SYMPOSIUM ON DIFFRACTION OF WAVES

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FROM September 26 to October 1, 1960, a combined symposium on the theory of wave diffraction was held in Odessa, called by the Commission on Acoustics of the U.S.S.R. Academy of Sciences, together with the Acoustics Institute of the Academy of Sciences and the Odessa Electrical Engineering Institute of Communications. More than 400 scientists took part in the work of the symposium, including upwards of seventy-five doctors of science. Some 100 papers were read on the following topics: rigorous and numerical solutions of boundary diffraction problems, asymptotic methods in boundary diffraction problems, nonstationary problems, Rayleigh waves, waves in heavy liquids, waves in layered media, gratings and corrugated surfaces, wave propagation, regular periodic waveguides, irregular waveguides.

The papers at the plenary sessions were chiefly either reviews of researches carried out in separate regions of diffraction, or generalizations of results obtained in different regions of physics on related questions.

Opening the symposium, Academician V. A. **Fock** noted the basic value of theoretical diffraction problems, the working out of which forms one of the chief directions of development of the theory of a number of branches of science and technology dealing with wave motions. Here, reference was made to such sciences as acoustics, optics, radio engineering, seismology, and also certain portions of hydrodynamics, nuclear physics, thermal physics, etc. Fock especially considered the development of asymptotic methods of investigation, emphasizing that the obtaining of the asymptote represents the appearance of a new "quality" which is inherent in the diffraction phenomenon. Representatives of regional organizations, scientific and educational institutes of Odessa warmly welcomed the participants in the symposium.

Reporting on researches on diffraction theory in the Mathematical-Physics section of the Physico-Technical Institute of the U.S.S.R. Academy of Sciences, G. A. **Grinberg** noted that they are part of more general investigations being carried out in the Physico-Technical Institute on the theory of the electromagnetic field and the development of general methods of mathematical physics. He spoke of the development of the method of integral transformations for the solution of different diffraction problems. Methods have been developed in the Physico-Technical Institute for the solution of integro-functional equations to which certain classes of problems of diffraction theory reduce. Grinberg also discussed the method of shadow currents and its application to problems of diffraction of waves by plane screens and on some other methods developed in the Physico-Technical Institute.

Some important problems of electrostatics and electrodynamics reduce to the solution of an integral equation which connects the value of the current (or charge) density on the surface of a hollow cylinder with the values of the vector (or scalar) potential on the same surface. **Fock** considered two methods of solving such an integral equation by transforming it into an infinite set of algebraic equations, and the properties of this set were investigated. For the case of very thin cylinders, a comparison was made with the results obtained by Kallen, Leontovich, and Levin by asymptotic methods.

L. A. **Vaĭnshteĭn** gave a review of researches on electromagnetic diffraction and boundary problems which were carried out in the U.S.S.R. in the period 1957-1960, and noted the important trends in the development of diffraction theory in recent years. The speaker divided problems on diffraction of electromagnetic waves by ideally conducting bodies (as regards mathematical methods and physical phenomena) into three regions, depending on the value of ka (k —wave number in the surrounding medium, a — characteristic dimension of the body): quasistatic ($ka \ll 1$), intermediate ($ka \sim 1$) and quasi-optical ($ka \gg 1$), and sketched the mathematical methods applicable in each of these regions. Speaking on the perspectives of development, Vaĭnshteĭn noted that further improvement in asymptotic methods will make it possible to penetrate deeper into diffraction problems characterized by the relation $ka \gg 1$. Also, application of computer techniques to calculations for the region $ka \sim 1$ makes it possible to decrease the volume of theoretical investigations necessary to obtain asymptotic formulas suitable in the region close to $ka \sim 10$.

In the paper on asymptotic laws of diffraction, G. D. **Malyuzhinets** spoke about the ideas of T. Young, and pointed out the inconsistency of the criticism of Young's ideas on diffraction by Fresnel. The speaker gave asymptotic description of the diffraction phenomenon as a process of diffusion of the wave amplitude along the propagating wave front, the definition of zones of effective diffusion and Fraunhofer's zones was introduced, and various aspects of the diffraction problem, treated in ray (Riemann) coordinates, were considered. In particular, new asymptotic formulas were derived which describe the diffraction field in the shadow behind a convex body, and which represent a generalization of the formulas of V. A. Fock, suitable for any distance from the body.

By considering the ray method of calculation of the density of wave fields, **A. S. Alekseev**, **V. M. Babich**, and **B. Ya. Gel'chinskii** noted that the wave fields in the vicinity of the front $t = \tau(x, y, z)$ are described when $(\mathrm{grad}\ \tau)^2 = 1/v^2$, by the function

$$u(x, y, z, t) = A(x, y, z, t)\, F(t-\tau) + B(x, y, z, t),$$

where $A(x, y, z, t)$ and $B(x, y, z, t)$ are analytic, and $F(t-\tau)$ is a nonanalytic function in this region; we can then apply the expansion

$$u(x, y, z, t) = \sum_{n=0}^{\infty} u_n(x, y, z)\, f_n(t-\tau) + B(x, y, z, t),$$

$$f_n'(t) = f_{n-1}(t), \quad f_0(t) = F(t)$$

for separation of the nonanalytic parts of the wave fields in the vicinity of the wave fronts; here $B(x, y, z, t)$ is an analytic function, $f_n'(t) = f_{n-1}(t)$, and $f_0(t) = F(t)$.

For terms of the series $u_n(x, y, z)$, in the case of hyperbolic equations, one can obtain a system of recurrent differential equations which is solved in quadratures (similar to the WKB method). The following problems were considered in the paper in connection with the use of this series for constructing the wave field in the vicinity of wave fronts: 1) the formal determination of the terms of the ray expansion, 2) the ray statement of problems of wave propagation, 3) boundary conditions in the ray method, 4) convergence of ray expansions in the vicinity of fronts, 5) the possibility of ray representations of the field in the vicinity of singularities, 6) the application of the analog of ray expansions in the region of geometric shadow.

In the paper of **M. D. Khaskind**, "Some Problems of Diffraction and Excitation of Waves in an Impedance Plane," the class of two dimensional problems was extended to the diffraction and radiation of hydrodynamic surface waves, which are present in problems of the hydrodynamics of ships on the sea swell; it is shown that their arrangement is identical with the corresponding acoustic and electromagnetic problems of the diffraction and excitation of waves in an impedance plane. The generalized Green-Kirchhoff formula makes it possible to determine the entire field approximately and to separate from it the field of surface waves, and also to calculate the energy characteristics. The method of special functional combination makes it possible, after reformulation of the boundary problems, to construct a rigorous solution to the problem of the diffraction of waves on slits and strips oriented perpendicular to the plane, on a strip located on the impedance plane, etc.

In the review paper of **E. L. Feĭnberg**, "Diffraction Problems in the Physics of Elementary Particles," the fundamental properties of wave fields describing particles of various types were set forth. The problems of the physics of the nucleus and of elementary particles were then considered. The theory of these connects them with the phenomena of diffraction in macroscopic electrodynamics and acoustics. The formal relations between these problems were specially emphasized. Here reference was made to the optical model of the nucleus for the scattering of fast nuclear particles both of charged and uncharged types. On the other hand, consideration was also given to inelastic diffraction processes of various types, such as those which find their analogy in macroscopic processes, and also the more specific cases for the physics of elementary particles.

L. D. Bakhrakh and **A. A. Pistol'kors** gave a review paper, "Urgent Diffraction Theory Problems in Centimeter-Wave Antenna Technology," pointing out the importance of the solution of the problem of the diffraction of a complicated wave front by a small mirror with account of its curvature, and also the diffraction by a cone of finite dimensions, and the determination of the amplitude and phase of currents on the cone for a given directivity pattern, as well as a number of other problems. It was noted that in the design of directional antennas of surface waves, the calculation of the diffraction of the surface electromagnetic waves on the impedance inhomogeneity has great importance; in particular, one must compute the necessary change of impedance for formulating the given pattern.

Problems of the theory of waves propagating in a heavy liquid were discussed in the paper of **N. N. Moiseev**.

Most numerous were the sections in which were considered rigorous and numerical solutions of boundary diffraction problems. **G. D. Malyuzhinets** and **A. A. Tuzhilin** obtained an exact solution in closed form for the electromagnetic field excited by an electric dipole in a wedge-shaped region with ideally conducting boundaries. Representing the solution in a Sommerfeld integral, the authors showed that the boundary conditions reduce, for the scalar components of the vector field, to a set of functional equations which are solved by means of Fourier integrals. The exact solution for the field at large distances reduces to a simple asymptotic formula. **V. Yu. Zavadskii** considered the problem of diffraction by a thin elastic plate, bounded by a free rectangular edge, and located on a liquid homogeneous half space, by the method of functional equations developed by Malyuzhinets. A solution was obtained in the form of a Sommerfeld integral in which the kernel of the integral is chosen in the form of a plane wave. The final results are expressed in terms of a special tabulated function and the Brewster angle of the elastic plate. An exact solution in the form of a Sommerfeld integral, which also reduces to the sum of a finite number of plane waves, was obtained by **I. A. Viktorov** in considering the problem of diffraction of a sinusoidal plane wave at a rectangular elastic wedge with free edges for the case of a hypothetical elastic medium with identical propagation velocities for the longitudinal and transverse waves. A number of problems of the mathematical theory of diffraction (diffraction of a monochromatic plane or spherical scalar or vector wave on spheres, a plane scalar wave diffracting on elliptical cylinders, waves irradiated by a dipole, two congruent spheroids with common axis of revolution) were reduced in the paper of **A. M. Rogov** and **E. A. Ivanov** to an infinite system of linear algebraic equations for the coefficients of the series in terms of elementary wave functions. With the help of addition theorems obtained by the authors, sets of equations were obtained for elementary functions of spherical scalar and vector waves, and elementary functions of scalar elliptical and spheroidal waves. **V. I. Dmitriev** obtained a rigorous solution of the scalar wave equation in a layered medium, for the presence in it of a vertical half plane, in the form of Fredholm's integral equation of the first kind with a kernel consisting of a singular part which depends on the difference in the arguments, and a regular contribution which depends on the sum of the arguments. By means of integral transformations in the complex plane, this equation is reduced to a Fredholm integral equation of the second kind, whose solution is obtained in the form of an absolutely and uniformly converging series which is obtained by the method of successive approximation. **G. V. Poddubnyĭ** spoke about the approximate solution of the problem of temperature waves in the ground under insulation of a freezing plant. He obtained a series of products of the type $V_n(\xi)W(\eta)$, where ξ, η are elliptical coordinates and $V_n(\xi)$ is a "tied-on" function, the form of which is so chosen that the solution constructed gives the exact solution of the stationary problem in the special case of the absence of temperature vibrations of the air. The numerical calculations that have been carried out make it possible to establish the behavior of the relative amplitude of the temperature at the center under the freezing plant as a function of the properties of the insulation and the temperature state of the air over the free surface of the ground. Approximate formulas describing the diffraction structure of the field of a converging cylindrical front were confirmed experimentally by **I. N. Kanevskiĭ**. These formulas are applicable for the case of an uneven distribution of amplitude over the front. It was shown that in the focal region the structure of the field is qualitatively the same as in axially symmetric focusing systems: the maxima of the potential alternate with the zeros. In the direction of the "optical axis" (away from the cylinder) the maxima of the potential alternate with minima which do not reach zero. **N. A. Bazhina** read a paper on "An Experimental Test of the Limits of Applicability of Geometric Methods in Acoustics." By means of the geometric theory of reflection, an additional nonuniformity in the frequency characteristic of the radiator was observed. This was due to the presence of a reflecting surface with finite absorption. The resultant nonuniformity was compared with that obtained experimentally in anechoic chambers upon the introduction of reflecting surfaces. The region of agreement of the results also shows, in the opinion of the author, the limits of applicability of geometric methods in the calculation of the sound field in rooms. **V. A. Borovikov** considered the "Three-Dimensional Problem of Diffraction of a Plane Wave on a Plane Screen with a Wedge-Shaped Opening." The determination of the wave function of the field satisfying the boundary conditions by a differential equation and by the radiation conditions at infinity is reduced in this research to the solution of the Dirichlet problem. The author pointed out the possibility of obtaining asymptotic formulas in different regions of space, and in particular, in the region of the geometric shadow and penumbra. In the last of the papers devoted to exact and numerical solutions of boundary diffraction problems, that given by **N. N. Govorun**, the "Integral Equations of Antenna Theory" were discussed.

A number of papers of the symposium were devoted to asymptotic methods in boundary diffraction problems. In the paper of **L. A. Vaĭnshteĭn** and **A. A. Fedorov**, "Scattering of Plane and Cylindrical Waves by an Elliptical Cylinder and the Concept of Diffraction Rays," a new, rigorous solution of this diffraction problem was given, by means of the method of separation of variables, in the form of a series and a contour integral which, upon substitution of the asymptotic expressions for the radial and angular functions of an elliptic cylinder, lead to the attenuation factors introduced by V. A. Fock. The resultant asymptotic solution corresponds to the concept of ray diffraction of J. B. Keller, and makes it possible to investigate the reciprocal transformation of diffraction and ordinary waves.

The report of **A. Ya. Povzner** and **I. V. Sukharevskiĭ** was devoted to finding "Asymptotic Expansions in Certain Problems of Short-Wave Diffraction" in an arbitrary region with ideally reflecting (and sufficiently smooth) boundaries, which is completely "illuminated" by the source. The method used was based on a theorem on the discontinuities in the Green's function in various nonstationary problems. The terms of a series which is found by formal integration of the integral equation corresponding to the boundary problem under consideration were used as generating functions in this method. The asymptotic formulas obtained correspond both to regular points and to caustics of various types.

Exact solutions of a number of diffraction problems show that in the diffraction of a plane wave by the edge of a half-plane, by the edge of a wedge, by an impedance discontinuity on a plane for large distances from the line where the properties change discontinuously, one can separate a cylindrical wave from the expression for the total diffraction field. This wave has the form it would have if it originated from this line. The question is raised: "is it possible to make a similar separation of a cylindrical wave from the diffraction field in the case in which the properties of the body or surface change smoothly?" In this connection, **Yu. P. Lysanov** considered the diffraction of a plane wave by an inhomogeneous surface the local coefficient of reflection of which changes smoothly from -1 to $+1$, while the character of the inhomogeneous surface is such that a comparatively rapid change of the reflection coefficient takes place only within the limits of a certain transition layer. It was shown that the diffraction field in the Fraunhofer zone, relative to the layer of most rapid change of reflection coefficients, consists of a specularly reflected plane wave and a cylindrical wave originating from the line on which the derivative of the reflection coefficient has a maximum value. **V. I. Ivanov** has investigated the asymptote of the two dimensional Green's function of the external problem for the parabolic cylinder. Asymptotic formulas were obtained for the field and for the induced currents, which are uniformly valid in regions of shadow and penumbra; he also obtained the asymptote of currents in the radiated region. A similar problem was solved for the axially-symmetric excitation of a paraboloid of revolution. In the paper of **P. Ya. Ufimtsev**, formulas were given for the calculation of the diagram of the "scattering of plane waves of arbitrary polarization by thin cylindrical surfaces." The scattered field was represented in the form of a sum of multiple diffraction fields. The scattering diagram is represented by the function $\psi(z)$ for the current in the conductor. This function was found by L. A. Vaĭnshteĭn by the method of slowly-varying functions. The calculation of the diagram is considerably simplified if an approximate expression which follows from a variational principle is used for the function. **E. N. Maĭzel's** and **P. Ya. Ufimtsev** investigated "Reflection of Circularly Polarized Electromagnetic Waves From Metals." These bodies were bodies of revolution of arbitrary form, and the authors showed that one could separate from the scattered field a "nonuniform" component which is determined by the curving of the surface. Numerical calculations agreed well with experiment.

By solving the problem "On the Diffraction of a Cylindrical Wave by the Inside of a Circular Cylinder" **B. E. Kinber** found that it is suitable as a model for the analysis of the adherence of the wave to the concave wall. For $kR \gg 1$, asymptotic formulas were obtained for the field components (multiply reflected rays and waves traveling along the concave side of a cylinder). In a second report of the same author, an "Approximate Solution of the Problem of Diffraction on a Parabolic Mirror of Finite Dimensions," was described. A solution was obtained in the form of a sum of terms whose phases satisfy the Fermat principle; i.e., they correspond to path extremal. The proposed method is a generalization of the method of Keller and takes into account the "rays" emanating from the points of the body. On the basis of this method (with accuracy up to terms of order kD^{-1}), a solution was obtained for currents and the field of a mirror with sharp edges. **P. I. Tsoĭ**, applying the method of Poincaré, obtained an asymptotic formula for the determination of the velocity potential of short sound waves in regions inside and outside a cone.

Among the "nonstationary problems" considered in the symposium, greatest interest was attached to the exact solutions obtained by various authors of problems connected with the propagation of waves in a different type of inhomogeneous media. **V. S. Boldyrev** and **I. A. Molotkov** studied the exact solutions of nonstationary diffraction problems in geometric shadow regions. The authors showed that in the vicinity of the first front of slipping (the first step into the shadow zone) the analytic part of the field is identically equal to zero. Therefore, they investigated only the nonanalytic part of the field, for which formulas were obtained which determined the dependence on the position of the point of observation and on parameters characteristic of the properties of the medium. **I. A. Molotkov** considered the nonstationary propagation of waves in an inhomogeneous space $z > 0$ in which the velocity of propagation depends only on the coordinate z, and constructed an exact solution of this problem, turning chief attention to the study of the field in the shadow zone in the vicinity of the front of slipping. The exact expression for the field was derived by him in the form of a sum of integrals over contours of the Airy type; he also obtained simple asymptotic formulas in the vicinity of the front of slipping and close to the boundary of the halfspace.

By the method of contour integrals and spherical vectors, developed by G. I. Petrashen', **V. S. Boldyrev** and **Z. Ya. Yanson** constructed an exact solution to the problem of nonstationary propagation in a spherical layer of SH waves generated by a rotary action applied to the outer surface of the layer. The solution has the form of a Fourier series in spherical vectors, whose coefficients — the contour integrals — depend on the coordinates of the point of observation and on the time. As a final result, it was possible to represent the wave field in the form of a sum of components, each of which is characterized by a definite frequency composition, with different phases and group velocities of wave propagation. The effect of the curvature of the layer on the process of propagation of interference waves is considered. The propagation of surfaces of discontinuity which arise in the instantaneous application of a distributed load (normal and tangential stresses) to the smooth, closed surface of an elastic solid with continuous curvature can be described by the ray method. However, in this case, it is necessary to determine the size of the discontinuities of various orders on the fronts of longitudinal and transverse waves at the initial instant of time, independent of the ray method. **N. V. Zvolinskii** found these quantities, expressing them in terms of the differential characteristics of the applied load. By making use of the formula of N. V. Zvolinskii for discontinuities which appear on the fronts of longitudinal and transverse waves brought about by stresses which are instantaneously applied to the surface of the elastic body, **V. A. Afanas'ev** investigated displacements along the front by means of the ray method in the case of stresses applied to a surface of an ellipsoidal cavity. The resultant asymptotic formulas were compared with the exact solution in the case of a spherical cavity. **L. M. Flitman** made use of the Wiener-Hopf method for the solution of the mixed boundary problem of elasticity theory. By assigning only normal stresses in the finite interval of the boundary of an elastic body occupying the half plane at a given moment, and taking it into account that the remaining part of the boundary is free from stresses, the speaker constructed a distribution of normal stresses on that part of the boundary where the displacements are given. As a result the problem is reduced to an integral equation of first order with a kernel which depends on the difference in the arguments; an exact solution has been obtained. The problem under consideration is analogous to the problem of the diffraction of an elastic wave on a thin strip of finite width. Taking a dipole as the source, **E. B. Khanakhbeĭ** solved the problem of the "propagation of electromagnetic pulses in a conducting medium," both unbounded, and bounded by a dielectric; this was done in the quasistationary approximation by the method of oscillating integrals. He also investigated the propagation velocity of the amplitudes and the pulse length at different distances, making clear the effect of the surrounding dielectric. Transition processes in an acoustic field produced by a plane piston membrane in a rigid convex baffle were studied by **O. G. Kozina**, who made it clear that if the membrane had an axis of symmetry in the plane of the screen and its contour was described by an analytic function, then the field in the vicinity of the wave fronts is described by special functions which are also applicable in certain arrangements of the points of observation and for rectangular and triangular membranes. **A. A. Kaspar'yants** considered the analogous problem for a piston (without a screen) which is turned on in some interval of time and then vibrates according to some harmonic law.

The section of papers devoted to Rayleigh waves was opened by the communication of **V. M. Babich**, "Rayleigh-Type Waves and the Ray Method." Even in 1954, I. G. Petrovskii had shown that if the displacement vector had a discontinuity along some curve on the surface of an elastic body of arbitrary shape, while regularity was preserved inside the body, then this curve is displaced along the surface and the normal displacement velocity of this curve will be equal to the velocity of the Rayleigh waves. It then followed that in the vicinity of the discontinuity curve, in first approximation, the displacement vector ought to behave in the same way as the nonstationary Rayleigh waves considered by S. L. Sobolev. The author considered the next approximations, and also constructed discontinuous solutions with the aid of the ray method, assuming them to be a generalization of the nonstationary Rayleigh waves to the case of a body of arbitrary shape. **V. Yu. Zavadskii** considered the properties of surface waves for an inhomogeneous elastic half space, in which the Lamé parameters are represented by piecewise-continuous differentiable functions of only a single coordinate. **G. I. Petrashen', I. A. Molotkov** and **P. V. Krauklis** represented solutions of problems on the propagation of nonstationary waves in media containing plane parallel (spherical or cylindrical) liquid (elastic) thin layers in the form of series or integrals of the Fourier type of contour integrals which contain a single space coordinate and the time. The account of branch points and poles of the integrand function of the contour integrals is suitable for the study of reflected, refracted and purely interference waves. This made it possible for them to develop the theory of propagation of waves in plates and rods, generalizing the prevailing approximate methods of calculation. **G. S. Pod"yapol'skii** spoke "On the Behavior of a Wave of the Rayleigh Type on the Boundary Between Two Elastic Media," in the case of a large difference between their elastic constants, for a point pulse source located in a less rigid medium, or close to the boundary in a more rigid medium. A displacement in the more rigid medium is the analog of a surface Rayleigh wave and goes over into the ordinary Rayleigh wave upon decrease in the rigidity of the other medium to zero. The displacement associated with it in the less rigid medium has characteristics close to those of frontal waves. The fundamental peculiarity of the wave is the damping in the propagation along the boundary because of radiation into the less rigid medium. **I. M. Khaĭkovich** spoke "On the Scattering of Waves at a Curvilinear Sloping Interface Between Two Media." By giving the scattered field in both media in the form of series, the first terms of which are solutions of the problem for a plane boundary $z = 0$, while the remaining terms are certain "additions," whose role is decreased with increase in the number of terms in the series, the speaker made use of the method of perturbation of boundary conditions proposed by **E. L. Feĭnberg** and transferred the boundary conditions on the curvilinear surface to the plane, which made it possible to determine the field values there. The terms of the series are determined successively since a knowledge of the lower approximations in both media makes it possible to calculate the values of the field on the plane boundary for the subsequent approximation. The resultant solution was illustrated by numerical calculations of the scattered field of a plane wave incident at various angles on a sinusoidal, two-dimensional surface. **Yu. A. Ukhanov** called attention to the fact that the propagation of flexural vibrations in a thin elastic shell approximately simulates the diffraction spreading (diffusion) of the amplitude along the wave front in the passage of sound through an aperture in a screen.

Two sessions of the symposium were devoted to a discussion of problems of the propagation of waves in a heavy liquid. By making use of Lagrangian variables, **Ya. I. Sekerzh-Zen'kovich** considered two dimensional free finite vibrations of the boundary between two layers of ideal incompressible heavy liquids of different density, demonstrating the method of complete solution of the problem in the form of series in powers of some small parameter without secular components. Obtaining an approximate equation for the boundary surface and an approximate equation connecting the wavelength, its amplitude, and the frequency of vibrations, the author noted the fundamental properties of the nonlinear vibrations under consideration. In the paper of **N. N. Moiseev**, a general method was proposed for reduction to integral-differential equations of the problem of steady-state waves in a stratified vortical liquid. He showed that the solvability of this problem follows directly from the theory of Lichtenstein for the single condition of the absence of particles with zero velocity in the liquid, and he made clear the conditions for the non-uniqueness of the solution. The speaker also proposed a scheme for the effective calculations of the wave parameters. **K. A. Bezhanov** spoke on "The Interaction of a Shock Wave with the Free Surface of a Liquid." He made use of Mellin transforms for the solution of the plane problem of gravitational waves on the surface of an incompressible liquid at a shore with an angle of inclination $\pi/2n$ (n is an integer). In the paper of **S. S. Voĭt**, "Diffraction of Waves, Formed on the Surface of an Infinitely Deep, Ideal Liquid by a Periodic Source, from a Half-Space" the solution was found by the Sommerfeld method for the case of immersion in the liquid of a flat, semi-infinite vertical wall. The asymptotic formulas characterizing the rise of the liquid are found and investigated. **V. V. Musatov** gave a review paper on the works of Kriz, which pertained to the propagation of waves in a rotating tank in the presence of solid walls. In the communication of **G. D. Malyuzhinets** and **V. I. Grishukhin**, "Solution of the Linearized Problem of the Diffraction of Gravitational Waves on a Water Surface with a Sloping Shore by the Method of the Sommerfeld Integral," it was shown that the problem considered by Peters and Rozo in 1952 of the oblique incidence of gravitational waves on a shore of arbitrary slope with certain additions is solved more simply by the method of the Sommerfeld integral which was applied earlier (1950) to the analogous acoustical or electromagnetic problem of the diffraction of a plane wave in the wedge-shaped problem with an impedance face. **B. N. Rumyantsev** obtained asymptotic formulas for the case in which the initial pulse or the initial rise of the surface is given in the form of a δ function, applied to the point of intersection of the edge with the free surface. For the case of a slope of the shore of 45 deg and shallow water, general formulas were derived, with the help of which one can solve the problem of the motion of the liquid under the action of an arbitrary distribution of pressure over the surface. **V. A. Magarik** applied a numerical method of solution to the nonlinear plane nonstationary problem of the breaking up and diffraction of long waves on a sloping obstacle. Using the exact solution obtained by M. D. Khaskind of the problem of vibrations of a floating contour on the surface of a heavy liquid, **A. Z. Sal'kaev** calculated the hydrodynamic forces acting on an elliptic cylinder floating on the surface of a disturbed liquid. This made it possible to calculate the rolling and the vertical and horizontal displacements of the center of gravity of ships of corresponding shape.

To study the maneuverability of ships on the sea, it is necessary to know the transverse and longitudinal forces and the yawing moment acting on a ship at sea. **D. M. Anan'ev** considered these quantities, due to regular waves of small amplitude for unlimited depth of channel, small displacement, and different angles of course of the ship. These forces and the moment are divided into two components, of which the first is determined according to the hypothesis of A. N. Krylov, while the second are additional forces brought about by the diffracting wave motion and are found by means of the hydrodynamic theory of rolling of ships developed by M. D. Khaskind. From the analysis of experimental data, it follows that the diffraction components of the longitudinal force can be neglected.

"Waves in Layered Media" was a topic of discussion both of acousticians and those interested in radiowave propagation. **L. M. Brekhovskikh** and **V. A. Eliseevin** considered, both exactly and in the ray approximation, the problem of the propagation of sound waves in a waveguide, the velocity of sound propagation in which changes in a certain part along the path of propagation, thus forming a transition zone. An expression is found for the reflection coefficient from the transition zone of an arbitrary normal wave and the equation of the ray is determined in the approximation of geometric acoustics. Expressions were found for the rays and group velocities of each normal wave, and the dependence of these velocities on the distance along the axis of the waveguide. It was pointed out that the theory thus developed is applicable for the analysis of propagation of acoustic and electromagnetic waves in natural wave guides over a great distance. In problems of radio wave propagation in inhomogeneous media, there is a very effective apparatus of the gauge equations. In a number of cases of propagation of electromagnetic waves in ionized inhomogeneous media, the calibration functions of the problem are the Whittaker functions $W_{\lambda\mu}(z)$. In the communication of **G. I. Makarov**, the propagation of radio waves is described in an ionized layer whose structure can be given by a certain arbitrary smooth function, which is determined from the experimental data. **É. M. Gyunniken** and **G. I. Makarov** spoke about new asymptotic representations of the Whittaker functions, which are applicable to the problems of radio wave propagation, and which were obtained on the basis of well-known integral representations with the aid of the method of steepest descent and use of the method of the gauge equation. In these cases, it is possible for $|\lambda| \gg 1$ to construct asymptotic formulas which are unified for the entire complex plane of the argument z, and which supplement each other very well. The paper of **L. M. Ponomarenko**, "The Role of Coherent Scattering in Long-Range Propagation of UHF Waves," was devoted to long-range tropospheric radio-wave propagation.

At the session devoted to "Gratings and Corrugated Surfaces," **M. I. Gurevich** delivered a paper discussing the simple derivation of formulas for the coefficients of acoustical admittance and reflection in the Rayleigh approximation for normal incidence of a sinusoidal plane wave. **G. D. Malyuzhinets** proposed a method for the solution of various problems of diffraction on transparent and opaque gratings (transmission of waves through a grating, diffraction on gratings of finite dimensions) under the action of arbitrary fields with low-frequency spectrum in space and in a grating by the method of separation of the mean (far) field. Boundary conditions are introduced on the median plane of the grating which make it possible to determine this field directly without the necessity of consideration of the near (inhomogeneous) field corresponding to it. This method is a generalization of the previous researches of the author on the calculation of the acoustic conductivity of perforated screens. It was shown by him that in coefficients of such a type the boundary conditions are expressions for the joined volumes computed by solution of the problem of the flow past a grating of given profile by a stream of an ideal incompressible fluid, or are determined experimentally, for example, with the aid of a galvanic analogy. In the paper of **A. I. Sivov**, "Oblique Incidence of a Plane Wave on the Plane of a Grating of Parallel Conductors," a solution was proposed of the electrodynamic problem for the determination of the coefficients of reflection and transmission in the oblique incidence of a plane electromagnetic wave on the plane of a grating in free space. The period of the grating is assumed to be small in comparison with the wavelength. The resultant formulas make it possible to establish the limiting transition for conductors infinitely close together (i.e., for the transition to a corrugation). The author found the field both far from the lattice and close to it, and obtained a set of equivalent boundary conditions for the grating. In the case in which the magnetic field vector is oriented parallel to the conductors, a solution of the problem is obtained with account of dielectric charging, which is homogeneous but generally different on the two sides of the grating. Equivalent boundary conditions were derived for the "far" field, and the propagation of electromagnetic waves was studied for a waveguide with a grating wall. **A. M. Model'** and **N. V. Talyzin** spoke on the results of an analysis of the diffraction of a plane wave on an infinite plane grating composed of separate vibrators. Such a grating possesses resonance properties, while the reflection coefficient increases rapidly at a frequency for which the grating spacing is equal to one half wavelength. **I. A. Urosovskii** spoke on "The Diffraction of Sound on a Periodically Irregular and Inhomogeneous Surface with a Given Normal Conductivity." In obtaining an exact solution of the problem, the author found that in the case of a plane wave incident at a sufficiently small angle of inclination, the surface reflected in the same manner as a perfectly soft surface. The phenomenon of "surface resonance" (the anomalous increase of amplitude of the gliding spectrum) was analyzed for the case in which the latter is especially strong. The amplitudes of all spectra except the gliding ones, for a sufficiently sloping and homogeneous surface can be determined with sufficient accuracy by means of Kirchhoff's principle. **R. G. Barantsev** solved the stationary problem of the scattering of a plane wave on an arbitrarily periodic surface and analyzed its solution, pointing out the connection of the resultant exact solution with approximations. **B. F. Kur'yanov** considered the scattering of sound on an irregular surface which is the superposition of fine and coarse irregularities. The height of the fine irregularities is assumed to be small in comparison with the incident wavelength, while the coarse irregularities are assumed to be such that one can apply Kirchhoff's principle to them. Fluctuation of the field was determined as a function of the measured characteristics of irregularities of both types under the assumption that the spatial correlation radius of the fine irregularities is much smaller than the correlation radius of the coarse irregularities. In the case of sufficiently smooth large irregularities, in directions which differ significantly from the mirror direction, the chief contribution to the scattered field is given by the fine irregularities. On the basis of the Kirchhoff approximation, **V. I. Aksenov** calculated the scattering of electromagnetic waves on two-dimensional periodic sinusoidal or sawtooth surfaces with finite conductivity.

Problems of "wave propagation" occupied an important place in the work of the symposium. Assuming generalized impedance boundary conditions for the solution of the problem, **M. D. Khaskind** investigated the excitation of electromagnetic waves formed from elementary electric and magnetic radiators in a vacuum above a homogeneous absorbing anisotropic (gyrotropic) medium with a plane surface. In the case of a transversely magnetized plasma and ferrite, it was very simple to find the attenuation function and the value of the reflection for different polarizations. In another paper, **M. D. Khaskind** discussed the problem of the radiation of electromagnetic waves by a thin gyrotropic layer. It was shown that the source always excites surface waves in transverse magnetization of a ferrite or plasma layer. This is explained by the fact that the magnetized layer "attracts," as it were, a field of the electric type to a field of the magnetic type, and vice versa. **Yu. K. Kalinin** and **A. D. Petrovskii** showed that the transition to a certain approximate boundary condition obtained from Snell's law makes it possible to analyze the contradiction in the representation of the field over the boundary section of two media in the form of a gliding plane wave and to solve certain diffraction problems in the case of a small difference in the properties of the bounding media. By considering the dipole field over a piecewise continuous path, sloping incline, shore line, etc., by means of approximate boundary conditions and Green's integral equation, **Yu. K. Kalinin** analyzed the experimental data for a range of wavelengths from centimeters to kilometers that have been published over the last two decades, and showed that account of the sphericity of the earth's surface makes it possible to remove the earlier discrepancies between theory and experiment, and to clarify some of these discrepancies. The paper of **V. V. Novikov** pertained to the calculation of the propagation of pulse signals irradiated by a vertical electric dipole over a plane homogeneous terrestrial surface, with account of displacement currents in the earth. In the paper of **E. M. Gyunniken, G. I. Makarov, A. V. Manankova**, the changes in shape of an electromagnetic pulse were considered in the process of propagation over the surface of a spherical earth, under the assumption that the source — a certical electric dipole — is turned on at some initial instant of time and begins to vibrate according to a harmonic law. **G. N. Krylov** obtained formulas for the calculation of the Hertz vector and the components of the electromagnetic field of a vertical electric dipole and of a vertical antenna with a sinusoidal current distribution in space over a plane homogeneous earth with finite conductivity, and investigated a number of interesting cases by means of calculations on an electronic computer. **G. D. Malyuzhinets, N. D. Vvedenskaya**, and **É. É. Shpol'** spoke about the numerical results obtained by them on an electronic computer of the solution of the Cauchy problem for an equation of the Schrödinger type $2ik\frac{\partial\varphi}{\partial x} = \frac{\partial^2 u}{\partial y^2} + k^2 v(x,y)u$, giving in many cases an excellent first approximation in problems of wave propagation in an inhomogeneous medium. In the paper of **V. N. Krasil'nikov**, it was remarked that, in low-frequency wave processes in a liquid half-space bounded by a thin elastic layer, an almost flexural deformation is generated in the latter. As a result, surface waves appear that are brought about by the joint vibrations of the elastic layer and the adjacent mass of liquid (the not too fortunate term "flexural" was used in the paper). Neglecting the mass of the plate and the compressibility of the liquid in the calculations, the author analyzed the behavior of these waves and compared them with capillary and gravitational waves and also found that the "flexural" waves possess a greater damping than the traveling wave, (which is explained by the relative smallness of the wavelength of the flexural wave as a consequence of which frictional losses and scattering appear).

One session of the section on waveguides was devoted to "regular waveguides." **M. G. Kreĭn** and **G. Ya. Lyubarskii** in their paper "The Theory of Pass Bands of Periodic Waveguides" also considered periodic waveguides in which the principal role is played by waves possessing the property $\varphi(x+l, y, z) = \varphi(x, y, z)e^{ikl}$, where l is the period of the waveguide. To each value of k $(0 \le k \le 2\pi/e)$ there corresponds a set of characteristic frequencies $\omega_n(k) = \omega_n(2n-k)$ for n = 1, 2, and the n-th pass band is an interval of values running through the frequencies $\omega_n(k)$ for change in k. The authors made clear a number of characteristics of such waveguides. In the first pass band the lowest frequency is $\omega_1(0)$ and the highest is the frequency $\omega_1(\pi/l)$; the absolute value of the group velocity $d\omega/dk$ does not exceed the upper value of the local velocity $c_{max}(x, y, z)$, whence it follows that the width Δ of any pass band is connected with the value of $c_{max}(x, y, z)$ by the expression $\Delta\omega \le (\pi/l)c_{max}$. The first pass band of an acoustic cylindrical waveguide for which the propagation velocity depends only on the axial coordinate coincides with the first value of the ω zone of stability of the ordinary differential equation of Hill $u'' + \frac{\omega^2}{c^2(x)}u = 0$. The locations established in the research are the analogs of a series of general locations of the theory of systems of ordinary differential equations with periodic coefficients of which not all are known. In the paper of **N. A. Kuz'min**, "Potentials and Variational Principle of Electrodynamic Equations Described in a Curvilinear Nonorthogonal Set of Coordinates," the properties of two special potential functions were discussed that are analogs of the Hertz potential, under the condition that the metric spatial coefficients are functions of two coordinates, and corrections are found for the propagation constant of surface waves of a cylindrical conductor with finite conductivity, which are weakly bent along the arc of the circle. **A. Ya. Yashkin** applied a new method of calculation of the propagation constant and the critical frequency of electromagnetic waves propagated in straight and curved waveguides possessing a cross section of complicated form. This method is based on the approximate integration of the wave equation, given on a complicated (step-like) region in an arbitrary orthogonal set of coordinates, thus permitting a separation of variables. The classical method of solution of problems of wave propagation is based on the transformation of the equations of propagation and the boundary conditions to such orthogonal coordinates in which the bounding surface is defined as a coordinate surface for constant values of some coordinate. In his paper, "On a Method of Analysis of Wave Propagation," **A. F. Osadchenko** described a method of determination of the equations of the problem for such bounding surfaces in which the ordinary method is inapplicable.

Another meeting of the session was devoted to "irregular waveguides." The method of transverse cross sections applied previously to problems on irregular electrodynamic waveguides was used by **B. Z. Katsenelenbaum** for the analysis of the field in irregular acoustic waveguides. According to this method, the field is expanded in a series in the fields of waves in regular waveguides, and a set of differential equations of first order is obtained for the coefficients of these series. For waveguides with slowly changing parameters, one can thus determine the amplitudes of all waves scattered by the irregular part. The method is applicable to waveguides with inserts of variable length, with variable cross section and with curves. **A. G. Sveshnikov** and **I. P. Kotik** spoke on effective methods using fast acting computing machines for calculation of wave propagation in arbitrary waveguides. **A. D. Lapin** gave a paper on "The Propagation of Sound in a Waveguide Possessing Rectangular Grooves on the Walls." Using expansions of the sound fields in the waveguide and in the grooves in the corresponding eigenfunctions, the speaker obtained an infinite set of algebraic equations with constant coefficients by joining these fields on the boundary of the waveguide and groove. These were solved numerically by the reduction method for certain values of parameters of the waveguide and the groove. The optimal value of the groove was found for which the largest reflection of sound is obtained and the frequency characteristics of the waveguide with the groove were determined. **A. A. Kovtun** and **G. I. Makarov** investigated "Nonstationary Processes that Arise in the Propagation of Pulses in a Circular Waveguide" by the method of incomplete separation of variables, or by contour integration, developed by G. I. Petrashen' for arbitrary parameters characterizing the electromagnetic property of the medium inside the waveguide and the walls of the waveguide. In the case in which the source is placed in the center of the waveguide, the conductivity of the medium is equal to zero and the conductivity of the walls of the waveguide is large, finite, or infinite, the solution is obtained in the form of a double integral of a combination of exponential and cylindrical functions which can be reduced to an infinite series over the residues at the poles of the integrand, and an integral over the cut. The first components of the series, which correspond in the stationary case to propagated waves, describe essentially the field far from the source, where they are received as separate pulses which propagate with different group velocities. Components which correspond in the stationary case to local waves describe the field in the vicinity of fronts of reflection of waves close to the source. In the paper of **M. S. Lifshitz** and **M. Sh. Flekser**, "Synthesis of a Transmission Line for a Given Law of Transformation of Waves," the properties of the so-called transmission matrix were investigated. This matrix characterizes the transformation of complex plane waves in the transition through a series connected dissipation-free linear passive inhomogeneity. Conditions are determined which the given matrix must satisfy in order that it be a transmission matrix, and it is shown how this can be realized in the form of a connecting chain of a finite or infinite number of certain simple quadrupoles and an inhomogeneous section of line with distributed constants. **S. S. Kalmykova** and **V. P. Shestopalov** considered the excitation of an E wave in a semi-infinite plasma waveguide, which is excited by a cylindrical waveguide of circular cross section with the same radius; they also consider E wave excitation in a waveguide with a housing, excited by a cylindrical waveguide with a radius equal to the radius of the housing, and excited by a coaxial waveguide with a radius of the inner conductor equal to the radius of the housing. They found the frequency characteristics of the input conductance for certain values of the parameter $(\omega a/c)^2$, where a is the radius of the plasma waveguide $\omega_0^2 = 4\pi Ne^2/m$ (N is the number of electrons with charge e and mass m), c is the velocity of light in vacuum. **Yu. N. Dnestrovskii** and **D. P. Kostomarov** investigated electromagnetic radiation which arises in the flight of a modulated beam of electrons past a plane waveguide with an infinite flange. A system of algebraic equations is obtained and solved numerically on the "Strela" electronic computer for a wide range of velocities.

A group of papers was presented at the symposium which were devoted to the problem of diffraction in optical instruments. **L. A. Vasil'ev** reported that after determination of the intensity and distribution of diffraction maxima in the image plane of a shadow apparatus, it is possible to measure the angle of departure of the wave front near the boundary of a nontransparent body, and the discontinuity in the density of the gas flow in plane shock waves; these measurements make it possible to investigate regions of flow which have not been accessible to date by shadow methods. In another paper, **L. A. Vasil'ev** considered "Diffraction Limitations of Sensitive Shadow and Interference Devices," Numerical estimates of the size of the error brought about by diffraction phenomena at the rim of the fundamental objectives and the diaphragm of the focal plane for the case of measurements of shifts or slopes of plane wave fronts have shown that the experimentally attainable sensitivity of these devices far from exhausts their possibilities. **L. A. Vasil'ev** and **O. M. Sineglazov** made clear also "Diffraction Limitations of the Phase Contrast Method and the Limits of Applicability of the Vector Theory," pointing out that the customarily used vector theory of phase contrast stems entirely from a diffraction calculation, and that the magnitude of the light intensity in the image of the object can be calculated by the vector theory with error of not less than 10%. Considering the "Comparative Characteristics of the Shadow Method and the Method of Phase Contrast from the Viewpoint of Diffraction Theory," **L. A. Vasil'ev** and **O. M. Sineglazov** showed that in the measurement of small shifts of the wave fronts the method of phase contrast is approximately one-and-one-half or two times as sensitive as the shadow method. However, it is fundamentally unsuitable for measurement of large angles of deflection of the light, and for creating a quantitative, nonstandard method of measurement. Furthermore, technical difficulties associated with the preparation of the phase plate considerably reduce the sensitivity of the method.

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The symposium on wave diffraction made it possible to sum up the large amount of work carried out in recent years by the scientists of the U.S.S.R. on the development of the theory of diffraction. This work has great scientific and applied value. In spite of the concrete differences of the problems under consideration, they are joined together by the common features involved in the theoretical approach and solution. The symposium was proof of the fact that Soviet diffraction theory continues to proceed successfully along the fruitful path of development of individual theoretical methods, applied to phenomena which differ in their physical nature, along the lines set forth by the Mandel'shtam-Papaleksi school. Along with this fact, it is evident that an organization for the continuous flow of information is necessary for the further successful joining together of diffraction studies in acoustics, optics, radio engineering, heat technology, seismology, hydrodynamics and other branches of science and technology dealing with wave motions. In this connection, the creation of a journal devoted to diffraction problems becomes essential. It would greatly facilitate the development of all the regions mentioned above.

The next symposium on diffraction is scheduled to meet in 1962 in Gor'kii.

Translated by R. T. Beyer