NUCLEAR REACTIONS IN COLD HYDROGEN
I. MESONIC CATALYSIS

Ya. B. ZEL'DOVICH and S. S. GERSHTEĬN

Usp. Fiz. Nauk **71**, 581-630 (August, 1960)

CONTENTS

1. Introduction . . . 593
2. Experimental Work on Catalysis of Nuclear Reactions by μ Mesons . . . 596
   a) Berkeley Experiments . . . 596
   b) Liverpool Experiments . . . 597
3. Kinetics of Catalysis of Nuclear Reactions by μ Mesons in Hydrogen . . . 598
4. Mesomolecular Processes in Hydrogen . . . 601
   a) Adiabatic Approximation . . . 601
   b) Improvement of the Adiabatic Approximation for Hydrogen Mesomolecules . . . 602
   c) Levels of Mesomolecules . . . 603
   d) The Process of Transfer of a μ Meson from a Light to a Heavy Isotope . . . 605
   e) Scattering of Mesonic Atoms by Nuclei . . . 605
   f) Formation of Mesomolecules . . . 607
   g) Transitions between Levels of Mesomolecules . . . 609
   h) Comparison with Experimental Data . . . 610
5. Nuclear Reactions in Mesomolecules . . . 611
   a) Reaction Constants . . . 611
   b) Penetration of Nuclei through the Barrier in a Mesomolecule . . . 612
   c) Nuclear Reaction in the pdμ Mesonic Molecule . . . 612
   d) Nuclear Reaction in the ptμ Mesonic Molecule . . . 614
6. Efficiency of Catalysis of Nuclear Reactions by Mesons . . . 616
7. Conclusion . . . 617
Appendices 1 — 3 . . . 618-622

1. INTRODUCTION

It is known that nuclear synthesis of hydrogen isotopes occurs at high temperatures in the so-called thermonuclear reactions. In 1957, in Berkeley, a group of experimenters under the direction of Alvarez discovered the phenomenon of catalysis of nuclear reactions in hydrogen by μ mesons. This phenomenon had been predicted earlier theoretically by Frank,[1] Sakharov,[2] and Zel'dovich,[3] and makes it possible to produce nuclear synthesis in cold hydrogen. Although almost immediately after the experiments of Alvarez it was established that μ catalysis cannot have technological application as a source of energy, the study of nuclear reactions produced by μ mesons in hydrogen is of unquestionable interest for mesonic and nuclear physics. Up to the present time there are three experimental[4,4a,5] and a large number of theoretical papers concerning various aspects of μ catalysis. It is thus possible to give a systematic survey of the experimental and theoretical results in this field. Along with problems of μ catalysis, it appears reasonable to include in the present summary also the treatment of another possibility for producing nuclear synthesis in cold hydrogen — the so-called "piezonuclear" reactions which occur at high pressures. It is known that the following reactions of nuclear synthesis which occur with energy liberation are possible among the various isotopes:

$$p+p=d+e^{+}+\nu \quad (2.2\ \text{Mev}), \tag{I}$$

$$p+d=\text{He}_3+\gamma \quad (5.4\ \text{Mev}), \tag{II}$$

$$d+d=\begin{cases} t+p & (4\ \text{Mev}), \\ \text{He}_3+n & (3.3\ \text{Mev}), \\ \text{He}_4+\gamma & (24\ \text{Mev}), \end{cases} \tag{III}$$

$$d+t=\text{He}_4+n \quad (17.6\ \text{Mev}), \tag{IV}$$

$$p+t=\text{He}_4+\gamma \quad (20\ \text{Mev}), \tag{V}$$

$$t+t=\text{He}_4+2n \quad (10\ \text{Mev}). \tag{VI}$$

In order for nuclear synthesis to occur, the reacting nuclei must approach to within a distance of the order of the radius of action of the nuclear forces, and this in turn requires that they overcome the Coulomb barrier between the charged particles. Therefore the reactions listed above can occur under ordinary conditions only if the nuclei are given sufficient kinetic energy: either in an accelerator tube, or by heating to

high temperature in an atomic explosion, or in a powerful gas discharge (for example, reaction III*).

It would be incorrect, however, to think that the nuclear reactions (II – VI) occur only when the kinetic energy of the nuclei exceeds the height of the Coulomb barrier. Actually, these reactions begin to occur with significant probability for much lower energies because of the quantum mechanical tunneling through the Coulomb barrier. Actually the height of the Coulomb barrier for reactions (II – VI) is some hundreds of kev, whereas, for example, reaction IV is observed in accelerators beginning already with an energy of the order of 1 – 10 kev and above.

In precisely the same way, thermonuclear reactions occurring, say, at a temperature $T = 10^8\,°K$ (which corresponds to a mean energy of the particles $\epsilon \approx 10$ kev), occur mainly through tunneling transitions, since at these temperatures the number of particles which, because of the temperature distribution, have kinetic energies exceeding the Coulomb barrier is insignificant.†

We should mention an important feature of sub-barrier transitions. The point is that the coefficient for penetration of particles through the barrier is much more sensitive to changes in the barrier width than to changes in its height. In fact, from the well-known formula for the penetration factor

$$B = \exp\left\{-\frac{2}{\hbar}\int_{x_1}^{x_2}\sqrt{2M(U(x)-E)}\,dx\right\}$$

$$= \exp\left\{-\frac{2}{\hbar}\sqrt{2M\bar{U}}\,(x_2-x_1)\right\} \qquad (1.1)$$

(where $\bar{U}$ is some average height of the barrier), we see that the exponent depends linearly on the barrier width and varies only as the square root of its height. This is the reason in particular that sub-barrier transitions play practically no part in chemical reactions. In fact, though the heights of the barriers for chemical reactions are much lower than for nuclear reactions, their width is so large that the penetration factor is negligible, and chemical reactions actually always occur over the barrier, i.e., under conditions where the particles are given the necessary energy for activation from heat sources or other sources.

Just because of this feature of the barrier factor, nuclear reactions begin to occur with reasonable probability even at energies where the height of the barrier has still been hardly reduced; however, the most difficult region for passage (the region at large distances) is overcome by virtue of the kinetic energy of the particles.

This feature of the barrier factor which we have pointed out also shows that reactions of synthesis in hydrogen could occur, generally speaking, even for extremely low kinetic energy of the nuclei, if one could bring the nuclei sufficiently close to one another by external pressure or by chemical forces.

It is easy to see, however, that for the usual interatomic separations in the hydrogen molecule a nuclear reaction is impossible. In fact, in this case the barrier factor is roughly speaking equal to $B \approx \exp\left\{-\frac{2}{\hbar}(2Me^2r_0)^{1/2}\right\}$ (where M is the reduced mass of the hydrogen nuclei, r_0 is the nuclear separation in the molecule).

Noting that r_0 is equal in order of magnitude to the Bohr radius for the electron, $a_e = \hbar^2/m_e e^2$, we obtain

$$B = \exp\left(-k\sqrt{\frac{M}{m_e}}\right) \qquad (1.2)$$

($\hbar$ and e cancel, k is a numerical factor around 3 – 3.3). Thus the coefficient for penetration through the Coulomb barrier in ordinary hydrogen molecules is extremely small, according to (1.2).

A computation of the probability of a nuclear reaction in the HD molecule gives, for 1 m^3 of liquid HD, 10^{-21} reactions per year. It is interesting in this connection to note that if the separation between the hydrogen nuclei were 5 – 10 times smaller than in the hydrogen molecule, the yield of the reaction could reach a quite significant magnitude. For example, in 1 kg of deuterium at a pressure of 600 million atmospheres (corresponding to a density of 80 g/cm^3 and a mean separation between the particles which is just half as great as in the H_2 molecule), one should observe one reaction per minute (cf. Part II).

At first glance it seems that the necessary reduction in distance between the hydrogen nuclei may occur in certain chemical compounds; actually, however, in all known compounds containing hydrogen the distance between the hydrogen nuclei is greater than in the H_2 molecule. Thus, interatomic separations in ordinary molecules are too large for the yield of sub-barrier nuclear reactions between hydrogen isotopes to have any significant magnitude.

From formula (1.2) we see that if the chemical

*It should be mentioned that the probability for reaction (I) is extremely small compared to that for the other reactions (II–VI). The reason for this is that it occurs only because of the conversion of a proton into a neutron as a result of the weak β interaction. Therefore reaction (I) could hardly be produced under terrestrial conditions. Nevertheless it plays a very important role in astrophysical processes, being one of the main sources of stellar energy. (Cf. also Part II of the present review.) The reaction $d + d = He_4 + \gamma$ is also extremely improbable. (In addition to the general reasons which apply to all reactions with γ quanta, namely the weakness of electromagnetic interaction, here the reaction is especially weak because of considerations of isotopic invariance and symmetry.)

†Actually, the most important region of the Maxwell distribution for the thermonuclear reaction is that region where including the Coulomb barrier factor $\exp\{-2\pi e^2 Z_1 Z_2/\hbar v\}$ (where v is the relative velocity and Z_1e, Z_2e are the charges on the particles) gives a maximum for the exponent $\left\{-\frac{Mv^2}{2kT} - \frac{2\pi Z_1 Z_2 e^2}{\hbar v}\right\}$, i.e., the neighborhood of the velocity $v_0 = (2\pi Z_1 Z_2 e^2 kT/M\hbar)^{1/3}$. The value of the "effective" energy $\xi_0 = Mv_0^2/2$ for the d + d reaction at $T = 10^8\,°K$ is 30 kev, which is considerably below the height of the Coulomb barrier.

binding in the hydrogen molecule were produced not by an electron, but by a particle with considerably higher mass, for example, a meson, the barrier factor would not be so small and the nuclear reaction could occur. It is obvious that for this purpose the only suitable particle is a negative meson which does not react with the nucleus, since any meson which reacts strongly with nuclei (for example, a π^- or K^- meson) will be captured long before it can give rise to a nuclear reaction. Of the particles known at present the only ones which are suitable are μ mesons. The fundamental process which determines the lifetime of the μ meson in hydrogen is its decay into an electron, neutrino, and antineutrino:

$$\mu \rightarrow e + \nu + \tilde{\nu},$$

which occurs with a decay rate $\lambda_0 = 0.45 \times 10^6$ sec^{-1} (lifetime 2.22×10^{-6} sec).

The probability for direct nuclear capture of a μ^- meson in hydrogen

$$\mu^- + p \rightarrow n + \nu$$

is $10^{-3} - 10^{-4}$ of the decay probability.[22,23] For this reason we shall in what follows not consider the process of capture of the μ^- meson.

We know that μ^- mesons entering matter slow down and, after a time which is much less than their lifetime, reach 1s states of the mesonic atom. The special feature of the mesonic hydrogen atoms, as contrasted with Heμ, Liμ, etc, is its electrical neutrality. The radius of the Bohr orbit of a μ-mesonic atom of hydrogen is $a_\mu = \hbar^2/m_\mu e^2 = 2.56 \times 10^{-11}$ cm, which is much less than atomic dimensions. Therefore the behavior of mesonic hydrogen atoms in matter is very much like the behavior of slow neutrons. The electrical neutrality of mesonic hydrogen gives rise to various specific effects associated with the possibility of very close approach of the mesonic hydrogen atom to other nuclei. When a hydrogen mesonic atom approaches a nucleus with $Z > 1$, a transition of the meson can occur to a mesonic atom orbit of this other nucleus, with transfer of the difference in the binding energies to the proton. Such transitions were already observed for π-mesonic atoms in the experiments of Panofsky,[24] who observed the transition $p\pi + Li \rightarrow Li\pi + p$ in LiH. In the collision of a mesonic atom pμ with protons, one can have μ-mesonic exchange between the protons (which leads in particular to the complete depolarization of μ-mesons in hydrogen, as pointed out by the authors),[25,26] while in the presence of deuterium or tritium impurities in the hydrogen one can have a transition of the μ meson to these isotopes, say, a reaction $p\mu + d \rightarrow d\mu + p$. As a consequence of the fact that the presence of the meson reduces considerably the width of the Coulomb barrier, the collision of dμ-mesonic atoms with protons or deuterons can lead to the nuclear reactions (II — III). In general, these reactions could occur in flight as well as via formation of the mesomolecular ions pdμ and ddμ. One may note, however, that the probability of reaction in flight is much less than the probability of a reaction from a state of the mesomolecule. In fact, if we disregard the presence of the Coulomb barrier, we may picture the nucleus in the mesomolecule as being bound in a potential well with dimensions of the order of the μ-meson Bohr radius a_μ; whereas for collisions of a free mesonic atom with nuclei, the number of collisions is determined by the mean distance between nuclei of the material, i.e., $1/N^{1/3} \approx a_e$ (where N is the number of nuclei per cc, and a_e is the Bohr radius of the electron). Thus the probability of nuclear reaction in flight should generally speaking be $(a_\mu/a_e)^3$ of the probability of a reaction in a mesomolecule. (More accurate estimates will be made later.)

The first to point out the possibility of nuclear reactions between hydrogen isotopes bound in a mesomolecule was Frank.[1] It is an interesting point that this idea was stated by Frank in connection with the experiments in which the π and μ mesons were first identified.

Until 1947 the π and μ mesons observed in cosmic rays were assumed to be the same particle. In 1947 Powell, Lattes, and Occhialini obtained several photographs in plates in which a new track, belonging to a meson with an energy in the neighborhood of 5 Mev (Fig. 1) emerged from the point of stopping of a meson. These cases were interpreted by Powell,

FIG. 1. Tracks of mesons in emulsion obtained by Powell, Lattes, and Occhialini in 1947.

Lattes and Occhialini on the hypothesis that there were two types of mesons; Powell et al. stated that the initial track belonged to a π meson which decayed, after stopping, into a μ-meson (with an energy around 5 Mev) and a neutral particle (neutrino). Realizing the importance of the assumptions that were made, Frank considered various other possibilities for interpreting the photographs of Powell et al., and, in particular, noted that if pd mesomolecules could be formed in the plates, there could occur in them a nuclear reaction $p + d \rightarrow He_3$ with a transfer of energy (5.4 Mev) to the meson by "internal conversion." However,

Frank pointed out that the probability of such a process in photographic plates is small because of the negligible content of deuterium in normal hydrogen,* and concluded completely correctly that the photographs are to be explained as a $\pi \rightarrow \mu + \nu$ decay.

Later, nuclear reactions produced by μ^- mesons in liquid deuterium were considered by A. D. Sakharov.[2] In particular, he treated the reaction $d + d + \mu \rightarrow He_4 + \mu$ and estimated the lifetime of the $dd\mu$ mesomolecule as 10^{-11} sec. Independently, in 1953, Ya. B. Zel'dovich came to the conclusion that it was possible to have catalysis of nuclear reactions by μ^- mesons, and made the first detailed estimates of the phenomena associated with this process.[3] In reference 3 it was shown that the probability of nuclear reaction in flight is small, whereas formation of the mesomolecule practically always leads to a nuclear reaction. The possible mechanisms were considered, and an estimate was made of the probability for formation of mesomolecules. Reference 3 also gave a rough determination of the levels of the hydrogen mesomolecule, and pointed out the possibility of "resonance," which increases the probability for formation of mesomolecules and of nuclear reaction in flight for the case where the mesomolecule has an excited state with low binding energy. Further theoretical papers[6-21] were stimulated by the experimental discovery of μ catalysis.

2. EXPERIMENTAL WORK ON CATALYSIS OF NUCLEAR REACTIONS BY μ^- MESONS

a) Berkeley Experiments[4]

The reaction $p + d \rightarrow He_3$, produced by a μ^- meson, was first discovered by Alvarez and co-workers[4] during an investigation of the interactions of K mesons in a liquid hydrogen chamber. Along with the K mesons a large number of π and μ mesons entered the chamber. Since the chamber was in a magnetic field (11,000 gauss), μ mesons with a range exceeding 10 cm could be satisfactorily identified by the curvature of their tracks. About 2,500 cases of stopping of μ mesons in the chamber were examined. Almost all of them gave the usual picture of $\mu \rightarrow e$ decay. However, in 15 cases the μ meson, after being stopped, again attained an energy of 5.4 Mev (which corresponds to a range of 1.7 cm), and the beginning of the secondary track was not infrequently displaced from the end of the primary track by a gap of up to 1 mm (cf. Fig. 2). At the end of the secondary tracks, electrons were observed (Fig. 2), with energies characteristic for the electrons from $\mu \rightarrow e$ decay. This is an additional confirmation of the fact that the secondary track is due to a μ meson.

*Actually, the process cannot occur in a photographic plate not so much because of the low deuterium content, as because of a transition of the μ^- mesons to nuclei with $Z > 1$.

FIG. 2. Photograph taken in a hydrogen bubble chamber by Alvarez et al.[4]

Considering that normal hydrogen always contains some deuterium impurity (one part in 6,000), the authors of reference 4 completely correctly concluded that the reason why the μ meson received an energy of 5.4 Mev after being stopped was the nuclear reaction $p + d \rightarrow He_3$ in the $pd\mu$ mesomolecule, i.e., the process described in the preceding paragraph.

Further experiments carried out in hydrogen[4] enriched in deuterium confirmed this assumption: with increasing concentration of deuterium, the yield of the reaction increased (cf. Table I). They also observed one case where the μ-meson gave rise to nuclear reactions twice.

An important point is that even at the highest deuterium concentration only a few cases were recorded of the reaction $d + d \rightarrow p + t$ produced by a μ meson. This fact is an experimental proof that the nuclear reaction does not occur in flight, but rather from a state of the mesomolecule. In fact, since the intrinsic probability of the $d + d$ reaction is 10^6 greater than for the $p + d$ reaction (cf. Sec. 5), if the nuclear reactions occurred in flight, the number of cases of the $d + d$ reaction in hydrogen enriched in deuterium should have far exceeded the number of cases of the $p + d$ reaction.

In the experiments of Alvarez et al. it appeared surprising at first glance that the reaction $p + d \rightarrow He_3$ has a considerable probability even in normal hydrogen with a negligible deuterium content. Another striking point was the peculiar dependence of the yield of the nuclear reaction on deuterium concentration: As we see from Table I, the reaction yield, which increases with increasing concentration of deuterium, clearly approaches saturation for a concentration of the order of 1%. As was noted in reference 4, these features of μ catalysis can be explained by a transfer of μ mesons from protons to deuterons, noting that, because of the difference in the reduced masses, the μ meson in the $d\mu$ atom has a binding energy which is 135 ev greater than in the $p\mu$ atom. In fact, the very first estimates given in the work of Zel'dovich and Sakharov[6] supported this assumption, and enabled

TABLE I. Yield of the reaction $p + d + \mu \rightarrow He_3 + \mu$ for various deuterium concentrations

Deuterium Concentration	Normal hydrogen		0.3%		4.3%	
	ref. 4	ref. 4a	ref. 4	ref. 4a	ref. 4	ref. 4a
Number of $\mu \rightarrow e + \nu + \nu$ decays	2541	2547	2959	5924	1269	1792
Number of $pd\mu \rightarrow He_3 + \mu$ reactions	15	18	57	139	32	45
Number of cases where the μ meson gave rise to two successive nuclear reactions	0	0	1	3	0	0
Yield of reaction in %	0.6	0.7±0.2	2	2.3±0.2	2.5	2.5±0.4

one to explain the dependence of the reaction yield on deuterium concentration (cf. also references 7 and 8). The transfer of the μ-meson from the proton to the deuteron also enables one to explain the occurrence of "gaps" between the end of the primary track of the μ meson and the beginning of the track of the μ meson which is converted in the $p + d \rightarrow He_3$ reaction (cf. Fig. 2). The presence of the gap is associated with the path of the neutral $d\mu$ atom which receives some energy as a result of the charge transfer. Such a "gap" is also observed between the point of stopping of a μ meson in a hydrogen chamber and the electron from the $\mu \rightarrow e$ decay. Experiments[4,4a] in hydrogen enriched in deuterium show that for low deuterium concentrations the frequency with which one observes a "gap" in the $\mu \rightarrow e$ decay increases with increasing concentration of deuterium. (No gaps are observed for a deuterium concentration of 4.3%. For the discussion of this question, cf. Sec. 4.) From the frequency of occurrence of the "gaps" in reference 4a, they measured the probability of transfer of the μ meson from the proton to the deuteron. This probability in normal hydrogen is (0.55 ± 0.29), while for a deuterium concentration of 0.3% it becomes (0.76 ± 0.16).

b) Liverpool Experiments[5]

In experiments with a bubble chamber one could record only those cases of the reaction $p + d \rightarrow He_3$ in which the energy of the reaction was transferred to a μ meson by conversion. It appeared unquestionable, however, that the $p + d$ reaction produced by a μ meson might also occur with transfer of the energy to a γ quantum. Catalysis by μ mesons of the nuclear reaction $p + d \rightarrow He_3 + \gamma$ was studied experimentally in reference 5. It should be mentioned that the measurement of the yield of γ radiation in μ catalysis is, from the point of view of experiment, a quite complicated problem because of the presence of a high background. Satisfactory detection of γ quanta from the $p + d$ reaction was achieved by the following: 1) measurements were made for a hydrogen target, both without deuterium enrichment and with enrichment to a concentration of 1.8% (which according to reference 4 corresponds to saturation of the reaction); 2) the γ quanta in the energy range from 3 Mev to 7.5 Mev were detected, i.e., near to the energy of the γ quanta from the reaction $p + d \rightarrow He_3$ (5.4 Mev); 3) the distribution of the γ quanta was measured as a function of time from the time when the μ meson hit the hydrogen target.

The overall arrangement of the apparatus used in reference 5 is shown in Fig. 3. Counter 4 was connected in anti-coincidence with counters 1, 2, and 3, while counter 5 was in anti-coincidence with counter 6.

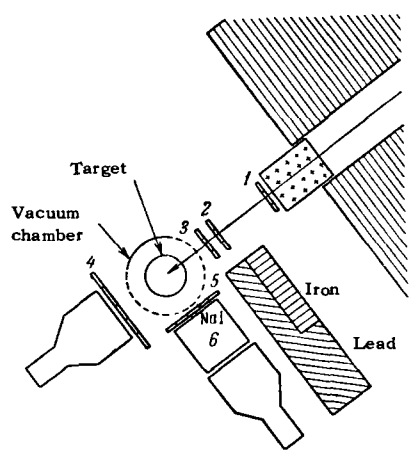

FIG. 3. Overall arrangement of equipment in the experiments of reference 5.

Tripping of the system 1234^-, indicating that a charged particle had stopped within the hydrogen target, was the starting reference of time for the tripping of the system 5^-6 which detects an uncharged particle (γ quantum). (If counter 5 were omitted, this arrangement could be used to measure the time distribution of electrons from $\mu \rightarrow e$ decay. These measurements were done to check the apparatus and to determine the number of μ mesons stopped in the target.)

The time distribution of quanta obtained in this way is shown in Fig. 4. A comparison of Fig. 4a and Fig. 4b, in which we show the time distribution of γ quanta after arrival of the meson in normal hydrogen and in

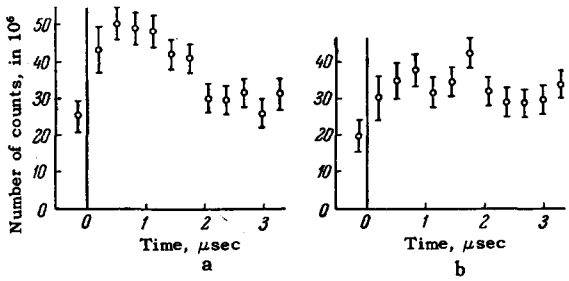

FIG. 4. Time behavior of γ quanta after stopping of μ mesons in a target: a) for hydrogen enriched in deuterium (1.8%); b) for normal hydrogen.

hydrogen enriched to 1.8% deuterium respectively, shows unquestionably that the recorded γ quanta are from the reaction $p + d \rightarrow He_3$. The time behavior of the γ quanta in Fig. 4a (after subtracting the background) reminds one very much of the time behavior of the activity of a daughter-product of radioactive decay, when we have a radioactive chain with two different lifetimes. This analogy is not accidental, since the time distribution of γ quanta from μ catalysis (for deuterium concentrations for which one has reached saturation of the yield of the p + d reaction) is determined by the probability for the process $p + d \rightarrow He_3 + \gamma$ occurring in the mesomolecule $pd\mu$, as well as by the probability of the process of formation of the mesomolecules $pd\mu$ themselves. Experimentally, one detects the occurrence of two processes: "fast" and "slow" probabilities, which are (after subtracting the probability for $\mu \rightarrow e$ decay)

$$0.19 \cdot 10^6 \text{ sec}^{-1} < \lambda_{\text{slow}} < 0.88 \cdot 10^6 \text{ sec}^{-1}, \quad (2.1)$$

$$0.55 \cdot 10^6 \text{ sec}^{-1} < \lambda_{\text{fast}} < 2 \cdot 10^7 \text{ sec}^{-1}. \quad (2.2)$$

It turns out that, on the basis of the very few experimental data, one cannot determine which of the processes refers to the formation of the mesomolecules and which to the nuclear reaction in the mesomolecule. The absolute yield of γ quanta per stopped μ meson is, according to the measurements,[5]

$$Y_\gamma = 0.34 \pm 0.06. \quad (2.3)$$

Comparing this value with the results of the experiments of Alvarez et al. (cf. Table I), the authors of reference 5 determined the coefficient for conversion of the μ meson in the reaction $p + d \rightarrow He_3$ in a mesomolecule to be

$$\alpha = 0.066 \pm 0.014. \quad (2.4)$$

The theory of the catalysis of nuclear reactions by μ mesons will be given in Secs. 3 — 5. In doing this it seems reasonable to consider separately mesomolecular processes in hydrogen (Sec. 4), and then the nuclear reaction in mesomolecules (Sec. 5).

3. KINETICS OF CATALYSIS OF NUCLEAR REACTIONS BY μ MESONS IN HYDROGEN

Let us consider the behavior of μ mesons in liquid hydrogen containing a negligible amount of deuterium. (In completely analogous fashion, one can treat the behavior of μ mesons in any other mixture consisting of a light isotope of hydrogen with some small impurity of a heavy isotope; for example, in hydrogen or deuterium enriched with tritium.)

The catalysis of nuclear reactions by μ mesons in hydrogen is related to the occurrence of a whole sequence of processes. The main such processes are:

1. Formation of $p\mu$ mesonic atoms (which includes the process of slowing down of the μ mesons in hydrogen, their capture into high levels of the mesonic atom, and their transition to the 1s state). All of these processes occur during a time which is much smaller than the lifetime of the μ meson, and we may therefore assume that the formation of the mesonic atoms occurs instantaneously. (According to Wightman,[27] a time of the order of 10^{-9} sec is required for slowing down a μ meson from an energy of 10 Mev to 100 kev in hydro-

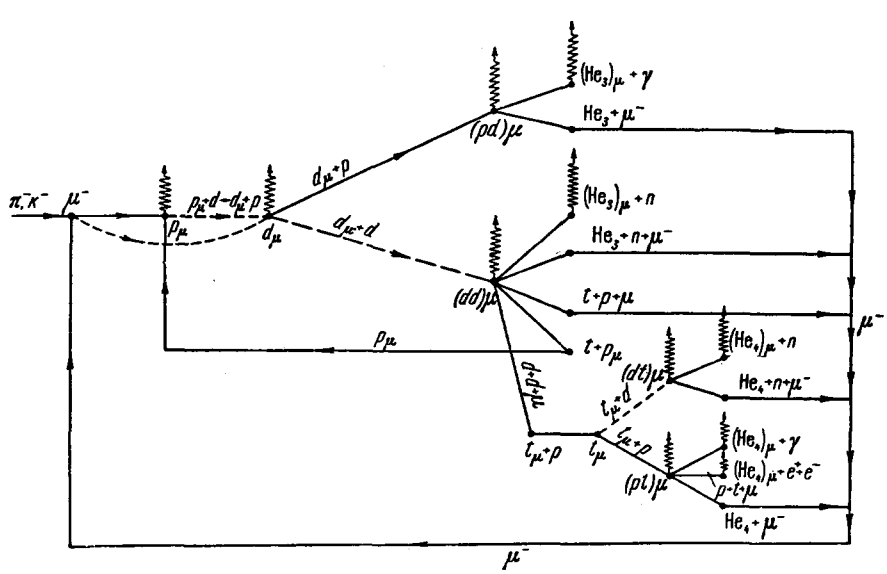

FIG. 5. Diagram of possible nuclear reactions induced by μ mesons in a mixture of hydrogen and deuterium. Processes proportional to the deuterium concentration are shown as dashed lines; the wavy line corresponds to the $\mu^- \rightarrow e^- + \nu + \nu$ decay.

gen at 20°K at normal pressure; for the further reduction of the velocity and capture by the hydrogen molecules, 10^{-12} sec is required, and 10^{-10} sec for the cascade transition to the K orbit.) Since the concentration of the heavy isotope is assumed to be small ($c \ll 1$), the number of mesonic atoms of the heavier isotope formed in the first stage of the process can be neglected. After this, the neutral mesonic atom $p\mu$ can participate in processes 2) and 3).

2) Formation of $pp\mu$ mesomolecules. We point out that a μ meson which has formed a $pp\mu$ mesomolecule is eliminated from participation in the process of catalysis. In fact, the $pp\mu$ system is no longer electrically neutral, so that a close approach to other nuclei and transfer of the μ meson to such nuclei is not possible. A nuclear reaction in the $pp\mu$ mesomolecule itself cannot occur during the lifetime of the meson because of the extremely low probability for the reaction $p + p \rightarrow d + e^+ + \nu$ (cf. Sec. 1); thus, the μ meson dies in the mesomolecule as a result of μ decay.

3) Transfer of the μ meson from the proton to a deuteron. It is obvious that the probability for this process is proportional to the deuterium concentration, but, as will be shown later on, for a concentration $c \approx 1\%$ the probability for transfer is already much greater than the probability for all other competing processes. It should also be pointed out that the process of transfer of the μ meson from a light isotope to a heavy isotope is irreversible because of the difference in the reduced masses of the μ meson. In fact, since the binding energy of the μ meson in a K orbit of a heavy isotope is greater than for a light isotope (cf. Table II), the reverse charge transfer cannot occur in liquid hydrogen on the basis of energy considerations.

4) Formation of $pd\mu$ and $dd\mu$ mesomolecules. The mesonic atom of deuterium obtained after charge transfer quite quickly slows down as a result of elastic collisions with nuclei of the material, and we can subsequently form the mesomolecules $pd\mu$ or $dd\mu$. (This last process, however, is less probable for low deuterium concentrations.)

The formation of $pd\mu$ mesomolecules could, in principle, also occur in collisions of the $p\mu$-mesonic atom with a deuteron. However, in this case a much more probable process is the competing one of transfer of the μ meson from the proton to the deuteron, so that the probability of formation of $pd\mu$ mesomolecules in $p\mu + d$ collisions can be neglected.*

5) Nuclear reactions in mesomolecules. In the hydrogen mesomolecules which have been formed, nuclear reactions can occur through the barrier. In addition to the reaction channels noted in the Introduction, other channels are also possible. For example, the energy of the reaction can be transferred to a conversion μ meson, be used for formation of electron-positron pairs, etc. In all cases, when the μ meson becomes free as a result of the reaction (or is bound in a hydrogen mesonic atom), it can once more participate in the process of catalysis. In those cases where the μ meson is captured in a helium nucleus which has been formed in the reaction, it is lost to the catalysis process, since the $He\mu$ mesonic atom is not electrically neutral and therefore can neither form mesomolecules with other nuclei, nor transfer the μ meson to hydrogen nuclei (cf. Sec. 6).

A competitor to all of these processes is the decay of the μ meson, $\mu \rightarrow e + \nu + \tilde{\nu}$, which occurs with a probability $\lambda_0 = 0.45 \times 10^6$ sec^{-1}.

The experimentally observed dependence of the yield of the nuclear reaction $p + d$ induced by μ mesons on the concentration of deuterium[4] is determined by the mesomolecular processes[2-4] which precede the nuclear reaction.[6,7,8] If we denote the probabilities per unit time of the processes:

$$p\mu + d \rightarrow d\mu + p \quad \text{by} \quad \lambda_e c, \tag{3.1}$$

$$p\mu + p \rightarrow pp\mu \quad \text{by} \quad \lambda_{pp}, \tag{3.2}$$

$$d\mu + p \rightarrow pd\mu \quad \text{by} \quad \lambda_{pd}, \tag{3.3}$$

$$d\mu + d \rightarrow dd\mu \quad \text{by} \quad \lambda_{dd} c, \tag{3.4}$$

(where c is the deuterium concentration), then for $c \ll 1$ the ratio of the number of mesonic atoms $d\mu$ produced as a result of the charge exchange (3.1) to the total number of μ mesons stopped in the chamber is equal to

$$Y_{d\mu} = \frac{\lambda_e c}{\lambda_0 + \lambda_{pp} + \lambda_e c}, \tag{3.5}$$

while the numbers of mesomolecules $pd\mu$ and $dd\mu$ which are formed (per μ meson) are respectively

$$Y_{pd\mu} = \frac{\lambda_e c}{\lambda_0 + \lambda_{pp} + \lambda_e c} \cdot \frac{\lambda_{pd}}{\lambda_0 + \lambda_{pd}}, \tag{3.6}$$

$$Y_{dd\mu} = \frac{\lambda_e c}{\lambda_0 + \lambda_{pp} + \lambda_e c} \cdot \frac{\lambda_{dd} \cdot c}{\lambda_0 + \lambda_{pd}}. \tag{3.7}$$

In deriving (3.6) and (3.7), the assumption is made that $\lambda_e \gg \lambda_{pp}$, λ_{pd}, λ_{dd} and $\lambda_{dd} c \ll \lambda_{pd}$. This assumption is confirmed by the computation (cf. Sec. 4.)

Suppose that in the $pd\mu$ mesomolecule the probabilities per unit time for nuclear reactions $p + d \rightarrow He_3$ with emission of a γ quantum and with transfer of energy to a μ-meson are equal respectively to λ_1 and λ_2. Then the yields of γ quanta (Y_γ) and conversion μ mesons (Y_μ) per μ meson (taking account of the fact that the converted meson can again participate in the catalysis process) are given by the expressions:

$$\frac{Y_\gamma}{1+Y_\mu} = Y_{pd\mu} \frac{\lambda_1}{\lambda_0 + \lambda_1 + \lambda_2}, \qquad \frac{Y_\mu}{1+Y_\mu} = Y_{pd\mu} \frac{\lambda_2}{\lambda_0 + \lambda_1 + \lambda_2}. \tag{3.8}$$

*The fact that the probability of the charge transfer $p\mu + d \rightarrow d\mu + p$ is much greater than the probability for formation of the $pd\mu$ mesomolecule is completely understandable qualitatively, since for the formation of the mesomolecule one must transfer the binding energy of the mesocule to a third particle (proton or a conversion electron), whereas for the charge transfer the difference in binding energy is given to the interacting atoms themselves.

However, since we know from experiment that $Y_\mu \ll 1$, we can write approximately*

$$Y_\gamma = Y_{pd\mu}\frac{\lambda_1}{\lambda_0+\lambda_1+\lambda_2}, \quad Y_\mu = Y_{pd\mu}\frac{\lambda_2}{\lambda_0+\lambda_1+\lambda_2}. \tag{3.9}$$

The dependence of Y_μ on deuterium concentration is given by the factor (3.5). According to (3.5), (3.6), and (3.9), $1/Y_\mu$ depends linearly on $1/c$ for low deuterium concentrations:

$$1/Y_\mu = A + B/c, \tag{3.10}$$

where

$$\frac{1}{A} = \frac{\lambda_{pd}}{\lambda_0+\lambda_{pd}}\cdot\frac{\lambda_2}{\lambda_0+\lambda_1+\lambda_2}, \tag{3.11}$$

$$\frac{B}{A} = \frac{\lambda_0+\lambda_{pp}}{\lambda_e}, \tag{3.12}$$

The experimental results of Alvarez et al. are satisfactorily described by formula (3.10), for example, with A = 39, B = 0.016 (cf. Table Ia),

$$B/A \approx 4\cdot 10^{-4}. \tag{3.13}$$

TABLE Ia. Number of cases of the reaction $p + d + \mu \rightarrow He_3 + \mu$ which should have been observed in the experiments of reference 4a, according to the formula $1/Y_\mu = A + B/C$ (cf. Table I)

Deuterium concentration	Normal hydrogen 1/6000	0.3%	4.3%
$1/Y=A+B/c$ $A=39$, $B=0.016$	19	137	47
Experiment[4a]	18	139	45

The errors lie within the statistical uncertainties.

From (3.10) we see that for

$$c \gg \frac{B}{A} = \frac{\lambda_0+\lambda_{pp}}{\lambda_e} \tag{3.14}$$

the yield of the reaction no longer depends on the concentration of deuterium, and one reaches a saturation of the reaction. Physically, this is completely understandable since, when (3.14) is satisfied, the process of transfer of the μ-meson from the proton to the deuteron becomes much more probable than the competing processes of formation of a $pp\mu$ mesomolecule and $\mu \rightarrow e$ decay. Thus for such concentrations of deuterium practically all the μ mesons are transferred to deuterons ($Y_{d\mu} \approx 1$) and the yield of the reaction is not changed by further increase in the deuterium concentration. It is important to note that, since $\lambda_e \gg (\lambda_0 + \lambda_{pp})$ [cf. (3.13)], condition (3.14) may already be satisfied for very low concentrations of deuterium, $c \ll 1$.

The absolute yield of the reaction $p + d \rightarrow He_3 + \gamma$ under conditions of saturation ($Y_{d\mu} \approx 1$), which was measured in reference 5, enables one to make some estimates of the probability for formation of $pd\mu$ mesomolecules and the probability of nuclear reaction in the mesomolecule. According to (2.3), (3.6), and (3.9),

$$\frac{\lambda_{pd}}{\lambda_0+\lambda_{pd}}\cdot\frac{\lambda_1}{\lambda_0+\lambda_1+\lambda_2} \approx \frac{1}{3}, \tag{3.15}$$

from which

$$\frac{\lambda_{pd}}{\lambda_0+\lambda_{pd}} > \frac{1}{3} \text{ and } \frac{\lambda_1}{\lambda_0+\lambda_1+\lambda_2} > \frac{1}{3},$$

i.e.,

$$\lambda_{pd} > \frac{\lambda_0}{2}, \tag{3.16}$$

$$\lambda_1 > \frac{\lambda_0}{2}. \tag{3.17}$$

The experimental data also make it possible to estimate the probability for formation of $dd\mu$ mesomolecules. As was already pointed out in references 2 and 3, the formation of $dd\mu$ mesomolecules practically always leads to nuclear reaction (cf. Sec. 5). Thus, the yield of the reaction d + d produced by a μ meson is determined entirely by expression (3.7). In reference 4 it is pointed out that, under conditions of saturation of the reaction $p + d \rightarrow He_3$ (i.e., for $Y_{d\mu} \approx 1$), several reactions $d + d \rightarrow t + p$ were observed. If by the word "several" one understands $\nu = 3 - 4$, then using the data of Table I for c = 4.3% and formula (3.7), we have

$$\frac{\lambda_{dd}}{\lambda_0+\lambda_{pd}} = \frac{\nu}{1260\cdot 0.043} = 0.05 - 0.08. \tag{3.18}$$

Considering Eq. (3.16), one can easily conclude that on the basis of the experimental data the probability of formation of $dd\mu$ mesomolecules is much less than the probability for formation of $pp\mu$. (It is possible that the probability for the d + d reaction is actually somewhat greater than the value which follows from the data of reference 4, since under the conditions in the liquid hydrogen chamber used in reference 4 cases of the reaction $d + d \rightarrow He_3 + n$ could not satisfactorily be observed.)

We shall now give a phenomenological description of the time dependence of the yield of γ quanta from the reaction $p + d \rightarrow He_3 + \gamma$, which was studied in reference 5.

We shall, for simplicity, assume that the nuclear

*We should note that the nuclear reaction in the $pd\mu$ mesomolecule is described in somewhat simplified fashion in Eqs. (3.8) and (3.9). Actually (cf. Sec. 5) the nuclear reaction occurs independently from the four different spin states of the $pd\mu$ mesomolecule, with probabilities for emission of a γ quantum (λ_{1s}) and of a conversion μ meson (λ_{2s}) which are characteristic for each of these states.

Thus in equations (3.8) and (3.9) we should make the replacements

$$\frac{\lambda_1}{\lambda_0+\lambda_1+\lambda_2} \rightarrow \sum_S c_S\frac{\lambda_{1s}}{\lambda_0+\lambda_{1s}+\lambda_{2s}}, \quad \frac{\lambda_2}{\lambda_0+\lambda_1+\lambda_2} \rightarrow \sum_S c_S\frac{\lambda_{2s}}{\lambda_0+\lambda_{1s}+\lambda_{2s}}$$

where c_S is the probability for formation of the mesomolecule in a given spin state S.

reaction $p + d \rightarrow He_3 + \gamma$ in the $pd\mu$ mesomolecule is characterized by a definite probability λ_1. (Actually, the nuclear reaction occurs independently from the four different spin states of the mesomolecule, with a probability characteristic for each of these states (cf. footnote on page 600.)

Let n_d, n_{pd}, and n_γ be respectively the number of mesonic deuterium atoms, the number of $pd\mu$ molecules and the number of γ quanta emitted up to the time t. Then under conditions of saturation (3.15), (assuming that the process of formation of $d\mu$ mesonic atoms is infinitely fast), we can write the equations:

$$\frac{dn_d}{dt} = -(\lambda_0 + \lambda_{pd})\, n_d, \tag{3.19}$$

$$\frac{dn_{pd}}{dt} = \lambda_{pd} \cdot n_d - (\lambda_0 + \lambda_1 + \lambda_2)\, n_{pd}, \tag{3.20}$$

$$\frac{dn_\gamma}{dt} = \lambda_1 n_{pd}. \tag{3.21}$$

[In Eqs. (3.19) — (3.21) we neglect for simplicity the formation of $dd\mu$ mesomolecules and the possibility of the reappearance in the cycle of a meson which has given rise to a p + d reaction.] From these equations it is easy to find that the yield of γ quanta per unit time per meson is:

$$\frac{dn_\gamma}{dt} = \frac{\lambda_1 \lambda_{pd}}{(\lambda_y - \lambda_x)} \{\exp(-\lambda_x t) - \exp(-\lambda_y t)\}, \tag{3.22}$$

where

$$\lambda_x = \lambda_0 + \lambda_{pd}, \qquad \lambda_y = \lambda_0 + \lambda_1 + \lambda_2. \tag{3.23}$$

The probabilities for the "fast" and "slow" processes measured in reference 5 [cf. (2.1) and (2.2)] are obtained by subtracting from λ_x and λ_y the probability λ_0 for the $\mu \rightarrow e$ decay. Since, however, formula (3.22) is symmetric in λ_x and λ_y, we cannot decide experimentally, as we have already pointed out, which process is the "fast" one, the formation of the $pd\mu$ mesomolecules or the nuclear reaction in the mesomolecule.

4. MESOMOLECULAR PROCESSES IN HYDROGEN

In this section we shall treat the various mesomolecular processes in hydrogen which are important for the course of nuclear reactions induced by μ mesons. The main processes are the transfer of the μ meson from a light isotope to a heavy isotope and the formation of mesomolecules. Various estimates concerning these processes are contained in references 3, 6 — 8, and 13 — 20; in the following presentation we shall follow reference 17.

a) Adiabatic Approximation

It is known that in computing the usual molecular processes one can apply successfully the so-called adiabatic approximation, whose validity is based on the smallness of the ratio of the electron mass to the mass of a nucleus. Because of the smallness of this ratio it is assumed that the electrons follow the motion of the nuclei adiabatically. Correspondingly, the wave function of a molecule is sought for as a product of a function describing the motion of the nuclei by a function describing the motion of the electrons for fixed separations between the nuclei.

Solving the Schrödinger equation for the electrons with immovably fixed nuclei, we determine the wave function of the electrons. The electron energy found in this way depends on the separation of the nuclei as a parameter, and, together with the Coulomb energy of the nuclei, gives the effective potential energy of interaction between the nuclei in the molecule. Then, solving the Schrödinger equation for the nuclei, one finds the rotational and vibrational levels of the molecule.

Thus, for example, the wave function of the H_2^+ molecular ion is equal in the adiabatic approximation to

$$\Psi(\mathbf{r}, \mathbf{R}_1, \mathbf{R}_2) = \Phi_n(\mathbf{R})\, \chi_n(R, \mathbf{r}), \tag{4.1}$$

where $\mathbf{r}$, $\mathbf{R}_1$, $\mathbf{R}_2$ are respectively the coordinates of the electron and the nuclei, $R = |\mathbf{R}_2 - \mathbf{R}_1|$, and the electronic terms are determined from the Schrödinger equation*

$$\left(-\frac{1}{2}\Delta_{\mathbf{r}} - \frac{1}{r_1} - \frac{1}{r_2} + \frac{1}{R}\right)\chi_n(R, \mathbf{r}) = E_n(R)\, \chi_n(R, \mathbf{r}), \tag{4.2}$$

where $r_1 = |\mathbf{r} - \mathbf{R}_1|$, $r_2 = |\mathbf{r} - \mathbf{R}_2|$, and the corresponding wave function for the motion of the nuclei is given by the equation

$$\left\{-\frac{1}{2M_{12}}\Delta_{\mathbf{R}} + E_{n(R)}\right\}\Phi_n(\mathbf{R}) = E\Phi_n(\mathbf{R}), \tag{4.3}$$

where M_{12} is the reduced mass of the nuclei.

The solutions of (4.2) have been studied by many authors and are well known (cf. the bibliography in reference 29). The solutions of (4.2), which when the nuclei are separated give an electron in the K orbit of one of the nuclei, are, according to the molecular notation, the functions $\Sigma_g(R, \mathbf{r})$ and $\Sigma_u(R, \mathbf{r})$. [Σ indicates that the projection of the angular momentum on the molecular axis is equal to zero; g denotes even (gerade), and u odd (ungerade) states with respect to interchange of the nuclei.]

For $R \rightarrow \infty$,

$$\Sigma_g \rightarrow \frac{1}{\sqrt{2}}(\psi(r_1) + \psi(r_2)), \qquad \Sigma_u \rightarrow \frac{1}{\sqrt{2}}(\psi(r_1) - \psi(r_2)), \tag{4.4}$$

where $\psi(r)$ is the hydrogen atom wave function for the 1s state,

$$\psi(r) = \frac{1}{\sqrt{\pi}} e^{-r}. \tag{4.5}$$

For $R \rightarrow 0$, Σ_g and Σ_u go over into the wave functions of the He^+ ion for the 1s and 2p states respec-

*In atomic units with $\hbar = 1$, $e = 1$, $m_e = 1$.

tively. The potential $E_u(R)$ in the Σ_u state gives a repulsion between the nuclei, whereas for the Σ_g state the potential $E_g(R)$ has a minimum for $R \approx 2$, whose depth (relative to the value at infinity) is approximately 0.1 atomic units. Thus a bound molecule can be formed only in the Σ_g state (we do not treat the formation of molecules for excited electronic states).

If we use the adiabatic approximation as the zeroth approximation for the hydrogen mesomolecules, the equation for the μ meson in the field of the nuclei at rest (in mesonic atomic units, with $m_\mu = 1$, $\hbar = 1$, $e = 1$) coincides exactly with Eq. (4.2) for the H_2^+ molecular ion. Thus in zero'th approximation the mesomolecular potentials are obtained from the potentials of the H_2^+ molecular ion by a simple similarity transformation.

Since m_μ/m_e is equal to 207, the depth of the potential well $E_g(R)$ at its minimum is around 580 ev, while the equilibrium separation between the nuclei is approximately $2a_\mu = 5 \times 10^{-11}$ cm. We should point out that for the vibrational and rotational levels of the mesomolecules there will be no similarity to the corresponding levels of the H_2^+ molecular ion. The sizes of vibrational and rotational quanta, whose ratios to the distance between electronic levels are, for ordinary molecules, of order of magnitude $(m_e/M_{12})^{1/2}$ and (m_e/M_{12}), for the case of mesomolecules are comparable with the depth of the potential well. Thus, hydrogen mesomolecules, unlike H_2^+, have only one or two rotational or vibrational levels.

The treatment given above is essentially qualitative, since for mesonic molecules the ratio of the mass of the μ meson to the mass of the nucleus is not so small as for electrons, and the application of the adiabatic approximation is far less justified than for ordinary molecules. It turns out that for mesomolecules (in particular, those consisting of different isotopes) the adiabatic approximation is not satisfactory for the treatment of various processes (for example, charge exchange, scattering of mesonic atoms, etc). For this reason, in the following subsection, we shall make some improvements on the adiabatic approximation.

b) Improvement of the Adiabatic Approximation for Hydrogen Mesomolecules[13-21]

The need for improving the adiabatic approximation for the case of mesomolecules with different nuclei is primarily related to the size of the corrections which take account of the motion of the nuclei in the isolated mesonic atoms. As $R \to \infty$ the potentials $E_g(R)$ and $E_u(R)$ tend to the value of the energy of a 1s level of the isolated mesonic atom with fixed nucleus, i.e.,

$$E_g(\infty) = E_u(\infty) = -\frac{1}{2}. \tag{4.6}$$

Taking account of the nuclear motion for $R \to \infty$ should lead to the result that the energy of the relative motion will be measured from the 1s level of the isolated mesonic atom, i.e., form $E_0 = -\frac{m_\mu^*}{2}$, where m_μ^* is the reduced mass of the μ-meson for a nucleus with mass M:

$$m_\mu^* = \frac{M}{M+1} \approx 1 - \frac{1}{M} \quad (M \gg 1).$$

Thus the corrections to the adiabatic approximation as $R \to \infty$ are of order of magnitude 1/M of the "large" energy of mesoatomic levels (as compared with molecular levels). For mesomolecules consisting of different nuclei, the difference between the levels of the mesonic atoms is comparable in order of magnitude to the size of the molecular potentials $E_g(R)$ and $E_r(R)$ themselves [we recall that the value of $E_g(R)$ at the minimum is 0.1]. Therefore in this case the approximation (4.1) is unjustified, and to describe the relative motion of the mesonic atom even in the zeroth approximation we need two functions.

In fact, as $R \to \infty$ the wave function of the system must have the form

$$\Psi \approx A(\mathbf{R})\,\psi_a(r_1) + B(\mathbf{R})\,\psi_b(r_2), \tag{4.7}$$

where the function $A(\mathbf{R})$ describes the motion of the second nucleus with respect to the mesonic atom of the first nucleus, while $B(\mathbf{R})$ gives the motion of the first nucleus with respect to the mesonic atom of the second; $\psi_a(r_1)$ and $\psi_b(r_2)$ are the wave functions of the μ meson of the first and second nucleus respectively. For a given total energy E of the system, the kinetic energies of relative motion in the states ψ_a and ψ_b are different and equal respectively to $(E - E_1^0)$ and $(E - E_2^0)$, where E_1^0 and E_2^0 are the energy levels of the first and second mesonic atoms

$$E_1^0 = -\frac{1}{2}\frac{1}{1+1/M_1}, \quad E_2^0 = -\frac{1}{2}\frac{1}{1+1/M_2}. \tag{4.8}$$

Recalling the behavior of the wave function for $R \to \infty$ [cf. (4.7)] and considering (4.4), we must in zeroth approximation try to find a wave function for the system of the form

$$\Psi = G(\mathbf{R})\,\Sigma_g(R, \mathbf{r}) + H(\mathbf{R})\,\Sigma_u(R, \mathbf{r}), \tag{4.9}$$

where the functions $G(\mathbf{R})$ and $H(\mathbf{R})$ are connected (as $R \to \infty$) with the functions $A(\mathbf{R})$ and $B(\mathbf{R})$ by the relations:*

$$A = \frac{1}{\sqrt{2}}\{G + H\}; \quad B = \frac{1}{\sqrt{2}}\{G - H\}. \tag{4.10}$$

As is shown in Appendix I, the choice of Ψ in the form (4.9) enables one to improve the adiabatic approximation to include terms of order 1/M. For the functions $G(\mathbf{R})$ and $H(\mathbf{R})$ [or for $A(\mathbf{R})$ and $B(\mathbf{R})$ defined according to (4.10)], one obtains a system of

*We note that the choice (4.9) automatically assures us that the center of gravity of the system will be at rest, since Ψ depends only on the difference in the coordinates of the particles.

two coupled equations which can be solved numerically.

Let us assume for definiteness that $M_2 > M_1$, i.e., $E_1^0 > E_2^0$. Then, depending on the total energy E, three types of motion are possible:

a) For $E \geq E_1^0 > E_2^0$ the motion is infinite, and the μ meson can be either on the heavy or on the light nucleus. In this region of energy there is a transfer of the μ meson from the light isotope to the heavy isotope.

b) For $E_1^0 > E > E_2^0$ the motion is again infinite, but the μ meson can only be at the heavier isotope. In this energy region, in the collision of a mesonic atom of the heavier isotope with the nucleus of the lighter isotope, there is elastic scattering or formation of a mesomolecule.

c) For $E < E_2^0 < E_1^0$ the motion is finite; the possible values of E form a discrete spectrum corresponding to the levels of the mesomolecule. As shown in Appendix I, for the approximation (4.9) one still preserves the classification of levels as rotational and vibrational.

Before proceeding to the study of processes occurring in these various regions of energy E, we note some special properties of the system consisting of a μ meson and two identical nuclei ($M_1 = M_2 = M$). The Hamiltonian of such a system is symmetric with respect to the two nuclei. Therefore, the wave function of the system cannot simultaneously contain states Σ_g and Σ_u, whose symmetry with respect to interchange of the nuclei is different. In accordance with this, in zero approximation, we may use the adiabatic approximation (4.1) (where χ_n is either Σ_g or Σ_u). The corrections to the adiabatic approximation (4.1) result in a shift of the potential curves $E_g(R)$ and $E_u(R)$, so that the values of the effective potentials for $R \to \infty$ (to an accuracy of the order of $1/M$) tend toward the energy of the isolated mesonic atom E^0 (and not to $-\frac{1}{2}$, as occurs in zeroth approximation), and in addition the shapes of the potential curves for finite values of R are changed* (cf. Fig. 6).

Thus, for identical nuclei, the treatment of molecular processes is simplified essentially since, in place of a system of coupled equations (1A.3), we need to solve two independent equations of the type (4.3). Since, in addition, the effective interaction potentials can be approximated by simple functions, in this case it becomes possible to give an analytic solution (cf. Appendix II).

Now we shall present the main data concerning levels of mesomolecules and probabilities for various mesomolecular processes.

c) Levels of Mesomolecules

In Table II we give the levels of mesomolecules calculated in references 14 and 17 – 19. The levels of mesomolecules with identical nuclei were calculated in reference 14 using an approximation of the effective interaction potentials by Morse functions.

*For ordinary H_2^+ molecules the corrections to the adiabatic approximation are treated in reference 30.

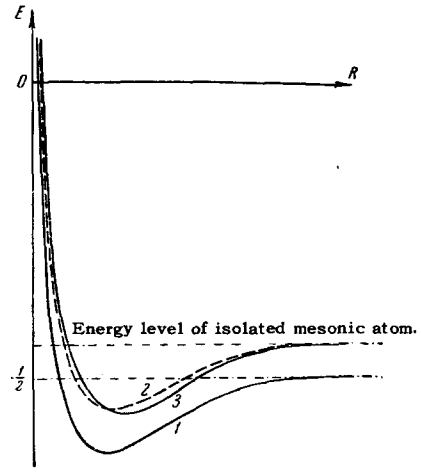

FIG. 6. 1 – Potential energy of interaction of nuclei $E_g(R)$ in the adiabatic approximation; 2 – potential energy taking account of the reduced mass in the isolated mesonic atom; 3 – potential energy including dynamical corrections to the adiabatic approximation.

TABLE II. Energy of 1s-level of mesonic hydrogen atoms (in ev)

For an infinitely heavy nucleus	$p\mu$	$d\mu$	$t\mu$
2800	2531	2666	2714

In Fig. 7 are shown potential curves for mesomolecules with identical nuclei (taking account of dynamical corrections to the adiabatic approximation) and the values of the approximating Morse potentials. The corresponding wave functions can be given analytically (Appendix II). For mesomolecules with different nu-

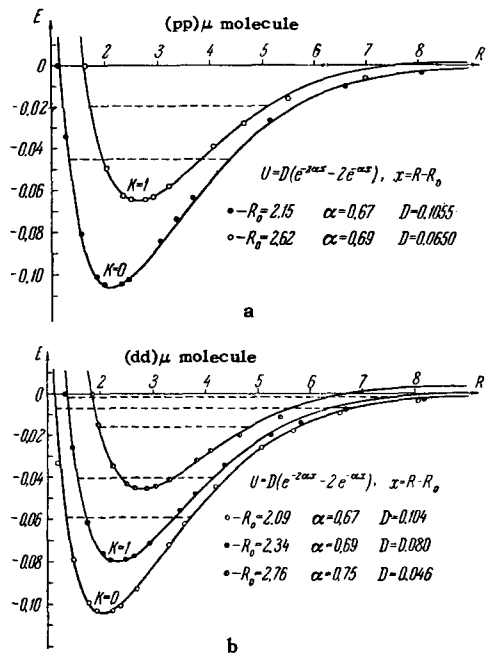

FIG. 7. Potential curves and energy levels for mesomolecules with identical nuclei. The points on the curves represent values calculated for Morse potentials with the parameters noted on the figures. The energy and separation are in meso-atomic units.

clei, the zeroth approximation was taken in reference 14 to be the adiabatic approximation $\Psi = G\Sigma_g$ (without including the Σ_u state). The corrections to the adiabatic approximation (to terms of order $1/M$) have the effect that the curve for the effective potential energy of interaction of the nuclei lies (for $R \to \infty$) midway between the levels of the isolated mesonic atoms (4.8) (cf. Figs. 8a and 8b). The energy levels calculated in this approximation are in fair agreement

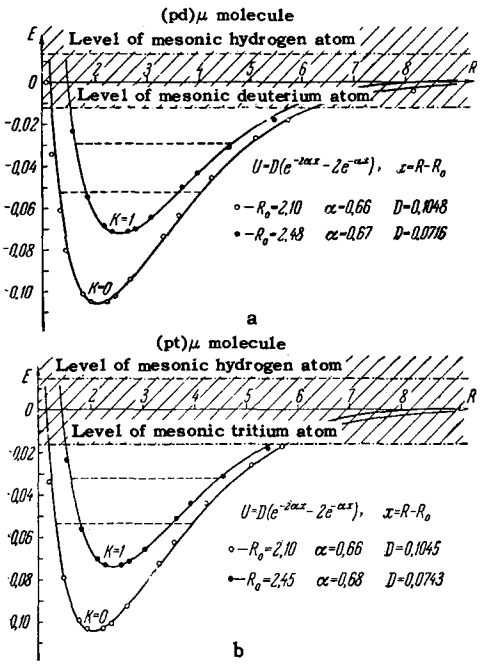

FIG. 8. Potential curves and energy levels for mesomolecules with different nuclei.

with the values obtained in references 17 and 18, where the Σ_u state was included. This fact shows that the Σ_u states are not very important for low-lying levels of the mesomolecule (cf. Fig. 9). Taking account of it in the bound state corresponds in a certain sense to the second approximation of perturbation theory. In reference 17, the levels of mesomolecules with different nuclei were found by a method which is described in Appendix I. In references 18 and 19 the

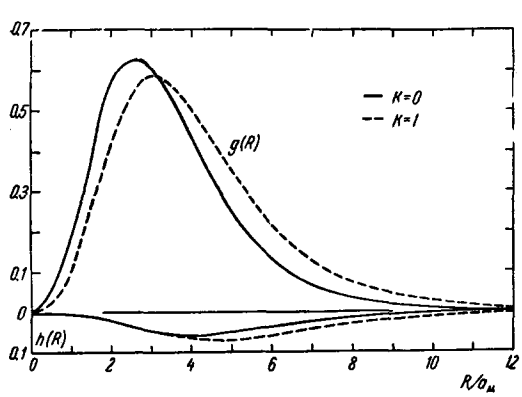

FIG. 9. Radial functions g(R) and h(R) [cf. (4.9) and (1A.10)] for the pdμ mesomolecule, according to reference 8.

levels were determined by a variational method. Wave functions corresponding to levels of mesomolecules with different nuclei were given in references 17a and 19 (cf. Fig. 9).

From Table III we see that there is a considerable discrepancy between the levels calculated in references 17 and 18. This circumstance is related to the fact that the approximations used in references 17 and 18 are valid up to an accuracy of the order of $1/M_{12}$ and differ from one another by terms of the order of $(1/M_{12})^2$. The difference between levels calculated by the two methods lies just at limits of uncertainty which exceed the accuracy of the starting approximations.*

TABLE III. Binding energy of mesomolecules in ev*

Rotational level	$K=0$		$K=1$		$K=2$	$K=3$
Vibrational level	$v=0$	$v=1$	$v=0$	$v=1$	$v=0$	$v=0$
$pp\mu$	252 [a] 241 [b] 248 [d]	—	106 [a] 93 [b]	—	—	—
$pd\mu$	223 [a] 214 [b] 220 [c]	—	95 [a] 90 [b] 90 [c]	—	—	—
$dd\mu$	330 [a] 322 [b] 317 [d]	40 [a] 181 [b]?	226 [a] 223 [b]	7 [a]? —	88 [a] 82 [b]	—
$pt\mu$	211 [a] 213 [c]	—	90 [a] 98 [c]	—	—	—
$dt\mu$	323 [a] 319 [c]	36 [a] 32 [c]	234 [a] 232 [c]	—	103 [a] 102 [c]	—
$tt\mu$	367 [a]	86 [a]	288 [a]	45 [a]	170 [a]	55 [a]

[a] Taken from ref. 14
[b] Taken from ref. 18, 19
[c] Taken from ref. 17
[d] Taken from ref. 28

*For the pdμ, ptμ and dtμ mesomolecules, the binding energy is given relative to the mesonic atom with the heavier nucleus.

It should be mentioned that in the case of the ppμ mesomolecule the depth of the potential well only barely misses enabling the existence of a bound vibrational level $K = 0$, $v = 1$ [cf. Appendix 2, formula (2A.4)]. Therefore for the ppμ mesomolecule there is a virtual level which leads to a resonance in the scattering of pμ mesonic atoms on protons (cf. below).

As already pointed out in reference 3, the presence of a bound or virtual level of a mesomolecule with an energy close to zero leads to a resonance in the collision of particles with low energy, which significantly increases the probability of formation of mesomolecules and the possibility of nuclear reaction in flight. As we see from Table III, among the possible reso-

*Except for the level $K = 0$, $v = 1$ of the ddμ mesomolecule, which is clearly determined incorrectly in reference 18.

nance levels one should mention the level K = 0, v = 1 in the $dd\mu$ and $dt\mu$ molecules, and the level K = 1, v = 1 in the $tt\mu$ mesomolecule.

d) Process of Transfer of a μ Meson from a Light to a Heavy Isotope

The transfer of the μ-meson from a light isotope to a heavy isotope is an inelastic process in which the difference in binding energies of the μ-meson (cf. Table II) is converted to kinetic energy of the relative motion of the charge-exchanging nuclei.

Considering that the velocities of the nuclei before the charge exchange are small, and that the cross section for inelastic processes at low velocities is inversely proportional to the velocity, we can estimate the cross section for transfer of the μ-meson from dimensional considerations:[6]

$$\sigma \approx 4\pi a_\mu^2 v^*/v, \tag{4.11}$$

where v and v* are the relative velocities of the mesonic atom and the free nucleus before and after charge exchange, respectively; a_μ is the Bohr radius of the mesonic atom. Such an estimate gives a result which is correct to order of magnitude. For the calculation of the effective cross section for this transfer one can use a method described in Appendix I. In doing this, since the relative velocity of the nuclei is extremely small before the transfer of the μ meson, one can limit oneself to the consideration of S waves. The effective cross section for the transfer can be represented in the following form:

$$\sigma = 4\pi f a_\mu^2 v^*/v, \tag{4.12}$$

where the coefficient f is determined by numerical computation.

Let N be the number of nuclei per cc (for liquid hydrogen $N = 4 \times 10^{22}$ cm^{-3}) and c be the concentration of heavy isotope, so that the probability for transition per unit time is $w_e = Nc\sigma v$.

Comparing with (3.1) we have

$$\lambda_e = N\sigma v = 4\pi f a_\mu^2 N v^*. \tag{4.13}$$

In Table IV we give values of f and the reaction probabilities (σv) for the possible transfers of a μ meson, as taken from reference 17. For the particular process $p\mu + d \rightarrow d\mu + p$, the effective cross section was also calculated in references 13, 14, and 16 – 21. The results obtained by the various authors for the quantity λ_e are given in Table V. As we see from Table V, the results of the computations of references 17 and 18 are very close to one another, but differ from the results of reference 16, although the calculation in reference 16 was done by a method analogous to that of references 17 and 18. For comparison we give the results of Jackson and Skyrme calculated by using the Born approximation; from Table V one sees that the value of λ_e in Born approximation is too high by a factor of 30 – 40. On the other hand, the results of references 20 and 21 are much too low compared with the results of references 13 – 19. This is related to the fact that in references 20 and 21, in solving the system of coupled equations (cf. Appendix I), a much cruder approximation was used than in references 17 and 19. A strange result is the anomalously small transfer in $d\mu + t \rightarrow t\mu + d$ obtained in reference 17 (Table IV).

TABLE IV. Cross section for transfer of μ meson from light to heavy isotope, $\sigma = 4\pi f a_\mu^2 v^*/v$ (according to the data of reference 17)

	$p\mu + d \rightarrow d\mu + p$	$p\mu + t \rightarrow t\mu + p$	$d\mu + t \rightarrow t\mu + d$
f	2,11	0,87	0,0067
σv cm³/sec	$3.42 \cdot 10^{-13}$	$1.54 \cdot 10^{-13}$	$1.2 \cdot 10^{-15}$

e) Scattering of Mesonic Atoms by Nuclei

The mesonic atom which has been formed as a result of the transfer of the μ meson from a light isotope to a heavy isotope possesses additional energy, as a result of which it traverses a definite path length in the material. According to reference 4 this explains the "gap" observed at the end of the track of the μ-meson (cf. Sec. 2). To calculate the range of the mesonic atom after the charge exchange one must determine the effective cross section for scattering of the mesonic atom of the heavy isotope by nuclei of the light isotope of hydrogen. In reference 17 the effective cross section for scattering of mesonic atoms by nuclei was determined for low values of the kinetic energy, when one can limit oneself to scattering in S-

TABLE V. Probability of transfer per unit time for the process $p\mu + d \rightarrow d\mu + p$ according to the data of various authors[6,7,8,13,14,16,17,18,19,20,21]*

	Crude estimates			13,14	16	17	18,19	20	21
	6	7	8						
λ_e (sec^{-1})	$1.6 \cdot 10^9$	$8 \cdot 10^{11}$	$3.6 \cdot 10^{11}$	10^{10}	$1.8 \cdot 10^{10}$	$1.36 \cdot 10^{10}$	$1.3 \cdot 10^{10}$	$7 \cdot 10^8$	$2 \cdot 10^9$

*Results are given for $N = 4 \times 10^{22}$ cm^{-3}, where N is the number of nuclei per unit volume in liquid hydrogen.

states (the energies of mesonic atoms obtained from charge exchange in practice satisfy this condition). The scattering lengths and effective cross sections calculated in reference 17 are shown in Table VI. Effective cross sections for the scattering of mesonic atoms also calculated in references 18 and 19, in which it was shown that, if one includes Van der Waals forces, a Ramsauer-Townsend effect occurs in the scattering $d\mu + p \rightarrow d\mu + p$ at low energies (i.e., the effective cross section goes to zero at some definite energy). However, it is easy to see that to explain the size of the "gap" it is not at all necessary to make use of the anomalously small value of the effective scattering cross section which is pointed out in references 18 and 19. In fact, in an elastic collision the incident particle transfers an energy E' to the particle at rest which is equal to[31]

$$E' = \frac{4M_1M_2}{(M_1+M_2)^2} E \sin^2 \frac{\chi}{2},$$

where χ is the scattering angle in the center of mass system, E is the energy of the incident particle in the laboratory system. Considering that the scattering occurs in S states, one can assert that the average fraction of the energy transferred per collision is

$$k = \frac{2M_1M_2}{(M_1+M_2)^2}. \tag{4.14}$$

If one considers collision of the $d\mu$ mesonic atom with protons and deuterons, the energy loss per unit length is

$$-\frac{dE}{dx} = N_p k_p \sigma_{dp} + N_d k_d \sigma_{dd}, \tag{4.15}$$

where N_p and N_d are the numbers of protons and deuterons per unit volume, $\sigma_{dp}(E)$ and $\sigma_{dd}(E)$ are the effective scattering cross sections for $d\mu + p \rightarrow d\mu + p$ and $d\mu + d \rightarrow d\mu + d$, while k_p and k_d are the fractional energies transferred in such collisions. The length of path of the mesonic atom in slowing down from the energy E_{max} to thermal energy E_{min} is

$$L \approx (N_p k_p \sigma_{dp}(0) + N_d k_d \sigma_{dd}(0))^{-1} \ln \frac{E_{max}}{E_{min}}. \tag{4.16}$$

In normal hydrogen and in hydrogen enriched in deuterium to 0.3%, the scattering of $d\mu$ by deuterons plays practically no part. Assuming that $E_{min} = 3 \times 10^{-3}$ ev (20°K) and $E_{max} = (1/3) \times 135$ ev = 45 ev, we find according to (4.16) and Table VI, $L \approx 0.2$ mm. This value apparently is smaller than the size of the "gap" observed in experiment. However, it is clear that to obtain the experimental value ($L \approx 1$ mm) one does not at all need an anomalously small value of σ_{dp}. In particular, the "uncorrected" value obtained in reference 19 for the scattering length for $d\mu + p \rightarrow d\mu + p$, equal to $0.8\,a_\mu$, gives, according to (4.16), a value $L = 1$ mm, which agrees very well with experiment.

The fact that at a deuterium concentration of 4.3% the "gap" is not observed (cf. Sec. 2), may be associated with the increase in importance of the scattering $d\mu + d \rightarrow d\mu + d$, whose effective cross section is considerably greater than that for $d\mu + p \rightarrow d\mu + p$. According to references 18 and 32 (cf. Appendix 2), the cross section $\sigma_{dd}(0) = 3.3 \times 10^{-19}$ cm^2, and if we take for $\sigma_{dp}(0)$ the "uncorrected" value $\sigma_{dp}(0) = 5.3 \times 10^{-21}$ cm^2, then according (4.18) the range of the $d\mu$ atom is 0.2 mm, so that the gap is considerably reduced. Here one should also take account of the fact that in the $d\mu + p \rightarrow d\mu + p$ scattering the maximum deflection of the $d\mu$ atom in the laboratory system is 30°, so that one may in the rough approximation of (4.16) assume that the $d\mu$ atom moves along a straight line; whereas for the $d\mu + d \rightarrow d\mu + d$ scattering the maximum deflection of the $d\mu$ atom is 90°, which should lead to a reduction of the "gap." It is possible that for more accurate estimates one should also include the fact that the scattering of the mesonic atoms occurs not on free nuclei, but on nuclei in molecules. The effective cross sections for scattering of the mesonic atoms in this case could be obtained by the pseudopotential method proposed by Fermi for treating the scattering of neutrons by molecules (cf., for example, reference 26). As pointed out in reference 32, it is possible that a "gap" will also be observed in pure hydrogen without deuterium impurity, since the effective cross section for the scattering process $p\mu + p \rightarrow p\mu + p$ at energies less than the energy of the hyperfine structure splitting in the $p\mu$ mesonic atom (0.18 ev) is anomalously small.

TABLE VI. Effective cross sections for scattering of mesonic atoms by hydrogen nuclei, omitting Van der Waals forces[17]

	$d\mu + p \rightarrow d\mu + p$	$t\mu + p \rightarrow t\mu + p$	$t\mu + d \rightarrow t\mu + d$
Scattering length in units $\hbar^2/m_\mu e^2$ . . .	2.03; (0.8)*)	2.66	6.7
Cross section in 10^{-20}cm^2	3.4; (0.53)*)	5.84	3.7

*The scattering length $0.8\,a_\mu$ was given in reference 19 as an "uncorrected" value (omitting Van der Waals forces).

f) Formation of Mesomolecules

In the collision of free mesonic atoms with nuclei of hydrogen molecules, formation of mesomolecules is possible. In such a process the binding energy of the mesonic molecule can, in general, be given off either as radiation or to the electron of the hydrogen molecule, or finally to a neighboring nucleus in the molecule. The last of these mechanisms might play an important role in the formation of mesonic molecules in excited states with a binding energy close to the dissociation energy of the hydrogen molecule. Since, however, there are no such levels in mesomolecules (cf. Table III), this mechanism need not be considered.

Since the dimensions of mesonic molecules are considerably smaller than the dimensions of electronic orbits, the process of transfer of energy to an electron during the formation of a mesonic molecule can be treated in exactly the same way as the conversion of an electron in nuclear transitions. The energy transferred to the electron in the formation of a mesomolecule lies in the range of tens to hundred of ev; for such energies the conversion coefficients are very large. Therefore, as was already pointed out in reference 3, the formation of mesonic molecules by conversion on an electron is much more probable than the radiative transition; in the case of so-called 0—0 transitions, it is the only means possible.

Since the collisions of mesonic atoms with nuclei leading to the formation of mesonic molecules occur for very small velocities (largely thermal velocities), we may assume that the mesonic molecules are formed from S waves of the continuous spectrum. In the final bound state there are rotational levels of the mesonic molecules with $K = 0$ and 1, while for the mesonic molecules $dd\mu$, $dt\mu$, and $tt\mu$ there are also levels with $K = 2$ and $K = 3$ (the last only for $tt\mu$). Thus, electric dipole transitions $E1\,(0 \rightarrow 1)$ are possible to rotational levels with $K = 1$, as well as electric monopole transitions $E0\,(0 \rightarrow 0)$ to the $K = 0$ level and, for the mesonic molecules $dd\mu$, $dt\mu$, and $tt\mu$, we can have electric quadrupole transitions $E2\,(0 \rightarrow 2)$ to the $K = 2$ level.

Let us first consider the E1 transitions. The dipole moment of the system consisting of two nuclei of hydrogen isotopes with masses M_1 and M_2 and a μ meson with respect to the center of gravity of the system (it is important to indicate the reference point, since the total charge of the system is not zero) is equal to

$$\mathbf{d} = -\frac{e}{2}\left\{\frac{M_2 - M_1}{M_2 + M_1}\mathbf{R} + (\mathbf{r}_1 + \mathbf{r}_2)\right\}, \qquad (4.17)$$

where $\mathbf{R} = \mathbf{R}_2 - \mathbf{R}_1$, $\mathbf{r}_1 = \mathbf{r} - \mathbf{R}_1$, $\mathbf{r}_2 = \mathbf{r} - \mathbf{R}_2$, $\mathbf{r}$, $\mathbf{R}_1$, and $\mathbf{R}_2$ are the coordinates of the μ meson and the nuclei. The first term in the dipole moment (4.17) is related to the asymmetry in the distribution of the nuclear charges with respect to the center of inertia, in the case where the masses of the nuclei are different. The second term in (4.17) is related to the distribution of charge of the μ meson with respect to the center of the molecule and, in the case where $M_1 = M_2$, completely determines the dipole moment of the system.

The probability for formation of a mesomolecule by a conversion transition E1 to a rotational level $K = 1$ can be calculated similarly to the problem of conversion for a nuclear transition. If, for the electron in the initial and final state, we choose exact hydrogenic Coulomb functions, omitting the fact that the mesonic atom is combined with a nucleus which is bound in an ordinary hydrogen molecule, we find for the probability of formation of the mesomolecule the expression[15]

$$W_{E1} = \frac{16}{3}(Na_e^3)\left(\frac{a_\mu}{a_e}\right)^5 \eta\zeta^2\left(\sum_{M_K} |\langle \mathbf{d}\rangle|^2\right)\frac{e^2}{a_e h}, \qquad (4.18)$$

where

$$\eta = \frac{e^2}{\hbar v_e}, \quad \zeta = \left[\frac{2\pi\eta}{(1+\eta^2)(1-e^{-2\pi\eta})}\right]^{1/2} \exp(-2\eta \cot^{-1}\eta); \qquad (4.19)$$

a_μ and a_e are the Bohr radii of the μ meson and the electron, v_e is the velocity of the conversion electron, N is the number of nuclei per unit volume with which the mesonic atom forms a mesomolecule. The matrix element of the dipole moment in (4.18) is taken between wave functions of the mesomolecule, corresponding to transition from an S wave of the continuous spectrum to a bound state with $K = 1$, and is expressed in mesoatomic units ($\hbar = 1$; $m_\mu = 1$; $e = 1$). The summation in (4.18) is taken over all possible projections of the orbital angular momentum in the final state, $M_K = 0$ and ± 1.

It is easily seen that the first term in the dipole moment (4.17) has non-zero matrix elements only between states with mesonic functions with the same parity $\Sigma_g \rightarrow \Sigma_g$ and $\Sigma_u \rightarrow \Sigma_u$, whereas the second term in (4.17) gives transitions only between meson functions of opposite parity $\Sigma_g \rightleftarrows \Sigma_u$.

For the formation of a mesomolecule with identical nuclei, the dipole moment of the system differs from zero only because of the second term in (4.17); thus, in this case only $\Sigma_u \rightarrow \Sigma_g$ transitions are possible, i.e., from repulsive states in the continuous spectrum. The state Σ_u is antisymmetric with respect to interchange of the nuclei, so that in the S wave of the relative motion of the nuclei there corresponds to it a state with odd spin of the two nuclei. The probability of formation of the mesomolecule (4.18) must thus, in the case of identical nuclei, be multiplied by the statistical weight of the state Σ_u, which is equal to $3/4$ for $pp\mu$ and $tt\mu$ and $1/3$ for $dd\mu$.

The wave functions describing the relative motion of identical nuclei can be determined analytically using an approximation for the molecular potentials (cf. Appendix 2).

For mesonic molecules consisting of different atoms the principal interest is in the calculation of the probability of formation of mesomolecules for the case where a collision occurs between a slow mesonic atom of the heavier isotope with the lighter nucleus; for example, $d\mu + H \rightarrow pd\mu + e$. For the collision $p\mu + d$, as we have already pointed out above, the much more probable process is the charge exchange $p\mu + d \rightarrow d\mu + p$. The probabilities of formation of mesomolecules by electric dipole E1 transitions, calculated in references 15, 17, 18, and 19, are given in Table VII.

TABLE VII. Probabilities of formation of mesomolecules of hydrogen, in units of 10^6 sec^{-1} ($N = 4 \times 10^{22}$ cm^{-3})

Type of transition	Sources	$pp\mu$	$pd\mu$	$dd\mu$	$pt\mu$	$dt\mu$	$tt\mu$
$E1$	As per data of reference 17 including Z_{eff}	2.5	1.2	0.01	0.4	0.002	0.65
	As per data of reference 19*	3.7	2.9	0.034			
$E0$	Estimate as per reference 3			~0.03		~0.03	

*The probabilities of formation of the mesonic molecules $pp\mu$ and $dd\mu$ given in references 18 and 19 are too high by a factor of two (cf. the text). In the table we give corrected values recomputed for $N = 4 \times 10^{22}$ cm^{-3}, as contrasted with reference 19, where the value $N = 3.5 \times 10^{22}$ cm^{-3} was used.

One point of interest is the fact that the probability of formation of mesomolecules consisting of heavier isotopes is, as a rule, considerably less than the probability of formation of mesomolecules of the lighter isotopes. This is related to the fact that the main contribution to the matrix element $\langle d \rangle$ comes from the classically forbidden region of motion of the nuclei. This is especially clear for mesomolecules consisting of identical nuclei. In fact, for large separations R between the nuclei the wave function for a bound state falls off as $\exp(-R\sqrt{2M_{12}E})$, where M_{12} is the reduced mass of the nuclei and E is the binding energy of the mesomolecule, whereas at small separations, the wave function of the continuous spectrum corresponding to the repulsive state Σ_u falls off exponentially within the barrier $E_u(R)$. Consequently, the main contribution to the dipole moment comes from some "intermediate" values of R, for which the product of the wave functions is a maximum and which correspond to the classically forbidden region of the motion. Since the falling off of the wave functions within the barrier depends essentially on the reduced mass of the particles, it is clear that the size of the matrix element of the dipole moment $\langle d \rangle$ should generally be considerably decreased when we go to heavier isotopes. An exception, which as one sees from Table VII is observed for the $tt\mu$ mesomolecule, is explained by the existence for this molecule of a rotation-vibration level K = 1, v = 1 with a quite low binding energy (cf. Table III).

The values for the probability of formation of a mesomolecule given in Table VII are correct only in order of magnitude. The main source of error is probably the use of electronic wave functions for an isolated hydrogen atom in the calculation of the conversion transition, whereas actually the collision of the mesonic atom occurs with a hydrogen molecule.

A correct treatment of the problem of conversion on electrons of a hydrogen molecule is quite complicated. Qualitative estimates show, however, that the probability of transition should be greater than for the hydrogen atom. In fact, if for the electron in the hydrogen molecule we use the variational wave function, then, according to Wang,[33] the effective charge $Z_{eff} = 1.193$. The probability of formation of the mesomolecule (4.18) should then be multiplied by Z_{eff}^3 which amounts to 1.7. An increase in the probability (4.18) occurs also because of the existence of heteropolar states of the molecule, as well as a screening of the Coulomb field of the nucleus by the second electron in the molecule. We note that, if there were complete screening of the nuclear Coulomb field and the wave function for the emerging electron were chosen to be a plane wave, we would have $\zeta = 1$ in (4.18); this would result in an increase in the probability of formation of the mesomolecule by approximately a factor of three.

Another possible source of error is the method used for treating the mesomolecules. This method is itself correct only to an accuracy of $(1/M_{12})$. For example, the methods used in references 17 and 18 differ only in terms of the order of $(1/M_{12})^2$; however, according to reference 18 one should insert in (4.18) not the Bohr radius of the μ meson a_μ, but some length a_μ^* corresponding to the reduced mass of the meson with respect to the nuclei M_1 and M_2: $a_\mu^* = a_\mu\left(1 + \frac{1}{M_1 + M_2}\right)$. Substitution of this value of a_μ^* in place of a_μ in formula (4.18) leads in the case of the $pp\mu$ mesomolecule to an increase of 30% in the probability of formation of the mesomolecule. Clearly, taking account of such terms, although it leads to a considerable change in the transition probability, is essentially beyond the limits of accuracy of the method which is used. The difference between the results of reference 17 and references 18 and 19 for the $pp\mu$ and $dd\mu$ mesomolecules is partially explained by the fact that in references 18 and 19 corrections are introduced for several of the effects listed above, and, in particular, the value Z_{eff} is used for the wave functions of the electron in the molecule. It should, however, be stated that the probabilities of formation of $pp\mu$ and $dd\mu$ are twice too large in references 18 and 19, since

the wave functions of the continuous spectrum are normalized not to unit probability density in the plane wave, but to $\sqrt{2}$. The values given in Table VII take account of this fact.*

Thus it is possible that actually the probabilities of formation of mesomolecules are 3 — 3.5 times greater than given in reference 17.

In this connection, it is very strange that there is a large difference in the values of the probability of formation of $dd\mu$ mesomolecules by E1 transitions, as obtained in reference 17 and references 18 and 19. Even if we accept the remark made above concerning the fact that one should increase the results of reference 17 by a factor of 3 — 3.5, the probability of formation of $dd\mu$ amounts to 2×10^4 sec^{-1}, which is still three times smaller than the value in references 18 and 19.

Let us now consider the formation of mesomolecules by an electric monopole transition E0. Such a transition can play an important role for the formation of mesomolecules in the vibrational-rotational state K = 0. In fact, for thermal velocities of the mesonic atoms, when the relative motion of the mesonic atom and the nucleus in a collision is described by an S wave, the formation of the mesomolecule in the K = 0 state is a 0—0 transition. For a 0—0 transition, emission of a single photon is not allowed, and the only possibility in this case is the formation of the mesomolecule by an electric monopole transition E0 with conversion on an electron of the hydrogen molecule.

The probability of formation of a mesomolecule by an E0 transition can be calculated completely analogously to the probability of conversion for a nuclear transition, and is determined by the monopole moment Q_0 of the system:

$$Q_0 = \Sigma e_i r_i^2, \tag{4.20}$$

where e_i are the charges and r_i the distances of the particles from the center of inertia of the system. In the case of a mesomolecule the sum (4.20) is taken over both nuclei and the μ meson.

If for the electron wave functions we take hydrogenic Coulomb functions, the probability of formation of mesomolecules will be equal to

$$W_{E0} = \frac{8\pi}{9}(Na_e^3)\left(\frac{a_\mu}{a_e}\right)^7 |\langle Q_0\rangle|^2 \frac{1}{(1-e^{-2\pi\eta})}\frac{e^2}{\hbar a_e}, \tag{4.21}$$

where a_μ and a_e are the Bohr radii of the μ-meson and the electron, $\eta = \frac{e^2}{\hbar v_e}$, v_e is the velocity of the conversion electron, N is the number of nuclei per unit volume, and $\langle Q_0\rangle$ is the matrix element of the monopole moment taken between the wave functions of the mesomolecule and expressed in mesoatomic units.

A comparison of (4.21) and (4.18) shows that the probability of formation of mesomolecules by an E0-transition is generally considerably less than for an E1 transition, since W_{E0} contains the ratio (a_μ/a_e) to a higher power than does W_{E1}. However, as was already pointed out in reference 3, the E0-transition probability may turn out to be quite large if the mesomolecule has a vibrational excited state (K = 0, v = 1) with low binding energy. The collision of a slow mesonic atom with the nucleus then occurs under conditions very close to resonance. In this case, the matrix element $\langle Q_0\rangle$ becomes very large since, first of all, the wave function of the bound state falls off slowly with distance between the nuclei, and, secondly, within the range of the interaction the amplitude of the wave function of the continuous spectrum increases (the latter also occurs for the case of a virtual level).

It is important that there are resonance levels K = 0, v = 1 in the $dd\mu$ and $dt\mu$ mesomolecules, for which the probability of formation by E1 transition is very small (cf. Table VII). The estimate made in reference 3 shows that the probability of formation of $dd\mu$ mesomolecules by E0 transition is 3×10^4 sec^{-1}. Thus, the formation of $dt\mu$ and $dd\mu$ mesomolecules must apparently occur to a large extent through 0—0 transitions.

It should be mentioned that the formation of mesomolecules in an E0 transition was also treated in a paper by Jackson.[8] However, he obtained a result which was too high by several orders of magnitude compared to that of reference 3. The error made by Jackson was that the wave functions which he chose for the initial (dissociated) and final (bound) states of the mesomolecule were not orthogonal. Thus, in computing the matrix element of the transition, a non-zero result was already obtained in the zeroth term of the expansion of the integrand (cf. reference 8, formula D.5) in powers of r_i/a_e and $k_e r_i$, whereas actually a non-zero result is obtained only in the second term of the expansion and reduces to the matrix element of the monopole moment $\langle Q_0\rangle$. (The zeroth term in the expansion goes to zero because of the orthogonality of the wave functions of the mesomolecule, and the first term is zero because both of the wave functions correspond to zero angular momentum.)

g) Transitions between Levels of Mesomolecules

The mechanisms for formation of mesomolecules which we have considered result in the mesomolecules being produced in excited states: either in a rotational state with angular momentum I = 1 in the case of E1 transitions, or in an excited vibrational state (with angular momentum K = 0) in the case of E0 transitions. Since the probability for a nuclear reaction in the mesomolecule depends essentially on the orbital

*In reference 18 the probability of formation of mesomolecules is given for a nuclear density $N = 3.5 \times 10^{22}$ cm^{-3}; in Table VII they have been recomputed for $N = 4 \times 10^{22}$ cm^{-3}.

angular momentum of the nuclei, we must make an estimate of the probability of transition of mesomolecules to the ground state.

For mesomolecules consisting of different nuclei, the transition from the $K = 1$ rotational level to the ground state can occur by an electric dipole transition E1. According to the well-known formula for electric dipole radiation, the probability for a radiative transition is equal to

$$W_r = \frac{4\,|\langle \mathbf{d}\rangle|^2 E^3}{3\hbar^4 c^3},$$

where E is the difference in energy between the first rotational state and the ground state, and $\langle \mathbf{d}\rangle$ is the matrix element of the dipole moment. Neglecting the admixture of Σ_u state, we have according to (4.17):

$$|\langle \mathbf{d}\rangle|^2 = \frac{e^2}{4}\left(\frac{M_2-M_1}{M_2+M_1}\right)|\langle \mathbf{R}\rangle|^2 \approx \frac{e^2}{4}\left(\frac{M_2-M_1}{M_2+M_1}\right)^2 a_\mu^2,$$

since $|\langle \mathbf{R}\rangle| \approx a_\mu$ in order of magnitude. Using values for E taken from Table III, we find for the probability of a radiative transition the value $W_r = 10^5 - 10^6\ \mathrm{sec}^{-1}$. Actually, however, since a mesomolecule is a system with charge $+e$, it should, during a time of the order of 10^{-11} sec, capture an electron as a result of charge exchange on hydrogen atoms and form a hydrogen-like atom.* Taking this fact into account, we may note that for mesomolecules consisting of different nuclei, the much more probable transition from the rotational $K = 1$ state to the ground state is via E1 conversion on an atomic electron. The difference in energy of the rotational and ground states is of order of magnitude 10^2 ev (cf. Table III). For such energies the coefficient for dipole conversion is very large $(10^6 - 10^7)$ and consequently the probability of a transition with conversion on an electron is $W_e = 10^{11} - 10^{12}\ \mathrm{sec}^{-1}$. Thus, mesomolecules consisting of different nuclei, which are formed in the rotational state, practically immediately make a transition to the ground state.

As for mesomolecules consisting of identical nuclei, their transition from the first rotational state to the ground state must be accompanied by a change in the total spin of the nuclei, since in the rotational state with angular momentum $K = 1$ the system of two identical particles must have odd spin, while in the state with angular momentum $K = 0$, the spin must be even. This transition can occur via the dipole moment Q'_{10} associated with a change in magnetic moment (cf. Blatt and Weisskopf, "Theoretical Nuclear Physics"). The probability of such a transition is apparently very, very small since, compared with the probability of the usual dipole transition, it contains a factor of order $\left(\frac{E}{M_n c^2}\right) \approx 10^{-14}$, where M_n is the mass of the nucleon, and E is the transition energy.

Thus, for mesomolecules consisting of identical nuclei, the rotational state with $K = 1$ is metastable, analogous to the situation for orthohydrogen, and the probability of transition to the ground state during the lifetime of the meson is extremely small. This fact is very important for the process of capture of a μ meson by a proton in the $pp\mu$ mesomolecule (cf. references 15 and 22). For $dd\mu$ mesomolecules which are in the metastable $K = 1$ state, the probability of a sub-barrier $d + d$ nuclear reaction will, because of the presence of the centrifugal barrier, be smaller than for the mesomolecules in the $K = 0$ state; but the conclusion drawn in references 3, 6 and 8 that the formation of the $dd\mu$ mesomolecule practically always leads to catalysis of the $d + d$ nuclear reaction is still correct (cf. Sec. V).

As already pointed out, in the formation of the $dd\mu$ and $dt\mu$ mesomolecules the E0-transition in which the mesomolecules are produced in an excited vibrational state $(K = 0,\ v = 1)$ may be important. From the estimates given above we may conclude that the mesomolecule $dt\mu$ during a time of the order of 10^{-11} sec can make an electric dipole transition, with conversion on an atomic electron, to the state $K = 1$ and then to the ground state. As for the $dd\mu$ mesomolecules, the most probable thing for them is the direct E0 transition to the ground state with conversion on an electron. The probability of such a transition, which is equal in order of magnitude to $10^8\ \mathrm{sec}^{-1}$, is much smaller than the probability of catalysis of the $d + d$ nuclear reaction (cf. Sec. V). Thus in the $dd\mu$ mesomolecules which are formed in the state $(K = 0,\ v = 1)$, the nuclear reaction should already occur in the excited level. To a certain extent this will also be true for the $dt\mu$ mesomolecules.

h) Comparison with Experimental Data

Let us compare the results of the calculation of the probabilities of various mesomolecular processes with the experimental data.

1) Dependence of yield of the $p + d \rightarrow He_3$ reaction on deuterium concentration. According to (3.10) and (3.12) the dependence of the yield of the reaction $p + d \rightarrow He_3$ on the concentration of deuterium is determined by the quantity $\frac{B}{A} = \frac{\lambda_0 + \lambda_{pp}}{\lambda_e}$. Using the values $\lambda_e \approx 10^{10}\ \mathrm{sec}^{-1}$ (Table V) and $\lambda_{pp} \approx 3 \times 10^6\ \mathrm{sec}^{-1}$ (Table VII), we obtain $B/A = 3.5 \times 10^{-4}$, which generally speaking is in fair agreement with the experimental value of B/A [cf. (3.13)], especially if we keep in mind the comments made above concerning the approximate character of the values given in Table VII.

2) Absolute yield of the $p + d \rightarrow He_3$ reaction. Both values of λ_{pd} given in Table VII satisfy the experimen-

*The probability of charge exchange of a mesomolecule on hydrogen atoms can be estimated from the formula $W = N\sigma v$, where $N = 4 \times 10^{22}\ \mathrm{cm}^{-3}$ is the number of nuclei per cc, $v \approx 4 \times 10^4$ cm/sec is the velocity corresponding to the thermal energy of $pd\mu$ mesomolecules at $T = 20°K$, and σ is the charge-exchange cross section which, for a rough estimate, can be taken equal to $10^{-16}\ \mathrm{cm}^2$.

tal inequality (3.16). If we assume that $\lambda_{pd} \approx 10^6$ sec^{-1}, we obtain for the quantity $\lambda_{pd}/(\lambda_0 + \lambda_{pd})$ which determines the number of mesomolecules $pd\mu$ formed at saturation by a single μ meson, the value

$$\frac{\lambda_{pd}}{\lambda_0+\lambda_{pd}} \approx 70\%.$$

Comparing this number with the quantity $Y_\gamma = 0.34$ [cf. (2.3)], we may conclude that the formation of mesomolecules is a much "faster" process [cf. (2.1) and (3.15)], than the nuclear reaction in the mesomolecule, even though the probabilities of both processes are close to one another in order of magnitude.

3) Yield of the d + d reaction. If we include E0 transitions, the probability of formation of $dd\mu$ mesomolecules can amount to $\lambda_{dd} \approx (0.04 - 0.06) \times 10^6$ sec^{-1}. Then the quantity $\lambda_{dd}/(\lambda_0 + \lambda_{pd})$ (for $\lambda_{pd} \approx 10^6$ sec^{-1}) takes the value

$$\frac{\lambda_{dd}}{\lambda_0+\lambda_{pd}} \approx (0.03 - 0.04),$$

which is close to the experimental value (3.18).

Thus the results of the calculations of mesomolecular processes agree quite well with the experimental data, at least in order of magnitude.

5. NUCLEAR REACTIONS IN MESOMOLECULES

a) Reaction Constants

Proceeding now to the consideration of nuclear reactions occurring between the nuclei of a mesomolecule, it is necessary to point out that under the conditions in a mesomolecule, in addition to the reaction channels (II – VI) listed in the Introduction, other channels may play a significant role, in particular, reactions with conversion of the μ meson and reactions with production of electron-positron pairs. To determine the probabilities for the individual reaction channels, just as to establish the total probability of the reaction, we must know the matrix elements of the transitions for a system containing several nucleons. The calculation of such matrix elements requires the use of a model of the structure of the nucleus and, in the present state of the theory of nuclear forces, cannot be carried out with any reasonable degree of certainty. Therefore, before we give a qualitative treatment on the basis of nuclear models, it is of interest to estimate the probability for nuclear reactions on the basis of experimentally measured effective cross sections for the reactions (II – VI). Such estimates were already made in reference 3. In our presentation we shall follow reference 8 closely.

At low energies of relative motion of the nuclei, the matrix elements of transitions, taken between wave functions of the compound nucleus, can be assumed to be independent of energy. The effective cross section for the reaction in this case is known to have the form

$$\sigma = C\,|\psi(0)|^2/v, \tag{5.1}$$

where $\psi(0)$ is the value of the wave function describing the relative motion of the nuclei for nuclear separations of the order of the radius of action of the nuclear forces, v is the relative velocity of the nuclei at infinity (the probability density in the incident wave is assumed normalized to unity). If we neglect the dimensions of the region of action of the nuclear forces, then for a reaction between charged particles at sufficiently low energy,

$$|\psi(0)|^2 = 2\pi\eta/(e^{2\pi\eta} - 1) \simeq e^{-2\pi\eta}2\pi\eta, \tag{5.2}$$

where

$$\eta = Z_1Z_2e^2/\hbar v \gg 1,$$

for nuclei of hydrogen $Z_1 = Z_2 = 1$, while the factor $\exp(-2\pi\eta)$ is the penetration coefficient through the Coulomb barrier. If the effective cross section for the reaction is measured in the low energy region and its energy dependence defined by using formulas (5.1) and (5.2), then from the experimentally known effective cross section one can compute the reaction constant C. According to reference 34, the energy dependence of the effective cross section for reaction III at energies below 50 kev follows formula (5.1). Here both channels for the reaction are almost equally probable, and the reaction constant is equal to

$$C_{\mathrm{III}} = 2\cdot 10^{-16}\ \mathrm{cm}^3/\mathrm{sec}. \tag{5.3}$$

In precisely this same way, the cross section for reaction IV for energies below 19 kev follows formula (5.1) with the reaction constant equal to

$$C_{\mathrm{IV}} = 2\cdot 10^{-14}\ \mathrm{cm}^3/\mathrm{sec}. \tag{5.4}$$

(For energies of the order of 80 kev, reaction IV has a resonance corresponding to the virtual state of He_5.)

As for reaction II, its cross section has not been measured at sufficiently low energies. This is related to the fact that the cross section for reaction II is very small, smaller by several orders of magnitude than the cross sections for reactions III and IV. The reaction cross section[35] measured for energies of the order of 1 Mev is not suitable for an estimate of the reaction probability in the $pd\mu$ mesomolecule. In fact, under the conditions of reference 35, one observes an electric dipole transition E1, which is confirmed by the angular distribution and polarization of the γ quanta. Consequently, the reaction $p + d \rightarrow He_3 + \gamma$ at these energies occurred from a P state, whereas the nuclear reaction in the $pd\mu$ mesomolecule, as pointed out in Sec. 4, should mainly occur from an S state. To estimate the order of magnitude of the probability of reaction II from the S state, we can use the known reaction cross section with the mirror nuclei

$$n + d = t + \gamma, \tag{II'}$$

measured for thermal neutrons. The cross section of reaction (II′) for slow neutrons has the form (5.1) with $|\psi(0)|^2 = 1$, since for neutrons naturally there is no Coulomb factor. In the case of thermal neutrons $v = 2.2 \times 10^5$ cm/sec and $\sigma = 0.57 \times 10^{-27}$ cm^2, the constant for reaction II is thus

$$C_{\mathrm{II}} = 1.25 \cdot 10^{-22} \text{ cm}^3/\text{sec}. \tag{5.5}$$

b) Penetration of Nuclei through the Barrier in a Mesomolecule

Knowing the reaction constants, we can estimate the probability of a nuclear reaction in a mesomolecule from the formula

$$w = C\,|G(0)|^2, \tag{5.6}$$

where C is the reaction constant and G(0) is the value at R = 0 of the wave function describing the relative motion of the nuclei in the mesomolecule [cf. (4.9)]. In principle, the value of G(0) can be calculated by exact integration of the system of equations for G(**R**) and H(**R**) (cf. Appendix 1), as was done, for example, in reference 18. However, for the purposes of a rough estimate one can use the method described in reference 8. The admixture of the Σ_u state for the bound state of a mesomolecule with different nuclei is small (in the case of identical nuclei, this admixture is completely absent if we omit spin interactions). Therefore, in place of the system of equations (1A.3) we can approximately treat the single equation for G(**R**) with the potential $V = E_g(R)$, neglecting the terms containing the function H(**R**). The potential $E_g(R)$ in the neighborhood of the minimum R_0 can, in crude approximation, be replaced by an oscillator potential. Then the radial function g(R) = RG(**R**) for the ground state of the oscillator must have the form

$$g = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{-\alpha (R-R_0)^2/2}, \tag{5.7}$$

where, in mesoatomic units,

$$\alpha = 2M_{12}E_0;$$

$M_{12} = M_1M_2/(M_1 + M_2)$ is the reduced mass of the nuclei, E_0 is the energy of the ground state measured from the bottom of the potential well, R_0 is the separation corresponding to the minimum in the potential energy. The wave function in the sub-barrier region can be determined quasiclassically using the well-known relation between the solutions of the wave equation inside and outside the potential barrier, respectively:

$$|Q(R)|^{-1/2} \exp\left(-\int_{R_1}^{R} |Q|\,dR\right) \rightleftarrows 2[Q(R)]^{-1/2} \cos\left(\int_{R_1}^{R} Q\,dR - \frac{\pi}{4}\right), \tag{5.8}$$

where R_1 is the classical turning point, and Q(R) is equal to

$$Q(R) = \sqrt{2M_{12}(E - V(R))}. \tag{5.9}$$

In order to normalize the solution (5.8), we can simply equate the value given by the quasiclassical formula (5.8) at the point of minimum potential energy R_0, to the exact value of the wave function (5.7). Comparison of (5.8) and (5.7) at the point $R = R_0$:

$$Q(R_0) = \sqrt{2M_{12}E_0}, \qquad \cos\left(\int_{R_1}^{R_0} Q\,dR - \frac{\pi}{4}\right) = 1,$$

shows that the solution (5.8) should be multiplied by the quantity

$$\frac{\alpha^{1/2}}{2\pi^{1/4}} = 0.38\alpha^{1/2}. \tag{5.10}$$

It can be shown that such a procedure for normalization of a quasiclassical solution is very rough, since the quasiclassical function for the ground state of an oscillator behaves entirely differently than the exact function (5.7). Because R_0 is so close to the turning point, the classical function increases (and does not decrease) as one moves away from the point R_0. A more rigorous treatment of the problem of normalization of the quasiclassical solution given by Furry[36] shows, however, that for the ground state of the oscillator both sides of (5.8) should be multiplied by the value

$$\frac{\alpha^{1/2}}{(2\pi)^{1/2}} \simeq 0.40\alpha^{1/2}, \tag{5.11}$$

which practically coincides with (5.10).

Thus, in the quasiclassical approximation the radial function under the barrier should have the form

$$g(R) = \left(\frac{\alpha}{2\pi}\right)^{1/2} |Q(R)|^{1/2} \exp\left(-\int_{R_1}^{R} |Q|\,dR\right). \tag{5.12}$$

In using the quasiclassical approximation for the radial function, the relation between the solutions (5.8) is maintained if we simply replace the centrifugal energy $l(l+1)/2M_{12}R^2$ by $(l+\frac{1}{2})^2/2M_{12}R^2$.

Such a replacement assures the correct phase of the quasiclassical function at large distances and also the correct behavior[38] near R = 0.

Since for small values of R the potential barrier $E_g(R)$ has the character of a Coulomb repulsion, it is important to note that when we make the replacement $l(l+1) \to (l+\frac{1}{2})^2$ the quasiclassical approximation can be applied with good accuracy to the repulsive Coulomb field down to $R \to 0$ and not just for $R > \hbar^2/M_{12}e^2$.*

*In fact, the exact Coulomb function (normalized so that at infinity it has the form $\Phi_{kl} \approx \sin\{[kR + \delta(R)]/kR\}$) can be written for $k \ll 1$, in the neighborhood of zero, in the form:[39]

$$R\Phi_{kl} \approx \frac{R^{l+1}e^{-\pi/k}}{\sqrt{k}} \frac{2^l\sqrt{2\pi}}{(2l+1)!}, \tag{5.13}$$

while the quasi-classical function for $R \to 0$ has the form

Thus in the sub-barrier region the radial function can be represented as

$$g(R) = \left(\frac{\alpha}{2\pi}\right)^{1/2} R \exp\left\{-\frac{1}{2}\lambda(R)\right\}, \qquad (5.15)$$

where the exponent of the barrier factor $\lambda(R)$ is

$$\lambda(R) = 2\int_R^{R_1} \left\{\sqrt{2M_{12}(E_g(R) - E_0) + \frac{1}{4R^2}} - \frac{1}{2R}\right\} dR + \ln R_1. \qquad (5.16)$$

The value of λ was determined numerically in the paper of Jackson for the pdμ, ddμ, and dtμ mesomolecules (cf. Table VIII). In this work the radius of the nuclear forces was taken in mesoatomic units to have the values $R_N = 0$, 0.02, and 0.05, corresponding to a sum of nuclear radii 0, 0.5×10^{-13} cm, and 1.2×10^{-12} cm.

TABLE VIII. Exponent of the barrier factor $\lambda(R_N)$[8]

R_N	$p+d$	$d+d$	$d+t$
0	6	8.3	9.5
$5\cdot10^{-13}$ cm	5.6	7.6	8.8
$1.2\cdot10^{-12}$ cm	5.0	6.9	7.9

Going over from mesoatomic to ordinary units, we can write the wave function in the region of action of the nuclear forces in the form*

$$G(0) = \frac{1}{(4\pi a_\mu^3)^{1/2}} \left[\frac{g(R)}{R}\right]_{R=R_N}. \qquad (5.17)$$

Thus, the reaction probability per unit time according to (5.6), (5.15), and (5.17) is equal to

$$w = \frac{C}{4\pi a_\mu^3}\left(\frac{\alpha}{2\pi}\right)\exp(-\lambda) \qquad (5.18)$$

or, using the numerical values of the parameters given in reference 8,

$$w = 0.7\cdot10^{30} C\left(\frac{M_{12}}{M_p}\right)^{1/2}\exp(-\lambda)\ (\text{sec}^{-1}), \qquad (5.19)$$

where M_p is the proton mass.

$$R\Phi_{kl} \approx \frac{R^{l+1}e^{-\pi/k}}{\sqrt{k}} \frac{e^{2l+1}}{2^{l+3/2}(l+1/2)^{2l+3/2}}. \qquad (5.14)$$

The expressions (5.13) and (5.14) coincide asymptotically only for $l \to \infty$. However, the numerical difference between the coefficients in (5.13) and (5.14) is already small for relatively small values of l. Thus, for $l = 0$ the coefficients in (5.13) and (5.14) are equal respectively to $\sqrt{2\pi} \approx 2.51$ and $e \approx 2.72$, while for $l = 1$, they are $\sqrt{2\pi}/3 \approx 0.84$ and $2e^3/27\sqrt{3} \approx 0.86$.

*We note that for the pdμ mesomolecule, according to the estimates made above, $|G(0)|^2 \approx 1.4 \times 10^{27}$, whereas in reference 6 the value $|G(0)|^2 \approx 6 \times 10^{27}$ is given, and in reference 18a, by integrating the Schrödinger equation, the value $|G(0)|^2 \approx 10^{28}$ is obtained, which agrees with the estimate by Skyrme.[7]

The probabilities for the reactions p + d, d + d and d + t per unit time, computed in reference 8 according to (5.19), are given in Table VIII. In order of magnitude, the values given in Table VIII coincide with the estimates already made in references 2 and 3 for the d + d reaction, and the estimates[3,6] of the p + d reaction.

The probability of a nuclear reaction in the mesomolecule is determined by the ratio

$$\frac{w}{w+\lambda_0} = \frac{w\tau}{1+w\tau}, \qquad (5.20)$$

where w is the probability of the nuclear reaction per unit time, $\tau = 1/\lambda_0 = 2.2 \times 10^{-6}$ is the lifetime of the μ meson.

As we see from Table IX, formation of the ddμ and dtμ mesomolecules should lead practically always to a nuclear reaction.

TABLE IX. Probability of nuclear reactions in mesomolecules[8] (in sec^{-1})

	$p+d$	$d+d$	$d+t$
0	$0.18\cdot10^6$	$0.35\cdot10^{11}$	$1.1\cdot10^{12}$
$5\cdot10^{-13}$ cm	$0.26\cdot10^6$	$0.7\cdot10^{11}$	$2.3\cdot10^{12}$
$1.2\cdot10^{-12}$ cm	$0.48\cdot10^6$	$1.41\cdot10^{11}$	$5.7\cdot10^{12}$

Such a conclusion can be made completely safely despite the crude character of the estimates, since w exceeds λ_0 by several orders of magnitude. As for the reaction in the pdμ mesomolecule, on the basis of the estimates given we can assert only that the probability of the p + d reaction is of the same order of magnitude as the probability for decay of the μ meson. This conclusion, as well as the estimate of the probability of formation of the pdμ mesomolecule, agrees qualitatively with the measurements[5] of the distribution of γ quanta in time and with the total yield of the $p + d \to He_3 + \gamma$ reaction produced by μ mesons (cf. Secs. 3 and 4).

A more precise conclusion concerning the value of the probability for nuclear reaction in the pdμ mesomolecule cannot be made on the basis of the estimates given above, since they are obtained by using the constants of the mirror reaction $n + d \to t + \gamma$, and do not take account of the dependence of the probability of the nuclear reaction on the relative orientations of the spins of the proton and deuteron.

Throughout all of the above it has been assumed that the nuclear reaction in mesomolecules occurs from S states. Concerning this assumption we should make several comments.

First of all, in the ddμ mesomolecules the rotational state with angular momentum K = 1 is meta-

stable (cf. Sec. 4), and therefore, for $dd\mu$ mesomolecules formed in this state via an E1 transition the nuclear reaction must occur from P states. However, it is known that, despite the presence of the centrifugal barrier in a P state, a P wave gives a sizeable contribution to the d + d reaction cross section down to the very lowest energies, as is witnessed by the angular distribution of the reaction products. This is explained by the fact that at low energies, in a repulsive Coulomb field, the ratio of the square moduli of the wave functions for P and S states at distances of the order of the radius of action of the nuclear forces do not depend on energy and, roughly speaking, are proportional to $(R_N/a_N)^2$, where R_N is the radius of action of the nuclear forces, $a_N = \hbar^2/M_{12}e^2$ is the Bohr radius of the nuclei with reduced mass M_{12} [cf. (5.13)]. Thus the probability of a nuclear reaction in the $dd\mu$ mesomolecule in the rotational state K = 1 cannot be reduced relative to the probability of the reaction in an S state by more than two or three orders of magnitude. Consequently, using the data given in the table, we can again assume that the formation of $dd\mu$ mesomolecules leads to catalysis of the nuclear reaction with a probability which practically does not differ from 100%.

In principle, reactions from P states could play an important role in the $pd\mu$ mesomolecule. The point is that the ground state of the $pd\mu$ mesomolecule contains an admixture of the state with the meson function Σ_u. At small separations of the nuclei, the Σ_u state goes over into a 2P state of the mesonic atom $He\mu$, and the wave function of the nuclei corresponding to this state is a P wave with zero projection of its angular momentum on the molecular axis. Since the nuclear reaction $p + d \rightarrow He_3 + \gamma$ occurring from a P wave via the electric dipole transition E1 is approximately 1000 times more probable than the $p + d \rightarrow He_3 + \gamma$ reaction from an S wave, transitions from P states should generally compete successfully in the $pd\mu$ mesomolecule with the nuclear reaction from an S state. However, the computation shows that, because of the small weight of the Σ_u state in the ground state of $pd\mu$, and the presence of the centrifugal barrier, the probability of the nuclear reaction $p + d \rightarrow He_3 + \gamma$ because of E1 transition from a P state in the mesomolecule $pd\mu$ is still considerably lower than the probability of nuclear reaction from an S state.[19]

Below we shall consider some features of catalysis of nuclear reactions in $pd\mu$ and $pt\mu$ mesomolecules.

c) Nuclear Reaction in the $pd\mu$ Mesomolecule

Since the admixture of the Σ_u state in the ground state of the mesomolecule is small, we may assume that the nuclear reaction mainly occurs from an S state. For an orbital angular momentum of zero, the $pd\mu$ system can have total angular momentum $3/2$ and $1/2$ and positive parity. Since the one bound state of He_3 has spin $1/2$ and positive parity, according to the selection rules the nuclear reaction p + d with emission of a γ quantum or transfer of energy to a conversion μ meson can go from the $3/2$ state via magnetic dipole (M1) or electric quadrupole (E2) transitions, or from the $1/2$ state via magnetic dipole (M1) and electric monopole (E0) transitions, where the last can occur only with conversion on an electron.

However, it should be pointed out that, in the $pd\mu$ mesomolecule, states with a definite total angular momentum of proton and deuteron will not always be eigenstates of the system when we include the interaction with the spin of the μ meson. When spin interactions are included, the ground state splits into four levels with total spins of J = 2, 1 (two levels), and 0 respectively. The separation between these levels is much greater than the level width Γ which is determined by the probability of the nuclear reaction: $w = 10^6\ \text{sec}^{-1}$; $\Gamma = \hbar w \approx 10^{-9}$ ev. Therefore, the nuclear reaction occurs independently from each spin state of the mesomolecule, but with different probabilities for each of these states.* Thus, in μ catalysis the distribution of γ quanta from the p + d reaction as a function of time should have, strictly speaking, not one but four characteristic times, corresponding to nuclear reactions in the $pd\mu$ mesomolecules. This fact could not be observed in the experiments[5] because of insufficient accuracy.

Let us consider the probability of a reaction with transfer of energy to the conversion μ meson. At the moment of the reaction when the nuclei are close to one another, we can, in the adiabatic approximation, assume that the meson is in an orbit corresponding to a helium ion (Z = 2).

The conversion coefficient for a magnetic dipole transition M1 is given by the expression[40]

$$\alpha = \frac{1}{2} Z^3 \left(\frac{e^2}{\hbar c}\right)^4 \left(\frac{2m_\mu c^2}{E}\right)^{7/2} \approx 4 \cdot 10^{-4}\ (E = 5.4\ \text{Mev}), \quad (5.21)$$

and for an electric quadrupole transition E2:

$$\alpha = \frac{1}{3} Z^3 \left(\frac{e^2}{\hbar c}\right)^4 \left(\frac{2m_\mu c^2}{E}\right)^{9/2} \approx 0.1. \quad (5.22)$$

From (5.21) we see that the magnetic dipole transition is not capable of giving the observed ratio of the probabilities of reaction with emission of a γ quantum and conversion on a μ meson (Sec. 2). As for the electric quadrupole transition, despite the fact that the value it gives for the conversion coefficient is close in order of magnitude to the experimental value, the probability for the E2 transition itself is small.[41] This is related to the fact that, for an E2 transition

*External fields ($\sim 10^4$ gauss) and the interaction with the electron spins ($\sim 10^{-5}$ ev) in the outer shell of the atom within which the $pd\mu$ mesomolecule exists are too small to mix the states with different J.

from an S state of the relative motion of proton and deuteron, there must be a participation of D states in He_3 and (or) the deuteron, and these states occur only as small admixtures to the ground state because of the non-central nature of the forces. In estimating the probability of the p + d nuclear reaction from the cross section for the mirror reaction n + d, it was implicitly assumed that the main transition is M1, since the quadrupole moments are considerably different for the two reactions.

We note that the assumption that the reaction in the pdμ mesomolecule occurs from a P state via an electric dipole transition (cf. above) also cannot explain the high probability for transfer of energy to the μ meson, since the conversion coefficient for the E1 transition is 2×10^{-4}.

To explain the relatively large probability observed for reaction with transfer of energy to the μ meson, the decisive role must be played[6,7,8] by the electric monopole transition E0. The E0 transition is especially important in the case of $0 \rightarrow 0$ transitions when, in general, the emission of a single γ quantum is impossible (for example, in the process $O_{16}^* \rightarrow O_{16} + e^+ + e^-$), and it can also occur for any $J \rightarrow J$ transition without change in parity. (The importance of E0 transitions for internal conversion in $J \rightarrow J$ transitions was pointed out by Church and Weneser.[43]). In the case of the p + d reaction an E0 transition is possible if the total spin of proton and deuteron before the reaction is equal to $\frac{1}{2}$. The estimate of the matrix element of the monopole moment $\sum_k e_k r_k^2$ is extremely difficult. To explain the order of magnitude, the following computation was made in reference 6. One charged particle was considered in the final (bound) state, with the wave function*

$$\psi_f = \frac{1}{(2\pi\lambda)^{1/2}} \frac{e^{-r/\lambda}}{r}. \tag{5.23}$$

The wave function in the initial state was chosen to be

$$\psi_i = G(0)(1 - \lambda/r), \tag{5.24}$$

where the value of G(0) was computed for the Coulomb barrier in the mesomolecule (cf. above). Expression (5.24) was chosen so that at r = 0 the wave function was subjected to the same phase condition $d \ln(r\psi)/dr = -1/\lambda$ as the function (5.23). Thus one was assured of orthogonality of the functions in the initial and final states. The matrix element $\langle er^2 \rangle$ is

$$\langle er^2 \rangle = 8\sqrt{2\pi} e G(0) \lambda^{7/2}, \tag{5.25}$$

and the probability of conversion on the μ meson is

$$w_{E0} = \frac{512\pi\sqrt{2}}{9} Z^3 \left(\frac{e}{\hbar c}\right)^5 \left(\frac{E}{m_\mu c^2}\right)^{1/2} \lambda^7 |G(0)|^2 \left(\frac{m_\mu c}{\hbar}\right)^4 \frac{m_\mu c^2}{\hbar}. \tag{5.26}$$

*The value $\lambda = \hbar/\sqrt{2M_{12}E}$ was used, where M_{12} is the reduced mass of proton and deuteron, $M_{12} = \frac{2}{3}M_p$, and $E = 5.4$ Mev is the binding energy of He_3.

After inserting Z = 2, $|G(0)|^2 = 6 \times 10^{27}$ cm^{-3}; $\lambda = 2.4 \times 10^{-13}$ cm, the value found in reference 6 was*

$$w_{E0} \approx 4 \cdot 10^5 \text{ sec}^{-1}. \tag{5.27}$$

In order of magnitude this value can explain the observed probability for reaction with transfer of the energy to the μ meson. However, the reliability of this result is very low, both because of the crudeness of the single-particle treatment (the deuteron, whose size is greater than that of He_3 and whose mass is twice as great as the proton mass, is replaced by a fixed point), as well as to the extreme sensitivity of the result to the parameter λ, which appears to the seventh power.

It is interesting to note the following fact: The dimensional quantities appear in the same way in the probabilities for monopole conversion E0 and electric quadrupole conversion E2.† Thus the estimate of the probability on the basis of dimensionality considerations should give the same value for E0 and E2. However, estimates on dimensional arguments cannot take account of orthogonality of the wave functions of the initial and final states, and in particular the fact that the D state which is necessary for the E2 transition is a small admixture to the ground state. Therefore, despite the dimensional arguments, the probability of conversion of the μ meson because of E0 can be considerably greater than for the E2 transition.

The important role of E0 transitions in the p + d reaction in the pdμ mesomolecule is related to the fact that the probability of M1 ($\frac{1}{2} \rightarrow \frac{1}{2}$) transitions with emission of γ quanta is relatively small, as manifested by the smallness of the reaction constant (5.5).‡

Along with conversion the electric monopole transition can occur with formation of electron-positron pairs. The ratio of the probability of pair formation to the probability of conversion can be found independently of the magnitude of the nuclear matrix element $\langle er^2 \rangle$. This ratio for the p + d reaction is of order 10^{-3} so that it is not easy to observe pairs. It is therefore very interesting to consider the p + t reaction

*A better value for the probability of the E0 transition, $w_{E0} \approx 5 \times 10^5$ sec^{-1}, was recently obtained in reference 19.

†In fact, in the nonrelativistic region the velocity of light should not appear in the expression for the internal conversion probability (since the transition is caused by direct Coulomb interaction). Therefore the dimensionality of the nuclear moment determines uniquely the powers of e, $\hbar$, m_μ, and E in the expression for the transition probability.

‡The M1 transitions in the $p + d \rightarrow He_3 + \gamma$ and $n + d \rightarrow t + \gamma$ reactions are small, since they must occur (just as the E2 reactions) because of the small admixtures to the ground state of He_3 which is a singlet state in the protons, or to the ground state of tritium which is a singlet state in the neutrons (cf. reference 42). We note that this fact causes a considerable production of pairs in the reaction $n + d \rightarrow t + e^+ + e^-$ via the E0 transition.

in the ptμ mesomolecule, where this ratio is of the order of unity.

d) Nuclear Reaction in the ptμ Mesomolecule

The barrier factor in the ptμ mesomolecule is less than for the pdμ mesomolecule by a factor of two. For a total spin of the p + t system equal to one, the transformation $p + t \rightarrow He_4 + \gamma$ from an S state is possible only by M1 (the transition $1 \rightarrow 0$ is forbidden for E2). Because of the large energy of the transition (around 20 Mev), with a matrix element of the same order as for the p + d reaction we may expect an increase of the reaction constant by a factor of 60. Then the probability of reaction with emission of a γ quantum in the ptμ mesomolecule during the lifetime of the μ meson will be close to 100%. The probability of transfer of the energy to a μ meson when the spin of p + t is equal to 1 is negligible, of the order of 10^{-5}.

In a state of p + t with spin 0, the emission of a γ quantum is impossible ($0 \rightarrow 0$ transition), while the probability for a reaction with transfer of the energy to a μ meson via an E0 transition is of the order of $10^5 - 10^6$ sec^{-1}. However, now the formation of electron-positron pairs competes successfully with conversion of the μ meson. As pointed out above, the ratio of the probabilities of these processes is of the order of unity for an energy of 20 Mev.

When we include spin interactions in the mesomolecule (Appendix 3), the ground state of the ptμ mesomolecules splits into three levels corresponding to different values of the total spin of the ptμ system, $J = 3/2$ and $J = 1/2$ (two states). The distance between these levels, as for the pdμ mesomolecule, is considerably greater than the level width. Therefore the p + t reaction must occur independently from each of the hyperfine structure levels of the ptμ mesomolecule, with a probability characteristic for each of these levels. In the state $J = 3/2$ the total spin of p + t is 1, so that the reaction from the $J = 3/2$ level must occur with emission of γ quanta via an M1 transition. Both $J = 1/2$ states are a mixture of states with total spin of p + t equal to 1 and 0. Thus, for these states there will be a competition with the M1 transition (for spin of p + t equal to 1) of the E0 transition with conversion of the μ meson or production of a pair (for a spin of p + t equal to 0).

The probability of an M1 transition is of course much greater than that for an E0 transition. However, for the E0 transition there is a certain favorable circumstance: namely, because the magnetic moments of proton and triton are close to one another, the state $J = 1/2$ will be close to a pure state with a definite spin of the p + t system. In the lower $J = 1/2$ state, the spin of p + t will be mainly 1, while the admixture of the state with a spin of p + t equal to 0 will be small. Conversely, in the upper $J = 1/2$ state the spin of p + t will be mainly 0, while the admixture of the state with total spin of p + t equal to 1 will be small. Thus, an M1 transition from the upper $J = 1/2$ state, competing with the E0, will be reduced because of the small probability of finding in this state a spin of the p + t system equal to 1.

It is interesting to note that, if the spin of the meson which forms the mesomolecule were equal to zero, the sum of the number of reactions with transfer of energy to the meson and with formation of pairs would amount to 25% of the total number of reactions, in accordance with the statistical weights for the spins of p + t equal to 0 and 1. Thus, the detailed study of the process of catalysis in the p + t system in principle could enable one not only to observe the heavy negative mesons which do not cause nuclear stars, but would also give information concerning their spins.

6. EFFICIENCY OF CATALYSIS OF NUCLEAR REACTIONS BY MESONS

After the experimental confirmation of the possibility of catalysis of nuclear reactions in hydrogen by μ mesons, the hope was frequently expressed that mesonic catalysis might be used for obtaining steady nuclear reactions between isotopes of hydrogen. In the case of μ catalysis, it was, of course, more or less obvious that such a possibility does not exist because of the fact that the probability of formation of mesomolecules is comparable with the probability for decay of the μ mesons, and, in some cases, is even considerable less than the decay probability. However, for a certain length of time the following question remained open: Let us assume that in nature there exists some negatively charged, long-lived meson, say, for example, a meson with mass 500. Is it possible with such a meson to have catalysis of a steady nuclear reaction in hydrogen? It has turned out that one can give a completely definite negative answer to this question.[9,8,7] The point is that for all the possible reactions between hydrogen isotopes (cf. the Introduction), in addition to the reaction I which is not of practical interest, there exist reaction channels in which formation of helium nuclei occurs. In the mesonic catalysis of nuclear synthesis there is therefore a definite probability that a meson giving rise to a nuclear reaction is linked with the helium nucleus which has been formed. Since the mesonic helium atom has a positive charge, other nuclei, including hydrogen nuclei, cannot approach this system to sufficiently small distances. Thus, the meson which is now in an orbit in a mesonic helium atom no longer participates in the catalysis of the nuclear reactions and, in this way, the efficiency of mesonic catalysis is self-limiting. A calculation of the probability of "attachment" of a meson to helium was done in reference 9 by a method analogous to that which was proposed by Migdal[44] for treating the ionization of an atom after β decay. When the nuclei of a mesomolecule approach to a distance

of the order of the radius of action of nuclear forces, the wave function of the meson, Σ_g, adiabatically goes over into the wave function

$$\varphi_0 = \left(\frac{Z^3}{\pi a_\mu^3}\right)^{1/2} e^{-Zr/a\mu}, \tag{6.1}$$

corresponding to the 1s state of the mesonic helium ion, $Z = 2$; $a_\mu = \hbar^2/m_\mu^* e^2$, m_μ^* is the reduced mass of the meson in a helium atom.

Suppose that the helium atom formed as a result of nuclear reaction receives a recoil energy E and is given a velocity $(-\mathbf{v})$. Then the wave function of the meson with respect to the helium atom at the initial moment after recoil has the form

$$\varphi_p = e^{i\mathbf{pr}/\hbar}\varphi_0, \tag{6.2}$$

where $\mathbf{p} = m_\mu^*\mathbf{v}$. Expanding the function (6.2) in eigenfunctions of the meson in the field of the helium nucleus, we can obtain the probability for capture of the meson into the various levels of the mesonic atom $He\mu$.

We may note that the most probable process is capture of the meson in the ground level of the $He\mu$ mesonic atom. In fact, the probability of capture to a state φ_f is given by the expression

$$K_f = \left|\int \varphi_f^* \varphi_p \, d\tau\right|^2 = \left|\int \varphi_f \varphi_0 e^{i\mathbf{pr}/\hbar} d\tau\right|^2. \tag{6.3}$$

If the momentum transfer is large, the function $e^{i\mathbf{p}\cdot\mathbf{r}/\hbar}$ oscillates rapidly, and the main contribution to the integral must come from small values of r $(r \lesssim \hbar/p)$. The probability of capture to a level $l \neq 0$ will be inversely proportional to a higher power of the momentum p than the probability of capture to the level $l = 0$, and will therefore be small. On the other hand, capture to excited levels with $n > 1$, $l = 0$ must also be small compared with capture to the ground state $(n = 1)$, since $|\varphi_f(0)|^2 \sim 1/n^3$ i.e., the probability of capture to excited levels will be less than 20% of the probability of capture to the ground level. In the other limiting case of small p, the probability of capture to excited levels will be small because of the orthogonality of the functions φ_f and φ_0 (for $n \neq 1$). The probability of capture of the meson into the ground level of $He\mu$ can also be calculated easily from (6.3), and is

$$K_0 = (1 + p^2 a_\mu^2/4Z^2\hbar^2)^{-4} = (1 + E/4\varepsilon_0)^{-4}, \tag{6.4}$$

where

$$\varepsilon_0 = \frac{MZ^2e^4}{2\hbar^2}; \tag{6.5}$$

E is the recoil energy, and M is the mass of the helium nucleus.

It is important to emphasize that the probability (6.4) does not depend on the meson mass, in the adiabatic approximation.

For the reaction $d + d \rightarrow He_3 + n$,

$$E = 0.8 \text{ Mev}, \qquad \varepsilon_0 = 0.3 \text{ Mev}, \quad K_0 \approx 0.13.$$

For the reaction $d + t \rightarrow He_4 + n$,

$$E = 3.5 \text{ Mev}, \qquad \varepsilon_0 = 0.4 \text{ Mev}, \quad K_0 \approx 0.01.$$

The probability that, during the further motion of the recoil nucleus, the meson will be torn out of the $He\mu$ mesonic atom as a result of collisions with electrons is not large and amounts, according to the estimates of reference 8, to about 4% for the d + d reaction and 20% for the d + t reaction.

Thus, even a stable meson cannot produce on the average more than 16 reactions in liquid deuterium and no more than 100 reactions in a mixture of deuterium and tritium; actually, in a mixture of d + t there will also be d + d and t + t reactions which reduce the meson yield. Consequently, the energy yield of the reaction will not compensate for the expenditure of energy in production of mesons by accelerators. The cosmic ray mesons, because of their low intensity, are not practically efficient.

7. CONCLUSION

Catalysis of nuclear reactions by μ mesons in hydrogen is a process in which the phenomena of mesoatomic and mesomolecular physics overlap in a very characteristic way with the problems of nuclear reactions between hydrogen isotopes. In this sense, the study of mesonic catalysis is of interest both for mesonic and for nuclear physics. Nuclear reactions in mesomolecules of hydrogen frequently occur under conditions completely different from the conditions under which one observes nuclear reactions in accelerators. The simplest example is the reaction $p + d \rightarrow He_3$ which occurs in the mesomolecule $pd\mu$ mainly from an S state via a magnetic dipole transition M1, whereas under laboratory conditions it occurs from a P state via an electric dipole transition E1. In order to observe, in an accelerator tube, the reaction $p + d \rightarrow He_3 + \gamma$ from an S state, it would be necessary to go to an energy of the order of 10 — 20 kev, which clearly is extremely difficult to reach because of the low cross section for the reaction. Thus the investigation of μ catalysis can give additional information concerning the matrix elements of various nuclear transitions.

On the other hand, the various mesomolecular processes in hydrogen associated with mesonic catalysis determine the specific behavior of μ mesons in hydrogen. Therefore these processes must be taken into account in experimental studies of weak interactions of μ mesons in hydrogen.[22,23,25]

The possibility is not excluded of using mesonic catalysis for discovering and investigating the properties of new, nuclearly inactive, negatively-charged mesons, if such are discovered.

It would be desirable to carry out further investigations of μ catalysis, in particular:

1) In hydrogen, enriched in deuterium to 2 – 3%, one should make more accurate measurements than those of reference 5 of the time distribution of γ quanta, and in particular find the various characteristic times corresponding to reactions from the different spin states. The precise determination of the absolute yield of γ quanta in such a mixture may be of interest for the study of the weak interaction of μ mesons with nucleons. In fact, since as a result of the $p + d \rightarrow He_3 + \gamma$ reaction, the meson is left in an orbit around the He_3 (cf. Sec. 6), there is a possibility of observing the capture of the μ meson by a He_3 nucleus: $\mu^- + He_3 \rightarrow t + \nu$. The matrix element for the transition can be established from data on the β decay of tritium $t \rightarrow He_3 + e^- + \nu$, and, thus, the study of this process makes possible a precise determination of the constant of the weak interaction $\mu^- + p \rightarrow n + \nu$ (cf. also reference 23).

2) At quite high concentrations of deuterium (of the order of 60 – 80%) one could, from the number of $p + d$ and $d + d$ reactions, establish the ratio between the probabilities of formation of $pd\mu$ and $dd\mu$ molecules.

In addition, in such a mixture, because of the reaction $d + d \rightarrow t + n$ in the mesomolecule $dd\mu$, there is a probability equal to about 2% for the μ meson to appear in an orbit around the triton. After this the mesonic atom $t\mu$ can form a $pt\mu$ mesomolecule, which will make it possible to observe the $p + t$ reaction, and in particular the production of e^+, e^- pairs in the reaction $p + t \rightarrow He_4 + e^+ + e^-$.

3) It is of interest to measure the absolute yield of reactions produced by μ mesons in pure deuterium and also the probability for "attachment" of a μ meson to the He_3 which is formed.

4) In hydrogen enriched in tritium to 0.01 – 0.1% it is entirely possible to observe the $p + t$ reaction in $pt\mu$ mesomolecules. Unfortunately, in this case one cannot use a bubble chamber, because of the tritium activity. However, the most beautiful phenomenon, the production of e^+, e^- pairs, can apparently be observed by using counters.

APPENDIX 1

IMPROVEMENT OF THE ADIABATIC APPROXIMATION

Let us consider a system consisting of two nuclei which are isotopes of hydrogen with masses M_1 and M_2 and a μ meson. Let $\mathbf{r}$, $\mathbf{R}_1$, and $\mathbf{R}_2$ be respectively the coordinates of the μ meson and the nuclei. The Hamiltonian of the system, when we neglect spin interactions, has the form:*

$$\hat{H} = -\frac{1}{2M_1}\Delta_{R_1} - \frac{1}{2M_2}\Delta_{R_2} - \frac{1}{2}\Delta_r - \frac{1}{r_1} - \frac{1}{r_2} + \frac{1}{R}, \quad (1A.1)$$

where

$$r_1 = |\mathbf{r} - \mathbf{R}_1|; \quad r_2 = |\mathbf{r} - \mathbf{R}_2|; \quad R = |\mathbf{R}_2 - \mathbf{R}_1|.$$

As stated in Sec. 4, the wave function of the system for the lowest mesonic term should have the form (4.9). Substituting (4.9) in the Schrödinger equation

$$\hat{H}\Psi = E\Psi \quad (1A.2)$$

with the Hamiltonian (1A.1), and noting that Σ_g and Σ_u are orthonormal functions satisfying Eq. (4.2), we can by multiplying (1A.2) by Σ_g and Σ_u and integrating with respect to the coordinates of the μ meson, obtain a system of equations for the functions $G(\mathbf{R})$ and $H(\mathbf{R})$:

$$-\frac{1}{2M_{12}}\Delta_R G + \left(E_g + \frac{1}{2M_{12}}K_{gg}\right)G + \frac{1}{2M_{12}}K_{gu}H - \frac{1}{M_{12}}Q_{gu}\nabla H = EG,$$

$$-\frac{1}{2M_{12}}\Delta_R H + \left(E_u + \frac{1}{2M_{12}}K_{uu}\right)H + \frac{1}{2M_{12}}K_{ug}G - \frac{1}{M_{12}}Q_{ug}\nabla G = EH, \quad (1A.3)$$

where $\frac{1}{M_{12}} = \frac{1}{M_1} + \frac{1}{M_2}$, and the quantities $\frac{1}{2M_{12}}K_{ij}$ and $\frac{1}{M_{12}}Q_{ij}$ are matrix elements, with respect to the meson functions Σ_g and Σ_u, of the operators

$$\frac{1}{2M_{12}}\hat{K} = -\frac{1}{2}\left(\frac{1}{M_1}\Delta_{R_1} + \frac{1}{M_2}\Delta_{R_2}\right),$$

$$\frac{1}{M_{12}}\hat{Q} = -\frac{1}{M_1}\nabla_{R_1} + \frac{1}{M_2}\nabla_{R_2}. \quad (1A.4)$$

These quantities in equations (1A.3) are dynamical corrections to the adiabatic approximation, which are dropped for ordinary molecules because $M_{12} \gg 1$. Using the symmetry properties of Σ_g and Σ_u with respect to interchange of the nuclei, we can separate out, in the quantities K_{ij} and Q_{ij}, the dependence on the masses M_1 and M_2:

$$K_{ii} = \int \Sigma_i(-\Delta_{R_1})\Sigma_i\, d\tau_\mu; \; i = g, u,$$

$$K_{ij} = \frac{M_2 - M_1}{M_2 + M_1}\int \Sigma_i(-\Delta_{R_1})\Sigma_j\, d\tau_\mu; \; i \neq j,$$

$$Q_{gu} = -Q_{ug} = Q\frac{\mathbf{R}}{R} = \frac{M_2 - M_1}{M_2 + M_1}\int \Sigma_g(-\nabla_{R_1})\Sigma_u\, d\tau_\mu. \quad (1A.5)$$

We note that the differential operators Δ_{R_1} and ∇_{R_1} act on the functions $\Sigma_g(R, r_1, r_2)$ and $\Sigma_u(R, r_1, r_2)$ via their arguments $r_1 = |\mathbf{r} - \mathbf{R}_1|$ and $R = |\mathbf{R}_2 - \mathbf{R}_1|$ for constant $\mathbf{r}$ and $\mathbf{R}_2$.

From the relations (1A.5) in particular it follows that for $M_1 = M_2$ the system of equations (1A.3) splits into two independent equations. The physical reasons for this were given in Sec. 4.

The quantities K_{gg} and K_{uu} were calculated in reference 30 by using the exact wave functions Σ_g and Σ_u.[29] The values of K_{ij} and Q, calculated using the functions Σ_g and Σ_u in the (LCAO) and (UA) approximations, are given in reference 17. Similar values were calculated in references 18a and 20 using

*In the following, we shall use mesoatomic units: $\hbar = 1$, $e = 1$, $m_\mu = 1$.

variational wave functions. Let us investigate the behavior of the system (1A.3) as $R \to \infty$. For this purpose we note first that as $R \to \infty$,

$$\int \Sigma_i (-\Delta_{R_1}) \Sigma_j \, d\tau_\mu \to \frac{1}{2},$$

$$\int \Sigma_i (-\nabla_{R_1}) \Sigma_j \, d\tau_\mu \to 0. \qquad (1A.6)$$

In fact, as $R \to \infty$, Σ_g and Σ_u no longer depend on R [cf. (4.3)], so that the action of the operators Δ_{R_1} and ∇_{R_1} reduces essentially to Δ_{r_1} and $(-\nabla_{r_1})$. Then the first of the integrals (1A.6) reduces immediately to the average value of the kinetic energy in the mesonic atom (i.e., to $1/2$), while the second integral vanishes because of the normalization condition.

Taking account of (1A.5) and (1A.6), we can express the limiting values of the potentials in (1A.3) in terms of the energy levels of the isolated mesonic atoms (4.8). To terms of order $(1/M_1)^2$ and $(1/M_2)^2$, we have

$$\left\{E_g + \frac{1}{2M_{12}} K_{gg}\right\}_{R=\infty} = \left\{E_u + \frac{1}{2M_{12}} K_{uu}\right\}_{R=\infty} = \frac{1}{2}(E_1^0 + E_2^0)$$

$$\frac{1}{2M_{12}} K_{gu}(\infty) = \frac{1}{2M_{12}} K_{ug}(\infty) = \frac{1}{2}(E_1^0 - E_2^0). \qquad (1A.7)$$

Thus, as $R \to \infty$, the system (1A.3) takes on the form

$$-\frac{1}{2M_{12}} \Delta_R G + \frac{1}{2}(E_1^0 - E_2^0) H = \left[E - \frac{1}{2}(E_1^0 + E_2^0)\right] G,$$

$$-\frac{1}{2M_{12}} \Delta_R H + \frac{1}{2}(E_1^0 - E_2^0) G = \left[E - \frac{1}{2}(E_1^0 + E_2^0)\right] H. \qquad (1A.8)$$

Together with the functions $G(\mathbf{R})$ and $H(\mathbf{R})$ it is convenient to introduce functions $A(\mathbf{R})$ and $B(\mathbf{R})$ defined by the relations (4.10). For $R \to \infty$, we obtain for the functions $A(\mathbf{R})$ and $B(\mathbf{R})$ according to (1A.8) the equations

$$-\frac{1}{2M_{12}} \Delta_R A + (E - E_1^0) A = 0, \quad -\frac{1}{2M_{12}} \Delta_R B + (E - E_2^0) B = 0. \qquad (1A.9)$$

Thus the corrections to the adiabatic approximation for $R \to \infty$, as was to be expected, include to an accuracy of the order of $1/M_1$ and $1/M_2$ the energy of the motion of the nucleus in the isolated mesonic atom. These corrections for finite R change the form of the molecular potentials and shift the mesonic terms Σ_g and Σ_u. The necessity for looking for the wave function in the zeroth approximation in the form (4.9) and not in the form (4.1) is completely clear from (1A.3), since for mesomolecules the cross terms $\frac{1}{2M_{12}} K_{gu}$ and $\frac{1}{2M_{12}} K_{ug}$ are of the same order as E_g and E_u. In (1A.3) we can separate out the angular dependence of the functions $G(\mathbf{R})$ and $H(\mathbf{R})$. In this sense, we may say that in the approximation (4.9) the classification of levels as rotational and vibrational is preserved. Setting

$$G(\mathbf{R}) = \frac{g(R)}{R} Y_{K, M_K}(\theta, \varphi), \qquad H(\mathbf{R}) = \frac{h(R)}{R} Y_{K, M_K}(\theta, \varphi), \qquad (1A.10)$$

where θ and φ are angles determining the orientation of the molecular axis with respect to the fixed system of coordinates, and $Y_{K,M_K}(\theta, \varphi)$ are the spherical functions, we obtain for $g(R)$ and $h(R)$ the equations

$$-\frac{1}{2M_{12}} \frac{d^2 g}{dR^2} + \left\{E_g + \frac{1}{2M_{12}} K_{gg} + \frac{K(K+1)}{2M_{12} R^2}\right\} g + \frac{1}{2M_{12}} K_{gu} h$$

$$- \frac{1}{M_{12}} QR \frac{d}{dR}\left(\frac{h}{R}\right) = Eg;$$

$$-\frac{1}{2M_{12}} \frac{d^2 h}{dR^2} + \left\{E_u + \frac{1}{2M_{12}} K_{uu} + \frac{K(K+1)}{2M_{12} R^2}\right\} h + \frac{1}{2M_{12}} K_{ug} g$$

$$- \frac{1}{M_{12}} QR \frac{d}{dR}\left(\frac{g}{R}\right) = Eh. \qquad (1A.11)$$

The functions $g(R)$ and $h(R)$ should obviously satisfy boundary conditions which guarantee the finiteness of the functions G and H for $R = 0$:

$$g(0) = h(0) = 0. \qquad (1A.12)$$

For the imposing of boundary conditions as $R \to \infty$ it is, however, more convenient to use functions $a(R)$ and $b(R)$ which are related to $A(\mathbf{R})$ and $B(\mathbf{R})$ in precisely the same way as g and h are related to G and H. According to (4.10) and (1A.10),

$$a(R) = \frac{1}{\sqrt{2}}(g + h), \qquad b(R) = \frac{1}{\sqrt{2}}(g - h). \qquad (1A.13)$$

For $R \to \infty$, the system of equations for $a(R)$ and $b(R)$ splits into two independent equations:

$$\frac{1}{2M_{12}} \frac{d^2 a}{dR^2} + (E - E_1^0) a = 0,$$

$$\frac{1}{2M_{12}} \frac{d^2 b}{dR^2} + (E - E_2^0) b = 0. \qquad (1A.14)$$

In the energy range $E \geq E_1^0 > E_2^0$ (to be specific we shall assume $E_1^0 > E_2^0$), the meson can be either on the light or on the heavy isotope. If we are considering the transfer of the μ meson from the light to the heavy isotope, the wave function $b(R)$ should not contain incoming waves. Correspondingly, the boundary condition for $R \to \infty$ is

$$b(R) \approx e^{ik_2 R}; \; k_2^2 = 2M_{12}(E - E_2^0). \qquad (1A.15)$$

In the energy range $E_1^0 > E > E_2^0$ the meson can only be on the heavier isotope. The wave function $a(R)$ should not contain any exponentially increasing term for $R \to \infty$:

$$a(R) \approx e^{-\varkappa_1 R}, \qquad \varkappa_1^2 = 2M_{12}(E_1^0 - E). \qquad (1A.16)$$

Finally, in the energy region $E < E_2^0 < E_1^0$ we must require that both of the functions $a(R)$ and $b(R)$ remain bounded for $R \to \infty$:

$$a(R) \approx e^{-\varkappa_1 R}, \qquad b(R) \sim e^{-\varkappa_2 R}, \qquad \varkappa_2^2 = 2M_{12}(E_2^0 - E). \qquad (1A.17)$$

In order to satisfy the boundary conditions (1A.15) — (1A.17), in the numerical integration of the system (1A.11) one can use the following method.

By integrating (1A.11) from $R = 0$, one can obtain

two linearly independent solutions by imposing upon them, for example, the conditions

$$\begin{aligned} &\text{I)}\ g(0)=h(0)=h'(0)=0, \quad g'(0)=1, \\ &\text{II)}\ g(0)=h(0)=g'(0)=0, \quad h'(0)=1. \end{aligned} \tag{1A.18}$$

Now forming a linear combination of these solutions, it is not difficult to get solutions such that, for sufficiently large R for which these solutions already have their asymptotic values the conditions (1A.15) — (1A.17) are satisfied. It is clear that conditions (1A.15) and (1A.16) must be satisfied for any value of the energy from the corresponding ranges ($E \geq E_1^0$ or $E_1^0 > E > E_2^0$), whereas conditions (1A.17) can be satisfied only for definite values of E_n forming the discrete spectrum of the molecule. In the numerical integration, the values of E_n are found from the condition of zero value of the determinant formed from the coefficients of the exponentially increasing terms in the linearly independent solutions given by conditions (1A.18) (cf. reference 17).

In conclusion we note that there is a relation between any pair of solutions of system (1A.3) $\{G_1, H_1\}$ and $\{G_2, H_2\}$, which is easily established by noting that

$$K_{gu}-K_{ug}=-2\,\mathrm{div}\,Q_{gu}.$$

The relation between the solutions has the form of a conservation law:

$$\mathrm{div}\{(G_2\nabla G_1-G_1\nabla G_2)+(H_2\nabla H_1-H_1\nabla H_2)+2Q_{gu}(H_1G_2-H_2G_1)\}=0. \tag{1A.19}$$

In particular, the law of conservation of flux follows from (1A.19). This relation is convenient to use for a check of the correctness of the numerical integration.

APPENDIX 2

COMPUTATION OF MESOMOLECULES WITH IDENTICAL NUCLEI

1) In the Σ_g state the effective interaction potential between the nuclei is

$$V_g=E_g+\frac{1}{M}K_{gg}, \tag{2A.1}$$

cf. (1A.3), $M = M_1 = M_2$ is the mass of the nucleus. The corresponding curves for the ground state and rotational state were shown in Figs. 7 and 8. The values $E_g(R)$ are taken from reference 29, the values of $K_{gg}(R)$ are gotten by recomputing from reference 30. The potential curves $V_g(R)$ in the neighborhood of the minimum are well approximated by the Morse potential

$$V_M=D\{e^{-2\alpha(R-R_0)}-2e^{-\alpha(R-R_0)}\}. \tag{2A.2}$$

Values computed from (2A.2) are shown in Figs. 7 and 8 by points: there we also give the parameters of the approximation (R_0, D, and α).

The energy levels for the potential (2A.2) are given by the formula

$$E_v=-D\left[1-\frac{\alpha}{\sqrt{MD}}\left(v+\frac{1}{2}\right)\right]^2, \tag{2A.3}$$

where v is an integer satisfying the condition

$$0<v<\sqrt{MD}/\alpha-\frac{1}{2}. \tag{2A.4}$$

For the ppμ mesomolecule there is only one vibrational level v = 0; for the ddμ and ttμ mesomolecules in the state K = 0, v = 0 and 1 are possible.

If we use the notation

$$s=\sqrt{M|E|}/\alpha,\ \delta=\sqrt{MD}/\alpha,\ \xi=2\delta e^{-\alpha(R-R_0)}, \tag{2A.5}$$

the radial functions corresponding to the levels v = 0 and v = 1 can be written as

$$v=0 \quad g=[\alpha/\Gamma(2s)]^{1/2}e^{-\frac{\xi}{2}}\xi^s, \tag{2A.6}$$

$$v=1 \quad g=[\alpha(2s+1)/\Gamma(2s)]^{1/2}e^{-\frac{\xi}{2}}\xi(1-\xi/(2s+1)), \tag{2A.7}$$

where $\Gamma(z)$ is the gamma function.

It should be noted that the Morse potential deviates essentially from the interaction potentials which it approximates both for small as well as for very large R. However, as one sees from Figs. 7 and 8, this deviation occurs in the sub-barrier region where the values of the wave functions are exponentially small. Therefore, the level shift produced by the deviation of the Morse potential from V_g

$$\delta E=\int g^2(V_g-V_M)\,dR \tag{2A.8}$$

is, as a rule, small. Because of this fact, the approximation using the Morse potentials gives a good result also for rotational levels; an exception is the level (K = 1, v = 1) in the ddμ mesomolecule, which is obtained formally according to (2A.4), but is extremely unsatisfactory.

2) The wave function of the continuous spectrum for the potential (2A.2) has the form

$$g=\frac{e^{-\frac{\xi}{2}}}{2i}\left\{e^{i\varphi}\xi^{-is}F\left(-\delta+\frac{1}{2}-is,\,1-2is,\,\xi\right)-e^{-i\varphi}\xi^{is}F\left(-\delta+\frac{1}{2}+is,\,1+2is,\,\xi\right)\right\}, \tag{2A.9}$$

where $F(\alpha, \gamma, z)$ is the confluent hypergeometric function, and the angle φ is determined by the relation

$$e^{2i\varphi}=\frac{\Gamma(1+2is)}{\Gamma(1-2is)}\cdot\frac{\Gamma\left(-\delta+\frac{1}{2}-is\right)}{\Gamma\left(-\delta+\frac{1}{2}+is\right)}. \tag{2A.10}$$

The asymptotic form of (2A.9) at large distances ($R \gg R_0$) is

$$g\approx\sin(kR+\omega),$$
$$k=\sqrt{ME},\ \omega=\varphi-kR_0-k\ln\delta/\alpha. \tag{2A.11}$$

For sufficiently small energies the function (2A.9) can

be represented in the region $R_0 \ll R \ll 1/k$ in the form:

$$g \approx R - a_g, \tag{2A.12}$$

where the scattering length a_g is equal to

$$a_g = \left[\psi\left(-\delta + \frac{1}{2}\right) - 2\psi(1) + \ln 2\delta + \alpha R_0\right]/\alpha, \tag{2A.13}$$

$[\psi(z) = \frac{d}{dz} \ln \Gamma(z)$ is the logarithmic derivative of the gamma-function].

3) In the repulsive state Σ_u, the effective potential energy of the interaction V_u is equal to

$$V_u = E_u + \frac{1}{M} K_{\omega i}; \tag{2A.14}$$

V_u is approximated with good accuracy in the range $3 < R < 8$ by the exponential

$$V_u \simeq U_0 \exp(-\beta R). \tag{2A.15}$$

For the radial function of the S state, the Schrödinger equation can be reduced by a change of variables to the Bessel equation with imaginary argument. The radial function for zero energy has the form:

$$h(R) = \frac{2}{\beta} K_0\left(\frac{2\sqrt{MU_0}}{\beta} e^{-\frac{\beta R}{2}}\right), \tag{2A.16}$$

where $K_0(z)$ is a Bessel function of the third kind. $h(R)$ falls off rapidly inside the barrier for small R, and for sufficiently large R goes over into the wave function for free motion

$$h(R) \approx R - a_u, \tag{2A.17}$$

where a_u is the scattering length for the potential V_u, equal to

$$a_u = \frac{2}{\beta} \ln \frac{C\sqrt{MU_0}}{\beta} \quad (C = 1.781 \text{ the Euler constant}) \tag{2A.18}$$

The cross section for elastic scattering $p\mu + p \rightarrow p\mu + p$ (disregarding the hyperfine splitting in the $p\mu$ mesonic atom) has the form

$$\sigma_{pp} = 4\pi\left(\frac{1}{4}\frac{a_g^3}{1 + k^2 a_g^2} + \frac{3}{4} a_u^2\right), \quad a_g = -17.3, \; a_u = 5.25 \tag{2A.19}$$

for low energies. In formula (2A.19) we include the possibility of resonance because of the presence in the $pp\mu$ system of a virtual level at low energy. For $E = 0$

$$\sigma_{pp}(0) \approx 8 \cdot 10^{-19} \text{ cm}^2. \tag{2A.20}$$

Since there is in the $dd\mu$ system no real or virtual S state with low binding energy, the scattering $d\mu + d \rightarrow d\mu + d$ should not have resonance character. It can be written for low energies as

$$\sigma_{dd} = 4\pi\left(\frac{2}{3} a_g^2 + \frac{1}{3} a_u^2\right). \tag{2A.21}$$

Using (2A.13), (2A.18), and the values of the parameters of the approximating molecular potentials, we can obtain $a_g = 6.67$ and $a_u = 5.73$, from which we get

$$\sigma_{dd}(0) \approx 3.3 \cdot 10^{-19} \text{ cm}^2. \tag{2A.22}$$

APPENDIX 3

SPIN STATES OF MESOMOLECULES

To illustrate the influence of the spin state of a mesomolecule on the probability of nuclear reaction, we shall consider a simple model of the spin interactions in the mesomolecule. In the Heitler-London approximation, we shall include the interaction of the nuclear magnetic moment with the magnetic moment of the μ meson, which is in a K orbit of this nucleus, and neglect the spin interactions of the nuclei. In this approximation the interaction of the nuclear spin with the spin of the μ meson will be described by the usual expression for the hyperfine structure

$$\frac{16\pi}{3} \beta_\mu \beta_N g_N |\psi(0)|^2 (\mathbf{s}\mathbf{i}), \tag{3A.1}$$

where β_μ is the mesonic and β_N the nuclear Bohr magneton, $\mathbf{s}$ is the spin of the μ meson, $\mathbf{i}$ the spin of the nucleus, g_N the gyromagnetic ratio for the nucleus, and $\psi(0)$ the value of the μ meson wave function at the position of the nucleus. Taking $\Sigma_g = \frac{1}{\sqrt{2}} \times (\psi_a + \psi_b)$, where ψ_a and ψ_b are wave functions of hydrogen atoms for the first and second nuclei $\left(\psi_a = \frac{1}{\sqrt{\pi a_\mu^3}} \exp\left\{-\frac{r}{a_\mu}\right\}\right)$, we can represent the spin interaction in the molecule in the form

$$\hat{H} = \gamma_1 (\mathbf{s}\mathbf{i}_1) + \gamma_2 (\mathbf{s}\mathbf{i}_2), \tag{3A.2}$$

where

$$\gamma_1 = \frac{8}{3} \frac{\beta_\mu \beta_N g_{N_1}}{a_\mu'^3}, \quad \gamma_2 = \frac{8}{3} \frac{\beta_\mu \beta_N g_{N_2}}{a_\mu''^3},$$

g_{N_1}, g_{N_2} are the gyromagnetic ratios for the first and second nucleus, a_μ' and a_μ'' are the reduced Bohr radii of the μ meson for the first and second nucleus: $a_\mu' = \frac{\hbar^2}{m_\mu e^2}\left(1 + \frac{m_\mu}{M_1}\right)$.

Let us first consider the $pd\mu$ mesomolecule. The proton spin $i_1 = 1/2$, the deuteron spin $i_2 = 1$; the mesomolecule $pd\mu$ can be in states with total spin $J = 2$, 1, 0, where there are two states with spin $J = 1$.

In the $J = 2$ state, the total spin of the proton and deuteron must be $3/2$, while the energy, as is easily verified, is

$$\varepsilon_2 = \frac{1}{4}\gamma_1 + \frac{1}{2}\gamma_2 \approx 0.034 \text{ ev}.$$

The energy is measured from the ground state of the mesomolecule calculated omitting the spin interactions. In the $J = 0$ state, the total spin of proton plus deuteron is $1/2$, and the energy of the level is

$$\varepsilon_0 = \frac{1}{4}\gamma_1 - \gamma_2 \approx 0.001 \text{ ev}.$$

Thus, the states J = 2 and J = 0 are states with definite total spin of proton and deuteron. In contrast to these, the state J = 1 is a linear combination of states corresponding to total spin of proton and deuteron equal to $1/2$ and to $3/2$. The spin function of the J = 1 state is

$$\chi_1 = C_{1/2}\chi_1^{(1/2)} + C_{3/2}\chi_1^{(3/2)},$$

where $\chi_1^{(1/2)}$ and $\chi_1^{(3/2)}$ are spin functions corresponding to a total spin of the system equal to unity, and a total spin of proton and deuteron equal to $1/2$ and $3/2$. From the condition $\hat{H}\chi_1 = \epsilon\chi_1$ we get a secular equation for $C_{1/2}$, $C_{3/2}$, and the energy. The lowest energy level ϵ_1 is

$$\varepsilon_1 = -\frac{1}{4}(\gamma_1+\gamma_2) - \frac{1}{2}\left(\gamma_1^2 - \gamma_1\gamma_2 + \frac{9}{4}\gamma_2^2\right)^{1/2} \approx -0.072 \text{ ev}$$

and the corresponding coefficients are

$$C_{1/2} \approx -0.41, \qquad C_{3/2} \approx 0.91.$$

The higher level with J = 1 is given by the formula

$$\varepsilon_1' = -\frac{1}{4}(\gamma_1+\gamma_2) + \frac{1}{2}\left(\gamma_1^2 - \gamma_1\gamma_2 + \frac{9}{4}\gamma_2^2\right)^{1/2} \approx 0.014 \text{ ev}$$

and the corresponding coefficients are

$$C_{1/2}' = 0.91, \qquad C_{3/2}' = 0.41.$$

Thus, when we include spin interactions, the ground state of the pdμ mesomolecule splits into four levels. The separation between these levels is 0.1 – 0.01 ev, which is much greater than the level width determined by the probability for nuclear reaction ($w \sim 10^6$ sec^{-1}, $\Gamma = \hbar w \sim 10^{-9}$ ev).

A similar treatment can also be given for the ptμ mesomolecule. In this case $i_1 = i_2 = 1/2$, $\gamma_1 = 0.091$ ev, $\gamma_2 = 0.123$ ev. The possible states have total spins of $J = 3/2$ and $J = 1/2$. The $J = 3/2$ state is an eigenstate of the total spin of the proton and triton, equal to 1, and the corresponding energy is

$$\varepsilon_{3/2} = \frac{1}{4}(\gamma_1+\gamma_2) \approx 0.053 \text{ ev}.$$

There are two energy levels corresponding to the total spin $J = 1/2$. The $J = 1/2$ state is a linear combination of states corresponding to total spin 1 and 0 of the proton and triton,

$$\chi_{1/2} = C_0\chi_{1/2}^{(0)} + C_1\chi_{1/2}^{(1)}.$$

The energy of the level and the coefficients C_0 and C_1 are determined from the secular equation.

The lower energy level is

$$\varepsilon_{1/2} = -\frac{1}{4}(\gamma_1+\gamma_2) - \frac{1}{4}\{(\gamma_1+\gamma_2)^2 + 3(\gamma_1-\gamma_2)^2\}^{1/2} \approx -0.109 \text{ ev}$$

and the corresponding coefficients

$$C_0 \approx 0.126; \qquad C_1 \approx 0.992.$$

The higher level has energy

$$\varepsilon_{1/2}' = -\frac{1}{4}(\gamma_1+\gamma_2) + \frac{1}{4}\{(\gamma_1+\gamma_2)^2 + 3(\gamma_1-\gamma_2)\}^{1/2} \approx 0.002 \text{ ev}$$

and the coefficients are

$$C_0' \approx 0.992, \qquad C_1' \approx -0.126.$$

It should be mentioned that, since the magnetic moment of tritium is close to the magnetic moment of the proton, the states $\epsilon_{1/2}$ and $\epsilon_{1/2}'$ will be almost pure states of the total angular momentum of proton and triton; the first state corresponds to a total spin 1, and the second to total spin 0.

[1] F. C. Frank, Nature **160**, 525 (1947).

[2] A. D. Sakharov, Report of the Physics Institute, Academy of Sciences, (1948).

[3] Ya. B. Zel'dovich, Dokl. Akad. Nauk S.S.R. **95**, 493 (1954).

[4] Alvarez et al., Phys. Rev. **105**, 1127 (1957).

[4a] Cresti and Gottstein, Private Communication (Cf. D. West, Rep. Prog. Phys. **XXI**, 271 (1958).

[5] Ashmore, Nordhagen, Strauch and Townes, Proc. Phys. Soc. (London) **71**, 161 (1958).

[6] Ya. B. Zel'dovich and A. D. Sakharov, JETP **32**, 947 (1957), Soviet Phys. JETP **5**, 775 (1957).

[7] T. H. R. Skyrme, Phil. Mag. **2**, 910 (1957).

[8] J. D. Jackson, Phys. Rev. **106**, 330 (1957).

[9] Ya. B. Zel'dovich, JETP **33**, 310 (1957), Soviet Phys. JETP **6**, 242 (1958).

[10] Gallone, Prosperi, and Scotti, Nuovo cimento **6**, 168 (1957).

[11] Hayashi, Nakano, Nishida, Suekane, and Yamaguchi, Progr. Theor. Phys. **17**, 615 (1957).

[12] H. Marschall and Th. Schmidt, Z. Physik **150**, 293 (1958).

[13] S. S. Gershteĭn, Dokl. Akad. Nauk S.S.R. **117**, 956 (1957), Soviet Phys.-Doklady **2**, 559 (1958).

[14] S. S. Gershteĭn, Dissertation, Institute for Physics Problems, 1958.

[15] Ya. B. Zel'dovich and S. S. Gershteĭn, JETP **35**, 649 (1958), Soviet Phys. JETP **8**, 453 (1959).

[16] Shimizu, Mizuno, and Izuyama, Progr. Theor. Phys. **20**, 777 (1958); **21**, 479 (1959).

[17] Belyaev, Gershteĭn, Zakhar'ev, and Lomnev, JETP **37**, 1652 (1959), Soviet Phys. JETP **10**, 1171 (1960).

[17a] B. N. Zakhar'ev, Dissertation, Joint Institute for Nuclear Studies (1960); Belyaev, Gershteĭn, Zakhar'ev, and Lomnev, Preprint, Joint Institute for Nuclear Research.

[18] Cohen, Judd, and Riddell, Phys. Rev. **110**, 1471 (1958).

[19] Cohen, Judd, and Riddell, UCRL 8389, 8390, 8391 (1959).

[20] Wu, Rosenberg, and Sandstrom, Nucl. Phys. **16**, 432 (1960).

[21] Burke, Haas, and Percival, Proc. Phys. Soc. (London) **73**, 912 (1959).

[22] Ya. B. Zel'dovich and S. S. Gershteĭn, JETP **35**, 821 (1959), Soviet Phys. JETP **8**, 570 (1959).
[23] H. Primakoff, Revs. Modern Phys. **31**, 802 (1959).
[24] For references to the original literature, cf. Usp. Fiz. Nauk No. 1, 253 (1953).
[25] S. S. Gershteĭn, JETP **34**, 463 (1958), Soviet Phys. JETP **7**, 318 (1958).
[26] S. S. Gershteĭn, JETP **34**, 993 (1958), Soviet Phys. JETP **7**, 685 (1958).
[27] A. S. Wightman, Phys. Rev. **77**, 521 (1950).
[28] Kolos, Roothaan, and Sack (Preprint).
[29] Bates, Ledsham, and Stewart, Phil. Trans. Roy. Soc. **246**, 215 (1953).
[30] A. Dalgarno and R. McCarroll, Proc. Roy. Soc. (London) **237**, 383 (1956).
[31] L. D. Landau and E. M. Lifshitz, Механика (Mechanics) Fizmatgiz, Moscow (1959).
[32] S. S. Gershteĭn, JETP **36**, 1309 (1959), Soviet Phys. JETP **9**, 927 (1959).
[33] S. C. Wang, Phys. Rev. **31**, 579 (1928).
[34] Arnold, Phillips, Sawyer, Stovall, and Tuck, Phys. Rev. **93**, 483 (1954).
[35] E. E. Salpeter, Phys. Rev. **83**, 547 (1952).
[36] W. H. Furry, Phys. Rev. **71**, 360 (1947).
[37] R. E. Langer, Phys. Rev. **51**, 669 (1937).
[38] H. A. Kramers, Z. Physik **39**, 828 (1926); L. A. Young and G. E. Uhlenbeck, Phys. Rev. **36**, 1154 (1930).
[39] L. D. Landau and E. M. Lifshitz, Квантовая механика Gostekhizdat, Moscow 1948; [Quantum Mechanics, Pergamon Press, 1958].
[40] A. I. Akhiezer and V. B. Berestetskiĭ, Квантовая электродинамика (Quantum Electrodynamics), 2nd Edition, Fizmatgiz, Moscow 1959.
[41] N. Austern, Phys. Rev. **85**, 147 (1952); **83**, 672 (1951).
[42] M. Verde, Helv, Phys. Acta **XXIII**, 453 (1950); Nuovo cimento **7**, 283 (1950).
[43] E. L. Church and J. Weneser, Phys. Rev. **103**, 1035 (1956).
[44] A. B. Migdal, JETP **9**, 1163 (1939).

Translated by M. Hamermesh
43