FLUCTUATIONS OF ELECTROMAGNETIC WAVES IN THE TROPOSPHERE IN THE PRESENCE OF SEPARATION BOUNDARIES

F. G. BASS, S. Ya. BRAUDE, É. A. KANER, and A. V. MEN'

Usp. Fiz. Nauk **73**, 89-119 (January, 1961)

CONTENTS

INTRODUCTION

THE propagation of radio waves in a real medium is usually accompanied by amplitude, phase, and frequency fluctuations which are due to the spatial inhomogeneity of the medium and to the time variation of its properties. The temperature, density, and water vapor tension fluctuations connected with the turbulent processes that take place in the troposphere manifest themselves in the case of electromagnetic waves in fluctuations of the dielectric constant ϵ. These fluctuations lead to such phenomena as the flicker of stars, fading of diverse kind, etc., and are due to scattering of the waves by the inhomogeneities.

The extent of the influence of fluctuations on the propagation of waves in an unbounded medium has been investigated for a long time. The first to call attention to this effect was Smoluchowski. Einstein has calculated the scattering by the fluctuations, and Rayleigh showed that it causes the blue color of the sky. A considerable number of investigations have been devoted to these problems.[1-14] In many cases, particularly in the propagation of electromagnetic waves in the troposphere, the separation boundary (the earth's surface) exerts an appreciable influence, for its presence gives rise to an interference structure, or, as it is sometimes called, a "lobe" structure of the radiation field in space, due to the reflection of the waves from the surface. The experimental data obtained in the presence of a separation boundary can therefore differ substantially from the results obtained by calculating the fluctuation effects for free space.

A more detailed examination of this question shows that the inclusion of the separation boundary leads to qualitatively new phenomena. We shall indicate two of these by way of illustration. If the separation boundary can be approximated by a highly-reflecting plane, the Fresnel coefficient of which is close to unity, then we know[15] that the regular component of the field vanishes (or is very small) at some points in space, owing to interference between the direct and reflected waves. In addition, at small glancing angles the tangential field components decrease much more rapidly at the start of the first interference lobe than in free space, namely in inverse proportion to the square of the distance, rather than to the first degree of the distance (the region of applicability of the so-called "quadratic formula" of Vvedenskii[15]).

In either case, the absolute fluctuations of the field do not increase appreciably compared with free space, whereas the relative fluctuations should obviously increase sharply.

An investigation of the fluctuations of electromagnetic waves above the separation boundary makes it possible to determine many important characteristics of the medium, such as the average dimension of the inhomogeneities, the mean square fluctuations, the correlation function of the dielectric constant, etc., and also to explain the influence of external factors on these characteristics.

Although fluctuations in unbounded space have been investigated rather thoroughly[1-14] and two monographs have been published on the subject,[1,112] the influence of the separation boundary on the fluctuations has been the subject of relatively few investigations, and this problem has not been considered in the review literature.

In the present review we discuss the results of theoretical and experimental investigations of fluctuations of the amplitudes, phases, and other characteristics of electromagnetic waves propagating above a separation boundary.

The first (theoretical) part of the survey is devoted to an examination of fluctuations above a plane separation boundary. Fluctuations in an unbounded medium are considered quite briefly, and principal attention is paid to those aspects of the problem which have not been sufficiently discussed in the literature. The second part contains a description of the procedure and the results of experimental investigations of the fluctuations.

Both theoretical and experimental results apply essentially to the case of the far zone ($L\lambda \gg l^2$, L — length of the path, λ — wavelength, l — characteristic dimension of the inhomogeneities), where the influence of fluctuations is most significant. Since $\lambda \ll l$ at the centimeter waves used in the experiments, the far zone corresponding to a troposphere with characteristic parameters $|\delta\epsilon| \sim 10^{-6}$, $l \sim 1$ to 10 m and $\lambda \approx 10$ cm amounts to $L \geq 1000$ m. Let us note that although we speak everywhere of electromagnetic waves, the theoretical results can be applied in most cases without modification to acoustic waves.

I. THEORY

1. Statistical Characteristics of the Electromagnetic Field

A complete statistical description of a random vector is given by the distribution function of its components. If the point of observation is separated from the source by a distance much greater than the characteristic radius of correlation l of the fluctuations $\delta\epsilon$ of the dielectric constant, then the distribution of the electromagnetic field is normal. Actually, as will be shown below, the random fluctuations of the electromagnetic field are a linear functional of $\delta\epsilon$. Dividing the entire path of propagation of the wave into segments on the order of the correlation radius, we can readily see that the fluctuations of the electromagnetic field at the point of observation are determined by the sum of the field fluctuations at each of the segments. Since the number of segments is large and the fluctuations on each segment are practically independent, the field at the point of reception is the sum of the large number of independent random terms, which has, in accordance with the limit theorem of probability theory, a normal distribution (see note added in proof at the end of this article).

Thus, the distribution function of each component of the field, $\mathscr{E} = \mathscr{E}_r + i\mathscr{E}_i$ ($\mathscr{E}_r$ and $\mathscr{E}_i$ — real and imaginary parts of $\mathscr{E}$) has the form

$$f(\mathscr{E}_r, \mathscr{E}_i) = A_0 \exp\left[-\left(\frac{\xi_r}{a}\right)^2 - \left(\frac{\xi_i}{b}\right)^2 + 2\frac{q\xi_r\xi_i}{ab}\right], \tag{1.1}$$

where

$$\xi_r = \mathscr{E}_r - E_r, \quad \xi_i = \mathscr{E}_i - E_i, \quad E = \langle\mathscr{E}\rangle,$$
$$a^2 = 2(1-q^2)\langle\xi_r^2\rangle, \quad b^2 = 2(1-q^2)\langle\xi_i^2\rangle,$$
$$A_0 = \frac{(1-q^2)^{1/2}}{\pi ab}, \quad q = \frac{\langle\xi_r\xi_i\rangle}{\pi ab}. \tag{1.2}$$

The angle brackets denote statistical averaging.

Thus, for a complete statistical description of the fluctuating electromagnetic field it is necessary to know the average and the mean square values of the real and imaginary parts of each field component, as well as their autocorrelation function $\langle\xi_r\xi_i\rangle$. The question of determining these quantities will be considered later on.

From formula (1.1) we readily obtain the distributions of the phase and absolute value of the quantity $\xi = \xi_r + i\xi$

$$\left.\begin{aligned} f(\varphi) &= \frac{A_0}{2}\left(\frac{\cos^2\varphi}{a^2} + \frac{\sin^2\varphi}{b^2} - \frac{2q\sin\varphi\cos\varphi}{ab}\right)^{-1}, \\ f(|\xi|) &= 2\pi A_0 \exp\left[-\frac{|\xi|^2}{2}(a^{-2}+b^{-2})\right]\times \\ &\times I_0\left(\frac{|\xi|^2}{2a^2b^2}[(a^2-b^2)^2+4q^2a^2b^2]^{1/2}\right), \end{aligned}\right\} \tag{1.3}$$

where $\varphi = \arg\xi$ and $I_0(x)$ is the modified Bessel function of zero order.

The equations in (1.3) have been described in detail in the literature (see, for example, references 16 and 17). By knowing the distribution function (1.1) we can determine any statistical characteristic of the fluctuating field.

Let us consider two limiting cases. The first corresponds to the case where the square of the average field is much greater than the square of the fluctuations ($|E^2| \gg \langle|\xi^2|\rangle$) and the second to the case when the average field can be neglected compared with the fluctuation field ($|E^2| \ll \langle|\xi^2|\rangle$).

In the first case we have the following formulas[18,19] for the statistical characteristics of the electromagnetic field:

1. The average phase $\langle\varphi\rangle$ is

$$\langle\varphi\rangle \equiv \left\langle \operatorname{arctg}\left(\frac{\mathscr{E}_i}{\mathscr{E}_r}\right) + \pi s(\mathscr{E}_r)\right\rangle$$
$$= \varphi_0 + (\langle\xi_r^2\rangle - \langle\xi_i^2\rangle)(\sin\varphi_0\cos\varphi_0 + \cos 2\varphi_0)|E^{-2}|, \tag{1.4*}$$

where φ_0 is the phase of the average field $E = |E| \times \exp i\varphi_0$, $s(x) = 0$ $(x > 0)$, $s(x) = 1$ $(x < 0)$, $\tan^{-1}x$ is defined in the interval $(-\pi/2, \pi/2)$, and the phase is defined in the interval $(-\pi/2, 3\pi/2)$.

2. Mean square phase fluctuation $\langle\delta\varphi^2\rangle$:

$$\langle\delta\varphi^2\rangle \equiv \langle(\varphi - \langle\varphi\rangle)^2\rangle = |E^{-2}|(\langle\xi_r^2\rangle\sin^2\varphi_0 + \langle\xi_i^2\rangle\cos^2\varphi_0 - \langle\xi_i\xi_r\rangle\sin 2\varphi_0). \tag{1.5}$$

3. Average amplitude $\langle|\xi|\rangle$:

*arctg = $\tan^{-1}$.

$$\langle |\mathscr{E}|\rangle \equiv \langle(\mathscr{E}_r^2+\mathscr{E}_i^2)^{1/2}\rangle = |E| + (\langle\xi_r^2\rangle\cos^2\varphi_0 + \langle\xi_i^2\rangle\sin^2\varphi_0 - \langle\xi_i\xi_r\rangle\sin 2\varphi_0\,(2|E|)^{-1}. \quad (1.6)$$

4. Mean square of the amplitude fluctuations $\langle\delta A^2\rangle$:

$$\langle\delta A^2\rangle \equiv \langle(|\mathscr{E}| - \langle|\mathscr{E}|\rangle)^2\rangle = \langle\xi_r^2\rangle\cos^2\varphi_0 + \langle\xi_i^2\rangle\sin^2\varphi + \langle\xi_r\xi_i\rangle\sin 2\varphi_0. \quad (1.7)$$

5. Autocorrelation of the amplitude and phase $\langle\delta\varphi\frac{\delta A}{|E|}\rangle$:

$$\left\langle\delta\varphi\frac{\delta A}{|E|}\right\rangle \equiv \left\langle(\varphi-\langle\varphi\rangle)\frac{(|\mathscr{E}|-\langle|\mathscr{E}|\rangle)}{|E|}\right\rangle = \frac{(\langle\xi_i^2\rangle-\langle\xi_r^2\rangle)\sin 2\varphi_0}{2|E^2|} + \frac{\langle\xi_r\xi_i\rangle\cos 2\varphi_0}{|E^2|}. \quad (1.8)$$

Thus, since $\delta\varphi$ and δA are linear functions of ξ_r and ξ_i when $\langle|\xi^2|\rangle \ll |E^2|$, they also have a normal distribution. Formulas (1.4) — (1.8) were obtained by expanding the corresponding expressions in powers of $\xi_i/|E|$ and $\xi_r/|E|$. A very important case is one in which $\langle\xi_r^2\rangle = \langle\xi_i^2\rangle = \frac{1}{2}\langle|\xi^2|\rangle$, and $\langle\xi_r\xi_i\rangle = 0$. In this case

$$\sigma = \langle\delta\varphi^2\rangle = \frac{\langle\delta A^2\rangle}{|E^2|} = \frac{\langle|\xi^2|\rangle}{2|E^2|}, \quad (1.9)$$

and the autocorrelation between the fluctuations of the amplitude and the phase vanishes.

In the second limiting case the inequality $\langle|\xi^2|\rangle > |E^2|$ holds. In order to simplify the analysis, we shall assume conditions (1.9) to be satisfied. Then the phase has a uniform probability distribution and the amplitude a Rayleigh distribution. The statistical characteristics of the electromagnetic field satisfy the following relations

$$f(\varphi) = (2\pi)^{-1},\ \langle\varphi\rangle = \frac{\pi}{2},\ \langle\delta\varphi^2\rangle = \frac{\pi^2}{3},$$
$$f(|\xi|) = 2a^{-2}|\xi|\exp\left(-\frac{|\xi|^2}{a^2}\right),\ \langle|\xi|\rangle = \left(\frac{\pi}{8}\langle|\xi|^2\rangle\right)^{1/2}. \quad (1.10)$$

Formulas (1.10) for $\langle\varphi\rangle$ and $\langle\delta\varphi^2\rangle$ are obviously related with the choice of the interval $(-\pi/2, 3\pi/2)$ in which the phase is defined.

The relative variation in amplitude is conveniently described by the following quantity

$$\langle(\ln|\mathscr{E}| - \langle\ln|\mathscr{E}|\rangle)^2\rangle \approx \langle(\ln|\xi| - \langle\ln|\xi|\rangle)^2\rangle = \pi^2/24. \quad (1.11)$$

2. Fluctuations of Electromagnetic Field in Unbounded Space

To solve the problem it is necessary to determine the dependence of the first two moments on the propagation conditions (length of the route, frequency, and polarization of radiation, etc.) and the statistical characteristics of the dielectric constant ϵ. Without loss of generality, we can assume the average dielectric constant ϵ to be unity:

$$\varepsilon = 1 + \delta\varepsilon, \quad (2.1)$$

where $\delta\epsilon$ is the fluctuation of the dielectric constant. The correlation of the fluctuations of the dielectric constant will be assumed to be a homogeneous function, i.e., we shall assume that it has the form

$$\langle\delta\varepsilon(\mathbf{r}_1)\,\delta\varepsilon(\mathbf{r}_2)\rangle = \langle\delta\varepsilon^2\rangle\,W(|x_1-x_2|, |y_1-y_2|, |z_1-z_2|). \quad (2.2)$$

In view of the statistical homogeneity of the medium, $\langle\delta\epsilon^2\rangle$ is independent of the coordinates, and the correlation coefficient W depends only on the moduli of the differences of the components of the vectors $\mathbf{r}_1$ and $\mathbf{r}_2$.*

As a rule, we shall not specify the details of the correlation function, since the final results depend little on this function.

The Maxwell equations that describe the propagation of electromagnetic waves from a point source in a medium with a fluctuating dielectric constant can be reduced to the form

$$\mathrm{rot\,rot}\,\mathscr{E} - k^2(1+\delta\varepsilon)\mathscr{E} = \mathbf{p}\delta(\mathbf{r}-\mathbf{r}_0). \quad (2.3)†$$

The time dependence is assumed in the form $\exp(-i\omega t)$, $k = \omega/c = 2\pi/\lambda$, $\mathbf{p} = 4\pi k^2\mathbf{d}$ ($\mathbf{d}$ is the dipole moment of the source and $\mathbf{r}_0$ the radius vector of the location of the source).

The fluctuations of the dielectric constant in the troposphere are small, so that we can use the perturbation method to determine the electromagnetic field. The use of the perturbation method in its usual form, corresponding to the Born approximation in scattering problems, leads to a contradiction with the energy conservation law.[8] To avoid this contradiction, we must either resort to renormalization of the solution,[8] or to take into account the second approximation in the method of smooth perturbations.[20] Suitable results can be obtained in the most consistent and clearest fashion by using the method of small perturbations in the form proposed in reference 21.

Let us average (2.3) and subtract the averaged equation from the unaveraged one. We obtain as a result the following system of equations for $\mathbf{E}$ and $\boldsymbol{\xi}$:

$$\mathrm{rot\,rot}\,\mathbf{E} - k^2(\mathbf{E} + \langle\delta\varepsilon\xi\rangle) = \mathbf{p}\delta(\mathbf{r}-\mathbf{r}_0),$$
$$\mathrm{rot\,rot}\,\xi - k^2\xi = k^2\mathbf{E}\,\delta\varepsilon. \quad (2.4)$$

In the derivation of the second equation of the system we discarded terms quadratic in $\delta\epsilon$.

The solution of the second equation of (2.4) in free space is known[21] to have the form

$$\xi_i(\mathbf{r}) = \frac{k^2}{4\pi}\int d\mathbf{r}'\,\delta\varepsilon(\mathbf{r}')\left[\left(\delta_{ik} + \frac{1}{k^2}\frac{\partial^2}{\partial x_i'\,\partial x_k'}\right)\frac{\exp ik|\mathbf{r}-\mathbf{r}'|}{|\mathbf{r}-\mathbf{r}'|}\right]E_k(\mathbf{r}'). \quad (2.5)$$

Substituting (2.5) into (2.4) we obtain the equation for the average field

*We shall not consider the time correlation due to the motion of the inhomogeneities.

†rot = curl.

$$\left[(\Delta + k^2)\,\delta_{ik} - \frac{\partial^2}{\partial x_i\,\partial x_k} \right] E_k(\mathbf{r}) + \frac{k^2\langle\delta\varepsilon^2\rangle}{4\pi} \int d\mathbf{r}' W(\mathbf{r}-\mathbf{r}')\,E_k(\mathbf{r}')$$
$$\times \left[\left(\delta_{ik} + \frac{1}{k^2}\frac{\partial^2}{\partial x_i'\partial x_k'} \right) \frac{\exp(ik\,|\,\mathbf{r}-\mathbf{r}'\,|)}{|\,\mathbf{r}-\mathbf{r}'\,|} \right] = -p_i\delta(\mathbf{r}-\mathbf{r}_0). \tag{2.6}$$

This integro-differential equation is readily solved by the Fourier method. We consider two limiting cases. We assume first that the distances over which the electromagnetic field changes substantially is much greater than the characteristic correlation radius l. In other words, the wavelength is large compared with l (small-scale fluctuations). We can then move $E_k(\mathbf{r}')$ in Eq. (2.6) outside the integral sign at the point $\mathbf{r}$, so that Eq. (2.6) assumes the form[22]

$$\left(\Delta\delta_{ik} + k^2\varepsilon_{ik} - \frac{\partial^2}{\partial x_i\,\partial x_k} \right) E_k = -p_i\delta(\mathbf{r}-\mathbf{r}_0), \tag{2.7}$$

where

$$\varepsilon_{ik} = \delta_{ik} + \frac{\langle\delta\varepsilon^2\rangle}{4\pi}\int \frac{d\varrho}{\varrho}\frac{\partial^2 W(\varrho)}{\partial\varrho_i\,\partial\varrho_k}$$
$$+ \frac{ik^3\langle\delta\varepsilon^2\rangle}{4\pi}\left[\delta_{ik}\int d\varrho W(\varrho) - \frac{1}{6}\int d\varrho\varrho^2 \frac{\partial^2 W(\varrho)}{\partial\varrho_i\,\partial\varrho_k} \right]. \tag{2.8}$$

The formula (2.8) for ϵ_{ik} was obtained under the assumption that $W(\rho)$ depends on all three components of the vector ρ. In the one-dimensional case, when $W(\rho)$ depends only on a single coordinate (say, x), we have

$$\varepsilon_{xx} = 1 - \langle\delta\varepsilon^2\rangle, \quad \varepsilon_{ik} = (1 + ikl\,\langle\delta\varepsilon^2\rangle)\,\delta_{ik}, \tag{2.9}$$

where i and k are not equal to x simultaneously. Here $l = \int_0^\infty dx\ W(x)$. It follows from (2.7) that the average electric field in a medium with random anisotropic homogeneous fluctuations of the dielectric constant is described by the same equations as in a single crystal, and consequently two waves with different phase velocities can propagate in such a medium.[23]

If $W(\rho)$ depends only on the modulus of ρ, it turns out[24] that

$$\varepsilon_{ik} = \left[1 - \frac{1}{3}\langle\delta\varepsilon^2\rangle(1 - 2ik^3\overline{l^3}) \right]\delta_{ik}, \tag{2.10}$$

where

$$\overline{l^3} = \int_0^\infty d\varrho\varrho^2 W(\varrho).$$

In the second limiting case, when the characteristic correlation radius is much greater than the distances over which the electromagnetic field changes appreciably (large-scale fluctuations, $kl \gg 1$), Eq. (2.7) does not reduce generally speaking to a differential equation. At considerably large distances from the source, however, the wave can be regarded as plane in the entire significant region of integration in Eq. (2.6), i.e., we can put $\mathbf{E}(\mathbf{r}') = \mathbf{E}(\mathbf{r})\exp\{ik(\mathbf{r}-\mathbf{r}')\}$. In order for such an approximation to be legitimate, the relation $kl^2 \ll L$ must be satisfied, where L is the distance from the source to the point $\mathbf{r}$; this distance is of the same order of magnitude as the radius of the wave front.

It will be shown later on that the effect of the fluctuations on the average field manifests itself only at large L, and therefore the assumption of a plane wave front is not an essential limitation.

Taking everything said into account, we readily obtain[24] for the average field, with $kl \gg 1$, the following equation

$$\left[\Delta + k^2\left(1 + \frac{1}{2}\langle\delta\varepsilon^2\rangle\,ikl + \frac{1}{4}\langle\delta\varepsilon^2\rangle \right) \right]\mathbf{E} = -\mathbf{p}\delta(\mathbf{r}-\mathbf{r}_0). \tag{2.11}$$

Thus, the influence of fluctuations of the dielectric constant on the average field is taken into account by introducing a complex effective dielectric constant

$$\varepsilon_{\text{eff}} = 1 + \frac{1}{2}\langle\delta\varepsilon^2\rangle\,ikl + \frac{1}{4}\langle\delta\varepsilon^2\rangle. \tag{2.12}$$

The real part of $\epsilon_{\text{eff}} - 1$ describes the variation of the phase velocity of the electromagnetic waves, $(v_{\text{ph}} - c)/c = -\langle\delta\epsilon^2\rangle/8$, while the imaginary added term in ϵ_{eff} is due to the transfer of energy from the average field by scattering on the fluctuations.[24]

Let us proceed to determine the second moments of the fluctuations of the electric field. We confine ourselves here only to the case $kl \gg 1$. If the radiation is from a point source, the average field away from the source has at a point R the form

$$\mathbf{E} = \left(\frac{\mathbf{p}}{4\pi R} \right)\exp\left\{ ik\left[1 + \frac{\langle\delta\varepsilon^2\rangle}{4}\left(ikl + \frac{1}{2} \right) \right] R \right\}. \tag{2.13}$$

Substituting (2.12) in (2.5) and averaging, we obtain for $\langle|\xi^2|\rangle$ at the point (L, 0, 0)

$$\langle|\tilde{\xi}^2|\rangle = \frac{p^2}{(4\pi L)^2}\left[1 - \exp\left(-\frac{1}{2}\langle\delta\varepsilon^2\rangle k^2 Ll \right) \right]. \tag{2.14}$$

On the other hand, we have at the same point for $|E^2|$

$$|E^2| = \frac{p^2}{(4\pi L)^2}\exp\left(-\frac{1}{2}\langle\delta\varepsilon^2\rangle k^2 lL \right). \tag{2.15}$$

From (2.14) and (2.15) it follows that

$$\langle|\xi^2|\rangle + |E^2| = \frac{p^2}{(4\pi L)^2}. \tag{2.16}$$

This is the law of conservation of energy: the sum of the energy of the fluctuating field and of the energy of the average field is equal to the energy of the field in the medium without fluctuations.

In order for the damping to play an essential role, the exponent in formula (2.14) should be on the order of unity, corresponding to $k^2Ll \sim 1/\langle\delta\epsilon^2\rangle \gg 1$. Thus, allowance for the attenuation of the average field becomes essential only at distances $L \lesssim 1/(k^2 l\langle\delta\epsilon^2\rangle)$.

If $\langle\delta\epsilon^2\rangle k^2 Ll \ll 1$, it follows from (2.14) and (2.15) that

$$\langle|\xi^2|\rangle \approx \frac{p^2\langle\delta\varepsilon^2\rangle k^2 l}{2(4\pi)^2 L}, \quad |E^2| = \frac{p^2}{(4\pi L)^2}, \tag{2.17}$$

from which we see that $\langle|\xi^2|\rangle \ll |E^2|$.

Substituting (2.17) in (1.9) we obtain the usual expressions for the mean square of the fluctuations of the amplitude and phase in the far zone (see, for example, references 3 and 11): $\langle|\xi^2|\rangle \approx p^2/(4\pi L)^2 \gg |E^2|$ when $\langle\delta\epsilon^2\rangle k^2 l L \gg 1$. As indicated in Sec. 1, at this ratio of the average field to the fluctuation field the phase of the latter has an equiprobable distribution.

In view of smallness of $\langle\delta\epsilon^2\rangle$ in the troposphere ($\langle\delta\epsilon^2\rangle \sim 10^{-12}$) the last case can be realized only for very short electromagnetic waves.

3. Influence of Boundaries on the Fluctuations of the Electromagnetic Field (Qualitative Discussion)

Before we embark on a rigorous solution of the problem of the influence of the separation boundary on the fluctuations of the electromagnetic field, it is advisable to clarify the main physical phenomena that are associated with the specific nature of this problem. It turns out that in many cases of practical importance the necessary result can be obtained by using an approximate method, in which the resultant field is represented as the sum of a direct wave and a wave reflected from the boundary. Using this procedure we can, in accordance with the results of references 25 and 26, find the connection between the phase fluctuations $\delta\varphi$ and the relative amplitude $\delta A/|E|$ of the summary field and the individual components $\delta\varphi_1$ and $\delta A_1/|E_1|$. This connection is

$$\left\langle\frac{\delta A^2}{|E^2|}\right\rangle = \frac{1}{2}\langle\delta\varphi_1^2\rangle \operatorname{ctg}^2\frac{\alpha_\pm}{2}(1-W_\varphi) + \frac{1}{2}\left\langle\frac{\delta A_1^2}{|E_1^2|}\right\rangle(1+W_A),$$
$$\langle\delta\varphi^2\rangle = \frac{1}{2}\langle\delta\varphi_1^2\rangle(1+W_\varphi) + \frac{1}{2}\left\langle\frac{\delta A_1^2}{|E_1^2|}\right\rangle\frac{1}{2}\operatorname{ctg}^2\alpha_\pm(1-W_A). \tag{3.1*}$$

Here W_φ and W_A are the coefficients of the correlation between the fluctuations of the phases and the amplitudes of the components, $\alpha_- = \gamma$, $\alpha_+ = \pi + \gamma$, $\gamma \simeq 2khh_0/L$ is the angle of the space lag between the interfering components, the plus and minus signs denote the normal and tangential components of the field, respectively, while h and h_0 are the heights of the corresponding points.

In analogy with the formulas for the average field[15] we can call (3.1) the "reflection" formulas for the fluctuations.

As follows from (3.1), $\langle\delta A^2\rangle/|E^2|$ and $\langle\delta\varphi^2\rangle$ tend to infinity near the interference minima, where $\alpha \approx 2\pi n$, although actually this growth has a finite limit at these points, as will be shown later (Sec. 4).

To develop the qualitative picture, we can assume in first approximation that the fluctuations of the amplitudes and phases of the individual components of the field are not greatly distorted by the presence of the boundary. Let us illustrate the result for a particular case. Let the correlation function of the pulsations of the dielectric constant have the form

$$\langle\delta\varepsilon_1\delta\varepsilon_2\rangle = \langle\delta\varepsilon^2\rangle \exp\left(-\frac{\varrho^2}{l^2}\right), \tag{3.2}$$

where ρ is the distance between the points in space. For the far zone we have[3,5,9]

$$W_\varphi = W_A = \frac{\pi^{1/2}l}{2z}\operatorname{erf}\left(\frac{z}{l}\right), \tag{3.3}$$

$$\langle\delta\varphi_1^2\rangle = \frac{\langle\delta A_1^2\rangle}{|E_1^2|} = \frac{1}{2}\pi^{5/2}\langle\delta\varepsilon^2\rangle\frac{lL}{\lambda^2}, \tag{3.4}$$

where

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x \exp(-x^2)\,dx,\quad z = \frac{2hh_0}{h+h_0}\left(1+\left(\frac{h-h_0}{L}\right)^2\right)^{-1/2}$$

Substituting (3.3) and (3.4) in (3.1) we get

$$\left\langle\frac{\delta A^2}{|E^2|}\right\rangle = \langle\delta\varphi^2\rangle = \frac{1}{4}\pi^{5/2}\langle\delta\varepsilon^2\rangle\frac{lL}{\lambda^2}\left\{\left[1-\frac{\pi^{1/2}l}{2z}\operatorname{erf}\left(\frac{z}{l}\right)\right]\operatorname{ctg}^2\frac{\alpha_\pm}{2} + \left[1+\frac{\pi^{1/2}l}{2z}\operatorname{erf}\left(\frac{z}{l}\right)\right]\right\}. \tag{3.5}$$

In the region of small angles, where the Vvedenskii quadratic formula is valid for the average field, we can simplify (3.5). Putting $\cot(\alpha_\pm/2) \approx 2/\alpha_\pm$ and

$$\left[1-\frac{\pi^{1/2}}{2}\frac{l}{z}\operatorname{erf}\left(\frac{z}{l}\right)\right]\left(\frac{Ll}{2\pi hh_0}\right)^2 \gg \left[1+\frac{\pi^{1/2}}{2}\frac{l}{z}\operatorname{erf}\left(\frac{z}{l}\right)\right],$$

we obtain

$$\left\langle\frac{\delta A^2}{|E^2|}\right\rangle = \langle\delta\varphi^2\rangle \approx \frac{\pi^{1/2}}{16}\langle\delta\varepsilon^2\rangle\frac{lL^3}{h^2h_0^2}\left[1-\frac{\pi^{1/2}}{2}\frac{l}{z}\operatorname{erf}\left(\frac{z}{l}\right)\right]. \tag{3.6}$$

To illustrate the relations obtained, Figs. 1, 2, and 3 show the calculated curves for $F(L)$, $F(\lambda)$, and $F(h_0)$, respectively:

$$F(L) = \frac{\langle\delta\varphi^2(L)\rangle}{\langle\delta\varphi^2(10\,\text{km})\rangle},\quad F(\lambda) = \frac{\langle\delta\varphi^2(\lambda)\rangle}{\langle\delta\varepsilon^2\rangle\, l}\cdot 4\cdot 10^{-12}$$

and

$$F(h_0) = \frac{\langle\delta\varphi^2(h_0)\rangle}{\langle\delta\varepsilon^2\rangle}\cdot 4\cdot 10^{-12}.$$

As follows from (3.5) and (3.6) the intensity of the fluctuation increases with increasing distance in the start of the first interference zone of the average field as L^x, where $1 \le x \le 3$, depending on the correlation of the fluctuations of the direct and reflected waves (see Fig. 1, on which are marked the experimental data obtained in the experiments described below). The presence of a boundary leads to an appreciable weakening of the frequency dependence of the fluctuations (see Fig. 2), compared with free space, where $F(\lambda) \sim \lambda^{-2}$. The dependence on height (see Fig. 3) is also quite distinct. Below the maximum of the first fringe we have $F(h) \sim h^{-2}$ for small values of the parameter l, and practically no altitude dependence for large l. It follows thus from the qualitative discussion that the presence of a smooth boundary can cause an appreciable change in the fluctuations compared with the case of free space. Although the foregoing discussion does enable us to visualize, in the main, the physical picture of the processes associated with the influence of the boundary on the fluctuations, nevertheless many important questions, such as the magnitude of the fluctuations at the minima of the

*ctg = cot.

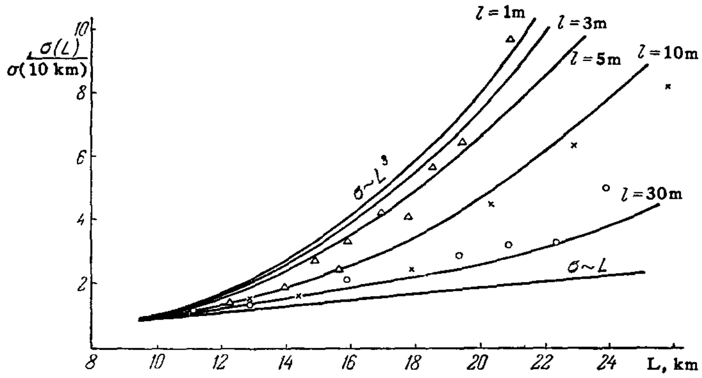

FIG. 1. Dependence of the intensity of fluctuations on the distance; σ (L) is normalized to its value at L = 10 km. h_0 = 10 m, h = 4 m, λ = 10 cm. o, x, Δ – experimental measurements.

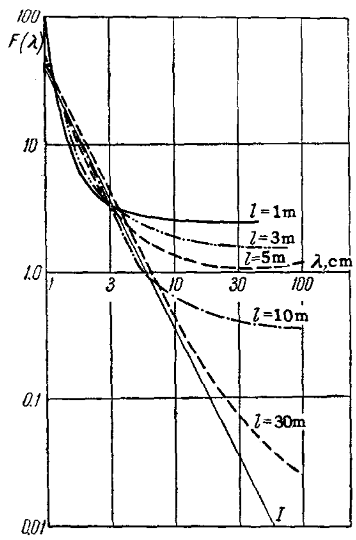

FIG. 2. Dependence of the intensity of the fluctuations on the wavelength. L = 30 km, h_0 = 40 m, h = 5 m, $F(\lambda) = [\sigma/\langle\delta\varepsilon^2\rangle l] \times 4 \times 10^{-12}$ deg²/m.

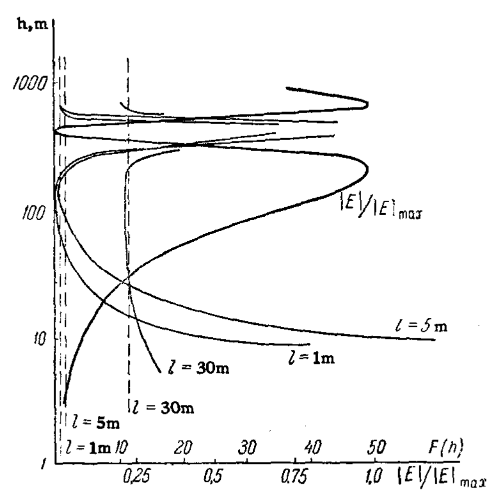

FIG. 3. Altitude dependence of the fluctuations L = 30 km, h = 5 m, λ = 10 cm, $F(h_0) = [\sigma/\langle\delta\varepsilon^2\rangle] \; 4 \times 10^{-12}$ deg²²

average field, the dependence on height at arbitrary heights h and h_0, etc. can be solved only within the framework of the rigorous theory, which will be considered in the following sections.

4. Fluctuation and Average Fields above the Boundary

In the presence of a boundary surface, Maxwell's equations (2.4) must be supplemented by the boundary conditions for the components of the average and fluctuation fields. We shall consider a plane surface. The coordinate system is chosen such that the Oz axis coincides with the outward normal to the surface z = 0, and passes through the point $\mathbf{r}_0$ with coordinates $(0, 0, h_0)$ where the source is located (h_0 is its height above the surface). The Ox axis is chosen along the projection of the ray joining the point of observation $R(L, 0, h)$ with the point r_0 on the surface.

We shall assume that the surface has infinite conductivity (allowance for the finite conductivity will be discussed later). Accordingly, the following boundary conditions are satisfied on the surface:

$$(\mathbf{E}_-)_0 = (\xi_-)_0 = \left(\frac{\partial E_+}{\partial z}\right)_0 = \left(\frac{\partial \xi_+}{\partial z}\right)_0 = 0. \tag{4.1}$$

The minus and plus signs, as in Sec. 3, correspond to the tangential and normal components. The subscript zero denotes that the corresponding quantities are taken at z = 0.

The solution of (2.4) for the random component can be readily written in explicit form by using the well known expressions for the Green's function of the operator $\Delta + k^2$ with boundary conditions (4.1).

This solution has the form[27]

$$\xi_\pm(\mathbf{R}) = k^2 \int\limits_{z' \geqslant 0} d\mathbf{r}' \varphi_\pm(\mathbf{R}, \mathbf{r}') \mathbf{E}_\pm(\mathbf{r}) \delta\varepsilon(\mathbf{r}'), \tag{4.2}$$

where

$$\varphi_\pm(\mathbf{r}, \mathbf{r}') = \varphi(|\mathbf{r} - \mathbf{r}'|) \pm \varphi(|\mathbf{r}_1 - \mathbf{r}'|), \quad \varphi(\varrho) = \frac{\exp ik\varrho}{4\pi\varrho}. \tag{4.3}$$

The point $\mathbf{r}_1$ is the mirror image of the point $\mathbf{r}$ in the plane z = 0. Integration in (4.2) is over the half space $z' \geq 0$ above the surface. When the average field $\mathbf{E}_\pm(\mathbf{r})$ is known, (4.2) yields a solution for $\xi_\pm(\mathbf{R})$ with the aid of which we can find all the interesting mean square quantities. To determine $\mathbf{E}(\mathbf{r})$ we obtain after substituting (4.2) into the first Eq. (2.4) an integro-differential equation analogous to (2.6)

$$(\Delta + k^2)\mathbf{E}_\pm(\mathbf{r}) + k^4\langle\delta\varepsilon^2\rangle \int\limits_{z' \geqslant 0} d\mathbf{r}' \varphi_\pm(\mathbf{r}, \mathbf{r}') \mathbf{E}_\pm(\mathbf{r}') W(\mathbf{r} - \mathbf{r}') = -\mathbf{p}_\pm \delta(\mathbf{r} - \mathbf{r}_0). \tag{4.4}$$

The integral term in (4.4), as in (2.6), determines the attenuation of the average field due to the transfer of energy from the average signal to the fluctuations.

All the subsequent analysis will be subject to the condition $kl \gg 1$ (large-scale fluctuations). In this case we can disregard the small corrections connected with the terms $(\partial/\partial x_i)\,\mathrm{div}\,\mathbf{E}$ in (2.6). Exact estimates show that the relative order of the discarded terms is $(kl)^{-1} \ll 1$.

Inasmuch as the attenuation is small and becomes significant only at sufficiently large distances from the source, we replace the integral term in (4.4) by its asymptotic expression for large r. (At small distances the form of this term is immaterial, since it can be neglected anyway.) It can be shown that at sufficiently large L, when $D = 2L/kl^2 \gg 1$, the following asymptotic formula holds true

$$\int\limits_{z'>0} d\mathbf{r}'\varphi_\pm(\mathbf{r},\mathbf{r}')W(\mathbf{r}-\mathbf{r}')\mathbf{E}(\mathbf{r}') = \int\limits_{-\infty}^{\infty} d\mathbf{r}'\varphi(|\mathbf{r}-\mathbf{r}'|) \times W(\mathbf{r}-\mathbf{r}')\mathbf{E}(\mathbf{r}')(1+O(1/D)). \tag{4.5}$$

Let us change over from the integro-differential equation (4.4) to an integral equation for $\mathbf{E}(\mathbf{r})$ using the corresponding Green's function

$$\mathbf{E}_\pm(\mathbf{r}) = \mathbf{p}_\pm\varphi_\pm(\mathbf{r},\mathbf{r}_0) + k^4\langle\delta\varepsilon^2\rangle \int\limits_{z'>0} d\mathbf{r}'\varphi_\pm(\mathbf{r},\mathbf{r}') \int\limits_{z''>0} d\mathbf{r}''\varphi_\pm(\mathbf{r}',\mathbf{r}'') \times W(\mathbf{r}''-\mathbf{r}')\mathbf{E}_\pm(\mathbf{r}''). \tag{4.6}$$

The solution of Eq. (4.6) by the usual method of iteration, corresponding to the Born approximation, is, as shown in Sec. 2, convenient only when $\langle\delta\epsilon^2\rangle k^2 L l \ll 1$, i.e., bounded on the side of large L.

Let us make the substitution $\mathbf{r}'' = \mathbf{r}' + \boldsymbol{\rho}$ in the integral

$$\int d\mathbf{r}''\varphi_\pm(\mathbf{r}',\mathbf{r}'')W(\mathbf{r}''-\mathbf{r}')\mathbf{E}_\pm(\mathbf{r}'') = \int d\mathbf{r}'\,W(\boldsymbol{\varrho})\mathbf{E}_\pm(\mathbf{r}'+\boldsymbol{\varrho})[\varphi(\boldsymbol{\varrho}) \pm \varphi(|2z'\mathbf{e}+\boldsymbol{\varrho}|)], \tag{4.7}$$

where $\mathbf{e}$ is a unit vector in the Oz direction.

An estimate of the integral with respect to $\mathbf{r}'$ in (4.6) by the stationary-phase method shows that at large r the main contribution is made by the region of values $|z - z'| \lesssim (L/k)^{1/2}$. Therefore, accurate to $W(\sqrt{L/k}) \ll 1$, we can replace the lower limit in (4.7) by $-\infty$, when $D > 1$, and we can neglect the second term [with an accuracy which is at any rate not less than* $O(1/\sqrt{D})$]. Consequently, at large distances Eq. (4.4) assumes the form

$$(\Delta + k^2)\mathbf{E}(\mathbf{r}) + k^4\langle\delta\varepsilon^2\rangle \int\limits_{-\infty}^{\infty} d\boldsymbol{\varrho}\,W(\boldsymbol{\varrho})\varphi(\boldsymbol{\varrho})\mathbf{E}(\mathbf{r}+\boldsymbol{\varrho}) = -\mathbf{p}_\pm\delta(\mathbf{r}-\mathbf{r}_0). \tag{4.8}$$

We shall be interested throughout in the case when the height of the source h_0 and of the point of observation h above the surface are small compared with the distance between them. In this case the average field $\mathbf{E}(\mathbf{r}+\boldsymbol{\rho})$ can be represented in the form $\mathbf{E}(\mathbf{r}) \times \exp(i\mathbf{k}\cdot\boldsymbol{\rho})$, where the vector $\mathbf{k}$ has the same direction as $\mathbf{r} - \mathbf{r}_0$. Thus, to find the average field above an ideally conducting plane it is enough to replace the propagation constant k by its effective value κ:

$$\mathbf{E}_\pm(\mathbf{r}) = \mathbf{p}_\pm\varphi_\pm^{(\varkappa)}(\mathbf{r},\mathbf{r}_0), \tag{4.9}$$

where κ is given by

$$\varkappa = k + \frac{1}{2}k^3\langle\delta\varepsilon^2\rangle \int\limits_{-\infty}^{\infty} d\boldsymbol{\varrho}\,\varphi(\boldsymbol{\varrho})W(\boldsymbol{\varrho})\exp(i\mathbf{k}\boldsymbol{\varrho}). \tag{4.10}$$

Let us note[18] that in the case when the correlation coefficient $W(\rho)$ depends only on $|\rho|$ we arrive at the same formula (4.10) for κ, but without the limitations $h, h_0 \ll L$ on the heights.

Equation (4.10) can be written for the effective dielectric constant $\epsilon_{\mathrm{eff}} = \kappa^2/k^2$:

$$\varepsilon_{\mathrm{eff}} = 1 + \langle\delta\varepsilon^2\rangle k^2(4\pi)^{-1}\int d\Omega_n \int\limits_0^\infty d\varrho\varrho W(\varrho\mathbf{n})\exp[ik\varrho(1-\mathbf{n}\mathbf{n}_0)]. \tag{4.11}$$

where n and n_0 are unit vectors in the directions of $\boldsymbol{\rho}$ and $\mathbf{k}$. In the case of large-scale fluctuations, which we are considering, the quantity $k\rho$ can be assumed to be large ($\rho \sim l$). It is easy to verify by the method of stationary phase that the main contribution to the integral over the angles is obtained from directions for which $\mathbf{n}\cdot\mathbf{n}_0 \cong 1$, i.e., $\mathbf{n} \cong \mathbf{n}_0$. After elementary integration we obtain

$$\varepsilon_{\mathrm{eff}} - 1 = \frac{1}{2}ik\langle\delta\varepsilon^2\rangle \int\limits_0^\infty d\varrho\,W(\varrho\mathbf{n}_0) = \frac{1}{2}ik\langle\delta\varepsilon^2\rangle l. \tag{4.12}$$

The attenuation coefficient of the field α in the presence of a boundary obviously coincides in the case of large-scale fluctuations with the coefficient for free space [compare with (2.12)]:

$$\alpha = \mathrm{Im}\,\varkappa = k\,\mathrm{Im}\,\varepsilon_{\mathrm{eff}}^{1/2} = \frac{1}{4}\langle\delta\varepsilon^2\rangle k^2 l, \tag{4.13}$$

where the effective radius of the correlation is

$l = \int\limits_0^\infty W(\xi, 0, 0)\,d\xi$, inasmuch as $\mathbf{n}_0 = (1, 0, 0)$.

5. Fluctuations of the Amplitude and Phase in the Far Zone[18,19]

It is most interesting to investigate the fluctuations in the far zone, where the following inequalities, which have been used above, are satisfied

$$\lambda\!\!\!^{-} \ll l \ll (\lambda\!\!\!^{-} L)^{1/2\,*} \quad \left(\lambda\!\!\!^{-} = \frac{\lambda}{2\pi} = \frac{c}{\omega} = \frac{1}{k}\right). \tag{5.1}$$

As usual, we assume the heights h and h_0 to be small compared with L. In the calculations we make use of the fact[19] that in the far zone $[l \ll (\lambda\!\!\!^{-} L)^{1/2}]$ the fluctuations of the relative amplitude and phase are equal to each other, with a relative accuracy on the order of $(\ln D/D) \ll 1$ $(D = 2L/kl^2 \gg 1)$, and are determined by formula (1.9).

Substituting (4.9) in (4.2) we obtain for $\langle|\xi^2|\rangle$:

*More accurate estimates show that the relative error due to neglecting this term is much less, namely on the order of $D^{-1/2} \times \exp(-k^2l^2/2)$ with $W(\rho) = \exp(-\rho^2/l^2)$.

*The meaning of the last inequality is that the dimension of the first Fresnel zone $(\lambda L)^{1/2}$ should be large compared with the average dimensions of the inhomogeneous zone.

$$\langle|\xi_{\pm}^{2}(\mathbf{R})|\rangle=\langle\delta\varepsilon^{2}\rangle k^{4}p_{\pm}^{2}\iint\limits_{z',z''\geqslant 0}d\mathbf{r}'\,d\mathbf{r}''\,\varphi_{\pm}(\mathbf{R},\mathbf{r}')\,\varphi_{\pm}^{(\varkappa)}(\mathbf{r}',\mathbf{r}_0)$$
$$\times\varphi_{\pm}^{*}(\mathbf{R},\mathbf{r}'')\,\varphi_{\pm}^{(\varkappa)*}(\mathbf{r}'',\mathbf{r}_0)\,W(\mathbf{r}'-\mathbf{r}''),\tag{5.2}$$

where the asterisk denotes the complex conjugate. Using the inequalities (5.1) we can calculate the integral in (5.2):

$$\langle|\xi_{\pm}^{2}(\mathbf{R})|\rangle=\langle\delta\varepsilon^{2}\rangle\left(\frac{p_{\pm}k}{4\pi}\right)^{2}L^{-1}\int\limits_{0}^{\infty}d\xi\int\limits_{0}^{1}dt\exp(-2\alpha Lt)$$
$$\times\Big\{W(\xi,0,0)+\tfrac{1}{2}W[\xi,0,2ht+2h_0(1-t)]$$
$$+\tfrac{1}{2}W[\xi,0,|2ht-2h_0(1-t)|]$$
$$\pm\cos\frac{2khh_0}{L}[W(\xi,0,2ht)+W(\xi,0,2h_0(1-t))]\Big\},\tag{5.3}$$

where

$$\alpha=\frac{1}{4}k^{2}l\langle\delta\varepsilon^{2}\rangle.$$

The square of the average field has under these assumptions the form

$$|E_{\pm}^{2}|=\frac{2p_{\pm}^{2}}{(4\pi L)^{2}}\exp(-2\alpha L)\,[1\pm(\cos 2khh_0/L)].\tag{5.4}$$

Let us consider several different limiting cases. At very large distances, when $\alpha L\gg 1$,

$$\langle|\xi_{\pm}^{2}(\mathbf{R})|\rangle\cong\frac{2|p_{\pm}^{2}|}{(4\pi L)^{2}l}\left(1\pm\cos\frac{2khh_0}{L}\right)\int\limits_{0}^{\infty}d\xi\,\{W(\xi,0,0)$$
$$+W(\xi,0,2h_0)\},\tag{5.5}$$

$$\frac{\langle|\xi_{\pm}^{2}|\rangle}{|E^{2}|}=\Big[1+(l)^{-1}\int\limits_{0}^{\infty}d\xi\,W(\xi,0,2h_0)\Big]\exp(2\alpha L)\gg 1.\tag{5.6}$$

As can be seen from (5.6) the fluctuating part of the field is much greater than the regular component, and consequently $\langle\delta\varphi^{2}\rangle=\pi^{2}/3$, and

$$\langle(\ln|\mathscr{E}|-\langle\ln|\mathscr{E}|\rangle)^{2}\rangle=\frac{\pi^{2}}{24}$$

[see (1.10)].

It should be noted that in the presence of a boundary the ratio $\langle|\xi^2|\rangle/|E^2|$ is not equal, generally speaking, to the ratio of the energy fluxes of the scattered and average fields — unlike the case of infinite space — inasmuch as the summary field is a superposition of direct and reflected waves.

Let us consider the case when the attenuation plays no role. In the far zone, away from the zeros of the average field, we have according to (1.9)

$$\sigma_{\pm}=\langle\delta\varphi^{2}\rangle=\frac{\langle\delta A^{2}\rangle}{|E^{2}|}=\frac{\langle|\xi_{\pm}|^{2}\rangle}{2|E^{2}|}=\frac{\langle\delta\varepsilon^{2}\rangle k^{2}L}{4(1\pm\cos(2khh_0/L))}$$
$$\times\int\limits_{0}^{\infty}d\xi\int\limits_{0}^{1}dt\,\Big\{W(\xi,0,0)+\tfrac{1}{2}W[\xi,0,2ht+2h_0(1-t)]$$
$$+\tfrac{1}{2}W[\xi,0,|2ht-2h_0(1-t)|]\pm\cos\frac{2khh_0}{L}[W(\xi,0,2ht)$$
$$+W(\xi,0,2h_0(1-t))]\Big\}.\tag{5.7}$$

Henceforth, in order to simplify the notation, we shall assume that $W(x,y,z)=W(x)W(y)W(z)$, with $W(0)=1$. Let us consider several particular cases.

On the edge of the first interference minimum, where $(2khh_0/L)^2\ll 1$, we have

$$\sigma_{-}=\frac{\langle\delta\varepsilon^{2}\rangle L^{3}l}{8(hh_0)^{2}}\Big\{1+\frac{1}{2}\int\limits_{0}^{1}dt\,[W(2ht+2h_0(1-t))$$
$$+W(|2ht-2h_0(1-t)|)$$
$$-2(1-2k^{2}h^{2}h_0^{2}L^{-2})(W(2ht)+W(2h_0t))]\Big\},\tag{5.8}$$

$$\sigma_{+}=\frac{1}{8}\langle\delta\varepsilon^{2}\rangle k^{2}Ll\,\Big\{1+\frac{1}{2}\int\limits_{0}^{1}dt\,[W(2ht+2h_0(1-t))$$
$$+W(|2ht-2h_0(1-t)|)+2W(2ht)+2W(2h_0t)]\Big\}.\tag{5.9}$$

Formulas (5.8) and (5.9) show that the fluctuations in a horizontal polarized wave are independent of the frequency and increase as the cube of the distance, whereas in a vertically-polarized wave the intensity of the fluctuations is proportional to $\omega^2 L$.

In the case of relatively large altitudes ($h, h_0\gg l_z$, where l_z is the radius of correlation along the normal to the surface) we have

$$\sigma_{-}=\frac{\langle\delta\varepsilon^{2}\rangle L^{3}l}{8(hh_0)^{2}},\quad\sigma_{+}=\frac{1}{8}\langle\delta\varepsilon^{2}\rangle k^{2}Ll,\quad\frac{\sigma_{+}}{\sigma_{-}}=\left(\frac{khh_0}{L}\right)^{2}\ll 1.\tag{5.10}$$

At small heights ($h, h_0\ll l_z$)

$$\sigma_{-}=\langle\delta\varepsilon^{2}\rangle\frac{L^{3}l}{60l_z^{4}}W^{\mathrm{IV}}(0),\quad\sigma_{+}=\frac{1}{2}\langle\delta\varepsilon^{2}\rangle k^{2}Ll,\tag{5.11}$$

where $W^{\mathrm{IV}}(0)=\left.\dfrac{d^4W(\xi)}{d\xi^4}\right|_{z=0}$ and it is assumed that $W(z)$ is an even function of $\zeta=z/l_z$; $W(\zeta)=W(z/l_z)$.

When either h or h_0 is much greater than l_z, and the other is much less, we have

$$\sigma_{-}=\frac{1}{4}\langle\delta\varepsilon^{2}\rangle k^{2}Ll\,\Big\{1+\frac{1}{3}\left(\frac{L}{kh_{\max}l_z}\right)^{2}W_{\zeta}''(0)\Big\},\quad\sigma_{+}=\frac{1}{4}\langle\delta\varepsilon^{2}\rangle k^{2}Ll,\tag{5.12}$$

where $h_{\max}$ is equal to the greater of the quantities h or h_0.

We can consider also other limiting cases[19] of the general formula (5.7).

Comparing the results of the approximate and rigorous theory, it should be noted that in a case when at least one of the heights h or $h_0\gg l_z$, it is quite permissible to neglect the influence of the boundary on the correlation functions W_φ and W_A, i.e., to assume them to be the same as in free space (see Sec. 1). In this case the results of the exact and approximate calculations coincide in the first approximation, and for calculations involved in engineering practice one can use the formulas and the curves given in Sec. 3, using for the values of the fluctuations at the field minima the expressions derived in the exact theory [(1.10) and (1.11)]. At low heights ($h, h_0\ll l_z$), and also in the higher-order approximations, the surface exerts an appreciable influence on the form of the functions W_φ and W_A, and the exact theory must be used.

As was already noted, the increase in the fluctuations near the minima of the regular field is due to the interference structure of the electromagnetic field in the space above the boundary. Interference effects are most clearly pronounced when the amplitudes of the direct and reflected waves are the same. If the modulus F of the coefficient of reflection $F \exp(i\psi)$ is different from unity, interference does not give rise to so strong a growth in the fluctuations. In the limiting case of small F, we can use the formulas obtained for unbounded space.

In the case of a boundary of finite but sufficiently large conductivity, the difference in the intensities of the fluctuations for the horizontal and vertical polarizations should disappear. This is due to the fact that the phase ψ of the reflection coefficient changes sharply from 0 to π with increase in elevation angle, whereas the amplitude F remains close to unity. Therefore the regular component of a vertically-polarized field decreases as L^{-2} and not as L^{-1}, as in the case of infinite conductivity ("the lobe rises"), and the fluctuations of the vertical component are described by the same relations as in horizontal polarization. Finally, in the intermediate region of values of F, the fluctuations should increase with the distance more rapidly than L, but more slowly than L^3.

Let us note that, as was indicated above, the sharp increase in the relative fluctuations near the interference minima and at large distances is connected not with the sharp increase in the absolute fluctuations of the field, but with the decrease in the regular component. From this point of view it is expected that when the curvature of the surface (for example, the spherical form of the earth) is taken into account the relative fluctuations outside the direct line of sight, where the regular field diminishes exponentially with the distance, should apparently increase exponentially. A detailed investigation of this question is highly desirable.

6. Correlation of the Fluctuations above the Boundary[28]

Along with fluctuations of the amplitude and the phase at one point, important statistical characteristics are the correlation relations of the fluctuations of the phases and amplitudes at two different points of space. The question of correlation of fluctuations in the case of infinite space has been investigated by many (see references 4, 5, 6, 9, 10, 11, and 13, for example). We shall therefore discuss only the singularities of the present problem, which are connected with the presence of the boundary.[28]

It can be shown that in the region where the fluctuating part of the field is small compared with the regular component, we have

$$K_\varphi \equiv \langle \delta\varphi_1 \, \delta\varphi_2 \rangle = \frac{\langle (\xi_{i1} E_{r1} - \xi_{r1} E_{i1})(\xi_{i2} E_{r2} - \xi_{r2} E_{i2}) \rangle}{|E_1 E_2|}, \tag{6.1}$$

$$K_A = \frac{\langle \delta A_1 \delta A_2 \rangle}{|E_1 E_2|} = \frac{\langle (\xi_{r1} E_{r1} + \xi_{i1} E_{i1})(\xi_{r2} E_{r2} + \xi_{i2} E_{i2}) \rangle}{|E_1 E_2|}. \tag{6.2}$$

The subscripts 1 and 2 denote two different points in space. It is also possible to determine the mixed amplitude-phase correlation; in the case of the far zone ($D = 2L/kl^2 \gg 1$) this correlation function is small and will be disregarded here.

As in the case of amplitude and phase fluctuations, the functions K_φ and K_A are equal in the far zone, inasmuch as the real and imaginary parts $\langle \xi_1 \xi_2 \rangle = \langle \xi_{r_1} \xi_{r_2} - \xi_{i_1} \xi_{i_2} \rangle + i \langle \xi_{i_1} \xi_{r_2} + \xi_{r_1} \xi_{i_2} \rangle$ are small compared with $|\mathrm{Re} \langle \xi_1 \xi_2^* \rangle|$ and $|\mathrm{Im} \langle \xi_1 \xi_2^* \rangle|$ in the ratio $\ln D/D$. Taking this fact into account, we obtain

$$K = K_\varphi = K_A = \frac{1}{2|E_1 E_2|} \mathrm{Re} \{ \xi_1 \xi_2^* \exp [i(\varphi_2 - \varphi_1)] \}, \tag{6.3}$$

where $\varphi_2 - \varphi_1$ is the phase difference of the regular components at the points R_2 and R_1.

With the aid of formula (4.2) for ξ we can calculate $\langle \xi_1 \xi_2^* \rangle$ and then find the correlation function K. We neglect here the attenuation of the average field, since we shall consider henceforth only small fluctuations of the amplitude and of the phase. We give the results of the calculations for two cases: a) transverse correlation and b) longitudinal correlation.

a) In the first case the points $R_1(L, -d/2, h - b/2)$ and $R_2(L, d/2, h + b/2)$ lie in the plane $x = L$.

Assuming, as before, $W(x, y, z) = W(x)W(y)W(z)$, we calculate the following value for the transverse correlation

$$\begin{aligned} K_\pm = \frac{\langle \delta\varepsilon^2 \rangle k^2 L l}{4[\cos(kh_0 b/L) \pm \cos(2khh_0/L)]} \int_0^1 dt\, W(td) \Big\{ \cos \frac{kh_0 b}{L} \\ \times \Big[W(bt) + \frac{1}{2} W(2ht + 2h_0(1-t)) \\ + \frac{1}{2} W(|2ht - 2h_0(1-t)|) \Big] \\ \pm \cos \frac{2khh_0}{L} \Big[W(2ht) + \frac{1}{2} W(2h_0(1-t) + bt) \\ + \frac{1}{2} W(|2h_0(1-t) - bt|) \Big] \Big\}. \end{aligned} \tag{6.4}$$

This general formula is valid under the same assumptions as the formulas of the preceding section [$\lambda\!\!\!^{-} \ll l \ll (\lambda\!\!\!^{-} L)^{1/2}$, $h, h_0 \ll L$]. It shows that transverse decorrelation of the phases and amplitudes in the direction of the z axis ($d = 0$) takes place when $b \sim l_z$, while the correlation in the direction of the y axis ($b = 0$) is significant when $d \sim l_y$. At large heights

$$K_\pm = \frac{\langle \delta\varepsilon^2 \rangle k^2 L l \cos\left(\frac{kh_0 b}{L}\right)}{4[\cos(kh_0 b/L) \pm \cos(2khh_0/L)]} \int_0^1 dt\, W(td)\, W(bt). \tag{6.5}$$

For large b or d ($d \gg l_y$, $b \gg l_z$), $K_\pm$ diminishes as $1/b$ or as $1/d$. A discussion of numerous limiting cases is contained in reference 28.

b) We shall take longitudinal correlation to mean the correlation of the fluctuations at two points $R_1(L, 0, h_0)$ and $R_2(L + \Delta, 0, h_0)$, located at the same height h_0 on a straight line parallel to the x axis, at a distance Δ apart ($\Delta \ll L$). In addition, we specify an explicit form for $W(r) = \exp\left(-\frac{x^2 + y^2}{l_x^2} - \frac{z^2}{l_z^2}\right)$

and we assume that $h_0 \gg l_z$. Then

$$K_\pm = \frac{\langle \delta\varepsilon^2 \rangle k^2 l L}{4\,[\cos(kh_0^2\Delta/L^2) - \cos(2kh_0^2/L)]} \operatorname{Re} \exp\left(-\frac{ikh_0^2\Delta}{L^2}\right) \times \int_0^1 dt \left[\left(1 + \frac{2i\Delta t^2}{kl_z^2}\right)\left(1 + \frac{2i\Delta t^2}{k/l_x^2}\right)\right]^{-1/2}, \tag{6.6}$$

where $l = (\pi)^{1/2} l_x/2$.

It is seen from (6.6) that longitudinal correlation takes place at distances on the order kl^2, i.e., approximately at the same distances as in the case of an infinite medium. In other words, the fluctuations are correlated in the direction of propagation of the wave over considerably greater distances than in the transverse direction. It can be shown[28] that this general conclusion is independent of the relation between h_0 and l_z.

In the case when $\Delta \gg kl^2$, the correlation function decreases as $\Delta^{-1/2}$. In the interference region K is an oscillating function of the distance Δ and vanishes when $\Delta = (L^2/kh_0^2)(n + 1/4)\pi$, $n = 1, 2, 3 \ldots$. The oscillations vanish on the first interference minimum ($kh_0^2 L^{-1} \ll 1$). In this case we have for $\Delta \gg kl^2$ and $l_x = l_z = l_0$

$$K_- = \langle \delta\varepsilon^2 \rangle \frac{\pi l L^3}{16 h_0^4} \left(\frac{k l_0^2}{\Delta}\right)^{1/2}, \quad K_+ = \frac{1}{16} \langle \delta\varepsilon^2 \rangle \pi k^2 l L \left(\frac{k l_0^2}{\Delta}\right)^{1/2}. \tag{6.7}$$

From an examination of various limiting cases we can draw the general conclusion that the correlation of the vertically-polarized component always increases in proportion to L, whereas the correlation function for horizontal polarization increases with the distance as L^3 (see above). The frequency and altitude dependences for the longitudinal and transverse correlations are different. The considerations expressed regarding the influence of the finite conductivity on the intensity of fluctuations, given in Sec. 5, pertain equally well to the correlation functions.

The foregoing results are valid away from the zeros of the regular part of the field. It is readily seen, however, that the decorrelation of the phases and the amplitudes near the interference minima occurs at distances not greater than in the case considered here. Actually, by virtue of the central limit theorem, ξ_r and ξ_i have normal distributions. At distances for which the fluctuations of the fields at different points of space are practically uncorrelated, the function of the simultaneous distribution of the quantities ξ_r and ξ_i breaks up into a product of the distribution functions $f(\xi_r)$ and $f(\xi_i)$.

II. EXPERIMENTAL INVESTIGATIONS OF THE FLUCTUATIONS

In investigating the conditions for the propagation of uhf waves, it has been noted many times that the radio signals passing through the troposphere are subject to intense fluctuations.[15,29-32] As the uhf wave became more widely used, more and more attention was paid not only to the average field intensities, but also to the fluctuations of the phases, amplitudes, angles of arrival, etc. at these wavelengths. Although such investigations began a relatively long time ago,[33-35] the experiments were set up until recently in a way that the results obtained reflected to a great degree the characteristics of the regions where these observations were carried out, and did not lead to general conclusions necessary for comparison with the theory.

The complexity and variety of the earth's topography, along with the difficulties connected with accounting for its effect on either the average field intensities or on their fluctuations, have made it necessary for most recent high-accuracy experiments[41,42] to be carried out with a beam "detached from the earth," by suitable choice of paths and apparatus. The data obtained in these experiments were practically the same as apply to unbounded space, and could be compared with the theoretical deduction obtained for that case.[11,12] It is obvious that for an experimental investigation of the influence of the boundary on the fluctuations, it would be most advantageous to perform such experiments over a smooth surface, particularly over the sea. Such investigations were begun only in recent times,[26,43,44] and the data obtained in these investigations permit, to some extent, a quantitative comparison with the theory developed to account for the influence of the boundary on the fluctuation of radio signals.

7. Procedure for the Measurement of Fluctuations of Radio Signals

In an investigation of fluctuations it is essential to be able to determine the laws of distribution of the amplitudes, phases, angles of arrival, etc. as well as the spectral characteristics and the space and time correlation functions and their dependence on the geometry of the path, the wavelength, the meteorological conditions, etc. These measurements must be carried out both in the "illuminated" zone, for which the theory has been developed (see Chap. I), as well as in the "penumbra" and "shadow" zones where, in spite of the lack of theoretical calculations, it is also extremely desirable to obtain experimental data.

As follows from the theoretical analysis, differences in the fluctuation characteristics are to be expected even in the "illuminated" zone, depending on whether the measurements are carried out in the near ($\sqrt{L\lambda} \ll l$) or far ($\sqrt{L\lambda} \gg l$) zones. However, in the case of waves with $\lambda > 1$ cm, practical interest attaches in most cases to measurements for the far zone, in which the fluctuations are maximal.

From the point of view of measurement techniques, a study of fluctuations is a much more complicated technical problem than the determination of the average values, since the influence of various errors (in particular, apparatus errors) on the measured effects increases sharply.

From the point of view of measurement procedure, the simplest is to determine the amplitude fluctuations, using linear or logarithmic amplifiers with balanced output (up to the recording system) to increase the accuracy, with the average value cancelled out (see, for example, reference 42). Thus, the quantity determined in the experiments is $\ln|\mathscr{E}| - \langle\ln|\mathscr{E}|\rangle$, or the quantity $|\mathscr{E}| - \langle|\mathscr{E}|\rangle$, which differs little from the former if the fluctuations are of low intensity.

A more complicated problem is the measurement of the phase fluctuations. Such measurements call for the design of rather complex precision apparatus.[41–45] Particularly great difficulties are encountered in the measurement of fluctuations of the "absolute" phases of the signals,* which characterize the variability of the "electric length" of the path of the radio waves. Such measurements call for a high relative accuracy (10^{-9} and better) in the carrier frequency and precise location of the corresponding points.[42] Much simpler is the differential procedure — the measurement of the phase difference $\varphi_i - \varphi_k$ in two (or more) points separated in space, a procedure frequently used in phase measurements. What is essentially determined in this case are the first spatial increments of the function that describes the fluctuations of the "absolute" phases. By measuring $\varphi_i - \varphi_k$ and determining the mean square of this quantity $\langle(\varphi_i - \varphi_k)^2\rangle$ (the so-called structural function), it is possible to determine both the intensity and the correlation function of the phase fluctuations. Such measurements are usually carried out in two variants, in which the phase differences are measured at reception points that lie on a line which is either parallel or perpendicular to the direction of propagation of the radio waves. In the former case one speaks of "transverse" correlation of the fluctuations, and in the latter of "longitudinal" correlation. It is possible to determine with such measurements the dependence of the fluctuations on the distance L between the corresponding points and the spatial separation of the reception points d.

In addition to these measurements, it is desirable to determine the height relationships, for comparison of experiment with theory. Such experiments can be carried out by varying the heights of the antennas in one (or several) of the corresponding points.

From the methodological point of view, it is best to set up experiments with fixed antenna positions and with a series of fixed values of L, d, h, and h_0, and to vary these parameters by switching.

As follows from the preliminary measurements, the fluctuations should be recorded without time delay by means of high-precision apparatus with long-time stability. Such apparatus should reproduce without distortion the spectral composition of the fluctuations in the frequency range from hundreds or tens of cycles down to the lowest frequencies (hundredths and thousandths of a cycle).

*What is meant here is the measurement of the phase at the receiving part of the apparatus, relative to the transmitter phase.

While the reproduction of the spectrum entails no particular technical difficulties on the high frequency side (hundreds of cycles), in the domain of very low frequencies (thousandths of a cycle) there are limitations due both to the insufficient precision of the apparatus and to the finite measurement time. It should be noted that not enough attention was paid in most investigations to the distortion of the spectrum of the fluctuations. Thus, apparatus subject to time delay was used in reference 41, and consequently the spectrum of the fluctuation frequencies is distorted starting with 0.1 — 1 cps. As will be shown below, such distortions of the spectrum lead to several incorrect conclusions, and in precision measurements of fluctuations serious attention must be paid to this aspect of the question.

In analyzing the measurement procedure it must be recognized that a specific feature of the fluctuations due to the turbulence of the troposphere is that they are essentially non-stationary. Consequently, when determining the space decorrelation of the fluctuations, the altitude dependence, etc., preference should be given to the method of simultaneous synchronized measurements over all the investigated paths. It became clear even in the preliminary measurements that in many cases it is practically impossible to compare non-simultaneous experiments, particularly prolonged ones. A certain approach to this procedure is the simultaneous measurement at least on two neighboring paths, so that one of the preceding measurements can be duplicated to take account of nonstationarity in subsequent measurements.[43,44]

8. Principal Characteristics of Fluctuations

As was already noted above, for a comparison with the theory it is necessary to use experiments on fluctuations on paths that lie wholly over the sea. Such experiments have been carried out at a frequency near 3000 Mc/sec, and particular attention was paid to fluctuations in the phase difference[43,44] for vertically-polarized radiation on a path 33 km long. The receiving and transmitting antennas were stationary, but provision was made for changing the height of the transmitting antenna by using alternately three transmitters with antennas 9, 18, and 35 m above sea level, respectively. The receiving antennas of the measuring apparatus were at a height h = 4 m along a line perpendicular to the direction of propagation of the radio waves. By switching it was possible to measure the transverse correlation of the fluctuations at distances d from 2 to 100 m between the measurements points. The stability of the transmitting and receiving apparatus was such that the fluctuation spectrum was reproduced within a range from 0.01 to 100 cps. This apparatus was used to measure a series of main characteristics of the phase-difference fluctuations. Figure 4 shows typical integral distributions of the modulus of the phase difference relative to the average value, plotted on a scale that linearizes the normal distribution, for an experiment performed in the summertime.

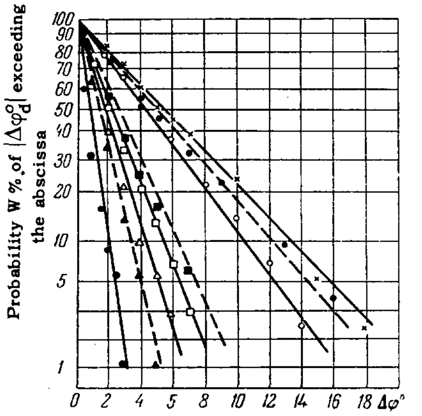

FIG. 4. Integral distributions of the fluctuations of the phase difference (absolute value) $|\varphi_1-\varphi_d|=\Delta\varphi_d$ for different distances d between receiving antennas. $h_0 = 35$ m, $h = 4$ m, $L = 33$ km, $\lambda = 10$ cm, $\Delta\varphi_d = \varphi_1 - \varphi_d$ is the deviation of the phase difference from the average value, in electrical degrees, W – probability that the fluctuations will exceed $\Delta\varphi^0$ (in absolute value). ●, Δ, □, o, x – d = 2, 5, 10, 30, 100 m, respectively, ▲, ■, ● – repeated measurements.

In these experiments the phase difference was measured relative to the first (reference) antenna. The data for distances d from 2 to 100 m are identified on the figure by different symbols. As seen from the figure, the experimental data, in agreement with theory, lie well on the straight lines corresponding to a normal distribution with different values of dispersion. In the experiments, the results of which are shown in Fig. 4, the antenna heights were $h_0 = 35$ m and $h = 4$ m, i.e., the corresponding points were in this case, in the zone of the line of sight. Qualitatively similar results were obtained for $h_0 = 18$ and 9 m, $h = 4$ m, i.e., in the "shadow" and "penumbra" regions.

A very important feature of the observed fluctuations is that they are not stationary. In measurements lasting 5 – 10 minutes, the nonstationarity was observed quite frequently, particularly in the low-frequency part of the spectrum. This nonstationarity increased with increasing height of the receiving antennas and with increasing distance between them. Although the low-frequency components are appreciably attenuated in measurements of phase-difference fluctuations (unlike in measurements of absolute-phase fluctuations), nevertheless[41,44] an increase in the spectral density with decreasing frequency is observed in this case, too. Such a tendency is noticeable down to frequencies 0.01 – 0.001 cps, which can still be handled by the apparatus used in these investigations. The running spectra of these fluctuations, unlike the spectra of stationary random processes, are characterized by an inhomogeneity of the spectral density and by time variability. For illustration, Fig. 5 shows the running intensity spectrum of the phase-difference fluctuations in the frequency interval from 0.03 to 0.36 cps with an equivalent analysis band of about 0.005 cps.

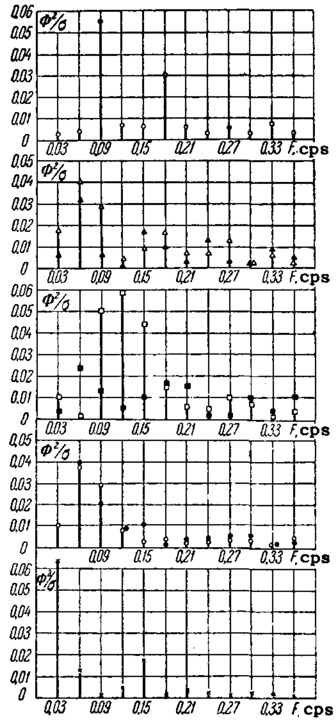

FIG. 5. Normalized "running" spectra of the intensity of phase-difference fluctuations, for different distances d between receiving antennas. Φ^2 – fluctuation power in a 0.005 cps band relative to the given frequency, σ – rms value of the total "power" of the fluctuations, $h_0 = 9$ m, $h = 4$ m, $L = 33$ km, $\lambda = 10$ cm, Δ, □, o, x – d = 2, 5, 10, 30, and 100 m respectively, ▲ ■, ● – repeated measurements.

The figure shows the values of the intensity of fluctuations for several discrete frequencies, referred to the total intensity, obtained by measurements with different distances d. Some experiments were performed twice (at d = 5, 10, and 30 m). As can be seen from Figs. 4 and 5, in repeated measurements performed only 5 minutes apart the nonstationarity causes the appearance of considerable (up to a factor of 2 or 3) changes in the intensity and the frequency spectrum of the fluctuations.

Even more appreciable are the oscillations in the fluctuation intensities (reaching 25 – 100 times on a fixed path) noticed in different experiments. Figure 6 shows the results of measurement of the intensity of the phase-difference fluctuations, obtained in summer and fall for distances d of 2 and 100 m between receiving antennas.*

It is interesting that the greatest changes in intensity were noted on different days at small distances

*In these experiments only the low-frequency part of the fluctuations spectrum ($F \lesssim 0.36$ cps) was reproduced.

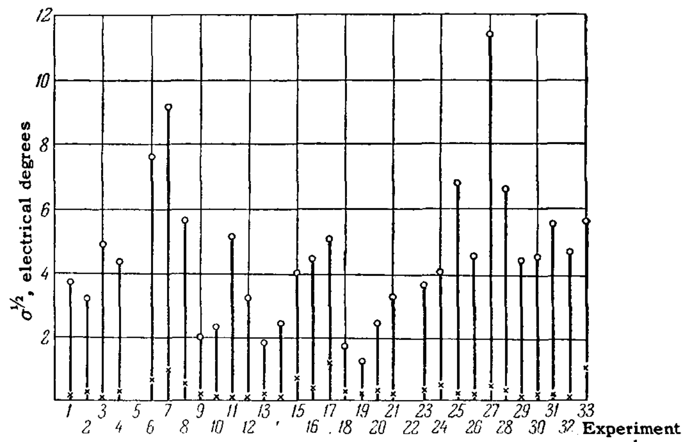

FIG. 6. Change in the intensity of fluctuations σ in measurements on a fixed path during the summer-fall season. The spectrum of fluctuations is reproduced down to $F \leq 0.36$ cps, $h_0 = 35$ m, $h = 4$ m, $L = 33$ km, $\lambda = 10$ cm. Experimental values: x, o – d = 2, 100 m.

between the measurement points. This effect, which is observed directly above the boundary, is expected in the far zone and in an unbounded inhomogeneous medium, since in the case of small d ($d < l$) the phase-difference fluctuations depend on the properties of the medium in a more complicated manner than in the case of large d. Actually, when $d < l$ and $R_d \sim 1$ (R_d is the coefficient of transverse correlation of the fluctuations at points located a distance d apart), the intensity of the phase-difference fluctuations is determined both by the fluctuations of the absolute phases of the signals and by the degree of their decorrelation

$$\sigma(d) = \langle(\varphi_i - \varphi_k)^2\rangle = 2\langle\varphi^2\rangle(1 - R_d); \quad \langle\varphi_i^2\rangle = \langle\varphi_k^2\rangle = \langle\varphi^2\rangle. \qquad (8.1)$$

In measurements with long base $d \gg l$ ($R_d \to 0$) the correlation of the fluctuations is insignificant, and the intensity of the fluctuations of the absolute phases, as shown in references 3 and 5, is practically independent of the explicit form of the correlation function for $\delta\epsilon$.

The noted tendency of the spectral density of the phase-difference fluctuations to increase with decreasing frequency becomes naturally more aggravated in the measurement of the fluctuations of the absolute phases. Thus, according to data obtained by continuously measuring the fluctuations for 40 hours,[45] the density changes as $F^{-2.8}$ in the frequency interval from 10 to 10^{-4} cps.

In spite of the presence of nonstationarity and of the difference in the intensities of the fluctuations and in the degree of their decorrelation in space, which were observed in prolonged measurements on fixed paths, it was possible to detect certain general laws common to all the experiments performed. In particular, one sees in the majority of the experiments a similar character of variation of the intensity of the phase-difference fluctuations $\sigma(d)$ as a function of the distance d, i.e., similar structural and correlation functions of the fluctuations of the absolute phases.

Much more variability is seen in the height function of the fluctuation intensity, $\sigma(h)$. Experiments have shown[43,44] that, depending on the singularities of this function and on the degree of nonstationarity, four qualitatively different types of phase-difference fluctuations can take place.

Let us consider each case separately.

9. Different Types of Phase Fluctuations

The phase fluctuations observed most frequently have characteristics that can be called standard and quasi-stationary. A characteristic of fluctuations of this type in the region below the maximum of the first interference zone is a reduction in intensity with increasing heights of the corresponding points. Measurements have shown that the fluctuation intensity σ varies as h^{-a}, where $a \leq 2$.

A standard quasi-stationary type of fluctuation is characterized by relatively high repeatability of both the intensity $\sigma(d)$ and of the time (spectral) characteristics.

To illustrate fluctuations of this type, Fig. 7a shows the height dependence of $\sigma(d)$. As can be seen from the figure, $\sigma \sim h^{-2}$ independently of d. Such a height dependence is in good qualitative agreement with the theoretical deductions.[18,19,25,26] It is impossible, however, to make a quantitative comparison of the experimental data with the theory, for in these experiments[44]

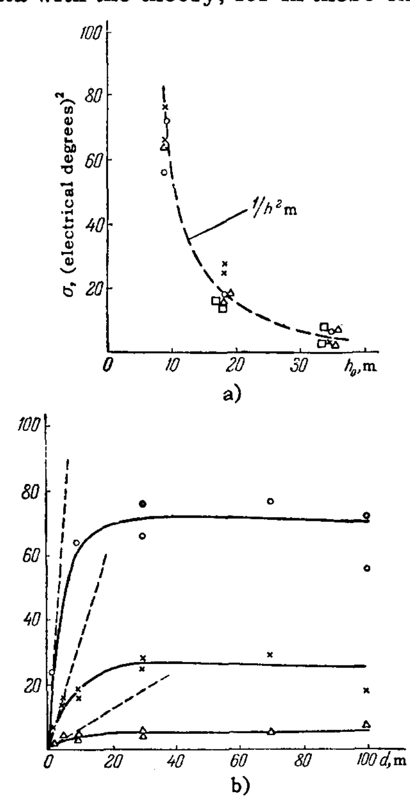

FIG. 7. Dependence of the structural function of the signal phase fluctuations $\sigma(d)$ on the height of the transmitter h_0 and on the distance between the receiving antennas d. $L = 33$ km, $h = 4$ m, $\lambda = 10$ cm. Experimental values: a) o, x, Δ, □ – d = 100, 30, 10, 5 m; b) o, x, Δ – h_0 = 9, 18, 35 m.

the height dependence was investigated not only for the illuminated zone, but also for the penumbral region. In particular, the quantity L/L_h (L_h — the distance of the radio horizon) ranged from 0.8 to 1.2. As is well known,[15,30] at these values of L/L_h it becomes necessary to take into account the curvature of the boundary.

Figure 7b shows the structural functions of the phase-difference fluctuations $\sigma(d)$ for different transmitter heights h_0. As can be seen from the figure, $\sigma(d)$ first increases rapidly with increasing d, but its rate of increase slows down at $d \geq 5-10$ m and then stops completely. A unique "saturation" of $\sigma(d)$ sets in, which is evidence of the decorrelation of the fluctuations in the separated antennas (i.e., $d \gg l$). The $\sigma(d)$ curves can be used to determine the correlation radii d_0. Using (7.1), we have

$$\frac{\sigma(d)}{\sigma(d_{\max})} = \frac{1-R(d)}{1-R(d_{\max})} \approx 1 - R(d), \tag{9.1}$$

where $d_{\max}$ is the distance at which "saturation" of $\sigma(d)$ is observed, and $R(d_{\max}) \approx 0$.

A plot of $R(d)$, based on data of Fig. 7b, is shown in Fig. 8, from which it follows that the correlation radii d_0 corresponding to $R(d) = 0.5$ are on the order of $4-8$ m for heights varying from 9 to 35 m. As can be seen from (9.1), the scale of the inhomogeneities can be determined by knowing the form of the function $R(d)$. Unfortunately, in the absence of experimental data for $d < 2$ m, the spread in the experimental points, due to the nonstationarity of the fluctuations,

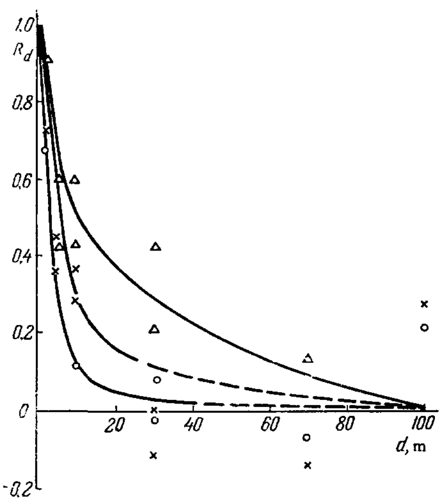

FIG. 8. Dependence of the transverse correlation of the fluctuations of the absolute phases on the distance d between the receiving antennas. L = 33 km, h = 4 m, λ = 10 cm. Experimental values: o, ×, Δ – h_0 = 9, 18, 35 m.

does not make it possible to determine from these experiments the exact form of the functional dependence $R(d)$, or the minimum distance $d_{\min}$ where the linear relation $\sigma^{1/2} = f(d)$ is violated. Estimates show that $d_{\min}$ does not exceed several meters, and that the region corresponding to the transition of this dependence from linear growth to saturation ranges from $1-2$ to $5-10$ m. These results are apparently not specific for measurements over a separation boundary, for a similar form of a structural function was obtained for the phase fluctuations[41] at λ = 3 cm when working with a beam "detached" from the surface. Unfortunately, the measurement procedure in that investigation did not provide for a rapid determination of the structural function for different values of d,* and in addition an appreciable distortion of the fluctuations spectrum took place, so that the data obtained in reference 41 are essentially only qualitative in nature.

By comparing with the calculations the results of the experimental measurements of the structural functions $\sigma(d)$, normalized beforehand to their maximum values corresponding to $d_{\max}$, we can determine the "scales of inhomogeneities" l. Although the results of such a determination of l depend on the specific form of the chosen correlation function, nevertheless, as shown in references 5 and 28, the differences for different correlation functions are small. In particular, for a Gaussian correlation function we have

$$\frac{\sigma(d)}{\sigma(d_{\max})} = \frac{1-\frac{\sqrt{\pi}}{2}\frac{l}{d}\,\mathrm{erf}\left(\frac{d}{l}\right)}{1-\frac{\sqrt{\pi}}{2}\frac{l}{d_{\max}}\,\mathrm{erf}\left(\frac{d_{\max}}{l}\right)}. \tag{9.2}$$

The values of the parameter l determined by the measurements of reference 44 are shown, in accordance with (9.2), for different experiments in Fig. 9, from which it follows that in the height interval from 9 to 35 m above the boundary the parameter l can, depending upon the meteorological conditions of the experiments, vary by a factor of $5-10$ times, amounting on the average to $3-6$ m. It is interesting to note that greater heights h_0 usually correspond to greater values of l.

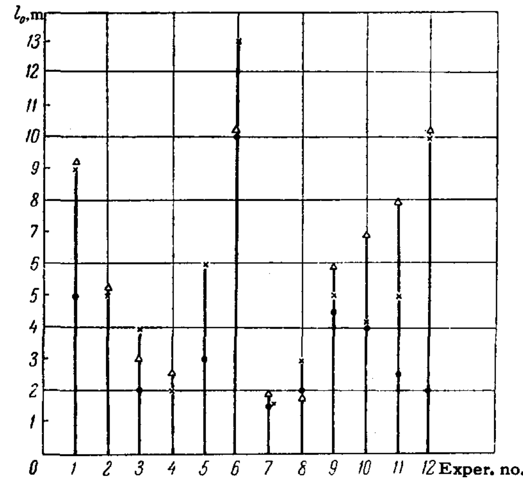

FIG. 9. Determination of the scales of inhomogeneity from measurements of the structural function of the phase fluctuation, made on a fixed path over the sea. L = 33 km, λ = 10 cm, h = 4 m. Experimental values: •, ×, Δ – h_0 = 9, 18, 35 m.

*The total time during which the structural dependence was measured in reference 41 was several hours.

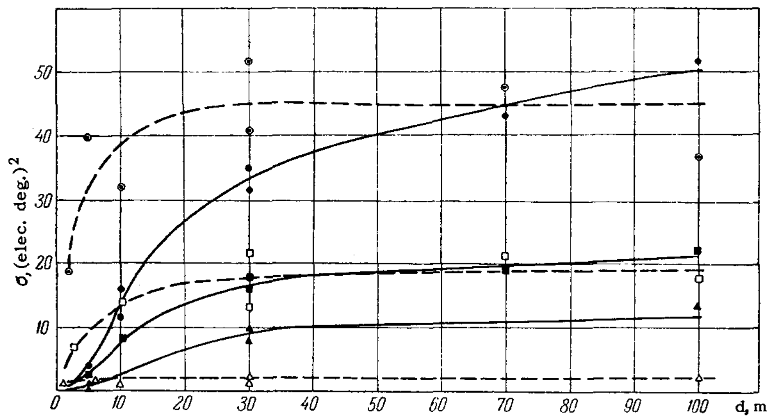

FIG. 10. Structural functions of the phase fluctuations $\sigma(d)$ for a partial reproduction of the fluctuation spectrum. $L = 33$ km, $h = 4$ m, $\lambda = 10$ cm. Measurement of low fluctuation frequency (< 0.36 cps): ●, ■, ▲ – $h_0 = 9$, 18, 35 m, respectively. Measurements of high-frequency fluctuations (> 0.36 cps): ⊙, □, △ – $h_0 = 9$, 18, 35 m.

It must be emphasized that the results of such an experimental determination of the scale of inhomogeneities are determined essentially by the degree of reproducibility of the measurements of the spectrum of the fluctuation frequencies.

The elimination of the high-frequency components of the spectrum leads to the slowing down of the growth of $\sigma(d)$ with increasing d, and to the contrary, the elimination of the low-frequency components of the spectrum causes a more rapid decorrelation of the fluctuations than in the case when the spectrum is completely reproduced. This premise can be illustrated by the data presented in Fig. 10, showing the results of individual measurements of the fluctuations in the high-frequency ($F_{lim} > 0.36$ cps) and low-frequency ($F_{lim} < 0.36$ cps) intervals of the spectrum. A series of experiments yielded, for a limiting frequency $F_{lim} = 0.36$ cps, $l_h = 1 - 3$ m (high-frequency measurements) and $l_l = 10 - 30$ m (low-frequency measurements). If F_{lim} is reduced, the experimentally measured value of l increases, and vice versa,[42,43] l decreases with increasing F_{lim}. Measurements of this kind show that, in agreement with the theory of turbulence,[11,12] the investigated fluctuations are due to "inhomogeneities" of different dimensions, and the high-frequency portion of the fluctuations spectrum is connected with the small-scale formations, while the low-frequency part is due to the large-scale ones. Thus, a possibility is afforded of investigating the spectrum of the inhomogeneity dimensions in the troposphere with the aid of suitable apparatus.

It follows from the foregoing that the influence of the high-frequency part of the spectrum of the phase-difference fluctuations is most appreciable at small distances d between antennas, where it comprises the main "energy" of the fluctuations. This result is also in agreement with direct determination of the spectra and the temporal correlation functions for different d. A similar effect is usually observed also when the heights of the antennas are changed. A reduction in the antenna height, like a reduction in the distance between receiving antennas, leads to a relative broadening of the fluctuation spectra.

In all cases, a tendency is observed towards a rapid reduction in spectral density with increasing fluctuation frequency. On the upper side, the energy spectrum of the phase-difference fluctuations is limited to frequencies on the order of 10 cycles when $2 \text{ m} < d < 100$ m. As already noted, a reduction in the fluctuation frequency is accompanied by an increase in the spectral density up to frequencies $\sim 10^{-3}$ cps. However, there are no more detailed data on the spectrum of the phase-difference fluctuations in the literature, and only fluctuations for the absolute phases are mentioned.[45]

The other type, which can be called standard nonstationary, includes fluctuations that are marked by sharp nonstationarity. Appreciable changes of qualitative character and in the intensity, reaching a factor of 2 or 3 and more, are observed in repeated measurements, shifted in time by five or ten minutes, and even during a single measurement. Experiments have shown that in spite of the nonstationarity, the main characteristics of the fluctuations remain in many experiments qualitatively the same as in the first case. In particular, the height dependence $\sigma_d(h_0)$, the structural function $\sigma(d)$ remain qualitatively the same as shown in Fig. 7.

Experiments carried out over the sea, in which the fluctuations had quasi-stationary and nonstationary standard characteristics, amounted to 70 percent (during the summer-fall season) to 90 percent (in the fall-winter season) of the total number of measurements.[43,44]

At the same time, height dependences of the fluctuations greatly different from standard are observed in many cases. At heights corresponding to points considerably below the maximum of the first zone, the fluctuations can increase monotonically with increasing altitude h_0, or pass through a maximum. Such nonstandard height dependences appear quite strongly in slow fluctuations, i.e., when the high-frequency components are not reproduced.* The data obtained in one of the experiments, in which such an anomalous type of fluctuations was observed, are

*If the fluctuation spectrum is completely reproduced, the anomalous character of the altitude dependence is less pronounced.

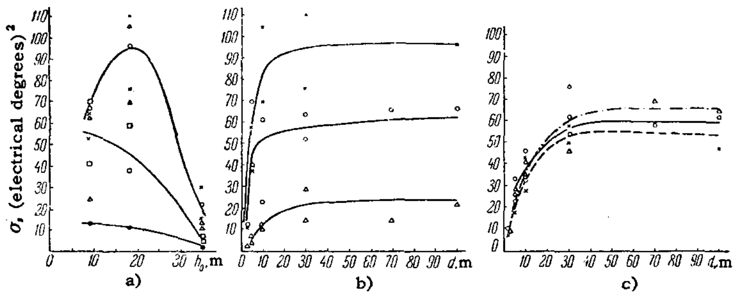

FIG. 11. Characteristics of phase fluctuations in the case of an anomalous height dependence: L = 33 km, h = 4 m, λ = 10 cm. a, b) The low-frequency spectral components have been reproduced (< 0.36 cps): o, x, Δ, $\square$ – d_0 = 100, 30, 10, 5, 2 m; o, x, Δ – h_0 = 9, 18, 35 m; c) the total fluctuation spectrum has been reproduced: o, x, Δ – h_0 = 9, 18, 35 m.

shown in Figs. 11a and b. Typical of fluctuations of this type is the fact that the anomalous height dependence is most sharply pronounced at maximal distances d, when the main fraction of the "energy" of the fluctuations is due to large inhomogeneities. As the distance d is decreased, the height dependence of the intensity of the fluctuations usually approaches standard.

It is typical that in such measurements the height dependence is close to standard for "fast" fluctuations, investigated without reproducing the low-frequency part of the spectrum. We note that measurements with sharply pronounced height anomalies are encountered relatively rarely. The cases much more frequently encountered are those with weakly pronounced or no height dependence. Such cases are apparently intermediate between the measurements of the first two types and experiments of the third type. An example of such measurements with degenerate height dependence, carried out with complete reproduction of the spectrum, may be Fig. 11c.

An even greater nonstationarity in measurements along a fixed path in regions considerably below the maximum of the first zone has been noted in individual cases in short-duration measurements, accompanied by an exceedingly sharp increase in the intensity of the fluctuations — the so-called "fluctuation flashes." Such flashes are usually preceded by large nonstationary fluctuations. During several minutes the phase fluctuations increase rapidly ($\delta\varphi > 2\pi$) and are accompanied by deep and frequent fading of the amplitude. The duration of such a state does not as a rule exceed several tens of minutes, after which the usual picture is restored.

The relatively good qualitative agreement in the experimental data and the theory enable us to suggest that the idealization of the turbulent medium, used in theory, by which the medium is assumed to be locally isotropic and statistically homogeneous, does not contradict the experiments made in the troposphere layers closest to the earth.

In spite of the fact that the real troposphere contains formations with different inhomogeneity scales, many regularities, such as the form of the altitude dependence, the structural functions, etc., can be explained by describing the medium with the aid of the correlation function of the pulsations of the dielectric constant, which has a single characteristic scale.

The situation is quite different in the case of the anomalous fluctuations. The presence of height characteristics, by which the intensity of the fluctuations may increase with increasing height, cannot be explained by the theory developed in Chapter I. From the experimental data given it can be concluded that to explain the observed effects in this case it is necessary to make use of the anisotropy and inhomogeneity of the large-scale formations in the vertical direction. Such an assumption is to some extent in agreement with the fact that the anomalous height characteristics are usually observed when the fluctuations are highly nonstationary.

The explanation of the phenomenon of fluctuation flashes is quite unique. It is obvious that a sharp increase in the fluctuations when the phase difference changes by more than 2π could be explained, if observed in an unbounded medium, only by an increase by some tens or even hundreds of times in the intensity of the fluctuations of the dielectric constant along the paths, which apparently is of little likelihood.

Allowance for the boundary shows that such an anomalous increase in the fluctuations may be due to the change in the average refraction along the path, leading to an interference minimum at the point of reception. The latter is in agreement with the fact that the "fluctuation flashes," if received in the region below the maximum of the first interference zone, are usually at reduced refraction, i.e., with rising of the zone.

10. Dependence of the Fluctuations on the Distance and on the Meteorological Conditions

The dependence of the fluctuations of the phases on the distance between the corresponding points in the direct line of sight and beyond the radio horizon was investigated recently[25,26] by measuring the fluctuations of the phase differences between the receiving antennas 10 m apart by the "moving transmitter" method. These experiments, which were carried out for fixed antenna heights h_0 = 10 m and h = 4 m, show that the character of the dependence of the intensity of the fluctuations on the distance does not remain unchanged in the various experiments, and deviates from the law $\sigma \sim L$, which would be expected in the case of an unbounded inhomogeneous medium.*

*We note that measurements carried out under conditions close to the case of an "unbounded medium" (for example in reference 42, where the fluctuations of the absolute phases were investigated) are in good agreement with this law.

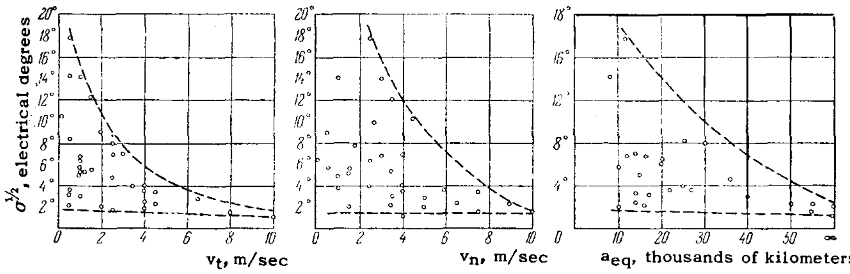

FIG. 12. Influence of the wind velocity v and radio refraction a_{eq} on fluctuations of phase differences (mean square values) in measurements above the sea. L = 33 km, h = 4 m, λ = 10 cm, v_t – wind component direction along the path (in m/sec), v_n – wind component transverse to the path (in m/sec).

Certain results of a series of such measurements were shown together with the calculated relationships in Fig. 1, which illustrates the dependence of the dispersion of the phase-difference fluctuations on the distance, normalized to its value for a distance of 10 km. As follows from the figure, within the zone of the direct line of sight (the standard distance to the radio horizon in these measurements was $L_h = 22$ km), this variation is always faster than L, and may reach L^3. Beyond the limits of the radio horizon, in the "penumbra" region, an even faster growth in the fluctuations was observed, reaching as much as L^6 for $1 \leq L/L_h \leq 2$.

We have already noted several times the variability of the fluctuation characteristics, both during the time of the experiment and from experiment to experiment. In this connection it is natural to attempt to seek a correlation between the fluctuations of the radio signals and the meteorological conditions. It should be pointed out that the results of radio measurements are as a rule integral in character, since they are determined by processes that occur in the entire region adjacent to the propagation path, whereas meteorological measurements are usually of local character. In view of this, it would be advantageous to carry out simultaneous meteorological measurements in different points of space. Unfortunately, there are no detailed data on this question in the literature. A certain simplification can be admitted in the analysis of very slow quasi-stationary tendencies, which can be estimated by carrying out measurments in the low-frequency part of the spectrum of the fluctuation frequencies, since it is expected in this case that the measurements in different points in space are significantly correlated. Such determinations of the fluctuations of the absolute phases at a frequency of 9400 Mc/sec and at a refractive index determined from recorded values of the temperature, pressure, and humidity have been carried out in reference 45 under conditions close to those of unbounded space, on a path 9.4 miles long, and have shown that an appreciable correlation, reaching 0.915, exists between them.

A comparison of the phase fluctuations, carried out in a considerably broader frequency band, with certain results of averaged meteorological measurements, carried out[44] on both ends of a path 33 km long, is shown in Fig. 12. As follows from the data of this work, it is impossible to establish a direct functional connection between the magnitude of the fluctuations and the meteorological measurements (temperature, pressure, humidity, etc.) on the ends of the path. Nonetheless, many of the measurements carried out above the surface of the sea do disclose certain tendencies. Thus, for example, an increase in the wind velocity, independently of the wind direction, and an increase in the waviness of the sea, are usually accompanied by a reduction in the intensity of fluctuations. A reduction in fluctuations was noted also, as a rule, at increased radio refraction, up to an equivalent earth radius $a_{eq} \to \infty$ during overcast and rainy weather. The greatest fluctuations were observed in quiet sunny days in the absence of waviness of the sea. Analogous results of preliminary measurements are given also in reference 45.

The influence of meteorological conditions can be given a natural explanation within the framework of the model proposed above, in which the boundary is taken into account. The presence of wind, the intensification in the waviness of the sea, etc. lead to a diffuse scattering of the radio waves and to a reduction in the effective coefficient of specular reflection. Therefore the interference effects, which cause an increase in the fluctuations, become less sharply pronounced and this in turn leads to a reduction in the fluctuations. From this point of view it is understandable why the fluctuations increase in the absence of wind and waviness of the sea.

CONCLUSION

As follows from a theoretical analysis and from the experimental data given in the present survey, the separation boundary has an appreciable influence on the fluctuations of the radio signals. The presence of the boundary leads to a faster decrease in fluctuation with the distance, changes the frequency dependence, causes the appearance of fluctuation flashes, leads to a sharp increase in the intensity of fluctuations in the minima of the average field, determines the unique height dependence, etc. Although the data obtained on the fluctuations of the signals above the boundary are of great interest, nevertheless many questions connected with this problem have not yet been answered, principally because a detailed investigation of these questions was started in fact only in recent years.

In view of the novelty of the problem and its urgency, it is advisable to point to several directions which we believe further research should follow.

On the theoretical end it is important to determine the influence of the curvature of the boundary and determine the fluctuations not only in the zone of the direct line of sight, but also in the "penumbra" and "shadow" regions, where the intensity of fluctuations becomes quite considerable and where their influence is of greatest importance in practice.

Another important theoretical problem is the fullest possible evaluation of the anisotropy and temporal instability of the medium. It would be desirable in particular, even in the development of the phenomenological theory, to take into account the existence of a spectrum of inhomogeneity scales in the turbulent medium.[12]

The experimental investigations should attempt a detailed determination of the frequency dependence of the intensity of fluctuations over the broadest possible range. In addition, it is desirable to set up experiments in which extensive meteorological investigations are made along with the radio measurements, so that a connection can be established between them, and the meteorological measurements can be used to forecast the character and intensity of the fluctuations of the radio signal.

Since the reproduction of various fluctuation spectra of the radio signals already enables us to judge the dimensions of the inhomogeneities, it will be advantageous in the future to attempt to investigate more consistently the radio methods for the study of physical processes that occur in the turbulent troposphere.

Note added in proof. The question of field distribution calls for a more detailed analysis. The amplitude distribution fits the normal Gaussian curve only in the case of small field fluctuations, when $\langle \xi^2 \rangle \gg |E^2|$, in other words, if the distance L is small compared with the "attenuation length" α^{-1} [α is the damping factor of the average field, due to scattering by inhomogeneities, see (2.12) ff]. In this case the fluctuations of the field and of the dielectric constant are actually linearly related. At large distances ($\alpha L \gg 1$) the field distribution apparently obeys the normal-logarithmic law,[3,11,12] as in the case of one-dimensional large-scale fluctuations, for which this can be rigorously proved by the WKB method. However, the method given for calculating the first two moments of the field distribution function[21] is essentially independent of the specific form of this function, and uses merely the law of energy conservation. It is consequently valid for all distances. As applied to the troposphere, this question is of purely academic interest, since the attenuation length is as a rule so great that the distribution is practically always normal.

[1] P. J. Bergmann, Phys. Rev. 70, 486 (1946).

[2] V. A. Krasil'nikov, Izv. Akad. Nauk SSSR, Ser. Geogr. i Geofiz. 13, 33 (1949).

[3] A. M. Obukhov, Izv. Akad. Nauk SSSR, Ser. Geofiz. 17, 155 (1953).

[4] L. A. Chernov, Doklady Akad. Nauk SSSR 98, 953 (1954).

[5] R. B. Muchmore and A. D. Wheelon, Pro. IRE 43, 1437 (1955).

[6] A. D. Wheelon and R. B. Muchmore, ibid. p. 1450.

[7] V. A. Krasil'nikov and A. M. Obukhov, Акустический журнал 2, 107 (1956), Soviet Phys.-Acoustics 2, 103 (1956).

[8] A. A. Chernov, ibid. 3, 192 (1957), transl. p. 203.

[9] V. N. Karavaĭnikov, ibid. 3, 165 (1957), transl. p. 175.

[10] V. I. Tatarskii, Doklady Akad. Nauk SSSR 120, 298 (1958).

[11] L. A. Chernov, Распространение волн в среде со случайными неоднородностями, (Propagation of Waves in a Medium with Random Inhomogeneities), AN SSSR, 1958.

[12] V. I. Tatarskii, Теория флуктуационных явлений при распространении волн в турбулентной атмосфере, (Theory of Fluctuations in the Wave Propagation in the Turbulent Atmosphere), AN SSSR, 1959.

[13] A. V. Men', Изв. вузов СССР (Радиофизика) (News of the Colleges, Radiophysics) 2, 395 (1959).

[14] A. D. Wheelon, J. Atmos. Terr. Phys. 15, 185 (1959).

[15] B. A. Vvedenskii and A. G. Arenberg, Распространение УКВ, (UHF Propagation), Svyaz'izdat, 1938.

[16] V. I. Bunimovich, Флуктуационные процессы в радиоприемных устройствах, (Fluctuating Processes in Radio Receivers), Soviet Radio Press, 1951.

[17] Теория случайных процессов и ее применение в радиотехнике (Theory of Random Processes and Its Application to Radio Engineering), Soviet Radio Press, 1957.

[18] É. A. Kaner and F. G. Bass, Doklady Akad. Nauk SSSR 127, 792 (1959), Soviet Phys.-Doklady 4, 847 (1960).

[19] É. A. Kaner and F. G. Bass, op. cit. ref. 13, 2, 553 (1959).

[20] T. A. Shirokova, op. cit. ref. 7, 5, 485 (1959), transl. p. 498.

[21] Lifshitz, Kaganov, and Tsukernik, Уч. записки ХГУ. Тр. физ.-мат. ф-та (Science Notes, Khar'kov State Univ., Phys. Math. Fac.) 2, 41 (1950).

[22] F. G. Bass, op. cit. ref. 13, 2, 1015 (1959).

[23] L. D. Landau and E. M. Lifshitz, Электродинамика сплошных сред (Electrodynamics of Continuous Media), Gostekhizdat, 1959.

[24] É. A. Kaner, op. cit. ref. 13, 2, 827 (1959).

[25] Men', Braude, and Gorbach, Doklady Akad. Nauk UkrSSR, No. 7, 740 (1959).

[26] Men', Gorbach, and Braude, op. cit. ref. 13, 2, 388 (1959).

[27] Al'pert, Ginzburg, and Feĭnberg, Распространение радиоволн (Propagation of Radio Waves), Gostekhizdat, 1953.

[28] F. G. Bass and E. A. Kaner, op. cit. ref. 13, **2**, 565 (1959).
[29] C. R. Burrows and S. S. Attwood, Radio Wave Propagation, Academic Press, N. Y., 1949.
[30] Распространение ультракоротких радиоволн, (Propagation of UHF Waves), Soviet Radio Press, 1954.
[31] Braude, Ostrovskii, Ivanchenko, Shamfarov, and Sanin, О распространении ультракоротких волн над морем, (Propagation of UHF above the Sea), Soviet Radio Press, 1949.
[32] Braude, German, Ostrovskii, Bezuglyi, Amosov, Bliokh, Sanin, Tsukernik, and Shamfarov, Распространение электромагнитных колебаний сантиметрового диапазона над морем при наличии "атмосферного волновода" и в условиях повышенной рефракции (Propagation of Electromagnetic Oscillations in the Centimeter Band over the Sea in the Presence of an ''Atmospheric Waveguide'' and under Conditions of Increased Refraction), Soviet Radio Press, 1951.
[33] Burrows, Decino, and Hunt, Proc. IRE **23**, 1507 (1935).
[34] K. G. Maclean and G. S. Wickizer, ibid. **27**, 501 (1939).
[35] A. Waynick, ibid. **28**, 468 (1940).
[36] G. S. Wickizer and A. M. Braaten, ibid. **35**, 670 (1947).
[37] W. M. Sharpless, ibid. **34**, 837 (1946).
[38] A. B. Crawford and W. M. Sharpless, ibid. **34**, 845 (1946).
[39] A. W. Stratton and J. R. Gerhardt, ibid. **36**, 916 (1948).
[40] A. W. Stratton, ibid. **37**, 808 (1949).
[41] A. P. Dean and B. M. Fannin, ibid. **43**, 1402 (1955).
[42] J. W. Herbstreit and M. C. Thompson, ibid. **43**, 1391 (1955).
[43] Men', Braude, and Gorbach, Doklady Akad. Nauk SSSR **125**, 1019 (1959), Soviet Phys.-Doklady **4**, 448 (1959).
[44] Men', Braude, and Gorbach, op. cit. ref. 13, **2**, 843 (1959).
[45] M. C. Thompson and H. B. Janes, J. Res. NBS **63D**, 45 (1959).

Translated by J. G. Adashko