SOVIET PHYSICS

USPEKHI

A Translation of Uspekhi Fizicheskikh Nauk

SOVIET PHYSICS USPEKHI (Russian Vol. 72, Nos. 3-4) MAY-JUNE 1961

RESONANCE SCATTERING OF GAMMA RAYS IN CRYSTALS

G. N. BELOZERSKIĬ and Yu. A. NEMILOV

Usp. Fiz. Nauk **72**, 433-466 (November, 1960)

TABLE OF CONTENTS

1. INTRODUCTION

RESONANCE scattering of γ rays makes it possible to obtain many data concerning nuclear properties. The knowledge of transition probabilities enables us to make judgements concerning nuclear structure, and the comparison of the experimentally obtained transition probabilities with theoretical computations enables us to draw conclusions concerning the validity of various nuclear models. It is especially important to measure lifetimes less than 5×10^{-11} sec., i.e., times which cannot be measured by delayed coincidence methods.[1] One of the methods* for measuring lifetimes of such transitions is resonance scattering of γ rays.

Resonance scattering of γ rays has its special features compared to resonance scattering in optics; the recoil energy in the emission and absorption of γ quanta is, as a rule, considerably larger than the line width itself. In the first experiments on resonance scattering of γ rays,[2-6] no methods were found for compensating the loss associated with the recoil energy, and therefore the experiments gave no positive results. Only in 1951 were the first experiments[7] done on resonance scattering of γ quanta by compensating the loss to recoil by producing a Doppler shift of the energy of the emitted quanta.

So far, in experiments on resonance scattering, the following four methods have been applied for compensating the loss to recoil in emission and absorption of the γ quanta:[8,9]

1. Ultracentrifuge method. The compensation is achieved by mechanical motion of the source relative to the absorber.

2. Doppler broadening of the line as a result of heating of the source.[10]

3. Use of the recoil from the preceding process of radioactive decay.

4. Use of the recoil received by an excited nucleus formed as a result of a nuclear reaction.

Experiments on resonance scattering enable us to

*Lifetimes less than 5×10^{-11} sec can also be studied by Coulomb excitation, by observing the Doppler broadening of the γ-ray line caused by the motion of the nucleus formed in the nuclear reaction.

obtain not only the transition probabilities, but also help in establishing decay schemes of various isotopes.

We should also point out the possibility of applying this method for studying solids: the study of the laws of slowing down of atoms in materials, the measurement of the distribution of velocities of atoms in the scatterer, the study of the behavior of molecules after β transformation of one of the atoms constituting the molecule, etc. A good illustration of the possible application of the method of resonance scattering of γ rays is the determination of the helicity of the neutrino.[11]

In 1958 a new method was discovered[12,13] for studying resonance scattering of γ quanta. In those cases where the emitting and absorbing nuclei are in a crystal lattice, in the emission of γ quanta the small energy of recoil can be transferred to the crystal as a whole. In this case the mass of the crystal can be considered to be infinite, and therefore the energy transferred to it will be zero.

Since the γ quantum practically loses no energy in recoil in emission and absorption, the resonance conditions are not violated. Theoretically this phenomenon was discussed on the basis of the theory developed by Lamb[14,15] for the capture of slow neutrons by atoms in a crystal lattice.

2. CAPTURE OF SLOW NEUTRONS BY ATOMS IN A CRYSTAL LATTICE

The dispersion formula gives the following expression for the capture of a neutron by a free nucleus at rest:

$$\sigma_c = \sigma_0 \frac{\Gamma^2/4}{(E-E_0)^2+\Gamma^2/4}. \tag{2.1}$$

Here σ_0 is the cross section at resonance, E_0 is the energy of the neutron at resonance, E is the energy of the incident neutron, Γ is the total width of the level.

If we consider the fact that the atoms are not free and at rest, but are in a gas at temperature T, the cross section for capture of neutrons can be written as follows:[14,15]

$$\sigma_c(\xi, E) = \sigma_0 \psi(\xi, x), \tag{2.2}$$

where $x = 2(E - E_0)/\Gamma$, $\xi = \Gamma/\Delta$, $\Delta = 2\sqrt{RkT}$ is the Doppler width,

$$\psi(\xi, x) = \frac{\xi}{2\sqrt{\pi}} \int_{-\infty}^{+\infty} \frac{e^{-\frac{1}{4}\xi^2 (x-y)^2}}{1+y^2}\, dy. \tag{2.3}$$

Actually, in most cases the source and absorber are solids, and therefore when the chemical binding is important we cannot apply the theory of Doppler broadening of a line in free atoms.

Lamb[14] obtained the expression for the probability of capture, W(E), of a neutron with energy E by atoms in a crystal lattice. As a result of the neutron capture, the crystal lattice makes a transition from one vibrational state to another, and the nucleus which is formed emits a γ quantum:

$$W(E) = \frac{2}{\Gamma} \mathrm{Re} \int_0^\infty e^{i\mu (E-E_0+i\Gamma/2)+g(\mu)}\, d\mu, \tag{2.4}$$

from which we obtain for $g(\mu)$, disregarding the dispersion of phonons and the anisotropy of the sound velocity,

$$g(\mu) = \frac{3Em}{M\theta^2} \int_0^\theta [(1+\bar{\alpha}_s) e^{-i\mu\varepsilon} + \bar{\alpha}_s e^{i\mu\varepsilon} - 2\bar{\alpha}_s - 1]\, \varepsilon\, d\varepsilon, \tag{2.5}$$

where $\epsilon = h\omega$, $\theta = h\omega_{max}$ is the Debye temperature, and $\bar{\alpha}_S = [\exp(h\omega_S/kT) - 1]^{-1}$.

In general, the integration in (2.5) cannot be carried out, and (2.4) can only be evaluated when the function $g(\mu)$ has been estimated.

For large energies of the incident neutrons only the region of small μ, $|\mu\theta| \ll 1$, is important in (2.4).

For such values of μ, we can expand $g(\mu)$ in powers of $\mu\theta$; stopping with quadratic terms we obtain for $g(\mu)$:

$$g(\mu) = -i\mu\theta \frac{R}{\theta} - (\mu\theta)^2 \frac{R\bar{\varepsilon}}{\theta^2}, \tag{2.6}$$

where $\bar{\epsilon}$ is the mean energy per vibrational degree of freedom, and $R = (m/M)E$ is the recoil energy of the nucleus.

From (2.5) and (2.6) we see that the region of small μ will be important in (2.4) when the condition

$$\frac{1}{2}\Gamma + (R\bar{\varepsilon})^{1/2} \gg \theta \tag{2.7}$$

("weak binding") is fulfilled.

Since usually $(R\bar{\epsilon})^{1/2} \gg \Gamma/2$, (2.7) can be rewritten in the form $R\bar{\epsilon} \gg \theta^2$, and consequently in (2.6) the coefficient in front of $(\mu\theta)^2$ is much greater than unity. For large μ the integrand in (2.4) will tend to zero.

In the case of "weak binding," we obtain for W(E)

$$W(E) = \frac{4}{\Gamma^2} \psi(\xi, x). \tag{2.8}$$

Consequently the cross section for neutron capture in a crystal is the same as for free atoms, but the Doppler width will now be equal to

$$\Delta = 2\sqrt{RT_{eff}}, \tag{2.9}$$

where $T_{eff} = \bar{\epsilon}$.

From the theory of solids it is known that the expression for the mean energy per vibrational degree of freedom can be written as follows:[16]

$$\varepsilon = 3T\left(\frac{T}{\theta}\right)^3 \int_0^{\theta/T} \left(\frac{1}{e^t-1} + \frac{1}{2}\right) t^3\, dt. \tag{2.10}$$

In Fig. 1 we show the dependence of T_{eff}/T on T/θ. For the two limiting cases we have

$$\left.\begin{aligned} T_{eff} &= \tfrac{3}{8}\theta, \quad \text{if} \quad T \ll \theta, \\ T_{eff} &= T, \quad \text{if} \quad T \gg \theta. \end{aligned}\right\} \tag{2.11}$$

From this it is clear that the Doppler width of an

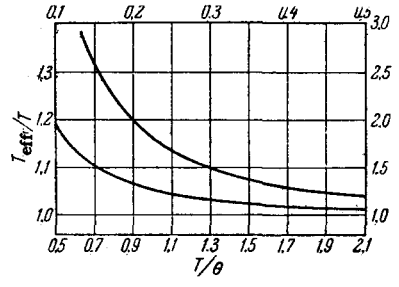

FIG. 1. Dependence of the effective temperature T_{eff}, which is used in computing the Doppler width of a line, on the crystal temperature T. The longitudinal and transverse Debye temperatures are assumed to be equal. The scales on the right and above refer to the upper curve; on the left and below, to the lower curve

absorption line in a crystal at low temperatures is greater than in a gas. This is caused by the presence of zero-point vibrations in the crystal.

Let us consider the capture of low energy neutrons, $\bar{\epsilon}R \ll \theta^2$. In this case the modulus of the function $g(\mu)$ is small, and the oscillating factors in (2.5) can be dropped. Consequently, we need consider only large values of μ, $|\mu\theta| \gg 1$. For $g(\mu)$ we obtain approximately

$$g(\mu) = g(\infty) = -\frac{6R}{\theta^3}\int_0^\theta \left(\bar{\alpha}_s + \frac{1}{2}\right)\varepsilon\, d\varepsilon. \qquad (2.12)$$

Substitution of (2.12) in (2.4) leads to the following simple expression for the capture probability:

$$W(E) = \frac{e^{g(\infty)}}{(E-E_0)^2+\Gamma^2/4}, \qquad (2.13)$$

$$\left.\begin{aligned} g(\infty) &= -6\frac{RT}{\theta^2}, \quad \text{if} \quad T \gg \theta, \\ g(\infty) &= -\frac{3}{2}\frac{R}{T}, \quad \text{if} \quad T \ll \theta. \end{aligned}\right\} \qquad (2.14)$$

In the case of very "strong" binding, $\theta \to \infty$, $g(\infty) \to 0$:

$$W(E) = \frac{1}{(E-E_0)^2+\Gamma^2/4}, \qquad (2.15)$$

i.e., we have obtained the normal absorption line distributed around $E = E_0$.

In the general case a certain amount of numerical integration is needed to find the shape of the absorption line. For illustration of possibilities of using the general equations (2.4) and (2.5), we show in Fig. 2 the shapes of resonance absorption lines in a crystal, 1, and in a gas, 2, for $T \ll \theta$.

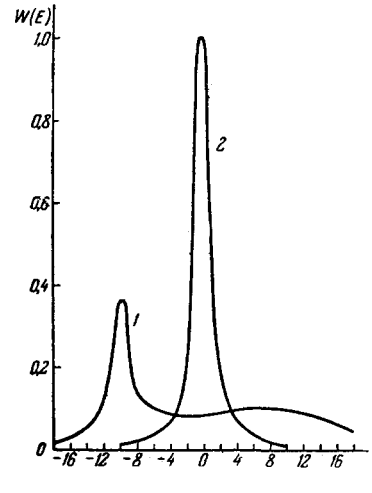

FIG. 2. Shape of neutron resonance absorption line: 1. in a crystal with Debye temperature $\theta = 210°$ K, $\Gamma = \theta/4$. For comparison, we give curve 2 for the absorption by atoms in a gas at the same temperature T. $x = 2(E - E_0)/\Gamma$ is measured in units of $\Gamma/2$.

3. APPLICATION OF LAMB'S THEORY TO RESONANCE SCATTERING OF GAMMA QUANTA IN CRYSTALS

Lamb's theory can easily be applied to the calculation of the emission line, and also for interpreting processes of resonance absorption and scattering of γ rays. In absorption or emission of a γ quantum with energy E by a free nucleus of mass M, the nucleus receives a recoil energy R

$$R = \frac{E^2}{2Mc^2}. \qquad (3.1)$$

Using the formulas of the preceding section, we can get the cross section for resonance absorption of the γ ray,

$$\sigma_{\text{res. abs.}}(E) = \frac{\Gamma^2}{4}\sigma_0 W_{\text{abs.}}(E), \qquad (3.2)$$

where $W_{abs}(E)$ determines the location and shape of the absorption line,

$$W_{\text{abs.}}(E) = \frac{2}{\Gamma}\,\text{Re}\int_0^\infty e^{i\mu(E-E_0+i\Gamma/2)+g_{\text{abs.}}(\mu)}\, d\mu, \qquad (3.3)$$

and $g_{abs}(\mu)$ is given by formula (2.5). The probability of emission of a γ quantum with energy E, $W_{em}(E)$ can be obtained from (3.3) by replacing $g_{abs}(\mu)$ by $g_{em}(\mu)$ and noting that $g_{em}(\mu) = g_{abs}(-\mu)$:

$$W_{\text{em}}(E) = \frac{2}{\Gamma}\,\text{Re}\int_0^\infty e^{i\mu(E-E_0+i\Gamma/2)+g_{\text{em}}(\mu)}\, d\mu, \qquad (3.4a)$$

$$g_{\text{em}}(E) = \frac{3R}{\theta^3}\int_0^\theta \left[(1+\bar{\alpha}_s)\, e^{i\mu\varepsilon} + \bar{\alpha}_s e^{-i\mu\varepsilon} - 2\bar{\alpha}_s - 1\right]\varepsilon\, d\varepsilon. \qquad (3.4b)$$

The functions $W_{em}(E)$ and $W_{abs}(E)$, and consequently the emission and absorption lines are mirror images of one another in the line $E = E_0$.

One can calculate $\sigma_{res.abs.}(E)$ for specific cases by using the Debye model.

In Fig. 3 we show[60] the graph of $\sigma_{res.abs.}(E)$ for the 129-kev γ ray in Ir^{191}. The crystal was assumed to be isotropic. In calculating $W_{abs}(E)$ the following parameters were used: $R = 4.6 \times 10^{-2}$ ev; $\Gamma = 4.6 \times 10^{-6}$ ev; $\theta = 316°$K. From Fig. 3 we see that the shape of the absorption curve at room temperature is similar to the Gaussian curve for a gaseous absorber. Such a broad distribution is caused by the fact that in absorption of the γ quanta there occurs a change in the vibrational state of the crystalline lattice.

As the temperature is lowered, there appears at $E = E_0$ a line with the natural width $\Gamma = 4.6 \times 10^{-6}$ ev. The height of this line depends strongly on temperature and reaches its maximum value at zero degrees. The part of the absorption curve lying in the region $E < E_0$ vanishes at a temperature equal to zero since there are no phonons in the crystal which could be absorbed, but the emission of phonons is possible.

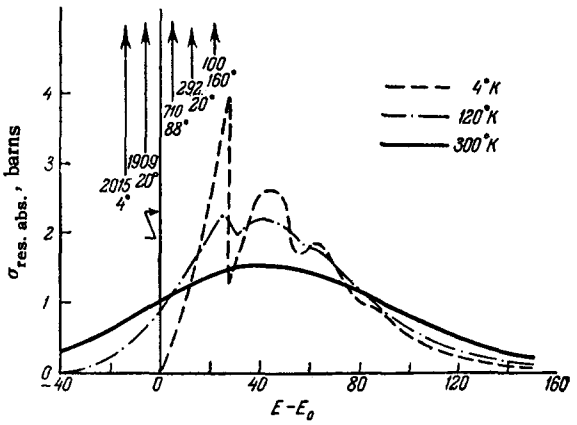

FIG. 3. Absorption cross section of monochromatic γ rays with energy 129 kev in a crystal of ordinary Ir^{191}. The curve corresponding to T = 120° K is similar to the curve for T = 4° K, except for the region $|E - E_0| < 3 \times 10^{-3}$ ev, and the reduced peak in the region $E - E_0 \sim \theta = 27.2 \times 10^{-3}$ ev. The arrow shows the strength of the absorption at T = 300° K. $E - E_0$ is in units of 10^{-3} ev.

The peak at $E - E_0 = \theta$ results from the fact that in the interval $\Gamma < E - E_0 < \theta$ one phonon is emitted in the absorption of the γ quantum. The maxima at larger values of $E - E_0$ are associated with the emission of two, three and more phonons.

In the resonance scattering of high energy γ rays satisfying the condition of "weak" binding,*

$$E \gg \theta \sqrt{\frac{2Mc^2}{\varepsilon}}, \tag{3.5}$$

we need to consider only small values of μ, and we then obtain

$$\left.\begin{aligned} g_{\text{em}}(\mu) &= i\mu R - \mu^2 R\bar{\varepsilon}, \\ g_{\text{abs.}}(\mu) &= -i\mu R - \mu^2 R\bar{\varepsilon}. \end{aligned}\right\} \tag{3.6}$$

In the resonance scattering of low energy γ rays, $\bar{\epsilon}R \ll \theta^2$, so that, using (2.11) and (3.1):

$$E \ll \sqrt{2Mc^2\theta}; \tag{3.7}$$

for temperatures which are small compared to θ, assuming only that $\mu \gg 1$, we obtain from (2.12)

$$g_\infty(T) = g_{\text{em}}(\infty; T) = g_{\text{abs}}(\infty; T)$$
$$= -6\frac{R}{\theta}\left\{\frac{1}{4} + \left(\frac{T}{\theta}\right)^2 \int_0^{\theta/T} \frac{t\,dt}{e^t - 1}\right\}. \tag{3.8}$$

In order to calculate the experimentally measured average cross section for resonance absorption $\bar{\sigma}_r(T_s, T_a)$, we must multiply $\sigma_{\text{res.abs.}}(E)$ in (3.2) by the normalized distribution function of the emitted quanta and integrate over all energies. (T_s and T_a are the temperatures of source and absorber.)

Using the normalization condition

$$\frac{\Gamma}{2\pi}\int_0^\infty W_{\text{em}}(E)\,dE = 1$$

and (3.2), we obtain

$$\bar{\sigma}_r(T_s, T_a) = \frac{\Gamma^3}{8\pi}\sigma_0 \int_0^\infty W_{\text{abs}}(E) W_{\text{em}}(E)\,dE. \tag{3.9}$$

If the transition to the ground state can occur both by emission of a γ quantum and by internal conversion, we must make a distinction between the cross section $(\sigma_0)_{\text{res. sc.}}$ for resonance scattering and $(\sigma_0)_{\text{res. abs.}}$ for resonance absorption.

For resonance scattering, according to reference 18, we have

$$(\sigma_0)_{\text{res. sc.}} = \frac{2I_a+1}{2I_g+1}\frac{\lambda_0^2}{2\pi}\frac{\Gamma_\gamma^2}{\Gamma^2}H = \frac{2I_a+1}{2I_g+1}\frac{\lambda_0^2}{2\pi}\frac{1}{(1+\alpha)^2}H, \tag{3.10}$$

where I_a and I_0 are the spins of the excited and ground states, H is the relative abundance of the resonant isotope, λ_0 is the wavelength of the resonance radiation, Γ is the total energy width of the resonance line, Γ_γ is the partial width for γ emission, α is the internal conversion coefficient.

In resonance scattering only the fraction Γ_γ/Γ of all transformations proceed via γ quantum emission. In resonance absorption all the transformations make a contribution to the cross section,

$$(\sigma_0)_{\text{res. abs.}} = \frac{\Gamma}{\Gamma_\gamma}(\sigma_0)_{\text{res. sc.}} = \frac{2I_a+1}{2I_g+1}\frac{\lambda_0^2}{2\pi}H\frac{\Gamma_\gamma}{\Gamma}. \tag{3.11}$$

To calculate the average cross section for resonance absorption according to (3.9), we must know $W_{\text{abs}}(E)$ and $W_{\text{em}}(E)$, which in general is difficult.

For resonance scattering of γ rays of low energy (3.7), we can calculate the line shape and thus compute $\bar{\sigma}_r(T_s, T_a)$.

1) In the region $|E - E_0| \gg \Gamma$, only values $\mu\theta \ll 1$ give a contribution to the integrals (3.3), (3.4); applying (3.6) we get

$$W_{\text{I}}(E) = \frac{4}{\Gamma^2}\psi(\xi, x), \tag{3.12}$$

which can be compared with (2.8).

2) In the region $E = E_0$, we obtain a good approximation for $W(E)$ if we break up the region of integration in (3.3) and (3.4) at $\mu\theta = 1$ and apply the approximations (3.6) and (3.8) respectively in the two regions:

$$W_{\text{II}}(E) = W_{\text{I}}(E) + \frac{e^{g_\infty(T)}}{(E-E_0)^2 + \frac{\Gamma^2}{4}}. \tag{3.13}$$

We thus obtain for the probability of emission or absorption of low energy γ quanta

$$\left.\begin{aligned} W_{\text{em}}(E) &= \frac{4}{\Gamma^2}\psi(\xi_{\text{em}}, x_{\text{em}}) + \frac{e^{g_\infty(T_s)}}{(E-E_0)^2+\Gamma^2/4}, \\ W_{\text{abs}}(E) &= \frac{4}{\Gamma^2}\psi(\xi_{\text{abs}}, x_{\text{abs}}) + \frac{e^{g_\infty(T_a)}}{(E-E_0)^2+\Gamma^2/4}. \end{aligned}\right\} \tag{3.14}$$

From (3.14) it follows that when we take account of crystal binding the emission and absorption lines have,

*Applying Lamb's theory for the case of "weak" binding to the experiments of resonance scattering of hard γ quanta, one can interpret correctly the results of experiments in Reference 17.

at the temperature T, almost the same shape and position as in an ideal gas at the temperature $T_{eff} = \bar{\epsilon}$ given by (2.10). At the position of the resonance, for $E = E_0$, at low temperatures there appears a line* with the natural width Γ, as was shown in Fig. 3 for Ir^{191}. The average value of the resonance absorption of low energy γ rays is obtained by substituting (3.14) in (3.9):

$$\bar{\sigma}_r = \frac{(\sigma_0)_{\text{res. abs.}}}{2}\Gamma\sqrt{\frac{\pi}{\Delta_a^2+\Delta_s^2}}\,e^{-\frac{4R^2}{\Delta_a^2+\Delta_s^2}} + \frac{(\sigma_0)_{\text{res. abs.}}}{2}e^{g_\infty(T_s)+g_\infty(T_a)}. \quad (3.15)$$

Remembering that

$$\Gamma = \frac{\hbar(1+\alpha)}{\tau_\gamma},$$

we can, for sufficiently high temperatures, neglect the second term in (3.16), and then

$$\bar{\sigma}_r \sim \frac{1}{\tau_\gamma}.$$

At low temperatures the second term, which does not depend on the lifetime of the level, predominates.

It is clear that the theoretical calculation of the shape of the emission and absorption line from the general formulas (3.4) is possible only in the limiting case (Debye distribution of frequencies in the vibrational spectrum of the crystal) which actually does not occur.

Lipkin[54] found a relatively simple expression for the probability of emission of quanta without recoil. He derived the probability $P(\{n_s\}, \{n_s\})$ for the emission of a γ quantum by a nucleus in a crystal lattice with no change in the vibrational state of the solid.

The only assumption which he made concerning the crystal, for simplifying the calculation, was that the forces acting between the atoms were assumed to be harmonic:

$$P(\{n_s\}, \{n_s\}) \approx \exp\sum\left(-(2n_s+1)\left[\frac{(\hbar K)^2}{2M\hbar\omega_s}\right]\alpha_{L_s}\right). \quad (3.16)$$

If the lattice is at the absolute zero of temperature, $T = 0°K$, then all the $n_s = 0$, and in the exponent (3.16) there stands the ratio of the energy of recoil of a free nucleus to the average energy of vibration of the lattice $E_{av} = \hbar\bar{\omega}$:

$$\hbar\bar{\omega} = \frac{1}{\sum_s \frac{\alpha_{L_s}}{\hbar\omega_s}},$$

where the α_{L_S} are certain coefficients. From (3.16) we see that the probability of emission of γ quanta without recoil decreases exponentially with increasing energy or with increasing temperature, since for $T > 0°K$, $n_s \neq 0$ and $P(\{n_s\}, \{n_s\})$ decreases very rapidly.

*The existence of this line was not noted in reference 14.

In the Debye approximation, $\alpha_{L_s} = \text{const}$,

$$\hbar\bar{\omega} = \frac{2}{3}\theta, \qquad P(\{n_s\}, \{n_s\})_{T=0°K} \approx \exp\left\{-\frac{3}{2}\frac{(\hbar K)^2}{2M\theta}\right\}.$$

Since the distribution of frequencies in the vibrational spectrum of a crystal is not a Debye distribution, we can introduce to characterize the phenomenon of resonance scattering without recoil,[63] a certain special temperature Φ:

$$\Phi = \left(\frac{K}{3N\hbar}\int_0^{\nu_{max}}\frac{g(\nu)}{\nu}d\nu\right)^{-1};$$

$g(\nu)d\nu$ is the number of states of phonons with frequency ν. The temperature Φ is simply related to the relative number of γ quanta emitted by the crystal without recoil:

$$\Phi = \frac{R}{\ln 1/f}.$$

4. USE OF TEMPERATURE CHANGE OF THE SOURCE AND MECHANICAL MOTION OF THE SOURCE RELATIVE TO THE ABSORBER FOR OBSERVING THE EFFECT OF RESONANCE ABSORPTION OF GAMMA RAYS WITHOUT RECOIL

It is frequently impossible to determine directly the cross section for resonance absorption by measuring the absolute attenuation coefficient, since the resonance absorption is small compared with the absorption by the atomic electrons. One of the methods for measuring lifetimes of excited states by using resonance scattering is a measurement of the dependence of the average absorption cross section on the temperature of the source. The cross section for resonance absorption is determined by a differential method in which one measures the absorption of the resonance line in a resonance absorber at various temperatures and under such conditions that the intensity change observed is directly related to the effective cross section for resonance absorption, and disturbing effects are eliminated. According to (3.15) the effective cross section depends on the temperatures of source and absorber. Since the number of atoms per square centimeter of surface of the absorber depends on the temperature, all the measurements are usually carried out at a fixed absorber temperature, and one changes only the temperature of the source. Self-absorption in the source also depends on temperature,[10] but it can be eliminated if we use a control absorber chosen so that it absorbs approximately the same as the resonance absorber. By studying the absorption in the resonance and control absorbers, one can eliminate the effect of self-absorption.

In Figs. 4 and 5 we show the arrangement of the experiment and the construction of the cryostat with the absorbers.

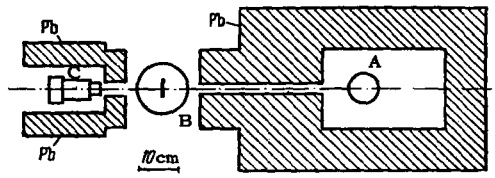

FIG. 4. Arrangement of experiment for measuring the magnitude of the resonance absorption of γ rays without recoil, at various source temperatures: A – Dewar in which the source is located, B – Dewar with absorber, C – counter.

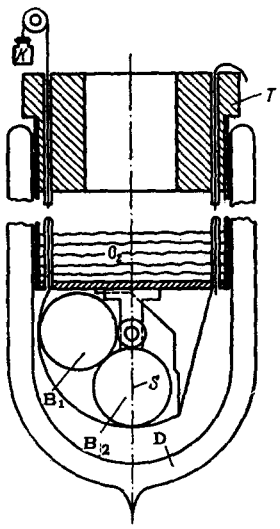

FIG. 5. Construction of Dewar in which are located the control B_1 and the resonance B_2 absorbers: D – Dewar, S – axis of γ ray beam, K – device for interchanging absorbers.

The control absorber B_1 and the resonance absorber B_2 (see Fig. 5) can, by means of a pulley system, be alternately placed in the beam S. During cooling the dimensions of the absorber can change freely.*

In all these measurements one requires a very high stability of the measuring apparatus, since the effect is very small and the total error in determining the intensity through the absorbers must be of the order of 0.05%.

It is most convenient to measure the relative intensity of γ rays of the resonance energy for a definite temperature T

$$\frac{I_{\text{res}}(T)-I_{\text{cont}}(T)}{I_{\text{cont}}(T)}.$$

Knowing the difference of the relative intensities M for source temperatures T_1 and T_2, and knowing which part K of the recorded γ quanta refers to the resonance line, we can easily calculate the lifetime of the level

$$M \approx Kn[\bar{\sigma}_r(T_2)-\bar{\sigma}_r(T_1)], \tag{4.1}$$

$$K=\frac{I_0(T_1)e^{-n\sigma_0(T_1)}}{I(T_1)}, \tag{4.2}$$

where n is the number of atoms of the resonance absorber per square centimeter, $\sigma_0(T_1)$ is the effective cross section for absorption of the resonance radiation in the resonance absorber at temperature T_1, $I_0(T_1)$ is the intensity of the resonance component at the source temperature T_1, $I(T_1)$ is the total intensity recorded in the channel for the resonance absorber, $\bar{\sigma}_r(T)$ is given by equation (3.15) and can be written as:

$$\bar{\sigma}_r=\frac{2Ia+1}{2I_g+1}\frac{h^2c^3}{4E_0^3}H\frac{\hbar}{\tau_\gamma}\sqrt{\frac{m}{2k\pi(\bar{\varepsilon}_s+\bar{\varepsilon}_a)}}\,e^{-\frac{R}{(\bar{\varepsilon}_s+\bar{\varepsilon}_a)}}. \tag{4.3}$$

All the quantities in (4.1), (4.2), (4.3) are either known or can be easily measured. From this we determine τ_γ and, knowing the conversion coefficient α, we find the lifetime of the level

$$\tau=(1+\alpha)\tau_\gamma.$$

In studying the phenomenon of resonance scattering we must know what fraction f of the resonance radiation is emitted without recoil and what fraction f′ is absorbed without recoil. Comparing the values found from experiment and by computation, we can determine the Debye temperature of the crystal.

Calculations based on the Debye model for the vibrating lattice give, for $T \ll \theta$,

$$f=e^{-\frac{3}{2}\frac{R}{k\theta}\left[1+\frac{2}{3}\left(\frac{\pi T}{\theta}\right)^2\right]}. \tag{4.4}$$

Let us denote by r(T) the ratio of the counting rate with the control absorber to the counting rate with the resonance absorber, as obtained at temperature T.

The quantity G = r(T)/r(300) depends only on the resonance cross section. The calculations give

$$G=1-f\left[1-I_0\left(\frac{1}{2}x\right)e^{-\frac{x}{2}}\right], \tag{4.5}$$

where $I_0(z) = J_0(iz)$ is the Bessel function of zero'th order and imaginary argument, $x = nf'\sigma_0 H$, n is the number of atoms per square centimeter in the absorber.

By constructing a graph of the dependence of G – 1 on x for various selected values of f and putting on the experimental data, we can determine f. In Fig. 6 we show a graph of the dependence of G – 1 on x for the 129-kev transition in Ir^{191} in which[9] the absorber was cooled to 4°K. From the graph one finds that f = 0.07 ± 0.015, f′ = 0.047 ± 0.015.

Of especial interest is the dependence of the cross section for resonance absorption on temperature. For the experimenter it is desirable to have the maximum possible absorption cross section, but working at low temperatures causes complication in the experimental apparatus and can be the cause of instability. In reference 19, they studied the dependence of G – 1 on temperature for Ir^{191}. The absorber thickness was 1.69×10^{21} atoms/cm^2.

From Fig. 7 one sees that at temperatures above 100°K the absorber can be regarded as "thin" for the

*If the dimensions of the absorber could not change freely then, as a result of the stresses produced in cooling, the thickness of the absorber at various places would be different, and the results of the experiment would not be reliable.

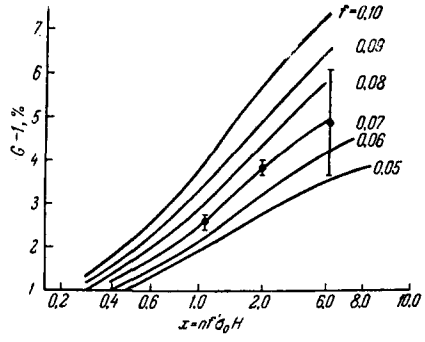

FIG. 6. Dependence on absorber thickness of the magnitude of the resonance absorption without recoil for 129-kev γ rays in Ir^{191}: f is the fraction of γ quanta emitted without recoil. The experimental results are given together with their errors.

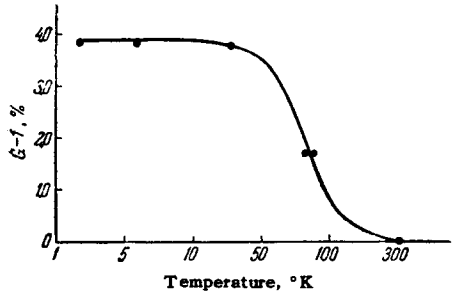

FIG. 7. Temperature dependence of magnitude of resonance absorption without recoil for γ rays of 129 kev in Ir^{191}. (Temperatures of source and absorber were equal.) The absorber thickness was 1.69×10^{21} atoms/cm^2.

resonance radiation of 129 kev. Below 30° K the absorber is "thick."* The authors of references 20 and 21 studied the temperature dependence of the resonance absorption cross section for the 14.4-kev γ rays in Fe^{57}. It was found that cooling the absorber to liquid air temperature essentially did not increase the effect, and therefore the experiments were done at room temperature in order to have higher stability. When the source was heated from 25 to 200° C, the reduction in resonance absorption was insignificant, and zero was reached only at a temperature around 550° C.

One must consider that the energy of the γ quanta emitted without recoil is necessarily always smaller than the difference between the energies of the first excited state and the ground state. Pound and Rebka[64] discovered and investigated the change in energy of quanta emitted without recoil as a function of the temperature. If we consider the crystal as a system of interacting atoms, the Hamiltonian of such a system is written as follows:[65]

$$H = \sum_i \frac{p_i^2}{2m_i} + V(r_1, r_2, \ldots).$$

Since the mass of the nucleus emitting the γ quantum decreases, this increases the frequency of the lattice vibrations. If the i-th atoms emits a γ quantum with energy E, the change in mass will be $\delta m_i = E/c^2$. The energy of the crystal then increases by δE at the expense of the energy of the emitted γ quantum:

$$\delta E = \delta \langle \frac{P_i^2}{2m_i} \rangle = \delta m_i \langle \frac{P_i^2}{2m_i^2} \rangle = \frac{ET_i}{m_i c^2},$$

where T_i is the kinetic energy of the i-th atom. Thus the relative reduction in energy of the γ quantum is

$$\frac{\delta E}{E} = \frac{T_i}{m_i c^2} \simeq \frac{c_p}{2c^2},$$

where $c_p(T)$ is the specific heat. For iron at 300° K, the relative change in energy of the γ quanta, i.e., $\delta E/E$, is equal to 2.2×10^{-15} per degree K.

Consequently, for relatively long-lived levels (like the 14.4-kev level in Fe^{57}) in interpreting the results we must take account of this "temperature shift." For γ transitions like the 129 kev transition in Ir^{191} this effect is less significant. From this it follows immediately that one cannot in general judge the magnitude of the Mössbauer effect by comparing the absorption at zero and at relatively high velocities.

In reference 64 it was discovered that, in addition to a change in the energy of the γ quanta with temperature, the width of the line also decreases as one cools the crystal.

Another method for measuring level widths consists in displacing the emission line relative to the absorption line by using the Doppler effect. In references 19, 22 — 24, and 61, the presence of narrow lines having the natural line width was shown by means of a motion of the source relative to the absorber. If the source moves toward the absorber with velocity v, the energy of the γ quanta will be equal to $E + (vE_0/c)$, and the emission line will appear to be shifted relative to the absorption line. The experimentally observed average value of the resonance absorption σ_r, (3.9), should then be written as follows:

$$\bar{\sigma}_r\left(\frac{v}{c}E_0\right) = \frac{\Gamma^3\sigma_0}{8\pi}\int W_{abs}(E) \times W_{em}\left(E + \frac{v}{c}E_0\right)dE = \frac{\Gamma^2\sigma_0 W'\left(\frac{v}{c}E_0\right)}{2}, \quad (4.6a)$$

where

$$W'\left(\frac{v}{c}E_0\right) = \frac{2}{\Gamma}\,\mathrm{Re}\int_0^\infty e^{i\mu\,(vE_0/c + i\Gamma) + g_{abs}(\mu) + g_{em}(\mu)}\,d\mu. \quad (4.6b)$$

If the structures of absorber and emitter are identical, then when we cool the source of absorber to liquid helium temperature, $W'(vE_0/c)$ can be written as

$$W'\left(\frac{v}{c}E_0\right) = W_0'\left(\frac{v}{c}E_0\right) + W_1'\left(\frac{v}{c}E_0\right) + \ldots, \quad (4.7)$$

where

$$W_0'\left(\frac{v}{c}E_0\right) = \frac{e^{2g(\infty,\,0)}}{\left(\frac{v}{c}E_0\right)^2 + \Gamma^2},$$

$$W_n'\left(\frac{v}{c}E_0\right) = \frac{e^{2g(\infty,\,0)}}{n!}\frac{\pi}{\Gamma}\left(\frac{2R}{3N}\right)^n \int_0^{\omega_{max}} \frac{N(\omega_1)}{\omega_1}d\omega_1 \ldots \int_0^{\omega_{max}} \frac{N(\omega_n)}{\omega_n}d\omega_n \delta\left(\frac{v}{c}E_0 - \omega_1 - \ldots - \omega_n\right)$$

*These results agree well with the theoretical computations[60] (cf. Fig. 7).

In Fig. 8 we show the theoretical curve for the dependence of $\bar{\sigma}_r(vE_0/c)$ on vE_0/c for the 129-kev γ ray of Ir^{191}. In the calculations, the same assumptions were made as for Fig. 3.

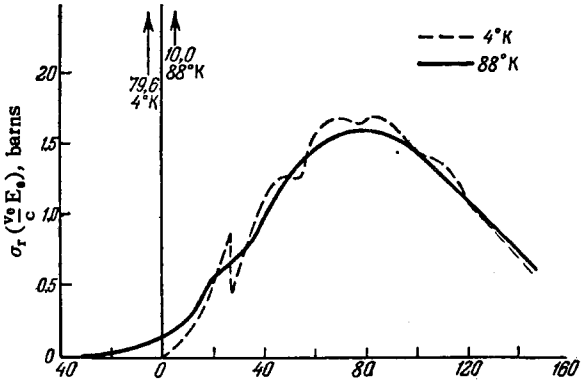

FIG. 8. Cross section for absorption of 129-kev γ rays in a crystal of Ir^{191} as a function of the velocity of motion of the source relative to the absorber. The abscissa is in units of 10^{-3} ev.

The measured intensity I(v) of γ quanta having the resonance energy can be written as:

$$I(v) = \text{const} \cdot \int_0^\infty W_{em}\left(E + \frac{v}{c}E_0\right) e^{-\sigma(E)n}\, dE. \qquad (4.8)$$

In the case of weak absorption ($n\sigma_0 e^{g_\infty(T_a)} \ll 1$), equation (4.8) can be written as follows:

$$I(v) = C_1 - \frac{C_2}{1 - \left[\left(\frac{v}{c}\right)\frac{E_0}{\Gamma}\right]^2}. \qquad (4.9)$$

In the case of strong absorption, the intensity distribution I(v) depends on the shape of the vibration spectrum of the absorber. For weak absorption the half-width of the intensity distribution does not depend on the shapes of the vibration spectra of absorber and emitter, and we can simply find the half width of the resonance line from (4.9). But the constant C_2 in (4.9), which determines the magnitude of the resonance effect, depends essentially on the shape of the vibration spectra and reaches its maximum value at a temperature of 0°K.

In Fig. 9 is shown a typical arrangement for determining the width of the resonance line. On the edge of a rotating disk there are three sources, so that the duty cycle amounts to about 30%. The error in the magnitude of the relative velocity is less than 10%.

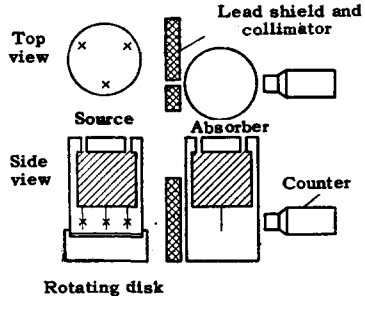

FIG. 9. Arrangement of an experiment for determining the width of the resonance level by using the shift of the emission line relative to the absorption line as a result of the Doppler effect. Horizontal and vertical sectional views are shown.

By measuring the counting rate for γ quanta of the resonance energy behind the absorber as a function of the relative velocity of the source, we can immediately determine the natural width of the line from (4.8), (of course, only if there are no electric or magnetic fields acting on the nucleus which could split or broaden the line).

In Fig. 10 is shown the dependence of the resonance absorption (in %) on the relative velocity.* For convenience we give the shift in energy corresponding to the velocity v. Thus the width of the 129-kev level in Ir^{191} was found. By averaging the results,[19,22,24] we have

$$\Gamma = (3.94 \pm 0.58) \cdot 10^{-6}\ \text{ev},$$
$$\tau = \hbar/\Gamma = 1.65 \cdot 10^{-10}\ \text{sec}.$$

From references 19, 22, 24, we see that the method of resonance absorption can be used for measuring lifetimes of levels in the range $10^{-10} - 10^{-11}$ sec and even shorter, i.e., in the region which is inaccessible to the delayed coincidence method.

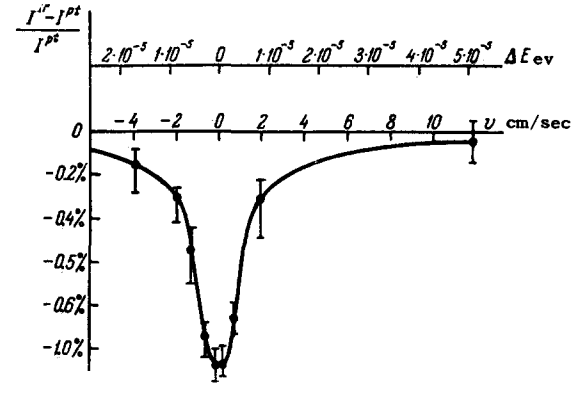

FIG. 10. Dependence of resonance absorption without recoil, for 129-kev γ rays in Ir^{191} on the velocity of motion of source relative to absorber. The ordinates give relative intensity in %.

5. HYPERFINE SPLITTING AND POLARIZATION OF LINES IN EXPERIMENTS ON RESONANCE SCATTERING OF GAMMA RAYS WITHOUT RECOIL

The discovery of the phenomenon of recoilless resonant scattering of γ rays makes it possible also to investigate phenomena in the domain of nuclear physics whose study a few years ago could not even be considered. The resolving power of the best spec-

*The measured width is twice as large as the actual width because of the overlapping of the emission and absorption spectra. In these experiments the case of weak absorption has not yet been realized. The curve in Fig. 10 which is compared with experimental data was obtained by using (4.6). Here the vibrational spectrum of the absorber is approximately represented by a Debye spectrum with $\theta = 285$°K. This approximation gives an added error in the determination of Γ.

trometers reaches a value $\Delta E/E$ of the order of 1,000. Therefore the observation of the Zeeman effect for γ rays was completely impossible earlier. Concerning this Frauenfelder[26] recently wrote: "In nuclear spectroscopy the Zeeman effect cannot be observed since the splitting of the energy levels of the nucleus, even in very strong magnetic fields, is only of the order of 10^{-8} ev."

With the discovery of the phenomenon of recoilless resonant scattering, it became entirely possible to observe the Zeeman effect for γ rays. It is known that the interaction of the nuclear moment with the magnetic field splits a level with spin I into $2I + 1$ sublevels, which are separated from one another by a distance $\Delta E = \mu H/I$. For $\Delta E > \Gamma$, in place of one line there are several lines, corresponding to transitions between these levels which result from the elimination of the degeneracy.

The magnetic fields can be either produced artificially (this is not always possible since for a significant splitting one sometimes requires fields in the hundreds of kilogauss), or by internal fields resulting from interaction with electron shells and from properties of the crystal.

It is known that the internal effective magnetic fields in various compounds can differ markedly. In reference 27 there was studied the effective magnetic field produced by the "hyperfine" interaction of a nucleus with an unpaired electron. As a result of measurements of the specific heat of the alloys CoFe and CoNi in the range $0.35 - 1.0°$ K, they found values for the effective magnetic field which are given in Table I.

TABLE I

Alloy (atomic %)			H_{eff} Kilogauss
60.0 Co	100 Co	40,0 Ni	161±3
91.5 Co		8,5 Fe	219±4
58,7 Co		41.3 Fe	223±4
17.2 Co		82,8 Fe	256±3
4.8 Co		85.2 Fe	293±10
	100 Fe		314±9
			—

Another method for obtaining sizable magnetic fields at the position of the nucleus is the use of the method of orientation of nuclei proposed by Gorter[28] and Rose.[29] In this method one uses the fact that an unfilled electron shell produces at the position of the nucleus of a paramagnetic ion a magnetic field which, from data on hyperfine structure, should be of the order of $10^5 - 10^6$ oe.

In those cases where the magnitude of the magnetic field is insufficient for a complete splitting of the lines, the field will broaden them. This leads to the fact that the width of the level measured by the method described in Sec. 4 will differ from the natural width for the free atom, and consequently one will obtain a too small value for the lifetime of the level.

One of the most suitable isotopes for observing the Zeeman effect is Fe^{57}. The small recoil energy (0.002 ev) in the emission of the 14.4-kev γ quantum and the high Debye temperature of iron allow one to obtain a sizable effect at room temperatures. The high cross section for resonance absorption enables one to carry out the experiment even with ordinary iron in which there is only 2.17% of Fe^{57}.

The magnetic field in iron, according to various estimates,[30,31,33] is of the order of one or several hundreds of kilogauss. Consequently, if in the excited state of Fe^{57} there is a sufficiently large magnetic moment μ^*, the splitting of the line will be large and one can observe the Zeeman effect, since the natural width of the 14.4-kev line is very small: $\Gamma = \hbar/\tau = 4.6 \times 10^{-9}$ ev. In fact, in the very first experiments[20,21,32] the hyperfine structure of the 14.4-kev line was found.

By using the Mössbauer effect one can study the polarization of γ rays in resonance absorption.[34,66]

The ferromagnetic properties of iron make possible a simple measurement of the polarization of the resonant 14.4-kev γ rays from Fe^{57} and a direct observation of the strong correlation in direction between the magnetization in the ferromagnetic domains and the internal magnetic field acting on the nucleus. For this purpose, they studied the resonance absorption of the 14.4-kev γ rays of Fe^{57} as a function of the direction of magnetization in source and absorber. The experimental arrangement is shown in Fig. 11. To one side of the magnet M there is clamped a yoke R.

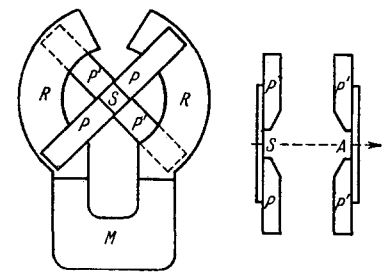

FIG. 11. Experimental arrangement for studying polarization of 14.4-kev γ rays in Fe^{57}.

The pole tips P close the circular gap produced by the yoke, and the source S is clamped in the gap between the pole tips.

An analogous construction is used for producing a magnetic field in the absorber A mounted on the other side of the same magnet M. The construction makes it possible to rotate the pole tips relative to one another so that the values of the fields in source and absorber are not changed, but one changes only the relative orientation of the fields. In this experiment the magnetic fields were perpendicular to the direction of propagation of the γ rays.

The magnetic field between the pole tips amounted to about 800 oe, which assured good saturation of the iron.

A photomultiplier was used as a detector and placed

in a magnetic shield; only the pole tips P which were far from the detector were rotated. Under these conditions the effects of changes in field on the detector were negligibly small. The polarization of the resonance γ radiation was studied by measuring the absorption of the γ rays by an absorber foil as a function of the orientation of the field in source and absorber. The results of the experiment are shown in Fig. 12.

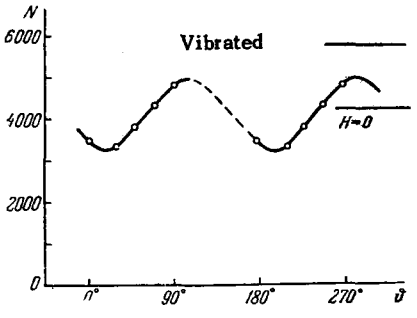

FIG. 12. Dependence of absorption on direction of magnetization in source and absorber. The abscissa gives the angle between the directions of magnetization in source and absorber; the ordinate gives the counting rate behind the absorber in relative units.

From Fig. 12 we see that the absorption depends strongly on the angle between the directions of magnetization of the source and the absorber. For comparison we show in Fig. 12 the absorption found with a vibrating source and the absorption found for an average field in source and absorber approximately equal to zero. The absorption found with transverse fields is somewhat greater than that found with a vibrating source. On the other hand, the absorption in the case of parallel orientation of the fields is considerably larger than in the "absence" of field.

The small asymmetry around 0° (90°) is due to an error in the experiment which was specially checked in the absence of any selected direction of magnetization.

The results of this experiment can be explained simply as follows: The alignment of the ferromagnetic domains in the source produces a corresponding alignment of the internal fields acting on the iron nuclei. Gamma radiation emitted in the transition from one magnetic sublevel to another is polarized with respect to this aligned field. Consequently, the magnetized source serves as a polarizer of the γ radiation. Analogously, the magnetized absorber serves as a polarization analyzer.

The study of the hyperfine structure of the ground and first excited states of Fe^{57} in the case of polarized γ rays is of great interest. For this purpose Hanna et al.[57] used the same experimental arrangement as in reference 34, but the source magnet could be moved relative to the absorber. As a source they used an iron foil on which Co^{57} was deposited. The absorber was an iron foil (76% Fe^{57}).

In Fig. 13a is shown a diagram for the splitting of these levels in metallic iron. The line is split into six components whose intensity ratios are given in Fig. 13c, in the absence of a preferred direction of

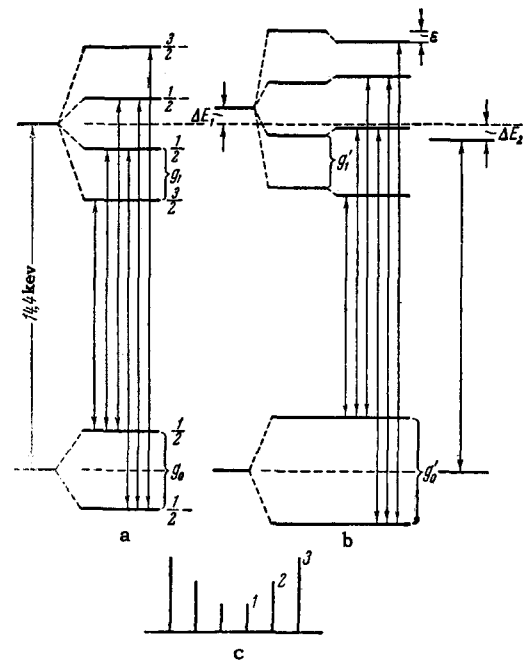

FIG. 13. a) Diagram of splitting of ground and first excited levels of Fe^{57} in ordinary iron. b) The same diagram but for the Fe^{57} nucleus in Fe_2O_3 (on the left) and in stainless steel (on the right). c) The ratio of intensities of components of the 14.4-kev line of Fe^{57} for the case of arbitrary orientation of the emitting nucleus.

magnetization. The separation between the sublevels of the ground state is $g_0 = \frac{\mu H}{1/2}$, where μ is the magnetic moment of the ground state.

The separations between sublevels of the first excited state is $g_1 = \frac{\mu^* H}{3/2}$ where μ^* is the magnetic moment of the excited state, H is the effective field acting on the nucleus. For a fixed source and absorber the resonance absorption will be maximum. If the source moves relative to the absorber, then as a result of the shift of the emission lines relative to the absorption lines the resonance conditions will be violated and the absorption will be reduced, reaching some minimum. With further increase in velocity we obtain a maximum absorption which is, however, smaller in size than the first. In Fig. 14 is shown the theoretical absorption spectrum for Fe^{57} predicted by means of the diagram of Fig. 13a.

Since the 14.4-kev line consists of six components, the spectrum should consist of eight lines, but actually there are two doublets which are denoted by 2 and 3 in Fig. 14.

If the source and absorber are magnetized, the situation changes. For parallel orientation of the magnetizations in source and absorber, the resonance absorption will be obtained only for overlapping of lines with the same direction of polarization. The theoretical spectrum for resonance absorption for this case is shown in Fig. 14, in the middle. When the internal

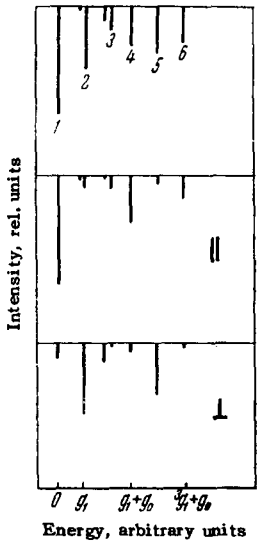

FIG. 14. Resonance absorption spectrum for the 14.4-kev γ ray of Fe^{57} in iron, constructed on the basis of Fig. 13 a and c. The upper figure is given for the case of arbitrary orientation of the emitting and absorbing nuclei. The middle figure corresponds to parallel magnetic fields in source and absorber. The lower figure corresponds to perpendicular directions of the fields in source and absorber. The abscissa gives the energy in arbitrary units, and the ordinate the intensity of the line in relative units.

magnetic fields in source and absorber are perpendicular, the resonance absorption will occur when there is overlapping of components with mutually perpendicular directions of polarization, for which the absorption spectrum is given in the lower part of Fig. 14. The intensities of lines shown in Fig. 14 correspond to the case of a thin absorber; here it is assumed that actually, because of incomplete alignment of fields in the source and absorber, lines which appear only in one orientation of the field will also be present in the other orientation with an intensity of the order of 10%. The resonance absorption spectra obtained in the experiment are shown in Fig. 15 and are in good agreement with those given in Fig. 14.

In order to determine the magnetic field which produces the hyperfine splitting and the magnetic moment of the first excited state of Fe^{57}, one must know the value of the splitting of the ground and first excited states. The value $g_1 = 1.07 \times 10^{-7}$ ev was determined by a careful measurement of the separation between the first and second absorption maxima. For the determination of g_0 they used the fact that the separation between components of the doublet in the third line is equal to $2g_1 - g_0$, and that one component will vanish for parallel orientation of the magnetic fields in source and absorber and the other will vanish for perpendicular orientation. By measuring the shift in the line we find the doublet splitting and consequently also g_0, since g_1 has already been measured: $g_0 = 1.9 \times 10^{-7}$ ev.

Since the magnetic moment of the ground state of Fe^{57} is known[56] [$\mu = (+0.090 \pm 0.0007)$ nuclear magnetons], we can by using the data in the experiment find $\mu^* = -(0.153 \pm 0.04)$ nuclear magnetons, and the effective magnetic field acting at the Fe^{57} nucleus is equal to $(3.33 \pm 0.10) \times 10^5$ oe.

In order to study the hyperfine splitting in more detail, one should introduce Fe^{57} atoms in different compounds. Thus, for example, in reference 68 the absorber was antiferromagnetic Fe_2O_3, and in preparing the source Co^{57} was deposited on a foil of stainless steel (25% Cr, 20% Ni), in which the emission line is not split.[69] In the graph obtained for the absorption of this unsplit line in a steel source, there was no symmetry around the zero velocity position. The location of the absorption lines is given in Table II.

TABLE II

Absorption line	Velocity of source relative to absorber, mm/sec
$^1/_2+ \rightarrow {}^3/_2+$	−7.89
$^1/_2+ \rightarrow {}^1/_2+$	−4.17
$^1/_2+ \rightarrow {}^1/_2-$	−0.76
$^1/_2- \rightarrow {}^1/_2+$	+1.92
$^1/_2- \rightarrow {}^1/_2-$	+5.36
$^1/_2- \rightarrow {}^3/_2-$	+8.59

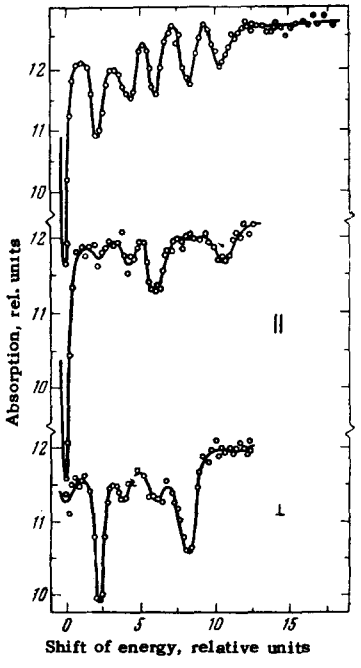

FIG. 15. Resonance absorption spectrum of 14.4-kev γ rays of Fe^{57} in iron. The upper figure is given for arbitrary orientation of the nuclei; the middle and lower figures are given for parallel and perpendicular orientations of the magnetizations.

The location of these lines corresponds to the splitting of the level given in Fig. 13b, where it is necessary to introduce* an energy shift of the emission line (stainless steel source) relative to the absorption line (Fe_2O_3 absorber), $\Delta E = \Delta E_1 + \Delta E_2$.

There is an additional energy shift ϵ due to quadrupole interaction

$$\varepsilon = \frac{eQ}{4I(2I-1)}\left(\frac{\partial^2 V}{\partial z^2}\right)[3m^2 - I(I+1)],$$

where Q is the quadrupole moment of the nucleus, $(\partial^2 V/\partial z^2)$ is the gradient of the electric field, and m is the magnetic quantum number.

For the particular source and absorber pair they found

$$\Delta E = 2.26 \cdot 10^{-8} \text{ ev}, \quad \varepsilon = \frac{1}{4}\left|eQ\left(\frac{\partial^2 V}{\partial z^2}\right)\right| = 5.75 \cdot 10^{-9} \text{ ev}.$$

*Supplementary experiments with a Co^{57} source in ordinary iron showed that ΔE_1 represents an increase and ΔE_2 a decrease.

The magnetic field acting at the Fe^{57} nucleus in Fe_2O_3 is 5.15×10^5 oe.

The shift ΔE which is caused by chemical binding can occur as a result of the following:

1. The "temperature" effect considered in Sec. 4. The γ-ray energy will always be less than the energy separation between the levels, and since the effect depends on the total specific heat, the reduction in energy of the γ quantum will be greater for nuclei in a lattice with a lower characteristic temperature.

2. The energy difference between the nuclear states, which will be reduced when we go over from a system of free atoms to a system of atoms bound in a crystal lattice. This reduction in energy is determined by the energy of the zero point vibrations and will be greater for materials having a higher characteristic temperature.

3. Change in chemical environment, as a result of which there can occur a nuclear isotope shift* which is caused by a change in the wave function of the electron in the region occupied by the nucleus. The direction of this shift is opposite to that resulting from the zero point vibrations, for Fe_2O_3.

By means of the Mössbauer effect one can measure local magnetic fields acting on iron nuclei in various compounds.[69]

By measuring the energy separation between the absorption maxima using different combinations of source and absorber having no magnetic moments in ground and first excited states, one can easily find the effective magnetic field. Thus, for example, at an Fe^{57} nucleus there act the following effective magnetic fields: In cobalt, 3.1×10^5 oe, in nickel, 2.6×10^5 oe, in n-type silicon, 3×10^4 oe.

6. USE OF THE PHENOMENON OF RESONANCE ABSORPTION OF GAMMA RAYS FOR STUDYING SOLIDS

The discovery by Mössbauer of the phenomenon of resonance absorption of γ quanta in crystals makes it possible to study the distribution of frequencies in the vibration spectrum for solids. One of the methods for studying the frequency distribution is to measure to resonance absorption cross section $\bar{\sigma}_r$ as a function of the temperature of the source, for a fixed temperature of the absorber. In Fig. 16 are shown the results of measurements on Ir^{191}. The deviation of the experimental results from the theoretical curve shows that the Debye model is an approximation, and that to obtain a correct theoretical curve one would have to know the detailed frequency distribution in the vibration spectrum of the crystal.

If the source and absorber have the same crystal structure, in certain cases one can, by studying the dependence of $\bar{\sigma}_r(vE_0/c)$ on the velocity of motion of the source relative to the absorber, find the energy distribution of the phonons in the vibrational spectrum of the crystal.[60]

*The nuclear isotope shift is similar to the isomeric isotope shift.[70]

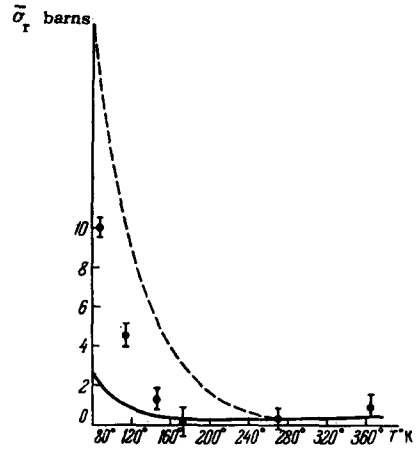

FIG. 16. The dependence of average absorption cross section on source temperature. The solid curve is the theoretical dependence of the average absorption cross section on temperature, in which the frequency distribution is proportional to the square of the frequency. The dashed curve is constructed on the assumption that the frequency distribution is proportional to ν^3. The limiting frequencies are equal in both cases.

If the isotope being studied is an isomer and the crystal in which it is located has the property that one can, in the expression (4.7) for $W'(vE_0/c)$, neglect terms with $n \geq 2$, then for $v \leq c\theta/E_0$ the measurement of the average cross section for resonance absorption in the range $\Gamma < vE_0/c < \theta$ enables us to find $N(\omega)/(vE_0/c)$, where $N(\omega)d\omega$ is the number of phonons with energy $\hbar\omega = vE_0/c$ in the energy interval $\hbar d\omega$. To carry out such an experiment one requires velocities of the order of 10^4 cm/sec and very low temperatures T, since no variation will be found in the energy distribution of the phonons with $\omega \leq kT/\hbar$. Such an experiment is possible only when

$$\frac{W'_{n+1}\left(\frac{v}{c}E_0\right)}{W'_n\left(\frac{v}{c}E_0\right)} \to 0.$$

If we assume that the frequency distribution in the vibrational spectrum of the solid is a Debye distribution, then $W'_2/W'_1 = R/2\theta$ and we must have a crystal with a high Debye temperature and the recoil energy must be small. One must also consider that for low energy γ quanta the non-nuclear absorption is large, which can have a much greater effect than the effect one is looking for.

From experiments on resonance scattering of γ quanta in crystals, one can determine the Debye temperature θ, as one sees, for example, from Figs. 6, 7, and 16. But here one must consider whether the quantity obtained from such experiment can be regarded as a Debye temperature, since one has made the assumption that the radiating nucleus is located at a lattice point in a crystal and that the transition being studied

was not preceded by processes which could have ejected the nucleus from its normal position in the crystal or influenced the atom at the moment of emission of the resonance γ-ray quantum. As an example, we may cite reference 35, where it is pointed out that the Debye temperature obtained from a study of resonance absorption without recoil is approximately 168°K, whereas that measured by other methods is greater than 200°K.

The magnitude of the resonance absorption cross section for γ rays can be studied by using as source and absorber different compounds containing one and the same isotope, and also comparing the size of the absorption obtained with an ordinary source and with a source which has been recrystallized after irradiation. In Table III are shown the maximum values of resonance absorption of γ rays without recoil[36] in percent, where the source and absorber were an iron foil and iron sulphate, $FeSO_4 \cdot 7H_2O$, at room temperature (r.t.) and at liquid nitrogen temperature (l.t.). The metallic absorber was a foil 2.5 mg/cm^2 of Fe^{57}. The metallic source was obtained by co-plating iron and radioactive Co^{57} from a weak solution of sulfuric acid. The error is ±5%.

From Table III we see that the resonance absorption is largest for a metallic source and absorber, but is also observed for a non-metallic source and absorber, especially at low temperatures. From this it is completely clear that the structure of source or absorber has a large influence on the magnitude of the effect because of the difference in the Debye temperature θ.

TABLE III

Source	Absorber		
	Metal (r.t.)	Metal (l.t.)	$FeSO_4 \cdot 7H_2O$ (r.t.)
Metal (r.t.)	52	75	
Metal (l.t.)			
$FeSO_4 \cdot 7H_2O$ (r.t.)	0		4
$FeSO_4 \cdot 7H_2O$ (l.t.)	10		8

It would be of considerable interest to do experiments in which the resonant γ quanta scattered inelastically from some crystal were analyzed by using a resonant absorber, and compensating the energy loss from the inelastic scattering.

If we were to study the angular energy distribution of inelastically scattered resonant γ quanta, then for the analytic expression of the differential cross section we would need to know the details of the vibration spectrum of the scatterer. By using the method of recoilless resonant scattering of γ quanta, one can determine the relation between the frequency, the wave vector k, and the polarization of the phonon s, i.e., $\omega = \omega_s(k)$.

In certain cases one can determine the distribution function of the vibration frequencies $g(\gamma)$ in a crystal lattice by studying the angular and energy distribution of inelastically scattered quanta, just as this is done by using slow neutrons or x rays.[37-40]

In the case of resonant γ rays, despite their high energy, a change in energy of the scattered quanta which can be detected is much less than in the case of slow neutrons. Using resonance γ rays, we have a beam of highly monochromatic γ rays for which one can detect changes in energy in the range from 10^{-1} to 10^{-8} ev and even lower.

Here we must mention that the theory developed for the use of slow neutrons in studying the dynamics of lattices[41-44] is also applicable to the case where we use resonant γ quanta.

7. EXPERIMENTAL TEST OF THE GENERAL THEORY OF RELATIVITY

Gravitational energy does not appear explicitly in the general expression for the energy of a system, but is taken into account by the use of gravitational potentials on which the properties of space and time depend.[45]

If the gravitational potential is denoted by u and we assume that $u \ll c^2$, then in first approximation the expression for the square of the interval is expressed as

$$dS^2 = (c^2 - 2u)\,dt^2 - (dx^2 + dy^2 + dz^2). \qquad (7.1)$$

The magnitude of the coefficient of dt^2 can be easily checked by comparing the frequency of electromagnetic vibrations emitted by two identical sources located at points with gravitational potentials u_1 and u_2. In this case it is easy to show that the difference between the periods of the electromagnetic oscillations is equal to

$$T_2 - T_1 = \frac{u_2 - u_1}{c^2} T_0. \qquad (7.2)$$

To check these relations it has been proposed[46,47] to use artificial earth satellites on which there are placed accurate clocks which make use of atomic or molecular processes. By comparing the times indicated by clocks located on the satellite and on the ground, one can determine the gravitational shift in frequency; but, for carrying out the experiment, one requires a very high precision of the clocks. The test would be considerably simplified if the experiment could be done under laboratory conditions on the earth. A radioactive isotope at a height H over sea level emits a γ quantum with energy $h\nu_H$, while the same isotope located at sea level emits a γ quantum $h\nu_0$.

The change in frequency $\Delta\nu_H$ is easily found from (7.2),

$$\frac{\Delta\nu_H}{\nu_0} = \frac{gH}{c^2} \frac{1}{1 + \frac{H}{r}} = H \cdot 1.09 \cdot 10^{-18}, \qquad (7.3)$$

where g = 981 cm/sec^2, r is the radius of the Earth

($= 6.36 \times 10^8$ cm), H is the height above sea level, and c is the velocity of light.

From (7.3) we see that if the source is located above the detector, the frequency of the emitted γ quanta is shifted toward the violet (a gravitational violet shift occurs). We may say that this shift is the result of "work" done by the photon with mass $h\nu/c^2$ in traversing the potential difference Δu.

The discovery of the effect of recoilless resonant scattering enables one to test experimentally the dependence of the frequency of spectral lines on the gravitational potential at the point where the emitting system is located. If, as a result of mechanical motion, we reduced the resonance absorption to half its maximum value, then there would occur a relative change in frequency of

$$Q = \frac{2\Gamma}{E} = \left[1.1 \cdot 10^{12} \cdot E\,(\text{Mev}) \cdot T_{1/2}\,(\text{nanosec})\right]^{-1}. \qquad (7.4)$$

The height necessary for a gravitational shift of the line by its half width can be found from formulas (7.3) and (7.4):

$$H_{1/2} = \frac{4.18}{E_{(\text{Mev})} \cdot T_{1/2\,(\text{nanosec})}}\ \text{km}. \qquad (7.5)$$

If the experiment is to be done with Fe^{57}, then $H_{1/2} = 2.9$ km. The use of such large differences is difficult because of the finite strength of the source, since the intensity falls off inversely proportional to the square of the distance, while the gravitational shift increases linearly with the distance, so that the statistical fluctuations will smear out the line.

The first work on measurement of the gravitational shift of a γ-ray line[74] was carried out with Fe^{57} with a difference in heights between absorber and source equal to 12.5 m. The observed shift in frequency of the γ quanta was 0.96 ± 0.45 of the expected value. The error is entirely due to statistical fluctuations. It should be mentioned that the authors did not take into account the "temperature" effect and the influence of chemical binding, whereas the difference in the temperatures of source and absorber was $\sim 0.6°$K and leads to this same magnitude of frequency shift. Somewhat more accurate measurements of the gravitational shift in the energy of γ quanta were made by Pound and Rebka.[75] They first carefully investigated the dependence of the energy of γ quanta on temperature and chemical binding. The difference in heights in their experiment was 22.54 m, while the temperature was controlled to an accuracy of 0.03° C. The gravitational shift was found to be $-(5.13 \pm 0.51) \times 10^{-15}$, which is in good agreement with the theoretically expected value -4.29×10^{-15}; this can be expressed as follows:

$$\frac{(\Delta\nu)_{\text{exp}}}{(\Delta\nu)_{\text{theor}}} = +1.05 \pm 0.10.$$

The plus sign means that the energy of a downward-moving quantum increases.

To study the gravitational shift in energy of γ rays, it would be interesting to use the 92-kev γ ray from Zn^{67}, since in this case one requires $H_{1/2} = 4.9$ m, but investigations[76] have shown that the resonance absorption is not shown for Zn^{67} for four different combinations of source and absorber.* The "temperature" effect will have an especially large influence in the case of Zn^{67}, since the relative width of the 92 kev line of Zn^{67} is of the order of 1×10^{-15}. The difference in Debye temperature between source and absorber leads to a relatively large shift in energy of the γ quanta because of differences in the zero-point energy. For a Debye spectrum of the lattice vibrations, the energy of the zero point oscillations is equal to $9\theta/8$ per atom, which for Zn^{67} leads to a partial reduction in frequency of 0.77×10^{-15} per degree increase in Debye temperature. To avoid such a frequency shift would be possible apparently only when the source and absorber are the same material.

By using the phenomenon of resonance absorption of γ rays in crystals one can set up an experiment to test the Einstein principle of equivalence,[78] i.e., the equivalence of gravitational and inertial mass. If the source and absorber rotate on circles of different radii R_1 and R_2, with angular frequency ω, then there should occur a shift in the energy of the γ quanta by an amount

$$\frac{R_1^2 - R_2^2}{2c^2}\,\omega^2.$$

In Fig. 17 is shown a graph of the dependence of the energy of γ quanta on frequency of rotation of the source. The theoretical curve is computed for the case

$$\frac{R_1^2 - R_2^2}{c^2}\,\omega^2 = 2.44 \cdot 10^{-20}\omega^2.$$

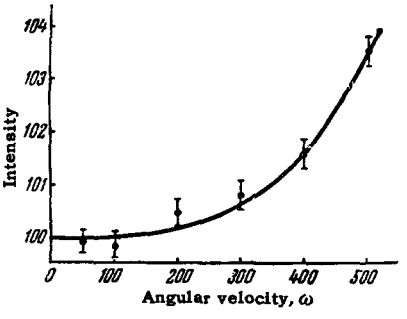

FIG. 17. Dependence of shift of emission line relative to absorption line on the acceleration of the source relative to the absorber.

8. PROSPECTS FOR USE OF THE PHENOMENON OF RESONANCE SCATTERING OF GAMMA RAYS

1. By means of the phenomenon of recoilless resonance scattering, one can study the lifetime of excited states in a region that overlaps the region of applicability of the method of delayed coincidences, and also even shorter times. This is especially important for

*Theoretical computations[77] also indicate that, if the resonance absorption exists at all for Zn^{67}, it is very small.

the study of collective excited states of nuclei, which decay about ten times as fast as single particle states of the same energy. By measuring the lifetime of the level with respect to γ decay, τ_γ, one can determine the suppression coefficients. Thus, for Ir^{191} it was found[12] that for the 129-kev level with an M1 transition there is a suppression by a factor of approximately 22 compared with the theoretical predictions of the single particle model, while the E2 transition occurs at least 64 times faster, as one should expect from the single particle model. Consequently, the excited state at 129 kev in Ir^{191} should be considered as a many-particle state. Knowing the transition probabilities B(E2) obtained by Coulomb excitation, and computing the lifetime τ_γ(E2), we can, in the case of mixed E2 + M1 transitions, compute the ratio of the intensities of M1 and E2 transitions from the experimentally determined τ_γ.

2. Recoilless resonance scattering can be used to check decay schemes, since resonant absorption without recoil is possible only for transitions to the ground state.

3. One of the methods for detecting the neutrino consists in observing the inverse β-decay process,[57] but the cross section for this process, for neutrinos with energies included in a wide interval, is small ($\sigma \sim 10^{-43}$ cm^2). For monochromatic neutrinos with energy precisely equal to the resonance energy, the cross section for the process will be several orders of magnitude greater. If atoms of type A (undergoing electron capture, $A \rightarrow B + \nu$) are in a crystal lattice, then for a neutrino energy $E_\nu \leq 100$ kev the recoil energy may be transferred to the crystal as a whole. But the neutrinos obtained will not be monochromatic, because the electrons are usually captured from the K shell and the width of this level is ~ 1 ev, while the natural width of the neutrino level may be of the order of magnitude 1×10^{-22} ev.*

In those cases where the crystal consists of atoms of type B, while the atoms of type A are an impurity, it is possible to have a capture of outer electrons from impurity levels which have a very small width and, as a consequence the emerging neutrinos will have energies equal to the resonance energy.[53,55] Fermi and Segre derived a formula for the probability that a valence S electron is inside the nucleus. The ratio of the probability of capture of a K electron to the probability of capture of a valence S electron is $z^{-2}n^{-3}$. The most favorable cases will be those where the K capture is energetically forbidden.

For isotopes with $z \sim 80$ and $E_\nu \sim 100$ kev, the cross section for the process may reach 10^{-25} cm^2. The Doppler broadening of the line as a result of the thermal motion will be much greater than the natural width of the line ($\sim 10^{-22}$ ev), even for a fixed source and absorber cooled to liquid air temperature. Therefore to carry out the experiment it is necessary that absorber and source be the same type of crystalline system, i.e., the phonons emitted at the location of the source must reach the absorber without any significant attenuation. All perturbations in such a system can be described by phonon excitation, which is taken into account in the theory of recoilless scattering. Thus, for example, if we choose a crystal of Tb^{159} weighing 50 grams, combine with 0.1 grams of Dy^{159}, and hold them at liquid helium temperature for two weeks, then after separation the previously pure crystal of Tb^{159} will contain Dy^{159}, which can be detected.

Actually, this quantity will be lower because of poorer monochromaticity of the neutrino as a result of the effects of other impurities in the crystal, interactions with conduction electrons and stray magnetic fields. But these disturbances can be estimated, while the change in energy because of the gravitational shift, which can be neglected for $\Gamma = 1 \times 10^{-22}$ ev only at distances of the order of 10^{-9} cm, can be compensated simply by placing the experimental equipment in a weightless state.

4. By using the phenomenon of recoilless resonance scattering of γ rays, one can investigate whether the radiation which is resonantly scattered by nuclei has coherence in phase.[62] If the γ radiation possesses phase coherence, the angular distribution will differ from the distribution as given by the usual theory of angular correlation, and one may observe the phenomenon of interference between the Zeeman sublevels. One also may observe diffraction phenomena just as this is done for the case of x rays. The phenomenon of coherence may be observed only in those cases where the scattering crystal consists almost entirely of resonant nuclei; otherwise the diffraction lines will be smeared out by inelastic scattering from the nuclei arranged in a disordered pattern.

Coherent resonant scattering of γ rays by resonant nuclei can easily be distinguished from coherent scattering by electron shells in the atom, if we compare the patterns obtained with fixed source and with moving source.

5. By means of the Mössbauer effect one can study the Rayleigh scattering of γ quanta in solids, because the detection of the γ quanta by means of a resonant absorption enables one to discard all the elastically scattered quanta. Debye and Waller[79] calculated the reduction in intensity of x rays scattered at the Bragg angle by a crystal at temperature T:

$$\varphi_T = \exp\left\{-\frac{3}{2}\frac{E_R}{k\theta}\left[\frac{1}{4}+\frac{1}{x^2}\int_0^x \frac{y\,dy}{e^y-1}\right]\right\},$$

where the energy transferred by the photons with energy E to a free nucleus when they are scattered

*The detection of neutrinos by using recoilless resonance absorption is only possible when the temperature effect, the effect of differences in zero-point vibrational energies, and other effects compensate one another, and do not lead to a breakdown of the resonance conditions.

through an angle θ is

$$F_R = \frac{E^2}{Mc^2}(1-\cos\theta); \quad x = \frac{\theta}{T}.$$

In reference 80 the authors studied the Rayleigh scattering of the 23.8 kev γ quanta from Sn^{119*} by graphite, paraffin, Pt, and Al. The experimental arrangement is shown in Fig. 18. The computed values of the factor φ_T are in agreement with the results obtained in the experiment.

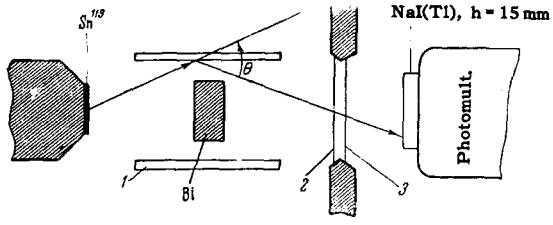

FIG. 18. Experimental arrangement for studying Rayleigh scattering of γ rays by using the Mössbauer effect: 1 – scatterer; 2 – absorber: 40 mg/cm² of Sn^{119} (71.5% Sn^{119}); 3 – Pd absorber, 62 mg/cm² (absorbs the x radiation of Sn), angle $\theta = 50 \pm 5°$.

6. Use of the Mössbauer effect made it possible to investigate for γ quanta a phenomenon which is analogous to the reversal of a spectral line in optics. If the 14.4-kev γ quantum which is emitted without recoil from Fe^{57} passes through an iron absorber, the recoilless resonance absorption will be greater for γ quanta with energies corresponding to the center of the line, and the absorption line will not have the classical resonance shape.[81]

In Fig. 19a is shown the decay curve for the 14.4-kev level of Fe^{57} when there is no resonance absorber. The decay occurs exponentially. In Fig. 19b is shown the decay curve for the 14.4-kev level of Fe^{57} when a resonance absorber is placed in front of the detector. Curve 1 is given for the case of a vibrating source. Curves 2 and 3 are for a fixed source, but with absorbers of varying thicknesses. From Fig. 19a it is clear that the absorption is smaller at the start and then increases. This increase in counting rate is an apparatus effect and can be explained by the fact that the resonance γ quanta absorbed by the nuclei are reradiated and some of them enter the detector, giving rise to an increase in the counting rate during a time of the order of the half-life of the level.[95] But from the increase in counting rate one can estimate the size of the absorption due to the Mössbauer effect. The deviation of the curve for the vibrating source from the exponential shape in the region of the first channels of the time analyzer is explained by the fact that the vibration was not sufficiently large to completely destroy the resonance conditions. For stronger vibrations curve 1 corresponds to an exponential decay.

7. It would be extremely interesting to investigate the question of whether or not there is a strict equivalence of inertial and gravitational mass. It is known that the direction of an acceleration coincides to a very high precision with the direction of the force, but the ratio of force to acceleration does not depend on the direction of the force, i.e., inertia is isotropic, and inertial mass is a scalar.

However, actually there may be a slight asymmetry in the distribution of matter relative to the Earth, and this would lead to an anisotropy of inertia which one could attempt to detect by means of the Mössbauer effect.[71,72]

The magnitude of the local anisotropy can be characterized by means of the following expression:

$$\Delta M = \frac{2}{3}(M' - M''),$$

where M′ is the inertial mass for a body accelerated toward the center of the galaxy and M″ is the inertial mass for an acceleration in the perpendicular direction.

In observing the Zeeman effect such an anisotropy of inertia must result in an additional shift and splitting of the lines. For γ rays, the shift is

$$\Delta E = \left(\frac{\Delta M}{M}\right)\overline{T}\overline{P},$$

where $\overline{T}$ is the mean kinetic energy of a nucleon in

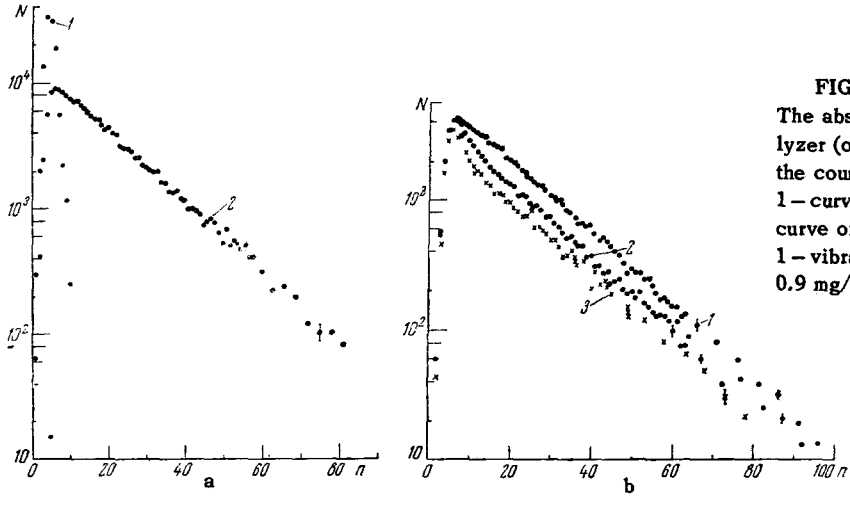

FIG. 19. Decay curve for the first excited state of Fe^{57}. The abscissa gives the channel number N of the time analyzer (one channel equals 8.5 nanosec); the ordinates give the counting rate N in arbitrary units: a) absorber absent; 1 – curve showing the true delayed coincidences, 2 – decay curve of the 14.4-kev level; b) decay curve of filtered γ rays, 1 – vibrating source with resonant absorber, 2 – absorber of 0.9 mg/cm² Fe^{57}, 3 – absorber of 2.7 mg/cm² Fe^{57}.

TABLE IV

Isotope	Percentage content	Spin of ground state	Energy of level, kev	Spin of level	Type of transition	Conversion coefficient	Parent nucleus	Half-life	Required Debye T°	Quadrupole moment	Magnetic moment	Lifetime of the level
$_{26}Fe^{57}$	2.17	$^1/_2-$	14.4	$^3/_2-$	$M1$	0.14	$_{27}Co^{57}$	270 days	11		0.09	$1\cdot10^{-7}$ sec
$_{28}Ni^{61}$	1.25	$^3/_2-$	71	$^5/_2-$	$M1$	large	$_{29}Cu^{61}$	3.3 hours	260		~0	
$_{30}Zn^{67}$	4.11	$^5/_2-$	92	$^3/_2-$	$M1+E2$	0.5	$_{31}Ga^{67}$	78 hours	390	0.18	0.88	$9.3\cdot10^{-6}$ sec
$_{32}Ge^{73}$	7.6	$^9/_2+$	13.5	$^7/_2+$		1300	$_{33}As^{73}$	76 days	8	−0.2	−0.88	$4.6\cdot10^{-6}$ sec
$_{33}As^{75}$	100	$^3/_2-$	67				$_{34}Se^{75}$	127 days	188	+0.3	+1.4	
$_{37}Rb^{85}$	72.1	$^5/_2-$	150	$^3/_2-$	$M1$		$_{38}Sr^{85}$	65 days	820	+0.3	1.34	
$_{44}Ru^{99}$	12.8	$^5/_2+$	89				$_{45}Rh^{99}$	15 days	245	−0.6	<0	
$_{44}Ru^{101}$	17	$^5/_2+$	127	$^3/_2+$	$M1+E2$	0.4	$_{45}Rh^{101}$	4.3 days	500		−0.69	$1.4\cdot10^{-9}$ sec
$_{50}Sn^{117}$	7.5	$^1/_2+$	161	$^3/_2+$	$M1+E2$		$_{50}Sn^{117m}$	14 days	690		−1.0	
$_{50}Sn^{119}$	8.6	$^1/_2+$	24	$^3/_2+$	$M1+E2$		$_{50}Sn^{119m}$	250 days	15		−1.0	$1.8\cdot10^{-8}$ sec
$_{51}Sb^{123}$	42.75	$^7/_2+$	153	$^5/_2+$			$_{50}Sn^{123}$	136 days	590	−0.7	+2.5	
$_{52}Te^{123}$	0.87	$^1/_2+$	159	$^3/_2+$	$M1+E2$	0.16	$_{52}Te^{123m}$	104 days	640		−0.73	$1.9\cdot10^{-10}$ sec
$_{52}Te^{125}$	4.6	$^1/_2+$	35	$^3/_2+$		160	$_{52}Te^{125m}$	58 days	30		−0.88	$1.6\cdot10^{-9}$ sec
$_{55}Cs^{133}$	100	$^7/_2+$	81	$^5/_2+$	$M1$		$_{56}Ba^{133}$	9.5 years	153	−0.003	+2.5	$6\cdot10^{-9}$ sec
$_{57}La^{139}$	99.91	$^7/_2+$	163	$^5/_2+$	$M1$	0.22	$_{58}Ce^{139}$	140 days	590	+0.27	+2.7	$1.5\cdot10^{-9}$ sec
$_{59}Pr^{141}$	100	$^5/_2+$	145	$^7/_2+$	$M1+E2$	0.37	$_{58}Ce^{141}$	33 days	460	−0.05	+4.0	
$_{62}Sm^{152}$	26.63		122				$_{63}Eu^{152}$	9 hours	305	5.7		$1.4\cdot10^{-9}$ sec
$_{63}Eu^{151}$	47.77		22		$M1$		$_{64}Gd^{151}$	150 days	10	+1.2	3.4	
$_{63}Eu^{153}$	52.23	$^5/_2+$	103	$^3/_2+$	$M1+E2$	1.2	$_{62}Sm^{153}$	47 hours	215	+2.5	+1.5	$4\cdot10^{-9}$ sec
			98	$^5/_2-$	$M1$		$_{64}Gd^{153}$	230 days	195			
$_{65}Tb^{159}$	100	$^3/_2+$	58	$^5/_2+$	$M1$		$_{66}Dy^{159}$	134 days	65	+7.3	+1.5	
$_{66}Dy^{160}$	2.3	0+	87	2+	$E2$		$_{65}Tb^{160}$	73 days	145			$1.8\cdot10^{-9}$ sec
$_{66}Dy^{161}$	18.8	$^5/_2+$	26				$_{65}Tb^{161}$	6.8 days	13	+1.1	−0.37	$2.7\cdot10^{-8}$ sec
			75	$^5/_2-$		0.064			110			$2,3\cdot10^{-9}$ sec
$_{67}Ho^{165}$	100	$^7/_2-$	95	$^9/_2-$	$M1$	2.9	$_{68}Er^{165}$	10 hours	170	2.0	+3.3	
$_{68}Er^{166}$	33.41	0+	80.6	2+		2.3	$_{67}Ho^{166}$	27 hours	129			$1.8\cdot10^{-9}$ sec
$_{68}Er^{168}$	27.07	0+	80	2+	$E2$		$_{69}Tm^{168}$	9.6 days	118			$1.8\cdot10^{-9}$ sec
$_{69}Tu^{169}$	100	$^1/_2+$	8	$^3/_2+$	$M1+E2$		$_{70}Yb^{169}$	32 days	2		−0.2	$1.8\cdot10^{-9}$ sec
			42						32			
			118	$^6/_2+$	$E2$	0.66	$_{68}Er^{169}$	9 days	252			
$_{70}Yb^{170}$	3.03	0+	84	2+	$E2$	4	$_{69}Tu^{170}$	129 days	128			$1.6\cdot10^{-9}$ sec
$_{70}Yb^{171}$	14.3	$^1/_2-$	67		$M1+E2$		$_{69}Tm^{171}$	680 days	81			
			76		$E2$				105	8.0*	0.43	$5\cdot10^{-7}$ sec
$_{70}Yb^{172}$	21.82	0+	79	2+	$E2$		$_{71}Lu^{172}$	6.7 days	113			
$_{70}Yb^{173}$	16.13	$^5/_2-$	79	$^7/_2-$			$_{71}Lu^{173}$	1.4 years	114	+3.9	−0.67	
$_{70}Yb^{174}$	31.84	0+	76.5	2+	$E2$		$_{71}Lu^{174}$	165 days	104			
$_{71}Lu^{175}$	97.4	$^7/_2+$	114	$^9/_2+$	$M1+E2$	2	$_{72}Hf^{175}$	70 days	230	+5.9		
$_{72}Hf^{176}$	5.21	0+	88	2+	$E2$		$_{73}Ta^{176}$	8 hours	136			$1.4\cdot10^{-9}$ sec
$_{72}Hf^{177}$	18.5	$^7/_2-$	113	$^9/_2-$	$E2$	2	$_{71}Lu^{177}$	6.75 days	220	+3	+0.6	$4.2\cdot10^{-10}$ sec
$_{72}Hf^{178}$	27.1	0+	93	2+	$E2$	0.5	$_{73}Ta^{178}$	145 days	150	6.4	~0	
$_{72}Hf^{180}$	35.22	0+	93	2+			$_{73}Ta^{180m}$	8.15 hours	149		~0	$1.4\cdot10^{-9}$ sec
$_{73}Ta^{181}$	100	$^7/_2+$	136	$^9/_2+$	$E2+M1$	0.5	$_{74}W^{181}$	145 days	317	+6.0	+2.1	
$_{74}W^{180}$	0.135	0+	102	2+	$E2$	5	$_{73}Ta^{180m}$	8.15 hours	180			
$_{74}W^{182}$	26.4	0+	100	2+	$E2$	4.5	$_{73}Ta^{182}$	115 days	170	6.1		$1.3\cdot10^{-9}$ sec
$_{74}W^{183}$	14.4	$^1/_2-$	46	$^3/_2-$	$M1+E2$	9	$_{73}Ta^{183}$	5 days	36		0.115	
			99	$^5/_2-$	$E2$	3.5	$_{75}Re^{183}$	71 days	167			
$_{74}W^{184}$	30.6	0+	1111	2+			$_{75}Re^{184}$	50 days	204	5.8*		$1.3\cdot10^{-9}$ sec
$_{74}W^{186}$	28.4	0+	123	2+	$E2$		$_{75}Re^{186}$	89 hours	252	5.8*		
$_{75}Re^{185}$	37.07	$^5/_2+$	125	$^7/_2+$	$M1+E2$		$_{76}Os^{185}$	94 days	262	+2.8	+3.14	
$_{75}Re^{187}$	62.93	$^5/_2+$	134	$^7/_2+$	$M1$	3.2	$_{74}W^{187}$	24 hours	298	+2.6	+3.17	$2\cdot10^{-9}$ sec
$_{76}Os^{186}$	1.59	0+	137	2+	$E2$	0.45	$_{75}Re^{186}$	89 hours	314			$5\cdot10^{-10}$ sec
$_{76}Os^{187}$	1.64	$^5/_2+$	135	$^7/_2-$			$_{77}Ir^{187}$	13 hours	300		+0.12	
$_{76}Os^{188}$	13.3	0+	155	2+	$E2$	0.4	$_{77}Ir^{188}$	41 hours	396			$7\cdot10^{-10}$ sec
$_{76}Os^{189}$	16.1	$^3/_2-$	135	$^5/_2-$			$_{77}Ir^{189}$	11 days	297	+0.6	+0.65	
$_{76}Os^{190}$	26.4	0+	187	2+	$E2$		$_{77}Ir^{190}$	11 days	570			$3.5\cdot10^{-10}$ sec
$_{76}Os^{192}$	41.0	0+	206	2+	$E2$	0.2	$_{77}Ir^{192}$	74 days	680			$2.8\cdot10^{-10}$ sec
$_{77}Ir^{191}$	38.5	$^3/_2+$	129	$^5/_2+$		2.1	$_{76}Os^{191}$	16 days	270	+1.5	+0.16	$1.4\cdot10^{-10}$ sec
			82.6	$^1/_2+$			$_{78}Pt^{191}$	3 days	110			
$_{77}Ir^{193}$	61.5	$^3/_2+$	73		$M1+E2$		$_{78}Pt^{193m}$	3 days	87	+1.0	+0.17	$5.7\cdot10^{-9}$ sec
			139	$^5/_2+$			$_{76}Os^{193}$	30 hours	310			
$_{78}Pt^{195}$	33.8	$^1/_2-$	99	$^3/_2-$	$M1+E2$		$_{79}Au^{195}$	185 days	157		+0.6	
			130	$^5/_2-$			$_{77}Ir^{195}$	2.3 hours	270			
$_{79}Au^{197}$	100	$^3/_2+$	77	$^1/_2+$	$M1+E2$		$_{78}Pt^{197}$	18 hours	93	+0.56	+0.136	$1.9\cdot10^{-9}$ sec
$_{80}Hg^{199}$	16.84	$^1/_2-$	158	$^5/_2-$	$E2$	2.3	$_{79}Au^{199}$	3.1 days	388		+0.49	$2.4\cdot10^{-9}$ sec
$_{80}Hg^{201}$	13.22	$^3/_2-$	32	$^3/_2-$			$_{81}Tl^{201}$	72 hours	16	+0.45	+0.6	
			167	$^1/_2-$					430			

the nucleus, $\bar{P}$ is a coefficient whose maximum value for the 14.4-kev level of Fe^{57} is $^1/_5$.

A careful investigation of the absorption spectrum of Fe^{57} by Sherwin et al.[73] during the period from March 2 to March 12, 1960, with varying orientation of source and absorber with respect to the direction toward the center of the galaxy, gave no evidence for any such anisotropy. From their work it follows that $\Delta M/M < 5 \times 10^{-16}$.

9. LIMITS OF APPLICABILITY OF THE METHOD OF RESONANCE SCATTERING OF GAMMA RAYS

The isotopes used for recoilless resonance scattering must satisfy the following conditions:

1. The recoil energy from the γ transition must be less than some definite value.

2. The lifetime of the state must be in a definite range.

The recoil energy R (3.1) of the γ ray must be of the order of θ or lower, since the density of phonons in the vibration spectrum for energies $E \leq \theta$ is relatively small and there is a reasonable probability for recoilless scattering.

With increasing R, a larger and larger number of phonons and their combinations can take up the recoil energy, and the intensity of the recoilless line decreases. Analytically this is given by equation (4.4). From this equation it is clear that with increasing recoil energy the probability of emission without recoil decreases exponentially. We may assume that the effect will be noticeable only for $R \leq \theta$ (in this case $f \geq 0.05$ for $T = 0°K$). From this we easily determine the Debye temperature of source and absorber which is necessary for obtaining a significant effect:

$$\theta \geqslant \frac{R}{2} = \frac{310E^2}{A}, \qquad (9.1)$$

where E is the γ ray energy (in tens of kev), A is the mass number of the nucleus under investigation.

In Table IV* are shown the isotopes for which one can observe the phenomenon of recoilless resonance scattering and which satisfy the requirement

$$E \leqslant \frac{A\theta}{310}. \qquad (9.2)$$

The lifetime of the excited state also determines the possibility for using a particular isotope for experiments on recoilless resonance scattering. The longest lifetime — $T_{1/2} = 10^{-7}$ sec — of all the isotopes investigated by this method is that for the 14.4-kev level of Fe^{57}. One could also work with narrower lines, but then one needs a very rigid mounting since the slightest vibrations destroy the resonance conditions.

Magnetic and electric fields, thermal motion, gravitational shifts, and other disturbances will smear out the lines and destroy the resonance condition. One possible method for observing recoilless resonance scattering is to have the absorber and source be a single crystalline system.[53,55]

There are limits to the applicability of the method of recoilless resonance scattering also from the side of short-lived transitions. This limit is determined by two factors:

1. In the case of very fast transitions there must be influences from residual effects of the radioactive decay processes which precede the process of interest.

2. The relations derived in Secs. 2 and 3 are valid for lines whose natural width can be neglected. Therefore if there are isotopes for which one can observe resonance scattering without recoil and which have a half-life less than 10^{-13} sec, the formulas in Secs. 2 and 3 must be altered.

*The table has been constructed mainly from the material in references 58 and 59. The Debye temperatures were computed according to equation (9.1) using the equality sign.

[1] R. E. Bell, in: Beta- and Gamma-Ray Spectroscopy, ed. Siegbahn, Interscience, 1955.
[2] W. Kuhn, Phil. Mag. 8, 625 (1929).
[3] L. Meitner and H. H. Hupfeld, Z. Physik 67, 147 (1931).
[4] K. Zuber, Helv. Phys. Acta 16, 407, 429 (1943).
[5] E. Pollard and D. E. Alburger, Phys. Rev. 74, 926 (1948).
[6] F. R. Metzger, Phys. Rev. 83, 842 (1951).
[7] P. B. Moon, Proc. Phys. Soc. (London) 64, 76 (1951).
[8] B. S. Dzhelepov, Usp. Fiz. Nauk 62, 3 (1957).
[9] F. R. Metzger, Progress in Nuclear Physics 7, 54 (1959).
[10] K. G. Malmfors, Arkiv fys. 6, 49 (1953).
[11] Goldhaber, Grodzins, and Sunyar, Phys. Rev. 109, 1015 (1958).
[12] R. L. Mössbauer, Z. Physik 151, 124 (1958).
[13] R. L. Mössbauer, Naturwiss. 45, 538 (1958).
[14] W. E. Lamb, Phys. Rev. 55, 190 (1939).
[15] A. Akhiezer and I. Pomeranchuk, Некоторые вопросы теории ядра (Some Problems of Nuclear Theory) Moscow, Gostekhizdat, 1950.
[16] F. Seitz, Modern Theory of Solids, McGraw-Hill, 1940.
[17] Rasmussen, Metzger, and Swann, Phys. Rev. 110, 154 (1958).
[18] J. D. Jackson, Can. J. Phys. 33, 575 (1955).
[19] P. P. Craig, et al, Phys. Rev. Letters 3, 221 (1959).
[20] R. V. Pound and G. A. Rebka, Phys. Rev. Letters 3, 554 (1959).
[21] G. De Pasquali et al., Phys. Rev. Letters 4, 71 (1960).
[22] R. L. Mössbauer, Z. Naturforsch. 14a, 211 (1959).
[23] P. P. Craig et al., Bull. Amer. Phys. Soc., ser. II, 4, 373 (1959).
[24] L. L. Lee, et al., Phys. Rev. Letters 3, 223 (1959).
[25] A. W. Sunyar, Phys. Rev. 98, 653 (1955).
[26] H. Frauenfelder, in: Beta- and Gamma-Ray Spectroscopy, ed. Siegbahn, Interscience, 1955.
[27] Arp, Edmonds, and Petersen, Phys. Rev. Letters 3, 212 (1959).
[28] C. G. Gorter, Physica 14, 504 (1948).
[29] M. E. Rose, Phys. Rev. 75, 213 (1949).
[30] B. Bleaney and K. W. H. Stevens, Reports on Progress in Physics 16, 108 (1953).
[31] W. Marshall, Phys. Rev. 110, 1280 (1958).
[32] J. P. Schiffer and W. Marshall, Phys. Rev. Letters 3, 556 (1959).
[33] A. C. Gossard and A. M. Portis, Phys. Rev. Letters 3, 164 (1959).
[34] G. J. Perlow et al., Phys. Rev. Letters 4, 74 (1960).
[35] A. I. Alikhanov and V. A. Lyubimov, Report to the Tenth Annual Conference on Nuclear Spectroscopy, Moscow, 1960.
[36] S. S. Hanna et al., Phys. Rev. Letters 4, 28 (1960).
[37] P. Olmer, Acta Cryst. 1, 57 (1948).

[38] H. Cole and B. E. Warren, J. Appl. Phys. **23**, 335 (1952).
[39] H. Cole, J. Appl. Phys. **24**, 482 (1953).
[40] C. B. Walker, Phys. Rev. **103**, 547 (1956).
[41] L. S. Kothari and K. Singwi, Solid State Physics **8**, 109 (1959).
[42] G. Placzek and L. Van Hove, Phys. Rev. **93**, 1207 (1954).
[43] L. Van Hove, Phys. Rev. **89**, 1189 (1953).
[44] J. C. Phillips, Phys. Rev. **104**, 1263 (1956).
[45] V. A. Fock, Теория пространства, времени и тяготения (The Theory of Space, Time, and Gravitation) Moscow, Gostekhizdat, 1955; translation, Pergamon Press, 1959.
[46] V. L. Ginzburg, Doklady Akad. Nauk S.S.S.R. **97**, 617 (1954).
[47] V. L. Ginzburg, JETP **30**, 213 (1956), Soviet Phys. JETP **3**, 136 (1956).
[48] R. V. Pound and G. A. Rebka, Phys. Rev. Letters **3**, 439 (1959).
[49] A. Abragam and M. H. L. Pryce, Proc. Roy. Soc. **A205**, 135 (1951).
[50] Solid State Physics, v. 9, ed. F. Seitz and D. Turnbull, New York, 1959.
[51] C. H. Townes, Phys. Rev. **71**, 909 (1947).
[52] R. V. Pound, Phys. Rev. **76**, 1410 (1949).
[53] W. M. Visscher, Phys. Rev. Letters **3**, 536 (1959).
[54] H. J. Lipkin, Annals of Physics **9**, 332 (1960).
[55] W. M. Visscher, Phys. Rev. **116**, 1581 (1959).
[56] G. W. Ludwig and H. H. Woodbury, Phys. Rev. **117**, 1286 (1960).
[57] F. Reines and C. L. Cowan, Phys. Rev. **113**, 273 (1959).
[58] B. S. Dzhelepov and L. K. Peker, Схемы распада радиоактивных ядер (Decay Schemes of Radioactive Nuclei), Moscow, Press of the Academy of Sciences, U.S.S.R., 1958.
[59] Strominger, Hollander, and Seaborg, Revs. Modern Phys. **30**, 630 (1958).
[60] W. M. Visscher, Annals of Physics **9**, 194 (1960).
[61] R. Barlouthaud et al., Compt. rend. **250**, 319 (1960).
[62] A. M. Kastler, Compt. rend. **250**, 509 (1960).
[63] D. Nagle et al., Phys. Rev. Letters **4**, 237 (1960).
[64] R. V. Pound and G. A. Rebka, Phys. Rev. Letters **4**, 274 (1960).
[65] B. D. Josephson, Phys. Rev. Letters **4**, 341 (1960).
[66] Barit, Podgoretskiĭ, and Shapiro, JETP **38**, 301 (1960), Soviet Phys. JETP **11**, 218 (1960).
[67] S. S. Hanna et al., Phys. Rev. Letters **4**, 177 (1960).
[68] O. C. Kistner and A. W. Sunyar, Phys. Rev. Letters **4**, 412 (1960).
[69] G. K. Wertheim, Phys. Rev. Letters **4**, 403 (1960).
[70] A. C. Melissinos and S. P. Davis, Phys. Rev. **115**, 130 (1959).
[71] G. Cocconi and E. E. Salpeter, Nuovo cimento **10**, 646 (1958).
[72] G. Cocconi and E. E. Salpeter, Phys. Rev. Letters **4**, 176 (1960).
[73] C. W. Sherwin et al., Phys. Rev. Letters **4**, 399 (1960).
[74] Cranshaw, Schiffer, and Whitehead, Phys. Rev. Letters **4**, 163 (1960).
[75] R. V. Pound and G. A. Rebka, Phys. Rev. Letters **4**, 337 (1960).
[76] R. V. Pound and G. A. Rebka, Phys. Rev. Letters **4**, 397 (1960).
[77] M. V. Kazarnovskiĭ, JETP **38**, 1652 (1960), Soviet Phys. JETP **11**, 1191 (1960).
[78] H. I. Hay et al., Phys. Rev. Letters **4**, 165 (1960).
[79] I. Waller, Ann. Phys. **79**, 261 (1926).
[80] C. Tzara and R. Barlouthaud, Phys. Letters **4**, 405 (1960).
[81] R. E. Holland et al., Phys. Rev. Letters **4**, 181 (1960).
[82] V. A. Lyubimov and A. I. Alikhanov, JETP **38**, 1912 (1960), Soviet Phys. JETP **11**, 1375 (1960).
[83] N. N. Delyagin et al., JETP **39**, 220 (1960), Soviet Phys. JETP **12**, 159 (1961).
[84] S. V. Maleev, JETP **39**, 891 (1960), Soviet Phys. JETP **12**, 617 (1961).
[85] N. N. Delyagin et al., JETP **39**, 894 (1960), Soviet Phys. JETP **12**, 619 (1961).
[86] Cordey-Hayes, Dyson, and Moon, Proc. Phys. Soc. **75**, 810 (1960).
[87] A. J. Boyle et al., Proc. Phys. Soc. **76**, 465 (1960).
[88] Barlouthaud, Picou, and Tzara, Compt. rend. **250**, 2705 (1960).
[89] S. S. Hanna et al., Phys. Rev. Letters **4**, 513 (1960).
[90] P. P. Craig et al., Phys. Rev. Letters **4**, 561 (1960).
[91] S. L. Ruby and D. I. Bolef, Phys. Rev. Letters **5**, 5 (1960).
[92] H. S. Snyder and G. C. Wick, Phys. Rev. Letters **5**, 118 (1960).
[93] J. G. Dash et al., Phys. Rev. Letters **5**, 152 (1960).
[94] S. Ofler et al., Phys. Rev. Letters **5**, 177 (1960).
[95] L. A. Khalfin, JETP, in press.

Translated by M. Hamermesh
57