### ELECTRIC BREAKDOWN OF GASES AT PRESSURES CLOSE TO ATMOSPHERIC PRESSURE\*

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#### I. INTRODUCTION

# 1.1. Classification of Types of Breakdown and Survey of Questions to be Considered

THE title "breakdown of gases at pressures close to atmospheric" takes in a wide class of transition processes in electric discharges in gasses and by itself is not a complete definition of a group of physical phenomena subject to uniform laws.

The information accumulated up to the present makes possible further distinctions between various types of breakdown, which are controlled by various mechanisms that are characteristic for the different types. Among these types we can name the following as having the greatest scientific interest or interest for applications:

A. Breakdown at voltages only slightly above the static breakdown voltage, not too long spark gaps (breakdown voltages up to several hundred kv), weak inhomogeneity of the field, and weak external ionization.

B. Static breakdown of extremely long spark gaps (such as lightning) and of gaps with inhomogeneous electric fields (also with not too strong external ion-ization).

C. Pulsed breakdown (breakdown resulting from the application of a voltage pulse of short duration to a gas-filled gap) with voltage in the pulse greatly exceeding the dc voltage required for static breakdown of the given gap.

D. High-frequency breakdowns (at voltage frequencies comparable with the frequency of collision of the particles of the gas).

E. Breakdown when there is intense ionization in the gap by an auxiliary low-power high-voltage source which produces a luminous channel in the gas.

From the point of view of basic understanding of the physics of electric discharges in gases, the problem of the first type of breakdown (type A) is the most important and of the most general interest; for a very long time it has received much attention from a wide circle of investigators. In recent years the study of the mechanism of this type of breakdown has given significant results, and one of the problems of this survey is to generalize and extend these results.

The problems of breakdowns of types B and C are

of a more specialized nature and are treated in special review articles.<sup>122,177,91</sup> A general treatment of these problems is difficult because there are many accessory parameters (the shape and polarity of the electrodes, the duration and magnitude of the voltage pulse, and so on). In the present paper these questions are touched upon only in connection with the consideration of the boundary regions between breakdowns of type A and types B and C.

The question of breakdowns of type D is an entirely isolated one and is not considered here, since it is the subject of quite a body of literature of very specialized interest.<sup>13,91</sup>

Finally, the problem of breakdowns of type E, which has great importance for applications to modern electronic gas-discharge devices (arrester spark gaps, pulsed lamps, some continuously operated gas-discharge tubes, etc.), has so far received very little attention. It seems desirable to include in this paper a brief discussion of present ideas about the physical mechanism of breakdowns of this type.

#### 1.2. The Historical Development of the Subject

At high pressures (above about 20 mm Hg) the transition from a non-self-sustaining to a self-sustaining gas discharge is always accompanied by a physical process which occurs with extreme rapidity and has received the general designation of "electric breakdown." The violent nature of this process, whose detailed mechanism can be quite different under different conditions, is due to two general properties of the electric conductivity of gases:

1. The exponential relation between the yields of most elementary processes of ionization (at the small absolute magnitudes that are characteristic at high pressures and small mean free paths) and the energy of the particles, or the parameters that determine this energy (electric field, temperature). Examples of such relations are the dependence of the coefficient  $\alpha$  for ionization of atoms by electron impact on the strength of the electric field, the similar dependence for the process of atomic excitation, whose intensity determines the effectiveness of stepwise ionization, the relations for photoelectric effects at the cathode and in the gas, and so on. At high presssures and temperatures of the gas, owing to the multiplicity of types of elementary processes of energy exchange between particles that is characteristic of these conditions, there is a tendency to

<sup>\*</sup>Breakdown at voltage slightly exceeding the static breakdown voltage, with weak inhomogeneity of the field of the electrodes and with weak or strong external ionization.

thermodynamic equilibrium of the gaseous plasma. The temperature dependence of the degree of ionization of such a plasma (which characterizes the intensity of all the ionization processes taken together) is also of the exponential form (being expressed by the so-called Saha formula).

2. The existence in the gas of multiply cyclic repeated processes with cycles of very short duration. which, in analogy with nuclear chain reactions, can be called "chain" processes. One example of a "chain" process is ionization by electron impact, during which at each new cycle (of duration a few millimicroseconds) the number of free electrons involved in the process is doubled, so that a so-called electron avalanche is formed. Another example is thermal ionization in a newly formed spark channel, which leads to a rise of the current density and an increase of the scattering power and temperature of the channel, which, in turn, raises the degree of ionization of the gas (as the duration of a cycle of such a process we may agree to take the time for establishment of thermodynamic equilibrium,  $\sim 10^{-9} \sec^{112}$ ).

These two properties, together with a wide variety of elementary processes of energy exchange between the particles, make the study of gaseous breakdown at high pressures, especially complicated, from both the experimental and the theoretical points of view.

The complexity of the experimental study of breakdown is primarily due to the following factors:

1) The short duration of the breakdown process. To record its time characteristics, oscillographs and photographic devices of the very highest speed are required.

2) The change of the parameters of the discharge (for example, the current, the intensity of radiation) by many orders of magnitude in the course of the breakdown. Devices used to measure these parameters must have a very wide range of sensitivity.

3) The highly critical nature of the conditions for the initiation of breakdown. Instruments for checking the parameters that determine these conditions (the voltage, the distance between the electrodes, the pressure and purity of the gas, the action of external sources of ionization, the electric field, and so on) must possess high absolute accuracy.

Knowing how much recent years have contributed to the perfecting of laboratory equipment in these respects, it is not hard to realize that only quite recently (in fact, during the last two decades) has experimental technique been able to provide any really serious advances in our knowledge of the phenomenon of gaseous breakdown at high pressures.

The difficulty of the theoretical explanation of breakdown phenomena is due (apart from the lack of basic experimental facts up until very recent times) to the fact that the two general properties of the electric conductivity of gases that have been mentioned, which account for the violent nature of the breakdown and the critical character of the conditions for its occurrence, could be satisfactorily explained by quite a number of different mechanisms for the increase of the conductivity. This circumstance has provided fertile soil for the production of ingenious but inadequately founded theoretical schemes of the breakdown mechanism.

These schemes have explained plausibly the transition of the main gap from a nonconducting to a conducting state and have given satisfactory quantitative estimates of the dependence of the breakdown voltage on the length of this gap and the density of the gas (this dependence is remarkably insensitive to the basic mechanism assumed). Subsequently, however, on the basis of new accumulations of experimental data, these schemes have had to be discarded or radically modified.

The schemes in question have tried in an inconsistent way to fit the extremely complex phenomenon of breakdown, over practically its whole duration, into the framework of some single isolated mechanism for example, the interaction of  $\alpha$  and  $\beta$  ionizations, or of  $\alpha$  ionization and thermal ionizations, or, finally, of  $\alpha$  ionization and the space-charge field of the head of the discharge, and so on. All of these schemes can be divided into the following two groups, for each of which there is a common space-time picture of the development of the breakdown, with various choices of the dominant elementary process:

a) Schemes involving the Townsend "swaying of ionization,"<sup>193-197,74</sup> in which the products of the primary electron avalanches (formed as the result of  $\alpha$ ionization, mainly near the anode) produce in one way or another secondary electrons near the cathode (as the result of  $\beta$  or  $\eta$  ionization in the volume – ioni– zation of atoms either by positive ions or by photons - or of  $\gamma$ ,  $\delta$ , or  $\epsilon$  processes at the cathode - ejection of electrons from the cathode, either by positive ions, by photons, or by excited atoms\*). These electrons are the initiators of secondary electron avalanches, which in turn give rise to a third generation of electrons at the cathode, and so on. If there is an active balance from a single cycle of this swaying (production of an average of more than one secondary electron per primary avalanche) the process leads to unlimited increase of the current without concentration of the discharge in a narrow channel.

b) So-called "streamer" schemes, in which an individual electron avalanche (produced by a single electron arising in the region near the cathode) acoumulates along its path such a large space charge at its head, that the local distortion of the field leads (through local photoionization<sup>120-122,99-103,140-143</sup> or thermal ionization<sup>65,119,181</sup>) to the formation of a narrow channel of highly ionized plasma (a so-called "streamer"), which quickly extends to the anode and

<sup>\*</sup>We employ the conventional symbols of the processes used by a number of authors; see, for example, the book by Llewellyn-Jones.<sup>91</sup>

the cathode. The growth of the streamer leads to a violent increase of the current in the discharge circuit and culminates in the formation of a narrow conduct-ing channel across the gap between the electrodes.

In the period in which these theoretical schemes appeared it seemed that the schemes of each group completely negated the physical reality of those of the other group. During the development of ideas about breakdowns of type A, however, the previously mentioned inconsistent approach to the phenomenon was overcome, and a single correct conception, now held by most authors, was developed, which combines the schemes of the two groups. According to the new conception, at different stages of the breakdown process the decisive part is played by the entirely different mechanisms peculiar to the various theoretical schemes.

The first steps in this direction were made in the experimental papers of Pedersen,<sup>133</sup> Burawoy,<sup>14</sup> Rogowski, Fleger, and Tamm,<sup>151-153,190</sup> Beams,<sup>7</sup> Torock,<sup>192</sup> and others, who measured the extremely short times of formation of breakdown under overvoltages, and also in the theoretical work of Rogowski,<sup>150,153,157</sup> Loeb,<sup>92,93</sup> and Hippel and Frank.<sup>73</sup> In these latter papers it was shown that the increase of ionization, after beginning in the Townsend "swaying" (owing to interactions  $\alpha$  and  $\beta^*$ , or  $\alpha$  and  $\gamma$ , or, finally, in  $\alpha$  and  $\delta$  processes<sup>154</sup> <sup>155</sup> <sup>172</sup>), can be greatly accelerated at a later stage by the action of the positive space charge formed during the swaying process (space charge distributed over the entire plane of the gap, unlike the concentrated space charge considered in the "streamer" schemes). An important part in the development of the new conception was played by a number of later papers by Rogowski and his collaborators, devoted to the effect of a plane space charge on the swaying of the ionization, 156, 158-162, 171, 16, 57-62 and also by analogous papers by Varney, White, Loeb, and Posin.<sup>203,135</sup>

In papers by White and Dunnington, who used a Kerr-cell shutter,<sup>32,208,209</sup> by Loeb and his collaborators, who observed the corona discharge,<sup>95,96,76, <sup>77,198,199</sup> and by Flegler and Raether, who observed tracks of delayed discharge pulses in a cloud chamber,<sup>50,137,138</sup> and also in accompanying theoretical papers by Meek, Raether, and Loeb,<sup>120,140,99</sup> studies were made on reciprocally reacting channels in breakdown. Thus the picture was developed of another very important link in the chain of processes acting at different stages of the development of the thin channel of highly ionized plasma known as a streamer. It was precisely these papers, however, that for a considerable period of time caused many authors to deny in general, on insufficient grounds,<sup>213,110,111</sup> the physical</sup> reality of the swaying mechanism for breakdown at high pressures.<sup>100-103,142,143,122</sup>

As a result of the first experiments, which showed that in a type A breakdown there are secondary processes at the cathode (influence of the material of the cathode on the breakdown voltage) and that the times for formation of breakdown are much larger than the time of passage of a single electron avalanche (at small overvoltages), Marshak in 1943 proposed<sup>110</sup> the modern conception of the development of type A breakdown. According to this conception, breakdown begins with Townsend swaying of ionization (evidently with the simultaneous action of several secondary mechanisms), which leads to the formation of a plane space charge. This accelerates the build-up of ionization and finally creates the conditions for the extension of a contracted channel of highly ionized plasma, which closes across the gap between the electrodes.

During the period since its first formulation, this conception has found confirmation in a large number of experimental papers by many authors, including investigations of:

a) The time of formation of breakdown (papers by Gänger,<sup>66</sup> Fisher, Bederson, Kachikas, and Lessin,<sup>37-48</sup> Morgan,<sup>127,128</sup> Aked, Bruce, and Tedford,<sup>2</sup> and by Raether, Köhrmann, Feldt, and Sohst.<sup>79-83,145-147,36,183</sup>).

b) The current in stationary non-self-maintaining discharge close to breakdown (papers by Llewellyn-Jones, Parker, Crompton, Dutton, and Haydon,<sup>90,33,21-23</sup> Wilkes, Hopwood, and Peacock,<sup>211</sup> and Fisher, De Bitetto, and Rose.<sup>29,30,163</sup>).

c) The effect of the cathode material on the breakdown voltage (papers by Trump, Cloud, Mann, and Hanson<sup>200</sup> and by authors named earlier<sup>33,29</sup>).

d) The growth of the current in the early stages of breakdown (papers by Bandel,<sup>4,5</sup> Raether, Schmidt, Pfaue, Vogel, Frommhold, Kluckow, and Schlumbohm,<sup>55, 56, 145-147,173-175,134,78,205,206</sup> and by Kojima and Kato<sup>84</sup>).

e) Luminescence in the early stages of breakdown (papers by Fisher, Kachikas, Lessin, and De Betitto,<sup>29</sup>, <sup>45,89</sup> Legler,<sup>146</sup> (sic) Saxe and Chippendale,<sup>167-169</sup> and Loeb and Hudson<sup>108</sup>).

In the last few years this conception has been further developed in theoretical papers and reviews by Loeb, Fisher, Bederson, and Kachikas<sup>42,43,45,48,49,106</sup>, <sup>108,109</sup> Raether, Köhrmann, and Legler, <sup>144–147,80,81,83,87,88</sup> Meek, <sup>123,124</sup> Llewellyn-Jones, Davidson, Davies, and Vick, <sup>25–28,34,91</sup> and Saxe.<sup>168</sup>

In the present paper we survey these experimental and theoretical papers, which confirm the modern conception of type A breakdowns, and enable us to formulate it in its most precise and clear form. These data also enable us to fix more accurately the limits that separate type A breakdowns from breakdowns of types B and C.

In the concluding part of the paper we present a theoretical scheme for type E breakdown and the ex-

<sup>\*</sup>Studies of the process of ionization by positive-ion impacts and of the energy distribution of the ions at breakdown field strengths has shown<sup>6,75,169,201,202,204</sup> that this mechanism is extremely ineffective at high pressures.

perimental basis of this scheme, as found in the first papers in this field by Marshak and Subbotin,<sup>116,117</sup> and also in previous papers on the characteristics of the channel of concentrated discharge by Abramson and Marshak.<sup>1,111-115</sup>

## II. CRITERIA FOR EXPERIMENTAL TESTS, AS OB-TAINED FROM THE THEORETICAL SCHEMES FOR TYPE A BREAKDOWN

#### 2.1. Schemes with Townsend Swaying of Ionization

2.1.1. The Essence of the Mechanism and the Fundamental Equation. Theoretical schemes using "swaying of ionization," the first of which was proposed by Townsend, <sup>193-197</sup> are based on the interaction between primary ionization by electron impacts and secondary processes in which the action of the products of  $\alpha$ ionization produce additional electrons in the region of the discharge near the cathode.\*

In the absence of secondary processes the current density j in a gas-filled gap of length d to which a field sufficient for  $\alpha$  ionization is applied is expressed by the equation

$$j = j_0 \exp \alpha d \tag{2.1}$$

 $(j_0$  is the density of the photoelectric current from the cathode, caused by its irradiation by an external source). The corresponding experimental plot of ln j vs. d is a straight line with slope  $\alpha$ .

When there are secondary processes with coefficients small compared with  $\alpha$  the expression for the current density takes the form<sup>91,110,159</sup>:

$$j = \frac{j_0 \exp ad}{1 - (\beta/a + \gamma + \delta/a + \varepsilon/a + \eta/a) (\exp ad - 1)} = \frac{j_0 \exp ad}{1 - \Gamma(\exp ad - 1)}$$
(2.2)

( $\Gamma$  is the summed coefficient that expresses the effectiveness of all the secondary processes taken together).

The expression (2.2) is the fundamental equation of theoretical schemes with swaying of ionization.

2.1.2. First Criterion – Curvature of the Plots of ln j. For small values of d the curve of the dependence of ln j on the length of the gap [given by Eq. (2.2)] is a straight line with slope  $\alpha$ , just like the curve corresponding to Eq. (2.1), which does not include secondary ionization effects.

For values of d approaching the distance at which breakdown sets in (for which the denominator in the right member of Eq. (2.2) begins to get small), the graph is a curved line with a rapidly increasing slope. Provided the absolute value of j is rather small (so that space charge that destroys the uniformity of the electric field<sup>24, 135,203</sup> is not formed in the gap) one can determine from the curved part of the graph the value of the total secondary-ionization coefficient  $\Gamma$  (the plot of ln j does not enable us to get separate estimates of the effectivenesses of the various secondary mechanisms). The existence of a curved part of the experimental graphs of ln j which corresponds to Eq. (2.2) with a constant coefficient  $\Gamma$  can serve as the <u>first</u> criterion for the experimental verification of the theory.

2.1.3. Second Criterion – Agreement between Calculated and Experimental Values of the Breakdown Voltage. From the point of view of schemes based on swaying of ionization, a condition that causes breakdown is an increase in  $\Gamma(\exp \alpha d - 1) = \mu$  (which is a measure of the multiplication of the electron avalanches through interaction of primary and secondary processes) up to the value unity, with corresponding unlimited growth of the current, which is interpreted as breakdown.\*

A family of graphs of ln j enables us to determine the coefficients  $\Gamma$  and  $\alpha$  as functions of the electric field strength  $\mathcal{E}$ , which is the ratio of the potential U applied to the gap to the gap length d. After substitution of expressions for  $\Gamma$  and  $\alpha$  in terms of U and d the equation

$$\boldsymbol{\mu} = \boldsymbol{\Gamma} \left( \exp \alpha d - 1 \right) = 1 \tag{2.3}$$

would take the form

$$U_{\rm S} = f(d_{\rm S}) \tag{2.4}$$

and would connect values of the breakdown voltage  $U_S$  with the corresponding distances d<sub>S</sub> between the electrodes. Thus measurements of stationary currents in non-self-sustaining discharges at voltages below the breakdown voltage should make it possible to calculate the breakdown voltage values by means of Eq. (2.3). Comparison of such calculated values of  $U_S$  with the corresponding experimental values can serve as a second criterion for the experimental testing of the theory.

2.1.4. Third Criterion – Dispersion of the Measured Values of the Breakdown Voltage. Primary and secondary ionization are statistical processes, and strictly speaking Eqs. (2.1) and (2.2) relate only to average values. On the basis of these equations, a number of papers<sup>11,12,72,86,210</sup> have considered the problem of the statistical deviations from the average values. The probability calculated by Wijsman for getting K secondary electrons from one avalanche is

<sup>\*</sup>Similar theoretical schemes with choices of the dominant secondary process different from that of the Townsend scheme were considered subsequently by quite a number of authors, among whom we may mention J. J. Thomson, W. Rogowski, L. Loeb, and others.<sup>94,191</sup>

<sup>\*</sup>Increase of the current to  $\infty$  with vanishing of the denominator of the expression (2.2) has in itself no physical meaning, since the schematic idea of the swaying of ionization and the derivation of Eq. (2.2) are applicable only to extremely small currents. Schemes with swaying of ionization cannot be applied at all to stages when the current is large, so that, as has been pointed out in a number of papers in which the present conception of the breakdown mechanism has been developed,<sup>23,91,110,111</sup> these schemes can claim only to provide a correct theoretical idea of the mechanism that fixes the condition for the <u>beginning of breakdown</u>, and not of the mechanism of the breakdown as a whole.

 $\omega_{\rm K} = \frac{\mu^{\rm K}}{(1+\mu)^{\rm K+1}}$ . The resulting<sup>86</sup> probability for breakdown caused by one electron formed at the cathode is P = 0 for  $\mu < 1$  and P =  $1 - \mu^{-1}$  for  $\mu \ge 1$ . Using the experimental dependence of  $\alpha$  on the field strength and the value of  $\Gamma$  calculated for the breakdown field strength from Eq. (2.3), one can get values of this probability P as a function of the voltage across the gap. An example of such a graph is shown as curve 1 in Fig. 1. As can be seen from the diagram,



FIG. 1. Distributions of the probability of breakdown P and its derivative dP/dU as functions of the voltage U across the gap (air, 760 mm Hg;  $d_s = 1 \text{ cm}$ ):  $1 - P = \mu^{-1}$  (mechanism with swaying of ionization); 2 and 3 - P and dP/dU, respectively, for the streamer mechanism, with the primary electrons arising in the volume; 4 and 5 - P and dP/dU for the streamer mechanism with the primary electrons arising at the cathode.

the probability of breakdown has an extremely steep rise near the breakdown voltage. By means of the expression for P one can estimate the amount of spread of the measured values of the breakdown voltage for a definite choice of values for the number N of ion pairs produced by cosmic radiation in 1 cm<sup>3</sup> of gas in 1 sec, the area S of the electrodes within which the field is uniform, and the time t during which the voltage is maintained at a given level in the measurements.<sup>110</sup> The probability of the onset of breakdown under these conditions is  $P_1 = PNSt\Delta$ , where  $\Delta$  is the thickness of the thin layer of gas at the cathode in which the electrons that cause breakdown must be produced. Assuming  $P_1 = 1$  for an estimate of the spread of the breakdown voltage, we find:

$$\left(1-\frac{1}{\mu_0+d\mu}\right)NSt\Delta=1,$$

where  $\mu_0 = 1$  is the value of  $\mu$  that corresponds to the minimum breakdown voltage that could be observed with infinitely long periods of waiting for breakdown at voltage levels infinitely close together, and  $d\mu$  is the increment of  $\mu$  associated with the maximum error in the determination of US (equal to the interval between the voltage levels) when one awaits breakdown during the time t at each level. From this we have

$$d\mu = \frac{1}{NSt\Delta - 1}, \qquad (2.5)$$

The inaccuracy  $\Delta$  in the determination of the thickness of the layer at the cathode in which the initial electrons must be produced is equivalent to an inaccuracy in the determination of  $\mu$  and US, and can be estimated by differentiating the expression for  $\mu$ , Eq. (2.3), with respect to the distance d. Thus we get  $\Delta = d\mu/(\mu_0 \alpha) = d\mu/\alpha$ . Inserting this expression for  $\Delta$  in Eq. (2.5) and using the fact that NSt/ $\alpha \gg 1$ , we find  $d\mu = (\alpha/\text{NSt})^{1/2}$ . Differentiating the expression for  $\mu$ , Eq. (2.3), with respect to the electric field  $\mathcal{E}$  and using the fact that  $\mathcal{E} \cdot d = U$ , we get  $d\mu = \mu_0 (d\alpha/d\epsilon) \cdot dU$ , from which we have

$$dU = \sqrt{\frac{\alpha}{NSt}} / \frac{d\alpha}{d\omega} . \qquad (2.6)$$

Taking, for example,  $N = 4 \text{ cm}^{-3} \sec^{-1}$ ,  $S = 10 \text{ cm}^2$ , t = 30 sec, d = 1 cm, and air at pressure 760 mm Hg, and using the appropriate experimental data<sup>90</sup> regarding the dependence of  $\alpha$  on  $\mathscr{E}$  [ $\alpha = 12.2$ ; (d $\alpha/d_{\beta}$ ) = 2 × 10<sup>-3</sup>], we find that the calculated maximum spread of the measured breakdown voltage must be dU = 50 v  $\cong$  0.2 percent of US, that is, it must be extremely small.

It must be even smaller under conditions of illumination by ultraviolet light, for which there is no spread  $\Delta$  in the position of the initial electron and the number of electrons capable of causing breakdown is much increased. In this case we must use instead of the term NSt $\Delta$  in the denominator of Eq. (2.5) the quantity NSt (N is the number of electrons ejected from 1 cm<sup>2</sup> in 1 sec). The expression for dU then takes the form:

$$dU = \frac{1}{(NSt-1)\,da/d\,\breve{g}}\,.\tag{2.7}$$

For N = 10<sup>4</sup>, S = 1 cm<sup>2</sup>, t = 10 sec,  $(d\alpha/d\mathcal{E}) = 2 \times 10^{-3}$  we get dU = 0.05 v  $\simeq 0.00015$  percent.

Agreement of the experimentally observed spread of the values of  $U_S$  with the estimate obtained by calculation can serve as a third criterion for the correctness of the theoretical scheme with swaying of ionization.

2.1.5. Fourth Criterion – Dependence of Us on the Cathode Material. The expression for  $\Gamma$  involves coefficients associated with processes at the cathode. Because of these processes Eq. (2.4) should change when one changes the material of the cathode, on which the coefficients  $\gamma$ ,  $\delta$ , and  $\epsilon$  must depend. If this is so, the breakdown voltages of gas-filled gaps of the same length but with cathodes of different materials should be unequal. It follows from Eq. (2.3) that the difference  $\Delta U_S$  between the values of Us for two cathodes for which the coefficients  $\Gamma$  differ by a factor a is given by:<sup>110</sup>

$$\Delta U_{S} = \frac{\ln a}{da/ds} \, .$$

For d = 1 cm, p = 760 mm Hg,  $(d\alpha/d\delta) = 2 \times 10^{-3}$ ,

we have  $U_S = 29.6 \text{ kv}$ .<sup>90</sup> From these data we get  $\Delta U_S / U_S = 1.7 \ln a$  percent.

According to the known data for low pressures, the values of the coefficient  $\Gamma$  for different metals differ as a rule by factors of 2 to 3. Thus the expected value of  $\Delta U_S$  must be of the order of 1-2 percent.

Experimental confirmation of the dependence of the breakdown voltage on the cathode material could be a <u>fourth</u> criterion for the correctness of the theory (unconnected with the determination of  $\Gamma$  from values of the current in non-self-maintaining discharges). At the same time, within the framework of this theory, it would give a more accurate idea of the importance of secondary processes localized at the cathode.

2.1.6. Fifth Criterion - Dependence of US on the Ionization by External Radiation. So far we have been considering Eq. (2.2) for the case of extremely small currents  $j_0$  which do not cause inhomogeneity of the electric field through the formation of space charge. Under these conditions the coefficients  $\alpha$  and  $\Gamma$  are constant quantities, independent of j. Rogowski and his co-workers<sup>156-162</sup> have examined the change of form of Eq. (2.2) when there are somewhat larger currents  $j_0$  and j that lead to the appearance of some distortions of the field by plane space charge (varying only along the length of the gap), and to a corresponding change of the coefficients  $\alpha$  and  $\Gamma$ . The effects of small distortions of the field can be reduced formally to an increase of the coefficient  $\Gamma$ alone (in first approximation the effect of j on  $\alpha$ can also be represented as a certain additional increase of  $\Gamma$ ). A detailed analysis of Eq. (2.2) shows that under these conditions it determines two values of j for a single value of  $\alpha \cdot d$ , and thus for a single value of U. The first value of j is so small that  $\Gamma$ is still very small. This value lies on the rising branch of the voltage-current characteristic (Fig. 2).



FIG. 2. Voltage-current characteristics of discharge at different values of  $j_0$ , according to Rogowski and Wallraff;<sup>158</sup> these curves are obtained from Eq. (2.2) when one takes into account in first approximation the effect of j on the coefficients in the equation:  $1 - j_0 = 0$ ;  $2 - j_0 = 10^{-11} \text{ a/cm}^2$ ;  $3 - j_0 = 10^{-11} \text{ xa/cm}^2$ .

The second value of j for the same U is so large that the term  $\Gamma \exp \alpha d$  is comparable with unity on account of the increase of  $\Gamma$ . This value lies on the descending branch of the voltage-current characteristic, which describes the process of violent increase of the current with simultaneous decrease of the voltage across the electrodes, i.e., the process of breakdown. It is natural to take as the breakdown voltage  $U_S(j_0)$  for a given  $j_0$  the voltage at which the rising part of the characteristic goes over into the falling part. As can be seen from Fig. 2, an increase of  $j_0$ leads to a decrease of US; the largest value of US is at the intersection of the common asymptote of the right-hand branches of the characteristics with the axis of ordinates and is the US for  $j_0 = 0$  (the voltage for so-called "one-electron breakdown" with practically complete absence of ionization in the gap by external sources).

A mathematical formulation of this approach to Eq. (2.2) enabled Rogowski and Fuchs to find the following theoretical dependence of the lowering of the breakdown voltage,  $\Delta U_S$ , on the current density  $j_0$  produced by external ionization in the gap:

$$\frac{\Delta U_S}{U_S} = K \sqrt{j_0} \cdot$$
 (2.8)

(K is a constant coefficient which can be expressed in terms of partial derivatives of  $\Gamma$  with respect to j and U.

Experimental confirmation of Eq. (2.8) can serve as a <u>fifth</u> criterion for the entire theoretical scheme with swaying of ionization.

2.1.7. Sixth Criterion - the Time of Formation of Breakdown. The time interval between the time the voltage across the gas-filled gap takes the value sufficient for breakdown and the time the breakdown itself begins (let us say, the time the voltage across the gap drops because of a sharp increase of the current) is called the breakdown delay. The breakdown delay t is made up of the time  $t_s$  required for the production of a free electron in the proper region of the gap (in a thin layer at the cathode) and the time  $t_f$  in which this electron can cause breakdown. The quantity ts is called the statistical delay, and was studied theoretically and experimentally quite a long time ago.<sup>85,214</sup> This quantity is not directly associated with any particular theory of the delay mechanism. On the other hand the quantity tf, called the time of formation of breakdown, is one of the most characteristic parameters of any theoretical scheme. Since by the definition of US the breakdown delay at precisely this voltage is theoretically infinite, we may speak of finite values of t in cases in which the voltage U instantaneously applied at the beginning of the time interval t exceeds US by a necessary amount  $\Delta U$ . The quantity  $\Delta U$  is called the overvoltage.

For small statistical delay  $t_s$  (for sufficiently intense ionization in the gap by external radiation) the quantity t is basically determined by the time of formation  $t_f$ .

The attention of investigators has been centered on the theoretical and experimental determination of the time of formation since the observation of extremely small values of  $t_f$  (less than  $10^{-7}$  sec) at considerable over voltages; these observations have given an impetus to all of the present more through study of breakdown.

Even without special calculations one can understand qualitatively that for small  $\Delta U$  (for which  $\mu$ is only slightly different from unity) a large increase of the current owing to swaying of the ionization (let us say an increase to values at which the process begins to be rapidly accelerated owing to the formation of space charge) requires many cycles of interaction of the primary and secondary mechanisms. With secondary processes involving positive ions or excited atoms ( $\beta$ ,  $\gamma$ , and  $\epsilon$  processes) the duration of a cycle is equal in order of magnitude to the time of passage of an ion from the anode to the cathod  $(10^{-5})$ sec for d = 1 cm, p = 760 mm Hg). With photoelectric secondary processes ( $\delta$  and  $\eta$  processes) is approximately equal to the time of passage of electrons across the gap (~  $10^{-7}$  sec). Thus for confirmation of theoretical schemes with swaying of ionization experiments with small overvoltages must give times of formation at least of the order of microseconds. As is shown by an analogous examination of the streamer theories (cf. Sec. 2.2.7), even in this qualitative form this estimate can serve as a sixth criterion for the correctness of the theoretical scheme with swaying of ionization.

Besides this, the large amount of theoretical work done in this field<sup>184-186,170,6,45,147,83,34,5,3,128,126,87,88,25-27a</sup> enables us to improve this criterion by introducing also a basis for a quantitative comparison of theoretical and experimental dependences of the time of formation on the overvoltage.

The most complete calculations of the time of formation have been made by Davidson and by Legler. By rigorously solving the system of continuity equations for the motion of electrons and ions in the presence of  $\alpha$ ,  $\gamma$ , and  $\delta$  ionization and a photoelectric current  $j_0$ produced by external radiation, Davidson<sup>25-27a</sup> obtained the space-time pattern of the increase of the statistical averages of the ionization and current for prescribed values of  $j_0$  and of the ratio of  $\gamma$  to  $\delta$ .

Davidson's solution shows that the growth of the ionization is sharply accelerated even before the appearance of sizable distortions of the field by space charge. Thus in the calculation of the time of formation one can neglect the last phase of the growth of the ionization, for which the solution takes a very complicated form, and take as the time of formation the time during which, with the field remaining uniform, the current increases to a value  $I = 10^{-5} - 10^{-7}$  amp. which can be registered on an oscillograph (because of the great speed of the growth of the current the exact choice of the final I does not affect the value of t<sub>f</sub> very much). An example of calculated curves of the dependence<sup>34</sup> of  $t_f$  on  $\Delta U$  is shown in Fig. 3. Comparison of the corresponding experimental plots with such curves would be a more complete quantitative test of the theory, and also would give an estimate of the relative effectivenesses of ionic and photoelec-

FIG. 3. Dependence of the time of formation  $t_f$  on the overvoltage  $\Delta U/U_s$ . The points are experimental values,42 and the curves are from calculations.<sup>31</sup> Curves 1, 2, 3 are for the pure  $\delta$ process; curve 4 is for  $\delta/\alpha = 0.9\Gamma$ . In case (1) the final current I at the cathode that is taken as an indication of completion of breakdown formation is 10<sup>6</sup> amp. In cases (2) and (4)  $I = 10^{-1}$  amp; in case (3)  $I = 10^{-9}$  amp.



tric secondary processes, so that at the same time it would make the theory more precise.

Legler<sup>87,88</sup> has analyzed the process of swaying of ionization from the statistical point of view, calculating the probability distribution of delays of formation (of course for cases with sufficiently small statistical delay associated with the irregularity in the times of appearance of the initial electrons). Examples of the probability distributions of delays of formation of various lengths are shown by the dashed lines in Fig. 4 (the statistical delay in the appearance of the electrons<sup>85</sup> would give instead of a curve a straight line with negative slope).

FIG. 4. Probability distribution of delays of formation. Nitrogen, pd = 1000cm · mm Hg.  $\Delta U/U_s = 4.7$ percent. The dashed curves are calculated for three values of j differing by ±20 percent from the central value  $j_0 = 6.7 \times 10^7$  electrons/sec; the stepped line is the experimental plot.<sup>183</sup>



Comparison between such curves and experimental plots would be still another type of quantitative test of the theory.

2.1.8. Seventh Criterion — the Increase of the Current in the Early Stages of Breakdown. Closely associated with the time of formation is the increase of the current in the very first stages of breakdown. Theoretical schemes with swaying of ionization allow us to predict a quite definite picture of this increase, which depends on the type of secondary process considered and the conditions of the external ionization.  $^{5,145,146,174}$ 

Let us consider a circuit consisting of a spark gap of length dp and with capacity C between its electrodes, connected to a storage condenser  $C_0$  through a large resistance R. Let x be the distance from the cathode reached by a primary electron avalanche, produced by a single electron that arises at the cathode, in the time  $t = x/v_{-}(v_{-})$  is the speed of transport of electrons in the uniform field).

The electron current at the time t is given by

$$I_e = \frac{e_0 v_-}{d} \exp \alpha v_- t$$
 ( $e_0 - is$  the charge of the electron)

The voltage drop across R (if it is infinitely large) during the time  $\tau = d/v_{-}$  is given by

$$\Delta U_e = \frac{\int I_e dt}{C} = \frac{e_0}{\alpha \cdot d \cdot C} \left( e^{\alpha d} - 1 \right)$$

After the passage of the electron avalanche the number of positive ions that are in the layer dx at distance x from the cathode is equal to  $\alpha \cdot \exp(\alpha x) \cdot dx$ . At the time  $x/x_{+}$  ( $v_{+}$  is the speed of the ions) the number of ions remaining in the gap, in the absence of a secondary process, is given by

$$N_{+}(t) = \int_{t}^{d} \alpha e^{\alpha x} dx = e^{\alpha d} - e^{\alpha v_{+}t},$$

and the ion current is

$$I_{+} = \frac{e_{0}v_{+}}{d}N_{+} = \frac{e_{0}v_{+}}{d}(e^{ad} - e^{av_{+}t})$$

The drop of the voltage across  $R = \infty$  at the time t on account of the ion current is given by

$$\Delta U_{\star}(t) = \frac{1}{C} \int_{0}^{t} I_{\star} dt = \frac{e_{0}}{Cd} \left[ v_{\star} t e^{ad} - \frac{1}{a} \left( e^{av_{\star} t} - 1 \right) \right].$$
 (2.9)

Thus the behavior in time of the voltage across  $R = \infty$ , associated with the passage of a single avalanche, must correspond to that shown in curve 1 of Fig. 5.



FIG. 5. Theoretical curves of the variation with time of the voltage across a resistance R connected in series with a spark gap of length d and with capacity C between its electrodes. 1 - single avalanche with  $R = \infty$ ; 2 - single avalanche with  $R = (\alpha Cd)^{-1}$ ; 3 - chain of avalanches with ionic secondary mechanism; 4 - chain of avalanches with photoelectric secondary mechanism.

If  $R \neq \infty$  (RC  $\gg d/v_{-}$ , but RC is comparable with  $d/v_{+}$ ), the electric processes in the circuit must obey the equation

$$R\frac{dI}{dt} + \frac{I}{C} = \frac{d}{dt} [\Delta U_{\star}(t)]. \qquad (2.10)$$

Substituting the value of  $\Delta U_{+}(t)$  from Eq. (2.9) and using the value  $I|_{t=0} = \Delta U_{e}/R = e_{0}(e^{\alpha d}-1)/(\alpha dRC)$ as initial condition, we find the relation between R and C for which I (and thus also the voltage drop across R) will remain constant over a considerable part of the time  $d/v_+$ :  $\alpha \cdot d \cdot R \cdot C = 1$ . The solution then takes the form:

$$I = \frac{e_{0}v_{+}}{d} \left( e^{ad} - \frac{e^{av_{+}t} + e^{-av_{+}t}}{2} \right)$$

This behavior in time of the voltage across R is shown by curve 2 in Fig. 5 (after the time  $t = d/v_+$ (2.10) ceases to hold and the further decrease of I is given by the solution of the corresponding homogeneous equation,  $I = (ev_+e^{\alpha d}/2d) \cdot e^{-t/RC}$ ).

When there is secondary ionization we can expect that for  $\mu \cong 1$ , at the end of the period  $d/v_+$ , when the majority of the ions reach the cathode, the ionic mechanism will give a new flash of voltage across R, corresponding to a secondary avalanche. The photoelectric secondary mechanism must give new flashes after time intervals of the order of  $d/v_-$ . In the case of  $\mu \cong 1$  and individual initial electrons, the chain of avalanches following each other because of the swaying of ionization must cause across the resistance R a voltage similar to curve 3 of Fig. 5 with the ionic mechanism, and one similar to curve 4 with the photoelectric mechanism.

If the number of electrons leaving the cathode simultaneously is large and  $\mu > 1$ , the pattern of the variation of the current in the very first stages of breakdown must be more complicated, but it too can be predicted.<sup>5</sup> It must have the same qualitative features as the pattern for individual avalanches. Owing to the fact that it is determined by the electronic component, the initial rise of the current must be very sharp. Later, just as in the case of individual avalanches, the ionic current begins to play a larger part (in the stationary case the ratio of its magnitude to that of the electron current would be equal to the ratio of the mean paths of ions and electrons in the gap, i.e., to  $(\alpha d - 1)$ : 1; for relatively slow increase of the current after the passage of the first electron avalanches there should also be a large excess of the ion current over the electron current). The increase of the current should then go approximately exponentially, corresponding to the process of swaying for  $\mu > 1$  (in the absence of a secondary mechanism the increase of the current would stop after a certain time). After the increase of the current to a point at which space charge begins to play a part and affect the increase of  $\mu$ , the further increase of the current will be violently accelerated, and end in the collapse of the voltage at breakdown. With larger voltages the curves should be steeper, corresponding to larger values of  $\mu$ .

The picture that has been given of the changes of the current in the very first stages of breakdown corresponds most closely to the process of swaying of ionization. Agreement of the experimental data with this picture could thus serve as a <u>seventh</u> criterion for the correctness of theoretical schemes based on swaying of ionization.

2.1.9. Eighth, Ninth, and Tenth Criteria – the Character of the Luminescence in the Early Stages of Breakdown, the Existence of Phases of Glow Discharge, and the Spatial Structure of the Discharge. Up to now we have been considering criteria based on quantitative comparisons between experimental and theoretical values of various parameters of the discharge. In conclusion let us examine some phenomenonological criteria.

With any mechanism of swaying of ionization, secondary electrons can arise in the region near the cathode at large distances from the axis of the primary avalanche. Owing to this fact the avalanches produced in the course of the swaying of the ionization should fill a wide volume in the space between the electrodes, and, at least in the early stages of breakdown, the discharge should be of a diffuse character. The presence or absence of a diffuse discharge in these stages could serve as an <u>eighth</u> criterion of the correctness of the theoretical scheme, and the experimental observation of diffuse luminescence would be a verification of this criterion.

Furthermore, at small pressures, at which the breakdown mechanism is undoubtedly due to swaying of the ionization, the diffuse discharge during breakdown also leads to the appearance of a diffuse glow discharge, of a nature that differs little in principle from that of the Townsend discharge (in both discharges there is interaction between  $\alpha$  and  $\gamma$  ionizations). In the glow discharge a large part is played by the space charge near the cathode, and it is just in the formation of this that the breakdown process produced by the swaying of ionization consists. The mechanism of swaying and the glow discharge are thus in a definite causal relation to each other.

Besides this, at high pressures breakdown results in the production of a discharge closely resembling an arc discharge. On the assumption that at high pressures the initial stages of breakdown are brought about by the mechanism of swaying of ionization, it is natural to expect that in this case also there will be signs of at least a brief transient phase reminiscent of a glow discharge.

The duration of such a phase will naturally be rapidly curtailed as the overvoltage leading to breakdown is increased. Therefore one could also expect that at a certain overvoltage the signs of a glow-discharge phase would begin to disappear, while at the same time the swaying mechanism would cease to play the decisive role.

Thus the glow-discharge phase is associated with still another, the <u>ninth</u>, criterion for the correctness of the theory.

In itself the diffuse character of the mechanism of swaying of ionization is not in agreement with the formation of a narrow, contracted breakdown channel at high pressures. In order for theoretical schemes based on this mechanism to agree with the spatial structure of the late stages of the discharge (we shall call this the <u>tenth</u> criterion), experimental confirmations must be found for processes of transition from the diffuse to the channel structure.

#### 2.2. "Streamer" Theoretical Schemes

2.2.1. The Essence of the Mechanism and the Fundamental Equation. In Sec. 2.1, examining the physical essence of theoretical schemes with swaying of ionization and the consequences of the fundamental equation for these schemes, we indicated ten criteria for the experimental testing of these schemes. At the end of the 1930's and during the 1940's many authors<sup>97-103</sup>, <sup>122,142,143</sup> reached the conclusion (though even at that time it was not adequately founded<sup>110</sup>) that the corresponding experimental information (primarily relating to criteria 1, 2, 4, 6, 7, 8, 9, and 10) indicated the untenability of the Townsend theory. Accordingly a new foundation was sought for the theory of the electric breakdown of gases at high pressures.

Theoretical schemes of the mechanism of breakdown based on the idea that breakdown occurs during the development of an individual (isolated from the other processes in the gas-filled gap) electron avalanche, which, after a critical space charge has been formed in its head, is transformed into a highly ionized channel (a so-called "streamer"), which grows with increasing speed toward the anode, were proposed in various versions by Slepian,<sup>181</sup> Raether,<sup>140</sup> Meek and Loeb,<sup>120,99,101,102</sup> and other authors.<sup>65,71,119</sup>

Phenomenological streamer theories were mainly based on observations of pulsed breakdowns at large overvoltages, which were made by means of cloud chambers<sup>50,137-139,149</sup> (uncompleted breakdowns) and Kerr shutters, <sup>32,208,209</sup> and also on observations of corona discharges.<sup>95,96,76,77,198,199</sup>

In these researches it was established that at large values of  $\alpha L$  (L is the path length of the avalanche) ordinary electron avalanches developing according to the exponential law and moving toward the anode at speeds ~  $10^7$  cm/sec (corresponding to the mobility of electrons) are transformed into anomalous avalanche-streamers, which are propagated almost simultaneously toward the anode and the cathode at a speed an order of magnitude higher than the usual one. Furthermore in such cases no secondary processes near the cathode were observed.

The authors in question assumed that this transformation of normal avalanches into anomalous ones is the most characteristic process in breakdowns at high pressures, and that the condition for the onset of breakdown must be associated with the possibility of such a transformation, and not with the swaying buildup of ionization owing to the interaction of  $\alpha$  and  $\Gamma$ processes.

Various authors suggested various more or less ingenious hypotheses about the concrete mechanism of the transformation of the avalanche. Raether, Meek, and Loeb connected it with a phase transport of the ionization because of the accelerated growth in the field at the head of daughter avalanches engendered by the photoelectric effect in the gas, 18, 20, 37, 63, 136, 138, 176, 179 Slepian and Gänger regarded the process as phase transport in the field at the head of a zone of high thermal ionization of the gas. Both kinds of explanation were of the nature of qualitative working hypotheses, not based on exact calculations or experimental evidence.\* While differing in their views on the physical nature of the process of transformation, the various authors are essentially unanimous in formulating the fundamental condition of the theory - the possibility of transformation of an avalanche into a streamer. The condition for this is that the space charge in the head of a normal electron avalanche that is capable of changing into a streamer must produce a local electric field comparable in order of magnitude with the external field from the electrodes.

The calculation of the local field of the head of the avalanche takes account of the  $\alpha$  ionization and the diffusion of the electrons during the development of the avalanche, owing to which the space charge is concentrated in a head of finite volume. Starting from somewhat different electrostatic analogies (sphere, cylinder) and equating the calculated maximum field strength to the field from the electrodes times an undetermined coefficient, these authors get as the condition for the transformation of the avalanche, and thus for the onset of breakdown, an equation of the type of ( $x_{cr}$  is the path of the avalanche before its transformation):

$$\frac{\exp ax_{cr}}{x_{cr}} = \text{const or } \frac{a \exp ax_{cr}}{V x_{cr}} = \text{const}, \qquad (2.11)$$

If we substitute here for  $x_{cr}$  the length dg of the spark gap, these equations give in implicit form the dependence of the breakdown voltage Ug (taken as the minimum voltage applied to the gap for which a streamer can occur in it) on dg. The exponential character of the dependence makes it insensitive to a change of the poorly defined constant in the right member and the power of x in the denominator of the left member. Formally these equations differ very little from Eq. (2.3), though they have an altogether different physical meaning.

If one completely neglects the variation of the de-

nominator in Eq. (2.11) in comparison with the rapid variation of the numerator, one can get a still simpler equation, which is used in the papers of Raether and others:\*

$$ad = 20. \tag{2.12}$$

The number 20 in this equation was chosen by Raether on the basis of his observations of the development of avalanches in a cloud chamber.

Let us now compare the "streamer" theoretical schemes with the criteria for testing the "swaying" theoretical schemes that were set up in Sec. 2.1.

2.2.2. First Criterion – Curvature of the Plots of  $\ln j$ . From the point of view of the streamer schemes curvature of the graphs of  $\ln j$  for stationary non-self-maintaining discharges could be due to:

a) Distortion of the field by space charge at larger currents.  $^{135,\,203}$ 

b) Influence of the space charge of the head on the  $\alpha$  ionization when it is insufficient for the formation of a streamer.

c) The presence of secondary processes that do not lead to breakdown but have some effect on the current in the non-self-maintaining discharge.

It is easy to exclude the first cause by comparing the shapes of plots of ln j for different values of  $j_0$ . The second and third causes seem improbable, since the transition phase must be narrow, and because of the large dispersion of  $U_S$  in the streamer schemes the approach of a small fraction of the avalanches to the threshold must sharply increase the probability of breakdown. In any case there is no basis for supposing that the curvature of the plot from the second cause would give a coefficient  $\Gamma$  constant along the graph. In fact, one can easily derive from Eq. (2.2) the following formula,<sup>110</sup> which determines the width of the range of variation of d in which curvature of the plot of ln j can be detected:

$$\varkappa = \frac{\Delta (1-\varkappa)}{ad_{\rm S}} - \frac{1}{ad_{\rm S}} \ln \left[ e^{\Delta (1-\varkappa)} - 1 \right]$$
 (2.13)

 $[\kappa = (d_{\rm S} - d)/d_{\rm S}$  is the width of the region in question, and  $\Delta$  is the minimum deviation of the graph from a straight line that can be detected in the presence of the experimental spread of the points]. Substituting known values of  $\alpha$  in Eq. (2.13) and taking the reasonable value  $\Delta = 0.1$ , we find that  $\kappa \sim 2$  percent.

Because of the large spread of breakdown voltages that follows from streamer schemes, the use of values of d so close to dg does not seem experimentally feasible. With the streamer mechanism the existence of  $\Gamma$  mechanisms in the immediate neighborhood of the breakdown threshold would mean that the two thresholds lie extremely close together, which also seems implausible.

Thus the streamer schemes seem scarcely compatible with the detection of curvature at values of d

<sup>\*</sup>In recent papers devoted to the problem of the development of the streamer, Francis and Engel<sup>53,54</sup> have pointed out that it may be . due to an intensification of  $\alpha$  ionization in front of the head because of the presence there of a large number of already excited atoms, which have as it were a lowered ionization potential, and also to an increased speed of transfer along the electric field of electrons that have just been removed from neutral atoms and do not yet have the energy that would correspond to the calculated electron temperature.

<sup>\*</sup>This equation has also been obtained as an empirical formula by Schumann.<sup>178</sup>

close to  $d_S$  on experimental plots of ln j for stationary non-self-sustaining discharges.

2.2.3. Second Criterion - Agreement between Calculated and Experimental Values of the Breakdown Voltage. Unlike the schemes with swaying of ionization, which would allow us to make calculations of the breakdown voltage from Eq. (2.3) guite accurately (provided measurements of the current in non-selfmaintaining discharges could give exact values of  $\alpha$ and  $\Gamma$ ), the streamer schemes do not allow exact calculations of  $U_S$ , because of the uncertainty in the choice of the constants in Eqs. (2.11) and (2.12). Thus the uncertainty in the choice of the constants does not contradict the conclusion that there is approximate agreement of the experimental and theoretical values of US, but does not allow us to use this agreement as a convincing criterion in support of the theoretical schemes. This conclusion is given double force by the uncertainty in the value of the breakdown voltage itself, which arises from the statistical spread in the development of the avalanches (cf. Sec. 2.2.4).\*

2.2.4. Third Criterion – Dispersion of the Measured Values of the Breakdown Voltage. Theoretical calculations made by Wijsman<sup>210</sup> and Legler<sup>86</sup> show that the distribution function for the number n of electrons in an electron avalanche that has traveled the distance x is  $\exp(-\alpha x - ne^{-\alpha x})$ . † In accordance with Eq. (2.12) we assume that an avalanche may be converted into a streamer under the condition  $n = \exp(\alpha_{Cr}d)$  (d is the length of the gap). For an avalanche originating in a layer dx at distance x from the anode (we assume that the breakdown occurs under the action of cosmic radiation, which produces electrons uniformly through the whole volume) the probability of reaching this value of n is given by

$$\frac{1}{\exp ax}\int_{\exp a_{\mathbf{cr}}\cdot d}^{\infty}e^{-\frac{n}{\exp ax}}\,dn=e^{-\exp(a_{\mathbf{cr}}d-ax)}.$$

For all layers dx from x = 0 to x = d the corresponding probability is  $P = \int_{0}^{d} \exp\left[-\exp\left(\alpha_{cr}d - \alpha x\right)\right] dx$ . This expression for P is a function of the parameter

 $\alpha$ , which in turn is a function of the electric field strength, i.e., of the voltage across the gap. A graph

<sup>†</sup>Legler's calculation is restricted by the conditions that  $\alpha_x$ does not exceed 10 and that the ratio of the field strength to  $\alpha$  must be larger than the ionization potential of the gas. The second condition is satisfied for breakdowns at high pressures, in any case. It is possible that the first condition is not always satisfied for large values of d, but this inaccuracy can scarcely be of any great importance for our estimate of the spread in the breakdown voltage. of P as a function of U for air and d = 1 cm is shown as curve 2 in Fig. 1.

We see that this curve is much flatter than the curve of the quantity  $1-1/\mu$ , which corresponds to the probability distribution for breakdown on the basis of the theoretical schemes with swaying of the ionization. An estimate of the probable spread of breakdown voltages predicted by streamer schemes is given by the derivative of the quantity P (curve 3 in Fig. 1). Figure 1 shows that in the case of occurrence of breakdown under the action of electrons produced by cosmic radiation the spread of the breakdown voltages must be larger than 3 ky.

If all of the electrons are ejected from the cathode by the action of ultraviolet light, the probability distribution<sup>101</sup> is given by  $P = \exp[-\exp(\alpha_{CT}d - \alpha d)]$ . The corresponding graphs of P and dP/dU are shown by curves 4 and 5 in Fig. 1. We see that although in this case the distribution is much narrower than when the primary electrons are formed throughout the volume, it still corresponds to a spread of the breakdown voltages around the mean value of the order of  $600v (\sim 2$ percent). This is also much larger than the spread expected on the basis of schemes with swaying of the ionization.

2.2.5. Fourth Criterion – Dependence of US on the Cathode Material. From the point of view of streamer schemes, which do not take into account any secondary processes at the cathode, a dependence of US on the material of the cathode can be explained only by extremely artificial assumptions, such as:

a) the existence of different dark currents for different materials (for example, because of the Paetow effect<sup>131</sup>), which affect the probability of breakdown for short waits at low voltage levels;

b) the existence, on account of the microscopic structure of the cathode surface, of local electric fields that are so large that they can change the value

of the quantity  $\int_{0}^{d} \alpha \, dx$ .

There are no experimental confirmations for such assumptions. Effects of such causes could produce differences of US for different cathodes that are negligibly small in comparison with the spread of  $U_S$  caused by the statistical nature of the breakdown mechanism. Therefore experimental evidence that US depends on the cathode material would be a weighty argument against streamer theoretical schemes of the mechanism of inception of breakdown.

2.2.6. Fifth Criterion – Dependence of US on the Ionization by External Radiation. According to streamer schemes, a dependence of US on  $j_0$  can be explained by the effect of  $j_0$  on the probability of breakdown for short waiting times at low voltage levels, and also, for large values of  $j_0$ , by the effect of distortions of the field by space charge on the quan-

tity  $\int_{0}^{\infty} \alpha \, dx$  (because of the positive value of  $d^2 \alpha / d \ell^2$ 

<sup>\*</sup>One of the authors of the streamer schemes (reference 97, page 424) has taken this spread into account and pointed out that with a short wait for static breakdown at each stage in a procedure for measuring U<sub>S</sub> the breakdown occurs in practice at voltages almost 15 percent higher than the minimum voltage at which transformation of avalanches into streamers is possible, as determined from his suggested second form of Eq. (2.11).

the distortion of the field increases this integral $^{203}$ ).

An estimate of the first effect has been given in Sec. 2.2.5. The second effect has not been examined in detail in the literature.

2.2.7. Sixth Criterion – the Time of Formation of Breakdown. From the point of view of streamer schemes the maximum duration of the entire process of formation of breakdown cannot be much longer than the time for a single passage of an electron avalanche from the cathode to the anode (since the speed of advance of a streamer exceeds by an order of magnitude the speed of a normal avalanche, the time of development of the streamer can be neglected). For a gap 1 cm long in air at atmospheric pressure the time of passage of an avalanche is about 0.08  $\mu$ sec. At overvoltages the streamer must be formed after the avalanche has moved a distance shorter than the length of the spark gap, and the time of formation must be even shorter.<sup>31</sup>

Thus to confirm the streamer schemes experiments at any overvoltages (no matter how small) would have to give times of formation of breakdown (which must not be confused with the statistical delay)  $\leq 10^{-7}$  sec. The occurrence of experimental values of the formation time of the order of the values of  $10^{-6}$  sec and longer that are predicted by schemes with swaying of ionization would show beyond question the untenability of streamer schemes of the inception of breakdown.

2.2.8. Seventh Criterion – the Increase of the Current in the Early Stages of Breakdown. According to Sec. 2.2.7, in the streamer schemes of the development of breakdown the increase of the current should not last more than  $\sim 10^{-7}$  sec, after which there should be a violent rise of the current, corresponding to a collapse of the voltage across the gap in a time  $\sim 10^{-8}$  sec. The streamer schemes do not allow any cyclic development of the current over a longer time.

2.2.9. Eighth, Ninth, and Tenth Criteria – the Character of the Luminescence in the Early Stages of Breakdown, the Existence of Phases of Glow Discharge, and the Spatial Structure of the Discharge. From what has been said it is clear that according to the streamer schemes the luminescence of the discharge from its earliest stages and up to its completion must be concentrated in a narrow channel. There can be no diffuse luminescence. Glow-discharge phases, corresponding to a diffuse nature of the discharge, are not to be expected. The channel spatial structure of the discharge follows naturally from the theoretical scheme.

### III. SURVEY OF THE EXPERIMENTAL MATERIAL ON TYPE A BREAKDOWN

#### 3.1. First Criterion – Curvature of the Plots of ln j

Up to 1952 repeated experimental measurements of the current in non-self-maintaining stationary discharges  $^{10,67-69,118,135,164,165}$  failed to detect any curva-

ture of the plots of ln j at gas pressures close to atmospheric pressure.\* This fact was one of the main arguments used by the authors of streamer theoretical schemes in their criticism of the schemes with swaying of the ionization. As early as 1943, however, it had been shown<sup>110</sup> that the accuracy of the experiments then being done was insufficient to detect curvature for large pressures. For example, in measurements of the increase of ln j with increase of the distance d between the electrodes (with constant field strength  $\mathcal{E}$ ), Bowls<sup>10</sup> obtained the last point before breakdown at  $\kappa = (d_S - d)/d_S = 2.3$  percent, and a spread of points corresponding to  $\Delta \cong 0.1$ , whereas for his values of  $\alpha$  Eq. (2.13) would have required measurements of j at  $\mathcal{E}/p \cong 40$  volt cm<sup>-1</sup> (mm Hg)<sup>-1</sup> (which corresponds to  $d_{S} = 1$  cm at atmospheric pressure) for  $\kappa$  smaller than 2 percent. For Bowls' case the value  $\kappa = 2.3$  percent corresponded to  $\mathscr{E}/p = 65$  volt  $cm^{-1}$  (mm Hg)<sup>-1</sup> [pd<sub>S</sub>  $\simeq$  65 cm (mm Hg)], which he found to be in fact the lowest value of  $\mathcal{E}/p$  that gave a curvature of the plot of ln j.

Work of much higher accuracy done in the last few years by Llewellyn-Jones and his co-workers, <sup>21-23,33,90</sup> and also work by Wilkes, Hopwood, and Peacock<sup>211</sup> and by De Bitetto and Fisher, <sup>29,30,49</sup> has demonstrated the curvature of plots of ln j quite beyond question (see, for example, Fig. 6) for air, nitrogen, hydrogen, and oxygen, and has made it possible to determine values



FIG. 6. Plots of ln j against d, taken from the paper of Llewellyn-Jones and Parker.<sup>90</sup> Air, p = 200 mm Hg. Values of  $j_0$  in amperes  $\cdot 10^{-15}$  and of &/p in volt/cm (mm Hg) are shown next to the curves.

of the coefficient  $\Gamma$  that are constant over a considerable section of the graph. In these experiments variation of the quantity  $j_0$  made it possible to exclude completely the possibility of curvature of the graphs because of distortion of the field by space charges.<sup>24</sup> In reference 90, for example, the beginning of the curvature of the graphs corresponds exactly to Eq. (2.13), if we substitute the values of  $\alpha$ , d<sub>S</sub> and  $\Delta$ taken from that paper. In reference 90 the stationary current of the non-self-maintaining discharge was

<sup>\*</sup>Except in the measurements of Paavola,<sup>130</sup> mentioned in Sec. 2.2.2., which have been regarded as erroneous.<sup>203</sup>

measured at voltages only 0.5 percent lower than the breakdown voltage, and in the work of De Bitetto and Fisher, at voltages only 0.05 percent below the breakdown voltage. Therefore explanation of the curvatures that were obtained by the causes b) and c) mentioned in Sec. 2.2.2 is completely implausible.

Thus the situation is now different from that at the time the streamer theories appeared; at present the test by this criterion gives extremely convincing confirmation of the theoretical schemes with swaying of the ionization.

### 3.2. Second Criterion – Agreement between Calculated and Experimental Values of the Breakdown Voltage

Before experimental values of  $\Gamma$  were obtained from the plots of ln j, this criterion was not suited for an accurate test of the two groups of theoretical schemes [because of the uncertainties in the coefficients occurring in the respective equations (2.3) or (2.4) on the one hand and (2.11) or (2.12) on the other]. The streamer schemes nevertheless gave acceptable calculated values of  $U_{S}$ , if in deriving Eq. (2.11) one used the equality of the field in the head of the avalanche and the external field. The complete absence of objective data even about the order of magnitude of the coefficient  $\Gamma$  for large values of pd, and the fact that the values of  $\Gamma$  obtained from measurements of In j for small pd gave calculated values of Us that were too low, served as arguments for preferring this criterion over the theoretical schemes with swaying of the ionization.

The recent experimental work cited in Sec. 3.3 has made it possible to get objective values of  $\Gamma$  for large pd, measured with satisfactory accuracy and in good agreement with each other. For example, the values of  $\Gamma$  obtained<sup>90</sup> for air and well cleaned nickel electrodes with  $\mathcal{E}/p$  from 39 to 45 volt cm<sup>-1</sup> (mm Hg)<sup>-1</sup> lie in the range from  $8 \times 10^{-6}$  to  $1.5 \times 10^{-4}$ . For nitrogen with similar electrodes and values of  $\mathcal{E}/p$  some authors<sup>33</sup> obtained values of  $\Gamma$  in the range from 1.3 to  $3.7 \times 10^{-4}$ , and others,<sup>29</sup> from 9 to  $10.5 \times 10^{-4}$ . In the case of hydrogen with the same electrodes and  $\ell/p$  from 20.3 to 25.1 volt cm<sup>-1</sup> (mm Hg)<sup>-1</sup>, and with d = 2 cm the values of  $\Gamma$  found from the data of De Bitetto and Fisher lie in the range from 1 to 2.4  $\times 10^{-3}$  and also agree in order of magnitude with the data of other authors<sup>23,211</sup> (for d = 1 cm and these same values of pd,  $\Gamma$  is smaller by about a factor 2). For oxygen and a nickel cathode De Bitetto and Fisher<sup>30</sup> got for  $\Gamma$  at  $\mathscr{E}/p = 35.4$  volt cm<sup>-1</sup> (mm Hg)<sup>-1</sup> the value  $\Gamma = 0.045$ , an order of magnitude larger than the values that were obtained for the other gases.

In a paper by Schmidt-Tiedemann<sup>175</sup> values of  $\Gamma$  for nitrogen and copper electrodes were obtained by an entirely different method — oscillographic observations of chains of avalanches (cf. Sec. 2.1.8). For pd ~ 50 cm  $\cdot$  (mm Hg) the value  $\Gamma \sim 1.5 \cdot 10^{-6}$  was

obtained for oxidized electrodes; for electrodes cleaned by annealing in hydrogen,  $\Gamma > 10^{-3}$ .

All of the authors named above found that a comparison of the breakdown voltages calculated from Eq. (2.3) (with use of experimental values of  $\alpha$  and  $\Gamma$ ) with the directly measured values of U<sub>S</sub> (for the same states of the gas and the electrodes) gave satisfactory agreement within the limits of the rather small experimental error. Such a comparison is illustrated by the plot in Fig. 7.



nd

720 cm : mm Hg

The higher value of  $\Gamma$  for oxygen compensates the loss in the multiplication of the electrons by the  $\alpha$ -ionization process owing to the production of negative ions<sup>70\*</sup> and leads to comparable values of U<sub>S</sub> for oxygen and nitrogen at pressures close to atmospheric pressure.<sup>30,49</sup>

400

560

We thus see that the test by this criterion now convincingly confirms the correctness of the theoretical schemes with swaying of the ionization.

# 3.3. Third Criterion – Dispersion of the Measured Values of the Breakdown Voltage

This criterion has not received enough attention in the literature so far, although, as was shown in Secs. 2.1.4 and 2.2.4, the difference between the spreads of the values of  $U_S$  predicted in the two groups of theoretical schemes is so large that one could expect that experiment would quite clearly support one group or the other. By this time, however, enough factual data has accumulated in the literature for a definite conclusion about this criterion. Some of the data are collected in Table I.

Fisher<sup>43</sup> has suggested that the actual dispersion of US, which could be obtained with sufficiently accurate regulation and measurement of the voltage, does not exceed 0.1v (~ 0.0003 percent).

Comparing these data with the calculations that we have given in Secs. 2.1.4 and 2.2.4, we can draw the

<sup>\*</sup>Geballe and Reeves<sup>54</sup> have allowed for the lowering of the ionization in electronegative gases owing to the production of negative ions, observed by Harrison and Geballe,<sup>70</sup> in their derivation of an improved formula to replace Eq. (2.2), which includes in addition to the coefficients  $\alpha$  and  $\Gamma$  a coefficient  $\eta$  that characterizes the process of production of negative ions.

Author	Literature reference	Gas	Pressure, mm Hg	d, cm	Conditions of ionization of gap	Mean breakdown voltage, kv	Fractional error in measure- ment of U <sub>S</sub> , percent	Dispersion of measured values of U <sub>S</sub> , percent
Marshak	110	Air	760	1	Natural background	~30.5	0.08	0.08
Fisher and Bederson	42	Air	760	1	Photoelectric current 10 <sup>-12</sup> amp/cm <sup>2</sup>	~30.0	0.01	0.01
Fisher and Kachikas	45	Nitrogen	730	1	Natural background	~30.0	0.007	0.007
Köhrmann	80	Air	450	1.7	Natural background	29.6	0.1	0.1
De Bitetto and Fisher	29	Nitrogen	300	2	Natural background	24.99	0.01	0.016

**TABLE I.** Data on the dispersion of measured values of thebreakdown voltage for breakdowns of type A

quite definite conclusion that the test by this criterion convincingly confirms the correctness of the theoretical schemes with swaying of the ionization.\* It must be noted that only owing to the extremely small actual spreads of the breakdown voltage has it been possible to get definite conclusions also with respect to criteria 1, 2, 4, 5, 6, and 7 (as we shall see, tests by the last three of these criteria would be impossible without the knowledge in the course of the experiments of an extremely accurate value of  $\Delta U$ , for which it is necessary to know Ug even more precisely).

Besides the experimental data on ordinary gases, shown in Table I, there have been published very recently papers by Pfaue and Raether<sup>134</sup> and by Franke<sup>54</sup> on breakdowns in organic vapors (alcohol, methane, methylal, and others), for which the coefficient  $\Gamma$  has extremely small values (~  $10^{-9}$ ). In the first of these papers the indefiniteness of the value of the breakdown voltage under conditions of pure streamer static breakdown is pointed out, and it is proposed to take as the value of U<sub>S</sub> under such conditions the voltage across the gap at which not more than 1 percent of the observable avalanches would lead to breakdown.

The experimental confirmation of an especially large spread of the breakdown voltages in the special case of streamer static breakdown is an indirect proof of the conclusion drawn above respecting this criterion for ordinary gases.

# 3.4. Fourth Criterion – Dependence of $U_S$ on the Cathode Material

Up to 1943 the accepted opinion in the literature was that for large pd the breakdown voltage does not depend on the cathode material. This view was based on very old researches (references 17 and 191, page 476), and was also used as one of the main arguments in the criticism of the theoretical schemes with swaying of the ionization.

In 1943 it was shown experimentally<sup>110</sup> that there is a difference, amounting to 0.7 percent, between the breakdown voltages of air gaps 1 cm long at p = 760mm Hg with cathodes of nickel and graphite. This difference agrees in order of magnitude with the theoretically expected value. Similar results were obtained in 1950 by Trump, Cloud, Mann, and Hanson<sup>200</sup> for gaps of lengths 0.6 - 1.8 cm in air at pressures of several atmospheres (electrodes of stainless steel and of . aluminum). Work by Dutton, Haydon, and Llewellyn-Jones<sup>33</sup> and by De Bitetto and Fisher<sup>29</sup> revealed differences of US depending on the state of the cathode surface, which correspond exactly to the differences of the coefficients  $\Gamma$  determined from the curvature of the plots of ln j. Recent work by Mielke has also detected<sup>132</sup> differences in  $U_S$  for copper and steel cathodes [hydrogen, pd ~ 500 cm  $\cdot$  (mm Hg)].

At present we can regard it as proved that there is a dependence of the breakdown voltage on the cathode material, which confirms the existence of secondary mechanisms at the cathode. Thus the test by the fourth criterion also supports the theoretical schemes with swaying of the ionization.

### 3.5. Fifth Criterion – Dependence of Ug on the Ionization by External Radiation

Experimental studies of the dependence of the breakdown voltage  $U_S$  on the photocurrent  $j_0$  from the cathode produced by irradiating it with a quartz lamp have been made on a large scale in work by Fuchs and other authors.<sup>16,57-62,171</sup> For various gases and over a very wide range of values of pd there has been confirmation of the dependence of  $U_S$  on small values of  $j_0$  (up to ~  $10^{-9}$  amp/cm<sup>2</sup>) as expressed by Eq. (2.8). Examples of the experimental curves for air are shown in Fig. 8.

<sup>\*</sup>The experimental information on the statistical nature of the value of the breakdown voltage includes work done in recent years by Kojima and Kato<sup>84</sup> and by Frommhold and Schlumbohm<sup>\$5,56,173</sup> to test calculations on the statistics of the growth of electron avalanches made by Wijsman and Legler<sup>96,210</sup> (which are the basis of the theoretical estimates of the dispersion of  $U_S$  for both schemes of the development of breakdown). This work has shown that up to  $\alpha d \sim 15.5$  the experimental distribution of the number n of electrons per avalanche corresponds strictly to the theoretical formula  $\exp(-\alpha d - ne^{-\alpha d})$ . Kojima and Kato found deviations from this formula for large values of pd. Schlumbohm and Frommhold established, however, that for  $\alpha d \lesssim 15.5$  the formula holds for pd up to at least about 1000 cm (mm Hg), provided there is no appearance of secondary avalanches (because of  $\Gamma$  processes), which must have had considerable effect in the work of Kojima and Kato. For  $\alpha d > 15.5$  (up to ~18.5), above which breakdowns often began) there is a relative decrease of the number of avalanches with large numbers of electrons, owing to effects of the space charge of the avalanche itself.



FIG. 8. Experimental curves of the dependence of  $\Delta U/U_s$  on  $j_0^{\frac{1}{2}}$  for air at p = 760 mm Hg, d = 0.6-2.6 cm, as obtained in work by Fuchs and Bongartz:<sup>5e</sup>  $1-U_s = 70.5 \text{ kv}$ ;  $2-U_s = 52.0 \text{ kv}$ ;  $3-U_s = 19.8 \text{ kv}$ .

Concrete ideas about particular  $\Gamma$  mechanisms allow one to calculate<sup>156-162</sup> the coefficient K that appears in Eq. (2.8). No single  $\Gamma$  mechanism can be chosen to give the right value of this coefficient. It comes out to be of the right order of magnitude, however.

Since a similarity transformation (increase of d and decrease of p by a factor a) corresponds to a decrease of  $j_0$  by a factor  $a^2$ , it follows that  $\Delta U/U_S$ should remain constant when K/d is a function of pd. Work by Fuchs, Schumacher, and Kettel<sup>57,59</sup> has shown that this law does not hold for nitrogen and hydrogen. This fact gave these authors reason to believe that in the gases in question an important part must be played by a  $\Gamma$  mechanism based on stepwise ionization.

Independently of agreement between calculated and experimental values of the coefficient K, we may grant that the results of the experimental test of the theory by the fifth criterion support the theoretical schemes with swaying of the ionization

# 3.6. Sixth Criterion — the Time of Formation of Breakdown

As has already been said, from 1923, when the extremely short formation time  $t_f$  (down to  $10^{-7}$  sec) was first observed, until 1943, the available experimental material on the value of tf for large pd provided one of the main arguments for rejecting the theoretical schemes with swaying of the ionization. In fact, during this period a rather large amount of data was accumulated on extremely small values of tf (down to a few nanoseconds) at large overvoltages<sup>52</sup>, 125,129,187,188 or for very large instantaneous photocurrents from the cathode, produced by irradiating it with an auxiliary spark.<sup>182,209</sup> These data agree with the later researches of Fletcher, Rose, and others, 51,89a who found  $t_f \sim 10^{-9}$  sec at large overvoltages. At the same time there was almost no information on tf under conditions close to static breakdown.\* The experimental data on extremely short values of  $t_f$  were in agreement with direct observations of the development of electron avalanches and streamers, which were also made at large overvoltages by means of Kerr shutters and cloud chambers.

Along with this we have the fact that the development of breakdown at large overvoltages, corresponding to  $\mu \gg 1$ , can be very different from its development for extremely slow increase of the voltage (static breakdown,  $\mu \cong 1$ ). In view of this, and because actual data on t<sub>f</sub> under conditions approximating those of static breakdown can be of decisive importance for the choice of the correct theoretical scheme, Marshak<sup>110</sup> in 1943 made measurements of the time of formation of breakdown in a plane air gap of length ~ 1 cm (p = 760 mm Hg) at small overvoltages (up to 1 percent). Examples of the curves obtained in this work are shown in Fig. 9.



FIG. 9. Dependence of the breakdown formation time  $t_f$  on the overvoltage  $\Delta U/U_S$  for small values of  $\Delta U/U_S$ , as obtained by Marshak.<sup>110</sup> Air, p = 760 mm Hg.

These curves clearly showed the existence of values of  $t_f$  at least an order of magnitude larger than the longest breakdown time consistent with the streamer theoretical scheme of the inception of breakdown (for d = 1 cm,  $t_f = 0.08 \ \mu \text{sec}$ ).

On the basis of these data on the dependence of  $t_f$  on  $\Delta U/U_S$  it was suggested<sup>110</sup> that there exists an upper limit on overvoltages, above which the time of formation becomes shorter than the time for a single

<sup>\*</sup>During this period there was published a paper by Bellaschi and Teague<sup>9</sup> on the pulse electric strength of air gaps of length 1 to 64 cm between spherical electrodes of diameter 6.25 to 20 cm

<sup>(</sup>p = 760 mm Hg), for pulse lengths  $t = 0.2 - 2 \mu sec$ . This work showed that already at  $t = 1 \mu sec$  the breakdown voltage is several percent larger than that measured at  $t = 2 \mu sec$ . Thus it indirectly indicated the existence of large values of tf at small overvoltages. During this period, however, this work was not taken into account by the majority of the authors who discussed the theory of breakdown. Attention must also be called to the work of Snoddy,182 who determined the time of formation of breakdown at a voltage practically equal to the static value, for a 4 mm gap between two spheres 6.5 mm in diameter (the field was not quite uniform). Measurements were made on the delay of appearance of luminescence in the gap relative to the start of the auxiliary spark, whose image was focused on the cathode by a quartz optical system. In spite of the extremely large values of  $j_0$ , which could lead to a rapid rise of  $\mu$ , the time of formation under these conditions was 0.2 µsec, i.e., approximately 5 times the maximum time for one-avalanche breakdown under the same conditions.

passage of an electron avalanche across the gap, and a pulsed breakdown begins to develop without swaying of ionization, through a pure streamer mechanism. From the data on  $t_f$  for gaps of various lengths obtained in this work and in work by Strigel,<sup>187</sup> Messner,<sup>125</sup> and Newman,<sup>129</sup> a curve has been constructed showing the dependence of the position of this upper limit on the gap length; this curve is shown in Fig. 10.



FIG. 10. Limit of the overvoltage  $\Delta U/U_S$  for spark gaps of various lengths d; above this limit breakdown develops in accordance with the streamer mechanism, and below it, with the mechanism, of swaying of ionization (curve for air). The curve is from the paper by Marshak;<sup>110</sup> I shows the region occupied by the experimental points taken from the paper by Köhrmann.<sup>81</sup> A schematic representation of the boundaries between breakdowns of types A, B, and C is shown in the upper right-hand corner.

This curve agrees with the data of White<sup>209</sup> on the place of inception of the streamer in a spark gap, which were obtained by observing breakdowns with a Kerr shutter (cf. Sec. 3.10). White shows that with increase of the gap length the overvoltage required for the initiation of pure streamer breakdown becomes smaller. In connection with this the suggestion was made that at very large gap lengths (for air at atmospheric pressure lengths  $\geq 10$  cm), for which the breakdown field strength ceases to depend on d (becoming equal to 26 kv/cm for air) and the breakdown voltage is hundreds of kilovolts or more, the limits for pure streamer breakdown and for static breakdown merge and the streamer theoretical schemes of the inception of breakdown begin to be justified.\*

At the end of the 1940's Fisher<sup>43</sup> also came to the conclusion that the most convincing choice of the correct theoretical scheme can be made on the basis of information about the time of formation of breakdown at small overvoltages.

In accordance with this, Fisher and his collaborators,  $^{38-42,44-48}$  and also, working independently, Köhrmann, Raether, and other authors<sup>2,36,66,78-83,127,145,146,183</sup> have made many measurements during the last ten years of the time of formation of breakdown at extremely small overvoltages (down to hundredths of a percent), and have studied the statistical nature of  $t_f$ and its dependence on the type of gas and the pressure, the length of the spark gap, and the ionization by external radiation.

The results of these investigations mainly confirm the data that were obtained in reference 110, and are in good agreement with each other. Figure 11 shows an example of the dependence of the time of formation on the overvoltage, as obtained in work by Köhrmann.<sup>82</sup> For dry air and with the same values of d and  $\Delta U/U_S$ , Köhrmann got values of t<sub>f</sub> about three times those found by Marshak. This difference can be explained by the difference in the humidity of the air, which, as Köhrmann has shown,<sup>82,83</sup> has a very large influence.

FIG. 11. Dependence of time of formation of breakdown on the overvoltage. The solid line is the experimental curve obtained by Köhrmann;<sup>82</sup> the dashed line is the curve calculated from formulas of Raether.<sup>144</sup> Dry air, d = 2 cm, p = 500 mm Hg.



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Köhrmann has found that there is a break in the curve of t<sub>f</sub> plotted against  $\Delta U/U_S$  with logarithmic scales; he ascribes this break to a change of the mechanism of development of breakdown (a transition from a mechanism with initial swaying of the ionization to a pure streamer mechanism). This result agrees qualitatively with the calculated curve shown in Fig. 11, which is obtained on the basis of approximate calculations made by Raether<sup>144</sup> for the two mechanisms (the position of the break corresponds to the time of passage of an electron avalanche across the gap). Köhrmann<sup>83</sup> explains the quantitative discrepancy between the calculated and experimental values of  $t_f$  as due to a delay of the  $\alpha$  ionization, which was detected recently in researches of Vogel<sup>206</sup> and Frommhold.<sup>56</sup> On the basis of the data of these authors Köhrmann<sup>83</sup> has obtained much better agreement between the calculated and experimental curves for moist air.

On the basis of measured values of  $t_f$  for gaps with various values of pd Köhrmann<sup>81,83</sup> and Raether,<sup>145,147</sup> in agreement with Marshak, have found a limit on the overvoltage, above which pulsed breakdown begins to develop without swaying of the ionization, by a pure streamer mechanism. Köhrmann's experimental points, which fix this limit, are shown in Fig. 10. As can be seen from this diagram, they agree satisfactorily with the corresponding curve found by Marshak.

<sup>\*</sup>Owing to the fact that static breakdown in a strongly nonuniform electric field also undoubtedly occurs in accordance with the streamer schemes, this gives a rational basis for defining type B breakdowns as static breakdowns whose inception is caused by purely streamer mechanisms (cf. the arrangement of boundaries between the three types of breakdown in Fig. 10).

For higher humidity of the air Köhrmann found a lowering of the overvoltage boundary between breakdowns of types A and C (the voltage for static breakdown, fixed by the swaying mechanism, increases with the humidity faster than the voltage for pulsed breakdown, at which the break in the logarithmic plot of t<sub>f</sub> against U/US occurs. Köhrmann associates this effect with a decrease of  $\Gamma$  with increasing humidity.\*) He relates the slow increase of the minimum voltage for pure streamer breakdown to a decrease of the coefficient  $\alpha$ , as well as  $\Gamma$ , with increasing humidity. The lowering of the boundary between breakdowns of types A and C means that with increasing humidity there is also a decrease of the gap length at which the transition from static breakdowns of type A to static breakdowns of type B sets in.

Examples of the experimental results of Fisher and Bederson<sup>42</sup> and of Fisher and Kachikas<sup>45</sup> are shown in Figs. 12 and 13, and are also represented by points in Fig. 3. In these author's work the absolute values of  $t_f$  are midway between Köhrmann's values for dry air and Marshak's for room air.



Figure 3 shows that experiment is in satisfactory agreement with a calculation based on the swaying mechanism, if one takes into account the lack of accuracy of our knowledge of the coefficients  $\alpha$  and  $\Gamma$  and

of the mobilities of the electrons and ions.

Figure 12, unlike the first data obtained in reference 110, shows that a decrease of the gap length at constant  $\Delta U/U_S$  leads to a shortening of  $t_f$ . According to the data of Fisher, Bederson, Kachikas, and Lessin,<sup>47</sup> at very small values of  $\Delta U/U_S$  this effect of the length is extremely large ( $t_f$  is proportional to d), which agrees with the simplified calculations of the authors. In these papers, however, the regions in which the effect of d on  $t_f$  is studied are different. An analysis of Köhrmann's data does not reveal any appreciable dependence of  $t_f$  on d as the latter varies from 1 to 2 cm.

It can be seen from Fig. 13 that in nitrogen  $t_f$  is about half the value in air, and is almost independent of the pressure (a weak dependence on p is observed only for very small  $\Delta U/U_S$  and large  $t_f$ ). Fisher and Bederson got this same result for air, and Fisher and Lessin<sup>47</sup> found it also for hydrogen, which gave values of  $t_f$  of the same order as those for air and nitrogen. Gänger<sup>66</sup> found no dependence of  $t_f$  on p in the pressure range from 25 to 760 cm Hg.

The work of Fisher and Kachikas<sup>45</sup> and of Sohst<sup>183</sup> also included measurements of the time of formation of breakdown when there was a sizable photoelectric current  $j_0$ . With corresponding decreases of the breakdown voltage the values of  $t_f$  found in reference 45 went as high as 100  $\mu$ sec (larger values of  $t_f$  could not be measured). In reference 183 it was found that increase of  $j_0$  at constant voltage led to shortening of  $t_f$ , in agreement with Legler's calculation.<sup>87,88,147</sup>

The papers of Fisher, Bederson, and Kachikas give a simplified treatment of the problem of the statistical spread of formation times, and reach the conclusion that the observed lack of constancy of  $t_f$  is entirely due to a small instability of the voltage in the measuring circuit.

As has been shown by Legler,  $^{87,88}$  this conclusion is erroneous. According to the scheme with swaying of ionization the process of formation of breakdown is a statistical one. The stepped curve shown in Fig. 4 gives an example of the statistical spread of the formation times obtained in work of Raether, Feldt, and Sohst,  $^{36,183}$  and agrees with the calculation of Legler.

Measurements of the time of formation, together with a correct theoretical calculation of the process of swaying of ionization, make it possible in principle to estimate the importance of the various secondary mechanisms in particular concrete cases. Attempts in this direction made in papers by Fisher, Bederson, Kachikas, and Lessin have been based on extremely simplified calculations. Nevertheless these authors make the assertion that in breakdowns in air, nitrogen, and hydrogen the main role is played by the  $\delta$  mechanism (photoelectric effect at the cathode). A more careful treatment of this problem by means of the approximate solution of Davidson<sup>25</sup> is given in a paper by Morgan,<sup>127</sup> who found that in breakdown in hydrogen

<sup>\*</sup>An analogous effect has been observed by Franke<sup>34a</sup> when vapor was added to methane. Data on the effect of dampness in air on U<sub>S</sub>, which agree with Köhrmann's work, have been obtained in references 38a and 52a.

at low values of  $\mathscr{E}/p$  [~ 40 v cm<sup>-1</sup> (mm Hg)<sup>-1</sup>; pd ~ 30 cm · mm Hg] 75 percent of the secondary electrons are formed through the  $\delta$  mechanism and 25 percent through the  $\gamma$  mechanism (ejection of electrons from the cathode by bombardment by positive ions). At high  $\mathscr{E}/p$  [~ 300 v cm<sup>-1</sup> (mm Hg)<sup>-1</sup>] the main part is played by the  $\gamma$  mechanism. Sohst<sup>183</sup> concludes that in breakdowns in hydrogen and nitrogen at pd ~ 1000 cm · mm Hg the main part is played by the  $\delta$  mechanism.

Thus all of the extensive material from measurements of formation times that has been accumulated since 1943 shows that:

a) At small overvoltages there are values of  $t_f$  that are clearly incompatible with the streamer theoretical schemes for the inception of static breakdown of type A.

b) Deductions from the theoretical schemes with swaying of the ionization agree (within the limits of accuracy of the calculations achieved so far) with the data on the dependence of the formation time on the overvoltage and with the statistical pattern of distribution of delays of formation.

c) There is a limiting value of the overvoltage, below which the beginning of pulsed breakdown is due to the swaying mechanism, and above which it is due to the immediate formation of a streamer; also there is a limiting length of a spark gap, below which the beginning of static breakdown is due to the swaying process, and above which it is due to the formation of a streamer For dry air at atmospheric pressure the second limit is at  $d \sim 10$  cm, and for gaps 1 cm long the first limit is at  $\Delta U/U_S = 6 - 8$  percent. For damp air both limits can be lower by about a factor two.

The experimental test by the sixth criterion thus confirms the correctness of the theoretical schemes with swaying of the ionization for breakdowns of type A, and enables us to set rational boundaries between this type of breakdown and the adjacent types B and C.

#### 3.7. Seventh Criterion — the Increase of the Current in the Early Stages of Breakdown

Before the development of the latest types of electronic apparatus no measurements were made on the increase of the current in the very earliest stages of breakdown in a uniform field at high pressures. In the last few years several papers have been published on measurements made under these conditions on the spread of the number of electrons in individual avalanches<sup>55,56,84,173</sup> (these measurements were mentioned in Sec. 3.3), on the development of chains of processes of primary and secondary ionization, <sup>134,145-147,174,175,</sup> <sup>205,206</sup> and also on the increase of the current up to the instant when the voltage collapses.<sup>4,5,78</sup>

The cited papers of Schmidt-Tiedemann, Vogel, Pfaue, and Raether present oscillograms of the voltage across a resistance R connected between the spark gap of length d and interelectrode capacity C and a condenser. In accordance with the calculation given in Sec. 2.1.8 the magnitude of the resistance R was chosen equal to  $1/(\alpha Cd)$ . Specimens of such oscillograms are shown in Fig. 14. From a comparison of these oscillograms with the calculated curves of Fig. 5 it is clear that the oscillograms are a direct proof of the existence of a swaying process with two different secondary mechanisms.



FIG. 14. Oscillograms of the voltage across the resistance  $R = 1/(\alpha Cd)$ , for passage across a gas-filled gap of a chain of electron avalanches with various secondary processes. a) Ejection of electrons from cathode by ion bombardment ( $\gamma$  process); b) photoelectric effect at the cathode ( $\delta$  process).<sup>146</sup>

On the basis of a study of such oscillograms for various gases, the last paper of Schmidt-Tiedemann<sup>175</sup> draws the conclusion that for  $pd \sim 50 \text{ cm} \cdot \text{mm}$  Hg the  $\gamma$  process predominates in nitrogen (the coefficient  $\Gamma$  $(= \gamma)$  is equal to  $1.5 \times 10^{-6}$  for an oxidized copper cathode, and  $\gamma > 10^{-3}$  for a cathode of copper annealed in hydrogen), and in hydrogen the  $\delta$  process predominates  $(\delta/\alpha = 5 \times 10^{-5} \text{ for oxidized copper and } 10^{-6} \text{ for}$ purified copper; the corresponding value of  $\gamma$  is about  $10^{-7}$ ). In oxygen both processes are about the same  $(\gamma \cong \delta/\alpha = 10^{-6})$  after oxidation of the copper cathode, but with freshly cleaned copper  $\gamma \simeq 10^{-7}$ . By using oxidized tungsten as cathode one can lower the value of  $\Gamma$  in oxygen to ~ 10<sup>-9</sup>, and thus raise the voltage for static breakdown so high that it will begin directly with the streamer mechanism (in the language of our classification, one can thus change the breakdown from type A to type B). The same thing was practically effected in work of Pfaue and Raether and of Franke, 134,54a who studied avalanches in various gases in which  $\Gamma$  is about 10<sup>-9</sup> (cf. Sec. 3.3).

In work of Vogel<sup>206</sup> it was noted that the development of single electron avalanches was slower than calculated from the mobility of the electrons (cf. Sec. 3.6). Vogel ascribes the slowing down to the fact that part of the electrons in the avalanche are produced not by ordinary  $\alpha$  ionization but through autoionization of doubly excited molecules (an effect known from spectroscopy), which occurs with a delay of about 10<sup>-8</sup> sec.

Papers by Bandel<sup>4,5</sup> and Kluckow<sup>78</sup> present curves of the increase of the current with time at various (small) overvoltages or, in the case of static breakdown, during the entire period of formation of breakdown (from 5 to 100  $\mu$ sec). Specimens of such curves are shown in Fig. 15. Each curve in Fig. 15a breaks off 1  $\mu$ sec before breakdown. An increase of the overvoltage leads to an increase of the steepness of the curve. For extremely small  $\Delta U/U_S$  ( $\mu \cong 1$ ) or for voltages several volts below U<sub>S</sub> one can sometimes



FIG. 15. Specimens of curves of increase of current with time during the period of formation of breakdown. a) From the work of Bandel,<sup>5</sup> various small overvoltages, air, d = 1 cm; b) from the work of Kluckow,<sup>78</sup> static-breakdown voltage, air, d = 1 cm, p = 105 mm Hg, single electron.

get curves like the dashed lines in Fig. 15a. Such curves correspond to breakdown that does not go to completion because there is a break in the chain of swaying of the ionization (because of the statistical nature of the process). For large  $j_0$  and not too small  $\Delta U/U_S$  Bandel found a small spread of behaviors of the curves under identical conditions. In his work a decrease of  $j_0$  led to an increase of the spread, while the average pattern remained the same, apart from the corresponding lowering of the initial part of the curve.

A comparison of experimental curves like those shown in Fig. 15 with the theoretically expected variation of the current at various stages of a breakdown developing in accordance with the scheme of swaying of ionization (cf. Sec. 2.1.8) shows that these curves are a direct demonstration of the process of swaying of the ionization during the period of formation of breakdown (before the violent increase of the current). From the straight-line parts of curves like that shown in Fig. 15b one can calculate corresponding values of the coefficient  $\Gamma$  (which in this case is equal to the coefficient  $\delta/\alpha$ ).

Thus the existing experimental material relating to the seventh criterion for testing the theoretical schemes gives extremely convincing confirmation of the schemes with swaying of the ionization.

### 3.8. Eighth Criterion — the Character of the Luminescence in the Early Stages of Breakdown

On the basis of the observations of Dunnington and White, made by means of Kerr shutters at short formation times  $t_{f}$ ,<sup>32,208,209</sup> it was believed up to 1952 that in the breakdown of molecular gases in uniform fields (at high pressures) the first luminescence that can be detected is produced in the formation of a streamer. The glowing volume had the shape of a segment of a filament, which rapidly extended across the entire gap. Exceptions to this behavior were helium and, possibly, argon.<sup>208</sup> In the former gas White noted a diffuse luminescence, which preceded the formation of the streamer.

The 1950's were marked by studies of breakdown using the newest electronic devices — electron-optical image converters, photoelectric multipliers, and extremely precise stabilizing devices, which made possible new tests of the character of the luminescence in the early stages of breakdown.

Studying the radiation from single avalanches by means of a photoelectric multiplier, combined with an oscillograph, Legler<sup>146</sup> registered chain processes of swaying of ionization with a photoelectric secondary mechanism at voltages several volts below the breakdown voltage. Furthermore he found that in the longwavelength region, beyond 220 m $\mu$ , one quantum of light is emitted on the average for each two ion pairs produced by  $\alpha$  ionization. It is believed that a large part of the radiated energy is concentrated in the part of the spectrum at shorter wavelengths.

Studying breakdown voltages for static breakdown with accuracies of tenths or hundredths of a percent, Fisher and Kachikas,<sup>45</sup> Lessin,<sup>89</sup> Bandel,<sup>5,108</sup> De Bitetto and Fisher,<sup>29,49</sup> and Shinohara<sup>180</sup> discovered a diffuse luminescence that covers the anode and sometimes fills the entire gap before the appearance of a filamentary spark. In the first of these papers, in which the voltage stabilization was evidently less precise, the diffuse luminescence was observed only in nitrogen at  $pd < 400 \text{ cm} \cdot \text{mm}$  Hg and with a sizable photocurrent  $j_0$  from the cathode, produced by external illumination.

In the papers of De Bitetto and Fisher (which also mention similar observations from an unpublished paper by Lessin<sup>89</sup>) there is no restriction on the range of values of pd in which this effect can appear, and it is also not stipulated that it occurs only for sizable  $j_0$ . In these papers it is stated that the diffuse luminescence corresponds to a self-sustaining discharge current of about 1  $\mu$ a. This current and luminescence are stable in nitrogen, and undergo fluctuations in hydrogen. The threshold for the occurrence of this kind of discharge is extremely close to the static breakdown voltage.

In work of Saxe and Chippendale<sup>167-169</sup> the breakdown of an air gap of length about 1 cm (p = 760 mm Hg) was studied by means of a shutter based on the use of an electron-optical image converter (closing time ~  $10^{-9}$  sec), and also by hindering the discharge with a high resistance that did not allow it to enter the high-current phase. In this work also it was noted that in static breakdown there is a diffuse discharge, which comes a few millimicroseconds before the production of the filamentary streamer channel. During the process of growth of the streamer the brightness of the diffuse discharge increases. From the spectrum of this discharge Saxe believes that the glow is produced by molecules of nitrogen bombarded by electrons of speed  $2 \times 10^8$  cm/sec (it is not clear why Saxe sees in this a contradiction with the earlier accepted value of  $1.3 \times 10^7$  for the speed of transfer of electrons in the field and even speaks of the inapplicability of the theory of mobility under these conditions). Saxe points out the important part played in the phenomenological picture of breakdown by the parameters of the power-supply circuit (the inductance, the external resistance, and the capacity of the gap).

The results of the recent work in which the luminescence in early stages of breakdown has been studied, which has given optical confirmation of the existence of chain processes of swaying of ionization during the period in which individual avalanches are passing across the gap, and also the results of studies of the diffuse discharge which precedes the formation of a narrow channel and corresponds to a secondary process at the entire surface of the cathode, thus convincingly confirm the theoretical schemes with swaying of the ionization.

# 3.9. Ninth Criterion - the Existence of a Phase of Glow Discharge

The discovery that preceding type A breakdown there is a diffuse discharge with its most intense luminescence at the anode is one of the elements in the proof that in the course of the breakdown there exists a phase reminiscent of a glow discharge.

Work of Rogowski,<sup>153</sup> Buss,<sup>15</sup> and, in particular, of Köhrmann,<sup>80-83</sup> on oscillographic observation of the voltage across the spark gap provide another important element of this proof. This work has established that in breakdowns of type A the voltage across the gap does not fall to a few tens of volts (corresponding to the voltage for arc discharge) immediately, but only after the passage through a plateau of short duration. Köhrmann<sup>80</sup> found, for example, that in air for  $U_S = 17.8$  kv the height of this plateau is about  $0.85 U_S$ , independent of the value of the overvoltage (it is evidently also independent of pd), and its duration is  $0.2 - 0.8 \ \mu sec$  (this value is very unstable, with a tendency to shorten with increase of the overvoltage). When  $\Delta U/US$  passes the limit that separates breakdowns of types A and C (defined by the break in the plot of tf against  $\Delta U/U_S$  and by the fact that tf becomes shorter than the time of passage of a single avalanche) the plateau disappears. Lowering of this limit by increasing the gap length or the humidity of the air leads to a corresponding lowering of the maximum voltage at which the plateau is observed. The transition range of variation of  $\Delta U/U_S$  in which the plateau disappears, which coincides on the average with the boundary between breakdowns of types A and C, has a rather large width (several percent of US), and is characterized by the fact that in it a plateau is

observed sometimes, but not always. Köhrmann believes that the existence of the plateau is a sign of a breakdown that begins with swaying of ionization, and that its absence is a sign of a pure streamer mechanism.

Observations of the stage of diffuse discharge and of the plateau of the voltage in type A breakdowns provide the missing link in the general sequence of processes in breakdown, in which the swaying of ionization culminates (the swaying itself is by its nature capable only of leading to a form of discharge reminiscent of glow discharge), and during which the transition is prepared to the contracted structure of the discharge which corresponds to its final form — the arc.

The discovery of this link strengthens the general complex of arguments in favor of the theoretical schemes with swaying of ionization.

## 3.10. Tenth Criterion – the Spatial Structure of the Discharge

The specific spatial structure of breakdown at high pressures, which is characterized by a contracted and sometimes sinuous channel, has been used by the authors of streamer theoretical schemes as one of the main arguments supposedly proving the inadequacy of the theoretical schemes with swaying of ionization.

The experimental material that has been accumulated by now gives ample confirmation of processes of transition from a diffuse to a channel structure of the discharge. Work by Saxe and Chippendale,<sup>166-169</sup> Hudson (briefly described in the report by Loeb<sup>108</sup>), and by Phillips and Allen (briefly described in the report by Meek<sup>124</sup>) has shown that after the diffuse discharge has existed for several nanoseconds in the gas-filled gap a bright filamentary streamer channel is formed, which then propagates toward both electrodes with speed  $(0.2-1) \times 10^9$  cm/sec. There are observations of the simultaneous formation of several streamers, and the products of the first streamer enable the streamers that start a bit later to grow faster. After a streamer reaches the anode, the brightness of the channel increases sharply, if the supply circuit is powerful enough. The spectrum of the streamer is essentially the same as that of the completed spark channel.

If the voltage is removed before the growth of the streamer is complete, or if there is an extremely large resistance in the discharge circuit, the streamer does not grow across the entire length of the gap. The part of the gap through which the streamer has not passed is filled with a diffuse azure glow.

The picture of static breakdown in air obtained in the most recent work agrees with the picture of breakdown at small overvoltages obtained much earlier by White, by means of a Kerr shutter, for breakdowns in various gases.<sup>208,209</sup> At the same time there is one more fact noted in White's work<sup>209</sup> that is extremely

important for an understanding of the processes of transition from the diffuse to the channel structure of the discharge. We refer to the change of the place of the first appearance of the glowing channel, as affected by the size of the overvoltage. For small values of  $\Delta U/U_S$  White, like Saxe, observed inception of the streamer at the cathode. With increase of  $\Delta U/U_S$ the place where the streamer started shifted gradually toward the anode, and for  $\Delta U/U_S \sim 12$  percent it touched the anode (see Fig. 16). The result of further increase of the overvoltage was that the place of inception of the streamer moved back to the middle of the gap. At overvoltages up to 12 percent, for a definite value of  $\Delta U/U_S$  there was a quite definite position of the start of the streamer, relative to the electrodes, but for higher overvoltages this position became unstable.



FIG. 16. Regions of start of the streamer at various overvoltages (in percent), according to White.<sup>209</sup>

Marshak<sup>110</sup> gave the following explanation of the behavior observed by White, which also agrees completely with the later observations of Saxe. At small overvoltages the inception of a streamer is possible only after the swaying of ionization has resulted in the formation of a rather large space charge, which increases the value of  $\int_{0}^{d} \alpha \, dx$ . Since almost the entire potential drop across the gap is then concentrated in a narrow region near the cathode, the conversion of one of the avalanches into a streamer must occur at the boundary of this region, i.e., not far from the cathode. As  $\Delta U/U_{\rm S}$  increases, less and less increase of  $\int_{0}^{d} \alpha \, dx$  on account of space charge is required, so

that there is a wider region of potential drop in the gap, and this means a greater distance from the cathode to the place where the conversion of an avalanche into a streamer occurs. At an overvoltage corresponding to the boundary between breakdowns of types A and C it becomes possible for pure streamer breakdown to occur, i.e., for a single avalanche to grow into a streamer. First of all this possibility applies to avalanches with a maximum number of electrons, i.e., to those that have travelled all the way from the cathode to the anode. Therefore, at the minimum overvoltage that corresponds to the transition from the mechanism with swaying to the pure streamer mechanism, the place where the streamer starts must come close to the anode. Further increase of the overvoltage creates conditions for the conversion into streamers of avalanches that have not moved all the way across the gap. Therefore the place the streamer starts can

move away from the cathode (sic) and must be unstable because of the instability of the place of appearance of the initial electron that forms the avalanche.

Thus we see that owing to the obtaining of experimental proofs that there are processes of transition from a diffuse to a channel structure of the discharge, the latter structure can no longer serve as an argument against the theoretical schemes with swaying of the ionization.

## IV. GENERALIZATION OF THE RESULTS OF STUD-IES OF TYPE A BREAKDOWN

As has been shown in Chapter III, the theoretical schemes with swaying of the ionization are in complete agreement with all the experimental information that has been accumulated in recent years. On the other hand, the streamer theoretical schemes of the initial development of breakdown with small overvoltages relative to the static breakdown voltage, gaps of not too great length, weak nonuniformity of the field, and weak external ionization, are in contradiction with many experimental facts and must be rejected (there are exceptions in the cases of organic vapors, and also, with a special choice of the cathode, for oxygen; in these cases the secondary-ionization coefficient  $\Gamma$  is particularly small).

The problem of the concrete secondary-ionization mechanisms that provide increasing reproduction of the electron avalanches during the swaying process (up to the large increase of the current that leads to the distortion of the field by space charge) is at present still not completely solved. The one of these mechanisms that plays the most important part is the photoelectric effect at the cathode, caused by the radiation from atoms that are excited during the development of the avalanches ( $\delta$  process). This statement is confirmed by:

a) the fact that the values of the time of formation of breakdown at small overvoltages are those corresponding to this mechanism;

b) the results of oscillograph studies of the chains of avalanches and the increase of the current in the early stages of breakdown;

c) the increase of the intensity of the diffuse luminescence during the advance of the streamer.

In all probability, in the majority of cases this secondary-ionization process is not the only one (especially at very small overvoltages, for which the time of formation of breakdown reaches hundreds of microseconds), but is supplemented by the process of ejection of electrons by the bombardment of the cathode by positive ions (the  $\gamma$  process), and possibly by other processes also.

The mechanism of the swaying of ionization acts only in the very first stages of the breakdown of a gas at high pressure. By means of it a sizable plane space charge is produced in the gas-filled gap, and this leads

to the concentration of the field in the region near the cathode. Simultaneously a diffuse discharge, closely similar in nature to an anomalous glow discharge, is established for a short period of time. After this the second stage of breakdown begins, in which the streamer ference between the electrodes of the spark gap no mechanism predominates. In this mechanism the main part is played by the concentration (almost to a point) of the space charge, which is formed at the head of one of the electron avalanches whose development is statistically most favorable. This charge produces such a large local distortion of the field that a phase transport of the front of enhanced ionization along the gap, toward the anode and cathode, becomes possible. A thin channel of the gas left behind this front is filled with highly ionized neutral plasma. This permits the passage of an extremely large electric current, as soon as the channel has closed the gap between the cathode and anode, and produces at the electrodes regions of the discharge that are of a nature closely similar to the corresponding regions of an arc.

The problem of the concrete nature of the phase transport of the front of enhanced ionization in the field of the concentrated space charge also remains an open one at the present time. This transport can be explained by: a) thermal ionization of the gas (under the action of all sorts of elementary processes of ionization of the atoms); b) photoionization of atoms at some distance in front of the head, so that the enhanced  $\alpha$  ionization produces at a short distance a daughter avalanche with a charge in its head equal to that in the head of the parent avalanche; c) acceleration of the electrons after their ejection from atoms until they have attained the kinetic energy corresponding to the electron temperature; d) impact ionization of excited atoms that are in front of the head of the avalanche.

Increase of the overvoltage in pulsed breakdown creates the conditions for the beginning of the second stage of the breakdown-the development of a streamer - at a smaller concentration of the field at the cathode, and consequently with less plane space charge produced by swaying of the ionization. This leads to a shortening of the formation time of the breakdown, more rapid growth of the current in the early stages of the discharge, and also a displacement of the place where the streamer starts toward the anode. At a certain value of the overvoltage the formation of a streamer limit that separates breakdowns of types A and C becomes possible without concentration of the field through the action of plane space charge. In this case the breakdown can start directly from the second stage - the development of a streamer - as was assumed in the streamer theoretical schemes. The transition to breakdown that begins directly from the second stage is characterized by a number of indications; some important ones are: a) the time of formation of the breakdown becomes approximately equal to the time of passage of a single electron avalanche across the gap; b) the curve of formation time against

overvoltage shows a break, owing to the change to an essentially new mechanism for the formation; c) the region where the streamer starts is displaced right up to the anode; d) oscillograms of the potential diflonger show the plateau corresponding to the phase of diffuse ("glow") discharge.

The voltage at which the development of breakdown begins directly from the second stage is the actual breakdown voltage from the point of view of the streamer theoretical schemes. It should obey all the laws that were derived from these schemes in Sec. 2.2, including the statistical law that gives the spread of the measured values of this voltage. According to the value found in Sec. 2.2.4 for the spread of the measured values of the breakdown voltage around its average value, which spread is about 600 volts (for emission of the initial electrons at the cathode, which corresponds to breakdown with the cathode illuminated with ultraviolet radiation), the limit on the overvoltage at which the stage of swaying of ionization disappears should not be sharp, but should have a statistical half-width of about 600 volts. This is indeed the size of the range of overvoltages in which oscillograms of the potential difference between the electrodes of the spark gap sometimes show the glowdischarge plateau and sometimes do not.<sup>81</sup>

A manifestation of the statistical nature of breakdown beginning directly from the second stage is the indefinite position of the region where the streamer starts at large overvoltages.

Pulsed breakdown of gaps of not too great length, with uniform field, at small overvoltages at which the discharge begins with swaying of the ionization, is qualitatively analogous to the static breakdown of such gaps. Pulsed breakdowns of such gaps at large overvoltages, at which the discharge begins with the immediate formation of a streamer, are qualitatively different from type A breakdowns, and form the separate type C.

The boundary between breakdowns of types A and C is thus a rather broad zone, within which in some cases the breakdown begins with swaying of ionization, and in others with the immediate formation of a streamer.

With increase of the gap length the overvoltage moves to lower values. For dry air at atmospheric pressure the amount of overvoltage that corresponds to the transition from breakdown of type A to that of type C becomes practically equal to the statistical dispersion when the gap length reaches about 10 cm. At and beyond this gap length, even static breakdown can begin with the immediated formation of a streamer. Static breakdown also begins in this way in strongly inhomogeneous electric fields. Therefore static breakdowns of long gaps (longer than about 10 cm for dry air) in uniform fields and breakdowns in strongly inhomogeneous fields can be assigned to the single type B, by the nature of their mechanism. The boundary between breakdowns of types A and B is also a rather broad zone, with a width fixed by the statistical variations in the development of the electron avalanches.

An increase of the humidity of the air or a special treatment of the cathode that lowers the value of the secondary-ionization coefficient  $\Gamma$  can lead to a decrease of the gap length at which the region of type B breakdowns begins, and also to a lowering of the fractional value of the overvoltage\* at which the region of type C breakdowns begins.

## V. BREAKDOWN OF A GAS-FILLED GAP IN WHICH THERE IS INTENSE IONIZATION BY AN AUXIL-IARY HIGH-VOLTAGE SOURCE WHICH PRODUCES A LUMINOUS CHANNEL IN THE GAS (TYPE E BREAKDOWN)

# 5.1. Ignition Voltage as the Analog of Breakdown Voltage

If by the term "breakdown" we mean a phenomenon in which a gas-filled gap changes from a state of low electric conductivity to a state of extremely high conductivity, then this concept also includes the process of ignition in a number of gas-discharge devices (pulsed lamps, protective spark gaps, ultrahighpressure lamps, and so on) which are controlled by means of an auxiliary high-voltage source (pulse transformer, coil, etc). When such a gas-discharge device is connected in a circuit like those illustrated in Fig. 17, the closing of the switching device 5 produces a high-voltage pulse on an auxiliary electrode



FIG. 17. Examples of circuits for devices that use type E breakdowns. a) circuit with external igniting electrode; b) circuit without auxiliary electrode. 1 – gas-discharge gap (lamp, protective gap); 2 – supply source for high-current discharge (charged capacitor, power line); 3 – pulse transformer; 4 – condenser to actuate pulse transformer (charged from 2 or from a separate source); 5 – switching element (synchronous switch, thyratron, auxiliary gap, etc.).

or on one of the main electrodes of the device, which is accompanied by the formation of a thin luminous channel between the main electrodes. The formation of this auxiliary channel is obviously a type C breakdown.<sup>†</sup> But in view of the small power and short dura-

\*Together with an increase of the absolute value of the breakdown voltage.

<sup>†</sup>In the case in which the auxiliary channel is produced by some means of an external igniting electrode (Fig. 17a), its formation must be due to charges on the walls; the mechanics of interaction with these must be analogous to the mechanism described by Warmolz.<sup>207</sup> tion of the high-voltage pulse this auxiliary breakdown does not by itself bring the gas filled gap into the state of high conductivity, since if the voltage of the source 2 is small the auxiliary breakdown is not followed by a major discharge supplied from this source.

If the voltage of the source 2 is gradually raised, while from time to time the auxiliary breakdown is repeated, the final result is that one of the auxiliary breakdowns goes over into a main breakdown with violent increase of the current in the circuit supplied by the source 2. It is this main breakdown that we call a type E breakdown.

The minimum voltage  $U_I$  of the source 2 for which such a main breakdown can occur (the so-called ignition voltage of the gas-discharge device) is the analog of the breakdown voltage for static breakdowns of types A and B, which were discussed in the preceding chapters. This voltage can be much lower than the voltage for ordinary static breakdown of the same gas-filled gap in the absence of the auxiliary high-voltage pulse (the so-called self-breakdown voltage  $U_{\rm Sb}$  of the device).

The present chapter is devoted to the problem of the physical criterion that fixes the magnitude of the ignition voltage for breakdowns of type E.

### 5.2. A Scheme for the Breakdown Mechanism and Criteria for Testing it Experimentally

The development of the present concept<sup>116</sup> of the condition for inception of type E breakdown has been based on consideration of the following features of this type of breakdown, which are known from the development and use of gas-discharge devices based on it: a) the definiteness of the value of the ignition voltage; b) the much lower values of  $U_{I}$  for devices filled with inert gases, as compared with those filled with molecular gases; c) the sharp increase of  $U_{I}$  when small amounts of a molecular gas are added to an inert gas; d) the lowering of  $U_{I}$  when the gas pressure is decreased or the distance between the electrodes is shortened, or when the power of the igniting pulse is increased; e) the lessening of the difference between the ignition voltage  $U_{I}$  of the discharge and its extinction voltage Ue (the residual voltage on the condenser supplying the current to the device) when the inside diameter of the discharge tube is made smaller. This difference approaches zero if the diameter of the tube approaches the diameter of the channel of the auxiliary discharge, which is determined by the streamer mechanism.

The mechanism of the extinction of a discharge fed from a condenser can be deduced from the idea of the energy balance in a shaped pulsed discharge confined by the walls of the discharge tube.<sup>113-115</sup> In the course of the discharge, as the electric field strength  $\mathcal{E}$  decreases because of the drop of the voltage across the condenser, the relative part played in the general energy balance by losses to the walls increases rapidly and approaches 100 percent. At low values of the field strength the electric power delivered to the channel of the discharge falls rapidly because of the characteristically rapid rise of the specific resistance of the plasma,<sup>115</sup> and at a certain value of  $\pm$  it ceases to balance the energy losses to the walls, owing to which there must be a sharp automatic cessation of the discharge (the cooling of the channel leads to increase of its resistance, and this leads to further decrease of the power and more cooling, and so on). Since the relative energy loss to the walls increases with decrease of the diameter of the channel, the voltage at extinction of the discharge in narrow tubes must be higher than in wide ones (as is actually observed experimentally).

The fact that  $U_I$  and  $U_e$  are close together for particularly narrow tubes suggests the idea that type E breakdown is the process inverse to the process of extinction of a pulsed discharge. The channel of ionized plasma between the main electrodes that is produced in the auxiliary breakdown may either rapidly lose its ionization, if the power of the current passing through the plasma under the action of the main voltage source is insufficient to cover the losses of energy into the surrounding space (case of negative power balance) or it can begin to expand, if the power of the current exceeds the losses (case of positive balance). The expansion of the channel leads to a still greater excess of the power received (proportional to the square of the diameter) over the power lost (proportional to the first power of the diameter). Thus the transition from negative to positive power balance is extremely critical, and leads to the precipitate rise of the current that is characteristic of all types of breakdown at high pressures. The condition for the existence of a positive power balance can be identified with the condition for the inception of type E breakdown.

This idea agrees completely with the features of type E breakdown mentioned earlier. In fact, the definiteness of the value of  $U_I$  arises from the definiteness of the power balance in the given plasma channel of the auxiliary breakdown and the critical character of the condition for the beginning of breakdown. The small cross section for scattering of the electrons by the gas atoms (Ramsauer effect<sup>148,149</sup>) explains the lower value of the electric field strength necessary for a positive power balance in an inert gas as compared with the case of a molecular gas, and thus explains the lower values of  $U_{I}$  in inert gases. Decrease of the distance between the electrodes makes it possible to get the same field strength with less voltage across the gap, and thus lowers the value of  $U_{I}$ . An increase of the power of the auxiliary pulse leads to an increase of the conductivity and diameter of the initial plasma channel, owing to which a positive power balance can be established with a smaller field strength.

The relationships that have been listed are evidence of a qualitative correspondence between theory and experiment. We must give particular attention to the dependences of  $U_I$  on the parameters that most strongly affect the value of the ignition voltage: the pressure of the gas and the amount of molecular gas mixed with the intert gas. These relations enable us to obtain also a quantitative test of the theory.

On the basis of the equation for the power balance in the plasma channel produced by the auxiliary breakdown one can derive<sup>116</sup> the following relations:

$$(U'_{i}/U_{i})^{2} = 1 + K\theta,$$
 (5.1)

$$K = \frac{1-x}{x} Q_1 \left/ \left( Q_1 + \frac{1-x}{x} Q_0 \right) \right.$$
 (5.2)

$$U_{i}^{2} = \operatorname{const} \cdot p. \tag{5.3}$$

Here  $U_I$  and  $U'_I$  are the ignition voltages in a pure inert gas and in the inert gas with a small admixture  $\theta$ (a small fraction) of a molecular gas; x is the degree of ionization of the plasma in the channel by the auxiliary breakdown;  $Q_0$ ,  $Q_1$ , and  $Q_i$  are the respective cross sections for scattering of an electron by an inert-gas atom, a molecular-gas atom, and an ion; and p is the initial pressure of the gas.

A case of particular interest is that in which the quantity  $xQ_i$  can be neglected in comparison with  $(1-x)Q_0$  (the case of low-power auxiliary pulse). Equation (5.2) then takes the form:

$$K = Q_1 / Q_0.$$
 (5.4)

In this case the slope of the straight line (5.1) is equal to the ratio of the cross sections for scattering of electrons by atoms of the molecular and inert gases.

Equations (5.1) and (5.3) can be tested experimentally, and are quantitative criteria for verifying the scheme of the mechanism of type E breakdown. By comparing the experimental values of the coefficient K for various combinations of inert and molecular gases one can test the validity of the relation (5.4), which is a further qualitative criterion for the correctness of the theory.

### 5.3. Experimental Tests of the Deductions from the Theoretical Scheme for Type E Breakdown

An experimental test of the relations (5.1) - (5.4) has been carried out in reference 117, in which studies were made of the ignition voltages of tubular pulsed lamps with external igniting electrodes (circuit of Fig. 17a). The lamps were filled with xenon, argon, and neon, to which various small amounts of nitrogen, hydrogen, and oxygen were added.

As an example, Fig. 18, shows a plot of the quantity  $(U'_{I}/U_{I})^{2}$  against  $\theta$  for xenon with small amounts of nitrogen added. For small  $\theta$  the experimental points lie well on a straight line, which is a confirmation of the theory.

Figure 19, shows curves of the dependence of the slope K of such lines on the primary voltage in the







FIG. 19. Dependence of the coefficient K on the voltage U<sub>I</sub> in the primary coil of the ignition pulse transformer, for six combinations of inert and molecular gases: 1 - neon + hydrogen; 3 - argon + hydrogen; 5-xenon + hydrogen; 2, 4, 6 - the same inert



FIG. 20. Dependence of the ignition voltage on the pressure (pure xenon).117

pulse transformer used for the ignition (i.e., on the power of the auxiliary pulse), for six combinations of inert and molecular gases.

The scale of ordinates for hydrogen (dashed curves) is smaller than that for the curves for nitrogen (solid curves) by a factor six. The curves for the various inert gases are drawn to the same scale.

The fact that all of the dashed curves in this diagram fit satisfactorily with the solid curves gives indirect experimental confirmation of Eq. (5.4). On the basis of this equation, we can conclude from Fig. 19 that the cross section for scattering of electrons of the energies in question (~1 ev) by a hydrogen atom is six times that for scattering by a nitrogen atom, and 200 to 400 times that for scattering by a xenon atom. The cross sections for neon and argon atoms are 5 to 15 times smaller than for the xenon atom.

An exact comparison of these conclusions with data is not possible, because there are no adequate data on the cross sections. The two kinds of data do, however, agree in order of magnitude.\*

Figure 20 shows an experimental curve of the dependence of the ignition voltage on the square root of the initial pressure of the gas (pure xenon). As can be seen from this diagram, the relation of these quantities agrees with Eq. (5.3).

Thus the experimental data existing at the present time confirm the existing scheme of the mechanism of type E breakdown.

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