PROPAGATION AND GENERATION OF LOW-FREQUENCY ELECTROMAGNETIC WAVES IN THE UPPER ATMOSPHERE

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Usp. Fiz. Nauk 72, 235-271 (October, 1960)

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INTRODUCTION

THERE has been an increased interest in recent years in natural low-frequency electromagnetic radiation and its propagation through the upper atmosphere. In this connection, attempts were also made to investigate the propagation of low-frequency electromagnetic signals broadcast by radio stations. The number of papers (for the most part experimental, strictly speaking observational in character) devoted to this group of problems is continuously increasing.

A considerable part of the natural radiation is due to atmospheric electric discharges, received in the form of whistling sound (whistling atmospherics). Another part is connected with the interaction between streams of charged particles from the sun and the ionized gas of the earth's upper atmosphere. Observation of this radiation was part of the program of the International Geophysical Year (IGY) and was carried out at numerous special stations. We refer here to frequencies 0.4 — 15 kcs (wavelengths 750 — 20 km), which are within the range of the human ear (16 — 20,000 cps) and are directly audible with a loudspeaker. This is a very favorable factor in the investigation of this radiation, for the primary observation can be simply carried out by ear.

Low-frequency electromagnetic radiation is also of considerable interest because it can pass through the ionosphere and reach interplanetary space. If there were no external magnetic field, the ionosphere layers would block all the low frequencies and the penetration of such radio waves through the upper layer of the atmosphere would be impossible. This can be seen from the expression for the index of refraction $\tilde{n}$ of ionized gas

$$\tilde{n}^2 = 1 - \omega_0^2/\omega^2,$$

where ω_0 is the plasma frequency (see below) and ω the circular frequency of the wave. It follows therefore that when $\omega < \omega_0$ we have $\tilde{n}^2 < 0$ and propagation of waves is impossible without attenuation. The presence of the earth's magnetic field H_0 makes the ionosphere transparent also to sufficiently low frequencies, the values of which will be given later (Sec. 2), so that

$$\tilde{n}^2 > 0.$$

The propagation of low-frequency waves usually has a unique character. In the upper atmosphere the radio signals propagate along definite trajectories, connected with the earth's magnetic field. The electromagnetic radiation may be trapped in a region located between magnetically-conjugate points on the earth's surface. The position here is reminiscent somewhat of the capture of corpuscular radiation by the earth's magnetic field (the earth's outer and inner radiation belts).

We can thus speak of the channeling of the low-frequency electromagnetic radiation. We note that analogous phenomena are frequently encountered in dealing with the propagation of radio and acoustic waves.[1,2]

Such propagation of the low-frequency waves is connected with the properties of ionized gas (plasma). In the case considered here the trajectory is a horseshoe curve, terminating at points of the earth's surface which are symmetrical about the magnetic equator. A low-frequency signal transmitted from the earth goes a considerable distance away from the surface and returns to the conjugate point; it may even bounce several times between these points. The low-frequency radiation generated in the upper atmosphere by various processes reaches the earth through channels of this kind.

An investigation of the low-frequency radiation may be quite useful to the study of the upper layers of the atmosphere. Data on the propagation of whistling atmospherics have already yielded much valuable information on the concentration of electrons at high altitudes above the earth and other useful data.

The present survey consists of three parts. In Section 1 we consider data obtained by observation of natural low-frequency emissions. In Sec. 2 we give the theory of propagation and generation of these emissions, and in Sec. 3 we consider low-frequency radio emission as one of the possible sources of information on the upper atmosphere.

1. OBSERVATIONAL DATA ON THE NATURAL LOW-FREQUENCY RADIO EMISSION

Low-frequency electromagnetic emission (we are essentially interested in the interval from 0.4 to 15 kcs, sometimes up to 32 kcs) produces audible sound in a loudspeaker. As already noted, this simplifies the primary observation of natural low-frequency signals. The receiving apparatus consists essentially of an amplifier fed through a vertical antenna (the vertical position ensures isotropic reception), the output of which is fed to a loudspeaker.[3,4] The necessary frequency interval is separated by suitable filters. The sound can be recorded on magnetic tape and simultaneously heard by the operator. The time of arrival of the whistler is recorded either by the operator or by time markers fed to the magnetic tape.

The tape is eventually replayed and the characteristic low-frequency signals are selected aurally. These can be copied on other tapes and subjected to frequency analysis. Usually the frequency of the low-frequency emission varies with time, but not too rapidly. It is therefore quite readily possible to determine the frequency corresponding to a given instant of time — the instantaneous frequency. The instantaneous frequency is determined from the maximum response of a suitable sharply-tuned resonator. In the interval of interest to us, up to 50 resonators are used. Storey[3] used 12 selective amplifiers with parallel inputs in his investigations. These amplifiers were provided with a tape recorder. In addition, suitably located neon bulbs connected to the resonators were used for monitoring purposes. The flashing bulbs were photographed on film. If the low-frequency signal had a gliding tone, it excited each of the amplifiers in turn and the dependence of the observed frequency f on the time t was directly obtained on the film.

A specially constructed analyzer, called sonograph, was used by an American group.[4]

If a definite dependence $f = f(t)$ is stipulated from theoretical considerations, the correctness of this dependence can be verified in the following manner. The bulbs are arranged in a rectangular grid, in which the vertical distances correspond to the proposed time dependence $f(t)$, and the horizontal spacing is uniform. If the proposed law is correct, the flashing bulbs will lie on a straight line.

Direct observations of the low-frequency signals thus make it possible to establish the time of their appearance and their duration, while subsequent spectral analysis gives their frequency composition; the frequency vs. time of arrival $f(t)$ curve can also be obtained.

Systematic observation of low-frequency signals has led to the conviction that although these signals are quite variegated, they can be distinctly separated into two groups, which we shall call whistlers[3,4] and ultra-low-frequency radiation.[5] Both groups appear in the same frequency region, but are of entirely different origin.

Whistlers have been reliably established to be due to atmospheric electric discharges. A broad frequency range is simultaneously generated in these discharges. If the discharge takes place near the receiver, the spectrogram displays first the arrival of all the frequencies (sferic), which the ear perceives as a crackle. This is sometimes followed by a signal which is spread out in frequency and in time. The simultaneous arrival of different frequencies is caused by the fact that the pulse passes over a sufficiently long path in a dispersive medium from the point of generation to the receiver. In the literature these noises are called whistlers or whistling atmospherics. We shall use here the shorter term, whistlers.

As regards the very low frequency radiation (VLF) emission, this noise was noted already in the early investigations of Storey,[3] who called attention to the fact that low-frequency signals clearly not due to discharges are observed. In these signals, the dependence of the frequency on the time of arrival $f(t)$ is not the same as in whistlers; they also differ in sound from the whistlers. By the sound analogy, Storey called them "dawn chorus." Following Storey, this term was applied to all signals which differed from whistlers in their properties. However, the "non-whistlers" were found to comprise an extensive category, consequently the term "dawn chorus" lost its definite connotation and it became necessary to discard this term in general. Following Gallet,[5] we shall call this noise VLF emission. The latter is not connected with the atmospheric discharges and is closely correlated with magnetic disturbances.

A third group of noise should also be mentioned. This noise is produced by interaction between whistlers and VLF emission. Examples of this group have been observed rarely and little is known about it.[5]

Later in this section we shall give details on the observation of whistlers and VLF emission. In the account of the observational results, the theory will be used only to the extent necessary for qualitative interpretation of the phenomena. A detailed theory of the propagation of low-frequency signals in the ionosphere and other theoretical problems are developed especially in Sec. 2 of this survey.

a) Whistlers

The first whistlers were apparently observed by Barkhausen in 1919. By 1933 it was firmly established that these signals were due to atmospheric discharges. This conclusion was based on the fact that certain whistlers were preceded by small sferics on the spectogram (heard in the form of crackles).* A systematic investigation of whistlers was first carried out by Storey.[3] Many features were disclosed by later workers.[4]

Dispersion. Figure 1 shows schematically typical spectrograms of whistlers. To the left of Fig. 1a is shown the solid vertical band corresponding to the arrival of a sferic (all frequencies arrive simultaneously). It is clear from the figure that after approximately one second a signal appears, with a frequency f that diminishes gradually with time. We shall discuss the subsequent signals later on. The spectrogram of Fig. 1a is characteristic of the so-called long whistlers, the appearance of which is preceded, as a rule, by a sferic. It is natural to assume that both the whistler and the sferic have a common origin. At the same time, some whistlers are not preceded by sferics. In spite of this, the same generation mechanism is assumed for such whistlers; the correctness of this assumption was finally confirmed in reference 3. It follows from this mechanism that all the frequencies are generated simultaneously at the point of discharge, and the relative delay of the different frequencies is due to peculiarities in the propagation of the low frequencies.

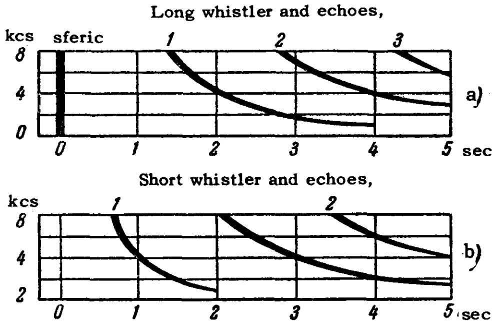

FIG. 1. Typical spectrograms of whistlers (schematic). The vertical axis represents the observed frequency and the horizontal axis the time of observation (of arrival): a) Vertical bar – sferic. This is followed by three consecutive whistlers of decreasing strength. This is a picture typical of a "long" whistler followed by an echo (see below); b) three successive whistlers without a preceding sferic. This is a typical picture of a "short" whistler with subsequent echo. As a rule, single whistlers are observed (with or without a sferic). The echo is relatively rarely observed.

*More detailed information on the history of the investigation of low-frequency radio emission can be found in references 3 and 4.

Through a study of a large number of spectrograms,[3] obtained in the frequency interval from 1 to 8 kcs, the following dependence of the frequency received at a given instant on the time t was established:

$$t = Df^{-1/2}, \tag{1.1}$$

where D is a coefficient called the dispersion. For a long whistler (see Fig. 1a), the time t is reckoned, from the instant of occurrence of the sferic. If the whistler is not preceded by a sferic, the choice of the time reference becomes more complicated. It can be established uniquely only in certain cases when an echo is observed (see below).

The plot of $f^{-1/2}$ vs. t for a whistler is a straight line, and the slope of this line relative to the f axis gives the value of the dispersion D (Fig. 2). The validity of relation (1.1) within a definite frequency interval is one of the characteristic features of whistlers. As will be shown in detail in Sec. 2, the dependence (1.1) may be fully due to the dispersion properties of the ionized gas in the frequency range from 1 to 8 kcs.* Thus, the observed law (1.1) is further evidence of the simultaneous generation of all the frequencies in the interval under consideration. Relation (1.1) distinguishes the whistles from other types of low-frequency emission.

Long and short whistlers. In simultaneous observation of signals of which some are accompanied by sferics and others have no sferics, Storey[3] established that they have different dispersions (Fig. 3). The dispersion of the signals of the first kind (long whistler) was as a rule twice the dispersion of the signals of the second type (short whistlers). This difference was explained as follows. There are theoretical (see Sec. 2, Storey's theorem) and experimental (see below) grounds for assuming that the motion of the low-fre-

*In some cases the dependence (1.1) may break down even when $f < 8$ kcs.

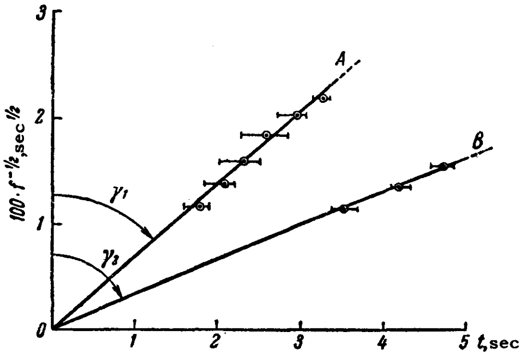

FIG. 2. Analysis of a long whistler and echo. The instant of arrival of the sferic is designated zero: A – first whistler, B – second whistler (echo), the dispersion of which is twice the dispersion of the first whistler. The dispersion is determined by the tangent of the angle relative to the vertical axis, i.e., by the angle γ.

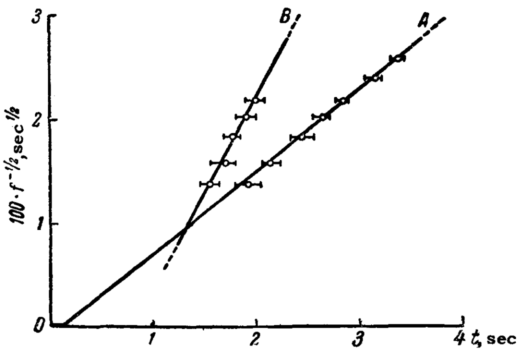

FIG. 3. Case when a short (B) and a long (A) whistler is observed. The difference in the dispersion of these two whistlers is obvious.

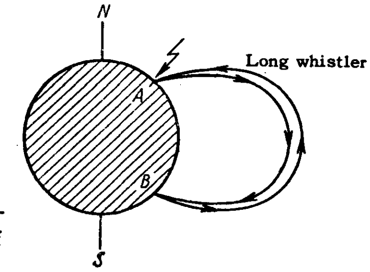

FIG. 4. Trajectories of a long whistler. The signal produced by a discharge at point A produces a sferic in the receiver (direct signal) and then makes the round trip from A to B and back. The dispersion of the first (indirect) signal is determined by twice the length of the arc AB.

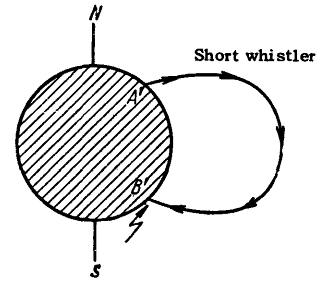

FIG. 5. Trajectory of a short whistler. The discharge occurs at the point B′ and travels over a certain trajectory to the point A′. The sferic is not received as a rule. The dispersion of the observed signal is determined by the length of the arc A′B′, covered by the signal.

quency electromagnetic signals follows certain trajectories connected with the force lines of the earth's magnetic field. If we accept this statement, we can explain naturally the behavior of the long and short whistlers. Both whistlers are produced by electric discharges in the atmosphere, which occur for all practical purposes on the earth's surface (the height of the discharge is negligibly small compared with the distance traversed by the whistlers). Let us assume that the discharge occurs in the northern hemisphere near the point of reception; then the receiver first detects the sferic, i.e., the direct signal from the discharge, followed somewhat later by a signal that has traveled along the trajectory to the point on the southern hemisphere symmetrical about the magnetic equator (the magnetically-conjugate point), and then returned to the receiver along the same trajectory. Such a signal makes a round trip over a horseshoe-like trajectory (Fig. 4).

On the other hand, the received signal may have been produced by an electric discharge in the southern hemisphere. This signal will travel only one way along the same trajectory, and its dispersion will be half the dispersion of the long whistler (Fig. 5).

If the initial pulse is sufficiently strong and the propagation conditions are sufficiently favorable, the signal may be reflected several times, and the receiver records not only the primary signal, but several subsequent signals (echo train). All the echoes pass through equal time intervals with decreasing amplitude. The dispersion of each subsequent echo increases.

The presence of echoes makes it possible to differentiate reliably between long and short whistlers. Naturally, if the whistler follows the sferic after several seconds then, except for random coincidences, we can assume that the whistler is long. But the sferic is not always registered, even if it is produced in the same hemisphere as the receiver. Storey believed that a sferic is registered by the receiver only when the discharge is not more than 2,000 km away from the receiver.[3] This conclusion, however, is not always correct (we shall give below more recent information).

In any case, in the presence of an echo, a long whistler is clearly distinguishable from a short one. For a long whistler the ratio of the dispersion of the initial signal to the dispersions of the subsequent echoes is 1 : 2 : 3 : 4 . . . (both the signal and the echo make the round trip). For a short whistler this ratio is 1 : 3 : 5 : 7 . . . (the signal goes one way, the echoes make the round trips). Figures 2 and 6 show the dispersions of the main signal and of the echo for short and long whistlers.

Thus, short and long whistlers differ in the magnitude of the dispersion. However, the only reliable way of distinguishing between the two is a comparison of the dispersions of the subsequent echoes.

Multiple whistlers. In addition to the whistler-echo combination, whistlers are observed also in different well-defined associations. Among these are whistlers corresponding to multiple discharges (multiple flashes). These whistlers are characterized by the

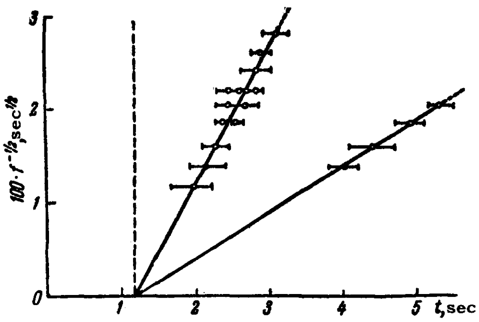

FIG. 6. Spectrograms of a short whistler and its echo. The dispersion of the echo is three times as large as the dispersion of the main signal.

fact that they follow each other at definite time intervals, and their dispersions and amplitudes[3,4] are more or less equal (Fig. 7). Very interesting are whistler pairs (Fig. 8). They differ from an echo in that the amplitude of the second whistler is not only not smaller, but is sometimes even greater than the amplitude of the first. Although the dispersion of the second whistler is indeed greater than the dispersion of the first, it is not twice as large. The dispersion lines of both whistlers converge to a single point, indicating that both components are produced in the same discharge. Apparently the appearance of whistler pairs is connected with the presence of two paths for the signal propagation.

Nose whistlers. The usual form of whistler spectrograms is as shown in Fig. 1. In 1956 Helliwell et al.[6] observed an unusual spectrum. Whereas the ordinary whistler spectrum is a decreasing function of the time,

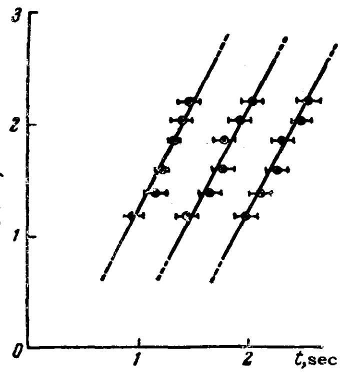

FIG. 7. Spectrogram of whistler in the form of a multiple discharge. The dispersion is the same in all whistlers.

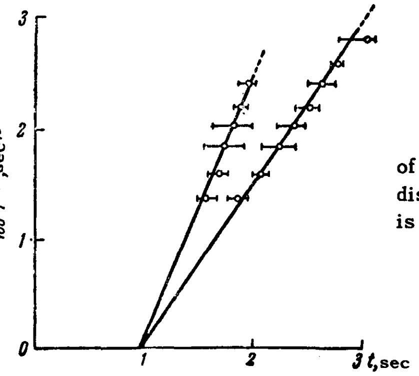

FIG. 8. Spectrogram of a whistler pair. The dispersion in the pair is not in an integer ratio.

in this case both a rising and descending branch of the spectrum were observed, converging to a single point at the so-called "nose frequency" (Fig. 9). Nose whistlers are frequently observed in the form of multiple whistlers, with decreasing nose frequencies of the subsequent signals. Nose whistlers are apparently the most common form of a whistler, the ordinary whistler being the descending branch of the nose-whistler spectrum.

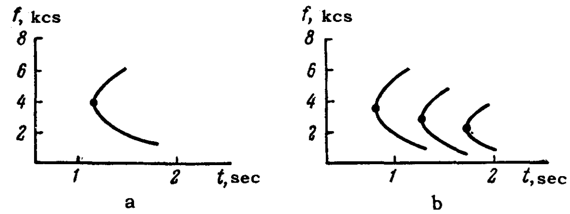

FIG. 9. Schematic representation of a spectrogram in the form of a nose: a) Single nose, b) multiple noses. The nose frequency is denoted by a bullet.

Discharges that generate whistlers. Storey sought a connection between the discharges in the vicinity of the receiver and long whistlers. He concluded that if the discharge is not more than 2,000 km from the receiver, a sferic is observed, and if the discharge is sufficiently strong, the sferic is sometimes followed by a whistler. The loudness of the whistler decreases with increasing distance between discharge and receiver. The spherics preceding the whistlers were always of the usual type, characteristic of an atmospheric discharge. At the same time, Storey indicated that sufficiently strong sferics were noted without being followed by a whistler. However, whenever whistlers were actively generated, loud sferics were accompanied by whistlers.

Helliwell et al.[7] investigated especially the features of discharges that preceded the whistlers. He found that sferics that gave rise to whistlers made up only a small part of the total observed sferics. They were characterized by a frequency spectrum with a maximum located near 5 kcs. Sferics of this kind occur more frequently over the sea than over dry land.

Most discharges form sferics with frequency spectra that have maxima near 10 kcs. Such sferics do not produce whistlers. The visible thunderstorm discharges accompanied by lightening are also frequently unaccompanied by whistlers. Why it is that precisely the discharges with maximum near 5 kcs give rise to whistlers has not yet been convincingly explained.

Experimental data on the trajectories of whistlers. The existence of long and short whistlers and particularly their echo is clear indication that the propagation of low-frequency signals is over certain horseshoe-like paths terminating on the surface of the earth. As will be shown in detail in Sec. 2 (from the theoretical point of view), low-frequency signals must propagate over definite trajectories.

Further evidence of the propagation of whistlers along trajectories that are closely related to the force lines of the earth's magnetic field is provided by the following experimental facts. Firstly, whistlers have never been observed on the equator.[8] Secondly, simultaneous reception of whistlers from one and the same source was registered in magnetically-conjugate reception points.[9]

At low geomagnetic latitudes, the trajectories of the whistlers are characterized in the equatorial region by relatively small distances from the earth's surface, so that the effect of the ionosphere becomes noticeable. This is confirmed by the establishment of a connection[10,11] between the dispersion D of whistlers and the critical frequencies of the F_2 layers. The observations reported in references 10 and 11 were carried at 24° and 32° northern magnetic latitude.*

Naturally, the question was raised of the transmission of man-made low-frequency signals from one conjugate point of the trajectory to the other. Such an experiment was actually carried out with a pulse transmitter, operating at 15.5 kcs in Annapolis.[12] Direct signals from this transmitter were heard at the conjugate point at Cape Horn. The echo of the man-made signal, with a time delay of 0.7 sec, was almost always heard at night. Simultaneous recording of the whistlers yielded precisely this time delay for the corresponding frequency. In some cases, when the echo was observed, the whistlers were not heard at all. This clearly indicates that the absence of signals is frequently due not to disturbances in the propagation conditions, but simply to the fact that the whistlers are not generated. Analogous experiments with a transmission frequency of 17.44 kcs were carried out between Tokyo and Tasmania.[13]

At the same time, the entire picture of propagation of low frequency pulses is much more complicated than motion along a single definite trajectory. Firstly, a whistler generated by one and the same source is observed at many stations which are sufficiently far from each other; secondly, one observes at a given station whistlers which are apparently generated by one and the same source, but which arrive to the receiver over different paths. Once the material obtained during the IGY is processed, a large number of cases will probably be disclosed, in which whistlers produced by the same source were simultaneously observed in many stations. However, even now there are isolated data to confirm this statement.

For example, a whistler from the same source was observed simultaneously in Hanover (long. 0°, north. lat. 55°) and Washington (long. 350°, north. lat. 50°).[4] Both observations yielded the same dispersion. Whistlers were observed simultaneously in Seattle (long. 295°, north. lat. 55°), Boulder (long. 315°, north. lat. 48°), and Stanford (long. 300°, north. lat. 45°). In the latter case, two circumstances are noteworthy. First, the whistlers appeared differently at different stations. A pure-tone whistler was observed in Stanford, i.e., a well pronounced single frequency was observed on the spectrogram at each instant of time, and the whistler itself had a definite musical tone; in Boulder and Seattle, the same whistler produced on the spectrogram a relatively broad band of frequencies, and no definite tone could be ascribed to this whistle by ear. Whistlers of this kind, which produce a sound with a decreasing average frequency, are called in the literature "swishy." Judging from the available data, the separation of whistlers into "pure-toned" and "swishy" has no relation to the nature of the whistler, and is wholly connected with the propagation conditions. Secondly, the dispersion of one and the same whistler in the three indicated stations was different. For the 4-kcs frequency, it was found to be 44 — 110, 80 — 120, and 92 — 120 $\sec^{1/2}$ in Stanford, Boulder, and Seattle respectively. It is seen therefore that the dispersion has a tendency to increase with increasing geomagnetic latitude. This is evidence in favor of stating that the trajectories of the signals, corresponding to the three points, are essentially different.

One of the interesting cases of observation, which can be explained by assuming two different propagation paths, is described in reference 14.

There are also other data indicating that the trajectories of the whistlers are not unique. These data follow from observation of nose whistlers. In Sec. 2 of this survey we shall show that the nose frequency, under certain limitations, is determined by the minimum gyrofrequency of the electrons along the path of propagation of the pulse. However, multiple nose frequencies show a decrease in the minimum frequency with increasing time of arrival. This indicates that the signals arriving at a given point move over different paths.[4]

Helliwell and Morgan[4] give an example in which the same whistler was recorded at ten stations, spaced a considerable distance apart. Eight of these stations were located in the northern hemisphere and two in the southern one. The dispersion of the main components in the northern hemisphere was half of that in the southern hemisphere. Consequently, this whistler was long in the southern hemisphere and short in the northern hemisphere. In contrast with earlier statements,[3] it was noted that the sferic generating the whistler, and also some other sferics, were identified simultaneously in both hemispheres. It is exceedingly remarkable that the whistler was observed in one hemisphere at stations spaced approximately 7,000 km apart. All this is evidence that, generally speaking, the pulse generating the whistler need not be at all located in the direct vicinity of the receiver or anywhere near the conjugate point of the trajectory.

Systematic variations of properties of whistlers. The changes in the frequency of appearance of whistlers depend on two circumstances — the frequency of

*We use geomagnetic coordinates throughout.

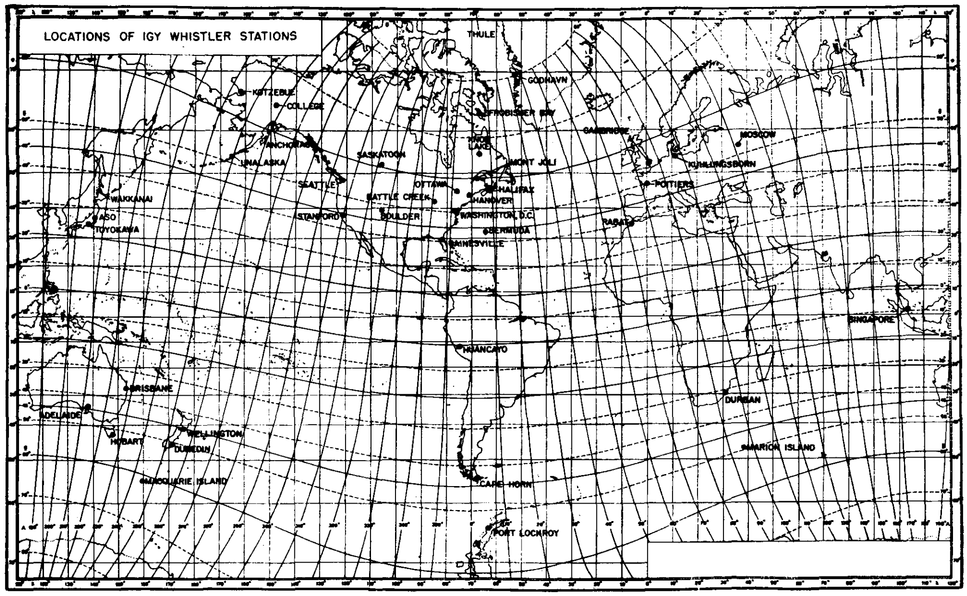

FIG. 10. Locations of stations which conducted observations on whistlers under the IGY program. The curves represent geomagnetic coordinates. [This figure was taken from Proc. Inst. Radio Engrs, 47, 205 (1959).]

occurrence of atmospheric discharges, and change in conditions on the path of propagation of the electromagnetic signals through the atmosphere. However, data on the frequency of occurrence of thunderstorms on earth are sufficiently scanty and this makes the interpretation of the data difficult. At the same time, some variations can be readily explained. Thus, for example, the average frequency of appearance of whistlers is greater during the night than during the day (local time). This is customarily associated with the daily variations of absorption in the D layer. The increase in the number of short whistlers and the decrease in the number of long whistlers indicates a change in the number of discharges in the conjugate hemisphere (the summer is characterized by a larger number of thunderstorms).[4]

As was already indicated, the changes in dispersion show a correlation with the variations of the critical frequency in the F_2 layer.

The connection between the properties of whistlers and geomagnetic phenomena is of great interest, but has been investigated little. In any case, there is no clear cut connection between the magnetic activity and whistlers. We shall see that this property also differentiates between whistlers and VLF emission.

Observation of whistlers in the IGY program.[4,15] Although whistlers can serve as a mean of investigating the upper layers of the atmosphere, the main purpose of the IGY program on whistlers was to study the properties of the whistlers themselves. For a systematic observation of whistlers and VLF emission, a network of special stations was organized; one of the stations was located below Moscow (see map, Fig. 10).

The territorial location of the stations was guided by the following principles:

a) Locating the stations at different geomagnetic latitudes from the equator to the pole, in order to disclose the latitude-dependent properties of whistlers.

b) Organization of pairs of stations, located at points symmetrical about the magnetic equator. The relations between whistlers observed on both ends of a single trajectory were investigated at these stations.

c) Placement of stations along the geomagnetic parallels. These stations made it possible to observe the longitude-dependent properties of whistlers and at the same time segregate the phenomena dependent on the local time from phenomena which are common for all stations independent of local time.

During the IGY, whistlers and VLF radiation were recorded on magnetic tape for 35 — 37 minutes of each hour, simultaneously with local-time signals. Owing to the large volume of the material obtained during the IGY, its processing and interpretation have not yet been completed and will apparently require a few more years.

b) Very Low Frequency Emissions

As already indicated in a), both whistlers and VLF emissions are observed in approximately the same frequency band. The VLF emission is distinguished from whistlers by its spectrograms. It has been possible to distinguish reliably VLF emissions from whistlers only after the start of 1957, when a large number of high grade spectrograms were obtained.[5] An analysis of these spectrograms has shown that the character of the spectrum is in many cases not at all connected with the features of the propagation of electromagnetic waves. We shall give below proof that the character of the propagation of VLF emissions is precisely the same as that of whistlers. At the same time, the generation of the VLF emission is of considerably greater interest than is the propagation of the signals. The generation of VLF emissions is closely related to the interaction with corpuscular streams from the sun, which penetrate the outer part of the atmosphere — the exosphere.* Therefore observation of the VLF emissions may yield information on these streams, which play an important role in many geophysical phenomena. In this connection, we shall emphasize in this description those properties of VLF radiation which are of significance to the understanding of the mechanism of production of this radiation. Before proceeding to describe the individual types of VLF emissions, we list the systematic classification proposed by Gallet[5] for the observed VLF noise, including whistlers. The further exposition of the data of observation of VLF emissions is also based on reference 5. The classification proposed there is based on the distinguishing features of the spectrograms and is as follows:

Systematic Classification of Observed VLF Noise

I. Whistlers
Produced by atmospheric discharges; radiation of all frequencies is simultaneous; shape of the spectrum is due to the dispersion of the pulse along the path.

II. VLF Emissions
Certainly produced by non-atmospheric discharges; very closely related with magnetic perturbations; subdivided into two principal parts:

1. Continuous radiation (a broad frequency spectrum is observed simultaneously); the radiation is continuous in both frequency and time. It corresponds to steady-state conditions and is observed as a hiss.

2. Discrete radiation (a narrow frequency spectrum is observed at each given instant of time); the radiation itself lasts relatively little and is characterized by a tendencey to repetition. It corresponds to a transient situation. A rather large number of classes is known (Fig. 11).

III. Interaction between whistlers and VLF emissions
This interaction includes both the continuous and the discrete VLF emissions. This group is encountered much more rarely than I and II.

It follows from this classification of VLF emissions that there exist stable and unstable states causing the generation of this radiation. The unstable states give rise to the discrete radiation, while the stable ones to hiss. Different classes of discrete radiation are shown in Fig. 11.

Certain features of discrete radiation. Numerous spectrograms indicate that a well defined frequency corresponds to each instant of time. The received average frequency varies with time in a definite manner.

*The exosphere is the region of the upper atmosphere higher than 800 or 1,000 km above the earth.

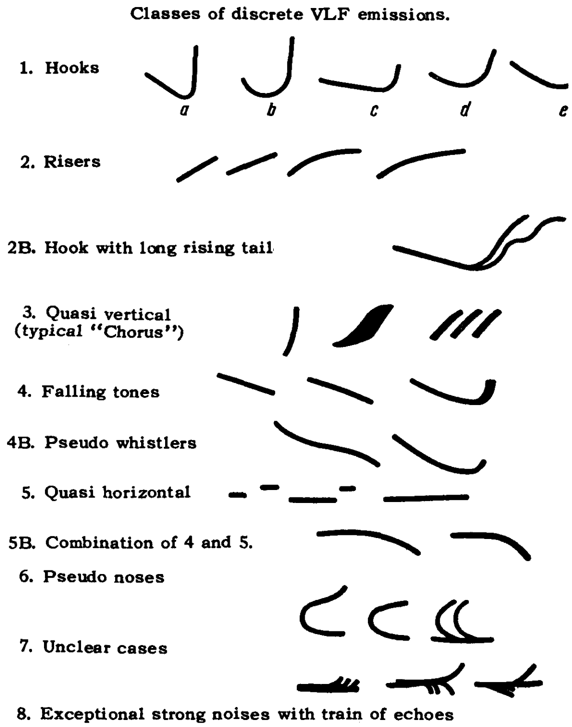

FIG. 11. Different classes of discrete VLF emissions.

The spectra repeat in time with a high degree of accuracy. The time intervals between the appearances of a spectrum of a given type can range from seconds to months. For example, during the time of magnetic storms the "hooks" are frequently repeated every few minutes or even seconds. More frequently, however, repetition of a class is observed rather than an identical repetition of an individual spectrum.

In any case, all the types of radiation noted, including the rarest, are observed at numerous times within a cycle of approximately two years. The reproducibility of the different types of radiation indicates that the radiation conditions are far from random, but correspond to certain definite states of the upper atmosphere, which repeats from time to time.

Method of propagation. The observations indicate with great degree of reliability that the VLF radiation does not propagate like the whistlers. The most convincing proof is obtained from observations of echoes that follow strong VLF radiation. In one of the cases it was possible to observe in addition to the initial signal also eleven succeeding echoes. In spite of the fact that a very small frequency interval was contained in the signal, it was possible to observe quite clearly and to measure the dispersion of the succeeding echoes. This dispersion was in full agreement with the dispersion of the whistlers observed at the same time in the same frequency interval.

On the other hand, additional features worthy of attention were observed in the same case. Firstly, the frequency interval is broadened in the echo. This indicates the presence of a certain nonlinear mechanism which transfers energy from one frequency to an adjacent one. Secondly, in some cases a clear amplification of the echo is observed, occurring somewhere along its propagation; the observed echo is found to be stronger than the signal that generates it.

Tendency to form horizontal or gliding tones. Frequently the signals (classes 5 and 5B in our classification) are of exceedingly pure tone, occupying a very narrow frequency band on the spectrogram. The usual duration of such a signal is 0.5 — 1 sec, but longer signals are also observed. For example, Gallet[5] gives a spectrogram of a signal lasting 2.5 seconds, the average frequency of which is near 12 kcs. Frequently slowly descending or even practically horizontal tones are separated on the spectrogram against the continuous noise band (hissing). These tones appear gradually inside the signal, indicating that such a signal consists actually of discrete lines. The fine structure of the hiss is described in detail by Watts.[16]

It must be emphasized that the VLF radiation frequently contains constant frequencies, emitted during a prolonged time interval. In the case of hissing, these constant frequencies appear as part of the overall radiation.

Connection with magnetic activity. The frequency of appearance of VLF radiation at a given place is clearly correlated with the magnetic activity. Almost every significant magnetic storm is accompanied by various kinds of VLF radiation. It was noted that the hissing appears most frequently during the abatement of a magnetic storm, although sometimes this noise is generated also during one or two hours of the maximum period.

There are at present many investigations carried out at different points, which may in time establish clearer and simpler laws.[17-19] We shall mention some of the latest data. Observations at the Geophysical Observatory of Kurina (Sweden),[17] located in the aurora region, have led to the conclusion that a correlation exists with the local micropulsations of the magnetic field H_0. During the time of observation, magnetic disturbances were fixed three times, October 12 — November 11, 1958, December 12, 1958 — January 11, 1959, and April 15 — May 14, 1959. A continuous VLF radiation, connected with the micropulsations of the magnetic field, was observed 11 times during these storms. This radiation was concentrated in two bands, 0.5 — 1.4 and 1.8 — 4.5 kcs. The average maximum frequency of the first band was relatively stable at 750 ± 150 cps; in the second band the maximum frequency was located in the interval between 2.5 and 3.4 kcs. The first band was narrower than the second. In all 11 cases, the magnetic recordings disclosed micropulsations with a period 0.5 — 6 minutes and a maximum amplitude 50γ. In four cases of the most prolonged VLF radiation, the first band was observed during the interval between two phases of a magnetic storm, during which micropulsations were noted. The authors of reference 17 indicate that both radiation bands were probably parts of one and the same signal.

A second group of observers[18] noted on November 27, 1959 a rare case, when a whistling sound, starting with a frequency of about 2 kcs, gradually became higher in pitch until it turned near 10 kcs into a hiss lasting for several minutes. Simultaneously, micropulsations of the magnetic field were observed, with periods of 10 — 20 seconds and with an amplitude that diminished gradually from 2.5 to 0.4γ within five minutes. Some time after the observation of the described hiss, a red aurora appeared on the sky and lasted for about a half hour.

The observed coincidence of the occurrence of VLF radiation with the micropulsations of the magnetic field is due to their having a common source, the streams of charged particles. Recently the question of the causes of magnetic pulsations have been investigated in detail in many works (see, for example, references 20 and 21).

Comparison of steady states (hissing) with transients (discrete radiation). The discrete VLF radiation corresponds to unstable phenomena, which are sharply outlined in frequency and in time. In contrast, the hissing appears as a continuous rather broad band of noise of considerable duration. The statistics of the visual observations of hissing spectra discloses that the average duration in 122 observed cases is 1.6 hours. The distribution of the duration decreases more slowly than the exponential function. Consequently cases are possible when the hissing lasts continuously for ten hours. The most common mean frequency is about 3.5 kcs. The width of the noise band is usually 2 — 3 kcs, but at the same time a continuous noise band in the interval 8 — 32 kcs has been noted.

Also noted were complicated cases of hissing, when two frequency bands appeared simultaneously. Individual singularities can be separated within the continuous band. It is obvious that the generation of the hissing takes place in stable states of radiating systems, and entire bands of frequencies are radiated simultaneously.

2. THEORY OF PROPAGATION OF LOW-FREQUENCY RADIO WAVES. GENERATION OF VLF EMISSIONS

In the preceding section we gave the basic experimental data obtained by observation of whistlers and VLF radiation. We shall develop here the theoretical foundations necessary for the interpretation of the observed laws. Since it has been firmly established that the whistler radiation is generated during atmospheric discharges, the only problem of actual interest is that of the passage of low frequency radio signals through the upper atmosphere of the earth. In the case of VLF radiation, problems in the propagation and absorption of signals are solved in exactly the same way as for whistlers; the central problem now becomes the determination of the mechanism by which various types of this radio noise come about.

We shall first consider problems connected with the propagation of waves, bearing in mind principally their application to whistlers. In the second part of this section we shall indicate possible mechanisms of generation of VLF radiation.

The interpretation of most experimental data, obtained by observation of whistling atmospherics, is based on the use of the deduction of the theory of propagation of electromagnetic waves in a plasma located in an external magnetic field (this field will henceforth be the earth's magnetic field $\mathbf{H}_0$). It is best to derive first several formulas that follow from this theory; these formulas are the starting point for all the exposition that follows.[1,22,23]

Of great importance to whistlers that contain a more or less broad set of frequencies is the determination of the group velocity. Connected with the solution of this problem is the important problem of determining the trajectories of the low-frequency signals. After analyzing the contents of several papers in which this problem is solved or in which allied questions are dealt with, we shall consider problems in absorption and interaction of normal waves.

Principal initial relations. We consider the propagation of normal waves in a homogeneous, magnetoactive, and unbounded plasma. In this case the electric field is in the form of a plane wave $\mathbf{E} = \mathbf{E}_0 \exp(i\omega t - i\mathbf{k}\cdot\mathbf{r})$, where $\mathbf{k}$ is the wave vector and $\mathbf{r}$ is the radius vector. We then obtain[1,22,23] for the square of the complex index of refraction, $\tilde{n}^2 = c^2k^2/\omega^2$,

$$\tilde{n}_{1,2}^2 = (n_{1,2} - iq_{1,2})^2 = 1 - \frac{2v(1-v-is)}{2(1-is)(1-v-is) - u\sin^2\alpha \mp \sqrt{u^2\sin^4\alpha + 4(1-v-is)^2 u\cos^2\alpha}}, \tag{2.1}$$

where n and q are the indices of refraction and absorption, α is the angle between the directions of propagation of the plane wave (defined by the wave vector $\mathbf{k}$) and the field $\mathbf{H}_0$,* and $s = \nu/\omega$ (ν is the frequency of collisions between electrons and other particles). In (2.1) the parameters v and u are defined as

$$v = \frac{\omega_0^2}{\omega^2}, \qquad u = \frac{\omega_H^2}{\omega^2}, \tag{2.2}$$

where $\omega_0 = \sqrt{4\pi e^2 N/m}$ is the plasma frequency at $\mathbf{H}_0 = 0$, $\omega_H = eH_0/mc$ is the gyrofrequency for electrons, e is the absolute value of the electron charge, m is the electron mass, and N is the electron concentration.

The upper sign in front of the square root in the denominator (2.1) corresponds to the propagation of extraordinary waves, called type "1" waves, while the lower one corresponds to ordinary waves (type "2" waves).

In considering the validity of relation (2.1), two circumstances must be kept in mind. Firstly, formula (2.1) is clearly unsuitable for very low frequencies $\omega \lesssim \Omega_H$ ($\Omega_H = eH_0/Mc$ is the gyrofrequency of the ions and M is their mass), for in this case allowance must be made for the motion of the ions.[1,22,23]

Only the motion of the electrons was taken into consideration in the derivation of (2.1). However, an investigation of the behavior of the waves near $\omega \sim \Omega_H$ is of certain interest as applied to whistlers, something to which we shall return later. Another circumstance which generally speaking limits the applicability of (2.1) is the inadequate account of the thermal motion of the electrons. The presence of this motion manifests itself in (2.1) only in the fact that $\nu \neq 0$. The use of the transport equation method[22,23] leads, however, to the conclusion that in certain regions thermal corrections can appreciably influence the character of propagation of waves even when $\nu = 0$. In particular, along with the extraordinary and ordinary waves, it becomes possible for a third type of normal waves, namely plasma waves, to propagate. The appearance of these waves, and also other effects that arise in the propagation of waves when thermal motion is taken into account, should apparently not play an important role when it comes to whistlers. We must add, however, that allowance for the thermal motion of the electrons will lead us to the conclusion that there exists in the plasma a specific absorption, not connected with collisions. Estimates of the value of this absorption will be given below.[24]

In the investigation of the propagation of low-frequency waves in a plasma, in the band corresponding to the whistler spectrum ($f = \omega/2\pi = 0.5$ to 20 kcs), considerable simplifications can be made in (2.1). It must be recognized first that in practice the following condition is satisfied along the entire path of propagation of the atmospherics:

$$v \gg 1 \qquad (\omega_0^2 \gg \omega^2). \tag{2.3}$$

Actually, $\omega_0^2 = 3.18 \times 10^9$ N. Even at relatively low electron concentrations, $N \simeq 10^2$, which is the order of magnitude characteristic of the D layer or interplanetary space, we have $\omega_0 = 5.65 \times 10^5$ sec^{-1}, whereas at f = 15 kcs the angular frequency is 9.4×10^4 sec^{-1}. Thus, even under conditions when the values of the parameter v (2.2) are close to minimal, we have an estimate $v \simeq 36$. The inequality (2.3) is much better satisfied at greater values of the electron concentration N, or at slower frequencies ω.

*To be specific, we shall assume everywhere in the discussion of the relations obtained below (unless otherwise stipulated) that $\cos\alpha > 0$, i.e., the vector $\mathbf{k}$ makes an acute angle with the magnetic field $\mathbf{H}_0$.

To determine the second parameter u in (2.2) it is necessary to find the value of the gyrofrequency with allowance for the variation of the magnetic field both in altitude and in latitude. In the dipole approximation of the geomagnetic field, the following relation holds:[25,26]

$$\omega_H = 2\pi f_1 (r_0/r)^3 (1+3\sin^2\vartheta)^{1/2}, \tag{2.4}$$

where r_0 is the earth's radius (6,370 km), r the distance from the center of the earth to the point under consideration, ϑ the geomagnetic latitude of this point, and $f_1 \simeq 0.8$ Mc/sec (Fig. 12). The latitude variations

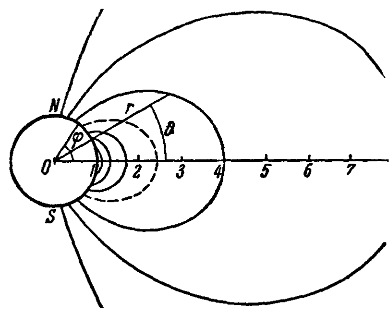

FIG. 12. Coordinates in which it is customary to write the equation for the force lines of the earth's magnetic field, assuming the earth to be a dipole: r and δ – polar coordinates relative to the earth's center (r_0 and φ are the same coordinates at the point where the force line makes contact with the earth's surface). The three-dimensional picture has an axis of rotation which coincides with the direction of the dipole.

of the magnetic field of the earth (the change in ϑ) lead to differences in the values of the gyrofrequency ω_H (at fixed r) by a factor of two at most. Depending on r, the gyrofrequency decreases as $1/r^3$. According to (2.4) we have $\omega_H = 5.02$ Mc/sec on the equator ($\vartheta = 0$) at the earth's surface ($r = r_0$). For the same latitude, the different values of ω_H at different altitudes above the earth ($h = r - r_0$) are listed in the table.

Estimates of the values of ω_H show that the condition $\omega_0 > \omega_H$ is always satisfied, except in the D layer. At the same time, the inequality $\omega_0 \gg \omega_H$ is most frequently satisfied. This is true, for example, in the F layer of the ionosphere or at very high altitudes, when $r \gtrsim 2r_0$. We can thus consider the following inequality

$$v \gtrsim u \tag{2.5}$$

satisfied (in many cases, as already indicated, $v \gg u$). If we use the recently obtained data on the electron concentration in the upper atmosphere (see, for example, reference 27), the approximate equality in (2.5) takes place tentatively at distances of 2500 – 3000 km above the earth's surface, where the electron concentration is appreciably decreased and approaches the interplanetary value ($N \simeq 200 - 600$ el/cm^3), but the magnetic field is still not greatly reduced. It can be shown that if

$$v^2 \gg \frac{u\sin^4\alpha}{4\cos^2\alpha}, \tag{2.6}$$

corresponding to the quasi-longitudinal approximation,[1,23,28] we obtain from (2.1) a simpler formula for the type "2" wave.

$$\tilde{n}_2^2 = 1 + \frac{v}{\sqrt{u}\cos\alpha - 1 - \frac{u}{2v}\sin^2\alpha + is}. \tag{2.7}$$

Except for very small values of cos α, the correctness of (2.6) is assured if (2.3) and (2.5) are satisfied. It must be noted that if the stronger inequality $v \gg u$ is fulfilled, then the condition (2.6) cannot be violated even for small values of cos α. We shall show later that a limitation (2.9) does exist. If, furthermore, $v \gg u$ then (2.6) reduces to the inequality (2.3) and the quasi-longitudinal approximation is always valid under these conditions. Usually in theoretical research on whistling atmospherics[25,26,29] the term $(u/2v)\sin^2\alpha$ in the denominator of (2.7) is disregarded. It will be omitted here, too, but the term must be borne in mind in the investigation of the propagation of the spectral components with sufficiently high frequency, when $\sqrt{u}\cos\alpha \simeq 1$.

Neglecting $(u/2v)\sin^2\alpha$ and taking the inequality (2.6) into account, from which it follows that the absolute magnitude of the second term in (2.7) is much greater than unity, we obtain the following expression for the index of refraction:

$$\tilde{n}_2^2 = \frac{v}{\sqrt{u}\cos\alpha - 1 + is}, \tag{2.8}$$

which has served as the basis for many investigations on the theory of propagation of whistling atmospherics. Neglecting collisions, we obtain $\tilde{n}_2^2 = v/(\sqrt{u}\cos\alpha - 1)$. It follows from this relation that the propagation of low-frequency ordinary waves is possible only if

$$\sqrt{u}\cos\alpha \equiv \frac{\omega_H\cos\alpha}{\omega} > 1, \tag{2.9}$$

when $\tilde{n}_2^2 > 0$. Connected with this condition is the establishment of the upper frequency limit in the whistler spectra. It follows from (2.9) that signals with frequencies ω exceeding ω'_H (the minimum value of the gyrofrequency along the path of propagation of the atmospheric) cannot penetrate through the plasma in the upper layers of the atmosphere. A more accurate determination of the limiting frequency calls for establishment of a minimum value of ω_H cos α over the entire trajectory of propagation of the whistling at-

h, km	1000	3000	6000	10 000	20 000	30 000	40 000
ω_H, sec^{-1}	$3{,}01\cdot10^6$	$1.59\cdot10^6$	$5{,}42\cdot10^5$	$2.96\cdot10^5$	$7{,}07\cdot10^4$	$2.7\cdot10^4$	$1.29\cdot10^4$

mospheric. If the condition (2.9) is violated, $\tilde{n}_2^2 < 0$ and it becomes impossible for waves of type "2" to penetrate into the ionized medium. As regards type "1" waves, at low frequencies we always have $\tilde{n}_1^2 < 0$; the propagation of these waves becomes impossible.

In connection with the foregoing, we note that many papers[3,29,30] contain the erroneous statement that the propagation of whistlers in the region (2.3) is due to type "1" waves. Many arguments can be cited, however, in favor of relating formula (2.8) to type "2" waves, as we do here. For example, it is well known from the theory of propagation of electromagnetic waves in a magnetoactive plasma that if (2.9) holds, the function $\tilde{n}^2(v)$ has a pole at the point $v = v_\infty$ only for the type "2" wave [when $v = v_\infty$ we have $n^2(v_\infty) \to \infty$].[1,22,23] Actually, it follows from (2.8) with $s = 0$ that $\tilde{n}_2^2(v_\infty) \to \infty$ if $\sqrt{u}\cos\alpha = 1$. To be sure, the last equation does not contain the parameter v, but this is due only to the approximate character of (2.8). If the term $(u/2v)\sin^2\alpha$ in the denominator of (2.7) is taken into account, we obtain $v_\infty = u\sin^2\alpha/2(\sqrt{u}\cos\alpha - 1)$. Since the difference $\sqrt{u}\cos\alpha - 1$ is small compared with unity in the region where $\tilde{n}^2$ is very large, we obtain $v_\infty = (u-1)/(u\cos^2\alpha - 1)$. The last is a well known equation[1,22,23] for the position of the pole of $\tilde{n}^2$. It is thus incorrect to relate Eq. (2.8) with the propagation of the extraordinary waves. The erroneous statement in references 3, 29, and 30, which is equivalent to an irrational reclassification of normal waves, is apparently connected with the fact that when $\alpha = 0$ the approximate formula $\tilde{n}^2 = v/(\sqrt{u} - 1)$ actually describes the propagation of extraordinary waves. However, the case $\alpha = 0$ is exceptional, and when $\alpha \neq 0$ (even for very small α) Eq. (2.8) corresponds to type "2" waves. It should be noted that in the case of the problems solved in references 3, 29, and 30, the error in the designation essentially did not affect the correctness of the results obtained.

Subject to the limitation

$$\sqrt{u}\cos\alpha \gg 1, \tag{2.10}$$

which is satisfied only at not too high frequencies, we obtain from (2.8) with $s = 0$ the well known formula

$$n^2 = \frac{v}{\sqrt{u}\cos\alpha} \equiv \frac{\omega_0^2}{\omega\omega_H\cos\alpha}, \tag{2.11}$$

from which Storey[3] developed the initial version of the theory of propagation of whistling atmospherics.

It was indicated above that when $\sqrt{u}\cos\alpha = 1$ (more accurately, at the values $v = v_\infty$ given above) we have $\tilde{n}_2^2 \to \infty$ if $s = 0$. It might have been expected that in this case the behavior of the type "2" wave would be determined by collisions, by virtue of resonance. It must be kept in mind here, however, that an appreciable effect can be produced in a rarefied plasma not only by collisions, but also by thermal motion of the electrons. If this motion is suitably taken into account, then $\tilde{n}_2^2$ will be finite[22,23] for $v = v_\infty$ even when $s = 0$. A gradual transition takes place here from a type "2" wave to a plasma wave. The behavior of normal waves in the region $v \simeq v_\infty$, as applied to the propagation of low-frequency signals in the upper atmosphere, has not yet been examined in detail from this point of view. A preliminary analysis, based on the use of deductions from the kinetic theory of propagation of electromagnetic waves in a magnetoactive plasma,[31] shows that under the conditions of the exosphere the character of wave propagation (i.e., the behavior of the index of refraction n) in the resonant region $v \simeq v_\infty$ is determined by thermal motion and not collisions. In this region, however, the collisions are quite significant in the determination of the absorption of waves.

To conclude this section we note that formulas (2.1), (2.7), and (2.8) have been derived under the assumption that the field $\mathbf{E}_{\text{eff}}$ acting on the electrons can be equated to the average macroscopic field $\mathbf{E}$. Generally speaking, $\mathbf{E}_{\text{eff}} \neq \mathbf{E}$, and we can assume in the simplest case that

$$\mathbf{E}_{\text{eff}} = \mathbf{E} + 4\pi a\mathbf{P}, \tag{2.12}$$

where $\mathbf{P}$ is the polarization and a is a certain coefficient. If we deal with plasma, we can consider it as proved at present[1,32] that we can set with great accuracy $a = 0$. We can add also that the data on the propagation of whistlers, most of which can be interpreted on the basis of relation (2.8), are also evidence in favor of the equality $\mathbf{E}_{\text{eff}} = \mathbf{E}$. Using relation (2.12) under the conditions by which the transition to (2.8) is determined, we can obtain the following formula[33,34]

$$\tilde{n}_2^2 = \frac{v}{\sqrt{u}\cos\alpha - 1 - av}. \tag{2.13}$$

In order to go over to the experimentally-confirmed formula (2.8) from the analogous relation (2.13) it is essential to have $a \ll \sqrt{u}\cos\alpha/v$. Taking inequalities (2.3) and (2.5) into account, we readily establish that the very fact that the whistlers propagate implies beyond any doubt that $a \ll 1$. What is important is that the last inequality demonstrates with relatively higher accuracy the smallness of the coefficient a. For example, in considering the passage of whistlers through the F layer we can readily obtain $a \lesssim 10^{-5}$ for sufficiently low frequencies, when $\sqrt{u}\cos\alpha \gg 1$ (2.10). In the opposite case we would get $\tilde{n}_2^2 < 0$ and propagation would be impossible.

Group velocity and determination of trajectories. Let us determine the group velocity V_{gr}, which characterizes the propagation of low-frequency signals. It is important that in a magnetoactive dispersive medium this velocity differs from the phase velocity both in magnitude and in direction.

We choose a coordinate system such that the magnetic field $\mathbf{H}_0$ is parallel to the z axis and the wave vector $\mathbf{k}$ is in the zy plane. We assume, to be specific, that the vector $\mathbf{k}$ is in the first quadrant (Fig. 13) so that $\sin\alpha > 0$ and $\cos\alpha > 0$. Using the relation $V_{gr} = \partial\omega/\partial\mathbf{k}$ and bearing in mind that $n = ck/\omega$, we obtain from (2.8), neglecting collisions,

$$V_{gr,y} = \frac{c}{n}\sin\alpha\left(1 - \frac{2}{\sqrt{u}\cos\alpha}\right),$$
$$V_{gr,z} = \frac{c}{n}\cos\alpha\left(1 + \frac{1}{\cos^2\alpha} - \frac{2}{\sqrt{u}\cos\alpha}\right). \qquad (2.14)$$

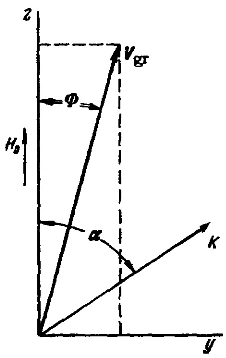

FIG. 13. Coordinate system used in determination of the magnitude and direction of the group velocity.

We note that it follows from (2.9), subject to the limitation (2.9), that $V_{gr,z} > 0$. Thus, the vector $\mathbf{V}_{gr}$ can lie only in the first or second quadrants (see Fig. 13).

For the absolute value of the group velocity we have

$$|V_{gr}| = \frac{c}{n}\sqrt{4\left(\frac{\sqrt{u}\cos\alpha - 1}{\sqrt{u}\cos\alpha}\right)^2 + \tan^2\alpha}. \qquad (2.15)$$

Relation (2.15) can be used to determine the nose-whistler frequencies. It was indicated in Sec. 1 that in many cases it is possible to separate in the spectrum of the atmospheric a certain average frequency $f = f_N$, corresponding to the smallest delay time (f_N — nose-whistler frequency). The group-delay time is determined by the relation $t = \int \frac{ds}{|V_{gr}|}$, in which the integration is carried out along the trajectory. A detailed determination of the minimum time of arrival t as a function of the frequency f calls for an analysis that takes the character of the trajectory into account. Certain tentative results concerning the nose frequency can be obtained, however, by finding the extremal values of f for sections of the trajectory on which the angle α can be assumed approximately constant. From the extremum condition

$$\frac{dV_{gr}}{d\omega} = 0$$

we arrive at the equation

$$16y^3 - 12\cos\alpha\, y^2 + 2\sin^2\alpha y - \sin^2\alpha\cos\alpha = 0, \qquad (2.16)$$

where $y = (\omega_H\cos\alpha - \omega_N)/\omega_H$ and $f_N = \omega_N/2\pi$ is the nose frequency. At $\alpha = 0$ we get from (2.16), for the root $y = 3/4$,

$$f_N = \frac{f_H}{4}. \qquad (2.17)$$

This result was obtained by Helliwell et al.[6] The possibility of detecting nose whistlers was actually predicted by Ellis[26] prior to their discovery. From (2.17) it is seen that from the measurement of the nose frequencies one can determine the value of the geomagnetic field at high altitudes above the earth. Ellis[26] states that one can assume $\alpha \simeq 0$ for the peak parts of the trajectories of whistler propagation. Thus, in these parts of the trajectories the propagation can apparently be considered longitudinal and relation (2.17) can be used. It is also necessary here that the main contribution to the group delay be due to the passage through the peak parts of the whistler trajectories. This, generally speaking, is incorrect. Relation (2.17) must therefore be considered as an approximation. In this connection, it is useful to estimate the degree of accuracy of (2.17) on the basis of the general equation (2.16). It can also be established that $\omega_N/\omega_H = 0.21$ when $\alpha = 30°$ and 0.17 when $\alpha = 60°$. At the same time we obtain from (2.17) with $\alpha = 0$ a value $\omega_N/\omega_H = 0.25$. The difference in the given values of ω_N/ω_H for different α is not very large in these examples. Using formula (2.17) to determine the frequency f_N, and from it the magnetic field $\mathbf{H}_0$, we can only undervalue the latter, since we obtain from (2.16), for $\alpha \neq 0$ and other conditions equal, higher values of the frequency f_N than when $\alpha = 0$.

Using relations (2.14), we can draw conclusions regarding the mutual orientation of the group velocity and the magnetic field $\mathbf{H}_0$. Denoting by Φ the angle between V_{gr} and $\mathbf{H}_0$ from (2.14), we obtain (see Fig. 13)

$$\tan\Phi = \frac{\sin\alpha\left(1 - \frac{2}{\sqrt{u}\cos\alpha}\right)}{\cos\alpha\left(1 - \frac{2}{\sqrt{u}\cos\alpha}\right) + \frac{1}{\cos\alpha}}. \qquad (2.18)$$

Let us determine the values of the angle $\alpha = \alpha'$, at which $\tan\Phi$ has an extremum. From the condition $d\tan\Phi/d\alpha = 0$ we get

$$\cos\alpha' = \frac{1}{\sqrt{u}} + \frac{1}{\sqrt{3u}}\sqrt{u-1}. \qquad (2.19)$$

These values, which satisfy the obvious requirement $\cos\alpha' < 1$, lie in the interval between $u = 4$ and $u = \infty$. When $u \gg 1$ we obtain from (2.19) $\cos\alpha' = 1/\sqrt{3}$. The corresponding maximum value is $(\tan\Phi)_m = 1/\sqrt{8}$, from which it follows that $\Phi_m = 19°29'$. Thus, when condition (2.10) is well satisfied, the possible values of the angle Φ lie within the limits

$$0 \leqslant \Phi \leqslant 19°29'. \qquad (2.20)$$

The limiting value $\Phi = 0$ is obtained from the limitation (2.9). The other value in (2.20) follows from (2.19). Analogous considerations are used also in later examples for the determination of the limiting angles Φ_m.

It is easy to establish that when the directions $\mathbf{k}$ and $\mathbf{H}_0$ make an acute angle, the vector V_{gr} is included between the vectors $\mathbf{k}$ and $\mathbf{H}_0$. When the angle between $\mathbf{k}$ and $\mathbf{H}_0$ is obtuse, the vector V_{gr} lies between the vectors $\mathbf{k}$ and $-\mathbf{H}_0$ (Fig. 14). It follows from the foregoing that at all directions of the phase velocity V_{ph} the velocity V_{gr} is contained in a cone with half-angle 19°29′ relative to the magnetic field. This statement was first derived by Storey[3] (the Storey theorem).

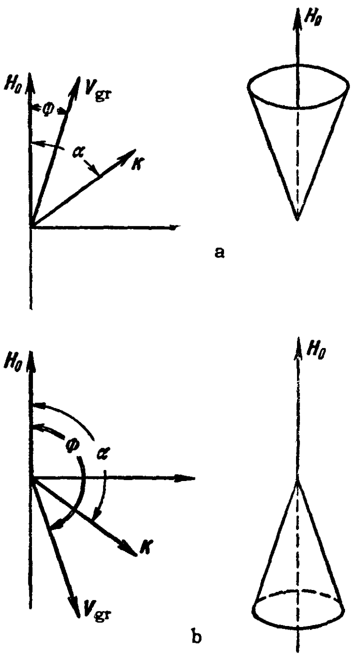

FIG. 14. Directions of the group velocity in the case when $\sqrt{u}\cos\alpha \gg 1$, corresponding to the maximum absolute values: a) Angle Φ at $|\alpha| < \pi/2$; b) angle $\pi - \Phi$ for $|\alpha| > \pi/2$. The cones drawn delimit the possible directions of the group velocity.

When the parameter u is not too large, the situation changes appreciably, and the Storey theorem just formulated becomes incorrect.[35] Thus, when $u = 4$ we readily obtain from (2.18) and (2.19) that the values of Φ lie in the range $-30° \le \Phi \le 0$. Compared with the limits established by (2.20), we have here not only a change in the possible absolute magnitude of the angles Φ but also a reversal of their signs* (although the vector $\mathbf{k}$ lies in the first quadrant, see Fig. 13). For the intermediate case, when $u = 10$, the angle Φ lies in the interval $-60°36' \le \Phi \le 17°$. Thus, on going from the case when $u \gg 1$ to other cases, for which the values of u are of order of magnitude unity, the orientation of the group velocity relative to the magnetic field $\mathbf{H}_0$ is greatly changed.[35] By virtue of this, a change is expected in the character of the trajectories on going from low frequencies to relatively high frequencies ($u \gtrsim 1$). This conclusion is confirmed by calculations carried out by Maeda and Kimura[29] and by Garriot.[36]

We shall now discuss briefly the contents of several papers,[29,30,36,37] in which the trajectories of whistling atmospherics are calculated. If it is considered that in the propagation of low-frequency signals the conditions of applicability of the approximations of geometric optics are usually well satisfied, there is no need to take diffraction effects and scattering into account, at least in the absence of strong perturbations.

The trajectories of the signals can be obtained from the equations

$$\frac{dx}{V_{gr,x}} = \frac{dy}{V_{gr,y}} = \frac{dz}{V_{gr,z}}. \tag{2.21}$$

Integration of these equations with use of certain simplifying assumptions was carried out by Gershman and Korobkov.[37] The only frequency region considered was that defined by condition (2.10), i.e., $\sqrt{u}\cos\alpha \gg 1$ [the calculations were based on formula (2.11)]. It was further taken into consideration that the values of the index of refraction are usually quite large in the propagation of whistlers, so that the inequality $n_2 \gg 1$ is satisfied. The latter can be considered as the consequence of conditions (2.3) and (2.5), i.e., $v \gg 1$ and $v \gtrsim u$.

In propagation in a medium with large indices of refraction, the phase velocity will approach the direction of ∇n. In reference 37 it was assumed from the very outset that the most significant changes of n_2 are in the radial direction. Under these limitations we can obtain an equation for the whistler trajectory, expressed in analytic form:

$$\frac{\cos^2\vartheta}{\cos^2\varphi} = \frac{3}{4}\frac{r}{r_0} + \frac{1}{4}\frac{r_0}{r}. \tag{2.22}$$

The corresponding notation was shown earlier in Fig. 12. We note that the equation of the magnetic force line that passes through the point where the atmospheric is generated (r_0, φ), has the form $\cos^2\vartheta/\cos^2\varphi = r_0/r$. From a comparison of this equation with (2.22) we can readily establish that the trajectories obtained always lie above the force lines with which they have common initial coordinates. Note that (2.22) calls for the trajectories to be symmetrical about the magnetic equator. If the deviations of the direction of the phase velocity from radial are taken into account,[30] this deduction becomes incorrect.

Unlike Gershman and Korobkov,[37] the authors of references 29, 30, and 36 determine the trajectories, in the final analysis, only by numerical methods. The calculation in references 29 and 30 is based on the Fermat principle. They introduce the so-called group index of refraction, determined by the relations $n_{gr} = n + \omega\,\partial n/\partial\omega$. This is followed by the use of a variational principle, which establishes directly the law of refraction for a group of waves. This law differs from the usual law of refraction of the phase front. In reference 30 it is assumed that the condition $\sqrt{u} \times \cos\alpha \gg 1$ is satisfied, whereas in reference 29 several trajectories are given also for relatively high frequencies. The aggregate of trajectories calculated in references 29 and 30 is shown in Figs. 15 and 16. All these trajectories are asymmetric to some degree relative to the magnetic equator. At the same time, a comparison of the trajectories given by (2.22) with

*In this example and those that follow a negative angle Φ denotes that the vector V_{gr} lies in the second quadrant (see Fig. 13).

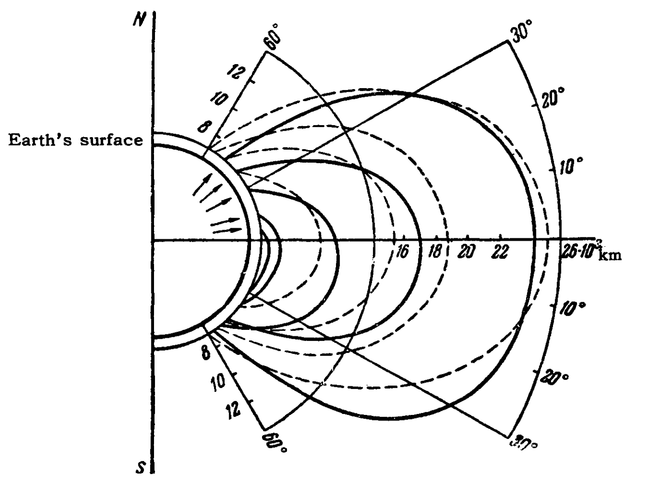

FIG. 15. Trajectories for whistlers (solid curves) with initial coordinates φ = 10, 15, 30, 40, and 50°, calculated by Maeda and Kimura[30] for the case $\sqrt{u}\cos\alpha \gg 1$. The dashed curves denote the lines of the earth's magnetic field.

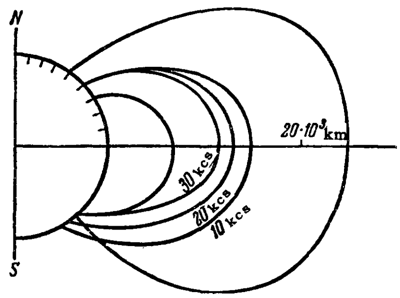

FIG. 16. Trajectories of whistlers, calculated by Maeda and Kimura.[29] The condition $\sqrt{u}\cos\alpha \gg 1$ is violated for three trajectories that leave a common point at the frequencies indicated. The two other trajectories (on which the frequencies are not marked) are plotted for the same case as the trajectories in Fig. 15.

the trajectories shown in Fig. 15 shows that the total path length and the maximum height reached by the atmospherics differ by not more than 10%.

The authors of references 29, 30, and 36 used in the calculation of the trajectories a definite distribution of the electron concentration N(r), chosen in most cases such as to make the values of the coefficient of dispersion D for whistlers agree with experimental data (see also Sec. 3). No specific distributions of the electron concentration with altitude have been used in the derivation of Eq. (2.22), in view of the assumption that the phase velocities are radial. However it is not necessary to make use of such distributions in the calculation of dispersion on the basis of (2.22).

It follows from the results of reference 26 that there is apparently no radial change in the index of refraction in the part of the trajectory farthest away from the earth's surface. If this is so, then Eq. (2.22) needs to be corrected in this region.

Communication 36 contains several trajectories constructed for relatively high frequencies, when even condition (2.3) is violated. The propagation of such frequencies is possible in principle only under special conditions, and in the case of whistlers, these frequencies ($f \sim 30-40$ kcs) are usually not contained in the received signals. We shall therefore not give the details of the calculation, and note only that the trajectories given in reference 35 have a very sharply pronounced asymmetry relative to the magnetic equator.

Knowledge of the propagation trajectories, along with a definite choice of the distribution of the electrons N, makes it possible to obtain an important integral characteristic of the propagation, namely the dispersion D, which we have already introduced in Sec. 1 on the basis of observation of whistlers. The dispersion coefficient can be obtained by analyzing the characteristics experimentally obtained in the form $t = t(f)$, where t is the delay time (see Sec. 1). It is obvious that this time is determined from the relation $t = \int \frac{ds}{|V_{gr}|}$, where ds is the element of path along the trajectory. Substituting the values of $|V_{gr}|$ (2.15) with condition (2.10) satisfied [in this case $|V_{gr}| = (c/n)\sqrt{4+\tan^2\alpha}$], we arrive at a relation in the form $t = Df^{-1/2}$, which we encountered in Sec. 1 [see (1.1)]. For the dispersion coefficient we have[3]

$$D = \frac{1}{\sqrt{2\pi}\,c}\int \frac{\omega_0\,ds}{\sqrt{\omega_H \cos\alpha\,(4+\tan^2\alpha)}}. \tag{2.23}$$

A comparison of the experimental and theoretical values of the dispersion will be made in Sec. 3.

Allowance for the ion motion. So far we have considered relations in which the motion of the ions was disregarded. It is well known, however, that for frequencies satisfying the condition

$$\omega \lesssim \Omega_H, \tag{2.24}$$

the motion of ions is very important to the propagation of the waves.[1,22,23] If the frequencies are so low, on the other hand, that $\omega \ll \Omega_H$, then magnetohydrodynamic waves should propagate in the rarefied plasma. The effect of ion motion on the propagation of whistlers was considered in references 25 and 38. Storey[25] indicated that it is possible to use observation of whistlers in order to ascertain whether the upper ionosphere contains ionized hydrogen. Certain important consequences drawn from the assumption of such a composition of the upper atmosphere at high altitudes were established in reference 39.

It must be borne in mind that real receivers record atmospheric spectra with components not lower than $f \simeq 200-400$ cps. Since the singularities connected with the effect of the ions begin to come into play when $\omega \leq \Omega_H$ (2.24), we can count on the detection of new phenomena only if the values of Ω_H exceed 1200 cps. Consequently, if the plasma in the upper atmosphere consists predominantly of protons and electrons, the influence of the former on the character of propagation of the wave can manifest itself in those regions where the gyrofrequency of the electrons, $\omega_H = 1.84 \times 10^3 \Omega_H$, exceeds $(2 \text{ or } 3) \times 10^6$. Using the formula (2.4), we can readily show that such values of ω_H are encountered approximately at $r_0/r \lesssim 0.7$ (see also table on p. 753). From the foregoing we can conclude[25] that effects connected with the presence of ions are relatively easier to detect by observation of whistlers at relatively small latitudes. References 25 and 38 do not contain formulas suitable for the description of the propagation of very-low-frequency waves in the general case.* However, from the relations given in reference 38 we can obtain the following expression for $\tilde{n}_2^2$, with allowance for the contribution due to the motion of the ions:

$$\tilde{n}_2^2 \simeq \frac{v}{\sqrt{uu_i}\left(1-\frac{\sin^2\alpha}{2}\right)+\sqrt{u\cos^2\alpha+\frac{uu_i}{4}\sin^2\alpha}}, \tag{2.25}$$

where $u_i = \Omega_H^2/\omega^2$. In the derivation of (2.25) collisions and thermal motion of charged particles were neglected. In addition, it was assumed that the condition

$$u \gg 1 \tag{2.26}$$

is satisfied. Thus, in the region of relatively high frequencies (say, in the vicinity of the nose frequency f_N and higher), the relation (2.25) does not hold. If we employ the limitation $u_i \ll 1$, which may not contradict (2.26), we arrive at the old relation (2.11), whereas the opposite requirement, $u_i \gg 1$, will give in conjunction with (2.25) the relation

$$n_2^2 = \frac{v}{\sqrt{uu_i}} = \frac{4\pi c^2 NM}{H_0^2}, \tag{2.27}$$

which determines the propagation of a magnetohydrodynamic wave. It must be noted that in the region where (2.27) is valid, n_2^2 is independent of the direction of $\mathbf{k}$, and the group velocity V_{gr} coincides in direction with the phase velocity V_{ph} (in addition, these velocities are now equal in magnitude, in view of the absence of dispersion). In this case no characteristic channel is produced for the passage of the whistler. At the same time, the region where $\omega \ll \Omega_H$ can hardly be attained, and in reality we can only speak of registering frequencies with $\omega \geq \Omega_H$. No detailed analysis of the behavior of the waves in the region $\omega \sim \Omega_H$ has been published, as far as we know. It is of interest to find the magnitude and direction of the group velocity V_{gr} and to determine the character of the trajectories in this region.

Hines[38] indicates that the ion motion can be significant not only at the lowest frequencies, but also at sufficiently high frequencies, if the direction of propagation is close to transverse $(\alpha \simeq \pi/2)$. In particular, if $u \ll 1$, then the limitation (2.9) must be modified, and in some cases it becomes altogether unnecessary. However the question as a whole is still insufficiently clear, since in the region $\sqrt{u}\cos\alpha \simeq 1$, which is actually the one referred to in reference 38, it is necessary to take into account the thermal motion of the electrons, and the specific absorption of waves can also be significant there.

Interaction of normal waves. Let us dwell briefly now on the interaction of normal waves. One of the features of whistlers is their relatively easy penetration into the ionosphere. The same can be said concerning the emergence of whistlers and VLF radiation from the ionosphere. At the same time, such penetration is very difficult for waves of higher frequency, say in the standard broadcast band, and the ionosphere behaves practically as a reflecting medium.

The ability of whistlers to penetrate in the ionosphere and to come out of it is undoubtedly connected with the interaction of normal waves, which occurs in an inhomogeneous magnetoactive plasma. In our case the wave of type "1" is transformed on striking the ionosphere into a wave of type "2" in the interaction region. This wave can now propagate further in the ionized gas. Many researches have been devoted to the theory of such an interaction, and their results have been extensively treated in the review by Gershman et al.[22] At normal incidence of radio pulses on the ionosphere, the interaction causes a tripling of the reflected signals.[1] For normal incidence, the region of interaction is located between the points $v = 1$ and $v = v_\infty$. In tentative estimates one can assume for oblique incidence that the position of this region remains practically unchanged. Since we speak here of the lower ionosphere, where the magnetic field $\mathbf{H}_0$ is large, it can be assumed that $u \gg 1$. Assuming that α is not too close to $\pi/2$, we obtain the following rough condition for the position of the interaction region:

$$v \simeq 1. \tag{2.28}$$

Using this condition, we estimate the electron concentration N. For frequencies f ranging from 400 cps to 10 kcs we obtain from (2.28) N = 0.002 to 1. These values of N are lower than in the ionosphere, but at the same time they are higher than under ordinary conditions at the earth's surface, where the electrons disappear rapidly by adhesion to the molecules of oxygen. Thus, the region of interaction is located between the ionosphere and the troposphere (approximately at altitudes from 20 to 70 km).

One might think that the penetrating ability is closely related with the influence of collisions. In the region under consideration, the number of collisions between electrons and molecules, ν_{em}, is much greater than the so-called critical number of collisions, ν_{cr}.[1,22,23]

*Storey[25] considers the previously analyzed case[1,22] of propagation in the direction of the magnetic field $\mathbf{H}_0$. Hines[38] concentrates his main attention on the propagation of waves in directions close to transverse $(\alpha = \pi/2)$.

The values of the coefficient of penetration can be quite large under these conditions. However, considerable absorption should take place simultaneously in the region of penetration. The influence of the absorption is apparently weakened by the fact that when $v \sim 1$ the values of the refractive index are low ($n_2 \sim 1$, the region of interaction is usually sufficiently remote from the point $v = v_\infty$). Since $n_2 \sim 1$, the wavelength in the medium is comparable with the wavelength in the free space λ_0. The wavelengths λ_0 amount to hundreds of kilometers and are comparable with or greater than the dimensions of the interaction region.

By way of a supplement to the information given in Sec. 1, we can indicate that whistlers are frequently produced above sea level in stormy weather. In this connection, a hypothesis has been advanced by Hill[40] that the increased number of whistlers during storms is due to the change in the conductivity at altitudes on the order of 10 — 20 km. The discharges produce free electrons, so that the efficiency of penetration of the radiation into the upper atmosphere is increased. However, no attempt is made by Hill[40] to base any quantitative conclusions on his hypothesis.

Absorption. Let us consider absorption of waves. Very little research has been done in this field as applied to the propagation of whistling atmospherics.[24,30] Let us discuss first the absorption due to collision.

As before, to determine this most significant part of the absorption it is sufficient to use the quasi-longitudinal approximation formula (2.8). At high altitudes, the principal role is played by collisions between electrons and ions. Using in this case the well known formula $\nu = \nu_{ei} = (5.5\,N/T^{3/2})\ln(220\,T/N^{1/3})$, we find that ν_{ei} ranges from $\sim 10^3$ to 10^{-2} sec^{-1} starting with the region of maximum of the F layer, up to altitudes that border on interplanetary space, where $N \simeq 200$ to 600. The inequality $\omega_H \gg \nu$ is well satisfied here. Assuming the limitation $|1 - \sqrt{u}\cos\alpha| \gg s$, which under the stipulations made above is necessary only in the vicinity $\sqrt{u}\cos\alpha \simeq 1$, we obtain from (2.8) the following expression for the absorption index:

$$q = \frac{sn}{2(\sqrt{u}\cos\alpha - 1)} = \frac{\nu}{2} n (\omega_H \cos\alpha - \omega)^{-1}. \qquad (2.29)$$

For the amplitude coefficient of absorption $\chi = \omega q/c$, we have from (2.29) with allowance for (2.9) the following formula:

$$\chi = \frac{\sqrt{\omega}\,\omega_0 \nu}{2c(\omega_H \cos\alpha - \omega)^{3/2}}. \qquad (2.30)$$

We see from this formula that the absorption increases at higher frequencies, particularly as resonance is approached ($\omega_H \cos\alpha \to \omega$). This apparently is the reason for the difficulties involved in observing the high-frequency part of the spectrum of atmospherics. At not too high frequencies, when the condition $\sqrt{u}\cos\alpha \gg 1$ is satisfied, we obtain from (2.30) $\chi = \sqrt{\pi/2}\,\frac{f^{1/2}\omega_0\nu}{c(\omega_H\cos\alpha)^{3/2}}$. Estimates based on this last formula show that under unperturbed conditions the losses due to collisions may reduce the amplitude of the field along the path of propagation of the atmospherics by one or at most two orders of magnitude. If, however, we use for the collision frequency ν the values that are characteristic of the collision of electrons with neutral particles, we can establish that noticeable absorption is possible in the lower ionosphere (in the D and E layers). This conclusion agrees with the predominant appearance of whistlers during nighttime (the D layer disappears at night). In reference 30, where the details of the calculations have been left out, it is noted that the absorption in the E layer is considerable. Attention is paid there, too, to the fact that to determine the intensity it is necessary to take into account the focusing action of the medium. It is indicated in reference 41 that as the whistlers pass through the perturbed upper atmosphere, it is quite possible for the signals to become amplified (experimental data indicating such a possibility were given in Sec. 1).

The value of the specific absorption produced in a magnetoactive plasma, not connected with the collisions, has been determined recently by several investigators.[42,43,44] In particular, the value of this absorption was calculated for slow ordinary waves, with which the propagation of whistlers is connected.[24,44] Estimates of the value of this absorption[24] lead to the conclusion that it is insignificant in the case of an unperturbed atmosphere. In the presence of corpuscular disturbances, this absorption is small at electron concentrations $N \lesssim 10^3$ in streams near the earth. On the other hand, if high electron concentrations are assumed ($N \sim 10^4$ to 10^5 cm^{-3}), strong damping of the waves is possible in the frequency range from 300 to 500 cps. Thus, observation of whistling atmospherics during time of disturbances may be of interest in connection with the question of the maximum concentration of electrons in solar corpuscular streams.

Mechanisms of generation of VLF emissions. The occurrence of VLF emissions is connected with the action of streams of charged particles, traveling with high velocities from the sun and entering the earth's outer ionosphere. A calculation of the radiation produced by such a beam on penetrating the magnetoactive plasma is quite complicated. Up to now, as applied to VLF noise near the earth, only the radiation of individual particles was considered, and the rather schematic case of propagation only along the magnetic field was considered. One version of the possible radiation mechanism was proposed by Gallet and Hellingwell;[45,5] This mechanism was called by Gallet[5] the "traveling-wave-tube mechanism." Another mechanism was proposed by MacArthur,[46] who considered the radiation from protons moving along a helix in a magnetic field, with allowance for the Doppler effect. Both mechanisms can be considered from a single point of view. This, and the final results obtained by these authors, will be given below.

We can obtain the spectral and angular distribution of the radiated energy for the radiation from a charge moving in a magnetoactive plasma. A general solution of this problem is given by Eĭdman.[47] It is found that this radiation can be broken up into magnetic bremsstrahlung (radiation due to acceleration in a magnetic field, proportional to the frequency of rotation of a charge around the field) and to Cerenkov radiation with a continuous spectrum.

In the general case, a charge moving with velocity V in a magnetic field H_0 directed along the z axis, radiates in a direction that makes an angle α with the z axis, the following frequencies:[47,48]

$$\omega = l\Omega_H + kV_z \cos\alpha \qquad (l = 0, \pm 1, \pm 2, \ldots), \tag{2.31}$$

where Ω_H is the gyrofrequency of the charge (we are interested in protons). We recall also that $k = \omega n/c$, where the refractive index $n_{1,2}$ in a magnetoactive plasma is determined for wave types "1" and "2" from (2.1) with $s = 0$.

For the case $l = 0$ we obtain from (2.31) the usual condition for the Cerenkov radiation, $\omega = kV_z \cos\alpha$, or $\cos\alpha = c/n(\omega)V_z$.

In the case $l \neq 0$, the radiated frequencies are determined from the conditions

$$\omega = \frac{l\Omega_H}{1 - \frac{V_z}{c} n \cos\alpha} \quad (l > 0), \quad \omega = \frac{l\Omega_H}{\frac{V_z}{c} n \cos\alpha - 1} \quad (l < 0). \tag{2.32}$$

When $l = \pm 1$ these formulas coincide with the formula for the Doppler effect in the medium. Actually, if the emitter radiates in its own reference frame at a frequency Ω_H and moves relative to a stationary medium with velocity V, then in the reference frame connected with the stationary medium the radiation has a frequency $\omega = \dfrac{\Omega_H\sqrt{1-\beta^2}}{|1-\beta n \cos\alpha|}$, where $\beta = V/c$, which corresponds to (2.32) in the nonrelativistic case ($V/c \ll 1$).

The radiation corresponding to $l = 0$ is considered in reference 45, while that corresponding to $l = 1$ is considered in reference 46. In both cases, only the radiation along the magnetic field ($\alpha = 0$) is considered.

Starting with the relation $V = c/n$, which is essentially the condition for the excitation of Cerenkov radiation in the direction of motion of the particle ($\cos\alpha = 1$), and using the expression $n_2^2 = v/(\sqrt{u} - 1)$ for the refractive index [see Eq. (2.8), where $\alpha = 0$*], we can obtain the frequencies radiated in this direction

$$\omega_{1,2} = \frac{\omega_H}{2}\left\{1 \pm \left[1 - \left(2\frac{\omega_0}{\omega_H}\frac{V}{c}\frac{1}{\sqrt{1-V^2/c^2}}\right)^2\right]\right\}. \tag{2.33}$$

Here V is the velocity of the corpuscular stream. Since many considerations[49] indicate for the corpuscular streams $V = 10^8$ cm/sec, we get $V/c \sim 10^{-2}$ and consequently we have from (2.33), with sufficient accuracy,

$$\omega_{1,2} = \frac{\omega_H}{2}\left\{1 \pm \left[1 - \left(2\frac{\omega_0}{\omega_H}\frac{V}{c}\right)^2\right]^{1/2}\right\}. \tag{2.34}$$

We see hence that in the longitudinal direction there are two radiated frequencies, the values of which depend on ω_0, ω_H, and the beam velocity V. One of these frequencies is greater than ω_H, and the other is smaller. The condition for the possibility of radiation is the inequality

$$2\frac{\omega_0}{\omega_H}\frac{V}{c} < 1. \tag{2.35}$$

An estimate[5] yields for distances not more than $4r_0$ away from the earth a value $(2\omega_0 V/\omega_H c)^2 \ll 1$, and then

$$\omega_1 = \left(\frac{V}{c}\right)^2 \frac{\omega_0^2}{\omega_H}, \qquad \omega_2 = \omega_H\left[1 - \left(\frac{V}{c}\frac{\omega_0}{\omega_H}\right)^2\right]. \tag{2.36}$$

It is obvious that the frequency ω_1 can remain constant as the beam moves, if the ratio ω_0^2/ω_H remains unchanged; this would mean that $N/H_0 = \text{const}$ along the trajectory. If we consider $H_0 \sim 1/r^3$, we obtain an electron concentration varying as $N \sim r^{-3}$. As has already been indicated, parts with constant frequency are sometimes observed in the VLF-radiation spectrum. The authors of reference 45 interpret the appearance of a constant frequency by assuming $\omega_0^2/\omega_H = \text{const}$ in the generation region. Changes in the frequency are explained as deviations from this law.

On the basis of Eqs. (2.34) — (2.36), calculations were made for the purpose of explaining different classes of VLF radiation (examples and the corresponding literature are given in references 5 and 46). In most calculations, values on the order of 10^3 km/sec are used for V, and it is assumed that $\omega_0^2/\omega_H = \text{const}$ for a definite portion of the exosphere. In particular, according to the statement made in reference 5, good agreement is obtained for spectrograms in the forms of hooks.

MacArthur[46] considered magnetic bremsstrahlung with allowance for the Doppler effect, and determined the radiated frequencies from the relation $\omega = \Omega_H/(1 - Vn/c)$, where n is again determined from (2.8) with $\alpha = 0$ and $s = 0$. He then obtains for the possible radiation frequencies ω the equation

$$\omega^3(1 - V^2/c^2) - \omega^2[\omega_H - (V/c)^2\omega_H + 2\Omega_H] + \omega[\Omega_H^2 + 2\Omega_H\omega_H + (V/c)^2\omega_0^2] - \Omega_H^2\omega_H = 0. \tag{2.37}$$

Since $V^2/c^2 \ll 1$ and $\Omega_H \ll \omega$, Eq. (2.37) simplifies to

$$\omega^3 - \omega^2\omega_H + \omega\left(2\omega_H\Omega_H + \frac{V^2}{c^2}\omega_0^2\right) - \Omega_H^2\omega_H = 0. \tag{2.38}$$

*It must be pointed out that such an approach is to some extent inconsistent, inasmuch as there is no Cerenkov radiation at all at $\alpha = 0$ in this approximation. Thus, this consideration becomes meaningful only if $\alpha \neq 0$ or when the conditions are changed (the thermal motion of the particle is considered, etc.). At the same time it must be emphasized that consideration of radiation precisely in the specified direction has no physical meaning.

Inasmuch as $\Omega_H \ll \omega$ for the usually considered case of whistlers, we obtain from (2.38) the quadratic equation

$$\omega^2 - \omega\omega_H + (V/c)^2 \omega_0^2 = 0,$$

the roots of which are

$$\omega_{1,2} = \frac{\omega_H}{2}\left\{1 \pm \left[1 - \left(2\frac{V}{c}\frac{\omega_0'}{\omega_H}\right)^2\right]^{1/2}\right\}. \quad (2.39)$$

This result agrees fully with (2.34).

However, the agreement of the limiting results in references 45 and 46 is accidental, for in principle different radiation components are considered.

The variation of the spectral composition of the radiation can be analyzed by considering waves traveling at a certain angle to the magnetic field. In this case. for $l = 0$, we have $\cos\alpha = c/n(\omega)V$ (the Cerenkov radiation condition) and $n^2(\alpha) = 1 + v/(\sqrt{u}\cos\alpha - 1)$ [see (2.8)]. This expression for the index of refraction corresponds to the quasi-longitudinal approximation and differs from the case $\alpha = 0$ in that ω_H is replaced by $\omega_L = \omega_H \cos\alpha$. It is easy to see that if we denote $V\cos\alpha$ by V_L, we obtain a relation similar to that for $\alpha = 0$ ($V = c/n$), but in the form $V_L = c/n^*$, where n^* differs from n when $\alpha = 0$ in that ω_H is replaced by ω_L. In other words, we obtain the same Gallet equations, but with different parameters V_L and ω_L ($V_L = V\cos\alpha$ and $\omega_L = \omega_H\cos\alpha$). The same holds for the case considered in reference 46.

The formulas obtained for the frequencies can be readily derived and will not be given here. We note only that all the relations in which the quantities V and ω_H are contained in the form of a ratio V/ω_H remained unchanged.

Summarizing, we can say that the proposed mechanisms for the radiation of the individual frequencies make it possible to explain certain frequency laws in the spectra of the VLF emissions. However, the problem of radiation produced by charged beams in the ionosphere has not yet been considered. Nor have there been considered problems connected with the calculation of the intensity of the arriving low-frequency radiation.

3. LOW-FREQUENCY RADIO WAVES AND THE INVESTIGATION OF THE UPPER ATMOSPHERE

In spite of the considerable expansion of investigations of the upper atmosphere by means of rockets and satellites, the principal role in systematic observations of the state of the upper atmosphere is still played by methods using earthside apparatus. Observations of whistlers and VLF emissions can play an important role in such methods. Although our information on whistlers and VLF emissions is at the present time in a state where the principal problem is the investigation of the radiation itself and of the character of its propagation, it is useful even now to speculate on certain possibilities of obtaining information concerning processes in the upper atmosphere from data on the low-frequency radiation.

Let us stop first to analyze briefly the possibilities that arise in the interpretation of data on the passage of whistlers through the atmosphere. We can point to several trends in the comparison of theory with the observational data, and to certain conclusions drawn concerning the structure of the earth's upper atmosphere.

Mention should first be made of comparisons of the values of the coefficient of dispersion D [see (1.1) and (2.23)]. The measured values of D, as noted in Sec. 1, can be obtained by using the relations $f(t)$ for the received frequencies. As far as the calculations go, they call for the use of specific models for the distribution of the electron concentration in the upper atmosphere. In addition, it is necessary to know the propagation trajectories of the atmospherics.

In his first paper on whistlers, Storey[3] determined by observation, that at the geomagnetic latitude $\varphi = 55°$, the dispersion D ranges from approximately 20 to 120 $\sec^{1/2}$ (see also reference 30). At the same time, a calculation of the values of D in the ionosphere only [the integration in (2.23) was carried out for only part of the path passing through the ionosphere] yielded $D \lesssim 2 \sec^{1/2}$. In order to obtain the considerably larger dispersions D derived from the experiments, it is necessary to assume that the electron concentration is sufficiently high over the entire path of propagation of the whistler.

In one of his later investigations, Storey[50] developed in detail a method for determining the electron concentration N in the upper atmosphere by determining the dispersion D. A shortcoming of this method is the need for a very accurate measurement of D. In addition, the method is based on a simplified representation of the character of the propagation trajectories, namely, that these trajectories coincide with the force lines of the earth's magnetic field $\mathbf{H}_0$. This is certainly not the case, as can be seen from examples given in Sec. 2.

A calculation of dispersion based on this inaccurate determination of the trajectories, made to coincide with some line of the geomagnetic field, was also carried out by Dangey.[39] He derived and used the following ion (and electron) concentration distribution at large altitudes.

$$N = N_0 \exp(2{,}5 r_0/r). \quad (3.1)$$

Formula (3.1) is based on considerations connected with the possibility of retaining the charge particles in the upper atmosphere of the earth. It was established in reference 39 that at $\varphi = 55°$ the values indicated above for the dispersions can be explained only if the quantity N_0 in (3.1), which is the electron concentration at very large distances from the earth ($r \gg r_0$), is $\sim 30 - 1200$ el/cm^3. It is concluded thus that at altitudes that border on interplanetary space $N \sim 10^2$ to 10^3 el/cm^3.

Rather detailed comparisons of the experimental and theoretical values of the dispersion D were made by Maeda and Kimura.[30] The distribution of the electron concentration was chosen to obtain the required value of dispersion D. As a result of this selection, Maeda and Kimura[30] obtained the following model for the distribution of the electrons at large altitudes (from 1,000 to 16,500 km):

$$N = 1.8 \cdot 10^5 \exp\left[-6.57 \cdot 10^{-4}(h - 300)\right] \text{ el/cm}^3 \qquad (3.2)$$

where h is the altitude above the earth in kilometers. When $h \simeq 16{,}500$ km, it is assumed that the electron concentration diminishes with altitude much more slowly than follows from (3.2). The assumed distribution, characterized by a sharp change in the dependence $N(h)$ at $h \simeq 16{,}500$ km [up to this altitude the electron concentration diminishes exponentially, and $N(h)$ varies little at $h > 16{,}500$ km], is apparently not a satisfactory approximation for large altitudes. Actually, if we put in (3.2) $h = 16{,}500$ km, we obtain $N \sim 36$ el/cm^3. Such a value of the electron concentration cannot be excluded, but in many cases it is much less than the concentrations indicated in other sources. In addition to the values of N given above, one can indicate according to references 3, 30, and 39 that it follows from measurements of the zodiacal lights that $N \simeq 600$ el/cm^3. It is probable that when $h \gtrsim 2r_0$ the decrease in the electron concentration can no longer be described by a relation of the form (3.2), which leads to a very sharp reduction in the values of $N(h)$. The fact that the distribution (3.2) was chosen to fit the experimental data on the dispersion D is not a decisive argument, since this choice, as is clear from the character of relation (2.23), is not unambiguous.

In those cases when the whistlers are registered at stations with relatively low geomagnetic latitudes, as was already indicated in Sec. 1, a clear cut connection is observed between the values of the coefficients of dispersion D and the state of the F layer of the ionosphere. Under these conditions, the values of the dispersion D can change substantially within several hours by a quantity on the order of 50%. A comparison of the values of the dispersions D, obtained both from observation and from calculation, was given in reference 11 on the basis of data on two reception points, located in Japan at geomagnetic latitudes 24.5° and 35.3°. It has been shown that for these latitudes the observed dispersion values cannot be explained by using only the distribution (3.1), in which it is assumed that $N_0 \simeq 600$ el/cm^3. This conclusion is connected with the contribution made to the overall dispersion by the passage of whistlers through the ionosphere and through the region directly adjacent to the F layer. In view of the presence of noticeable variations in the electron concentration in this region, we can establish the degree of its influence. In the final analysis, we can determine from the dispersion data the electron density in the region of the upper atmosphere at altitudes 1300 – 3200 km, for which the daily variations of D are small. Reference 11 gives for these altitudes an average value of $N \simeq (2.1 \text{ to } 2.4) \times 10^4$ el/cm^3 for the electron concentration.

Observations of whistling atmospherics open up certain possibilities of determining the values of the earth's magnetic field at considerable altitudes. In particular, the correctness of the dipole approximation for this field can apparently be estimated. This possibility has not yet been fully investigated at present, and the literature[4,6] contains only individual comparisons, based on the use of the relation $f_N = f_H/4$, (2.17), which connects the nose frequency f_N with the gyrofrequency $f_H = \omega_H/2\pi$. It is assumed here that the values of f_H must be taken near the part of the trajectory which is farthest away from the earth's surface. Neither this statement nor relation (2.17) itself can be considered sufficiently well founded (see Sec. 2). The experimental data, which are far from complete, apparently indicate that the ratio f_N/f_H exceeds the value $f_N/f_H = 0.25$, subject to the stipulations made concerning the values of the frequency f_H. This disagreement cannot be attributed to deviations of α from zero (it is assumed in references 4 and 6 that $\alpha = 0$), since it follows from the analysis of Eq. (2.16), to the contrary, that $f_N/f_H < 0.25$ when $\alpha \neq 0$. Reference 4 indicates two possible ways of eliminating this difficulty. First, the earth's magnetic field may decrease more slowly than in the dipole approximation, used everywhere in the calculations. Furthermore, this discrepancy may be due to the fact that the whistler arrives at the point of reception by the waveguide path, coming out of the ionosphere farther south of the place of observation (it is assumed that the latter is in the northern hemisphere).

It must be added to the foregoing, however, that even the theoretical determination of the nose frequency given in references 4 and 6 is not convincing. The experimental data cited there can be explained without resorting to the assumptions indicated above, if it is found upon a more detailed determination of the nose frequency that its values are connected not only with the apex part of the trajectory.

We can expect valuable information on the processes in the upper atmosphere to be obtained from an analysis of the singularities in the reception of the low-frequency components in the spectra of whistlers (with frequencies $f = 1{,}000 - 400$ cps), when the hydrogen ions begin to exert an influence. Definite information can apparently be obtained by analyzing the intensity of the whistlers, and particularly by determining their absorption. These problems, however, are at present merely raised, and have no definite solution as yet.

As regards VLF radiation, the following remark can be made here. We have indicated that many spectra of radio signals of this type disclose radiation at a constant frequency. From the point of view of the mechanism proposed in Sec. 2 (and this mechanism certainly works, although it possibly is not the only mechanism) this frequency permits an estimate of the ratio N/H_0 along the trajectory. This ratio should remain constant, so that still another possibility of estimating the electron density is found. On the other hand, knowledge of the constant frequencies makes it possible to obtain the ratio V/c, i.e., to estimate the velocity of the corpuscular streams that penetrate into the earth's upper atmosphere (see Sec. 2).

In addition, the VLF radiation has a certain feature distinguishing it from whistlers, which may yield additional data. We have in mind the fact that the VLF radiation is generated at considerable altitudes. If an echo is observed in addition to the main signal, then the delay time of this echo can obviously give an idea of the length of path of the signal from the point of generation to the observer.

CONCLUSION

From the analysis of the data on the propagation and generation of low-frequency waves in the upper atmosphere it follows that at present we can consider established the main laws of propagation of these waves and the factors that cause the occurrence of whistlers and VLF radiation. On the basis of the observations performed, we can establish several important laws. However, the interpretation of many peculiarities in the propagation and generation is still qualitative in character or is completely lacking. We wish to mention again briefly the basic unsolved problems which perhaps will serve as the objects of further research.

Of considerable interest in the case of whistlers is a more thorough experimental study of the nose whistlers, and also of the region of frequencies higher than the nose frequency. In particular, it would be essential to determine the limiting frequencies, above which propagation of whistlers becomes impossible. In this connection, a more accurate theoretical determination of the nose frequency is necessary, with allowance for the contribution of all the parts of the whistler trajectory. In all probability, absorption must be taken into account in calculating the limiting frequencies. We must add, incidentally, that we did not touch upon any problems connected with the calculation of intensity of radiation of whistlers with simultaneous allowance for focusing and absorption.

The low frequency domain ($f \lesssim 1$ kcs) is interesting in that the motion of ions in the plasma may become influential. A preliminary analysis shows that the channelling of the radiation should decrease at low frequencies, and this may place a low-frequency limit on the whistler spectra. This question needs a detailed analysis from both the theoretical and experimental points of view.

Many phenomena connected with multiple-path propagation must be explained (the appearance of whistler pairs, combined propagation paths, the presence of multiple nose whistlers with gradual decrease in the nose frequency). It is essential to explain the conditions under which several channels are produced, and to relate the appearance of these channels with definite states of the upper atmosphere.

The question why discharges with a maximum near 5 kcs in the spectrum are the most favorable for the generation of whistlers is still unanswered.

Many problems of theoretical nature arise for VLF emissions, since only the first steps have been made in the existing researches. It is necessary to establish first the cause of the differences in the origins of radiation with continuous spectrum (hissing) and radiation with a discrete spectrum. The theory will progress apparently along the path of investigating the interaction between streams of charged particles and plasma in the presence of an external magnetic field, and determination of the singularities in the radiation from such systems.

It is necessary also to improve the methods by which the data on the propagation and generation of low-frequency waves can be used to obtain information on the construction of the upper atmosphere. Of great importance in problems of this kind is the determination of the trajectories of propagation of low-frequency signals under different conditions.

Note added in proof. Recently several papers have appeared dealing with the subject of the present review. Gallet et al.[52] discussed the possibilities of investigating the propagation of microwaves of the whistling atmospheric types in a dense laboratory plasma. This question may be of interest also for plasma diagnostics. Experiments using the "Zeta" apparatus have been started. Dinger[53] gives a survey of the data obtained in the U.S.A. by observation of whistlers, starting with the beginning of 1955. Interesting data connected with the reception of whistlers and VLF radiation in Greenland for a point located 150 km away from the geomagnetic pole are given by Ungstrup.[54] It is interesting that a cutoff is observed in the spectra of whistlers at relatively low frequencies. The corresponding minimal frequencies, however, are quite high (from 2.3 to 6.2 kcs).

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Translated by J. G. Adashko