INVESTIGATION OF THE THERMAL STRUCTURE OF HELIUM II BY SCATTERING OF COLD NEUTRONS

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EVER since Landau postulated a definite form for the spectrum of the thermal oscillations of helium II, the question of the thermal structure of this liquid has been the most vital problem in the theory of superfluidity. In fact, all our ideas on the behavior of helium II are based on the idea that there are two types of thermal excitations, making up its normal component. We have, however, no direct data on these thermal excitations; all we know about them has been initially postulated by Landau. Although the hypotheses concerning their nature do not contradict many of the experimental facts, a direct confirmation of the existence of such thermal excitations is nevertheless quite essential.

Let us recall briefly the nature of the spectrum of thermal excitations of helium II, proposed by Landau as long ago as in 1947.

Of the two types of excitations, which were called phonons and rotons, the phonons have the greater wavelength. The connection between the energy of such a quasi-particle and its momentum is given by the linear law

$$E = cp, \qquad (1)$$

where c is the velocity of ordinary sound in helium II, approximately 240 m/sec, while E and p are the energy and momentum of the quasi-particle, respectively. Essentially these are none other than sound waves propagating in helium II, quantized in a suitable manner, and regarded as wave packets. The phonons have a continuous energy spectrum.

The second type of thermal excitations — rotons — has a shorter wavelength than the phonons. It is postulated that the connection between the energy and momentum has in this case a parabolic character:

$$E = \Delta + \frac{(p-p_0)^2}{2\mu}. \qquad (2)$$

In this equation Δ denotes the energy gap between the main unexcited state and the first excited level, p_0 is the momentum possessed by the roton while in the unexcited state, and μ is the effective mass of the roton (Fig. 1). The rotons have a continuous spectrum above the energy gap, and the most populated levels are located near the minimum of the curve. Feynman visualizes rotons as vortex rings similar to smoke rings. Then the momentum of the stationary roton is due not to its translational motion as a whole, but to a motion around a cylindrical surface bent into a closed ring. According to the latest calculations of Khalatnikov[2] the roton parameters have the following numerical values:

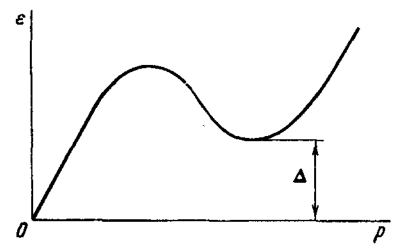

FIG 1.

$$\frac{\Delta}{k} = 8.9^\circ\,\mathrm{K}, \quad \frac{p}{\hbar} = 1.99\,\mathrm{A}^{-1}, \quad \mu = 0.26\, m_{\mathrm{He}}.$$

Landau has established that the number of phonons increases with the temperature as T^3, while the number of rotons increases exponentially with an exponent $-\Delta/kT$. Thus, phonons predominate at low temperatures, while at a temperature on the order of 0.6°K rotons begin to predominate and are responsible for practically all the physical properties of helium II at temperatures above 1°. We know that Landau[3] has developed a thermodynamic theory of helium II by calculating the values and the temperature behavior of all the most important thermodynamic functions of both the roton part and the phonon part of the normal component.

Let us indicate once more that both types of thermal excitations form the normal component of helium II. The overall density of these excitations determines the effective density of its normal component, which must be treated phenomenologically as that part of liquid helium II, which is involved in the thermal motion at a given instant and at a given temperature.

Unlike the normal component, the superfluid component of helium II does not contain any thermal excitations, either phonons or rotons, and represents a medium in which the quasi-particles of both types are distributed in a discrete manner. In such a description we can say that the temperature of the superfluid component, containing no thermal-energy quanta, is zero independently of the temperature of the helium II. The density of the normal and superfluid components was measured at different tempera-

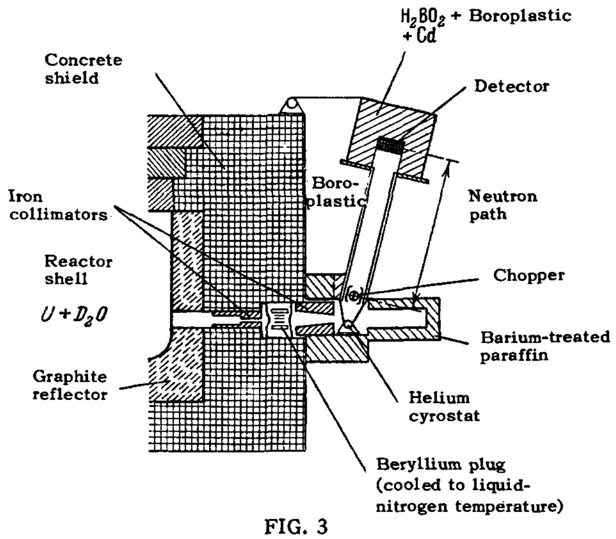

FIG. 3

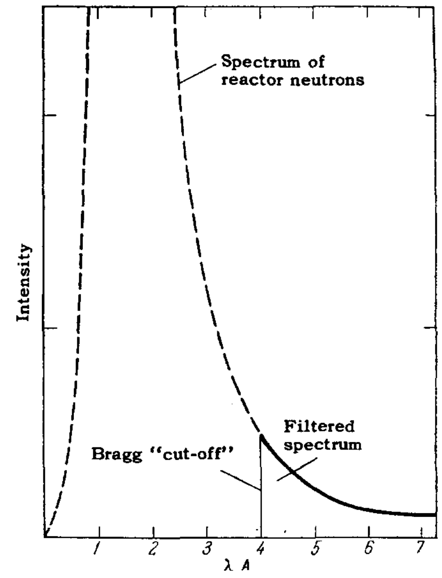

FIG. 5

transparent to wavelengths shorter than 3.95 A.

The sharpness of the Bragg cutoff, shown in Fig. 5, is determined in practice by the resolving power of the spectrometer. After the helium II has scattered the primary beam with the Bragg cut-off, the secondary beam, scattered at a definite angle to the primary one, should duplicate the Bragg cut-off of the primary beam. However, this cut-off will be shifted towards the longer waves. The magnitude of the shift will depend on the angle at which the scattered beam is observed. This is illustrated in Fig. 6, taken from the article by Larsson et al.[10] The procedure described is essentially analogous to that proposed by Feynman to investigate the scattering of a monochromatic neutron beam.

Different procedures were used in different laboratories to analyze the scattered beam. Larsson, Palevsky, et al.[10] used for this purpose a "chopper" with curved slits, which could be rotated together with the three-meter beam used to mount the proportional counters, about the sample with the helium II.

The group headed by Yarnell[12] used a spectrometer of the crystal-monochromator type with a rock-salt single crystal. The monochromator and the sample were located 1.5 meters apart, and a multiple-slit collimator was located between the two. After being reflected from the crystal, the neutrons covered a distance of 1.4 meters and were registered by the counters. The distribution of the intensities along the spectrum of the scattered neutrons at different temperatures, obtained by Larsson, is shown in Fig. 7.

The results of the experiments are illustrated by E vs. p curves. Henshaw[11] gives (Fig. 8) the expected shape of the dispersion curve and compares it with his own measurements. Larsson[10] et al. give (Fig. 9) the portion of the curve near the maximum of

Experimental setup (dimensions in cm)

Neutron spectrometer

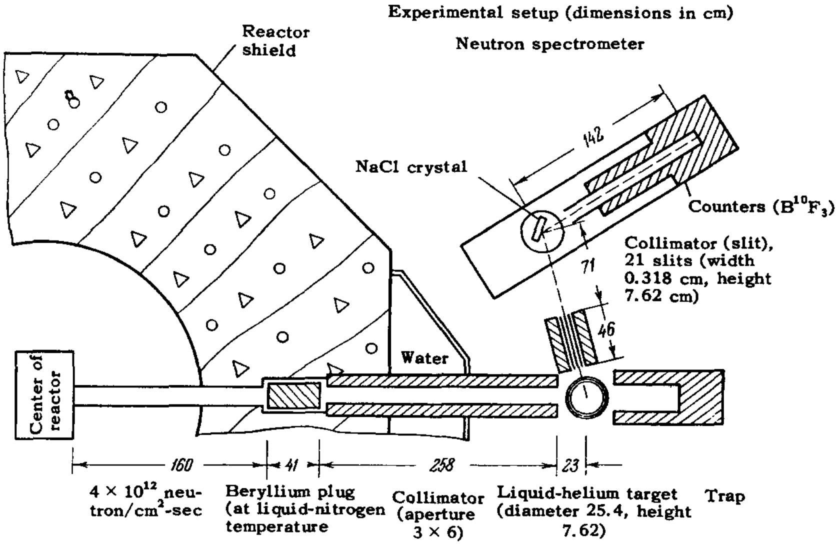

FIG. 4

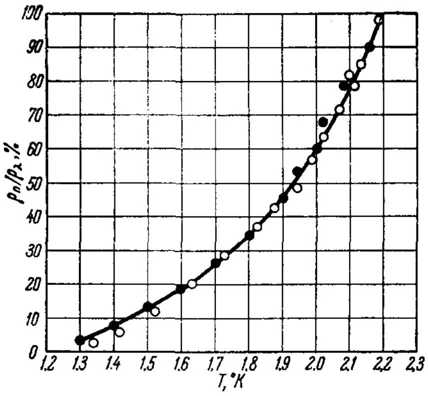

FIG. 2

tures in direct and indirect experiments.[4,5] The temperature dependence of the ratio of the density ρ_n of the normal component to the density ρ_λ of helium at the λ point is shown in Fig. 2. The dots in this figure represent the results obtained by Peshkov from measurements of the velocity of second sound, while the circles are the direct results measured by Andronikashvili.

Attempts to determine the internal structure of helium II by one method or another began quite some time ago. One such method, used by various investigators, is based on neutron-diffraction analysis. Harst and Henshaw,[6] as long ago as in 1955, used for this purpose a beam of monochromatic neutrons of 1.04 A wavelength. The scattering of neutrons by helium was investigated by Egelstaff and London,[7] and also by Sommers, Dash, and Goldstein.[8] Apparently, however, these experiments yielded no interesting physical results.

A major shift occurred when Feynman and Cohen[9] pointed out the possibility of determining the energy spectrum of the thermal excitations of helium II, and outlined the principal features of an experiment aimed at investigating inelastic scattering of cold neutrons by helium II.

Feynman and Cohen have shown that in the interaction between cold neutrons and helium II at liquid temperatures, on the order of 2° K and below, the predominant role is played by the quanta of thermal excitations produced by the neutrons. This process takes place, however, at neutron wavelengths shorter than 16.5 A, since longer-wave neutrons cannot generate anything.

It was found that neutrons at wavelengths on the order of 10 A could generate only long-wave phonons. Only neutrons with wavelengths less than 10 A are capable of generating phonons of high energy and rotons.

However, in order for the generation of one excitation to be a two-particle process, it is necessary that the energy of the incident neutron not be too high. Only then will the conservation laws of the energy and momentum make it possible to determine the dependence of the energy of excitation E (p) on its momentum p.

Following these theoretical investigations, highly significant results were obtained in many researches, and have led literally to a triumph of the Landau theory, along with representing in themselves a high point in neutron-physics research.

I have in mind the work by Larsson, Palevsky, et al.[10] performed with the Stockholm reactor, the work by Henshaw[11] performed with the Chalk River (Canada) reactor, and finally the work by Yarnell, Bendt, et al.[2] carried out with the Omega-West reactor in Los Alamos.

The principal part of the work was the same in all the investigations. A beam of cold neutrons of known wavelength was inelastically scattered by the helium II. Ultimately the roton, produced by the interaction between the neutron and the group of atoms, moves away from the scattered neutron. The kinematics of the scattering is governed by the conservation laws:

$$\frac{\hbar}{2m}(p_i^2 - p_f^2) = E(p), \qquad \hbar(\mathbf{p}_f - \mathbf{p}_i) = \mathbf{p}. \tag{3}$$

Here $\hbar p_i^2/2m$ is the energy of the incident neutrons, $\hbar p_f^2/2m$ the energy of the scattered neutron, E (p) the energy of the excitation produced in helium II, and p the momentum connected with this excitation.

Thus, by measuring the energy and the momentum of the neutrons scattered at a definite angle to the direction of the primary monochromatic beam, or, what is the same, by measuring the wavelength of the primary and of the scattered neutrons, we determine the energy and momentum of the roton produced as a result of the inelastic scattering. The connection between the energy and momentum, empirically determined in this experiment, should confirm or refute the Landau relation. The same experiments should yield all three parameters of the roton: Δ, p_0, and μ.

The cold neutrons are obtained in all three investigations from a nuclear reactor operating at 600 — 800 kw thermal power with a total neutron flux on the order of 10^{12} neutron/sec-cm^2 on the inside of the outlet channel.

The experimental setups for the Swedish and Los Alamos reactors are shown in Figs. 3 and 4. The neutron flux from the active zone of the reactor passes in both cases through a beryllium plug, which serves as a filter, and then through a collimator to the specimen, i.e., to an aluminum Dewar vessel filled with liquid helium II, cooled to a low temperature.

If the energy spectrum of the neutrons is represented by a Maxwellian curve, then the polycrystalline beryllium filter will cut off all the neutrons of wavelength shorter than 3.95 A (Fig. 5). Almost all these neutrons experience Bragg reflection from the corresponding crystallographic planes. The quantity 3.95 A is twice the largest interplanar distance in the beryllium crystals. The beryllium filter is practically

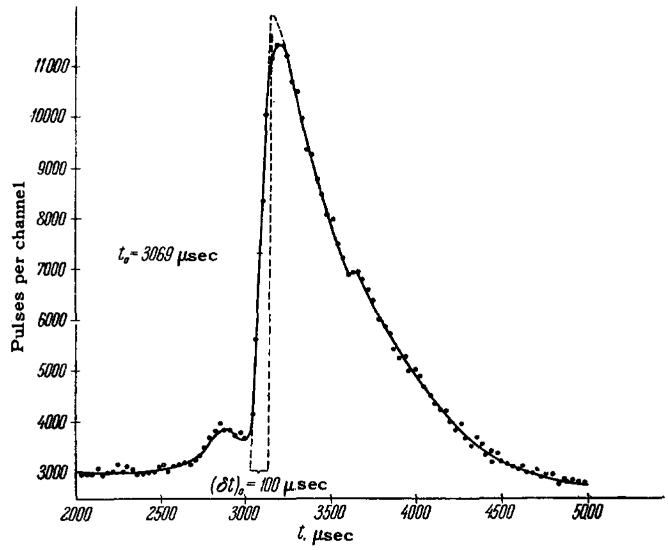

FIG. 6

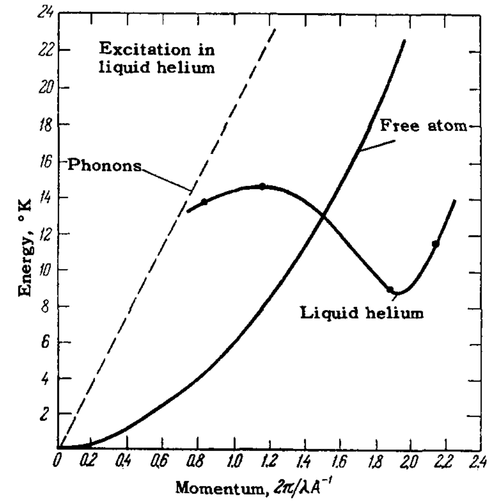

FIG. 8

the roton part of the spectrum. Finally, Yarnell and co-workers[12] give (Fig. 10) the complete dispersion curve, covering the momentum interval from 0.5 A^{-1}, corresponding to the end of the phonon part of the curve, up to 2.5 A^{-1}, in the region far past the minimum of the roton part.

Yarnell: $\frac{\Delta}{k} = 8.65°\,K$, $\frac{p_0}{\hbar} = 1.92\ A^{-1}$, $\mu = 0{,}16\ m_{He}$;

Larsson: $\frac{\Delta}{k} = 8.1°\,K$; $\frac{p_0}{\hbar} = 1.90\ A^{-1}$, $\mu = 0.16\ m_{He}$.

The width of the Δ/k gap is 8.65° K at T = 1.1° K after Yarnell and 8.1° K at T = 1.5° K after Larsson. Thus, the agreement is excellent not only between the two experiments, but even between experiment and theory. It must be noted, nevertheless, that Larsson's

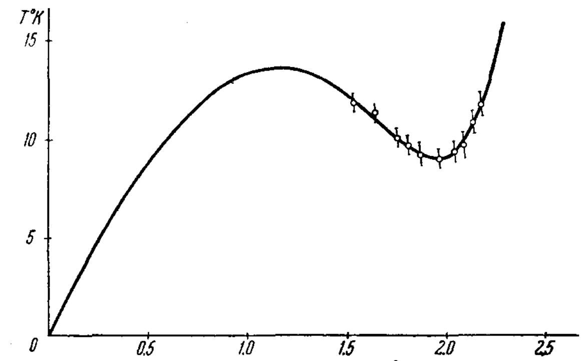

FIG. 9

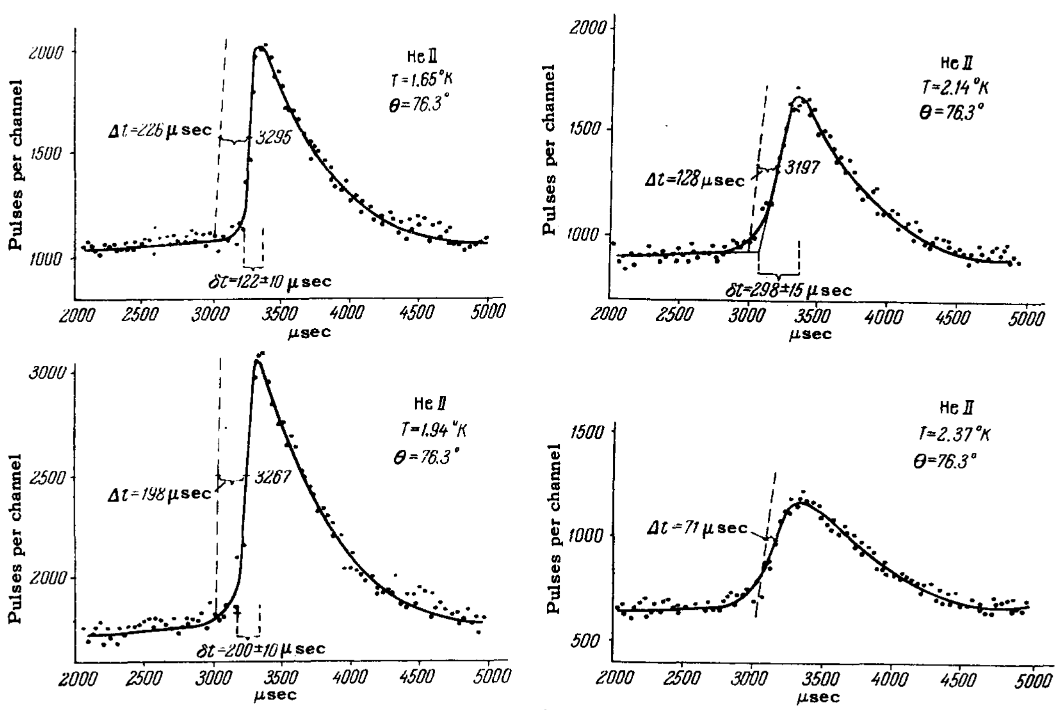

FIG. 7

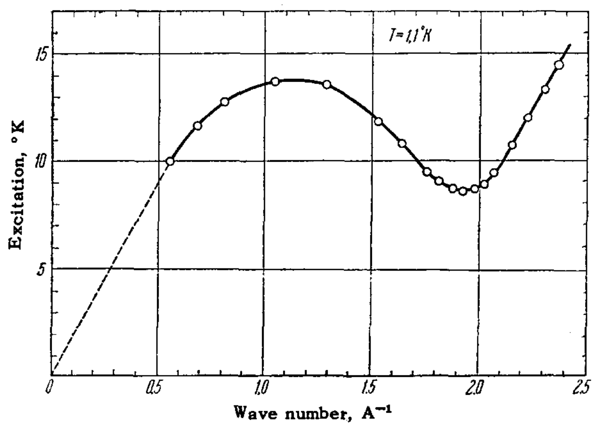

FIG. 10

value of the gap width is somewhat too low compared with that given by Yarnell.

Further investigation of the dependence of the width of the gap on the temperatures has led Larsson to plot the curve shown in Fig. 11. Yarnell, in turn, gives an empirical temperature dependence of the gap width in the form $\Delta/k = (8.68 \pm 0.0084)T^7$ deg K.

Let us consider now in greater detail the temperature-dependence curve of the gap width. As can be seen from the experimental data, the energy of roton production decreases as the temperature approaches the λ point. Thus, the probability of formation of a roton increases not only because the temperature T increases in the exponent, but also because the gap width decreases. A notable change in the gap width begins at temperatures 1.8 — 1.9° K, i.e., in the region where, according to other data, the rotons begin interacting with each other.[13] At the same time, the width of the dispersion curve also increases sharply.

Can we assume that the investigations reported in this survey have resolved completely the problem of the character of the Landau curve? Not so long ago Pitaevskiĭ[14] investigated the question of how the Landau dispersion curve terminates in the region of high momenta. It turned out that at high energies and momenta there exists a certain stability threshold for roton-type oscillations. We designate it p_k. The roton should decay beyond the stability threshold. It can do so, however, in various ways, neither of which

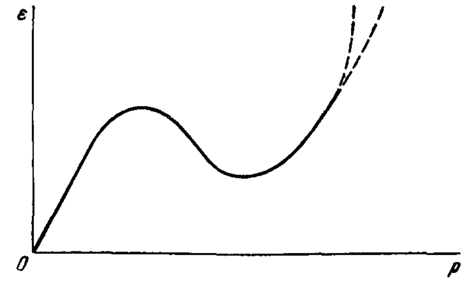

FIG. 12

can be given theoretical preference. To solve this problem he therefore proposed the use of a neutron beam. The behavior of the scattered neutron beam should depend on which of the decay channels is actually realized.

If the slope of the curve at the point of decay exceeds the slope of the curve near the origin, i.e., if the roton moves with a velocity higher than the velocity of ordinary sound in helium II, it can emit a photon just as an electron moving faster than light in a given medium emits Cerenkov radiation. The emission is produced within a time proportional to $1/(p - p_k)^3$. In this case the stable part of the curve terminates with a slope equal to c, i.e., a slope equal to the velocity of ordinary sound reached at the point $p = p_k$. This is followed by a diffuse region which represents unstable excitations (Fig. 12).

Another possibility is the decay into two excitations with identical directions of motion and final momenta. The decay occurs at an energy equal to twice the width of the gap. In this case the E(p) curve has no continuation at all beyond $p = p_k$, denoting an instantaneous decay (Fig. 13).

In the third possible case, a decay into two excitations that are scattered in different directions occurs, again after reaching an energy equal to twice the width of the gap. Here, too, the E(p) curve has no continuation beyond $p = p_k$. However, the tangent to the final point is horizontal in this case (Fig. 14).

An investigation of these interesting questions undoubtedly still awaits a solution in the nearest time.

The problem of scattering of cold neutrons by liquid has recently attracted the known Chinese scientist Lee, who discovered parity nonconservation in weak interactions. Together with Mohling[15] he

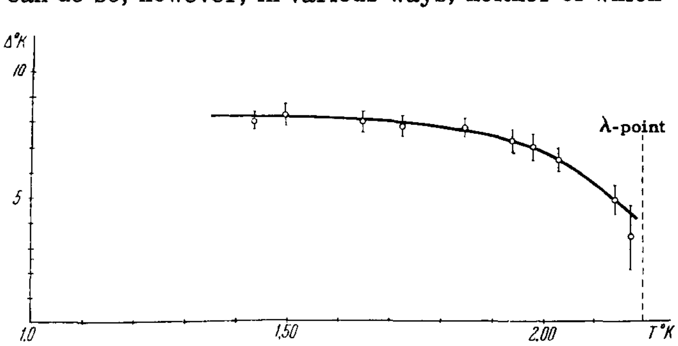

FIG. 11

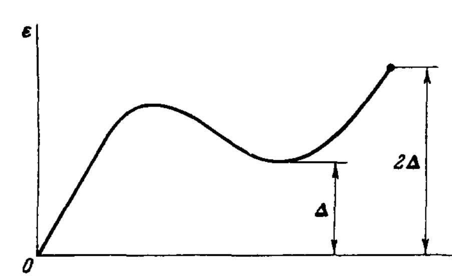

FIG. 13

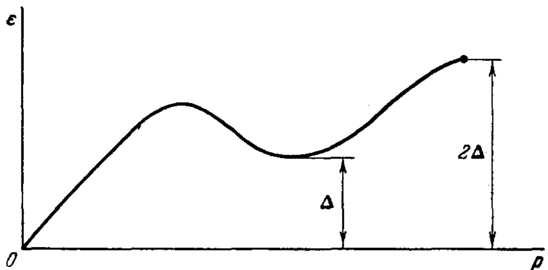

FIG. 14

pointed out the possibility of direct determination of the chirality of the rotons, i.e., a quantum number on par with the other parameters, describing the state and the nature of this quasi-particle.

By chirality, as is well known, is meant the quantum number that results from projecting the angular-momentum vector on the direction of the momentum vector **p**. The chirality of a system should be conserved in interaction processes.

When slow neutrons are scattered by helium II atoms, there is no spin-orbit interaction. This means that the neutron cannot generate a particle with chirality other than zero without changing its direction of motion. It follows hence that the effective neutron forward scattering cross section should be zero if the chirality of the roton differs from zero.

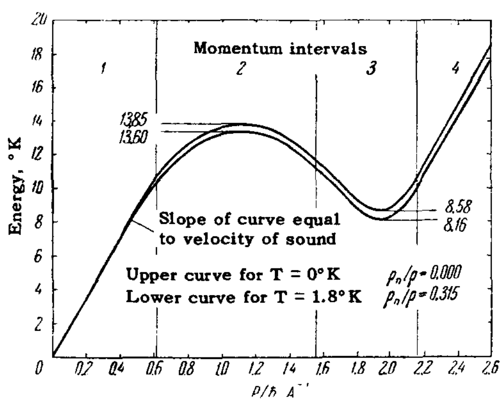

FIG. 15

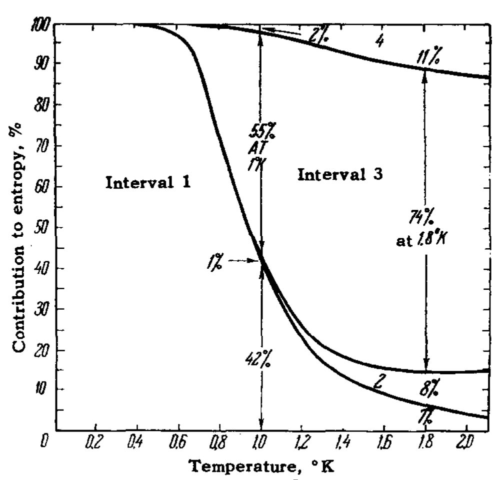

FIG. 16

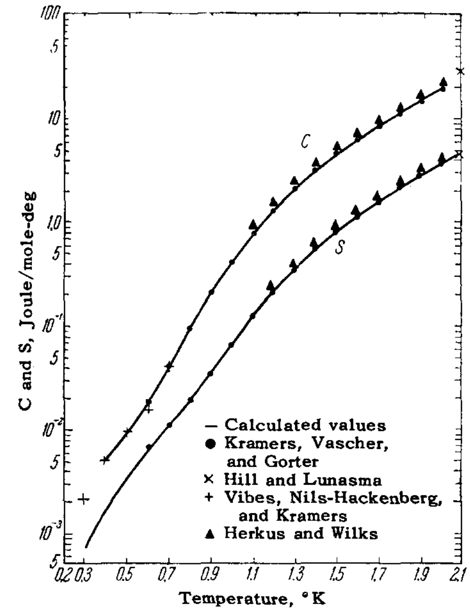

FIG. 17

Thus, by studying the angular dependence of the intensity of scattered neutrons, one can conclude whether the rotons have their own angular momentum or not. If, for example, it is found that the chirality of the rotons is equal to zero and they do not have angular momentum, then there would be no grounds for speaking, from the physical point of view, of the existence of two types of thermal excitation in helium I. Actually, if the phonons, which certainly have chirality equal to zero, and the rotons have no properties which distinguish one from another, there is no sense in subdividing the excitation quanta into two types. In this case one could assume that they merely pertain to different portions of one and the same dispersion curve.

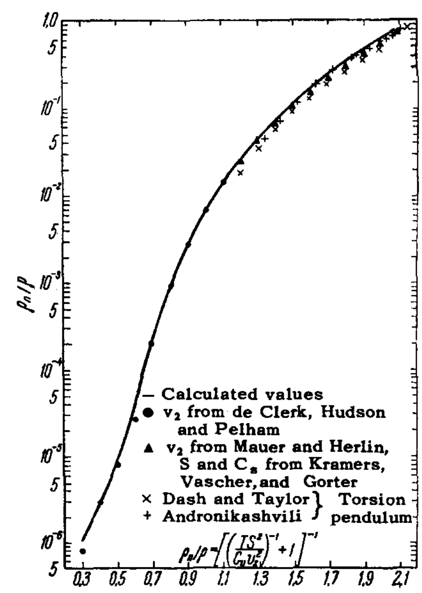

FIG. 18

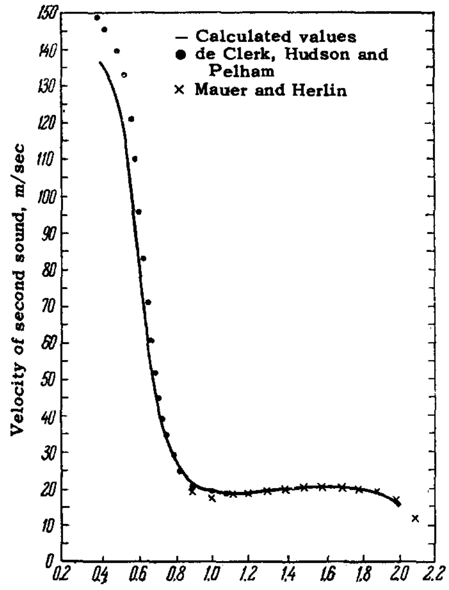

FIG. 19

It goes without saying that a clarification of these problems is of exceeding importance for the understanding of the nature of thermal excitations.

To conclude this survey, a few words should be said regarding the temperature dependence of several thermodynamic functions, calculated on the basis of the dispersion curve of Yarnell et al.[16] Figure 15 shows how the authors have subdivided the dispersion curve into sections (intervals): 1 — phonon section, 2 — transition section, corresponding to the maximum, 3 — roton section — low-energy rotons, 4 — high-energy rotons.

It is seen from Fig. 16 that at temperatures up to 0.5° K only phonons come into play. At temperatures above 1° K the main contribution to the entropy is made by rotons of interval 3. Only near the λ point do the high-energy phonons and the high-energy rotons assume a definite significance.

For specific heat and entropy there is excellent agreement with data of other experiments over the entire investigated temperature interval shown in Fig. 17.

No less excellent an agreement is obtained when the values of the density of the normal component, calculated by the authors, are compared with the values determined in several other experiments. Only the points obtained by Dash and Taylor at the lowest temperature lie somewhat below the continuous curve (Fig. 18), and in the comparison of the results for the velocity of second sound there is a certain discrepancy between the data by these authors and the experiments made by Pell and coworkers (Fig. 19).

Note added in proof. Henshaw has recently found experimentally that the dispersion curve terminates as indicated in Fig. 14. However, being unacquainted with Pitaevskiĭ's work, Henshaw assumes without justification that he encountered a second maximum.

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