$

is conserved modulo 4. It is tempting to take N to be the baryon number, $(a-b)$ for the hypercharge Y, and the difference $(\beta-\alpha)$ for the strangeness; we then arrive at Eq. (2.4):

$$N = Y - S.$$

It must be emphasized that according to this point of view the baryon number is not conserved precisely, in contradiction with the usual assumption. To be sure, the baryon number will be conserved with high probability, since four baryons are required for an annihilation, and this process will be hindered because of other conservation laws. Recently, on the basis of astronomical considerations, Wheeler[10] has suggested that the existence of a rather sharp upper limit on masses indicates the possibility of nucleon annihilation on a cosmic scale.

In the light of what has been said it is natural to try to characterize leptons by "normal" spinors, assigning the various factors ± 1, i, γ_5 to the particles e, ν, μ, and baryons by the "mixed" spinors $\Psi(1)$ and $\Psi(2)$. Very similar proposals as to the possibility of describing baryons by anomalous spinors have been made by Ogievetskiĭ and Chou Kuang-Chao,[70] Salam,[71] Taylor,[72] and McLennan.[73]

A somewhat different type of classification of particles by means of anomalous spinors has recently been proposed by G. A. Sokolik,[74] who had in view a special connection with an empirical classification of particles made by V. I. Gol'danskiĭ.[75] The essential point is that from the two anomalous (ψ^{12}, ψ^{21}) and two normal (ψ^{11}, ψ^{22}) four-component spinors considered above one can construct two anomalous scalar doublets, one of which (together with its adjoint) is assigned to the K-meson doublets $\begin{pmatrix} K^+ \\ K^0 \end{pmatrix}$, $\begin{pmatrix} \tilde{K}^0 \\ K^- \end{pmatrix}$, and the other (and its adjoint) to the following combinations of pions and the hypothetical ρ meson (the σ meson of Schwinger[22]):

$$\begin{pmatrix} \pi^+ \\ \frac{\pi^0 + \varrho^0}{\sqrt{2}} \end{pmatrix} \text{and} \begin{pmatrix} \frac{\pi^0 + \varrho^0}{\sqrt{2}} \\ \pi^- \end{pmatrix}.$$

The direct products of the two normal spinors and the two anomalous scalars give four eight-component anomalous spinors, which are assigned to the four baryon isodoublets of Gell-Mann, Eq. (3.1). On the other hand, Gell-Mann's hypothetical baryon isosinglets Ω^+ and Z^- are associated with normal spinors, and the hypothetical mesons ω^+ and ω^-, with normal scalar representations. Finally, for the lepton family, μ is given by a normal four-component spinor, and e and ν, by an anomalous eight-component spinor.

Despite the fact that attempts to construct a classification of particles on the basis of anomalous and normal spinors are still only of a preliminary nature, such a possibility of managing a description of the intrinsic properties of particles without any isotopic space, in the framework of just ordinary four-space, is an attractive one. It must be used both in the formation of an invariant Lagrangian for the four-fermion weak interaction (β decay and other processes), and also for the interaction of fields in nonlinear field theory.

5. THE CLASSIFICATION OF PARTICLES IN NONLINEAR SPINOR THEORY AND UNIFIED GEOMETRIZED THEORY

a. Nonlinear Spinor Theory

Let us now turn to the classification of elementary particles that arises on the basis of the unified nonlinear spinor theory. Without going into details, we recall that evidently one of the promising attempts to pass beyond the bounds of existing relativistic theory and construct a unified theory of all particles, while also removing difficulties with divergences, is that found in the nonlinear generalization of field theory. To speak briefly, we shall start from an idea of de Broglie,[76] who has suggested that matter is based on a field with the minimum spin $s = \frac{1}{2}$. This idea is to some extent a translation into modern language of an idea of Kelvin and Helmholtz, who tried to construct matter from rotating structures in the ether. The roots of such a theory of matter even go back to the "vortices" of Descartes. By the combination of spins one can hope to get arbitrarily large spins or zero spins. For obtaining the functions of compound particles de Broglie suggested the method of "fusion," in which the ψ function is set equal to $\psi_1\psi_2$, and certain supplementary conditions are imposed on ψ_1 and ψ_2. A graphic example is the attempt of de Broglie, which was developed by Kronig,[77] A. Sokolov,[78] and others, to construct a neutrino theory of light by combining pairs of neutrinos to make photons. There was a weak point in the method of fusion, owing to the absence of an energy of interaction between the particles.

If we take the point of view of the unified spinor theory, then we must introduce into an equation of the Dirac type a nonlinear term to describe the interaction of the spinor field with itself. In fact, since there are no other fields, there is nothing left for the basic field to interact with except itself! Possible forms of nonlinear added terms have been indicated by one of the present writers and A. M. Brodskiĭ.[79]

This idea received an important development when Heisenberg[80] dropped the mass term from the nonlinear Dirac equation, on the assumption that the masses of the elementary particles must arise as a consequence of the self-interaction. As the quantization rules, Heisenberg proposes changes in the values of the anticommutators, which evidently make it possible to avoid the divergences characteristic of the linear theory. Recently D. F. Kurdgelaidze[81] and Mitter[82] have proposed to take for the propagator in the nonlinear case a radially symmetrical four-dimensional solution of the nonlinear equation. Referring to the literature[88] for details, we only remark that within the framework of this scheme one can get finite values of the mass of the fundamental particle, the nucleon, a series of values for meson masses, as yet in only rough agreement with experiment, and also finite values for the electric and mesonic charges. Although there is so far only rough correspondence with the experimental data, we have here an impressive attempt at the construction of a unified theory of matter.

Here, however, we are interested in the general relationships of the nonlinear spinor theory as regards the classification of particles, and to a certain extent these relationships do not depend on the details of the formalism.

The cornerstone of the theory is the spinor equation with the mass term omitted and a nonlinear term of pseudovector character added:

$$\gamma_\mu \frac{\partial \psi}{\partial x_\mu} \pm l^2 \gamma_\mu \gamma_5 \psi (\bar{\psi} \gamma_\mu \gamma_5 \psi) = 0. \tag{5.1}$$

The choice of the pseudovector invariant [and not the simplest scalar $\psi(\bar{\psi}\psi)$] from among all the possibilities is made on the basis of the requirement of invariance not only under space rotations and Lorentz transformations, but also under the Pauli—Gürsey transformations [54,56]

$$\begin{aligned} \psi &\to a\psi + b\gamma_5 C^{-1}\bar{\psi}^T, \\ \bar{\psi} &\to a^*\bar{\psi} + b^*\psi^T C\gamma_5, \end{aligned} \tag{5.2}$$

where

$$|a^2| + |b^2| = 1, \quad \bar{\psi} = \psi^*\gamma_4, \quad C\gamma_\mu C^{-1} = -\gamma_\mu^T, \quad C^T = -C,$$

and the T *transposition affects the Dirac indices*, and under the Salam—Touschek transformation,[51,52,53]

$$\psi \to e^{i\alpha\gamma_5}\psi, \quad \bar{\psi} \to \bar{\psi} e^{i\alpha\gamma_5}. \tag{5.3}$$

The first transformation is isomorphic to a rotation group in three-dimensional space and determines two independent quantum numbers J and J_3 ($J^2 = J_1^2 + J_2^2 + J_3^2$), which are identified with the vector isospin and the third component of the isospin. The second transformation leads to the conservation of $2J_N$, the number of fermions minus the number of antifermions. The connections between the numbers J, J_3, and J_N and the charge Q, the strangeness S, the baryon number N, and the lepton number L are given by the formulas:

$$Q = J_3 + \frac{l_Q}{2}, \tag{5.4a}$$

$$N = J_N + \frac{l_N}{2}, \tag{5.4b}$$

$$L = J_N - \frac{l_N}{2}, \tag{5.4c}$$

$$S = l_Q - l_N, \tag{5.4d}$$

where l_Q and l_N are to take arbitrary positive and negative values, whereas, in accordance with the experimental data, S takes the values $S = 0, \pm 1, 2$; that is, S is fixed modulo 4. We note that the basic nonlinear equation is invariant under a one-parameter continuous group of conformal transformations which is a simple dilation of space-time:

$$x_r \longrightarrow \eta x_r, \qquad \psi \longrightarrow \eta^{3/2}\psi(x\eta,\ l\eta) \tag{5.5}$$

(η is real).

Because of the invariance of the fundamental equation and of the commutation relations under these transformations, we can define in the Hilbert space a "semiunitary" (since it multiplies the eigenvectors by real coefficients) operator O_η such that

$$O_\eta \psi(x,\ l)\, O_\eta^{-1} = \eta^{3/2}\psi(x\eta,\ l\eta). \tag{5.6}$$

For an infinitesimal transformation Λ (sic)

$$O_\eta = \eta^\Lambda;$$

Λ is a new quantum number with integral and half-integral eigenvalues, which are identified with the quantum number $l_N/2$. Then, as can be seen from Eqs. (4.5a) and (4.5d), l_Q has the meaning of the isospin.

Thus it is possible to classify all the elementary particles according to the values of the quantum numbers J_3, J_N, $l_N/2$, and $l_Q/2$, as is shown in Table V.

b. Gravitation and the Unified Nonlinear Spinor Field Theory

The prospect of obtaining all ordinary matter, i.e., the elementary particles, on the basis of a nonlinear theory leads us to pose the question of including gravitation in such a scheme. Obviously we must distinguish the question of the possibility of constructing gravitons as quanta of the weak gravitational field, having the spin $S = 2$, in terms of excited states of the fundamental spinor, from the problem of constructing a unified theory that reduces the complete field of the $g_{\mu\nu}$ to spinors. The starting point of the arguments must be the expression for the covariant derivative of a spinor,

$$\nabla_\mu \psi = \left(\frac{\partial}{\partial x_\mu} - \Gamma_\mu\right)\psi, \tag{5.7}$$

where for the present case the parallel-displacement coefficients Γ_ν introduced by V. A. Fock and one of the writers replace the Christoffel symbols. This enables us to write the Dirac equation in generally covariant form and take into account the effects of gravitation on fermions.

It is important to note that the coefficients Γ_μ are not determined uniquely in terms of the corresponding metric quantities γ_μ, the generalized Dirac matrices, which in this case are functions of the coordinates and the time. As we have pointed out previously, one can include the vector potential A_μ of the electromagnetic field in the expression for the covariant derivative.

Besides this, as Kita[85] has remarked, the covariant derivative can be complemented with a vector and a pseudovector that are bilinear in ψ, so as to take into account the effects of strong and weak couplings. By constructing the A_μ from ψ we get typical nonlinear terms of the form ψ^3 to be added to the nonlinear equation. As Kita points out, however, it is impossible to construct the $g_{\mu\nu}$ or the γ_μ themselves from the ψ,

Table V. Classification of particles on the basis of the nonlinear theory

Particles	γ	e^+	$\tilde{\nu}$	ν	e^-	μ^+	μ^-	π^+	π^0	π^-	K^+	K^0	$\tilde{K}^0$	K^-	p	n	$\tilde{n}$	$\tilde{p}$	Λ^0	Σ^+	Σ^0	Σ^-	Ξ^0	Ξ^-
J_3	0	1/2	−1/2	1/2	−1/2	0	0	1	0	−1	1/2	−1/2	1/2	−1/2	1/2	−1/2	1/2	−1/2	0	1	0	−1	1/2	−1/2
J_N	0	−1/2	−1/2	1/2	1/2	−1/2	1/2	0	0	0	0	0	0	0	1/2	1/2	−1/2	−1/2	1/2	1/2	1/2	1/2	1/2	1/2
$\frac{l_Q}{2}$	0	1/2	1/2	−1/2	−1/2	1	−1	0	0	0	1/2	1/2	−1/2	−1/2	1/2	1/2	−1/2	−1/2	0	0	0	0	−1/2	−1/2
$\frac{l_N}{2}$	0	1/2	1/2	−1/2	−1/2	1/2	−1/2	0	0	0	0	0	0	0	1/2	1/2	−1/2	−1/2	1/2	1/2	1/2	1/2	1/2	1/2
$Q=J_3+\frac{l_Q}{2}$	0	1	0	0	−1	1	−1	1	0	−1	1	0	0	−1	1	0	0	−1	0	1	0	−1	0	−1
$N=J_N+\frac{l_N}{2}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	−1	−1	1	1	1	1	1	1
$L=J_N-\frac{l_N}{2}$	0	−1	−1	1	1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$S=l_Q-l_N$	0	0	0	0	0	1	−1	0	0	0	1	1	−1	−1	0	0	0	0	−1	−1	−1	−1	−2	−2

so that for the time being we must retain a dualistic point of view, distinguishing between the geometrical quantities $g_{\mu\nu}$ and the quantities that describe ordinary matter. Kita proposes to take as fundamental three fields, using along with gravitation and the field of the strongly interacting particles (baryons) also neutrinos, electrons, and photons, and hopes to construct all other particles from them. Recently Kita and Predazzi[86] have tried to treat muons as excited states of electrons.

In this connection we must deal briefly with a very recent attempt to construct a geometrized picture of the world, which has been put forward by Wheeler. As is well known, the success of the Einstein theory of gravitation, in which gravitation is associated with the curvature of space-time in accordance with Riemannian geometry, has aroused the hope of also explaining electromagnetic, mesonic, and other fields geometrically. For this purpose many attempts to generalize the Riemannian geometry in various directions were made in the 1920's: the unsymmetrical metric (Einstein), generalization of the affine-connection coefficients (Eddington, Weyl), use of a twisted, not merely curved, space (Cartan), introduction of a fifth coordinate (Kaluza, later Jordan and Tiri), and other generalizations. The additional geometrical quantities arising from an extension of the geometry were used for the description of the electromagnetic field, and even of the mesonic field (Schrödinger). Despite the mathematical elegance of many of these researches, they did not lead to any physical results.

Rejecting all attempts of this sort and remaining within the framework of Riemannian geometry, Wheeler, along with Misner and other collaborators,[103,87,88] is now making an attack on the topology, and bringing in the modern quantum treatment of fields and particles. These authors show that at the very smallest distances, $r \approx (h\kappa/c^3)^{1/2} \approx 10^{-32}$ cm the quantum vacuum fluctuations of the gravitational field, or of the metric, must reach large values, owing to which space will be distorted in various ways. Wheeler tries to associate holes in space, connected by tubes, with the classical model of electric charges, and sketches a preliminary quantum interpretation of charges and the electromagnetic field. In addition to a number of particular difficulties, the conversion of such a "geometrodynamics" into a unified geometrized theory of space-time, gravitation, and ordinary matter encounters a fundamental difficulty, owing to the necessity of including fermions in a picture based on the Bose field of the $g_{\mu\nu}$. Be that as it may, Wheeler's series of papers, which contain many interesting ideas and results, are a rather impressive attempt at a revenge from the side of a unified geometrized field theory, and perhaps the only such attempt that is possible. For details the reader is referred to the papers of Wheeler, the most important of which will be published in two collections devoted to the latest problems of gravitation.

6. THE HYPOTHESIS OF COMPOUND PARTICLES

Besides the attempt to construct the wave functions of particles from the ψ functions of other particles taken as basic, along the lines of L. de Broglie's method of fusion or of the unified spinor theory of matter, there have been proposed a number of models of compound particles of more intuitive, and also cruder, sorts. Fermi and Yang were the first to note[80] that pions can be regarded as formed from nucleons and antinucleons, on the assumption that at the very smallest distances there is some enormous binding energy, of unknown origin. The connection of such ideas with those of the method of fusion and of the unified spinor theory is obvious. Goldhaber[90] proposed taking as the basis nucleons and K mesons. The one of these models that has attracted most attention is that of Sakata,[91] which has been developed by Maki,[92] L. Okun',[93] M. A. Markov,[1,94] I. Polubarinov,[95] and others. It is proposed to take as the basis nucleons and Λ hyperons, together with their antiparticles. From these one obtains all other hyperons, and also mesons, for example,

$$\pi \equiv N+\overline{N}, \quad K \equiv N+\overline{\Lambda}^0, \qquad \Sigma \equiv N+\overline{N}+\Lambda^0. \tag{6.1}$$

Thus here, as in other such models, hyperons are regarded as excited states of nucleons and Λ hyperons. A contact interaction of the basic baryons is constructed according to the general rule of four-fermion interactions; here, as usual, one can take a coupling of scalar type, vector type, etc., with some constant of the Fermi type. Concrete calculations have shown that on such assumptions one can approximately reconstruct the various particles in the domain of strong interactions, and also arrive at some possible new particles. For all details we refer the reader to the literature.

Developing these considerations, Sakata[96] and his collaborators propose to include leptons also in a unified system of matter. As the basis they take the three leptons ν, e^-, and μ^- and some new form of matter B^+. The nature of the field B^+ is so far not exactly specified. The question even remains open as to whether it is like ordinary matter or is of the nature of charge, or indeed whether it is to be understood only by going further beyond the framework of usual theory. Then the three main baryons can be represented as combinations of the field B^+ with the various leptons:

$$P \equiv B^+\nu; \quad n \equiv B^+e^-; \quad \Lambda^0 \equiv B^+\mu^-. \tag{6.2}$$

The mesons and the other hyperons can be constructed as combinations of p, n, and Λ, according to the original Sakata model. The disintegration of a baryon into

B^+ and a lepton is extremely difficult, if it is possible at all. It is further assumed that all the corpuscular properties of the baryons (spin, statistics, etc.) are due to the bare lepton, but the field B^+ in the baryon provides its mass. In the generalized Sakata model the effective Hamiltonian of the strong coupling is of the form

$$H_s = \sum_a g_a (\bar{\chi} O^A \chi)(\bar{\chi} O^A \chi), \tag{6.3}$$

where the baryon wave function is

$$\chi = \begin{pmatrix} p \\ n \\ \Lambda \end{pmatrix}.$$

It is proposed to construct the theory of the weak interaction on the basis of a product of currents in the style of Feynman and Gell-Mann:[97]

$$H_w = g_\mu g_\mu^+, \tag{6.4}$$

where

$$g_\mu = j_\mu + J_\mu,$$

$$J_\mu = f\{(\bar{n}\gamma_\mu(1+\gamma_5)p) + (\bar{\Lambda}\gamma_\mu(1+\gamma_5)p)\},$$

$$j_\mu = f\{(\bar{e}\gamma_\mu(1+\gamma_5)\nu) + (\bar{\mu}\gamma_\mu(1+\gamma_5)\nu)\}.$$

The Hamiltonian of the strong interaction is charge invariant, and also symmetrical in the three main baryons (p, n, and Λ). This model provides directly for the important symmetry

$$p \longleftrightarrow \nu, \quad n \longleftrightarrow e^-, \quad \Lambda \longleftrightarrow \mu^-, \tag{6.5}$$

noted by Gamba and by Marshak and Okubo.[98]

In concluding their interesting, although extremely preliminary, considerations, Sakata and his collaborators indicate the following obvious possibilities for describing the coupling of the field B^+ with the leptons L:

a) B^+ and L are points (atomlike model);

b) $B^+(L)$ is a point and $L(B^+)$ forms a cloud (model of the type of nucleons);

c) B^+ and L are interpenetrating fluids;

d) model of the vessel type: the lepton plays the role of the vessel, which can be filled with the field B^+.

We note, finally, the possibility that Sakata's field B^+ may be the result of a tight combination caused by the nonlinear interaction of the field ψ. There is in essence a close connection between the Sakata model and Thirring's proposal[99] to take as the basis for the strong particles three spinor (Weyl or Majorana) fields. By forming products of such fields (without rest masses) one can obtain[99,100] the functions for the various elementary particles and conservation of the baryon number N, the isospin T, the hypercharge Y, and the combined parity PC. The invariance group of the strong interactions, which has as generators N, T, and Y, is isomorphic to the product of the two-dimensional unitary unimodular group and two independent one-dimensional unitary groups, i.e., translations; in other words, it is isomorphic to the motions of a plane. On the other hand, the full three-dimensional unitary group, which is the natural generalization of the strong interactions, leads to doubly charged particles and to a number of other states which are not observed, and according to Thirring it must be rejected.

7. THE PROBLEM OF THE LEPTONS

In connection with the discovery of parity nonconservation in weak interactions there is now intensive study of problems of the weak interactions, and in particular of processes involving leptons. It is now generally recognized that weak interactions of the four-fermion type occur through a V-A coupling, as has been indicated by Marshak and Sudarshan[101] and also by Gell-Mann and Feynman.[97] These facts have led to the appearance of new classifications of leptons in which they are given isotopic characteristics such as isospin and strangeness, as well as lepton number and neutrino charge. The situation is not yet clear, however, with regard to these points. Moreover, a number of authors doubt in general the very possibility of such a classification, believing that leptons do not have isotopic properties. The difficulty of treating the isotopic properties of leptons is evidently due to the fact they are mainly characterized by the weak interactions, in which isospin and strangeness are not conserved, even for strongly interacting particles.

In place of these properties a new one takes the primary position, namely the helicity, which is most clearly manifested in the neutrino, the case $m_\nu = 0$.

In spite of this, a number of attempts at the classification of leptons have been put forward from various quarters, which deserve attention, although they are generally regarded as far from conclusive.

We have already considered the dynamical scheme of Schwinger, in which the leptons appear in a natural way, and interaction between them, and also that between baryons and leptons, occurs through hypothetical intermediate vector Z particles, the mass difference of e and μ being explained by a special interaction with hypothetical scalar π_0^0 mesons. The introduction of intermediate particles (both vector and scalar), following an idea found in the classic paper of Yukawa, is a characteristic feature of many systems for classifying leptons. These hypothetical bosons are used in attempts to explain the large mass difference $\mu - e$, and also to deal with the weak interactions.[21,44,102–109] In particular, the suggestion has been made that the universal V-A interaction is realized by means of a charged vector boson coupled with fermion pairs. Such a mechanism forbids processes between four neutral or four charged fermions, which leads to agreement with experiment by suppressing the processes $\mu \to 3e$ and $\mu \to e + \gamma$. The probability of this process has been calculated as a function of the mass of the intermediate boson and its magnetic moment; for sufficiently large values of the boson mass the decay in question is extremely improbable.

Besides, as has already been stated, leptons have been included both in the attempts at a systematics based on anomalous spinors and in the nonlinear spinor field theory.

On the other hand, there are a number of attempts to construct a more phenomenological classification of the leptons,[75,110-114] as a more or less natural extension of the Gell-Mann—Nishijima scheme to these particles.

Sachs[115] has tried to apply the concept of an attribute a, which is essentially the same as the isoparity, A = −U, to leptons as well as other particles. The interesting suggestion that we should admit two types of neutrino particles — the neutrino arising from the decay of the neutron, and a neutretto arising from the decay of the muon — has been developed by a number of authors, following Cini and Gamba.[116] According to Nishijima[55] this is possible with a four-component treatment of the neutrino. Then in the case of the four-component neutrino the leptons are e_-, μ_+, whereas for the two-component neutrino they are e_- and μ_-. A. M. Brodskiĭ[117] has pointed out that the neutrino and neutretto can have different effective kinematic magnetic moments, which are predicted on the basis of the formalism of current interactions.

Moreover, independently of the lepton number, one often introduces the neutrino charge, associated with the helicity of the neutrino. Independently of this the concept of a neutrino charge has been used by Ya. P. Terletskiĭ[118] for a classification of particles, which somewhat in the style of the later Sakata model[96] tries to construct particles by combining various charges. In a paper by Umezawa and others[119] an attempt is made to extend the concept of neutrino charge to all particles. It then turns out that the neutrino charge of baryons and mesons coincides with the hypercharge, and for leptons it coincides with the attribute used by Sachs. Here we see a curious example of the transfer of typical leptonic properties to baryons.

Thus in spite of the incomplete nature of the treatment of the leptons, the work of the last few years has undoubtedly brought nearer the possibility of their legitimate inclusion in the general classification of the particles.

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We call attention to the following article:
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