# ON SOME PROBLEMS OF ISOMERISM OF ATOMIC NUCLEI 

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## INTRODUCTION

A
quarter of a century has passed since the appearance of the classic paper of I. V. Kurchatov and his collaborators, which laid the foundation for the detailed study of isomerism of atomic nuclei, that is, for the study of excited metastable levels with extremely long lifetimes, up to hundreds of years.

Like all great discoveries, that of nuclear isomerism has not decreased in importance with time; on the contrary, its importance has constantly increased. Data on this phenomenon have played an important part in the construction and development of the nuclear shell model. The creation of a generalized model of atomic nuclei has also led to the necessity of a comprehensive analysis of our information about nuclear isomerism, and this has given important support to the new model.

Quite recently the discovery of the Mössbauer effect has opened the way to the use of nuclear isomers as the active elements in the world's most accurate clocks, which have made it possible to measure the gravitational red shift of the frequency of $\gamma$ rays even under terrestrial conditions.

There are many review articles and books on the phenomenon of nuclear isomerism - we may name, for example, the 1954 monograph ''Isomerism of Atomic Nuclei"' by M. I. Korsunskii, and the review by L. I. Rusinov and G. M. Drabkin in the January, 1958 issue of Uspekhi Fizicheskikh Nauk. The present issue contains an article by L. I. Rusinov (see page 282) which outlines the history of the discovery of the phenomenon and reveals the outstanding part played by I. V. Kurchatov in the study of nuclear isomerism.

Our article is intended only to supplement these general reviews with a detailed discussion of some more special problems of nuclear isomerism.

## 1. ON ISOMERIC TRANSITIONS NEAR THE MILLISECOND RANGE ( $10^{-5}-1 \mathrm{sec}$ )

a) Introduction. After twenty years of investigation of the isomerism of artificial radioactive elements we know of the existence of many dozens of isomers with lifetimes less than $\sim 10^{-5} \mathrm{sec}$ or more than 1 sec . On the other hand, up to the end of 1955 there were only ten known examples of isomeric transitions with durations from $10^{-5}$ to 1 sec , which we shall call by convention millisecond transitions.

As has been remarked with justice by O. I. Leippunskii, ${ }^{1}$ the lack of knowledge of the existence of millisecond isomers was to a large extent due simply to
the absence of attempts to excite such levels by means of powerful accelerators with pulsed operation. Another cause which is of course of some importance is the fact that the range of times from $10^{-5}$ to 1 sec lies in the gap between lifetimes that are so short that they can be studied by means of instantaneous or delayed coincidence apparatus and those that are long enough to be determined by means of ordinary methods for registering activities.

At the present time, owing to the rapid development of experimental technique, the situation in this region has changed decidedly; searches for "millisecond" isomers have been made on a wide scale (Fig. 1, a), and the number of cases is increasing rapidly. ${ }^{1-11}$ This increase can be seen clearly from Fig. 1, b, which represents the data on the numbers of isomeric states with various lifetimes. The shaded area in this diagram contains the ''millisecond'' isomers that were known before 1955.

In accounting for the relative rarity of millisecond isomers we must, however, take into account not only methodological questions but also the physical peculiarities of these isomers.

As is well known, in most cases isomeric states of nuclei pass into other states by the emission of $\gamma$-ray quanta or conversion electrons.

If in estimating the lifetimes of isomeric levels against emission of radiation we use the one-particle transition formulas of Weisskopf and Moszkowski, ${ }^{12}$ we can easily show that in the overwhelming majority of cases transitions with $\mathrm{T}_{1 / 2}=10^{-5}-1 \mathrm{sec}$ and energy 100 to 500 kev must be octupole transitions (of types E3 and M3) or else magnetic quadrupole transitions (of type M2).

Transitions of the types E3, M3, and M2 are observed both in nuclei with odd A and in odd-odd and even-even nuclei. Let us examine in more detail the properties of isomeric states responsible for such transitions in nuclei with odd A.
b) Isomeric Levels in Nuclei with Odd A. As is well known, there has been splendid success in the description of isomeric levels in nuclei with odd $A$, mainly owing to the development of the nuclear shell model, at first the simple one-particle model, and later also the generalized shell model. One of the main achievements of the one-particle model was the explanation of the observed "islands of isomerism."

Figure 2 shows a portion of the diagram of the oneparticle levels in a spherical nucleus (Goeppert-Mayer diagram ). It can be seen from the diagram that the levels that are closest together are


$$
p_{1 / 2}-g_{9 / 2}, \quad d_{3_{/ 2}}-s_{1 / 2}-h_{11 / 2}, \quad f_{3 / 2}-p_{3 / 2}-p_{1 / 2}-i_{13 / 2}
$$

If one of the levels of any of these groups is the ground level, the other levels of that group are lower than any other one-particle level. Therefore, strictly speaking,


FIG. 2. Portion of diagram of oneparticle levels for a spherical nucleus (Goeppert-Mayer scheme). The numbers marked at the right indicate the values of Z and N that bound the "islands of isomerism."

FIG. 1. a) List of elements that have been bombarded with neutrons, protons, and gamma rays in the search for "millisecond" isomers; b) Distribution of isomeric states over ranges of values of $\mathrm{T}_{1 / 2}$ (the abscissa is $\log \mathrm{T}_{1 / 2}$ ). The shaded area includes the "millisecond" isomers known up to 1955.
within the framework of the ideas of the one-particle shell model one should find in the 'islands of isomerism'' only isomeric transitions with $\Delta I \geq 4$ and a change of parity, i.e., transitions of the types M4, E5, and higher multipole characters, and transitions of types M2 and E3 are impossible.

In deformed axially symmetrical nuclei with odd A each of the one-particle levels with a given value of $j$ (corresponding to the total angular momentum of the nucleon) is split up into ( $2 \mathrm{j}+1$ )/2 sublevels, which are characterized by definite values of $\Omega$, the projection of the total angular momentum $j$ of the nucleon along the axis of symmetry of the nucleus.

Figure 3 shows portions of the diagram of oneparticle levels of deformed nuclei (Nilsson scheme). ${ }^{14,15}$ It can be seen from the diagram that in such nuclei the density of one-particle levels is much larger than the density of levels in spherical nuclei, and there can be closely spaced levels 1 ) with $\Delta I=2,3$, and 2 ) with the same parity.

It follows from this that according to the generalized shell model in deformed nuclei there can be isomeric transitions of any type, including the types M2,


FIG. 3. Portions of the diagram of one-particle levels in deformed axially symmetrical nuclei (Nilsson scheme) for the regions $Z=50-$ 82 and $N=82-126$. Near each level there are shown at the right the values of the quantum number $\Omega$ and the parity, and, in square brackets, the values of the asymptotic quantum numbers $N, n_{z}$, and $\Lambda$. The regions in which the deformation parameter has values $\delta \geq 0.15(\eta>3)$ correspond to "classical" deformed nuclei (with clearly marked rotational bands).

E3, M3, E4, and so on, and for such nuclei the concept of islands of isomerism loses its meaning. ${ }^{16}$ According to the Nilsson scheme, for deformation parameters $\delta \geq 0.2$ characteristic of deformed nuclei the two levels $1 / 2^{-}[523]$ and $1 / 2^{+}[411]$ are adjacent.

The numbers in square brackets are the Nilsson asymptotic quantum numbers $N, n_{Z}, \Lambda$. In the odd isotopes of Ho ( $Z=67$ ) the ground state is described as $7 / 2^{-}[523]$. Therefore it can be expected that in $\mathrm{Ho}^{161}, \mathrm{Ho}^{163}, \mathrm{Ho}^{165}$, and so on there are isomeric levels of the type $1 / 2^{+}[411]$ and isomeric transitions of type E3. In fact, such levels and transitions have been discovered in all three nuclei, although the lifetime, $T_{1 / 2}=0.8 \mathrm{sec}$, has so far been measured only for the
$1 / 2^{+}$level in $\mathrm{Ho}^{163}$. It must be about the same in $\mathrm{Ho}^{161}$. In $\mathrm{Ho}^{165}$ there is a one-particle level $3 / 2^{+}[411]$ at 361 kev that is between the two levels already mentioned, and this partly destroys the isomerism caused by the level $1 / 2^{+}[411]$ (because this level can go to the 361 kev level by a fast transition of type M1). The $3 / 2^{+}$ [411] level is itself isomeric, however, and has lifetime $1.6 \times 10^{-6} \mathrm{sec}$, in agreement with the multipole character of the $\gamma$-ray transition of type M2.

Figure 4 shows the decay schemes ${ }^{15,17}$ of the isomeric levels of a number of other deformed nuclei with transitions of types E3, M3, and M2.

Even in cases in which the first excited one-particle state has a spin that is 3 units larger than that of the ground state, however, so that the conditions for isomerism exist, the isomerism is actually destroyed owing to the presence of lower rotational levels. In particular, we encounter such cases in the nuclei ${ }_{69} \mathrm{Tm}_{100}^{169}$ (cf. Fig. 4) and ${ }_{69} \mathrm{Tm}_{102}^{171}$, where between the ground state of type $1 / 2^{+}[411]$ and the one-particle excited level $7 / 2^{+}[404]$ or $7 / 2-[523]$ there are levels of a rotation band with spins $3 / 2^{+}, 5 / 2^{+}, 7 / 2^{+}$, etc. Naturally, under these conditions nuclei in the level $7 / 2{ }^{+}[404]$ will not go directly to the ground state, but will go to one of these levels, because the corresponding radiative transition is of a lower multipole order (M1, E2, not E3).

Unfortunately this fact decidedly limits the possibilities of predicting the lifetimes of isomeric levels in odd deformed nuclei.

Let us now consider isomeric transitions of types M2 and E3 which are encountered outside the limits of the region of deformed nuclei $A=150-190$ and A $>222$.

It has already been pointed out that such transitions cannot be explained within the framework of the ordinary one-particle shell model. In this connection their study is of great interest and can lead to further developments in our ideas about nuclear structure.

What are the cases that lead to the appearance of isomeric transitions of the types M2 and E3 in odd nuclei? There can be several such causes.

There is a group of odd nuclei in which isomeric transitions of the type E3 are often encountered. These nuclei belong to an "island" of isomerism in which the one-particle levels $\mathrm{p}_{1 / 2}$ and $\mathrm{g}_{9 / 2}$ are being filled simultaneously and $\mathrm{Z}, \mathrm{N}=41 ; 43 ; 45 ; 47$ (Z and N are the numbers of protons and neutrons).

In most of these nuclei a level $7 / 2^{+}$is observed in addition to the $\mathrm{g}_{9 / 2}$ level (in ${ }_{32} \mathrm{Ge}_{41}^{73}$ there is a level $5 / 2^{+}$). Transitions of type E3 occur between the levels $p_{1 / 2}$ and $72^{+}$, and transitions of type M2 between $p_{1 / 2}$ and $5 / 2^{+}$. The data on these transitions are presented in Fig. 5. In their explanation of the origin of the $7 / 2^{+}$levels Goldhaber and his coworkers ${ }^{18}$ started from the fact that in the nuclei with $\mathrm{Z}, \mathrm{N}=39$ and 49 , in which there can be only one nucleon (or hole) in the $\mathrm{g}_{9 / 2}$ level, there are no levels of type $7 / 2^{+}$. For


FIG. 4. Isomeric transitions of types E3, M3, and M2 between levels of deformed odd nuclei. In addition to the excitation energies, for the one-particle levels all of the asymptotic quantum numbers $\Omega\left[\mathrm{N}, \mathrm{n}_{\mathrm{z}}, \Lambda\right]$ are indicated. For the rotational levels only the values of the spin and parity are indicated. The energies are given in kev.



FIG. 5. Isomeric transitions of type E3 (and M2)
in nuclei with $Z, N=41-47$ between levels
$7 / 2+\overrightarrow{\text { ® }} 1 / 2-(5 / 2+\overrightarrow{\text { }} 1 / 2-$ ).


FIG. 6. Isomeric transitions of types E3 and M2 in odd nuclei which are outside the "classical" regions of deformed nuclei. For $\mathrm{As}^{75}, \mathrm{Br}^{79-81}, \mathrm{Ir}^{191-193}, \mathrm{Au}^{197}$, and $\mathrm{T}^{203-205}$ levels which are well verified by the method of Coulomb excitation are indicated by cross hatching.
$\mathrm{Z}, \mathrm{N}=41-47$ there can be 3,5 , or 7 nucleons (or holes) in the $g_{9 / 2}$ level. Only under the very special conditions assumed in the simple shell model does the total angular momentum of a system (configuration) of 3,5 , or 7 nucleons have to be equal to the angular momentum $\mathbf{j}$ of the odd nucleon. In actual nuclei a configuration $\left(\mathrm{g}_{9 / 2}\right)^{3,5,7}$ can have a set of states with different total angular momenta (spins) 1 , in particular with the angular momenta $I=7 / 2^{+}$and $\frac{5}{2}{ }^{+}$(see the table on page 301).

To distinguish these states from the one-particle states, it is convenient to call them configuration states.

It is possible that it is precisely the configuration states that are responsible for the isomeric transitions of type M2 in ${ }_{32} \mathrm{Ge}_{41}^{73}$ and of type E3 in the group of nuclei we have been considering.

It is interesting to note that in these transitions the total angular momentum I of the nucleus changes by 3 (or 2) units, and $j$ changes by 4 units, i.e., $\Delta I<\Delta j$. A so-called $j$ forbiddenness arises. This leads to a decided slowing down of the isomeric E3 and M2 transitions. Such a slowing down is actually observed experimentally. ${ }^{19,20}$

Isomeric transitions of types M2 and E3 are observed in many other odd nuclei besides those we have indicated. We can try to explain these transitions in the following simple way. ${ }^{21}$ As is well known, the oneparticle shell model is an approximate model and cannot pretend to give an exact description of the energies of one-particle levels.

Therefore, although in the majority of cases the most closely spaced groups of levels are $\mathrm{p}_{1 / 2}, \mathrm{~g}_{9 / 2}$; $\mathrm{d}_{3 / 2}, \mathrm{~s}_{1 / 2}, \mathrm{~h}_{11 / 2} ; \mathrm{p}_{1 / 2}, \mathrm{f}_{5 / 2}, \mathrm{p}_{3 / 2}, \mathrm{i}_{13 / 2}$, the possibility is not excluded that other one-particle levels can come close to them. This can result in the existence of isomeric transitions of type M2 or E3. It may be this phenomenon that we encounter in the study of the shortlived isomeric states of the nuclei ${ }_{33} \mathrm{As}^{73-77},{ }_{35} \mathrm{Br}^{77-81}$, ${ }_{37} \mathrm{Rb}^{81,85}$. The ground states of these nuclei are of the type $3 / 2^{-}$(for $\mathrm{Rb}^{85}$ it is $5 / 2^{-}$). According to the experimental data the isomeric levels found in these nuclei have spin and parity $9 / 2^{+}$(Fig. 6). Obviously the simplest assumption is that in the framework of the one-particle model the ground states and the isomeric levels can be described as $p_{3 / 2}$ and $g_{9 / 2}$.

In the isotopes of As and in $\mathrm{Br}^{81}$ a $5 / 2$ level is inserted between these levels; this can be interpreted as
a one-particle $f_{5 / 2}$ level. Therefore in these nuclei the isomeric transition is of multipole type M2 and the isomeric states have much shorter lifetimes than those of $\mathrm{Br}^{77}, \mathrm{Br}^{79}$, and $\mathrm{Rb}^{81}$, in which the transition is of type E3.

Similar situations may cause the appearance of isomeric levels $\mathrm{h}_{11 / 2}$ in the odd isotopes ${ }_{46} \mathrm{Pd}_{59}^{105}$, ${ }_{46} \mathrm{Pd}_{61}^{107}$, and ${ }_{46} \mathrm{Pd}_{63}^{109}$, whose ground states can be described as $d_{5 / 2}$. Between the levels $h_{11 / 2}$ and $d_{5 / 2}$ in ${ }_{46} \mathrm{Pd}_{59}^{105}$ there is a level which can be most simply interpreted as a one-particle $g_{7 / 2}$ level. Therefore the isomeric transition in this nucleus is of type M2 and the lifetime of the level is much smaller than in the isotopes $\mathrm{Pd}^{107}$ and $\mathrm{Pd}^{109}$, where isomeric transitions of type E3 occur.

From Fig. 6 (left-hand part) it can be seen that there is a $d_{5 / 2}$ level between the levels $s_{1 / 2}$ and $h_{11 / 2}$ in the nucleus ${ }_{48} \mathrm{Cd}_{63}^{111}$, owing to which the isomeric transition occurs between the levels $h_{11 / 2}$ and $d_{5 / 2}$ and is of the electric octupole type.

The nuclei ${ }_{63} \mathrm{Eu},{ }_{79} \mathrm{Au}$, and ${ }_{81} \mathrm{Tl}$ are of particular interest in connection with the question of the regrouping of the levels in the Goeppert-Mayer scheme.

We begin with the fact that in the odd isotopes $E u^{147}$, $\mathrm{Eu}^{149}$, and $\mathrm{Eu}^{151}$ a level with spin $7 / 2$ is inserted between the isomeric level with spin $11 / 2$ and the ground state, which has spin $5 / 2$. Therefore in these nuclei along with the isomeric transition of type E3 there is observed a transition of type M2, and the levels between which these transitions occur can be interpreted as oneparticle levels $h_{11 / 2}, g_{7 / 2}, d_{5 / 2}$.

In its turn the level $5 / 2$ occurs between the levels $11 / 2^{-}, 3 / 2^{+}$, and $1 / 2^{+}$in ${ }_{57} \mathrm{Xe}^{125-127}$, $\mathrm{Ir}^{191}$, and the odd isotopes of ${ }_{79} \mathrm{Au}$ and ${ }_{81} \mathrm{Tl}$. In these cases also the levels can be interpreted in principle as one-particle levels $h_{11 / 2}, d_{5 / 2}, d_{3 / 2}$, and $s_{1 / 2}$. This gives an explanation of the isomeric transitions of type E3 observed in these nuclei (Fig. 6, right-hand part). Let us look in more detail at the isomers in the odd isotopes of ${ }_{81} \mathrm{Tl}$. There is information in the literature about isomers ${ }_{81} \mathrm{~T}_{114}^{195 m}$ and ${ }_{81} \mathrm{Tl}_{116}^{197 \mathrm{~m}}$. It can be seen from the diagram that in these nuclei the interval between the levels $\frac{11}{2}-$ and $5 / 2^{+}$increases with increase of the number of neutrons.* Therefore if the analogous isomeric level ${ }^{11} / 2^{-}$exists in the nuclei ${ }_{81} \mathrm{Tl}_{118}^{199},{ }_{81} \mathrm{Tl}_{120}^{201},{ }_{81} \mathrm{Tl}_{122}^{203}$, and ${ }_{81} \mathrm{Tl}_{124}^{205}$, because of the increase of the interval $11 / 2^{-}-5 / 2^{+}$the lifetime of this level will decrease rapidly with increase of the number of neutrons in the nucleus.

In this connection we must call attention to the work of Leipunskii and Yampol'skii and their coworkers; ${ }^{4}$ they bombarded Hg with fast protons and found in the resulting Tl two short-lived activities with $\mathrm{T}_{1 / 2}=0.042$ $\sec \left(E_{\gamma}=370 \mathrm{kev}\right)$ and $T_{1 / 2}=5 \times 10^{-3} \mathrm{sec}$. Since except for the isotope $\mathrm{Hg}^{204}$, whose abundance is only 6.85

[^0]percent, all of the stable isotopes of Hg have $\mathrm{A} \leq 202$, and at the proton energies used in this work there could occur only the reactions ( $p, n$ ) and ( $p, 2 n$ ), the mass numbers of the resulting new isomers of Tl must be equal to or less than 202. Isomers are well known in the Tl isotopes with $\mathrm{A}=194,195,196,197,198,202$. Therefore it is reasonable to suppose that the lifetimes found by the authors of reference 4 belong to new odd isomers ${ }_{81} \mathrm{Tl}_{118}^{199 \mathrm{~m}}$ and ${ }_{81} \mathrm{Tl}_{120}^{201 \mathrm{~m}}$, with the smaller value of $\mathrm{T}_{1 / 2}$ corresponding to the larger value of A. As regards the isomer ${ }_{81} \mathrm{Tl}_{118}^{199 \mathrm{~m}}$ this assumption is also favored by the agreement of the $\gamma$-ray energies 370 kev (observed in the decay of $\mathrm{Tl}^{\mathrm{m}}$ ) and 367 kev (observed in the decay of $\mathrm{Pb}^{199}$ ).

Recently there have been observed two more extremely interesting cases of isomerism, in the nuclei ${ }_{78} \mathrm{Pt}_{121}^{199}$ and ${ }_{82} \mathrm{~Pb}_{123}^{205}$, which are located in a classical island of isomerism.

In the nucleus $\mathrm{Pt}^{199}$ a level $\frac{7 / 2}{}$ - with excitation energy 32 kev occurs between the isomeric state $\mathrm{i}_{13 / 2}$ and the ground state $f_{5 / 2}$. There is no such level in the other nucleus with the same number of neutrons, ${ }_{82} \mathrm{~Pb}_{121}^{203}$. This level is most simply interpreted as a one-particle $\mathrm{f}_{7 / 2}$ level, although this interpretation encounters great difficulties.

Indeed, in this case the interval between the levels $f_{5 / 2}$ and $f_{7 / 2}$ caused by the spin-orbit interaction is only 32 kev , whereas in the nearby nucleus ${ }_{82} \mathrm{~Pb}_{125}^{207}$ it is $\sim 1770 \mathrm{kev}$.

The properties of the isomeric state in ${ }_{82} \mathrm{~Pb}_{123}^{205}$ are quite unusual. Whereas in all the other odd isotopes ${ }_{82} \mathrm{~Pb}^{195-207}$ there are isomeric states with $\mathrm{T}_{1 / 2}=0.95$ $\mathrm{sec}-17 \mathrm{~min}$, with transitions of type M 4 , in $\mathrm{Pb}^{205}$ the lifetime of the isomeric state is only $4.8 \times 10^{-3}$ sec.

The decay scheme of this state is shown at the right in Fig. 6. The authors indicate that $\mathrm{T}_{1 / 2}=4.8 \times 10^{-3}$ sec most likely belongs not to the level at 987.8 kev , but to a level lying higher up (within 100 kev ).

In this case the peculiarities of the isomerism in $\mathrm{Pb}^{205}$ can be explained by assuming that the levels at 284 kev and 987.8 kev are one-particle levels $\mathrm{f}_{7 / 2}$ and $h_{9 / 2}$ inserted between the levels $i_{13 / 2}$ and $h_{9 / 2}$. The isomeric transition responsible for the observed lifetime is then of type M2 ( $\left.i_{13 / 2} \rightarrow h_{9 / 2}\right)$.

It must be kept in mind, however, that also in $\mathrm{Pb}^{205}$ the interpretation of the level $284 \mathrm{kev} \mathrm{f}_{7 / 2}$ considered above encounters the same difficulty as the interpretation of the 32 kev level in $\mathrm{Pt}^{199}$.

Therefore the possibility is not excluded that the levels $7 / 2^{-}$in $\mathrm{Pt}^{199}$ and $\frac{7}{2}{ }^{-}, 9 / 2-$ in $\mathrm{Pb}^{205}$ are "configuration" levels of the same type as the $7 /{ }^{+}{ }^{+}$levels in nuclei with $\mathrm{N}, \mathrm{Z}=41-47$.

Thus if we admit the possibility of a regrouping of the one-particle levels in the Goeppert-Mayer scheme and take configuration levels into account, we can interpret practically all of the "anomalous'' isomeric states and transitions of types E3 and M2.

There are, however, many difficulties in the interpretation of the levels responsible for isomeric transitions as pure one-particle states of the one-particle shell model or simple configuration levels.

In some of the nuclei in question $-\mathrm{As}^{75-77}, \mathrm{Br}^{77-79}$, $\mathrm{Pd}^{105}, \mathrm{Eu}^{147-151}, \mathrm{Ir}^{191-193}, \mathrm{Au}^{193-197}, \mathrm{~Pb}^{205}$, and others - there are many low-lying excited levels, which can certainly not be described in the framework of the simple one-particle model for spherical nuclei.

The question arises as to why there are nevertheless two or three of these levels, and precisely the ones that are associated with isomeric transitions, which are so simple that they can be described as strictly one-particle levels.

Earlier we already called attention to another difficulty. In these nuclei the observed intervals between the levels which are interpreted as $d_{3 / 2}-d_{5 / 2}, f_{5 / 2}-f_{7 / 2}$ are too small in comparison with the values that were to be expected.

Intervals of this type between strictly one-particle levels depend on the magnitude of the spin-orbit interaction, and in the case of strong spin-orbit coupling, which is the basis of the shell model, these intervals must be ${ }^{13}$ of the order of $1000-1500 \mathrm{kev}$. In the nuclei in question they do not exceed 400 kev .

In connection with this and a number of other difficulties, some papers attempt a different interpretation of the levels between which isomeric transitions of types E3 and M2 occur. First of all we must note the fact that the cross sections for Coulomb excitation of the levels $5 / 2^{-}$in $\mathrm{As}^{75}$ and $\mathrm{Br}^{81}, 5 / 2^{+}$in $\mathrm{Ir}^{191}$ and $\mathrm{Au}^{197}$, and $3 / 2^{+}$in $\mathrm{Tl}^{203}$ and $\mathrm{Tl}^{205}$ are much larger than those for excitation of other levels in these same nuclei (cf. reference 22), and also larger than the cross sections to be expected on the basis of the idea of pure one-particle levels. There is no doubt that the properties of the intermediate levels shown in Fig. 6 in the nuclei $\mathrm{As}^{77}, \mathrm{Au}^{193-195}$, and $\mathrm{TI}^{195-201}$ are similar. It is interesting to note that levels of this same type (obtained by the method of Coulomb excitation) in the nuclei $\mathrm{Br}^{79}$ and $\mathrm{Ir}^{193}$ lie somewhat above the isomeric states ${ }^{22}$ and do not affect their decay. Therefore the multipole characters of the isomeric transitions and the lifetimes of the isomeric levels are quite different in the pairs of nuclei $\mathrm{Br}^{79}, \mathrm{Br}^{81}$ and $\mathrm{Ir}^{191}$, $\mathrm{Ir}^{193}$, although the odd number of protons is the same in the nuclei of each pair (cf. Fig. 6).

Besides this, all of the excited levels indicated above have the spin $I=I_{0}+1$, where $I_{0}$ is the spin of the ground state. In these same nuclei higher levels with $\mathrm{I}=\mathrm{I}_{0}+2$ can also be obtained easily, by the method of Coulomb excitation.

In addition to the nuclei that have been mentioned, levels with the spins $I=I_{0}+1$ and $I=I_{0}+2$ are readily obtained by Coulomb excitation ${ }^{22}$ of the nuclei ${ }_{45} \mathrm{Rh}_{58}^{103}{ }_{47} \mathrm{Ag}_{60}^{107}$, and ${ }_{47} \mathrm{Ag}_{62}^{109}$.

All of these facts can serve as a basis for the assumption that the levels with spins $I_{0}+1$ and $I_{0}+2$
considered above are not one-particle levels, but collective levels, and that they form a rotation band associated with the ground state. ${ }^{22 *}$ If this assumption is correct, the nuclei in which these levels are observed must be deformed to a larger or smaller extent.

At first glance it seems that this contradicts the basic ideas of the generalized shell model, because all of the nuclei enumerated are outside the well known regions of deformed nuclei. Besides this, the eveneven nuclei adjacent to them definitely lack rotational structure and are spherically symmetrical. According to existing ideas, however, the equilibrium shapes of nuclei are determined by a competition between the polarizing action of the outside nucleons, which try to deform the nucleus, and the pair forces, which favor a spherical equilibrium shape. In some ranges of nuclei these opposing effects almost completely compensate each other, and the shape of the nucleus is in a state of unstable equilibrium. Naturally in these regions the even-even nuclei, in which all of the nucleons are paired and the pair forces are more effective, have the spherical equilibrium shape, and the odd nuclei, in which there is an odd nucleon, are either spherical but very easily deformable (on the slightest excitation), or else deformed. In both cases rotation band levels can be excited in such odd nuclei. Thus there is an explanation for the interpretation assumed above for the levels $I_{0}+1$ and $I_{0}+2$.

The assumption that the equilibrium shape of the nucleus is deformed does not merely lead to the appearance of rotational levels. For such nuclei the Goeppert-Mayer scheme of one-particle levels must be replaced by the Nilsson scheme.

In particular, this has the result that the $7 / 2$ levels in $\mathrm{Rh}^{103}, \mathrm{Ag}^{107}, \mathrm{Ag}^{109}$, and possibly in other nuclei of this group, and also the $5 / 2^{+}$level in $\mathrm{Ge}^{73}$ (cf. Fig. 5), which are usually interpreted as configurational levels, can be ordinary one-particle levels of deformed nuclei.

Naturally in this case the probabilities of isomeric transitions can be affected not only by the selection rules for the total angular momentum (spin) and the parity, but also by the specific selection rules for the asymptotic quantum numbers ${ }^{14,15} \mathrm{~N}, \mathrm{n}_{\mathrm{z}}, \Lambda, \Sigma$. In the group of nuclei with the isomeric level $7 / 2^{+}$this can lead to a decided slowing down of the E3 transitions, which is usually interpreted as a consequence of forbiddenness in $\mathbf{j}$.

In summarizing our discussion of the properties of the short-lived isomeric levels of odd nuclei we must, however, emphasize that at the present time we cannot give decisive preference to either of the approaches we have considered to the explanation of isomeric transitions of types M2 and E3 outside the known regions of strongly deformed nuclei.

[^1]It is possible that neither of these approaches will by itself give a complete solution of the problem. There are already many signs of the necessity of a synthesis of the two approaches, i.e., of a simultaneous consideration of the collective and one-particle properties of nuclear levels.

## 2. ISOMERIC STATES IN ODD-ODD NUCLEI

a) General Characteristics. Odd-odd nuclei differ from odd nuclei by the presence of an additional unpaired nucleon. Owing to this the total angular momenta of the nuclear states are determined by the angular momenta $j_{p}$ and $j_{n}$ of the odd proton and the odd neutron. In addition they depend on the relative orientation of $j_{p}$ and $j_{n}$. All of this leads to much complication in the level schemes of odd-odd nuclei, because to each configuration ( $\mathrm{j}_{\mathrm{p}}, \mathrm{j}_{\mathrm{n}}$ ) there can correspond a set of levels (a so-called multiplet) with spins $\left|j_{p}-j_{n}\right|,\left|j_{p}-j_{n}\right|+1, \ldots,\left|j_{p}+j_{n}\right|$ and the same parity. ${ }^{13}$

It is not hard to see that in odd-odd nuclei we shall encounter more cases than in odd nuclei of levels with large and small values of the spin, corresponding to addition or subtraction of $j_{p}$ and $j_{n}$, and consequently there is a large probability that close to the ground state there will be one or more levels with decidedly different spins.

Therefore on the most general considerations we must expect that long-lived isomeric states will be encountered much more often in odd-odd nuclei than in nuclei of other types (odd and even-even), and that for these states there are no such regularities as, for example, a concentration in "islands of isomerism."

From tables of isotopes, ${ }^{23,24}$ which give the data on the long-lived isomers of odd-odd nuclei that are known at present, it can be seen that the number of such nuclei is much larger than 100.

Attempts to apply the ideas of the shell model to the analysis of these isomers encounter great difficulties, because the character of the interaction between the unpaired nucleons is almost unknown, and therefore we know practically nothing about the order of succession of the levels in a multiplet with given values of $j_{p}$ and $j_{n}$.

To find the spin I of the ground state of a multiplet in a specific nucleus one uses the semiempirical rules of Nordheim. ${ }^{13}$ According to these rules, I is determined in the following way:

$$
\begin{array}{ll}
I=\left|j_{p}-j_{n}\right|, & \text { if }
\end{array} j_{p}=l_{p} \pm \frac{1}{2} ; j_{n}=l_{n} \mp \frac{1}{2}, ~ 子 \quad j_{p}=l_{p} \pm \frac{1}{2} ; j_{n}=l_{n} \pm \frac{1}{2}, ~ \$
$$

where $l_{\mathrm{p}}$ and $l_{\mathrm{n}}$ are the orbital angular momenta of the odd proton and neutron.

The second of these rules, however, does not give unambiguous results, and although the first enables us to predict uniquely the spins of odd-odd nuclei, its
value is much diminished by the existence of numerous exceptions.

Only in the rare cases in which the shell structure of the nucleus is simplest* do the theoretical methods ${ }^{25}$ existing at present enable us to interpret with some reliability the properties (including the spins) of the lowest excited levels.

Thus even if the configuration (the values of $j_{p}$ and $\mathrm{j}_{\mathrm{n}}$ ) of the multiplet is known, in many cases it is hard to explain the spins, not only of the isomeric state, but also of the ground state of an odd-odd nucleus. As an example to illustrate these difficulties, we may mention the odd-odd nuclei ${ }_{19} \mathrm{~K}_{21}^{40}$ and ${ }_{19} \mathrm{~K}_{23}^{42}$. They are both characterized by the same values $j_{p}=3 / 2^{+}$and $j_{n}=7 / 2^{-}$, but have different spins, $4^{-}$and $2^{-}$, respectively. ${ }^{13,24}$

To the difficulties we have mentioned we must add the difficulty of determining the configuration of the ground state (the values of $\mathrm{j}_{\mathrm{p}}$ and $\mathrm{j}_{\mathrm{n}}$ ). Evidently the interaction between the odd proton and the odd neutron can sometimes change the order in which the one-particle levels are filled, as it is known as characteristic of nuclei with odd numbers of nucleons. This can be seen easily from the example of the odd-odd nuclei of ${ }_{49}$ In, for which long-lived isomeric states are known.

The spins and parities of the ground states of these nuclei are as follows: ${ }^{23,24}$

$$
{ }_{49} \operatorname{In}_{61}^{110}-2+,{ }_{19} \operatorname{In}_{63}^{112}-1+,{ }_{49} \operatorname{In}_{65}^{114}-1+,{ }_{49} \operatorname{In}_{67}^{116}-1+\text { etc. }
$$

The Goeppert-Mayer level scheme and the data on the spins of the odd isotopes with $\mathrm{Z}=49$ indicate that $j_{p}=9 / 2^{+}\left(g_{9 / 2}\right)$.

On the same sort of evidence we can assume that for spherical nuclei with $N=65,67$, and $>67$

$$
\dot{j}_{n}=\frac{3}{2}+\left(d_{3 / 2}\right), \quad \frac{1}{2}+\left(s_{1 / 2}\right), \quad \frac{11}{2}-\left(h_{11 / 2}\right)
$$

It is not hard to see that no possible combination of these values of $j_{p}$ and $j_{n}$ can give the spin and parity $1^{+}$which is observed for $\operatorname{In}^{114}$ and $\operatorname{In}^{116}$.

A number of such examples could be given. It is the more surprising that when the situation is so complicated the behavior of the isomeric levels of certain odd-odd nuclei manifests regularities analogous to those found in the properties of the one-particle isomeric levels of nuclei with odd $A$.

As an example let us consider the properties of the isomeric levels of the odd-odd nuclei ${ }_{81} \mathrm{Tl}$ :

$$
{ }_{81} \mathrm{Tl}_{113}^{191}, \quad{ }_{81} \mathrm{Tl}_{115}^{196}, \quad{ }_{81} \mathrm{Tl}_{117}^{198}, \quad{ }_{81} \mathrm{Tl}_{121}^{202},{ }_{81} \mathrm{Tl}_{123}^{204}
$$

These nuclei are similar in that they have identical spins and parities in their ground and isomeric states: ${ }^{23,24,26} 2^{-}$and $7^{+}$.

These values of the spins can be explained if we assume that in the ground state $j_{p}=1 / 2+\left(s_{1 / 2}\right)$ and $j_{n}$ $=5 / 2^{-}\left(f_{5 / 2}\right)$. The multiplet for this configuration con-

[^2]

FIG. 7. Dependence of the interval $\Delta \mathrm{E}$ (kev) between one-particle levels $7^{+}$and $2^{-}\left(i_{13 / 2}\right.$ and $f_{5 / 2}$ in odd-odd nuclei ${ }_{81} T 1$ and odd nuclei ${ }_{80} \mathrm{Hg}$ and $8_{2} \mathrm{~Pb}$ on the number of neutrons.
tains only two levels, with the spins $2^{-}$and $3^{-}$.
The first of these levels is the ground state, and the second must exist among the excited levels and have approximately the same excitation energy in the various isotopes of ${ }_{81} \mathrm{Tl}$. In the isotopes $\mathrm{TI}^{196}$ and T1 ${ }^{198}$ levels with the expected properties have actually been observed ${ }^{26}$ (in $\mathrm{Tl}^{196}$ a $3^{-}$level at 275 kev and in $\mathrm{Tl}^{198}$ a $3^{-}$level at 283 kev ).

The isomeric state $7^{+}$can be formed as the result of the transition of the odd neutron from the $f_{5 / 2}$ level to an $i_{13 / 2}$ level, because after this transition the configuration is $j_{p}=1 / 2^{+}\left(s_{1 / 2}\right)$ and $j_{n}=13 / 2^{+}\left(i_{13 / 2}\right)$.

Therefore the interval between the levels $7^{+}$and $2^{-}$in ${ }_{81} \mathrm{Tl}$ is actually just the interval between the oneparticle levels $f_{5 / 2}$ and $i_{13 / 2}$. This interval can be compared with the interval between the one-particle levels $\mathrm{f}_{5 / 2}-\mathrm{i}_{13 / 2}$ in the odd nuclei ${ }_{80} \mathrm{Hg}$ and ${ }_{82} \mathrm{~Pb}$.

In Fig. 7, where this comparison is made, we see the quite analogous behavior of the size of the interval in the odd-odd nuclei ${ }_{81} \mathrm{Tl}$ and the odd nuclei ${ }_{80} \mathrm{Hg}$ and ${ }_{82} \mathrm{~Pb}$. This confirms the interpretation we have given for the levels of the odd-odd isotopes of Tl .

The further accumulation of experimental data will presumably make it possible to find more cases of such analogies.
b) Isomeric States of Deformed Odd-odd Nuclei. The problems of interpreting the excited levels of odd-odd nuclei are somewhat simpler in the case of deformed nuclei. In such nuclei the integral of the motion is not the angular momentum $j$ of a nucleon, but its projection on the axis of symmetry of the nucleus, $\Omega=\Lambda \pm \Sigma$, where $\Lambda$ and $\Sigma=1 / 2$ are the projections of the orbital and spin angular momenta of the odd nucleon on the axis of symmetry of the nucleus. ${ }^{14,27}$

The result is that a level with any given value of $\Omega$ can contain only one odd nucleon (and not 1,3 , $5, \ldots$, as in spherical nuclei), and the total angular
momentum of an odd group of nucleons is the angular momentum $\Omega_{\mathrm{p}(\mathrm{n})}$ of this nucleon.*

Therefore one can always find the value of $\Omega_{p(n)}$ from the Nilsson scheme of one-particle levels or by using experimental data on the spins of odd nuclei with the same odd nucleon.

Another simplification is that when there is deformation the multiplet of a given configuration always reduces to a doublet, whose terms have ${ }^{27,28}$ the spins $\mathrm{I}=\mathrm{K}=\left|\Omega_{\mathrm{p}} \pm \Omega_{\mathrm{n}}\right|$.

Gallagher and Moszkowski have proposed a semiempirical rule for the determination of the relative position of the levels in this doublet. ${ }^{29}$ According to this rule the level with $K=\left|\Omega_{p}+\Omega_{n}\right|$ is lower if the states of the odd proton and neutron are such that the signs of the spin projections $\Sigma_{p}$ and $\Sigma_{n}$ are the same ( the spin angular momenta are parallel). In the opposite case (when $\Sigma_{p}$ and $\Sigma_{n}$ have different signs, i.e., the spin angular momenta are antiparallel), the ground state of the doublet is the level with $K=\left|\Omega_{p}-\Omega_{n}\right|$.

At present the experimental data confirm this rule in most cases.

In all doublets

$$
\Delta K=\left|\Omega_{p}+\Omega_{n}\right|-\left|\Omega_{p}-\Omega_{n}\right|=2 \Omega,
$$

where $\Omega$ is the smaller of the numbers $\Omega_{\mathrm{p}}$ and $\Omega_{\mathrm{n}}$.
Except for configurations with $\Omega_{p(n)}=1 / 2, \Delta K=3$, 5,7 , etc.

Since in most cases $K=I$, the spins of the two levels differ by not less than 3 units. Therefore it can be expected that if the first excited level has the same configuration as the ground state it should be a typical long-lived isomeric level. Depending on the value of $\Delta \mathrm{I}$ in the doublet, this level will decay by $\beta$ or $\alpha$ decay (for $\Delta I>4$ ), or for $\Delta I=3,4$ by the emission of $\gamma$ rays and conversion electrons, with transitions of multipole character M3 and E4.

Observed isomeric levels that are evidently of this type are found in

$$
{ }_{11} \mathrm{Na}^{24},{ }_{13} \mathrm{Al}^{26},{ }_{71} \mathrm{Lu}^{176},{ }_{73} \mathrm{Ta}^{180},{ }_{95} \mathrm{Am}^{242}
$$

In the first of these nuclei $\Delta \mathrm{I}=3$ and an isomeric transition of type M3 is observed ( $\mathrm{E}_{\gamma}=472 \mathrm{kev}$, $\left.T_{1 / 2}=0.02 \mathrm{sec}\right)$. In the other nuclei $\Delta I>3$ and the isomeric states decay practically entirely through $\beta$ decay.

The properties of the last three of these isomers are of particular interest. In them the ground or isomeric state has $K=0^{-}$and $I=1^{-} .31$

This means that in these states the rotational angular momentum $R$ of the nucleus is different from zero ( $I^{2}=K^{2}+R^{2}$; if $K=0, I=1$, then $R=1$ ), and the nucleus rotates as a whole around an axis perpendicular to the axis of symmetry. Recently such a rotation has

[^3]been observed directly ${ }^{32}$ in the ground state of ${ }_{95} \mathrm{Am}^{242}$. Since we are concerned with the ground state or a longlived isomeric state, such a nucleus can be said to be "eternally rotating."

This peculiarity of levels with $\mathrm{K}=0$ must put its stamp on the properties of isomeric transitions in oddodd nuclei, since in this case $\Delta \mathrm{I}=\Delta \mathrm{K}-1$. The result is that the multipole index of the isomeric transition is lowered by 1 , and consequently the lifetime of the level is sharply diminished.

This peculiarity must be manifested particularly clearly in nuclei in which the ground and isomeric states have the configuration $\Omega_{p}=3 / 2$ and $\Omega_{n}=3 / 2$ ( $\mathrm{K}_{1}=3, \mathrm{~K}_{2}=0$ ). In such isomeric transitions $\Delta \mathrm{K}=3$, but $\Delta \mathrm{I}=2$, and the multipole character will be E 2 (and not M3, as in doublets with $\Delta K=3$ but $K \neq 0^{-}$).

Since transitions of type E2 are fast,* the lifetimes of such isomeric levels must be very short, and they cannot be observed by means of the usual methods for detecting activities.

Many nuclei of this type are now known:

$$
{ }_{63} \mathrm{Eu}^{154},{ }_{65} \mathrm{~Tb}^{156},{ }_{65} \mathrm{~Tb}^{158},{ }_{65} \mathrm{~Tb}^{160},{ }_{77} \mathrm{Tr}^{188},{ }_{77} \mathrm{Ir}^{190} \text { etc. }
$$

In none of these cases has a long-lived isomeric state been observed that has the same internal structure ( configuration) as the ground state. As we have seen, this is in agreement with the results of the generalized shell model.

A comparison of the properties of the nuclei ${ }_{63} \mathrm{Eu}_{89}^{152}$ and ${ }_{63} \mathrm{Eu}_{91}^{154}$ is particularly interesting in this connection. These two nuclei have the same spin and parity $3^{-}$in the ground state, and evidently have almost the same deformed equilibrium shape. At the same time, as has been noted, $\mathrm{Eu}^{154}$ has no long-lived isomeric level, but in $\mathrm{Eu}^{152}$ there is a well known isomeric state with $\mathrm{T}_{1 / 2}=9.2 \mathrm{hr} .{ }^{23,24}$ The spin and parity of this state $^{33}$ are $0^{-}$.

This difference in the properties of the adjacent nuclei $E u^{152}$ and $E u^{154}$ is due to the fact that, unlike $E u^{154}$, $E u^{152}$ is right on the boundary between spherical and deformed nuclei. Some years ago it was shown that the equilibrium shape of this nucleus must be in a state of "unstable equilibrium.",34 Therefore we may suppose that with even a slight excitation there is a change of the equilibrium shape of the nucleus, and in the isomeric state the nucleus ${ }_{63} \mathrm{Eu}_{89}^{152}$ is spherical. ${ }^{34}$

Therefore the isomeric transition in ${ }_{63} \mathrm{Eu}_{89}^{152}$ is due not only to a change of the spin by 3 units, but also to an important change of the equilibrium shape. This must greatly slow down the isomeric transition. In spite of careful searches, this transition of type M3 with energy 50 kev has not yet been detected. In comparison with ordinary one-particle proton transitions it is slowed down by a factor of more than $5 \times 10^{6}$, which fully corresponds to our expectations.

[^4]Besides isomeric levels that have the same configuration as the ground state, in odd-odd nuclei one often encounters isomeric levels associated with a change of the state of one or both odd nucleons. The parity of such a level can differ from that of the ground state. Therefore no restriction is placed on the multipole character of transitions from such levels. They can be of types E3, M3, E4, M4, etc.

Isomeric levels of this type are observed in ${ }_{65} \mathrm{~Tb}^{156}$, ${ }_{65} \mathrm{~Tb}^{158},{ }_{67} \mathrm{Ho}^{160},{ }_{73} \mathrm{Ta}^{182},{ }_{75} \mathrm{Re}^{188},{ }_{93} \mathrm{~Np}^{240},{ }_{99} \mathrm{Es}^{254}$, etc. ${ }^{23,24}$ The experimental data are inadequate, however, for a more detailed analysis of these levels.

## 3. CONFIGURATION ISOMERIC LEVELS AND POSSIBILITIES FOR THEIR EXCITATION BY MULTIPLY CHARGED IONS

a) Introduction. As is well known, excited energy levels of electrons in atoms can arise not only through transitions of electrons from one shell to another but also by various ways a given number of electrons can be accommodated within unfilled shells. For example, there are the levels ${ }^{1} \mathrm{D}_{2},{ }^{3} \mathrm{P}_{2,1,0}$, and ${ }^{1} \mathrm{~S}_{0}$ that can be formed when there are two $p$ electrons in the outer shell. The energy difference of such levels, which arises from the interactions within the shell, is relatively small ( $\sim \mathrm{ev}$ ), and is about an order of magnitude smaller than the energy difference of electrons in different shells.

Naturally in nuclei to which the shell model applies there can also be observed not only levels corresponding to transitions between shells, but also levels arising from the intrashell interaction of nucleons - the configuration levels, which have already been mentioned in the first sections of our review. The possible values of the total angular momentum of configuration levels when there are various numbers ( $k$ ) of nucleons in shells with $j=3 / 2-11 / 2$ are listed in the table, which is taken (with some emendations) from the book by M. Goeppert-Mayer and J. Jensen. ${ }^{13}$

When there are only two or three nucleons (or holes) in the shell, shifts of one nucleon (or hole) can lead to the realization of any of the configuration levels enumerated in the table. If, on the other hand, the shell contains $k \geq 4$ nucleons (or holes), one-nucleon transitions can excite only those levels which are realized with two (for even $k$ ) or three (for odd $k$ ) nucleons in the given shell. Not all of these levels, however, can be realized in one-nucleon transitions - some of them occur as the result of a mixture of one-nucleon transitions and transitions of two or more nucleons. For example, one-nucleon transitions from the ground state ( $J=0$ ) of four nucleons in the shell with $j=9 / 2$ cannot give the configuration levels with angular momenta 9, 10, and 12.

The differences of the excitation energies of the configuration levels enumerated in the table evidently amount to tens and hundreds of kev, and basically these levels are almost equidistant. Because of the necessity

Possible values of the total angular momentum for configuration levels ( j$)^{\mathrm{k}}$

| $\begin{gathered} j=3 / 2 \\ k=1 \\ 2 \end{gathered}$ | $3 / 2$ 0.2 |
| :---: | :---: |
| $\begin{gathered} j=5 / 2 \\ k=1 \\ 2 \\ 3 \end{gathered}$ | $\begin{aligned} & 5 / 2 \\ & 0,2,4 \\ & 3 / 2,5 / 2,9 / 2 \end{aligned}$ |
| $\begin{gathered} j=7 / 2 \\ k=1 \\ 2 \\ 3 \\ 3 \\ 4 \end{gathered}$ | $\begin{aligned} & \begin{array}{l} 7 / 2 \\ 0,2,4,6 \\ 3 / 2,5 / 2,7 / 2,9 / 2,11 / 2,15 / 2 \\ 0,2,2,4,4,5,6,8 \end{array} \end{aligned}$ |
| $\begin{gathered} j=9 / 2 \\ k=1 \\ 2 \\ 3 \\ 4 \\ 4 \\ 5 \end{gathered}$ | $\begin{aligned} & 9 / 2 \\ & 0,2,4,6,8 \\ & 3 / 2,5 / 2,7 / 2,9 / 2,9 / 2,11 / 2,13 / 2,15 / 2,17 / 2,21 / 2 \\ & 0,0,2,2,3,4,4,4,5,6,6,6,7,8,8,9,10,12 \\ & 1 / 2,3 / 2,5 / 2,5 / 2,7 / 2,7 / 2,9 / 2,9 / 2,9 / 2,11 / 2,11 / 2,13 / 2,13 / 2, \\ & 15 / 2,15 / 2,17 / 2,17 / 2,19 / 2,21 / 2,25 / 2 \end{aligned}$ |
|  | $\begin{aligned} & 11 / 2 \\ & 0,2,4,6,8,10 \\ & 3 / 2,5 / 2,7 / 2,9 / 2,9 / 2,11 / 2,11 / 2,13 / 2,15 / 2,15 / 2,17 / 2,19 / 2, \\ & 21 / 2,23 / 2,27 / 2 \\ & 0,0,2,2,2,3,4,4,4,4,5,5,6,6,6,6,7,7,8,8,8,8,9, \\ & 9,10,10,10,11,12,12,13,14416 \\ & 1 / 2,3 / 2,3 / 2,5 / 2,5 / 2,5 / 2,7 / 2,7 / 2,7 / 2,7 / 2,9 / 2,9 / 2,9 / 2,99 / 2 \\ & 11 / 2,11 / 2,11 / 2,11 / 2,11 / 2,12 / 2,13 / 213,13,13 / 2,15 / 2,15 / 2,15 / 2, \\ & 15 / 2,15 / 2,17 / 2,17 / 2,17 / 2,17 / 2,19 / 2,19 / 2,19 / 2,19 / 2,212, \\ & 21 / 2,21 / 2,23 / 2,23 / 2,23 / 2,25 / 2,25 / 2,27 / 2,27 / 2,29 / 2,31 / 2,35 / 2 \\ & 0,0,0,2,2,2,2,3,3,3,4,4,44,4,4,4,5,5,5,6,6,6,6, \\ & 6,6,6,7,7,7,7,8,8,8,8,8,8,9,9,9,9,10,10,10,10, \\ & 10,11,11,12,12,12,12,13,13,14,14,15,16,18 \end{aligned}$ |

for breaking up pairs of nucleons, however, the excitation energy of the first of the levels can be considerably larger ( $\sim \mathrm{Mev}$ ). Therefore the entire "fine" structure of the configuration levels will by no means necessarily lie below all of the levels from intershell transitions, as is the case with rotational levels. It is possible for a level from an intershell transition to lie below the group of configuration levels (if the energy of this transition is less than the pairing energy), or for it to fall within the range of the group. This, however, does not change the general pattern of the transitions between the configuration levels. These transitions must occur with the emission of E2 or M1 radiation or conversion electrons. For an energy difference of $20-100 \mathrm{kev}$ between the levels the time for an E2 transition, calculated (without taking conversion into account) for a one-proton transition, ${ }^{12,36}$ is $\tau$ $=\left(6 \times 10^{-4}-2\right) \mathrm{A}^{-4 / 3} \mathrm{sec}$. Inclusion of the effect of electron conversion can lower the upper limiting value of $\tau$ by two or three orders of magnitude. It is obvious, however, that when conversion is taken into account there is a quite real possibility of detecting isomeric E2 transitions between configuration levels. As is clear from the table, such transitions (which, by the way, must in large part fall in the "millisecond"
region $\mathrm{T}_{1 / 2}=10^{-5}-1 \mathrm{sec}$ ) are most probable in shells in which the number of nucleons (or holes) is 2.
b) Cross Sections for the Excitation of Configuration Levels by Multiply Charged Ions. While they sometimes have angular momenta very different from that of the ground state, configuration levels of the intrashell interaction are at the same time characterized by relatively small excitation energies.

This causes a specific feature of their excitation: the most characteristic intrashell levels (with large angular momenta) cannot be obtained by bombardment of nuclei with elementary particles, nor as the result of $\beta$ decay or $\gamma$ radiation from a level of another type.

For example, even for the relatively small angularmomentum change $\Delta \mathrm{j}=4$ [for instance, in the production of $\mathrm{O}^{18 *}\left(4^{-}\right)$by the inelastic scattering of neutrons] one must bombard $\mathrm{O}^{18}$ nuclei with neutrons with energies of the order of tens of Mev (which corresponds to the condition $R_{O^{18}} \sim 4 \lambda_{n}$ ). It is clear that at such an energy all sorts of nuclear reactions will predominate, and the excitation of intrashell levels is extremely improbable. Therefore, as has been pointed out in reference 37 , the most effective way of exciting configuration levels with large angular momenta must evidently be by interaction with complex nuclei, i.e., by bombard-
ing nuclei with multiply charged ions with sufficiently large values of the angular momentum of the outer neutron shells.* As examples we can name the ions ${ }_{8} \mathrm{O}^{18}$ (neutrons $\mathrm{d}_{5 / 2}^{2}$ ), $\mathrm{Ne}^{20}\left(\mathrm{~d}_{5 / 2}^{2}\right),{ }_{18} \mathrm{~A}^{40}\left(\mathrm{f}_{7 / 2}^{2}\right),{ }_{36} \mathrm{Kr}^{88}\left(\mathrm{~g}_{9 / 2}^{7}\right)$, ${ }_{36} \mathrm{Kr}^{84}\left(\mathrm{~g}_{9 / 2}^{8}\right)$. The excitation of configuration levels can occur both in inelastic scattering, for example,

$$
{ }_{18} \mathrm{Ar}^{40}(0)+{ }_{36} \mathrm{Kr}^{83}(9 / 2+) \rightarrow{ }_{18} \mathrm{Ar}^{40 *}\left(6 \mathcal{\gamma}^{-}\right)+{ }_{36} \mathrm{Kr}^{83 *}(21 / 2+),
$$

and also sometimes in neutron transfer processes of the type

$$
{ }_{36} \mathrm{Kr}^{83}(9 / 2+) \div{ }_{38} \mathrm{Sr}^{87}(9 / 2+) \rightarrow{ }_{36} \mathrm{Kr}^{84 *}(8+)+{ }_{38} \mathrm{Sr}^{86 *}(8+)
$$

Neutron-transfer processes have, it is true, been rather thoroughly analyzed theoretically in a number of papers (cf. e.g., references 38 and 39), without a special analysis of the possibility of excitation of configuration and other levels. The theory of the excitation of configuration levels in inelastic scattering of complex nuclei is given in references 37 and 40 .

In reference 37 one of us has treated in the quasiclassical approximation an interaction between complex nuclei 1 and 2 in which there is an actual exchange of two neutrons: one neutron goes from nucleus 1 to nucleus 2 , and the other goes from 2 to 1 . The cross section for this neutron-exchange interaction is proportional to the exponential factor

$$
\begin{equation*}
W=\exp \left\{-\frac{2}{\hbar} \frac{Z_{1} Z_{2} e^{2}}{E_{0}}\left(\sqrt{2 m I_{1}}+\sqrt{2 m I_{2}}\right)\right\} \tag{1}
\end{equation*}
$$

where $Z_{1}$ and $Z_{2}$ are the charges of the nuclei, $I_{1}$ and $I_{2}$ are the binding energies of a neutron in nuclei 1 and $2, \mathrm{~m}$ is the mass of the neutron, and $\mathrm{E}_{0}$ is the energy of the relative motion of the nuclei. The calculation of the exponential for the exchange of the neutrons was made in complete analogy with the calculation of E. M. Lifshitz ${ }^{38}$ for the process of transfer of a neutron from one complex nucleus to another. The coefficient of the exponential was estimated from a comparison of the Lifshitz formula with experimental data on the transfer of neutrons from $\mathrm{N}^{14}$ nuclei. In this way the energy dependence of the cross section for processes of interaction by direct neutron exchange was obtained, and estimates were found for the size of such cross sections near the limiting barrier energy of the colliding nuclei - values $10^{-30}$ to $10^{-29} \mathrm{~cm}^{2}$. Recently papers have appeared ${ }^{41,42}$ reporting the observation of "exchange transfer" of neutrons and other particles, for example ( $p \rightleftarrows \mathrm{n}$ ); the cross section for this process at the top of the Coulomb barrier is $2 \times 10^{-30} \mathrm{~cm}^{2}$, in agreement with the estimate. ${ }^{37}$ Thus experimental data obtained from the study of reactions of heavy ions confirm the reasonableness of this sort of approach to the probability of the observation of many new configuration isomeric levels. In reference 40 the theory of the excitation of such levels has been further extended to the

[^5]case in which the process is not due to the actual exchange of two neutrons. The formula (1) can be rewritten in the form
\[

$$
\begin{equation*}
W=\exp \left\{-2 R\left(\mu_{1}+x_{2}\right)\right\}, \tag{2}
\end{equation*}
$$

\]

where $R=Z_{1} Z_{2} e^{2} / E_{0}$ is the minimum possible distance between the centers of the colliding nuclei, as calculated classically from the Coulomb interaction, and $\kappa_{i}$ $=\left(2 \mathrm{mI}_{\mathrm{i}} / \hbar^{2}\right)^{1 / 2}$ is the reciprocal of the relaxation length of the wave function outside the nucleus, computed for a neutron with binding energy $\mathrm{I}_{\mathrm{i}}$. This must be the form of the expression for the probability of excitation of one-nucleon neutron levels, not only in the case of actual neutron exchange, but also when the possible virtual processes are considered; the only difference is that instead of $I_{1}$ and $I_{2}$ one inserts in Eq. (2) the values I and I*, the binding energies of the neutron in the ground and excited states of the nucleus. In fact, we can then interpret Eq. (2) as the exponential factor in the square of the absolute value of the matrix element for the transition of a neutron from the state of energy I to that of energy I* owing to a perturbation localized at the distance R from the nucleus.

In the calculation of the cross sections for excitation of neutron levels by such a mechanism made in reference 40 account was also taken of some features not considered in the approximate calculations, namely the finite size of the nucleus and the nonadiabatic nature of the process (the necessity for transfer of part of the energy of the colliding nuclei to excitation of the level). These two effects act in opposite directions, but cannot completely compensate each other, because the correction to the coefficient of the exponential associated with the finite size of the nucleus does not depend on the energy, whereas the effect of the nonadiabatic nature of the process gives a factor which falls off exponentially for small energies of the colliding nuclei (which also causes the predominance of excitations of levels with small energies, for example configuration levels). Calculations ${ }^{40}$ made in the quasi-classical approximation (i.e., for the classical Coulomb trajectory of the colliding particles) gave the following expression for the cross section for excitation of neutron levels:

$$
\begin{align*}
& \sigma_{\text {exc }}=\sigma_{\text {exc }}^{0} \exp \left\{-2(R-\varrho)\left(x+x^{*}\right)\right. \\
& \left.\quad-2 R \frac{\omega}{v_{0}} \operatorname{arctg} \frac{\omega}{v_{0}\left(x+x^{*}\right)}\right\}, \tag{3}
\end{align*}
$$

where $\sigma_{\text {exc }}^{0}$ is a quantity which varies rather slowly with the energy and is close (in order of magnitude) to the geometrical cross section, $\rho$ is the sum of the radii of the colliding nuclei, $\omega=\left(\mathrm{I}-\mathrm{I}^{*}\right) / \hbar$ is the frequency of the transition, and $v_{0}$ is the relative velocity of the motion of the colliding nuclei at infinity.

On the other hand, it can be shown that the cross section for the transfer of a neutron from one nucleus

[^6](with binding energy $\mathrm{I}_{\alpha}$ for the neutron) to another (with binding energy $I_{\beta}$ for the neutron in the final nucleus) can be written, for the case $\mathrm{I}_{\beta}<\mathrm{I}_{\alpha}$, in the form
\[

$$
\begin{equation*}
\sigma_{\text {trans }}=\sigma_{\text {trans }}^{0} \exp \left\{-2(R-\varrho) \kappa_{\beta}-2 R \frac{\omega^{\prime}}{v_{0}} \operatorname{arctg} \frac{\omega^{\prime}}{v_{0} x_{\beta}}\right\} \tag{4}
\end{equation*}
$$

\]

where $\kappa_{\beta}=\left(2 \mathrm{mI}_{\beta}\right)^{1 / 2}, \omega^{\prime}=\left(\mathrm{I}_{\alpha}-\mathrm{I}_{\beta}\right) / \hbar$, and $\sigma_{\text {trans }}^{0}$ is a quantity of the order of magnitude of the $\sigma_{\text {exc }}^{0}$ for the final nucleus. Therefore the excitation cross section of a nucleus with N neutrons and Z protons is

$$
\begin{align*}
& \sigma_{\mathrm{exc}}(N, Z) \approx \sigma_{\mathrm{trans}}(N-1, Z) \exp \{-2(R-\varrho) x(N, Z) \\
& \quad-2 R \frac{I(N, Z)-I^{*}(N, Z)}{v_{0}} \operatorname{arctg} \frac{I(N, Z)-I^{*}(N, Z)}{1 v_{\mathrm{n}}\left[x(N, Z)+x^{*}(N, Z)\right]} \\
& \left.\quad+2 R \frac{I(N-1, Z)-I(N, Z)}{V_{0} v_{0}} \operatorname{arctg} \frac{I(N-1, Z)-I(N, Z)}{1 \sigma_{0}[x(N-1, Z)+x(N, Z)]}\right\} \tag{5}
\end{align*}
$$

where $\sigma_{\text {exc }}$ and $\sigma_{\text {trans }}$ are to be taken at the same speed of the relative motion. The numerical values of the excitation cross sections are close to the values obtained in reference 37 for the direct neutron-exchange interaction.

The detection of inelastic scattering by the subsequent radiation of millisecond duration is of course more difficult than the detection, already accomplished, of $p \rightleftarrows n$ exchange processes, which lead to the formation of relatively long-lived active products. Work with heavy ions is, however, coming to be done on a wider and wider scale, and there is no doubt that its further development will in the next few years make it possible to establish the existence and properties of many new configuration isomeric levels.

Note added in proof. The candidate's dissertation of A. M. Morozov (Institute of Chemical Physics, Academy of Sciences, U.S.S.R., and Moscow Engineering Physics Institute) gives a complete list of elements which have been bombarded (either in the pure state or in compounds) with $19.2-\mathrm{Mev}$ protons in the search for 'millisecond"' isomers, in various researches in which the author of the dissertation has taken part (see list of literature). This list provides us with 38 elements that have been bombarded with protons, in addition to those indicated in Fig. 1a; their atomic numbers are $Z=3,5,9,11,14,15,16,17,19,23,24,25,26$, $27,35,37,44,45,46,51,52,53,55,56,58,59,63,64$, $65,66,67,68,70,71,72,76,79,90$.

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## Supplementary Literature

(Papers on individual isomers, $1958-1961$, not cited in review)
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Translated by W. H. Furry


[^0]:    *As is well known, this feature of the intervals is also observed for isomeric transitions of type M4 (intervals $\mathrm{P}_{1 / 2}-\mathrm{g}_{9}, \mathrm{~d}_{3} / 2 \mathrm{~h}_{11 / 2}$, $\mathrm{f}_{5 / 2}-\mathrm{i}_{13 / 2}$ ).

[^1]:    *We emphasize that in the nuclei $\mathrm{Pd}^{105}$ and $\mathrm{Eu}^{151}$ the cross sections for the Coulomb excitation of the levels with $I=I_{0}+1$ are small, ${ }^{22}$ and these levels cannot be interpreted as collective levels.

[^2]:    *For example, in nuclei ${ }^{25}$ of the type of ${ }_{81} \mathrm{Tl}_{125}^{206}$ and ${ }_{83} \mathrm{Bi}_{127}^{210}$, in which the configuration of the outer nucleons includes either one each of neutrons and protons, or one each of their "holes."

[^3]:    *We recall that in spherical nuclei the total angular momentum of an odd group of nucleons is not always equal to that of the last odd nucleon.

[^4]:    *The slowing down of these transitions owing to the weak K selection rule, which comes into play in cases with $\Delta K>\Delta I$, is of no practical importance.

[^5]:    *It can also not be excluded that there is production of nuclei with excited intrashell-interaction levels among the fission fragments.

[^6]:    $* \operatorname{arctg}=\tan ^{-1}$.

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