UNDISCOVERED ISOTOPES OF LIGHT NUCLEI

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1. INTRODUCTION

At present about three hundred isotopes of light nuclei ($Z \lesssim 36$) are known, and considerable experimental material concerning their properties has been accumulated. Even a superficial analysis of the available data shows that in the region of light nuclei there should exist hundreds of as yet undiscovered isotopes, stable with respect to heavy particle emission. In the present paper we shall give a summary of the main properties of isotopes whose existence is conjectured and, in conclusion, discuss very briefly the question of the boundaries of the region of stable nuclei.

In Sec. 2 we shall consider neutron-deficient isotopes with $Z > N$. For the prediction of their properties we use the fact that (as it follows from charge invariance) the properties of two isotopically conjugate nuclei [a nucleus (A, Z_1, N_1) is said to be isotopically conjugate to the nucleus (A, Z_2, N_2), if $Z_1 = N_2$ and $N_1 = Z_2$] coincide except for Coulomb corrections and corrections for the neutron-proton mass difference. (For a bibliography on this question see reference 2.) Using the fact that these corrections are easily taken into account, we can establish the main properties of the whole series of isotopes with $Z > N$ from the known properties of the experimentally studied nuclei with $N > Z$. New phenomena should be observed in the neutron-deficient nuclei — proton and two-proton radioactivities; Sec. 3 is devoted to this problem, where we roughly delineate the region of nuclei where these phenomena may occur and also consider the main properties of two-proton radioactivity.

Section 4 is devoted to nuclei with a large neutron excess. The procedure described above no longer gives anything new, but one can still make various predictions by basing oneself on the empirical and semiempirical regularities in the binding energies of neutrons and protons, and the locations of levels with different values of the isotopic spin T.

The general question of limits of stability of nuclei is discussed briefly in Sec. 5.

2. NEUTRON-DEFICIENT ISOTOPES

The total energy of the nucleus (A, Z, N), or, briefly, (A, Z) can be written as

$$ {}_ZM_N^A c^2 = E^A(T) + E_c(A, Z) + c^2(Zm_p + Nm_n), \qquad (1) $$

where $E^A(T)$ is that part of the energy which is due to the specifically nuclear interaction between the nucleons in the nucleus; $E_c(A, Z)$ is the energy of Coulomb interaction of the protons, and m_p and m_n are the mass of the proton and neutron respectively. Because of the charge invariance of nuclear forces, $E^A(T)$ depends only on A and on the isotopic spin T of the ground state of the particular nucleus. Usually $E^A(T)$ increases with increasing T. Therefore the ground state corresponds to the minimum possible value of T for given A and Z, i.e., $T = \frac{|Z-N|}{2}$. The only exceptions from this rule are the nuclei with $A \gtrsim 34$, for which $E^A(0)$ is greater by ~ 0.5 Mev than $E^A(1)$. This part of the total energy does not change when we go from the nucleus (A, Z) to the isotopically conjugate nucleus $(A, A-Z)$. In first approximation the Coulomb energy is

$$ E_c(A, Z) \simeq Q\frac{Z(Z-1)}{A^{1/3}}, \quad Q \simeq 0.6 \text{ Mev} \qquad (2) $$

and represents the energy of electrostatic interaction of the Z protons with one another; each of the protons interacts with the $(Z-1)$ other protons; the numerical coefficient corresponds to the assumption that all the protons are distributed with equal probability over the interior of a sphere of radius $R \approx 1.45 \times A^{1/3} \times 10^{-13}$ cm, and that the probability of finding a proton at a given point in space is independent of the positions of the other protons; the expression $0.6\frac{Z(Z-1)}{A^{1/3}}$ is the average value (mathematical expectation) of the Coulomb energy under these assumptions.

We mention that direct measurements of the charge density distribution in the nucleus from electron scattering by nuclei[4] give a considerably smaller effective radius of the nucleus — about $1.05\ A^{1/3} \times 10^{-13}$ cm; one therefore might expect that the factor in the expression for the Coulomb energy would be 0.83 Mev in place of the value 0.6 Mev, which is in agreement with data on the energies of light nuclei. Actually, there are all sorts of factors which reduce the average value of the Coulomb energy compared with the value given above from elementary computations. These are the so-called exchange and correlation corrections.[3,5] The first of these is related to the fact that the protons are identical particles, and tends to reduce the Coulomb energy. The correlation correction arises from the effect of nuclear and Coulomb forces on the shape of the wave function for the two protons which are located close to one another. The sign of this correction depends on the detailed form of the nuclear interaction.

As a general conclusion we may state that the expression (2) for the Coulomb energy is approximate; it is therefore natural that the coefficient Q in it should be chosen empirically from data on energies of nuclei and not from the nuclear radius; it is also natural that even after this is done there should remain notable discrepancies between the formula and experiment. Thus, for example, for $Z = 3$ (cf. the two right-hand columns of Fig. 1) the Coulomb energy per proton is 0.6 Mev smaller than according to formula (2). Knowing the energy of Li^8 and applying formula (2) to the computation of the energy of B^8, we come to the conclusion that B^8 is unstable. Actually B^8 is stable, its existence can be observed in experiment, and it has a binding energy for the proton of ~ 0.2 Mev (instead of − 0.4 Mev according to the formula).

Z \ N	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	$\frac{1.2(Z-1)}{(2Z-1)^{1/3}}$	ΔE_{np} aveg.
H 1																							
He 2	0,76	0,76																					0,76
Li 3		0,84	0,84	0,83																		1,42	0,84
Be 4		1,63	1,64	1,64	1,70	1,78																1,91	1,68
B 5			1,90	1,86	1,85	2,02																2,33	1,91
C 6			3,01	2,94	2,76	2,76	2,64															2,73	2,82
N 7					2,88	3,01	3,01	3,02	3,20													3,10	3,02
O 8						3,55	3,54	3,54	3,25													3,45	3,47
F 9						3,50	3,26	3,55	3,55	3,43												3,79	3,46
Ne 10								3,91	4,05	4,05	(5,55											4,10	(4,0)
Na 11									(5,81)	4,30	4,30											4,40	(4,3)
Mg 12											4,86	4,86	5,00									4,70	4,89
Al 13											5,21	5,08	5,08									4,98	5,12
Si 14													5,61	5,61	(4,99)							5,26	5,39
P 15													5,09	5,75	5,75							5,54	5,53
S 16															6,25	6,25						5,80	6,25
Cl 17																5,98	5,98					6,05	5,98
Ar 18																	6,76	6,76				6,32	6,76
K 19																		6,88	6,88			6,56	6,88
Ca 20																			7,43	7,43	7,31	6,80	7,39
Sc 21																			6,63	6,74	6,74	7,04	6,71

So we see that the second term in (1) can be computed without difficulty, while the third term is unknown. This enables us from the experimentally known mass ${}_{Z}M_{N}^{A}$ to obtain the value of the nuclear energy $E^{A}(T)$ and then, using it, to calculate the mass ${}_{N}M_{Z}^{A}$ of the isotopically-conjugate nucleus. Thus, charge invariance makes it possible to determine the total energy of the neutron-deficient nucleus (A, Z) (with $Z > N = A - Z$), if the mass of the nucleus (A, A − Z) is known. Moreover, it follows from charge invariance that the wave functions of the nuclei (A, Z) and (A, A − Z) coincide provided that one interchanges neutrons and protons. Therefore, the two nuclei have very similar properties, and consequently, from the known properties of one of them, one can predict the properties of the other.

All these simple considerations are applicable only to light nuclei ($A \lesssim 50$), in which the Coulomb energy is still small compared to the nuclear energy and where, consequently, one can still use the idea of charge invariance.

In analyzing experimental data it is more convenient not to use formula (1), but rather the formula for the binding energy of the nucleons which is easily obtained from it.[6] If we denote by $E_n(A, Z)$ and $E_p(A, Z)$ the binding energies of the neutron and proton in the nucleus (A, Z), then it follows from (1) and (2) that the binding energy of the Z'th proton in (A, Z) is expressed in terms of the binding energy of the Z'th neutron in the isotopically conjugate nucleus (A, A − Z) as follows:

$$E_p(A, Z) = E_n(A, A-Z) - \Delta E_{np}, \tag{3}$$

where

$$\begin{aligned} \Delta E_{np} &= [E_c(A, Z) - E_c(A-1, Z-1)] \\ &\quad - [E_c(A, A-Z) - E_c(A-1, A-Z)] \\ &= 2Q\frac{Z-1}{(2Z-1)^{1/3}}\left[1 + \left(\frac{A-2Z}{3A}\right)^2\left(1 - \frac{5}{A-2Z}\right) + \ldots\right] \\ &\approx 1.2\frac{Z-1}{(2Z-1)^{1/3}} \text{ Mev.} \end{aligned} \tag{4}$$

The second term in the square brackets is very small for real nuclei (no more than a few percent), and we shall neglect it throughout.* Remarkably, ΔE_{np} turns out to be independent of A and N. Therefore, for all of the isotopes of a given element, one can use the same value for the quantity ΔE_{np}, determined, for example, from the difference ΔE_0 of the energy for removing a neutron and a proton in the even self-conjugate nucleus (2Z, Z):

$$\Delta E_{np} \approx \Delta E_0 = E_n(2Z, Z) - E_p(2Z, Z). \tag{4'}$$

Formulas (4) and especially (4′) are in beautiful agreement with the present known values of E_n and E_p, as one sees from Fig. 1. Comparing the experimental data with the computations from (4) (the second on the right of the graph in Fig. 1), we see that qualitatively formula (4) is valid for all the light nuclei, while for obtaining quantitative results, it can be used beginning with $Z = 6$. However, it is better always to use the average experimental values, (the last column in Fig. 1) or the values of ΔE_0 (which are indicated by the thick dark squares in Fig. 1). The points for Ne^{21} and Na^{20} which are not contained in this general picture show the inaccuracy of the present value for the mass of Na^{20}.

Sometimes it proves to be more convenient to use the following formula for the mass difference of distant mirror nuclei:

*For two mirror nuclei with $|Z - N| = 1$, formula (4) goes over into the well-known relation $\Delta E_{np} = 1.2(Z-1)A^{-1/3}$. In the general case of isotopically conjugate nuclei with $|Z - N| > 1$ one must take into account the fact that the denominator in (4) is equal to $(2Z-1)^{1/3}$, and not to $A^{1/3}$, i.e., it does not depend on N.

$$_{Z}M_{N}^{A}-{}_{N}M_{Z}^{A}\approx(Z-N)\Delta M_{0},\tag{5}$$

where

$$\Delta M_{0}={}_{\frac{A+1}{2}}M^{A}_{\frac{A-1}{2}}-{}_{\frac{A-1}{2}}M^{A}_{\frac{A+1}{2}}\quad\text{for odd } A,$$

and

$$\Delta M_{0}=\frac{1}{2}\{{}_{\frac{A}{2}+1}M^{A}_{\frac{A}{2}-1}-{}_{\frac{A}{2}-1}M^{A}_{\frac{A}{2}+1}\}\quad\text{for even } A.$$

Using (3) — (5), one can, on the basis of the extensive experimental data concerning E_n for nuclei with $N > Z$, find the binding energies of the protons in the conjugate nuclei with $Z > N$ and establish whether the latter are stable with respect to emission of heavy particles. (It is clear that, as a rule, for $Z > N$ the threshold for emission of a proton should lie lower than the threshold for emission of n, d, α etc.) In addition, one is able also to establish other properties of these nuclei: mass defects, energies and periods of β decay, the occurrence of proton and two-proton radioactivities.

All of the neutron-deficient isotopes and their properties which are predicted in this way are collected in Table I, which is given at the end of the paper and which contains also the series of neutron-rich nuclei concerning which we will talk later. We illustrate the method of construction of the table with an example.

It is known that the nucleus ${}_{11}Na^{25}_{14}$ exists with a binding energy $E_n(Na^{25}) = 8.84$ Mev. The nucleus which is isotropically conjugate to Na^{25} is the as yet unknown isotope ${}_{14}Si^{25}_{11}$. By means of (3) we find the binding energy of a proton, $E_p(Si^{25}) = E_n(Na^{25}) - 5.2 = 3.64$ Mev., i.e., this isotope is stable with respect to proton emission. Using the tabulated data[1] we calculate the mass defect of this nucleus: 11 Mev. We obtain the energy of the β^+ decay, $Si^{25} \rightarrow Al^{25}$ (ground state) from the known masses of $Al^{24,25}$: $E_{\beta^+} = M(Al^{24}) + m_H - 3.64 - M(Al^{25}) - 1.02 = 11.6$ Mev.

If the masses of these nuclei were known, the maximum energy of the β^+ decay could be estimated crudely from the formula

$$E_{\beta^+}({}_{Z}M_{N}^{A})-E_{\beta^-}({}_{N}M_{Z}^{A})\approx\left[1.2\,\frac{A-1}{A^{1/3}}-2.6\right]\text{ Mev}\tag{6}$$

[here $2.6 \text{ Mev} = 2(m_e + m_n - m_H)c^2$], which relates the end points of the β decays in isotopically conjugate nuclei and follows directly from (1) and (2). According to (4) we obtain for Si^{25}, $E_{\beta^+} = 4.0 + 7.3 = 11.3$ Mev [it is known that $E_{\beta^-}(Na^{25}) = 4.0$ Mev], which differs by only 3% from the value of 11.6 Mev given above.

The half-life of Si^{25} can also be evaluated starting from data on the decay of the conjugate nucleus Na^{25}. To do this we must take the experimental log ft (=5.2) for Na^{25} and, using the value obtained above for the β^+ decay energy of Si^{25}, find the half-life of this nucleus. (The quantity ft is determined by the shape of the wave function, and, if the considerations of charge invariance are correct, the ft values for transitions of the same type in conjugate nuclei should be identical.) In this way one obtains $T_{1/2}(Si^{25}) \sim 0.5$ sec.

In estimating β decay periods one must keep in mind the possibility of "superallowed" transitions without change of isotopic spin. The point is that one of the consequences of charge invariance is the existence of similar states in nuclei with the same value of A, but with different relative numbers of neutrons and protons [for example, (A, Z) and (A, Z − 1)]. The spatial parts of the wave functions $\psi(A, Z)$ and $\psi(A, Z-1)$ of the similar states are identical with one another, and the states themselves are obtained from one another by replacing one of the protons in the nucleus (A, Z) by a neutron. The value of log ft for transitions between similar states (called "superallowed" transitions) has the minimum possible value $\simeq 3.5$, since in such a transition there is practically no readjustment of the nuclear wave function.

The β^+ decay energy for "superallowed" transitions $(A, Z+1) \rightarrow (A, Z)$ is given very simply:

$$E_{\beta^+}(\Delta T=0)\approx\left(1.2\,\frac{Z}{A^{1/3}}-1.8\right)\text{ Mev}\tag{7}$$

[where $1.8 \text{ Mev} = c^2(2m_e + m_n - m_H)$] and changes very little with change in A for a given value of Z. If E_{β^+} exceeds the threshold given by (7), the "superallowed" transitions begin to compete with transitions to the ground state. As an illustration, we show in Fig. 2 the decay schemes of the isotopically conjugate

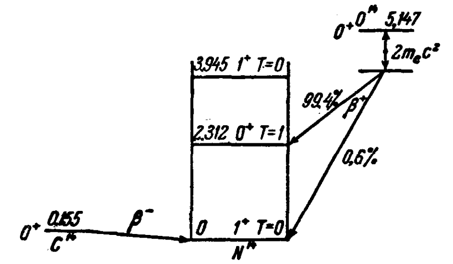

FIG. 2. Decay scheme of the isotopically conjugate nuclei C^{14} and O^{14}.

nuclei ${}_{6}C^{14}_{8}$ and ${}_{8}O^{14}_{6}$. The first of these can decay only to the ground state of N^{14} (for this transition, log ft ≈ 9.0). The O^{14} nucleus is raised considerably above N^{14} because of the additional Coulomb energy as compared with C^{14}. For this reason, the transition to the excited $T = 1$ state of N^{14} (at 2.31 Mev) is allowed energetically, this state being the analog of the ground states of C^{14} and O^{14}. Because of the small value of log ft (=3.5), this transition is dominant (99.4% of all decays). Many (and beginning with $Z > 20$, practically all) of the neutron-deficient isotopes predicted here should undergo β^+ decay with $\Delta T = 0$ and subsequent cascade emission of γ rays. In such cases the values of E_{β^+} given in Table I characterize the sum of the maximum energy of the positrons and the energies of the succeeding γ transitions. The values of the maximal energy of positrons for "superallowed" transitions vary from ~ 5 Mev for calcium to ~ 8 Mev for selenium, while the half-lives (for $\log ft \approx 3.5$) vary from ~ 0.5 to ~ 0.07 sec.

A confirmation of this is the set of values of the half-lives of the series of β^+-active isotopes already known with $Z = 13 - 25$, for which $E_{\beta^+} > (1.2\,Z/A^{1/3} - 1.8)$ Mev (for example, Al^{24}, Ca^{38}, V^{46}, and Mn^{50}).

For lighter nuclei, because the energy is low, "superallowed" transitions are quite slow (for example, ~ 1 min for O^{12}), and therefore more often the transitions to the ground state are most important.

The example of Si^{25} considered above gives a good illustration of the method for constructing the table, and we shall not belabor this point any further.

Below we give a list of stable isotopes which are located, according to our estimates, on the limits of stability with respect to proton emission:

Li^6, Be^7, B^8, C^9, N^{12}, O^{13}, F^{17}, Ne^{17}, $Na^{20\,(19?)}$, $Mg^{19(18?)}$, Al^{23}, $Si^{23(22?)}$, P^{27}, S^{26}, Cl^{31}, Ar^{30}, K^{35}, $Ca^{35(34?)}$, $Sc^{40(39?)}$, Ti^{39}, $V^{43(42?)}$, Cr^{43}, Mn^{46}, Fe^{45}, $Co^{50(48?)}$, Ni^{48}, Cu^{54}, $Zn^{55(54?)}$, $Ga^{60(59?)}$, Ge^{59}, $As^{64\,(?)}$, $Se^{62\,(61?)}$.

Comparison of this list with the table of isotopes known at present[1] leads to the conclusion that it is possible to discover approximately ninety new neutron-deficient isotopes of the light nuclei.

All of the neutron-deficient isotopes which are predicted by the method described above are collected in Table I, in which there are also given some nuclei predicted on the basis of the data of Sec. 4 (cf. below). The general appearance of the region of stable nuclei is apparent from the upper part of Fig. 3.

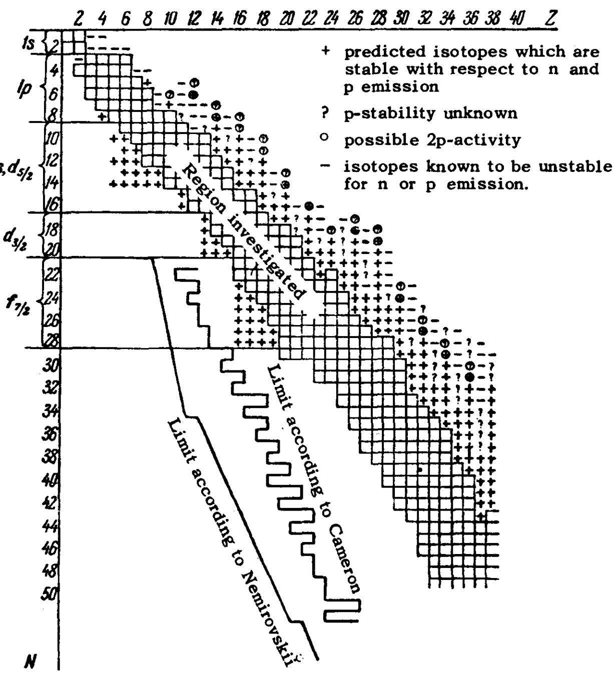

FIG. 3

With regard to the method applied above for determining the stability of neutron-deficient isotopes, we should make the following remark: many of the nuclei predicted (and, in particular, all those lying on the limits of the stability region) are located right near the threshold for proton emission.

The wave functions of these nuclei should therefore extend to considerably greater distances than is the case for their isotopically conjugate colleagues which are located far from the limit of stability. (At large distances, $\psi \sim e^{-\kappa r}$, where $\kappa = \sqrt{2\mu E/\hbar^2}$, E is the binding energy of the last nucleon.) The wave functions of the conjugate nuclei (A, Z) and (A, A – Z) are in this case rather different from one another. (This manifests itself very clearly when the energy of the nucleus which is being considered is close to the threshold for emission of an S neutron. In the case of neutrons with $l \neq 0$ or in the case of protons, the wave function spreads out only slightly because of the influence of the centrifugal or Coulomb barrier which tries to hold it back.) In this connection there arises the question of the applicability of the idea of isotopic invariance for estimating the energies of these more or less stable nuclei. In addition one can question the validity of (2) – (5) for nuclei with $Z > 10$, where the Coulomb energy becomes comparable with the nuclear energy. However, it is easy to see that estimates using (2) – (5) give too low values for the binding energy of a proton in neutron-deficient nuclei; consequently, if the nucleus is stable according to our computations, it is certainly stable in actuality. As a matter of fact, the procedure actually used is that in considering the neutron-deficient nucleus (A, Z) one takes the wave function ψ of the conjugate nucleus (A, A – Z) and uses it to calculate the energy (1) of the nucleus (A, Z): $M(\psi, H\psi)$, where H is the Hamiltonian of this nucleus. In doing this one actually has to calculate only the second and third terms in (1), since the value of the first term is taken from data concerning the nucleus (A, A – Z). But there is a rigorous theorem that the matrix element $(\psi, H\psi)$ calculated with inexact wave functions (ψ in our case) is always greater than the exact value $(\varphi, H\varphi)$ calculated with the exact wave function of the nucleus (A, Z). The statement given above follows from this.

In conclusion we mention that the best practical means for obtaining neutron-deficient isotopes of light nuclei are the (p, xn) and (He^3, xn) reactions, as well as reactions using multiply charged ions. It should however be remembered that because of the low Coulomb barrier the boiling off of protons will not be hindered and that therefore the cross section for formation of neutron-deficient isotopes should be small.

3. PROTON AND TWO-PROTON RADIOACTIVITY

At the limit of stability of neutron-deficient nuclei with respect to decay with proton emission one should observe a new physical phenomenon: protonic radioactivity.[6a] However, the probabilities for detecting this phenomenon are quite small.[6a] For a lifetime with respect to p-decay greater than $1-10$ sec, the effect will be strongly masked by β^+ decay. On the other hand, for half-lives for p-radioactive nuclei less than 10^{-12} sec, it seems to be impossible to observe the delayed emission of the proton even by using thick-layered emulsions or Wilson chambers. The half-life of proton-radioactive isotopes can be determined approximately from the formula

$$\log T_{1/2}\,(\text{sec}) \approx 0.43Z^{2/3}f(x)-22, \tag{8}$$

where x is the ratio of the energy of the emitted proton to the height of the Coulomb barrier; for $x \ll 1$

$$f(x)=0.6x^{-1/2}\left[\arccos x^{1/2}-x^{1/2}(1-x)^{1/2}\right]\approx 0.6\left(\frac{\pi}{2}x^{-1/2}-2\right). \tag{9}$$

To the interval $T_{1/2} = 10^{-12}-10$ sec there correspond energies of the emitted protons up to 0.04 Mev for $Z = 10$; $0.1-0.35$ Mev for $Z = 20$; $0.2-0.7$ Mev for $Z = 30$, and $0.35-1.1$ Mev for $Z = 40$. The accuracy of all these computations of masses of neutron-deficient nuclei is, of course, insufficient for predicting for precisely which of the isotopes the half-lives with respect to proton radioactivity will fall between the limits given above. However, it is clear that this phenomenon must be sought close to the stability limits given above (or among the various heavier nuclei which are not considered here).

Because of the very high binding energy of neutrons for these isotopes, all of them are stable with respect to α decay from the ground state.

There is also the possibility of observing another new effect which is much more interesting than proton radioactivity. We are speaking of the phenomenon of two-proton radioactivity[6] which is characteristic for the nuclei of elements with even Z which are located near the stability limits. Because of pairing effects, for such nuclei, even when there is still a positive binding energy for a single proton, there can already occur an instability with respect to simultaneous emission of two protons. Such an instability can give rise to two-proton radioactivity of various isotopes which are stable with respect to proton and α decay.

Let us give an example. The nucleus F^{15} is unstable with respect to the decay $F^{15} \to O^{14} + p$, which occurs with the liberation of an energy of 2.3 Mev. However, the neighboring even nucleus Ne^{16}, as one sees from Table I, is stable with respect to the decay $Ne^{16} \to F^{15} + p$, which requires, as can be shown on the basis of Sec. 2, the expenditure of an energy between 0.5 and 1.8 Mev. Consequently, the $Ne^{16} \to Fe^{15} + p$ decay is energetically impossible, while the $Ne^{16} \to O^{14} + 2p$ decay occurs with liberation of an energy between 0.5 and 1.8 Mev, which is much less than the height of the Coulomb barrier of the Ne^{16} nucleus for a doubly charged particle (~ 6 Mev). Thus the nucleus Ne^{16} should have the property of two-proton radioactivity.

Other nuclei which may also be 2p-radioactive are:

$Si^{21(22?)}$, $S^{25(24?)}$, $Al^{29(28?)}$, $Ca^{33(34?)}$, Ti^{48}, Cr^{42}, $Fe^{44\,(43?)}$, $Ni^{46(47?)}$, $Zn^{53(54?)}$, $Ge^{59(58?)}$, $Se^{63(62?)}$, $Kr^{67(66?)}$ etc.

The probability of a two-proton decay is much less than for one-proton decay since it contains a product of two penetration factors of the Coulomb barrier. In fact, it is not hard to see that probability of two-proton decay in which the protons carry off energies ϵ and $E-\epsilon$ must have the form:

$$w(\varepsilon, E-\varepsilon)=\frac{a}{\sqrt{\varepsilon(E-\varepsilon)}}\left|\psi_\varepsilon(R)\,\psi_{E-\varepsilon}(R)\right|^2,$$

where a is a constant and $\psi_\epsilon(R)$ is the value of the Coulomb function F_0, corresponding to the proton with energy ϵ at the nuclear radius R. (It is assumed that the protons are emitted with zero orbital angular momentum.) For protons which are far below the barrier,

$$\frac{1}{\sqrt{\varepsilon}}\left|\psi_\varepsilon(R)\right|^2\sim\exp\left\{-\frac{\pi}{\hbar}Ze^2\sqrt{\frac{2m}{\varepsilon}}\right\}, \tag{10}$$

where Z is the charge of the nucleus A, m is the proton mass, and

$$w(\varepsilon,\ E-\varepsilon)=a\exp\left\{-\frac{\pi}{\hbar}Ze^2\sqrt{2m}\left(\frac{1}{\sqrt{\varepsilon}}+\frac{1}{\sqrt{E-\varepsilon}}\right)\right\}.$$

As we see from this formula, the most probable case is the emission of the two protons with the same energy $\epsilon = E/2$, since then the expression in the numerator of the exponential is a minimum. From this it follows immediately that the total decay probability

$$w(E)=\int_0^E d\varepsilon\cdot w(\varepsilon, E-\varepsilon)\sim a\exp\left\{-\frac{\pi}{\hbar}(2Z)\,e^2\sqrt{2(2m)}\,\frac{1}{\sqrt{E}}\right\} \tag{11}$$

has exactly the same form as if we emitted a doubly charged particle with mass $2m$ — a diproton for which the Coulomb barrier is twice as high as for the proton [cf. (11) and (10)]. Therefore, the lifetime of isotopes with respect to two-proton radioactive decay, for a much larger energy interval than for proton radioactivity, falls in the region convenient for detection ($T_{1/2} = 10^{-12}$ to 10 sec). Two-proton radioactivity can be observed successfully (for example, by using thick emulsions or Wilson chambers) also in those cases where the decay occurs almost instantaneously ($T_{1/2} \gtrsim 10^{-19}$ sec), since there should be a strong correlation between the energies of the two emitted protons. Thus, the probability of 2p decay in which one of the protons obtains the energy fraction $(0.5 + \kappa)$ and the other proton the fraction $(0.5 - \kappa)$ is proportional to $\exp\left\{-\frac{6\pi Ze^2\sqrt{m}}{\hbar\sqrt{E}}\kappa^2\right\}$. Consequently, to demonstrate the two-proton mechanism of radioactive decay it is sufficient to show that the distribution of differences of energy of the two sub-barrier protons does not have statistical character, but is concentrated in the neighborhood of a zero value for this difference.

The interaction of the pair of protons which are emitted by 2p-radioactive nuclei, not only inside the nuclear potential well, but also under the barrier, must of course result not only in an energy correlation, but also in the associated definite angular correlation of the protons.

Two-proton radioactivity must, of course, be distinguished from the usual chains of successive p decays of the type:

$$\begin{array}{cccccccccccc}
Zr^{69} & \xrightarrow{p} & Y^{68} & \xrightarrow{p} & Sr^{67} & \xrightarrow{p} & Rb^{66} & \xrightarrow{p} & Kr^{65} & \xrightarrow{p} & Br^{64} & \xrightarrow{p} Sc^{63} \\
\downarrow \beta^+ & & \downarrow \beta^+ & & \downarrow \beta^+ & & \downarrow \beta^+ & & \downarrow \beta^+ & & \downarrow \beta^+ & \\
Y^{69} & \xrightarrow{p} & Sr^{68} & \xrightarrow{p} & Rb^{67} & \xrightarrow{p} & Kr^{66} & & (Br^{65}) & & Sc^{64} &
\end{array}$$

However, as Fig. 4 illustrates, the transition from such a successive emission of protons to two-proton radioactivity occurs smoothly. Suppose that the energy $E_{p\,\mathrm{odd}}$ of binding of the odd $(2m + 1)$st proton is negative, i.e., that the nucleus with $Z = 2m + 1$ is unstable. So long as the binding energy of the next even $(2m + 2)$nd proton $E_{p\,\mathrm{even}}$ is also negative and only a little less than $E_{p\,\mathrm{odd}}$, we have the usual chain of successive acts of p-decay. When $E_{p\,\mathrm{even}}$ becomes such a small negative quantity that the binding energy for two protons $|E_{pp}| > 8\,|E_{p\,\mathrm{even}}|$ (Fig. 4a), 2p-decay begins to compete noticeably with the usual sequence of p-decays. If the binding energy of the $(2m + 2)$nd proton $E_{p\,\mathrm{even}}$ is a positive quantity, but less than the half-width of the level from which the emission of the $(2m + 1)$st proton occurs, the processes are "simultaneous" and "successive" emission of two protons is already indistinguishable (Fig. 4b). As an example, we give the decay scheme of the nucleus Be^6 which is investigated in reference 7

$$(Be^6 \rightarrow p + Li^5 \rightarrow p + p + He^4).$$

Finally, in the case where $E_{p\,\mathrm{even}}$ is such a large positive quantity that the levels of emission of the $(2m + 2)$nd and $(2m + 1)$st protons do not overlap, we come to the "pure" case of two-proton radioactivity (Fig. 4c).

In concluding this section, we note that one can, in the case of neutrons, also observe phenomena analogous to two-proton radioactivity. Since there is no Coulomb barrier here, the centrifugal barrier is present only for $l \neq 0$ and hinders the emission of neutrons only slightly. Consequently, the levels from which the emission of the $(2m + 2)$nd and $(2m + 1)$st neutrons occurs are, as a rule, very broad and overlap. Therefore, for neutrons one can hardly ever have the variant of 2n decay corresponding to that shown in Fig. 4c, but it is entirely possible to have the successive emission, but with overlapping of levels, or the variant of type Fig. 4b. Multiple evaporation of neutrons from highly-excited nuclei (especially heavy nuclei) is well known. Such an evaporation proceeds in several stages in which the successively evaporated neutrons do not interact with one another, and the levels which occur before and after the evaporation of each of the neutrons do not overlap. In our case, however, we are talking precisely about simultaneous emission of a pair of neutrons, correlated in angle, and consequently also in energy by virtue of the interaction of the neutrons with one another. It is precisely such a correlation which must be investigated in the experimental observation of the phenomenon which we are considering, in order to distinguish it from the trivial cascade "evaporation" transitions. A convenient method for detecting two-neutron correlation may be the study of coincidences of delayed neutron pairs emitted by nuclei which have an excess of neutrons after a preceding β decay.

4. NUCLEI WITH A NEUTRON EXCESS

In this case we can no longer use the extremely simple procedure of Sec. 2, since, for nuclei with $N > Z$, it gives nothing new compared with that which is already known. It is physically clear that nuclei, with an excess of neutrons and with energy close to the threshold, must have very much looser structure than the nuclei in the middle of the stability region. Because of this, many of the properties of the nuclei

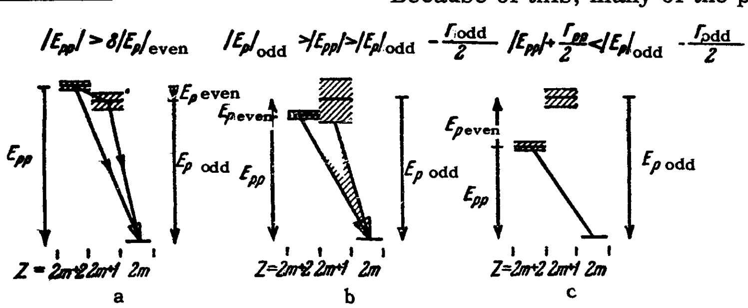

FIG. 4. Possible 2p-decay schemes.

located near the limits of the stability region may differ markedly from the properties of their more stable colleagues. In particular, the use of the self-consistent potential inside the nucleus may already be a poor approximation. This makes much more difficult any attempts at theoretical calculations in this region, and justifies the use of methods based on the extrapolation of empirical regularities.

The first method for prediction is based on regularities in the energies of excitation of levels with different isotopic spins.[8]

Making use of the known data concerning binding energies of light nuclei, one can construct curves on which are marked the energy differences of the lowest levels with isotopic spins $T = 0, 1, 2, 3 \ldots$ for nuclei with even mass numbers A (cf. Fig. 5). The smooth

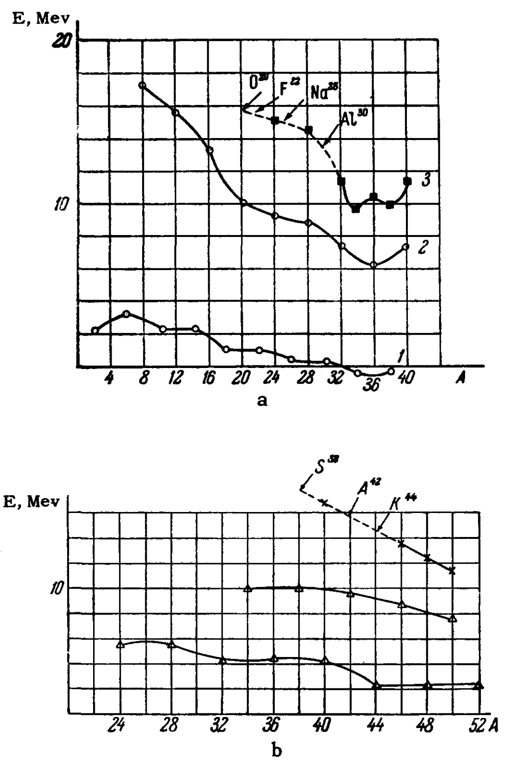

FIG. 5. Difference in energy of levels with different isotopic spins. a) o – energy differences of first states with T = 1 and T = 0; ■ – energy differences of first states with T = 2 and T = 0; b) x – energy differences of first states with T = 3 and T = 1; Δ – energy differences of first states with T = 2 and T = 1.

character of the curves thus obtained (the solid lines) enables us to extrapolate the curves into the regions where there are as yet no experimental data. (The extrapolated portions of the curves are shown as dashed lines.) By means of such an extrapolation one can predict the binding energies of certain as yet undiscovered isotopes. Thus, for example, from the curve (Fig. 5a) it is clear that for nuclei with mass number A = 30, the first state with T = 2 should lie at approximately 13.7 Mev above the first state with T = 0, and approximately 13.2 Mev above the first state with T = 1; the first level with T = 0 in the system of 30 nucleons corresponds to the ground state of the nucleus P^{30} (N = Z), the first level with T = 1 to the ground state of the nucleus Si^{30} (N – Z = 2), and finally the first level with T = 2 corresponds to the ground state of the nucleus Al^{30} (N – Z = 4). Thus, for A = 30 the nuclear part of the energy for the state with T = 2, $E^{A=30}(2)$ [cf. (1)] is 13.7 Mev greater than $E^{30}(0)$. In order to find the actual difference in energies of the nuclei Al^{30} and Si^{30}, one would have to include the differences in Coulomb energy and the mass difference of neutron and proton.

When this is done one finds that the total energy of Al^{30} is greater than the energy of Si^{30} by approximately 9 Mev. It is easy to see that the nucleus Al^{30} is stable with respect to decay into heavy particles. Consequently, Al^{30} should be a β emitter. Similarly, one evaluates the energies of decay of various other isotopes F^{22}, Na^{26}, S^{38}, P^{26}, Cl^{30}, which are given in Table I.

A second method[9] is based directly on extrapolation of the binding energies of neutrons in nuclei with the same value of A. Here it is important to take into account the following simple considerations. Nuclei with an excess of neutrons do not exist if all the discrete levels are already filled with neutrons. The number of levels, when one takes account of the neutron spin, is always even. Therefore, if there exists a nucleus containing an odd number of neutrons, (2m + 1), then there is also room for the next (2m + 2)nd neutron. Still, the absence of a nucleus with an odd number of neutrons does not exclude the existence of the neighboring isotope with an even number of neutrons. (For example, He^6 exists, even though He^5 is unstable by 1 Mev.) Because of the "pairing" of the neutrons, the binding energy of the (2m + 2)nd neutron is always greater than the binding energy of the preceding (2m + 1)st. This rule is rigorous within the limits of the shell model. It can be violated only when this model becomes meaningless, as, for example, in the case of the very lightest nuclei of the type H^4, H^5, etc. (if they exist).

In Fig. 6 are shown curves for the binding energy of the last neutron $E_n(Z, N)$ as a function of the number of protons Z for a fixed number of neutrons N. From Fig. 6 one sees that $E_n(Z, 2n+2) > E_n(Z, 2n+1)$ always. Therefore, for example, from the existence of Be^{11} (Z = 4, N = 7) and C^{15} (Z = 6, N = 9) it definitely follows that Be^{12} and C^{16} exist. Extrapolating, we find that the estimated binding energy of the neutron in Be^{12} is ~ 2 Mev and the β decay energy 12 – 13 Mev, while for C^{16} we obtain respectively ~ 3 Mev and ~ 8 Mev.

In order to proceed even further in taking account of the number of neutrons, we must refer to some of the data of the shell model.

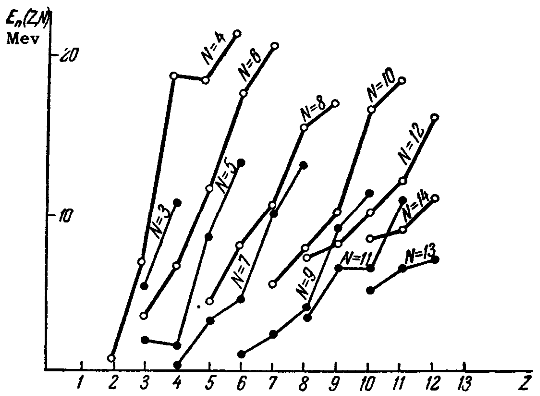

FIG. 6. Binding energy $E_n(Z,N)$ of a neutron in a nucleus as a function of the number of protons Z, for fixed values of N.

If we compute the energy of pairing interactions of several neutrons which are in the same shell, on the assumption that the radius of the neutron-neutron interaction is much less than the shell radius, we obtain the following simple result:[10] if the interaction of two neutrons in this shell is B, the interaction of three neutrons is also equal to B, while the interaction of four neutrons is 2B, i.e., the neutrons behave as if they were linked in pairs. The physical meaning of this result is entirely clear: if the radius of the forces between neutrons is small, this means that the neutrons can interact with one another only when their relative orbital angular momentum is equal to zero, i.e., when they are in an S state with respect to one another. But, according to the Pauli principle, no more than two neutrons can be in an S state with respect to one another, and consequently the neutron can interact only with some one other neutron among those present in the particular shell. Thus, if the number of neutrons in the shell is even (2m), the energy accumulated from the interaction energies of the Bm pairs is mB. If now we add to the shell another neutron, it cannot find a companion since all the neutrons are already joined in pairs. Therefore in this case the total energy of interaction remains equal to its previous value mB.

For the validity of all these arguments it is necessary that: a) the shell model be a good approximation for the particular nucleus; b) the binding energy of neutrons not be large (above it was already pointed out that this is the condition that the orbit radius be relatively large, cf. Sec. 2); and c) that there be no protons in the particular shell, since in this case the picture would be seriously complicated because of the neutron-proton interaction.

Now let us look at the experimental data. As one sees from the examples of isotopes of oxygen with $A > 16$ and isotopes of Ca with $A < 40$ (the neutrons fill the $d_{5/2}$ and $f_{7/2}$ shells respectively), the combining of neutrons into pairs around the two magic nuclei is carried out in exact accordance with the recipe given above; for example, for the oxygen isotopes the neutron binding energy $E^A(N)_{d_{5/2}}$ depends on the number N of neutrons in the $d_{5/2}$ shell as follows:

$E^{17}(1)$,	$E^{18}(2)$,	$E^{19}(3)$,	$E^{20}(4)$
4.15	8.07	3.96	7.65,

i.e., the energy of the neutron in the $d_{5/2}$ shell in the field of the magic nucleus O^{16} is equal to ~ 4.15 Mev, while the pairing energy for the neutrons is $\sim 8.07 - 4.15 \simeq 4$ Mev.

If the proton shell is not closed, then within the limits of this neutron shell there occurs a very marked drop in E; one can imagine that the first neutrons can interact with the "free" protons (above a closed shell) (once again one neutron with each proton), while the succeeding neutrons can no longer do this. As an example, we consider the $d_{5/2}$ shell in Ne^{18}, the nucleus with two protons beyond O^{16}:

$E^{19}(1)$,	$E^{20}(2)$,	$E^{21}(3)$,	$E^{22}(4)$,	$E^{23}(5)$,	$E^{24}(6)$
11.4	16.9	6.8	10.4	5.2	8.9.

When one needs 1, 2, or 3 protons to close a shell, the binding energy of the neutrons is reduced compared with the binding for the closed shell. However, within a particular neutron shell (with holes in the proton shell), E changes very little, in contrast to the case of the presence of excess protons.

We give an example of the filling of the $f_{7/2}$ shell by neutrons in the isotopes of K, where the proton shell is not closed:

$E^{40}(1)$,	$E^{41}(2)$,	$E^{42}(3)$,	$E^{43}(4)$
7.9	10.0	7.4	10.8 .

On the basis of these remarks we can formulate the following rough rule:[9] if there exists a nucleus with one neutron in a particular shell (for example, C^{15} in which there is one $d_{5/2}$ neutron), then there also exist the heavier isotopes of this element in which there is a filling of the whole neutron shell (C^{16}, C^{17}, C^{18}, C^{19}, and C^{20}).* Referring to Fig. 2, in which each box corresponds to a nucleus (N, Z), we immediately see that, from the experimental fact of the existence of N^{16} and N^{17} there follows the existence of the heavy isotopes N^{18}, N^{19}, N^{20-23}, O^{21-24} and F^{22-25} etc. Extrapolation of the curves of Fig. 6 with N = 11 and 12 enables us to estimate the binding energies of neutrons in N^{18} (~ 1.5 Mev), N^{19} (~ 5 Mev), F^{22} ($\sim 3-4$ Mev), and F^{23} ($6-8$ Mev).

Extrapolation of the curve with N = 10 shows the possible existence of B^{15} with two neutrons in the $d_{5/2}$ shell and consequently, on the basis of the principle of constancy, the existence also of B^{17} and B^{19}. The existence of the odd neutron nuclei B^{14}, B^{16} and B^{18} is less probable. One can, with considerable assurance,

*From the examples given here it follows that this rule can be violated only in nuclei where the filling of the next shell has begun.

assert that the odd-neutron nuclei $Be^{13,15,17}$, Li^{10}, and He^9 do not exist.

Concerning the heavier nuclei with neutron excess, one can assert, by applying the principles formulated above, that there exist many such nuclei. (Some of them are indicated by the crosses in Fig. 3.) In actuality, however, the region of nuclei with neutron excess seems to extend much further. This is indicated by the computations of P. É. Nemirovskiĭ[11] and A. Cameron.[12] We shall give more detail concerning the computations by Nemirovskiĭ later on, and concerning the limit of stability obtained by him and shown in Fig. 2. Cameron in his computations used an extended Weizsäcker formula in which, to a certain extent, one takes account of the shell structure of the nucleus. His result is also shown in Fig. 2. Although the computational results concerning the limits of the stability region can obviously not pretend to high accuracy, they still show the existence of a large number of as yet undiscovered neutron-rich nuclei.

It is interesting to note that the limit obtained by Cameron for the region of stability of neutron-deficient nuclei* actually coincides with that obtained above (cf. Sec. 2) from considerations of charge invariance.

All the nuclei predicted in this paragraph, for which one can make an estimate of binding energy, lifetime, etc., are recorded in Table I. The number of such nuclei, as one sees from Fig. 3, is only a small part of a total number of stable neutron-rich nuclei.

We should separately consider the interesting question of the existence of certain of the lightest nuclei: n^2 (dineutron),[13] n^4 (tetraneutron), H^5 (cf. references 14 — 17), and He^8 (cf. references 9, 17).

At present it is generally considered that the dineutron does not exist. This conviction is based on the fact that there is no bound $^1S_0$ state of the n-p system (the energy of the virtual state is +69 kev). By virtue of the charge invariance of nuclear forces, this should also be the case for the n-n system. If, however, we admit the possibility of deviations from charge invariance, then the question of the dineutron is no longer so obvious. It is easy to show that increasing the radius of the n-n interaction by only $\sim 3\%$ compared with the n-p interaction (and retaining the same magnitude of the interaction) leads to the appearance of a bound state for two neutrons.

It is known at present that the hypothesis of charge invariance is satisfied in the low energy region to an accuracy of the order of one or a few percent. Thus, it is still not possible with certainty to assert that the dineutron does not exist. Attempts to estimate the binding energy of the other nuclei enumerated above by using various extrapolations do not permit one to make any definite statements except that, if these nuclei exist, then they have only a slight stability. The problem of the existence of all of these nuclei can only be solved experimentally.

*This limit is not shown on Fig. 3.

One general approach to the problem consists in determining the location of levels with large values of isotopic spin T in the systems with 4, 5, 7, and 8 nucleons, in particular in the already known nuclei[18] with low A. For example, if one knows the position of the level with $T = 3/2$ in Li^7, then by using the method described at the beginning of this paragraph one could determine the energy of decay of the assuredly unstable nucleus He^7. Experimentally the simplest method for studying levels with large T is to look for narrow resonances in nuclear reactions. Let us consider, for example, the question of the tetraneutron n^4.* If such a nucleus exists, then this means that for the system of four nucleons there is a level with $T=2$, and it should occur in various nuclear processes, say, in the scattering of neutrons by tritium, $H^3(n, n)H^3$. The distinction between the resonance associated with the level with $T = 2$ (if it exists at low energies) from possible resonances with $T = 1$ will consist in the anomalously small width of the $T = 2$ resonance. In fact, the width of a resonance is directly proportional to the probability of decay of the state; all the states with $T = 1$ should, in the case we are considering, decay extremely rapidly, since the number of nucleons which constitute the "intermediate nucleus" is very small. On the other hand, the states with $T = 2$ should decay much more slowly (by a factor of 100 — 10,000) since the decay (state with $T = 2$) $\rightarrow n + H^3$ is associated with a change of isotopic spin by one unit (n and H^3 each have $T = 1/2$) and, consequently, can occur only as a result of a deviation of nuclear interactions from precise charge invariance. This of course is true only for relatively low energies, so long as final states with $T = 2$ are energetically impossible (for example, break-up into four particles, $3n + p$). For $n + H^3$, the threshold for break-up into four nucleons is 8.49 Mev (in the c.m.s.) and, if there is a level with $T = 2$ in this interval, it should appear as a narrow resonance (with a width of 0.1 — 10 kev). The discovery of such a resonance would show very convincingly the existence of the tetraneutron n^4. Precisely the same considerations are applicable to the reactions ($D + H^3$, $Li^7 + n$) etc. If in these reactions one detects narrow resonances in the energy range (in the c.m.s.) $\lesssim 2.2$ Mev, and $\lesssim 10.8$ Mev, this will be a very strong argument in favor of the existence of H^5 and He^8 respectively. Since the existence of He^8 seems to be quite probable, the study of the $Li^7 + n$ reaction would be especially interesting.

In addition to such experiments, one can attempt to observe directly the formation of these nuclei (if they exist) in various nuclear reactions.

*We note that n^4 may exist even if the super-heavy isotope of hydrogen H^4 does not. The point is that the only means for decay of n^4 is the break-up into four neutrons, $n^4 \rightarrow 4n$, and n^4 is stable for any arbitrarily small positive binding energy. In the case of H^4 this is no longer the case, since the decay $H^4 \rightarrow H^3 + n$ is possible; H^4 is stable only when its binding energy with respect to the break-up $H^4 \rightarrow p + 3n$ exceeds the binding energy of H^3, i.e., 8.49 Mev.

One can look for the dineutron in various ways (cf. also reference 19). For example, by preparing the dineutron (if n^2 exists) in the reaction $n + Be^9 \rightarrow 2\alpha + n^2 + 2$ Mev, one can attempt to observe the formation of the short-lived activity B^{12} in the reaction

$$N^{14} + n^2 \rightarrow B^{12} + \alpha + 3.2 \text{ Mev}$$

[One can also use other (n^2, α) reactions which should occur on very many light nuclei.] Another method for detecting n^2 may be experiments in which one measures the angular correlation between the neutrons as a function of the distance between the target and the detectors (Fig. 7). The point is that because of the very large radius of the dineutron there must be a high probability of diffraction break-up of $n^2 \rightarrow 2n$ in the passage of n^2 through matter. Therefore, if n^2 exists, it will break up in passing through a scatterer (Fig. 7),

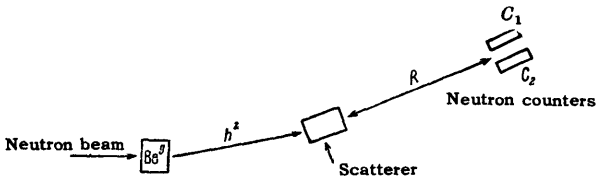

FIG. 7.

and the neutrons formed should be highly correlated in direction. The probability of recording coincidences of two neutrons by counters C_1 and C_2 should be proportional to $1/R^2$. If n^2 does not exist, one will record only chance coincidences, and the counting rate will be proportional to $1/R^4$. Thus, by measuring the number of counts as a function of R, one can satisfactorily isolate the effect of formation of n^2.

Experiments searching for H^5 can also be carried out by looking for delayed neutrons in the reactions $Li^7(\gamma, 2p)H^5$, $H^3(H^3, p)H^5$, $Li^6(\pi^-, p)H^5$, and by observing these reactions in emulsions where, for example, in the case of $Li^6(\pi^-, p)H^5 \xrightarrow[\beta^-]{} He^5 \rightarrow He^4 + n$, one should see tracks of p and H^5 (with an energy around 20 Mev and a range of $600\,\mu$) emerging from the point of stopping of the π^-, at an angle of 180° with respect to one another, and one should also see an electron track at the end of the range of the H^5.

A good method for searching for He^8 may be the study of the stopping of π^- (or a study of the n, 2p reaction) in emulsions filled with Be^9 nuclei. In the case of the $Be^9(\pi^-, p)He^8 \xrightarrow[\beta^-]{} Li^8 \xrightarrow[\beta^-]{} Be^{8*} \rightarrow 2\alpha$ reaction, one should observe, starting from the point of stopping of the π^- meson and directed at an angle of 180° to one another, tracks of p and He^8 (where the energy of the He^8 nucleus is around 12 Mev, and its range $60\,\mu$), and at the end of the track of the He^8 nucleus (if the β decay goes to the ground state of Li^8) one should see two electron tracks and two oppositely directed and identical tracks of α particles.

It is true that the more probable case is the β decay $He^8(0^+) \rightarrow Li^{8*}(1^+)$ (excited state of Li^8 at 3.22 Mev), and not the formation of Li^8 in the ground (2^+) state. In such a case there would be emitted at the end of the He^8 track only one electron, which however would not prevent a completely definite identification of the $Be^9(\pi^-, p)He^8$ reaction. In addition, such a decay of He^8 opens a path for its identification by the appearance of delayed neutrons $(Li^{8*} \rightarrow Li^7 + n)$, for example in the reactions $Be^{10}(\gamma, 2p)He^8$, $B^{11}(\gamma, 3p)He^8$ or the reactions given above.

One should, of course, remember that a capture of the type (π^-, p) is extremely improbable. As A. T. Varfolomeev reported, having observed altogether twelve cases of the reaction $Be^9(\pi^-, n)Li^8$ in 6,000 captures of π^- by Be^9 nuclei in emulsions,[20] he saw no events which could be interpreted as the reaction $Be^9(\pi^-, p)He^8$. (It is true that his emulsions were sensitive not only to electrons, but also to high-energy protons.)

The case of $Be^9(n, 2p)He^8$ (with the decay $He^8 \rightarrow Li^8 \rightarrow Be^8$) is also very characteristic and could easily be identified. The threshold for this process is close to 40 Mev, and it is most reasonable to use neutrons with an energy up to 100 Mev. The cross section for the $Be^9(n, 2p)He^8$ reaction in this energy range is apparently $10^{-28} - 10^{-27}$ cm^2, judging from the fact that the total cross section for formation of all the isotopes of beryllium in the reactions $C^{12}(n; 2p, xn)Be$ (where $x = 0 - 4$) under the action of 90 Mev neutrons[21] is equal to $(6.3 \pm 1.5) \times 10^{-27}$ cm^2.

5. SOME REMARKS CONCERNING THE LIMITS OF THE STABILITY REGION

Before concluding this summary, let us consider briefly what is known at present concerning the limits

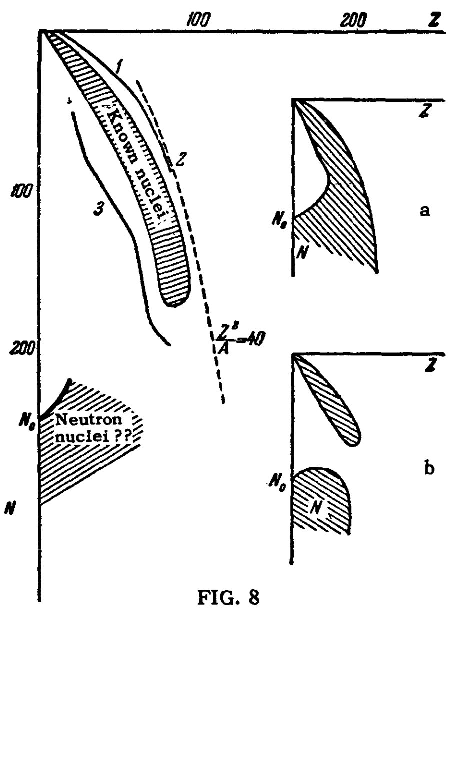

FIG. 8

	Element	N	A	$(M-A)$, Mev	E_p, Mev	E_n, Mev	E_β, Mev	$T_{1/2\beta}$, sec	Remarks
2	Helium	5	7	29.4±0.5		−1.6			
		6	8	<36.1±3		>1.7	12.8	0.01	$T_{1/2}$ given for the transition to the 3.2 Mev level of Li^8 (1^+), cf. also references 9, 17; (β^-)
4	Beryllium . . .	8	12	<30.3		>1.5	<13.4	0.01	(β^-)
5	Boron	2	7	31.5±0.5	−3.6				
6	Carbon	2	8	<40.4±4	−(<1.3)				
		10	16	<17.5		>4.6	<4.5	Depends on level of N^{16}	(β^-)
7	Nitrogen	4	11	28.6	−2.5				
8	Oxygen . . .	4	12	<38.3	−(<2.1)				
		5	13	27.3	1.4	<19.4	17.1	0.01	
9	Fluorine	13	22			~4.0	12.0	0.1	
10	Neon	6	16	27.7—29	$0.5<E_p<<1.8$		11.1—12.4	Depends on level of F^{16}	
		7	17	21.4	1.8	<16	13.4	0.05	$T_{1/2}$ given for transitions to the $1/2^-$ level of F^{17} (3.1 Mev).
11	Sodium	8	19	18.3	−0.3		(9.9)	(0.7)	$T_{1/2}$ given for transitions to the $5/2^+$ level of Ne^{19} (0.2 Mev)
		15	26			3.8	11.3	0.1	(β^-)
12	Magnesium . . .	8	20	23.1	2.8		9.4	0.7	
		9	21	17	3.4	14.5	12	0.3	
		10	22					5	
14	Silicon	10	24	17.9	3.3		9.7	0.2	$T_{1/2}$ given for the transition to the 1^+ level of Al^{24} (0.5 Mev)
		11	25	11	3.7	15.2	11.6	0.4	
		12	26	0.6	5.5	18.8	4.1	1.7	
		19	33	−(<12.4)		<6.1	>4.2	?	(β^-)
15	Phosphorus . .	11	26	—			15.6	?	
		12	27	7.5	0.7		10.9	0.3	$T_{1/2}$ given for the transition to the $1/2^+$ level of Si^{27} (0.9 Mev)
16	Sulphur	12	28	12.1	2.2		10.5	0.2	$T_{1/2}$ given for the transition to the ~1.5 Mev level of P^{28}
		13	29	5	3.1	15.5	12.4	0.4	$T_{1/2}$ given for the transition to the $3/2^+$ level of P^{29} (~1.4 Mev)
		14	30	−5.2	4.4	18.5	5.1	2	$T_{1/2}$ given for the transition with $\Delta T=0$
		23	39	−(<12.7)		<5.5	>5.6	?	
17	Chlorine	13	30		0.9		~17.0	?	
		14	31	1.8	0.6		10.6	2	
18	Argon	14	32	6.9	2.5		<20	1	$T_{1/2}$ given for the transition with $\Delta T=0$
		15	33	0.2	3.3	15.1	10.8	0.7	
		16	34	−8.6	4.6	17.2	4.7	1	$T_{1/2}$ given for the transition with $\Delta T=0$
		24	42	−(>20)		>6.5	<2.6	?	(β^-)
19	Potassium . . .	14	33	>15.3	−(>0.8)		(14.1)	(<0.1)	
		15	34	8.1	−0.3	>15.6	(15.7)	(0.1)	
		16	35	−1.1	0.1	17.6	10.6	0.7	
		17	36	−6.7	1.7	14	<12	0.8	$T_{1/2}$ given for the transition with $\Delta T=0$
20	Calcium	16	36	4	2.5		9.7	0.5	$T_{1/2}$ given for the transition with $\Delta T=0$
		17	37	−2	2.9	14.4	10.9	0.5	
		18	38	−10.6	4.4	17	5.8	(0.7)	
21	Scandium	16	37	14	−0.7		(15)	(0.05)	$T_{1/2}$ given for the transition to the ~2.7-Mev level of Ca^{37}
		17	38	6.3	−0.2	16.1	(15.9)	(0.5)	
		18	39	−2.9	0.6	17.6	11.7	0.5	$T_{1/2}$ given for the transition with $\Delta T=0$
22	Titanium	16	38	21	0.6		13.7	0.3	For all the following isotopes, $T_{1/2}$ assumed to refer to transitions with $\Delta T=0$
		17	39	13.1	0.8	16.3	15	0.3	
		18	40	2.0	2.6	19.5	10.2	0.4	
		19	41	−4.4	2.8	14.8	11.6	0.4	
		20	42	−13.7	4.2	17.7	5.6	0.5	
		21	43	−17.1	4.3	11.8	5.5	(0.6)	
23	Vanadium . . .	16	39	>29.6	−(>1)				
		17	40	22.2	−1.7	>16.4			
		18	41	10.9	−0.6	19.6	14.3	0.2	
		19	42	3.2	−0.1	16.1	16	0.2	
		20	43	−6.5	0.4	18.2	9.5	0.3	
		21	44	−11.5	2.2	13.5	12.1	0.3	
		22	45	−19	1.9	15.8	5.7	0.3	

TABLE (Continued)

	Element	N	A	$(M-A)$, Mev	E_p, Mev	E_n, Mev	E_β, Mev	$T_{1/2\beta}$, sec	Remarks
24	Chromium	18	42	<18	−(<0.2)		(<13.8)	(0.2)	
		19	43	9	1.9	<17.4	14.5	0.2	
		20	44	−2.3	3.4	19.7	8.2	0.2	
		21	45	−7.6	3.6	13.7	10.4	0.2	
		22	46	−16.8	5.2	17.6	5.7	(1.1)	
		23	47	−21.2	5.3	12.8	6	(0.4)	
25	Manganese	19	44	16.6	0		(17.9)	(0.2)	
		20	45	5.9	−0.6	19.1	(12.5)	(0.2)	
		21	46	−0.8	0.8	15.1	15	0.2	
		22	47	−10.1	0.9	17.7	10	0.2	
		23	48	−16.1	2.5	14.4	11.8	0.2	
		24	49	−23.8	2.5	16.1	6.2	(0.4)	
26	Iron	20	46	11.3	2.2		11.1	<0.2	
		21	47	4.4	2.4	15.3	13.5	<0.2	
		22	48	−5.8	3.4	18.6	9.3	<0.2	
		23	49	−11.9	3.4	14.5	10.8	0.2	
		24	50	−20.9	4.7	17.4	6.3	0.2	
		25	51	−25.7	5.1	13.2	6.4	0.2	
27	Cobalt	20	47	20	−1.1				
		21	48	12.2	−0.2	16.2	(17)	(<0.2)	
		22	49	2.1	−0.3	18.6	(13)	(<0.2)	
		23	50	−5.2	0.9	15.4	14.7	<0.2	
		24	51	−14.1	0.8	17.3	10.6	<0.2	
		25	52	−20.2	2.1	14.5	12	<0.2	
		26	53	−(<27.4)	>1.9	15.5	6.6	<0.2	
28	Nickel	20	48	25.8	1.4		12.6	0.1	
		21	49	18.2	1.5	16	15.1	0.1	
		22	50	7.1	2.3	19.5	11.3	0.1	
		23	51	−0.1	2.4	15.6	13.2	0.1	
		24	52	−10	3.4	18.3	9.2	0.1	
		25	53	−16.2	3.4	14.6	<10.2	<0.2	
		26	54	−24.8	5	17	6	<0.2	
		27	55	−30	6.6	13.5	6.9	<0.2	
29	Copper	20	49	37	−3.7				
		21	50	27.9	−2.2	17.5			
		22	51	17.1	−2.5	19.2			
		23	52	9	−1.5	16.5			
		24	53	−1.6	−0.9	19	(13.6)	(0.1)	
		25	54	−8.8	0.1	15.6	15.0	0.1	
		26	55	−17.8	0.4	17.4	11.2	0.1	
		27	56	−23.6	1.1	14.2	<13.2	0.1	
		28	57	−31.8	>1.7	16.6	7	(0.18)	
30	Zinc	23	53	16.2	0.4		16.8	<0.1	
		24	54	5.3	0.7	19.3	13.1	<0.1	
		25	55	−2.4	1.2	16	14.4	<0.1	
		26	56	−12.4	2.2	18.4	10.2	<0.1	
		27	57	−18.7	2.7	14.7	12	<0.1	
		28	58	−26.8	2.7	16.5	<6.8	<0.1	
		29	59	−30.6	>3.6	12.2	7.3	<0.1	
31	Gallium	24	55	16.1	−3.2				
		25	56	7.2	−2	17.3			
		26	57	−3.2	−1.6	18.8			
		27	58	−10.1	−1	15.3	(15.7)	(<0.1)	
		28	59	−18.9	−0.3	17.2	(10.7)	(<0.1)	
		29	60	−23.7	0.7	13.2		<0.1	Also stable are the still undiscovered isotopes with A=62-63
		30	61	−29.5		14.2		<0.1	
32	Germanium	25	57	15.6	−0.8		(17.8)	(<0.1)	
		26	58	3.9	0.5	20.1	13	<0.1	
		27	59	−3.5	1	15.8	14.4	<0.1	
		28	60	−13.2	1.9	18.2	9.5	<0.1	
		29	61	−18.4	2.3	13.6	10.1	<0.1	Also stable are the still undiscovered isotopes with A = 63-65
		30	62	−25.5	3.6	15.5		<0.1	
33	Arsenic	26	59	14.5	−3				
		27	60	6.3	−2.2	16.7			
		28	61	−3.7	−1.9	18.4			
		29	62	−10	−0.8	14.8	(14.5)	(<0.1)	
		30	63	−17.4	−0.5	15.7	15.7	<0.1	The isotopes with A = 64-67 are also apparently stable

TABLE (Concluded)

	Element	N	A	$(M-A)$, Mev	E_p, Mev	E_n, Mev	E_β, Mev	$T_{1/2\beta}$, sec	Remarks
34	Selenium	27	61	14.7	−0.8		(17,4)	(<0,1)	
		28	62	3.2	0.7	20	12,2	<0,1	
		29	63	−3,3	0.9	15	13,1	<0,1	Also stable are the still undiscovered isotopes with A = 65-69
		30	64	−11.7	1.9	16,8		<0,1	
35	Bromine	27	62	25.3	−3				
		28	63	14.1	−3.3	19.7			
		29	64	6,5	−2,2	16.0			Now known are isotopes starting with A = 73
		30	65	−2	−2.1	16,9			
36	Krypton	28	64	22,3	−0.6		(14,8)	(<0,1)	
		29	65	14,5	−0.4	16.3		(<0.1)	Now known are isotopes starting with A = 76
		30	66	4.9	0.7	18		(<0.1)	
37	Rubidium . . .	28	65	34.3	−4,4				Now known are isotopes starting with A = 81
		29	66	25,5	−3,4	17,2			
		30	67	15,9	−3,4	18			
38	Strontium. . . .	29	67	34,6	−1.5				Now known are isotopes starting with A = 81
		30	68	24	−0.5	19		(<0,05)	
39	Yttrium	30	69	36	−4,4				Now known are isotopes starting with A = 82
40	Zirconium . . .	30	70	45,4	−1.8				Now known are isotopes starting with A = 86

of the region of stable nuclei. Nuclei with too great a proton excess certainly do not exist. We can therefore confidently assert that to the right of a certain line 1 (Fig. 8) there is not a single stable nucleus. The exact location of the limiting line 1 is unknown. In the region of low A this curve must pass through the proton- and two-proton-radioactive nuclei enumerated in Sec. 2. For large values of A the fundamental means of decay of nuclei with excess Z are α decay and fission. We can very roughly assume that the nucleus (A, Z) is stable against fission if $Z^2/A \lesssim 40$ (curve 2 in Fig. 8). Curves 1 and 2 should give a correct general picture of this region.

A considerably more complicated question is that of the left-hand limit of the stability region, since fission and α decay no longer have a decisive role, and the principal means for decay at this edge of the diagram should be the emission of neutrons.

There are various indications that there can exist nuclei with relatively very large values of the ratio N/Z. In favor of this view, there speaks, for example, the fact of the existence of neutron-rich isotopes of uranium in nuclear explosions (for example, U^{256} which then goes over by a chain of β decays into the isotopes of Fm and Es), as well as the recently obtained data on observations of isotopes of californium in the explosions of supernovae.[21a] Apparently the only paper in which the question of stability of nuclei with respect to neutron emission has been treated sufficiently correctly is the paper of P. É. Nemirovskiĭ.[11] His conclusions are based on the available information concerning the magnitude of the "optical" potential U(r) describing the interaction of a neutron with a nucleus. Choosing the magnitude, shape, and isotopic dependence (i.e., dependence on A and Z) of the potential U(r) from experiment, P. É. Nemirovskiĭ calculated the energies of the bound states for a neutron, taking account of U(r). The total number of bound states obviously immediately gives us the maximum number of neutrons in a nucleus with given Z. The results of the computation are shown in Fig. 8 by the kinked curve 3. The limit of stability predicted in this fashion should be close to correct for sufficiently heavy nuclei. The accuracy gets worse when one goes to light nuclei. For large values of Z and N, Nemirovskiĭ's curve practically coincides with that computed by Cameron[12] using the extended Weizsäcker formula. Some years ago Wheeler,[22] considering the question of the number of neutrons which can be attached to a heavy nucleus, came to the conclusion that for $Z \sim 90-100$ there can exist nuclei with $A = 500-600$. It was pointed out by Nemirovskiĭ that the usual Weizsäcker formula, which Wheeler used, is not valid in the neighborhood of the limits of stability. Moreover, it cannot be extrapolated to such large values of A. For this reason, Wheeler's conclusions are not trustworthy.

Another approach to the problem of super-heavy nuclei was made in reference 9, where the limiting case of a very heavy nucleus consisting of neutrons alone is considered. If such nuclei exist, their density is certainly less than the density of ordinary nuclei, and we may speak of a Fermi gas with low density. In such a gas the kinetic energy of the colliding particles is small, and the main role is played by the interaction of neutrons colliding in S states; thus, interaction occurs only for pairs of neutrons with opposite spins.

Under these conditions the interaction in an S-state is determined entirely by the scattering length, which for the n-n system should be around $a = -20 \times 10^{-13}$ cm. The sign of a corresponds to a mutual attraction of the two neutrons. The cross section is then given by the familiar formula

$$\sigma = 4\pi a^2 \frac{1}{1 + a^2 p^2},$$

where a is the scattering length, p is the momentum of the relative motion of the two neutrons. The scattering of the particles affects the average energy of the system like a weak interaction potential, for which

$$VR^3 \sim -\frac{\hbar^2}{m}\sqrt{\sigma}.$$

In the limit of very low density $\rho < 0.001\rho_0$ (ρ_0 is the nuclear density)

$$p \ll \frac{1}{a} \text{ and } \sigma = 4\pi a^2 = \text{const},$$

the average potential energy of the particle is of the order of $-\hbar^2/ma\rho$; it is less than the kinetic energy of the particle, which is proportional to $\rho^{2/3}$; in this isolated state, liquid is surely not formed. For $\rho > 0.001\rho_0$, $\sigma \sim 4\pi\hbar^2/p^2$ and, considering that $p \sim \rho^{1/3}$, we find that the potential energy is of the same order and also depends on the density in the same way as the kinetic energy ($\sim \rho^{2/3}$). Including only the pair interactions numerically gives the result that the total energy is negative; $U_{pot} = -\frac{4}{3}\bar{E}$; $U_{pot} + \bar{E}_{kin} = -\frac{1}{3}\bar{E}_{kin}$; however, as pointed out in reference 9, this conclusion (from which there would follow the existence of a neutron liquid) is by no means conclusive; the influence of triple collisions and collisions with higher multiplicity can easily change the sign of the energy; their calculation is an extremely difficult problem which has not been solved as yet. It therefore remains an open question whether there exist nuclei consisting solely of neutrons.

However, let us assume that such nuclei are possible. Because of the surface tension there must exist a definite critical size of the neutron drop, i.e., a minimum number of neutrons for which one can have a neutronic nucleus. If they exist, beginning with some value of the number of neutrons N_0, then obviously there exists a whole region of super-heavy nuclei, located on Fig. 8 in the neighborhood of the neutron axis. Then, depending on the size of this region, it either touches the region of nuclei known to us (Fig. 8a) or forms a separate island (Fig. 8b).

However, it may be that more exact computations will show the impossibility of the existence of neutronic nuclei. In this case, the whole set of stable nuclei will be limited by the small, cigar-shaped island in the (N, Z) plane shown in Fig. 8.

Note added in proof: Recently Brueckner, Gammel, and Kubis [Phys. Rev. 118, 1095 (1960)] discussed the question of a neutron liquid. Taking into account at low density only pair interactions and the dependence of the scattering length on energy, the authors come to the conclusion that a neutron liquid does not exist. This conclusion is not final, since in this region, the density, although it is small compared to the usual nuclear density, is not small compared to $1/a^3$.

[1] F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. **11**, 1 (1959); V. A. Kravtsov, Usp. Fiz. Nauk **65**, 451 (1958); Strominger, Hollander, and Seaborg, Revs. Modern Phys. **30**, 585 (1958).
[2] A. I. Baz' and Ya. A. Smorodinskiĭ, Usp. Fiz. Nauk **55**, 215 (1955).
[3] D. C. Peaslee, Phys. Rev. **95**, 717 (1954).
[4] R. Hofstadter, Revs. Modern Phys. **28**, 214 (1956).
[5] N. Swamy and A. Green, Phys. Rev. **112**, 1719 (1958).
[6] V. I. Gol'danskiĭ, JETP **39**, 497 (1960); Nucl. Phys. (in press).
[6a] B. S. Dzhelepov, Izv. Akad. Nauk S.S.S.R., Ser. Fiz. **15**, 496 (1951).
[7] G. F. Bogdanov, et al., Атомная энергия (Atomic Energy) **3**, 204 (1957).
[8] A. I. Baz', ibid. **6**, 571 (1959).
[9] Ya. B. Zel'dovich, JETP **38**, 1123 (1960), Soviet Phys. JETP **11**, 812 (1960).
[10] M. G. Mayer, Phys. Rev. **78**, 22 (1950); D. Kurath, Phys. Rev. **88**, 804 (1952).
[11] P. É. Nemirovskiĭ, JETP **36**, 889 (1959) and **33**, 746 (1957), Soviet Phys. JETP **9**, 627 (1959) and **6**, 573 (1958).
[12] A. G. Cameron, Chalk River Report CRP-690, (1957).
[13] M. L. Pool et al., Phys. Rev. **82**, 305 (1951); K. W. Allen, et al., Phys. Rev. **82**, 262 (1951); F. W. Fenning and F. R. Holt, Nature **165**, 722 (1950).
[14] C. Blanchard and R. Winter, Phys. Rev. **107**, 774 (1957).
[15] R. Cence and B. Moyer, Reports of the Washington Conference on Photonuclear Reactions, 1958.
[16] G. Tautfest, Phys. Rev. **111**, 1162 (1958).
[17] V. I. Gol'danskiĭ, JETP **38**, 1637 (1960), Soviet Phys. JETP **11**, 1179 (1960).
[18] Ya. B. Zel'dovich, JETP **38**, 278 (1960), Soviet Phys. JETP **11**, 202 (1960).
[19] A. I. Baz' and Ya. A. Smorodinskiĭ, Proceedings of the International Conference on Nuclear Physics, Paris (1958).
[20] A. T. Varfolomeev, JETP **25**, 90, 224 (1953).
[21] D. A. Kellogg, Phys. Rev. **90**, 224 (1953).
[21a] A. Ghiorso et al., Phys. Rev. **99**, 1048 (1955).
[22] J. Wheeler in "Niels Bohr and the Development of Physics," Pergamon (1955); F. Werner and J. Wheeler, Phys. Rev. **109**, 126 (1958).

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