INTERACTION CROSS SECTIONS OF HIGH-ENERGY PARTICLES

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1. INTRODUCTION

A considerable amount of new data on the interaction cross sections of high-energy particles have recently been obtained, mainly by means of three great accelerators: the Brookhaven cosmotron, the Berkeley bevatron, and the proton synchrotron in Dubna. The maximum energy E_{max} of the protons accelerated by these machines is 3, 6.2, and 10 Bev respectively, and the maximum energy of π mesons produced in the collisions of these protons with target nuclei is approximately 2.5, 5.5, and 9 Bev respectively. The high densities of the accelerated particle beams make precise measurements possible.

Only a negligible part of the data have been obtained in cosmic-ray experiments. The difficulties of such measurements lead to a relatively low accuracy of the results obtained. The measured values are, as a rule, averaged over large energy intervals. However, this small amount of data is extremely important, since it provides information on the interaction of particles at the enormous energies of hundreds and thousands of Bev. While the proton synchrotron in Dubna enables us to penetrate into matter down to a distance $\lambda \sim 1/E_{max}^{1/2} \approx 10^{-14}$ cm, the cosmic-ray experiments will evidently, for a long time to come, provide the only means for studying objects with dimensions on the order of $\lambda \sim 10^{-15}$ to 10^{-18} cm.

Experimental data on the interaction cross sections of protons and neutrons with protons at energies $E = (0.01 - 6.2)$ Bev were reviewed by Hess.[1] However, many new data are now available, especially in the high-energy range $E \gtrsim 1$ Bev, which permit us to draw certain important theoretical conclusions. Experimental results on the interaction cross sections of other particles are dispersed among many original papers and are therefore difficult to use.

In the present paper, experimental data on the interaction cross sections of different particles at energies $E \geq 0.8$ Bev, as well as their theoretical interpretation, will be discussed in detail. We shall limit ourselves to the discussion of the high-energy region; the phenomena in that region exhibit a number of characteristic features which are common to the interactions of various particles.

2. EXPERIMENTAL DATA

Various definitions of the interaction cross sections are being used in the different original papers. Before discussing the experimental data, we therefore have to decide how to define the different interaction cross sections, and introduce the necessary notation.

The total interaction cross section σ_{tot} consists of the cross section for the elastic scattering σ_e plus the cross section for the inelastic scattering σ_i:

$$\sigma_{tot} = \sigma_e + \sigma_i .$$

In turn,

$$\sigma_e = \sigma_d + \sigma_p + \sigma_{e,i} + \sigma_{dp}.$$

where σ_d is the cross section for the diffraction scattering. This component of the elastic scattering is fully determined by inelastic reactions, and vanishes for $\sigma_i \to 0$.

σ_p is the cross section for the elastic "potential scattering." An example of such scattering is the Coulomb scattering of two charged particles, or the scattering of nucleons with a meson exchange. It is essential that the interacting particles preserve their individuality, at any moment of time.

Primary particles may also lose their individuality during the interaction at a certain moment of time, producing a single compound particle which, in a particular case, may again decay into the same two particles. We shall characterize such an elastic scattering by the cross section $\sigma_{e,i}$ where the subscript i denotes its inelastic origin. In certain cases (e.g., in calculating the decay probabilities of compound particles), it is necessary to include $\sigma_{e,i}$ in the cross section for inelastic processes σ_i. In the discussion to follow, however, it will always be understood what is meant by the cross sections σ_e and σ_i.

σ_{dp} is the cross section for the scattering due to the interference between the diffraction scattering and that part of the potential scattering coherent with it. It is clear that elastic scattering with a compound-particle state is incoherent with diffraction scattering.

The cross section for an inelastic interaction σ_i is the sum of the cross sections for all possible inelastic reaction channels:

$$\sigma_i = \sum_j \sigma_i^{(j)}.$$

Special cases of inelastic scattering are the "charge-exchange" elastic scattering characterized by the cross section σ_{ce}, and the spin-flip elastic scattering with cross section σ_s. However, in the high-energy region, all cross sections measured so far are averaged over the spins of the interacting particles; in such a case, σ_s is included in the experimental values of the elastic scattering cross section σ_e.

TABLE I. pp-interaction

Kinetic energy in the laboratory system E, Bev	Method of measurement of the cross section	σ_e, mb	σ_i, mb	σ_{tot}, mb
0.8	Counters[1,2]	21.5+2.0	25.5+2.8	47±2
0.8	Counters[3]	—	—	$47.5^{+2.2}_{-1.1}$
0.81	Diffusion chamber[4]	24±2	24±2	48±2
0.83	Counters[5]	—	—	$47.8^{+1.6}_{-1.2}$
0.85	"	—	—	$47.6^{+1.7}_{-1.2}$
0.91	Counters[6]	—	—	46.1+0.5
0.92	Counters[3]	—	—	$47.7^{+3.0}_{-1.1}$
0.925	Emulsion[7]	17 ±3	33+3	50±6 *)
0.93	Bubble chamber[8]	25.9±1.9	20.8±1.5	47.7±3.4 *)
0.94	Counters[9]	26±3	23±3	49±5
0.97	Diffusion chamber[10]	25.8±0.9	20.9±1.3	46.7±0.5 *)
1.0	Counters[1,2]	19.2^{+3}_{-2}	28.8±3.2	48.0+3.5
1.03	Counters[3]	—	—	$46.5^{+2.0}_{-0.6}$
1.075	Counters[5]	—	—	$48.3^{+1.6}_{-1.1}$
1.17	Counters[3]	—	—	$46.3^{+3.2}_{-0.8}$
1.275	Counters[5]	—	—	$47.5^{+1.6}_{-1.2}$
1.295	"	—	—	$49.4^{+1.6}_{-1.1}$
1.3	Counters[3]	—	—	$45.0^{+3.0}_{-1.0}$
1.490	Counters[5]	—	—	$47.2^{+2.6}_{-1.2}$
1.5	Diffusion chamber[11]	20±2	27±3	47 ±5 *)
1.7	Counters[3]	—	—	$45.6^{+1.9}_{-0.7}$
2.0	Counters[3]	—	—	$41.4^{+3.2}_{-1.4}$
2.17	Counters[3]	—	—	$45.1^{+0.9}_{-0.5}$
2.24	Counters[1,12]	16.9+2.5	—	—
2.6	Counters[5]	—	—	$41.6^{+4}_{-1.6}$
2.75	Diffusion chamber[13]	15±2	26±3	41±5 *)
2.75	Counters[3]	—	—	$43.3^{+0.6}_{-0.5}$
3.0	Emulsion[14]	8.9±1.0	35.6	44.5
3.17	Counters[3]	—	—	42.4±0.6
4.4	Counters[1,12]	9.0±1.4	—	—
5.3	Diffusion chamber[1,15]	5.6±2.3	26.8±5.5	32.4±6
5.7	Emulsion[16]	13±6	—	—
6.15	Counters[1,12]	6.9±1.0	—	—
6.2	Emulsion[17]	8.8±2	22.6±5.3	31.4±5.1
6.2	Emulsion[18]	—	25	—
6.2	Not given[19]	8	—	—
8.5	Emulsion[20]	8.6±0.8	—	—
9	Emulsion[21]	10 ±3	21	31+3

*See footnote on this page.

We shall now survey the existing experimental data.

2.1. Interactions of Nucleons

Experimental data on the interaction cross sections of nucleons, taken from references 1 to 30, are shown in Tables I to III.*

The cross sections for pn interactions found from direct measurements of the interaction of neutrons with protons were obtained only at 1.4 and 4.5 Bev. All other cross sections were obtained by the difference method from experiments with deuterium and hydrogen. It was assumed that the absorption or scattering of the incident proton by a nucleon of the deuteron decreases if this nucleon falls within the shadow region of the other nucleon (screening effect).[54] In the energy range E ≈ 1 − 3 Bev, the corresponding corrections increase the total cross section for the pn interaction $\sigma_{tot}(pn) \equiv \sigma_{tot}(pd) - \sigma_{tot}(pp)$ by approximately 20%, varying somewhat with the choice of the deuteron wave function. The values of $\sigma(pn)$ are shown in Table II with the screening correction already applied.

*The errors in the total cross section, $\delta\sigma_{tot}$, if not otherwise specified by the author, were assumed to equal the sum of the absolute errors of the cross sections σ_i and σ_e. The errors $\delta\sigma_i$ and $\delta\sigma_e$ were calculated in an analogous way (provided the experimental errors of two other cross sections were known). The errors calculated in this manner are denoted in the tables by an asterisk.

TABLE II. pn-interaction*

Kinetic energy in the laboratory system E, Bev	Method of measurement of the cross section	σ_{tot}, mb
0.8	Counters[5]	32.5
0.91	Counters[6]	39.2 ± 3.1
0.97	Diffusion chamber[10]	37.6 ± 3.9
1.1	"	34.4
1.3	"	38.8
1.4	Counters[22]	42.4 ± 1.8
1.5	Counters[5]	40.8
2.0	"	40.9
2.6	"	37.4
4.5	Counters[23]	33.6 ± 1.6

*For energies $E > 0.8$ Bev, the only known values of σ_e and σ_i are $\sigma_e = 15.5 \pm 3$ and $\sigma_i = 22 \pm 3$ mb, measured by means of a diffusion chamber at $E = 0.97$ Bev.[10]

TABLE III. pN-interaction

Kinetic energy in the laboratory system		Range in emulsion L, cm*	σ_i, mb	σ_{tot}, mb
Energy range E, Bev	Mean energy E, Bev			
—	4.5 [24]	—	—	35 ± 3
—	5.7 [16]	37.6 ± 5.3	—	29^{+16}_{-7}
—	6.2 [25]	34.7 ± 3.4	—	$35^{+13.5}_{-6.5}$
—	6.2 [17,26]	$\geqslant36.4$	—	$\geqslant31$
—	9 [21,27]	37.3 ± 0.3	—	30 ± 0.5
>9	~20 [28]	—	—	32 ± 10
0.9—34	20 [29]	—	—	32 ± 3
28—58	37 [30]	—	$28+4$	—
58—121	77 [30]	—	$21+4$	—
121—387	178 [30]	—	25^{+18}_{-7}	—
—	200 [73]	42 ± 10	—	22^{+24}_{-13}

*As was shown by comparative calculations, the *mean free paths in the emulsions NIKFI-R and Ilford* G-5 are practically identical (see Appendix).

The values of σ_{tot} and σ_i, averaged over pp- and pn interactions and obtained by the reduction (using the optical model) of the experimental values for the mean free path of protons in photographic emulsions and the experimental values of the cross section for the interaction of protons with nuclei,[24,28–30] are shown in Table III (see Appendix).* The mean energy for each of the three intervals listed last in the table is calculated taking the energy spectrum of protons in the atmosphere into account. It can be seen that the resulting cross sections are in good agreement with the values of σ_{tot} and σ_i in Tables I and II.

For better visualization, the basic experimental data are summarized in Fig. 1. The curves shown in this figure can be used for interpolation of the experimental data.

Considerations of charge symmetry and of the invariance with respect to charge conjugation lead to the equality of the cross sections*

$$\sigma(pp) = \sigma(nn) = \sigma(\tilde{p}\tilde{p}) = \sigma(\tilde{n}\tilde{n}), \qquad \sigma(pn) = \sigma(\tilde{p}\tilde{n})$$

independently of the type of cross section (σ_{tot}, σ_i, or σ_e).

It can be seen from the experimental data presented above that, with increasing energy, the cross sections for pp interactions tend towards the constant values $\sigma_{tot} \approx 30$ mb, $\sigma_i \approx 22$ mb, and $\sigma_e \approx 8$ mb. As shown by cosmic-ray measurements, the nucleon-interaction cross section remains constant, within the limits of experimental error, up to energies of the order of several hundred Bev. An analysis of experimental data on extensive air showers shows that the cross sections apparently remain constant up to $E \sim 10^9$ Bev.

It can also be seen from Tables I to III and from Fig. 1 that, with increasing energy, the total cross sections for pp and pn interactions become equal. One might think that the cross sections σ_i and σ_e would also be equal at high energies. The measurements of Brenner and Williams[30] have shown that, in the 28 — 387 Bev energy range, the cross sections for the inelastic interaction of protons and neutrons with iron nuclei do not differ, within the limits of experimental error ($\sim 30\%$). This confirms the equality of the inelastic cross sections for pp- and pn interactions at these energies.

2.2. Interactions of Pions with Nucleons

Experimental values of the cross sections for πN interactions are presented in Tables IV to VI. The main results are summarized in Fig. 2. The values of σ_{tot} in Table VI are calculated, according to the optical model, from the experimental values of the mean free path of π mesons in emulsion (see footnote to preceding column).

From considerations of charge symmetry and of the invariance with respect to charge conjugation, the following equations apply for the cross sections σ_{tot}, σ_i, and σ_e:

$$\sigma(\pi^+p) = \sigma(\pi^-n) = \sigma(\pi^-\tilde{p}) = \sigma(\pi^+\tilde{n}),$$
$$\sigma(\pi^-p) = \sigma(\pi^+n) = \sigma(\pi^+\tilde{p}) = \sigma(\pi^-\tilde{n}),$$
$$\sigma(\pi^0p) = \sigma(\pi^0n) = \sigma(\pi^0\tilde{p}) = \sigma(\pi^0\tilde{n}),$$
$$\sigma(\pi^0n) = \sigma(\pi^0p) = \sigma(\pi^0\tilde{n}) = \sigma(\pi^0\tilde{p}).$$

Moreover,

$$\sigma_{ce}(\pi^0p) = \sigma_{ce}(\pi^-p) = \sigma_{ce}(\pi^0n) = \sigma_{ce}(\pi^0\tilde{p}) = \sigma_{ce}(\pi^+n) = \sigma_{ce}(\pi^+\tilde{p}) = \sigma_{ce}(\pi^-\tilde{n}) = \sigma_{ce}(\pi^0\tilde{n}).$$

It can be seen from Tables IV to VI and from Fig. 2 that the cross sections for π^+p and π^-p interactions already become practically equal, and independent of energy, for $E \approx 2 - 3$ Bev. Moreover, within the limits of experimental error, the limiting value of σ_{tot} coin-

*The errors shown in Table III correspond to experimental errors in the values of the mean free path and in the interaction cross sections of protons with nuclei.

*It is understood that we neglect the electromagnetic interactions, which is certainly not correct for extreme peripheral collisions leading to small-angle scattering. However, such collisions contribute very little to the cross sections σ_{tot}, σ_i, and σ_e.

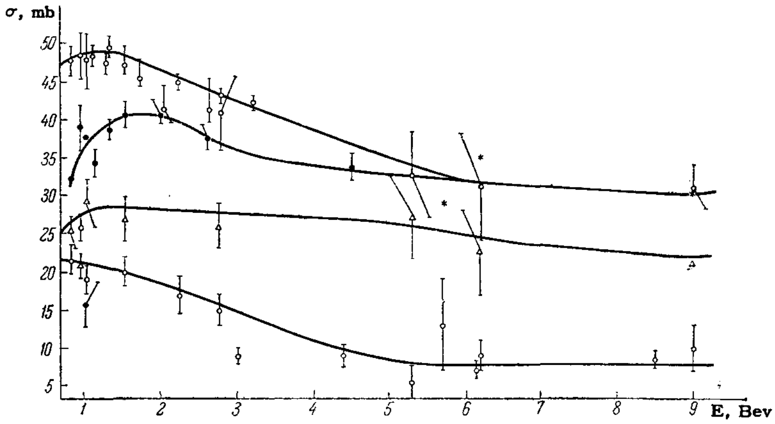

FIG. 1. Interaction cross sections of nucleons. ○ – cross section σ_{tot}(pp) and σ_e(pp); △ – σ_i(pp); ● – σ_{tot}(pn) and σ_e(pn); * – σ_{tot}(pN) calculated from experimental interaction cross sections of protons with nuclei.

TABLE IV. π^-p-interaction

Kinetic energy in the laboratory system E, Bev	Method of measurement of the cross section	σ_e, mb	σ_{ce}, mb	σ_i, mb*	σ_{tot}, mb
0.8	Counters	16.4	7.3	24.6	41
0.8	Bubble chamber[34]	21±1	10	32.3±1.5	53.3±2.4
0.819	Counters[31]	—	—	—	47.9±1.9
0.84	"	—	—	—	54.6±2.1
0.86	Counters[32]	—	—	—	40±1.3
0.868	Counters[31]	—	—	—	58.6±2.4
0.89	"	—	—	—	57.8±2.2
0.9	Diffusion chamber[35]	18±3	10±3	27.5±3	46.1±3
0.9	Counters[33]	—	—	—	44.4±2.3
0.9	Counters[32]	—	—	—	48.5±1.3
0.915	Bubble chamber[44]	19.8	—	—	—
0.918	Counters[31]	—	—	—	54.5±2.4
0.94	Counters[32]	—	—	—	51±0.9
0.943	Counters[31]	—	—	—	50.4±2.6
0.95	Bubble chamber[36]	19.1±1.6	—	26±2.5	45±3
0.96	Diffusion chamber[35]	20±3	8±5	26.3±2	46.3±2.7
0.97	Counters[33]	—	—	—	45.1±2.7
0.97	Counters[32]	—	—	—	49.5±0.9
0.972	Counters[31]	—	—	—	44.7±2.2
1.0	Counters[33]	—	—	—	46±3
1.014	Counters[31]	—	—	—	39.6±2.0
1.03	Counters[32]	—	—	—	40.5±0.9
1.076	Counters[31]	—	—	—	35.9±2.0
1.08	Counters[33]	—	—	—	36.3±2.6
1.08	Counters[32]	—	—	—	37.5±0.9
1.12	"	—	—	—	35±0.9
1.15	Counters[31]	—	—	—	35.5±2.0
1.17	"	—	—	—	33.5
1.25	Counters[33]	—	—	—	29.2±3.7
1.3	Bubble chamber[37]	10±0.8	—	19	—
1.35	Counters[33]	—	—	—	30.1±2.8
1.37	Diffusion chamber[38]	10.0±0.8	—	20.3±3.4 **)	30.3±2.5
1.38	Counters[33]	—	—	—	30.8±2.8
1.4	Emulsion[39]	9±1	7±1	—	—
1.4	Not given[40]	10±2	—	24	—
1.47	"	—	—	—	31.4±1.8
1.5	"	—	—	—	30.0±2.0
1.67	"	—	—	—	31.4±3.9
1.85	Diffusion chamber[44]	11.1±2.3	—	20.3±3	31.4±5.3 ***)
1.9	Counters[33]	—	—	—	31.3±1.6
4.3	Counters[42]	—	—	—	28.7±2.6
4.4	Counters[43]	—	—	—	30±5
4.5	Emulsion[44]	4.5±1	2±0.7	24.5±2.4	29±2.6
4.7	Not given[45]	6±1.5	—	22	—
5	Diffusion chamber[46]	4.7±10	—	19.7±2 ****)	24.5±2.4 ****)
5.2	Not given[47]	—	—	—	29.1±2.9
6.8	Bubble chamber[48]	5.5±0.5	—	—	30±5
14	Counters[90]	—	—	—	33±4

*Cross section σ_{ce} is included in σ_i.

**σ_{tot} is obtained by interpolating data from reference 31. Corresponding cross sections are σ_e, σ_{ce}, and σ_i, calculated from the data of references 32 and 33.

***See footnote on p. 690.

****The value σ_{ce} = 2 mb is taken from reference 44.

TABLE V. π^+p-interaction

Kinetic energy in the laboratory system E, Bev	Method of measurement of the cross section	σ_{tot}, mb	Kinetic energy in the laboratory system E, Bev	Method of measurement of the cross section	σ_{tot}, mb
0.8	Counters[32]	17±1.3	1.46	Counters[49]	38.5±1,4
0.827	Counters[31]	21,36±0.81	1.50	Counters[33]	35.3±2.5
0.847	"	22.42±0.83	1.58	Counters[49]	38.2±1.2
0.872	"	21.85±0.86	1.60	Counters[3]	35.8±0.9
0.95	Counters[32]	21.5±1.3	1.67	Counters[33]	32.6±1.8
1.0	Counters[33]	23.5±1.4	1.67	Counters[49]	36±0.7 −1.1
1.07	"	27.3±3.7			
1.1	Counters[32]	27.5±1.3	1.73	Counters[3]	30.1±0.5
1.15	Counters[33]	31.3±1.7	1.89	"	284±0.6
1.23	Counters[49]	30±1.3	2.05	"	27.8±0.6
1.25	Counters[33]	38.8±2.5	2.47	"	29.0±0.6
1.28	Counters[49]	33±1.2	2.76	Counters[50]	28±4
1.33	"	34.9±1.1	2.97	Counters[3]	29.2±0.5
1.36	"	37.2±1.4	3.58	"	29.2±0.4
1.38	Counters[33]	41.4±3.0	4.00	"	29.3±0.4
1.4	Counters[3]	39,4±0.6	4.3	Counters[42]	28±4
1.46	"	39.1±0.8	14	Counters[90]	26±4

TABLE VI. π^-N-interaction

Kinetic energy in the laboratory system E, Bev		Range in emulsion L, cm*	σ_{tot}, mb
1.0	[51]	38±3	29^{+6}_{-5}
1.5	[52,53]	35±1	35±2.5
3	[16]	35.5±5	$34^{+18.5}_{-11}$
4.2	[74]	38.7±3.5	27^{+8}_{-5}
4.3	[75]	33.7±4.7	39^{+21}_{-12}
5.7	[75]	41±6	24^{+11}_{-6}
6.8	[53]	⩽40	⩾25
7.5	[91]	38±2	29±4
50	[77]	37±6	30^{+20}_{-8}

*Average over the range 1–200 Bev

cides with the corresponding one for NN interactions; the limiting value of σ_e is evidently somewhat lower, and the limiting value of σ_i higher, than for NN interactions. Measurements of greater precision will be necessary in order to draw final conclusions.

At the present time, there are no direct experimental data on the interaction of π^0 mesons with nucleons at high energies. However, the total cross section for this interaction can be determined from charge-symmetry considerations:

$$\sigma_{tot}(\pi^0 p) = \frac{1}{2}[\sigma_{tot}(\pi^- p) + \sigma_{tot}(\pi^+ p)].$$

It can be seen from Fig. 2 that at high energies this cross section is very close to the interaction cross section of charged π mesons with nucleons. From theoretical considerations concerning the weak isotopic spin-dependence of high-energy interactions (see below), one should expect that the corresponding cross sections $\sigma_i(\pi^0 p)$ and $\sigma_e(\pi^0 p)$ will also be close to the corresponding cross sections for the interaction of $\pi^\pm$ mesons with protons.

2.3. Interactions between Nucleons and Antinucleons

Experimental data on the interactions between antinucleons and nucleon are poorer than those available on NN and πN interactions, and are limited to the energy range E ≤ 2 Bev. Presently-available values for

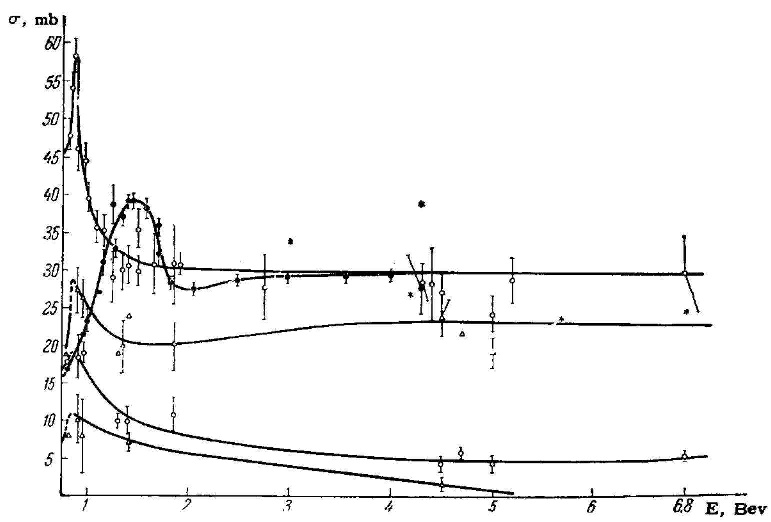

FIG. 2. Cross sections for the interaction of π mesons with nucleons. o – cross sections $\sigma_{tot}(\pi^- p)$ and $\sigma_e(\pi^- p)$; Δ – $\sigma_i(\pi^- p)$ and $\sigma_{ce}(\pi^- p)$ ● – $\sigma_{tot}(\pi^+ p)$; • = $\sigma_{tot}(\pi^- N)$, calculated from the mean free path of π mesons in emulsion.

TABLE VII. $\tilde{p}p$-interaction

Kinetic energy in the laboratory system E, Bev	Method of measurement of the cross section	σ_e, mb	σ_{ce}, mb	σ_i,* mb	σ_{tot}, mb
0.81	Counters[9]	37±5	7±2	67±7 **)	105±6
0.95	"	33±3	8±2	64±5 **)	96±3
1.0	Not given[55]	33	8	69	102±3
1.07	Counters[9]	30±3	7±1	65±4 **)	96±4
1.25	Not given[55]	—	8	—	90±3
2.0	"	23	7	60	83±3

*σ_{ce} is included in the cross section σ_i.
**See footnote on p. 690.

the cross sections of $\tilde{p}p$ interactions[9,55] are shown in Table VII and in Fig. 3. From charge-symmetry considerations we have, of course, $\sigma(p\tilde{p}) = \sigma(n\tilde{n})$.

The measurements[55] show that, in the $E \approx 0.5 - 1.1$ Bev energy range, the total cross section for the interaction of antiprotons with deuterons remains approximately constant and equal to 170 — 180 mb. Hence, neglecting the nucleon screening effect in the deuteron, we have $\sigma_{tot}(\tilde{p}n) = \sigma_{tot}(\tilde{p}d)$, $\sigma_{tot}(\tilde{p}p) \approx 70 - 80$ mb. If the corrections for screening are taken into account, the value increases by 10 to 20%. The cross sections $\sigma_{tot}(\tilde{p}p)$ and $\sigma_{tot}(\tilde{p}n)$ are then close to each other.

It can be seen from the above data that, at energies of several Bev, the cross sections for the $\tilde{p}p$ interaction are considerably greater than the cross sections for NN interactions. This can be explained as follows:

Since the antinucleon is, in many respects, similar to the nucleon, it, just like the nucleon, can be represented as consisting of a hard core and a relatively soft meson shell. Using this model, it is natural to assume that the annihilation occurs in the collision of the cores of a nucleon and an antinucleon. Such a collision will be characterized by strong absorption and,

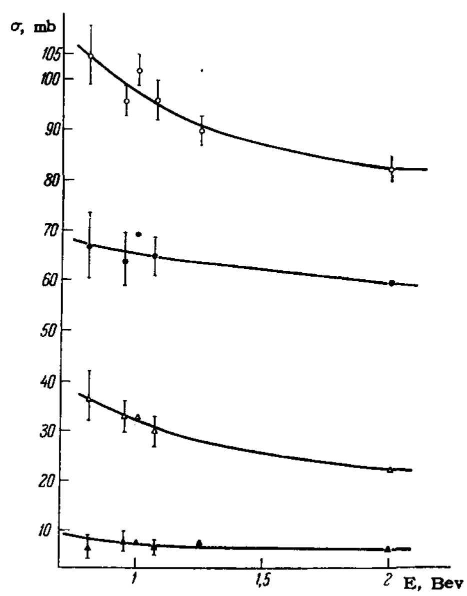

FIG. 3. Cross sections for the interaction of antiprotons with protons. ○, ●, △, ▲ — cross sections for $\sigma_{tot}(\tilde{p}, p)$, $\sigma_i(\tilde{p}p)$, $\sigma_e(\tilde{p}p)$, and $\sigma_{ce}(\tilde{p}p)$, respectively.

consequently, a larger cross section than in the case of NN interactions. In the collision of cores with a peripheral meson cloud, elastic or inelastic nucleon and antinucleon scattering occurs with the accompanying production of new particles, similarly as for NN collisions. For energies not greater than several hundred Bev, such a model of $\tilde{N}N$ collisions corresponds to an interaction potential which, at distances $r > 2a$ (where $a \approx \hbar/Mc \approx 2 \times 10^{-14}$ cm is the characteristic radius of the core), differs only in sign from the unknown NN potential of Gartenhaus, Signell, and Marschak,[56] and which is characterized by strong absorption at small distances $r \lesssim a$.* The estimates of Koba and Takeda,[57] and the more accurate calculations of Ball, Chew, et al.[58] show that the experimental data are well explained by such a phenomenological theory.

For higher energies, no theory exists so far which would make it possible to calculate the cross section for the interaction of nucleons with antinucleons. However, one should expect that, with increasing energy, the cross sections for NN and $\tilde{N}N$ interactions will become equal (see Sec. 3.2). One of the reasons for this is the fast increase with energy of the number of possible inelastic channels in the collision of two nucleons (in other words, the increase in the "opacity" of the nucleon core).

2.4. Interactions of K Mesons with Nucleons

The experimental interaction cross sections of charged K mesons with nucleons are presented in Tables VIII to X and in Fig. 4.[59,60]

The interaction cross sections of K^+ mesons with neutrons are obtained by the difference method from measurements of the interactions of K^+ mesons with deuterium and hydrogen. The values of the cross sections presented in Table IX do not take the correction for the nucleon screening in the deuteron into account. However, this correction is small. The curves in Fig. 4, which approximate the cross sections $\sigma_{tot}(K^+p)$ and $\sigma_{tot}(K^-p)$, are taken from the paper of Alvarez.[60]

From considerations of charge symmetry and in-

*It should be mentioned that the π-meson charges of nucleons and antinucleons differ in sign. Therefore, in the exchange of an odd number of π mesons, the signs of the potentials for NN and $\tilde{N}N$ interactions also differ in sign.

TABLE VIII.
K^+p-interaction

Kinetic energy in the laboratory system E, Bev	Method of measurement of the cross section	σ_{tot}, mb
0.96	Counters[59]	18.8±0.8
1.1	"	16.4±0.7
1.1	Counters[60]*	17.5±1.3
1.23	Counters[59]	18.3±0.8
1.28	Counters[60]	19.5±1.3
1.45	Counters[59]	15.7±0.7
1.45	Counters[60]	18.75±0.6
1.6	Counters[59]	15.5±1.1
1.6	Counters[60]	16.3±0.8
1.76	"	18.3±0.8
1.95	Counters[59]	13±1
1.95	Counters[60]	15.4±0.6
2.26	"	15±1.50
2.42	"	13.1±0.8

*It is not known to us whether there is a connection between the data obtained in reference 59 and in the review paper by Alvarez.[60]

TABLE IX.
K^+n-interaction

Kinetic energy in the laboratory system E, Bev	Method of measurement of the cross section	σ_{tot}, mb
0.9	Counters[59]	16.4±2
1.2	"	16.8±3.7
1.4	"	18.2±4
1.9	"	12.5±3.5

TABLE X.
K^-p-interaction

Kinetic energy in the laboratory system E, Bev	Method of measurement of the cross section	σ_{tot}, mb
0.9	Counters[60]	52±9
0.94	Counters[59]	36±5
1.17	Bubble chamber[60]	48±5
1.23	Counters[59]	44±5
1.4	Counters[59]	36±5
1.7	"	44±5
2.8	"	20±5

variance with respect to charge conjugation, the total cross sections, as well as the cross sections for elastic and inelastic interactions, should be:

$$\sigma(K^+p) = \sigma(K^0n) = \sigma(K^-\tilde{p}) = \sigma(\tilde{K}^0\tilde{n}),$$
$$\sigma(K^+n) = \sigma(K^0p) = \sigma(K^-\tilde{n}) = \sigma(\tilde{K}^0\tilde{p}),$$
$$\sigma(K^-p) = \sigma(\tilde{K}^0n) = \sigma(K^+\tilde{p}) = \sigma(K^0\tilde{n}),$$
$$\sigma(K^-n) = \sigma(\tilde{K}^0p) = \sigma(K^+\tilde{n}) = \sigma(K^0\tilde{p}).$$

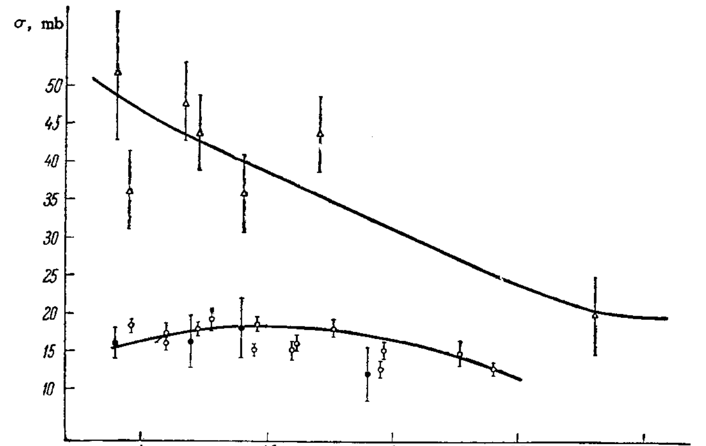

FIG. 4. Total cross sections σ_{tot} for the interaction of K mesons with nucleons. ○, ●, Δ – cross sections for $\sigma_{tot}(K^+p)$, $\sigma_{tot}(K^+n)$, and $\sigma_{tot}(K^-p)$, respectively.

At high energies, the charge symmetry and invariance with respect to charge conjugation have not been tested experimentally for K mesons, but their validity is very probable.

It can be seen from the above experimental data that the cross sections for the interaction of K^+ and K^0 mesons with protons for energies $E \gtrsim 1$ Bev are the same within the limits of experimental error, which indicates a weak isotopic-spin dependence of the interaction. At high energies, the interaction cross sections of K^+ and K^- mesons with protons also become similar. The cross sections are smaller by roughly one third than the corresponding cross sections for NN and πN interactions. It is, however, not clear whether this cross section will change substantially at energies $E > 2.5$ Bev.

2.5. Interactions of π^- and K Mesons with π Mesons

Information concerning these interactions can at present be obtained only from an analysis of indirect experimental data.

The phase analysis of πN and NN interaction shows that the value of the cross section for the $\pi\pi$ interaction is close to that of the cross section for πN and NN interactions. The analysis of the angular asymmetry of particles produced in the collision of fast π mesons with nucleons[61] leads to the same result, as does the study of multiple-particle production in cosmic rays at very high energies.[62]

These results can be illustrated by the following very qualitative considerations. Let us write the cross sections for inelastic NN and πN interactions in the form:

$$\sigma(NN) = 4\pi r_N^2, \quad \sigma(\pi N) = \pi(r_N + r_\pi)^2.$$

At high energies, where the wave length of the interacting particles is very small ($\lambda\!\!\!^{-} \ll r_N, r_\pi$), the quantities r_N and r_π can be considered as the effective dimensions of the nucleon and the meson. Since $\sigma(NN) \approx \sigma(\pi N)$ for $E > 1$ Bev, we have $r_\pi \approx r_N$, and the cross section for inelastic $\pi\pi$ interactions is

$$\sigma(\pi\pi) \approx 4\pi r_N^2 \approx \sigma(\pi N).$$

The cross sections for elastic scattering will also be close to each other, $\sigma_e \approx \sigma_d$. (It should be noted that the differential scattering is fully determined by inelastic processes.)

As has been mentioned,[72] the lower limit for the $\pi\pi$ interaction cross section $\sigma_i(\pi\pi) > 5$ mb can be obtained if we consider π mesons as consisting of a point ("bare") nucleon and a point antinucleon. If we take the effective dimensions of these virtual particles into account, the cross section $\sigma_i(\pi\pi)$ increases several times.[89]

Using the double-dispersion relations, Mandelstam and Chew[63] have recently succeeded in formulating a system of equations determining the amplitude of elastic $\pi\pi$ scattering up to energies $E \sim 0.5$ Bev. However, the solution to this system of equations has not as yet been published.

Available information on the interaction between π and K mesons is even more meager. The very existence of such an interaction is at present problematic.

It has been shown[64,65] that, if we take the interaction

$$H_{\text{int}} = \lambda(\widetilde{K}^+K^+ + \widetilde{K}^0K^0)(\pi^+\pi^- + \pi^-\pi^+ + \pi^0\pi^0),$$

into account, where π, K, and $\widetilde{K}$ are operators of the corresponding particle fields and λ is the coupling constant, we can explain a number of experimental facts concerning the interaction of K mesons with nucleons and nuclei at low energies.

If K^+ and K^0 mesons have different parity, then a triple interaction of the form $H_{\text{int}} = \lambda\widetilde{K}\pi K$ is, in principle, possible.

In the high-energy region, we can expect to obtain information on the interaction between K and π mesons from the study of peripheral interactions of fast K mesons with nucleons,[66] and also from the analysis of the angular distribution of particles produced in collisions of very fast π mesons with nucleons.[61] The interaction between π and K mesons should, in the latter case, lead to a definite angular asymmetry of the strange particles that are produced. Preliminary results obtained by Wang et al.[67] at an energy $E = 6.8$ Bev indicate the presence of such asymmetry, although the interpretation of such preliminary results is, of course, still very problematic.

III. THEORETICAL INTERPRETATION OF THE EXPERIMENTAL DATA

3.1. Constant Value of the Interaction Cross Sections at High Energies

From the experimental data listed above, it can be seen that the cross sections σ_{tot}, σ_i, and σ_e remain constant at high energies, at least for NN and πN interactions.

It should be mentioned that this conclusion is correct only within the limits of experimental error, especially since only a limited energy range is accessible to either theoretical or experimental investigation.

There is, at present, no exact theory that could explain the behavior of the cross sections even at those high energies that are accessible to modern experimental procedure. Moreover, the fact that the interaction cross sections remain constant for $E \gg 1$ Bev seems to contradict the contemporary field theory in which, when calculating many physical quantities (particle masses, magnetic moments, etc.), it is necessary to make various assumptions to suppress the interactions at high energies. It is, however, possible to cite examples (e.g., reference 68) in which even very weak interactions lead to large cross sections, for increasing energy. The weakening of the interactions can, in that case, compensate for the increase in the interaction cross section. It is possible that an analogous situation prevails in the general case.

The theoretical analysis of the properties of the propagation functions using the local field theory yields, under very general assumptions, the following limiting estimate for the energy dependence of the cross section for the interaction of π mesons with nucleons in the laboratory system[69]

$$\frac{\text{const}}{E} < \sigma_{\text{tot}}(E) < E\cdot\text{const}.$$

This does not contradict the experimental results.

3.2. Equality of the Interaction Cross Sections of Particles and Antiparticles

Assuming that the interaction cross section of protons is constant at high energies, one can demonstrate the equality of the total cross sections for pp and $\widetilde{p}$p interactions. An analogous conclusion holds for the total cross sections for the interaction of π^- and π^+ mesons with protons, for the total cross sections for the K^+p and K^-p interactions, etc., and in the general case, for interactions involving alternatively a particle or the corresponding antiparticle.[70]

In order to prove this theorem, let us consider the dispersion relation for the elastic scattering amplitude A(E) for the angle $\theta = 0$:

$$\operatorname{Re} A(E) = \frac{(E-E_0)^2}{4\pi^2}p \times \int_0^\infty \frac{dE'}{\lambda'}\left[\frac{\sigma_{\text{tot}}(E')}{(E'-E)(E'-E_0)^2} + \frac{\widetilde{\sigma}_{\text{tot}}(E')}{(E'+E)(E'+E_0)^2}\right] + a + bE,$$

where a and b are constant coefficients,[71] $\lambda\!\!\!^-$ is the nucleon wave length, and E_0 an arbitrary energy value.*

*For simplicity, we shall not take the spin- and isotopic-spin dependence of the amplitude A(E) into account. All arguments can easily be repeated for the general case also. We shall also neglect electromagnetic interactions, which contribute only small corrections.

It is significant that the dispersion integral always includes the interaction cross sections both for particles σ_{tot} and for antiparticles $\tilde{\sigma}_{tot}$.

For $E \to \infty$, we obtain, keeping only the largest terms,

$$\operatorname{Re} A(E) \sim E^2 p \int_{\mathcal{E}}^{\infty} \frac{dE'}{E'} \left[\frac{\sigma_{tot}(\infty)}{E'-E} + \frac{\tilde{\sigma}_{tot}(\infty)}{E'+E} \right],$$

where the energy $\mathcal{E}$ is chosen so that $\mathcal{E} \gg E_0$ and, for $E' \geq \mathcal{E}$, $\sigma_{tot}(E') = \sigma_{tot}(\infty)$, $\tilde{\sigma}_{tot}(E') = \tilde{\sigma}_{tot}(\infty)$. Evaluating the integral, we find

$$\operatorname{Re} A(E) \sim E \ln E [\tilde{\sigma}_{tot}(\infty) - \sigma_{tot}(\infty)]. \quad (1)$$

However, the inelastic scattering amplitude A(E) cannot increase as E ln E. In fact, at high energies, where $l \gg 1$,

$$|A(E)| = \left| \frac{\lambda}{2} \sum_{l=0}^{\infty} (2l+1)(1 - e^{2i\eta_l}) \right| \leqslant \frac{1}{\lambda}$$

$$\times \int_0^{\infty} \varrho \, d\varrho \, |(1 - e^{2i\eta\left(\frac{\varrho}{\lambda}\right)})| \leqslant E \cdot \text{const}, \quad (2)$$

since[72]

$$\frac{1}{\lambda} \sim E$$

and

$$|(1 - e^{2i\eta\left(\frac{\varrho}{\lambda}\right)})| \leqslant 2.$$

Clearly, the relations (1) and (2) are compatible only if

$$\sigma_{tot}(\infty) = \tilde{\sigma}_{tot}(\infty). \quad (3)$$

It is sometimes maintained that Eq. (3) results only from the properties of the dispersion relations. However, from the proof given above, it is evident that Eq. (3) has been obtained by means of the equivalent assumption that the interaction cross sections remain constant for $E \to \infty$. If Eq. (3) is considered as the basic one, then, from the dispersion relations, we find that the cross sections σ_{tot} and $\tilde{\sigma}_{tot}$ are constant for $E \to \infty$.*

If a variation of the cross section (e.g., a slow decrease) should occur only for extremely high energies of $E \gg 100 - 1000$ Bev, then this will introduce only a small correction to the dispersion relations for lower energies, and all the above arguments, and in particular Eq. (3), will remain approximately correct for energies $E \lesssim 100 - 1000$ Bev.

3.3. Isotopic Spin Dependence of the Cross Sections

At energies $E \lesssim 1$ Bev, the particle interactions depend markedly on their isotopic spin. This can be seen from Figs. 5 and 6, where the energy dependence of the experimental cross sections of π mesons and nucleons is shown for states with definite values of isotopic spin.

*D. V. Shirkov has first drawn the author's attention to this problem.

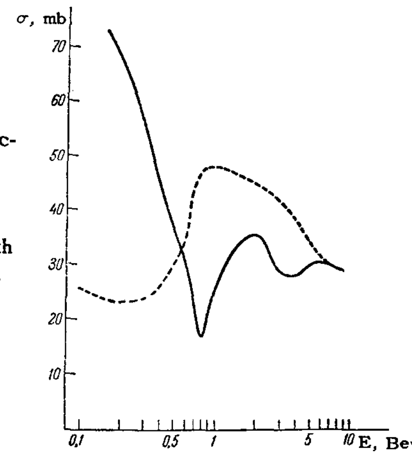

FIG. 5. Total cross sections for the interaction of nucleons in the state with isotopic spin T = 0 (solid curve), and in the state with isotopic spin T = 1 (dotted curve).

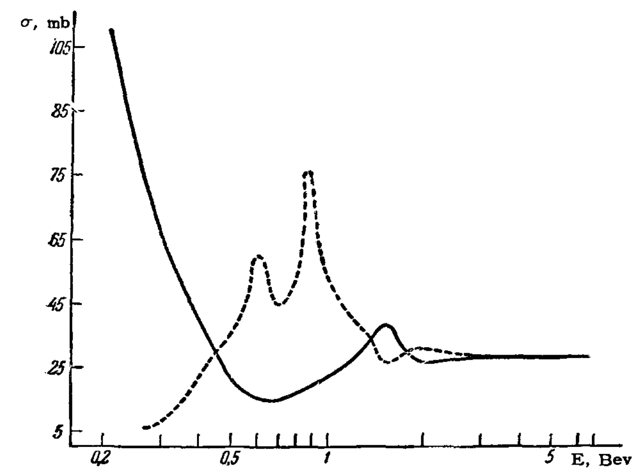

FIG. 6. Total cross sections for the interaction of π mesons with nucleons in the state with isotopic spin T = 3/2 (solid curve), and in the state with isotopic spin T = 1/2 (dotted curve) The dotted curve denotes the cross section $\sigma_{tot}(\pi^0 p)$.

With increasing energy, however, the number of possible inelastic-reaction channels increases rapidly, while the cross section $\sigma_i \to$ const., or at least does not increase. Moreover, the cross section for each of the inelastic channels, including the charge-exchange scattering σ_{ce}, becomes constantly smaller: $\sigma_{ce}/\sigma_e \to 0$.* In other words, the contribution of interactions involving the reorientation of the isotopic spin of colliding particles becomes negligibly small. This leads to the result that the high-energy cross sections become independent of the isotopic spin.

We shall illustrate this on the example of π-meson and nucleon scattering on nucleons.[78]

If F_1 and F_3 are the amplitudes of π-meson scattering on protons in states with isotopic spin $T = 1/2$ and $T = 3/2$ respectively, then the differential cross sections for π-meson scattering can be written as

*It is of course assumed that the charge-exchange cross section shows no resonance at high energies. This agrees with present views on the mechanism of particle interactions at energies $E \gg 1$ Bev.[72,79]

$$\sigma_1 \equiv \sigma(\pi^+ p \to \pi^+ p) = |F_3|^2,$$
$$\sigma_2 \equiv \sigma(\pi^0 p \to \pi^0 p) = \frac{1}{9}|2F_3 + F_1|^2,$$
$$\sigma_3 \equiv \sigma(\pi^0 p \to \pi^+ n) = \frac{2}{9}|F_3 - F_1|^2,$$
$$\sigma_4 \equiv \sigma(\pi^- p \to \pi^- p) = \frac{1}{9}|F_3 + 2F_1|^2,$$
$$\sigma_5 \equiv \sigma(\pi^- p \to \pi^0 n) = \sigma_3.$$

From the condition that the charge-exchange cross sections vanish, $\sigma_3 = \sigma_5 \approx 0$, it follows that $F_1 = F_3$, i.e.,

$$\sigma_1 \approx \sigma_2 \approx \sigma_4. \tag{4}$$

As can be seen from Figs. 2 and 6, the cross sections already become independent of the isotopic spin at energies of 2 – 3 Bev, within the limits of experimental error.

Since, at high energies, Eq. (4) is satisfied for any scattering angle θ, the corresponding scattering-amplitude phase shifts η_e are also found to be equal:

$$A(\theta) = \frac{\lambda}{2i}\sum_{l=0}^{\infty}(2l+1)(1-e^{2i\eta_l})P_l(\cos\theta)\,^*).$$

This leads, in particular, to the equality of the cross sections σ_i and σ_{tot}.

For the case of scattering of protons on nucleons, the differential cross sections can be written as

$$\sigma_1 \equiv \sigma(pp \to pp) = |F_1|^2,$$
$$\sigma_2 \equiv \sigma(pn \to pn) = \frac{1}{4}|F_1 + F_0|^2,$$
$$\sigma_3 \equiv \sigma(pn \to np) = \frac{1}{4}|F_1 - F_0|^2,$$

where F_1 and F_0 are the scattering amplitudes of states with isotopic spin $T = 1$ and $T = 0$ respectively.

From the condition $\sigma_3 = 0$, it follows that $F_0 \approx F_1$, i.e.,

$$\sigma_1 \approx \sigma_2.$$

Also, the phases of the elastic scattering amplitude η_e, and the cross sections σ_i and σ_{tot}, become equal.

From Figs. 1 and 5 it can be seen that, within the limits of accuracy of the experiment, the isotopic-spin dependence of the interaction cross section disappears at energies $E > 5 - 6$ Bev. This energy is approximately twice the corresponding energy for the case of πN interactions.

Similarly, one can also consider the interaction of other types of particles. For the interaction of antinucleons with nucleons, the isotopic spin dependence apparently becomes negligible for somewhat higher energies than for the NN interaction. For E = 2 Bev, the charge-exchange cross section $(p\tilde{p} \to n\tilde{n})$ still amounts to about 30% of the elastic-scattering cross section.

As can be seen from Fig. 4, there is no marked isotopic spin dependence of the interactions of K^+ mesons with nucleons at energies $E \gtrsim 1$ Bev.

3.4. Dependence of the Cross Sections on the Spins of Colliding Particles

At high energies, where the main role is played by the orbital quantum numbers $l \gg 1$, the phase of the elastic-scattering amplitude depends only on the energy and on the total spin of the colliding particles S, since

$J = |l - S|; \ldots; |l + S| \approx l$ and $l' = |J - S|; \ldots; |J + S| \approx J \approx l$:

$$\eta_{ll'}(E;\ J;\ S) \approx \eta_l(E;\ S).$$

In the energy range $E \gtrsim 1$ Bev, not a single experiment with polarized particles is known to have been performed up to the present. We therefore have no experimental data on the spin dependence of fast-particle interactions. We can, however, assume that the interactions will be practically spin-independent at energies $E \gg 1$ Bev.

For the interactions of particles of various types, e.g., π or K mesons with nucleons, this can be explained by similar considerations as for isotopic spins. The spin-flip scattering cross section σ_s will then decrease rapidly with increasing energy.

Similar considerations apply to the interactions of nucleons, where there are no transitions between singlet and triplet states, and $\sigma_s \equiv 0$.* However, according to present views on the mechanism of inelastic interactions at high energies (the compound particle model,[80] and the statistical theory of central and peripheral collisions[72,79]), one should expect that also in this case $\eta_l(E;\ S) = \eta_l(E)$. (It should be remembered that, for $E > 1$ Bev, $\sigma_e \approx \sigma_d$ and is fully determined by inelastic processes.)

Clearly, all the considerations concerning the spin and isotopic-spin dependence are also applicable to interactions with nuclei.[81]

IV. CONCLUDING REMARKS. PROBLEMS FOR EXPERIMENTAL INVESTIGATION

From the experimental and theoretical data presented above, a rather clear qualitative picture emerges of the interaction cross-section behavior of nucleons, antinucleons, and π and K mesons with nucleons in the energy range $E \approx 1 - 10$ Bev. Quantitative data, however, are in many cases still insufficient. This is especially true for the interaction of antinucle-

*This can easily be seen if Eq. (4) is written for the scattering amplitude multiplied by the Legendre polynomial $P_l(\cos\theta)$, and integrated over all values of $\cos\theta$ from –1 to +1.

It should be noted that Eq. (4) can lose its validity in the region of very small angles $\theta \sim 0$, where there is a contribution from distant peripheral collisions with a small energy transfer. However, this contribution diminishes rapidly with increasing energy.

*In the case of pp interactions (and, in general, in the case of the interaction of any two identical particles), $\sigma_s \equiv 0$ because of the parity conservation of the identical-particle system. In the case of pn interactions, $\sigma_s \equiv 0$ because of charge symmetry.[82]

ons and K mesons with nucleons. The interaction of any particles with neutrons has been poorly investigated, and the interaction of polarized particles represents a completely blank spot. More exact measurements would, in themselves, be interesting, and would also be important for a quantitative check of the theoretical schemes and models. Any data concerning the $\pi\pi$ interaction, the interactions of π and K mesons, and the interactions of hyperons, would be of great interest.

The qualitative picture will most likely remain unchanged in the energy range up to several tens of Bev.

We can, at any rate, point out two basic problems of high-energy physics. One is the investigation of up to what energy, for $E \to \infty$, the cross sections σ_{tot}, σ_i, and σ_e remain constant. If experiments should indicate that these cross sections remain constant up to very high energies, then, in the collisions of very high-energy particles, showers may be produced in which the mass of the newly produced particles may attain a macroscopic size. Figuratively speaking, the stars that are produced in such a case may not be stars as the word is understood in the emulsion laboratory, but as it is used in astronomy.[83] On the other hand, the variation of the asymptotic behavior of interaction cross sections would lead to very important theoretical conclusions.

Cosmic-ray experiments are at present, and apparently will remain in the near future, the only source of information on particle interactions at extremely high energies.

The second basic problem is the study of possible violations of the equality of interaction cross sections of particles and antiparticles $\sigma = \tilde{\sigma}$. Such a violation would indicate the invalidity or incorrectness of the dispersion relations and of the principles based upon them, and first of all, of the causality principle in small regions of space-time. Important results in this direction might be obtained at energies attainable in accelerator experiments.

APPENDIX

MEAN FREE PATH OF FAST PARTICLES IN EMULSION

In the analysis of fast-particle tracks in photographic emulsion, the elastic scattering events of these particles on nuclei are, as a rule, not considered. Elastic scattering at high energies is almost exclusively diffractive, and is characterized by very small angles, which are the smaller the greater the dimensions of the nuclei. A special method is necessary in order to measure small deflections of the tracks in elastic nuclear scattering.[86]

In all events of interactions of nucleons and π mesons with photographic emulsions, for which the values of the mean free path are presented in Tables III and IV, only elastic scattering on hydrogen at relatively large angles has been taken into account, in addition to inelastic interactions.* The mean free path is, in that case, given by

$$L = \frac{1}{\{\sum_i N_i \sigma^i_{\mathrm{i}} + N_H \sigma_{\mathrm{tot}}\}}, \tag{A}$$

where N_H is the number of hydrogen nuclei per cm³ of the emulsion, N_i is the number of nuclei of other elements per cm³ of the emulsion (see Table XI), σ_{tot} is the total interaction cross section of the primary particle with hydrogen, and σ^i_i is the cross section for inelastic interactions of these particles with other nuclei.

TABLE XI. Composition of the emulsion (number of nuclei per cm³, $N_i \times 10^{-22}$)

Emulsion \ Element	H	C	N	O	Br	Ag
NIKFI-R	2.93	1.39	0.37	1.06	1.02	1.02
Ilford G-5	3.37	1.36	0.29	1.02	1.02	1.02

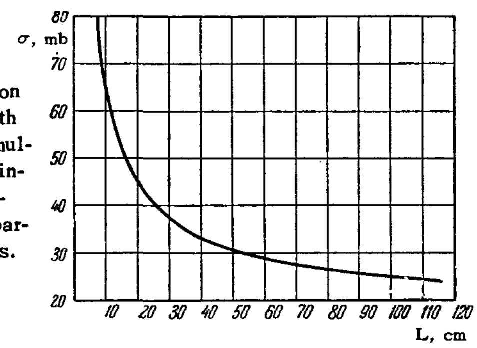

FIG. 7. Variation of the mean free path L of particles in emulsion with the total interaction cross section σ_{tot} of these particles with nucleons.

At high energies, where the wavelength of the particle interacting with the emulsion is much smaller than the dimensions of the nuclei, the optical-model formula can be applied with success[27,72]

$$\sigma^i_{\mathrm{i}} = 2\pi \int_0^\infty r\left[1 - e^{-2k_i \int_0^\infty \varrho_i\left(\sqrt{r^2+s^2}\right) ds}\right] dr. \tag{B}$$

where $\rho_i(r)$ is the distribution of matter in the nucleus of the i-th type, as determined in experiments on the scattering of fast electrons on nuclei.[87] The absorption coefficient $k_i = d_i \sigma_{tot}$, where

$$d_i = \frac{A_i}{\int \varrho_i(r)\, d^3x}$$

is the mean density of nucleons in the nucleus with

*Those elastic-scattering events on hydrogen where the particles were emitted at very small angles have, of course, not been measured. However, as is shown by estimates, the contribution of such interactions that have not been accounted for cannot change the magnitude of the mean free path appreciably. At energies E > 10-15 Bev, one should exchange σ_{tot} for σ_i in Eq. (A).

atomic number A_i. Equation (B) is correct for energies $E \gtrsim 1$ Bev. The validity of this equation increases with increasing energy.

The calculated values of L are shown in Fig. 7 as a function of the cross section σ_{tot}. As has been shown by calculations, the difference in the mean free path $L(\sigma_{tot})$ in the emulsions Ilford G-5 and NIKFI-R is negligible.[88] The curve $L(\sigma_{tot})$ in Fig. 7 is applicable for both emulsion types. The values of the cross sections σ_{tot} shown in Tables III and VI have been obtained by means of these curves.

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