CERTAIN THEORETICAL ASPECTS OF RADIATION DUE TO SUPERLUMINAL MOTION IN A MEDIUM

V. L. GINZBURG

Usp. Fiz. Nauk, 69, 537--564 (December, 1959)

CONTENTS

INTRODUCTION

FUNDAMENTALLY new effects arise when a radiator moves in a medium, or close to a medium, with a velocity which exceeds the phase velocity of light in this medium ("superlight" or superluminal motion). Thus, Cerenkov radiation is produced, the nature of the Doppler effect is modified in a fundamental way and, in certain cases, oscillations of the radiating particle can be excited, causing instability in a beam of such particles.

The theory of the Cerenkov effect for a charge that moves in an infinite isotropic medium is now well known (cf. reference 1 and the reviews in references 2 and 3). However, the same cannot be said for a number of other problems: for example, Cerenkov radiation of dipole moments in an infinite medium and in slits or channels, Cerenkov absorption, the radiation reaction in superlight motion, and instabilities of superlight particle beams. Some of these problems have been investigated only recently and are as yet not always clearly understood.

Below we consider the theory of radiation due to motion at superlight velocities, emphasizing the physical ideas and the general approaches being used in this field. Our presentation will, to an appreciable degree, be rather incomplete. We shall omit many of the details of the calculations and make only brief mention of certain problems which are either discussed in the literature [2--3a] or require detailed analyses (collective effects, instabilities of particle beams, generation of radio waves [4] etc.).

1. CHARACTERISTIC FEATURES OF RADIATION DUE TO SUPERFLIGHT MOTION (CLASSICAL THEORY)*

In most cases superlight radiation can be considered within the framework of classical theory. The radiation condition for the Cerenkov effect is given by [1--3]:

$$\cos\theta_0 = \frac{c}{n(\omega)v}, \tag{1}$$

where θ_0 is the angle formed by the particle velocity v and the Cerenkov wave vector k and $n(\omega)$ is the index of refraction at the frequency being considered ω ; the medium is assumed to be isotropic.

The condition given in (1) is a kinematic (interference) relation, and, for this reason, is always valid, regardless of the nature of the radiator (charge, dipole, etc.); this condition applies in an anisotropic medium,[5] but $n(\omega)$ must be replaced by $n_j(\omega, \mathbf{k}/k)$, the refractive index for the characteristic wave denoted by $j = 1, 2$ which is propagated in the k direction in the case of motion along the axis of a uniaxial crystal or along a magnetic field in a plasma, $n_j = n_j(\omega, \theta_0)$) and the condition given in (1) is then a relation for determining $\theta_0(\omega)$].

*§§ 1--3 of the present review are based on the text of a paper[4a] given at the Ministry of Higher Education of the III All-union Conference on Radio Electronics (Kiev, January 1959).

The energy radiated per unit time by virtue of the Cerenkov effect for a point charge which moves uniformly in an isotropic medium is given by [1--3]

$$\frac{dW}{dt}=\frac{e^2v}{c^2}\int\limits_{c/n(\omega)\leq v}\left[1-\frac{c^2}{n^2(\omega)v^2}\right]\omega\,d\omega. \qquad (2)$$

If a radiator that moves in the medium radiates at a frequency ω_i in its own coordinate system then, because of the Doppler effect, the frequency of the radiation in the reference system fixed in the medium is given by $\omega(\theta)$:

$$\omega=\frac{\omega_i\sqrt{1-\beta^2}}{|1-\beta n(\omega)\cos\theta|},\qquad \beta=\frac{v}{c}. \qquad (3)$$

When $\beta n < 1$, i.e., at "sublight" velocities, Eq. (3) is the familiar expression for the Doppler effect in a medium. However, if $\beta n > 1$ (superlight velocity) the radiation must be considered separately inside and outside the Cerenkov cone.*[6] Inside the cone ($\theta < \theta_0$), cf. Fig. 1) the Doppler effect is "anomalous" (superlight) and the frequency ω increases as the angle θ increases; if n = const, $\omega \to \infty$ as $\theta \to \theta_0$. Outside the cone ($\theta > \theta_0$) the Doppler effect may be called "normal" since the frequency ω diminishes as θ increases.

Equation (3) reflects a general characteristic feature of radiation in a medium -- the role of the quantity $\beta = v/c$ for vacuum is played by the quantity βn in a medium; thus, whereas the extreme relativistic case in vacuum is indicated by the condition $\beta \to 1$, in a medium, as far as the nature of the radiation is concerned, the extreme relativistic case corresponds to $\beta n \to 1$. Furthermore, whereas in a vacuum at $\beta \to 1$ the radiation is highly directive in the direction of the velocity, in a medium the preferred direction is given by the Cerenkov cone. For a medium characterized by $n < 1$ (isotropic plasma), however, the radiation at any velocity does not exhibit the features characteristic of radiation in the extreme relativistic case in vacuum. Under certain conditions this last feature can become extremely important. Thus, in radio astronomy the "magnetic bremsstrahlung" (synchrotron) radi-

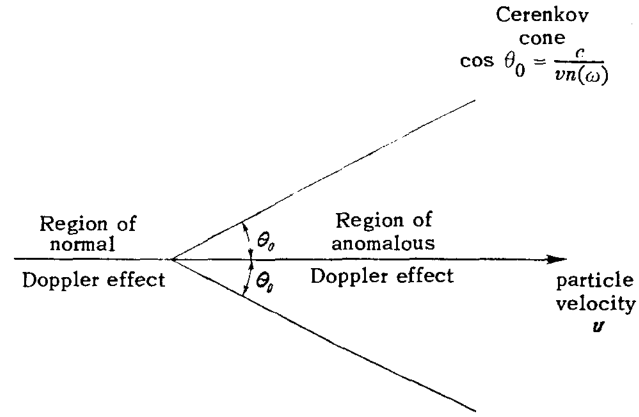

FIG. 1.

ation of relativistic electrons that move in weak magnetic fields is of extreme importance. The nature of this radiation is modified in an important way at low frequencies if account is taken of the effect of the medium, i.e., the interstellar or stellar plasma, whose refractive index is given by $n=\sqrt{1-\frac{4\pi e^2N}{m\omega^2}}$ (usually the effect of the magnetic field on n can be disregarded).

The deviation of n from unity can be neglected for $\beta \to 1$ if

$$|1-n(\omega)|\ll\left(\frac{mc^2}{E}\right)^2\sim(1-\beta),\qquad 1-\beta\ll 1. \qquad (4)$$

In the case given by (4), $|(1-\beta n)-(1-\beta)| = \beta|1-n| \ll (1-\beta)$ and the factor n in Eq. (3) is not important. To a certain extent this remark also applies to other expressions for radiation intensity, which contain the same factor $(1-\beta n\cos\theta)$ in the denominator (cf. Sec. 3). In the case of an isotropic plasma, when $1-n \ll 1$ the condition in (4) assumes the form $2\pi e^2N/(m\omega^2) \ll (mc^2/E)^2$ [here, as in (4), E is the total energy]. A discussion of this criterion as it applies to cosmic radio waves is given in reference 7.

Although a radiator that moves through a dense medium suffers high losses, it is important to note that the Doppler effect in a medium can still be of importance for the following two reasons. Firstly, the characteristic features of the superlight Doppler effect are maintained for motion in narrow slits or channels in a medium,[8--11] or for motion near a medium or an artificial slow-wave system. Secondly, the effect is of interest in connection with motion in a magnetoactive plasma,[12-14a] in which case the losses are small. These remarks apply also to the Cerenkov effect, although the latter can be observed even in the case of motion through a dense continuous medium.

*For anisotropic media (including gyrotropic media) $n(\omega)$ is replaced in Eq. (3) by $n_j(\omega, \mathbf{k}/k)$. In the text, for simplicity, it is assumed that the medium is isotropic. When dispersion [$n = n(\omega)$] is taken into account, Eq. (3) may yield several values of ω for a given value of θ ("complex" Doppler effect, cf. reference 6); when dispersion must be considered, the Cerenkov angle θ_0 depends on ω and thus, for a given value of $\theta < \pi/2$, it is possible in general to have radiation at both the normal and anomalous Doppler frequencies [these are distinguished by the sign of the quantity $1-\beta n(\omega)\cos\theta$]. In the present review dispersion is not considered.

2. QUANTUM THEORY OF RADIATION AND ABSORPTION DUE TO SUPERLIGHT MOTION

Elementary quantum-mechanical ideas are found to be very fruitful in the analysis of various problems dealing with radiation, absorption, and amplification of electromagnetic waves associated with the motion of charges or "systems" (atoms, particle bunches, antennas) in a medium. It is interesting to note that this is the case in spite of the fact that the analysis of the problem is essentially classical and that to this accuracy, the final formulas are independent of the quantum constant $\hbar$.

The point of departure in the quantum-mechanical analysis is the notion of photons in the medium; the energy of these photons is $\hbar\omega$ and the momentum is $\hbar \mathbf{k} = \frac{\hbar\omega n(\omega, s)}{c}$ where $\mathbf{k} = k\mathbf{s}$ is the wave vector and n is the refractive index for a given characteristic wave that propagates in the medium being considered (in the general case the medium can be anisotropic and gyrotropic). The quantization procedure has been carried out for the case of an isotropic medium in reference 15 and the generalization of the results to an arbitrary medium can be made immediately by the use of the plane-wave expansion.[16-17a] Obviously this approach is valid only when a phenomenological theory can be used. It should also be kept in mind that the momentum of the photon in the medium is the total momentum; this includes the momentum of the field as well as the momentum communicated to the medium in the radiation of the wave (cf. references 18--19 and below).

From the quantum-mechanical point of view, the radiation kinematics, i.e., the constraints imposed on the frequency and direction of the radiation, are determined by conservation of energy and momentum (the same constraints obviously also apply in absorption). For example, if a "system" (electron, atom, antenna) has an energy E_0 before radiating, and E_1 after radiating (with corresponding momenta $\mathbf{p}_0$ and $\mathbf{p}_1$), these quantities are governed by the conservation laws:

$$E_0 - E_1 = \hbar\omega, \tag{5}$$

$$\mathbf{p}_0 - \mathbf{p}_1 = \hbar\mathbf{k} \equiv \frac{\hbar\omega n}{c}\frac{\mathbf{k}}{k} \equiv \frac{\hbar\omega n}{c}\mathbf{s}. \tag{6}$$

For a system that moves uniformly in a vacuum (i.e., for n = 1), radiation without a change of the internal state of the system is impossible (for example, an electron that moves uniformly in a vacuum cannot radiate). This well-known result also follows from Eqs. (5) and (6) because with n = 1, for a particle without internal degrees of freedom, the only solutions for these equations is $\quad = 0$. However, if n = 1, substituting $E_{0,1} = \sqrt{m^2c^4 + c^2p_{0,1}^2}$ in Eqs. (5) and (6), where $p_{0,1} = \frac{mv_{0,1}}{\sqrt{1 - v_{0,1}^2/c^2}}$, the radiation condition for the case in which there is no change in the internal state is given by :

$$\left.\begin{aligned} \cos\theta &= \frac{c}{n(\omega)v_0}\left(1 + \frac{\hbar\omega(n^2-1)}{2mc^2}\sqrt{1 - \frac{v_0^2}{c^2}}\right), \\ \hbar\omega &= \frac{2\frac{mc}{n}\left(v_0\cos\theta - \frac{c}{n}\right)}{\sqrt{1 - \frac{v_0^2}{c^2}\left(1 - \frac{1}{n^2}\right)}}. \end{aligned}\right\} \tag{7}$$

When $\hbar\omega/mc^2 \ll 1$ this condition becomes the classical radiation condition (1), as is to be expected (if $\hbar\omega/(mc^2) \ll 1$, the recoil associated with the radiation of the photon can be neglected.* It follows from Eq. (7) that radiation is possible (i.e., $\cos\theta < 1$ and $\omega > 0$) only for superlight motion, that is, only when the inequality $V_0 n/c = \beta n > 1$ is satisfied.

When the results do not contain $\hbar$, the quantum-mechanical analysis is of methodological value only. Essentially, we are then using the conservation laws, which are broad in scope, and which can be applied without introducing quantum-mechanical ideas. Specifically, we assume from the classical theory of the electromagnetic field in a medium that the relation between the energy $\mathscr{E}$ and the total radiation momentum of the medium P is given by $\mathbf{P} = \mathscr{E}n\mathbf{s}/c$.** Furthermore, in the free motion of a charge, if the changes in energy and momentum are small we have $\Delta E = \mathbf{v}\cdot\Delta\mathbf{p}$ since $\frac{dE}{d\mathbf{p}} = \frac{d}{d\mathbf{p}}\left(\sqrt{m^2c^4 + c^2p^2}\right) = \frac{c^2\mathbf{p}}{E} = \mathbf{v}$. Thus, using the conservation laws (5) and (6) and replacing $\hbar\omega$ by $\mathscr{E}$ we obtain $\Delta E = \mathscr{E} = \mathbf{v}\cdot\Delta)\mathbf{p} = \mathscr{E}n \quad \mathbf{s}\cdot\mathbf{v}/c \quad \cos\theta_0 = c/nv$, the condition given in

*It is clear from Eq. (7) that for very large values of n the condition which must be satisfied in order for the classical analysis to apply is somewhat different, namely:

$$\frac{\hbar\omega n^2}{2mc^2}\sqrt{1 - \frac{v_0^2}{c^2}} \ll 1.$$

**As has been shown in reference 18, the momentum of the field is $P_f = \mathscr{E}/nc$ while the momentum communicated to the dielectric (because of radiation) is

$$P_d = (n^2-1)P_f = \frac{\mathscr{E}(n^2-1)}{nc}.$$

Hence the total momentum is given by the expression $P = P_f + P_d = \mathscr{E}n/c$.

However, the introduction of a photon of energy $\hbar\omega$ makes the analysis simpler* so that this procedure is fruitful in both the classical and quantum-mechanical cases. This is the technique which will be used here.

If we are not concerned with the motion of a particle, but the motion of a system whose internal energy can change, then $E_0 = \sqrt{(m+m_0)^2c^4 + c^2p_0^2}$ and $E_1 = \sqrt{(m+m_1)^2c^4 + c^2p_1^2}$ where $(m+m_0)c^2 = mc^2 + \epsilon_0$ is the total energy in the lower state and $(m+m_1)c^2 = mc^2 + \epsilon_1$ is the total energy in the upper state. Obviously, $\epsilon_1 - \epsilon_0 = \hbar\omega_i > 0$ 0 is the energy difference between the two states of the system (atom, etc.).

Now, applying the conservation laws (5) and (6) with $\hbar\omega/(mc^2) \ll 1$ we obtain the Doppler condition (3).[20] In this approach, however, we throw light on a situation which is completely obscured in the classical derivation of Eq. (3).[6] In particular, for the normal Doppler effect, i.e., when

$$\beta n(\omega)\cos\theta > 1, \qquad (8)$$

radiation corresponds to a transition of the system from the upper state characterized by an energy ϵ_1, to the lower state, characterized by an energy ϵ_0 (the direction of the transition is uniquely determined by the fact that the energy of the radiated photon must be positive, i.e., the formal requirement $\omega > 0$). However, if the photon is radiated inside the Cerenkov cone, that is to say, if the Doppler effect is anomalous

$$\beta n(\omega)\cos\theta < 1, \qquad (9)$$

the radiation of the photon means a transition of the system from the lower state ϵ_0 to the upper state ϵ_1. The energy of the photon and the energy which goes into excitation of the radiating system is, in this case, derived from the kinetic energy of the forward motion of the system.

It is apparent from this example that in the quantum-mechanical analysis, as contrasted with the classical analysis, the radiation conditions themselves determine the direction of the process (transition in the upward or downward direction). It is because of this feature, and the relative simplicity of the induced emission calculation (cf. below), that the quantum-theoretical calculations are valuable in obtaining the radiation conditions, conditions for amplification (instability) of waves in beams, and so on.

If the system has only two discrete states, 0 and 1, when $\beta n < 1$ (sublight motion) under stationary-state conditions the radiator is in the lower state, 0, (it is assumed that the system moves, say, in a channel in a medium and that there are no external sources). In other words, if state 1 is excited, after a certain time the system radiates, making a transition to state 0. However, if $\beta n > 1$ (superlight motion) under stationary-state conditions there is a finite probability for finding the system in state 1 and it continues to emit both normal and anomalous Doppler radiation. The populations of states 1 and 2 and the intensity of the normal and anomalous radiation are determined by the ratio of the total emission probabilities for the normal and anomalous radiation. In a many-level system[21] the emission of anomalous Doppler radiation, with an upward transition of the system, implies the excitation of "transverse" oscillations and, for example, ionization of an atom. To be more precise, two cases must be considered.[22] In the first case, the mean energy associated with the transverse oscillations of the system is reduced in the course of the motion. This means that if one considers a wave packet made up of wave functions characterized by slightly different energies (for example, energies of the transverse oscillatory motion of an electron which moves in the direction of a magnetic field) the center of gravity of the packet (on the energy scale) is lowered. In this case the difference between the sublight and superlight motions lies in the different rates of change of the mean energy and in the nature of the spreading of the packet. Thus, for sublight velocities it is reasonable to expect no occupation of states with energies higher than those corresponding to the initial spectrum of the packet. For superlight velocities, however, in spite of the reduction in mean energy, there is a finite probability for finding the system in some higher state, consistent with (9) (here we are obviously considering an ensemble of systems).

In the second case the system is unstable even "in the mean," i.e., its mean energy (the oscillation energy) increases with time, regardless of

*Suffice it to say that the problem of the classical expression for the field momentum P_f and the total momentum P is an extremely complicated one which has been discussed for several decades and was finally resolved only recently.[18,19] At the same time the relation $p = \hbar\omega n/c = \mathscr{E}n/c$, used in quantization, can be obtained immediately.[15] Actually, it follows from the fundamental concepts of quantum mechanics that the total momentum of the radiation is $\hbar\mathbf{k}$ where $\mathbf{k}$ is the wave vector which appears in the Fourier expansion of the field or the vector potential ($E = \Sigma$ const $\cdot \exp(i\mathbf{k}\cdot\mathbf{r})$ etc). On the other hand, it follows from the field equations that $k^2 = \omega^2 n^2/c^2$. Whence, $\mathbf{p} = \hbar\mathbf{k} = \frac{\hbar\omega n}{c} s$.

the way in which the packet spreads.

In order to determine which one of these cases applies one must calculate the actual transition probabilities. In this respect, the quantum-mechanical calculation has no advantage over the classical calculation and generally classical radiation theory is used. This question will be considered further below (cf. Sec. 3).

At this point we may note that quantum-mechanical considerations such as those used above are also useful in the analysis of the problems involving the attenuation and growth of waves in particle beams (streams). In general the instability of a beam is due to growing waves. Using our approach it is easy to obtain the stability criteria for a beam of particles which move in an isotropic plasma (cf. reference 23 and Sec. 4 below). It is also clear that in general, in superlight motion of streams of systems characterized by two or more states, we will not have absorption (reabsorption), but amplification (negative absorption) of the anomalous radiation.[24] This result follows from the fact that in the absorption of a photon in the anomalous Doppler region (i.e., a photon emitted at an angle $\theta < \theta_0$ with respect to the velocity of the system) the system does not make a transition from a lower state to a higher state (as in the normal effect) but from the higher state to a lower state.* Now, however, an upward transition of the system corresponds to induced emission which, for the normal Doppler effect, corresponds to a downward transition of the system. Hence, if all the systems (atoms, electrons in a magnetic field) in a superlight beam are in the lower state, the normal Doppler radiation emitted by one of the systems is absorbed by the beam while the anomalous radiation is amplified, i.e., in propagating it causes other systems to make upward transitions, accompanied by induced emission; the net result is the emission of an additional anomalous Doppler photon.

If both the lower and upper states, 0 and 1, are occupied the absorption coefficient of the beam for normal Doppler radiation is (cf. references 23 and 24):

$$\mu_n = \frac{dI_\omega}{I_\omega} = A_1^0 \frac{8\pi^3 c^2 N_1 \left(\frac{N_0}{N_1} - 1\right)}{\omega^2 n^2}, \quad I_\omega = I_\omega(0)\, e^{-\mu z}, \tag{10}$$

where $A_1^0(\theta)$ refers to the probability for a spontaneous transition $1 \leftrightarrow 0$ with the radiation of a photon at an angle θ with respect to the velocity (per unit solid angle). N_1 and N_0 are the concentrations of particles in the beam for states 1 and 0 correspondingly, n is the refractive index of the medium at the frequency ω for propagation at an angle θ (for simplicity we assume that for all particles the dipole moment for the $1 \leftrightarrow 0$ transition is parallel to the velocity). In order for the normal Doppler waves to be amplified the number of particles in the upper state 1 must exceed the number in the lower state 0 (in this case $N_0/N_1 < 1$ and $\mu < 0$). A state distribution of this kind is not characteristic of thermal equilibrium and in general, the production of such a distribution involves definite difficulties. However the situation is changed in anomalous Doppler radiation, in which case the emission of a photon is characterized by the transition $0 \to 1$ while absorption is associated with the transition $1 \to 0$. For the anomalous effect

$$\mu_{a_n} = A_1^0 \frac{8\pi^3 c^2 N_0 \left(\frac{N_1}{N_0} - 1\right)}{\omega^2 n^2} \tag{11}$$

and $\mu < 0$ when $N_1/N_0 < 1$. It also follows that for the anomalous Doppler effect (i.e., $\beta n > 1$) a beam in which all particles are in the lower state 0 is characterized by negative absorption, and the waves radiated by the individual particles are amplified. This situation would appear to be very favorable from the point of view of using beams of particles moving in a dielectric slit or a slow-wave system for the generation and amplification of microwaves.[25]

As has been indicated, the anomalous Doppler radiation system can be electrons which oscillate under the influence of an applied field or which move along helical paths in a magnetic field parallel to the axis of the beam. At small amplitudes, if we neglect Cerenkov radiation, these electrons radiate in the same way as oscillators moving with a velocity v, given by the projection of the electron velocity $v_{||}$ in the direction of the beam axis.

In an electron beam the distribution of transverse velocities $v_\perp$ is usually such that the distribution function $f(v_\perp)$ is a decreasing function of $v_\perp$ (for example $f(v_\perp) = \text{const} \cdot \exp\{-mv_\perp^2/(2\kappa T)\}$. In this case the normal Doppler waves are attenuated because of reabsorption in the beam; the anomalous Doppler waves, on the other hand, are amplified. Amplification of the waves in an electron beam implies that the amplitude of the oscillations increases and the beam loses stability. Under these conditions the electrons generally become bunched and coherent radiation

*Since absorption is the inverse process to emission, this statement follows immediately from the calculations carried out for emission. The terms "higher" and "lower" are used here in terms of energy.

is produced. The quantum-mechanical criterion for beam instability ($\beta_{||} n(\omega) > 1$; $\beta_{||} = v_{||}/c$) coincides with the criterion obtained by solving the classical problem of stability of a beam of electrons in a magnetic field.[24] This instability of electron streams, which is characteristic of a magnetoactive plasma, is of interest as a possible explanation of sporadic radio radiation from the sun.[26]

As we have indicated, the radiation condition in (1) is an interference expression and is universal for waves of all kinds (obviously the phase velocity of light $c/n(\omega)$ must be replaced by v_p, the phase velocity of the particular wave being considered, for example, acoustic waves, capillary waves, etc.). The same remark applies to the results which have been obtained on the basis of the conservation of energy and momentum, both classically and quantum mechanically. The second approach (the introduction of photons) is much simpler not only for optical waves, but also for plasma waves (cf. reference 23 and § 4) and sound waves. In the latter case the energy of the acoustic photon (phonon) is $\epsilon = \hbar\omega$ and its momentum is $\mathbf{p} = \hbar\mathbf{k}\ (\epsilon/u)\mathbf{s}$, where u is the acoustic velocity (acoustic dispersion is usually negligible and there may be no difference between the phase velocity and the group velocity). By analogy with the electrodynamic case, we may assume that in "supersound" motion an acoustic system which emits anomalous Doppler radiation will make transitions in the upward direction (i.e., will be excited) and will thus become "unstable" to some degree.[26a]

In concluding this section we indicate another interesting point, which stems from the fact that the directions of the phase velocity and group velocity of a wave do not coincide, for instance, in an anisotropic medium, or when spatial dispersion is taken into account.*

If the projection of the group velocity $d\omega/d\mathbf{k}$ in the direction perpendicular to the velocity of the particle (i.e., the quantity $d\omega/dk_r$ where k_r is the projection of k perpendicular to v) is negative, it would appear that energy is not emitted by the radiator, but is absorbed. In this case, however, we must use the advanced potentials rather than the delayed potentials.[27,29] If k is always taken parallel to the phase velocity, when $d\omega/dk_r < 0$ for Cerenkov and Doppler radiation[14,29] points toward the particle trajectory, while the energy necessarily is propagated away from the trajectory. The difference between the cases $d\omega/dk_r > 0$ and $d\omega/dk_r < 0$, for Cerenkov radiation is shown in Fig. 2. When $d\omega/dk_r < 0$, as before the angle θ_0 is determined by (1) as is clear for the direction of k chosen from interference considerations and from the conservation laws (5) and (6). The latter remark follows from the fact that we use plane waves $e^{i(\omega t - \mathbf{k}\cdot\mathbf{r})}$ and the momentum of the corresponding photons in the medium is $(\hbar\omega n/c)(\mathbf{k}/k)$; when plane waves are used the direction of k is the same in Figs. 2a and 2b because (in terms of plane waves) the positions of the wave fronts are the same in both cases (all are wave fronts with k parallel to the Cerenkov cone). Equation (3) remains valid when $d\omega/dk_r < 0$. The physical difference between these two cases is quite significant, however, and stems from the difference in the direction of the group velocity. In an isotropic medium (Fig. 2a) the group velocity is usually parallel to k. In the case shown in Fig. 2b, however, the group velocity vector $d\omega/d\mathbf{k}$ is antiparallel to k and forms an obtuse angle $(\theta_1 = \pi - \theta_0)$ with the particle velocity vector v. In this case, if a k particle passes through a slab of finite thickness, the Cerenkov radiation is emitted from the rear surface of the slab and is also refracted in an unusual way at this surface (the latter point follows from the results of reference 27).

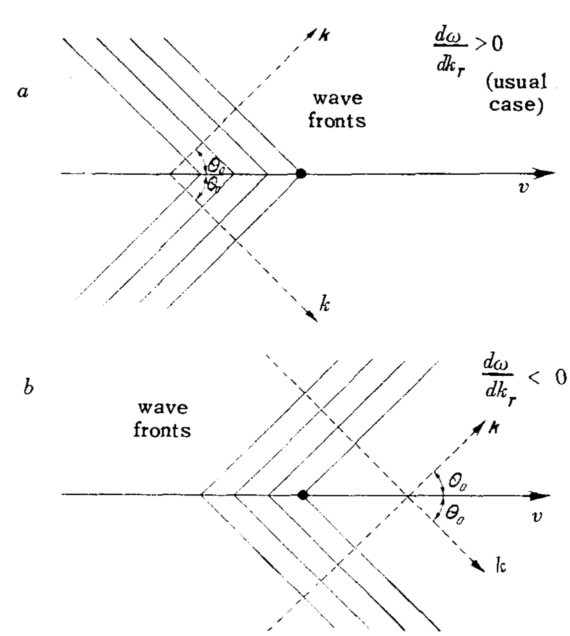

FIG. 2

*Cf. reference 27 and reference 28 (in Figs. 2 and 3 of reference 28 the functions $\tilde{n}^2(\omega)$ are introduced; in certain regions these functions are characterized by $\frac{d}{d\omega}(\tilde{n}\omega) < 0$ so that the phase velocity and group velocity are different in sign.

3. RADIATION REACTION FORCE FOR MOTION OF A CHARGE IN A MEDIUM

In considering electrons that move with superlight velocities in a plasma or slow-wave system in the presence of a magnetic field, or in considering related cases of oscillatory electron motion, one is usually interested only in the classical region, that is to say, the case in which the quantum numbers corresponding to the transverse motion are large. Under these conditions problems involving radiation of waves and attenuation or amplification of transverse oscillations of electrons can (and in practice, must) be solved by classical calculations. In essence, these calculations lead to the computation of the radiation reaction force due to the motion of the charge in a medium.

We shall consider this problem in somewhat broader terms.

Inasmuch as the presence of a medium can cause a radical modification of the electromagnetic radiation produced by a moving particle, it is clear that the radiation reaction force in a medium also undergoes important changes. As an example, an oscillator characterized by a frequency ω in an isotropic plasma with a refractive index $n = \sqrt{1 - \frac{4\pi e^2 N}{m\omega^2}}$ will not radiate, in general, when $\omega_0^2 = 4\pi e^2 N/m > \omega^2$, in which case $n^2 < 0$; in a magnetoactive plasma, in the nonrelativistic approximation no radiation is produced by an electron which rotates in a magnetic field H_0 at a frequency $\omega_H = eH_0/mc$ (here we are considering radiation propagated at an angle to the field, cf. reference 30). In both of these cases there is obviously no radiation reaction; on the other hand, in vacuum the radiation reaction is

$$\mathbf{f}_0 = \frac{2e^2}{3c^3}\frac{d^2\mathbf{v}}{dt^2}. \tag{12}$$

In the case of uniform motion in a medium, if the condition $v > c/n(\omega)$ is satisfied at some frequency, there is a Cerenkov radiation force $\mathbf{f}_C$, which performs work given by $\mathbf{f}_C \cdot v = -dW/dt$. Hence, from Eq. (2) it is clear that

$$\mathbf{f}_C = -\frac{e^2\mathbf{v}}{c^2 v}\int_{\frac{c}{n}\leqslant v}\left[1-\frac{c^2}{n^2(\omega)v^2}\right]\omega\,d\omega. \tag{13}$$

In light of those remarks it would appear that the problem of calculating the radiation reaction force for arbitrary motion of a charge in an arbitrary medium is of great interest. Nevertheless, to the best of our knowledge, this problem has not been treated at the present time.

This lack of attention would seem to be explained by the fact that for motion in a medium the radiation force is usually appreciably smaller than the braking force due to ionization losses. Thus, the loss due to Cerenkov radiation, which may be regarded as a radiation force, is only a small fraction of the total loss even in a transparent (dense) medium. In general this is also the case for nonuniform motion of a charge.

As has been noted, however, there are interesting cases of practical interest in which the radiation forces for motion in a medium become quite important (motion in channels and slits, motion close to a medium and in a magnetoactive plasma).

An expression for the radiation reaction force in a medium has been obtained in reference 22. Here we shall give only a brief sketch of the calculation.

For a point charge, with a density given by $\rho = e\,\delta(\mathbf{r}-\mathbf{R})$, $\int\delta\,d\mathbf{r} = 1$, the field equations and the equations of motion are:

$$\left.\begin{aligned}\mathrm{curl}\,\mathbf{H} &= \frac{4\pi}{c}e\mathbf{v}\delta(\mathbf{r}-\mathbf{R}) + \frac{1}{c}\frac{\partial\mathbf{D}}{\partial t},\\ \mathrm{div}\,\mathbf{D} &= 4\pi e\delta(\mathbf{r}-\mathbf{R}),\\ \mathrm{curl}\,\mathbf{E} &= -\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t},\quad \mathrm{div}\,\mathbf{H} = 0,\end{aligned}\right\} \tag{14}$$

$$\frac{d}{dt}\left(\frac{m\mathbf{v}}{\sqrt{1-\frac{v^2}{c^2}}}\right) = e\left\{\mathbf{E}^{(0)} + \frac{1}{c}\mathbf{v}\times\mathbf{H}^{(0)}\right\} + e\int\left\{\mathbf{E}(\mathbf{r}) + \frac{1}{c}\mathbf{v}\times\mathbf{H}(\mathbf{r})\right\}\delta(\mathbf{r}-\mathbf{R})\,d\mathbf{r}. \tag{15}$$

Here $\mathbf{R}(t)$ is the radius vector for the position of the charge ($\mathbf{v} = d\mathbf{R}/dt$); $\mathbf{E}^{(0)}$ and $\mathbf{H}^{(0)}$ are the external fields and $\mathbf{E}$ and $\mathbf{H}$ are the fields produced by the charge itself (for simplicity the medium is assumed nonmagnetic).

For the case of an arbitrary medium the only effective method for solving the problem is expansion of the fields into plane waves.

This procedure yields

$$\left.\begin{aligned}\tilde{D}_\alpha^{(\omega)} &= \varepsilon_{\alpha\beta}(\omega)\,\tilde{E}_\beta^{(\omega)} \qquad (\alpha,\beta = 1, 2, 3),\\ \mathbf{E} &= -\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t} - \nabla\varphi,\quad \mathbf{H} = \mathrm{curl}\,\mathbf{A},\\ \mathbf{A} &= \sqrt{4\pi}\,c\sum_{\lambda,\,j=1,\,2}\frac{q_{\lambda j}(t)\,\mathbf{a}_{\lambda j}}{n_{\lambda j}}e^{i\mathbf{k}_\lambda\mathbf{r}},\end{aligned}\right\} \tag{16}$$

$$\varepsilon_{\alpha\beta}\frac{\partial\tilde{A}_\alpha}{\partial x_\beta} + \mathrm{c.\,c.} = 0, \tag{17}$$

where the condition in (17) is chosen for convenience, repeated indices indicate summation, and the index ω indicates that we take the Fourier components while the real fields are given by $\mathbf{D} = \tilde{\mathbf{D}} + \tilde{\mathbf{D}}^* \equiv \tilde{\mathbf{D}} + \mathrm{c.\ c.}$, $\mathbf{E} = \tilde{\mathbf{E}} + \mathrm{c.\ c.}$ and so on.

In Eqs. (16) and (17) $n_{\lambda j}$ is the refractive index and $a_{\lambda j}$ is the complex polarization vector for the j-th plane wave. The equations for the potentials which are obtained from Eqs. (14), (16), and (17) are

$$\left.\begin{aligned} &\Delta \mathbf{A} - \operatorname{grad} \operatorname{div} \tilde{\mathbf{A}} - \frac{1}{c^2} \varepsilon_{\alpha\beta} \frac{\partial^2 \tilde{A}_\beta}{\partial t^2} \mathbf{e}_\alpha - \frac{1}{c} \varepsilon_{\alpha\beta} \frac{\partial^2 \tilde{\varphi}}{\partial t\, \partial x_\beta} \mathbf{e}_\alpha \\ &\qquad + \mathbf{c.c.} = -\frac{4\pi}{c} \mathbf{j}_e = -\frac{4\pi}{c} e \mathbf{v} \delta (\mathbf{r} - \mathbf{R}). \\ &\varepsilon_{\alpha\beta} \frac{\partial^2 \tilde{\varphi}}{\partial x_\alpha \partial x_\beta} + \mathbf{c.c.} = -4\pi e \delta (\mathbf{r} - \mathbf{R}). \end{aligned}\right\} \quad (18)$$

where $\mathbf{e}_\alpha$ is the unit vector associated with the α axis and $\mathbf{j}_e = e\mathbf{v}\delta(\mathbf{r} - \mathbf{R})$ is the current density corresponding to the particle being considered.

Substitution of the expansion in (16) in Eq. (18) yields a system of oscillator equations for the field amplitudes $q_{\lambda j}$ (cf. references 13, 16, 17 and Sec. 4). The solution of this system is elementary. The fields determined in this way are then substituted in the equation of motion (15). As a result we obtain (cf. reference 22):*

$$\begin{aligned} \frac{d}{dt} \left(\frac{m\mathbf{v}}{\sqrt{1 - \frac{v^2}{c^2}}} \right) = \mathbf{F}^{(0)} - \frac{e^2}{2\pi^2} \sum_{j=1,2} \int_0^t \int_0^{\mathbf{k}_{max}} \Bigl\{ \frac{\mathbf{a}_j (\mathbf{v}'\mathbf{a}_j^*)}{n_j^3} \cos \omega_j (t - t') - \\ [i\mathbf{v} \times \mathbf{k} \times \mathbf{a}_j] \frac{(\mathbf{v}'\mathbf{a}_j^*)}{n_j^2 \omega_j} \sin \omega_j (t - t') \Bigr\} e^{i\mathbf{k}(\mathbf{R} - \mathbf{R}')} dt\, d\mathbf{k} + \mathbf{c.c.} = \mathbf{F}^{(0)} + \mathbf{f}_{rad}, \end{aligned} \quad (19)$$

where

$$\mathbf{R}' = \mathbf{R}(t'), \quad \mathbf{v}' = \mathbf{v}(t') \quad \text{и} \quad \mathbf{F}^{(0)} = e \left\{ \mathbf{E}^{(0)} + \frac{1}{c} \mathbf{v} \times \mathbf{H}^{(0)} \right\}.$$

The method of computing the radiation reaction force used here is convenient for a number of cases including isotropic media or vacuum). For instance, the radiation damping force which acts on a particle moving with nonrelativistic velocity in vacuum can be obtained by using Eq. (19) to obtain Eq. (12) (cf. references 31 and 32). On the other hand, in the case of a particle which moves uniformly in an isotropic medium characterized by a refractive index $n > c/v$, using Eq. (19), we can obtain Eq. (13) for the braking forces due to Cerenkov radiation.

An analysis of superlight motion of an oscillator has been given in reference 22. For an oscillator which oscillates parallel to the forward velocity $\mathbf{v}_0$ in an isotropic medium, we have

$$\left.\begin{aligned} &\mathbf{R} = \{0,\ 0,\ v_0 t + R_0 \sin \Omega t\}, \quad \mathbf{v} = \{0,\ 0,\ v_0 + v_\infty \cos \Omega t\}, \\ &v_\infty = R_0 \Omega, \quad \mathbf{a}_1 = \{1,\ 0,\ 0\}, \quad \mathbf{a}_2 = \{0,\ \cos\theta,\ -\sin\theta\}, \\ &\mathbf{k} = \{0,\ k\sin\theta,\ k,\ \cos\theta\}. \end{aligned}\right\} \quad (20)$$

For simplicity we consider only the case in which

$$kR_0 = \frac{\omega}{c} n(\omega) R_0 \ll 1. \quad (21)$$

Under these conditions, we obtain from Eq. (19) the following expression for the work performed by the radiation field on the particle in a time T:

$$A = \int_0^T \mathbf{v}\mathbf{f}_{rad} dt = v_0 \int_0^T f_{rad,\,z}\, dt + v_\infty \int_0^T \cos \Omega t f_{rad,\,z}\, dt = A_0 + A_\infty. \quad (22)$$

$$\begin{aligned} A = -\frac{e^2 R_0^2 T}{4c^3 \beta_0} \Bigl\{ \int_{\beta_0 n(\omega) \cos\theta < 1} \omega^3 \left[1 - \frac{1}{\beta_0^2 n^2(\omega)} \left(1 - \frac{\Omega}{\omega} \right)^2 \right] d\omega + \\ + \int_{\beta_0 n(\omega) \cos\theta > 1} \omega^3 \left[1 - \frac{1}{\beta_0^2 n^2(\omega)} \left(1 + \frac{\Omega}{\omega} \right)^2 \right] d\omega \Bigr\} \end{aligned} \quad (23)$$

$$\omega = \frac{\Omega}{|1 - \beta_0 n(\omega) \cos\theta|}, \qquad \beta_0 = \frac{v}{c}. \quad (24)$$

If the dispersion function is a step function, i.e., if

$$\left.\begin{aligned} &n(\omega) = n = \text{const} \quad \textbf{for} \quad \omega < \omega_c, \\ &n(\omega) = 1 \quad \textbf{for} \quad \omega > \omega_c, \end{aligned}\right\} \quad (25)$$

Eq. (23) can be written in the form:

$$A = -\frac{e^2 \Omega^4 R_0^2 n T}{4c^3} \int \frac{\sin^3\theta\, d\theta}{|1 - \beta_0 n \cos\theta|^5}, \quad (26)$$

where, for the anomalous Doppler effect we have $0 \ll \theta \ll \arccos \frac{1}{\beta_0 n}(1 + \frac{\Omega}{\omega_c})$, while for the normal Doppler effect

$$\arccos \frac{1}{\beta_0 n} \left(1 - \frac{\Omega}{\omega_c} \right) \ll \theta \ll \pi.$$

*It should be noted that in the calculations account must be taken of the fact that the charge is not a point charge, but has a definite radius r_0. However, instead of introducing a form explicitly in the integration over $\mathbf{k}$, it is sufficient to introduce an upper limit $k_{max} \sim 2\pi/r_0$. Since the radiation force which is of interest to us is independent of r_0 (in contrast to the electromagnetic mass), no complications arise and the results of the calculation are unique.

The quantity $W = -A (> 0)$ is the energy radiated by the particle (cf. reference 6). The work done by the radiation field in amplifying or attenuating the particle oscillations is obtained from Eq. (22):

$$A_\infty = A - A_0 = \frac{e^2\Omega R_0^2 T}{4\beta_0 c^3}\left\{\int\limits_{\beta_0 n(\omega)\cos\theta > 1}\omega^2\left[1 - \frac{1}{\beta_0^2 n^2(\omega)}\left(1+\frac{\Omega}{\omega}\right)^2\right]d\omega - \right.$$
$$\left. - \int\limits_{\beta_0 n\cos\theta < 1}\omega^2\left[1 - \frac{1}{\beta_0^2 n^2(\omega)}\left(1-\frac{\Omega}{\omega}\right)^2\right]d\omega\right\}. \quad (27)$$

For the case given by (25),

$$A_\infty = \frac{e^2\Omega^4 R_0^2 n T}{4c^3}\left\{\int\limits_0^{\arccos\frac{1}{\beta_0 n}\left(1+\frac{\Omega}{\omega_c}\right)}\frac{\sin^3\theta\, d\theta}{(1-\beta_0 n\cos\theta)^4} - \right.$$
$$\left. - \int\limits_{\arccos\frac{1}{\beta_0 n}\left(1-\frac{\Omega}{\omega_c}\right)}^{\pi}\frac{\sin^3\theta\, d\theta}{(1-\beta_0 n\cos\theta)^4}\right\}. \quad (28)$$

Thus, the radiation outside the Cerenkov cone, corresponding to the second integral in Eqs. (27) and (28), causes damping of the oscillations; the radiation inside this cone (anomalous Doppler effect), corresponding to the first integral in Eqs. (27) and (28), causes amplification of the oscillations.* This result is in complete agreement with the quantum mechanical results (cf. Sec. 2). It will be apparent that the second integral in Eqs. (27) and (28) is larger than the first; $A_\infty < 0$, as is the total work A [cf. Eqs. (23) and (26)]. Thus, in an isotropic medium the oscillations are always damped and $A_\infty \to 0$ only if $\beta_0 n(\omega) \to \infty$ in the actual range of integration.

In reference 22, an analysis is also made of the case in which the oscillator oscillates perpendicularly to its forward velocity V_0. As in the preceding case, in an isotropic medium the oscillations are always damped.

To understand certain features of superlight motion of charges in anisotropic media it is convenient to consider the motion of an oscillator along the optical axis of a uniaxial nongyrotropic crystal; the electron is assumed to be oscillating in the same direction.

In this case

$$\mathbf{R} = \{0,\ 0,\ v_0 t + R_0\sin\Omega t\},\quad \mathbf{k} = \{0,\ k\sin\theta,\ k\cos\theta\},$$
$$\mathbf{a}_2 = \{1,\ 0,\ 0\},\ \mathbf{a}_1 = \{0,\ \cos\theta + K_1\sin\theta,\ -\sin\theta + K_1\cos\theta\},$$
$$K_1 = \frac{(n_1^2 - \varepsilon_\perp)\cos\theta}{\varepsilon_\perp\sin\theta},\quad \frac{1}{n_1^2} = \frac{\sin^2\theta}{\varepsilon_\parallel} + \frac{\cos^2\theta}{\varepsilon_\perp},\quad kR_0 \ll 1,$$

where n' is the refractive index for the extraordinary wave, which is the only one in the present case. The quantity K' is the ratio of the components of the electric field in the extraordinary wave which are parallel and perpendicular to the vector **k**; the electric vector is parallel to the polarization vector $\mathbf{a}_1$, whose length satisfies the condition

$$(\varepsilon_\parallel a_{z1}^2 + \varepsilon_\perp a_{y1}^2)\ \frac{1}{n_1^2} = 1 \text{ (cf. ref. 5).}$$

We can now obtain expressions that correspond to Eqs. (23) and (27):

$$A = -\frac{e^2 R_0^2 T}{4\beta_0 c^3}\int\limits_{L_1+L_2}\omega^3\frac{\varepsilon_\perp^2(\omega)\sin^2\theta\, d\omega}{[\varepsilon_\perp(\omega)\sin^2\theta + \varepsilon_\parallel(\omega)\cos^2\theta]^2\left|1 - \frac{\cot\theta}{n_1}\frac{\partial n_1}{\partial\theta}\right|}, \quad (29)$$

$$A_\infty = \frac{e^2 R_0^2\Omega T}{4\beta_0 c^3}\left\{-\int\limits_{L_1}\omega^2\frac{\varepsilon_\perp^2(\omega)\sin^2\theta\, d\omega}{[\varepsilon_\perp(\omega)\sin^2\theta + \varepsilon_\parallel(\omega)\cos^2\theta]^2\left|1 - \frac{\cot\theta}{n_1}\frac{\partial n_1}{\partial\theta}\right|} + \right.$$
$$\left. + \int\limits_{L_2}\omega^2\frac{\varepsilon_\perp^2(\omega)\sin^2\theta\, d\omega}{[\varepsilon_\perp(\omega)\sin^2\theta + \varepsilon_\parallel(\omega)\cos^2\theta]^2\left|1 - \frac{\cot\theta}{n_1}\frac{\partial n_1}{\partial\theta}\right|}\right\} \quad (30)$$

The regions of integration, L_1 and L_2, are determined by the Doppler relations:

$$1 - \beta_0 n(\omega,\ \theta)\cos\theta = \frac{\Omega}{\omega} \quad (31)$$

for the normal Doppler radiation (L_1), and

$$\beta_0 n(\omega,\ \theta)\cos\theta - 1 = \frac{\Omega}{\omega} \quad (32)$$

for the anomalous Doppler radiation (L_2). It is apparent that both integrals in Eq. (30) are always positive. Thus the normal Doppler radiation

*The positive work in A_∞, or part of it, corresponds to excitation of oscillations since A_∞ is the work done on the particle by the radiation force.

[first integral in Eq. (30)] corresponds to damping of the oscillations while the anomalous Doppler radiation corresponds to amplification of the oscillations.

This division, however, is only provisional, since the physically meaningful quantity is the work given by the difference of the two integrals.

In contrast with the case of an isotropic medium, in an anisotropic medium there may not only be a reduction in the attenuation, but even amplification (if one considers the sign of the total work A_∞ rather than its components). Suppose, for example, that $\epsilon_{||}$ and $\epsilon_\perp$ are independent of frequency with $\epsilon_{||} < 0$ and $\epsilon_\perp > 0$. In this case $n_1^2(\theta_\infty) \to \infty$ at an angle θ_∞ which is determined by the following condition (cf. the expression for n_1^2 given above):

$$\varepsilon_\perp \sin^2\theta_\infty + \varepsilon_\perp \cos^2\theta_\infty = 0. \tag{33}$$

In a medium of this kind the extraordinary waves can propagate at an angle $|\theta| < |\theta_\infty|$; at angles $\pi/2 > \theta > \theta_\infty$, however, $n_1^2 < 0$ and wave propagation is impossible. Furthermore, n_1^2 is a minimum and equal to $\epsilon_\perp$ when $\theta = 0$. Now, if $\beta_0 \epsilon_\perp > 1$, it is always possible to choose $\epsilon_{||}$ in such a way that the Cerenkov angle θ_0 is larger than θ_∞ (here $\beta_0 n \cos\theta_0 = 1$).). Under these conditions, in general there is no Cerenkov radiation (the angle θ_0 corresponds to values $n_1^2 < 0$)); in the forward direction (for $\theta < \pi/2$, actually $\theta < \theta_\infty$) only the anomalous Doppler radiation is produced. In the backward direction ($\pi - \theta < \theta_\infty$) there is normal Doppler radiation but here $(1 - \beta_0 n_1 \cos\theta) = (1 + \beta_0 n_1 |\cos\theta|)$ and the total work A_∞ is positive. One is easily convinced of this by using Eqs. (31) and (32) in Eq. (30) to transform to integration over θ; this procedure yields

$$A_\infty = \frac{e^2\Omega^4 R_0^2 T}{4c^3}\left\{ \int_0^{\tan^{-1}\sqrt{\frac{|\varepsilon_{||}|}{\varepsilon_\perp}}} \frac{n_1^5(\theta)\sin^3\theta\, d\theta}{\varepsilon_{||}^2[1-\beta_0 n_1(\theta)\cos\theta]^4} - \int_0^{\tan^{-1}\sqrt{\frac{|\varepsilon_{||}|}{\varepsilon_\perp}}} \frac{n_1^5(\theta')\sin^3\theta'\, d\theta'}{\varepsilon_{||}^2[1+\beta_0 n_1(\theta')\cos\theta']^4} \right\}, \tag{34}$$

where $\theta' = \pi - \theta$. Here, $A_\infty > 0$, since the first integral in Eq. (34) is larger than the second and the oscillations are amplified in this case.

The motion of charges in a magnetoactive plasma has been considered in reference 33. In this work it has been shown that under certain conditions the oscillations grow; more precisely, there is an "uncurling" of the helical line along which the particle moves in the magnetic field. Thus, amplification can take place if

$$\omega_0^2/\omega_H^2 = \beta_0 \ll 1 \quad (\omega_H = eH_0/mc, \quad \omega_0^2 = 4\pi e^2 N/m),$$

where H_0 is the uniform magnetic field applied to the plasma and N is the electron concentration of the plasma. Amplification obtains for the following values of the parameters (results obtained by numerical integration):

$$\beta_0 = 0.01, \quad \omega_0^2/\omega_H^2 = 10 \text{ and } \beta_0 = 0.99, \quad \omega_0^2/\omega_H^2 = 10.$$

However, if $\beta_0 = 0.99$ and $\omega_0^2/\omega_H^2 = 0.01$, for example, the transverse motion of the particle is damped. If the oscillations build up, there is a transfer of energy from the forward motion (in the present case, motion along the field) into the energy of the transverse motion. As a result the forward velocity v_0 is reduced and the amplification ceases when the velocity v_0 falls to the critical velocity c/n_{max} (the smallest value of the propagation velocity in the medium).

The difference in the sign of the force which act on the oscillatory motion of a particle in normal and anomalous Doppler radiation is in complete agreement with the results obtained in Sec. 2. In the isotropic case there is a weakening of the "friction" (in some cases it can disappear completely), but the oscillations can never grow (the quantum "build-up" of oscillations due to the spreading of the packet in "energy space" for superlight radiation obviously does not occur in an isotropic medium; cf. Sec. 2). In the anisotropic case, however, particularly in the case of a magnetoactive plasma, growing oscillations are possible.

It will be apparent that the instability of superlight particle beams which is found in the classical approximation even in an isotropic medium, is closely related to the radiation reaction on a single particle.

4. CERENKOV RADIATION AND ABSORPTION OF WAVES IN AN ISOTROPIC MAGNETOACTIVE PLASMA

Because of the great interest being shown in plasma physics at the present time, we shall consider briefly certain aspects of this field which are related to the theory of radiation at superlight velocities.

In an isotropic plasma, i.e., in the absence of an external magnetic field H_0, the index of refraction for the transverse waves is

$n_{1,2}^2 = 1 - \frac{4\pi e N}{m(\omega^2 + \nu_{eff}^2)} < 1$ (the phase velocity of the wave is $v_p = (c/n) > c$ and Cerenkov radiation can not be excited. However, because of the thermal motion in an isotropic plasma, longitudinal plasma waves* can be propagated; the refractive index for these waves is:[34–38]

$$n_3^2 = \frac{c^2k^2}{\omega^2} = \frac{1-\frac{\omega_0^2}{\omega^2}}{3\beta_T^2}, \quad \beta_T^2 = \frac{\varkappa T}{mc^2}, \quad \omega_0^2 = \frac{4\pi e^2 N}{m} \tag{35}$$

Here e and m are the charge and mass of the electron, N is the electron concentration, κ is the Boltzmann constant, and T is the absolute temperature. Eq. (35) is equivalent to the dispersion equation $\omega^2 = \omega_0^2 + \frac{3\kappa T}{m} k^2$, which yields the following expressions for the phase and group velocities:

$$v_{\text{ph}} = \frac{\omega}{k} = \frac{c}{n_3} = \frac{\sqrt{3\frac{\varkappa T}{m}}}{\sqrt{1-\frac{\omega_0^2}{\omega^2}}}$$

$$v_{\text{gr}} = \frac{d\omega}{dk} = \frac{3\varkappa T}{m\omega} k = \sqrt{\frac{3\varkappa T}{m}} \sqrt{1-\frac{\omega_0^2}{\omega^2}}. \tag{36}$$

The plasma waves are one of three equally important groups of characteristic waves in a plasma. The phase velocity of the plasma waves can be smaller than c, the velocity of light in vacuum, so that a Cerenkov effect is possible for these waves. This form of the Cerenkov effect arises when the motion of charged particles in the plasma is such that the energy lost by the particles as a result of "remote" collisions goes into Cerenkov radiation of plasma waves. By virtue of this radiation, a particle with charge e_1 and velocity v (appreciably greater than the thermal velocity $v_t = \sqrt{\kappa T/m}$) loses the following energy per unit time[39]

$$\frac{dE}{dt} = -\frac{e_1^2\omega_0^2}{2v} \ln\left(1 + \frac{2v^2}{v_T^2}\right)$$

The excitation of plasma waves by a moving particle is not usually called the Cerenkov effect. Obviously the question of terminology is not very important, being more or less a matter of taste. Nevertheless, it would appear that in the case of plasma waves (in contrast, say, to acoustic waves) it is more meaningful to call the phenomenon in question the Cerenkov effect. In the first place, as has been indicated, in a plasma the high-frequency longitudinal (plasma) waves are of equal importance with the electromagnetic (transverse) waves. In the second place, and this is perhaps more important, in a magnetoactive plasma (i.e., in the presence of an external magnetic field H_0) in the general case there are three characteristic waves and these are neither longitudinal nor transverse. The classification of plasma waves under these conditions is necessarily rather arbitrary.[37, 38] According to the conventional procedure one considers the waves produced by motion of a charge in a magnetoactive plasma as Cerenkov waves, electromagnetic waves, and plasma waves. However, as the external field H_0 becomes vanishingly small (transition to isotropy) so-called Cerenkov waves [12, 26] do not vanish but are converted, in continuous fashion, into the plasma waves indicated above.[26]

*The ion motion is neglected so that we can neglect the quasi-acoustic (low frequency) longitudinal waves (cf. for example, references 37 and 38). Absorption due to collisions is also neglected so we may set the collision frequency $\nu_{eff} = 0$.

These remarks apply not only to a gas plasma, but also to other media in which plasma waves can be propagated. For instance, a medium which is analagous to a magnetoactive plasma is an optically anisotropic medium (crystal).[28, 38]

In solids and liquids the plasma frequency $\omega_0 = \sqrt{4\pi e^2 N/m}$ is always high (ultraviolet region of the spectrum). Quantization is important and one introduces the notion of photons associated with the plasma waves -- plasmons,[40] with energy $\hbar\omega \cong \hbar\omega_0$ (the medium is assumed to be isotropic). The difference between plasmons and photons associated with the electromagnetic field in the medium used in Sec. 2 corresponds to the difference between transverse and longitudinal waves (cf. above). In an anisotropic medium, in general there is no such difference and the so-called discrete energy losses which arise in the passage of electrons through thin layers [40] can, with complete justification, be regarded as the result of the Cerenkov effect.[28] In investigations of these discrete losses it is important to take account of the momentum of the photons or plasmons.[40]

In a gas (rarified) plasma the frequency ω_0 is relatively low ($h\omega \ll \frac{Mv^2}{2}$ and $h\omega_0 \ll \kappa T$, where M is the mass and v the velocity of the radiating particle, and T is the plasma temperature) and it is not necessary to use a quantum-mechanical approach. However, in these cases, just as in the case of electromagnetic waves (cf. § 2), the application of the quantum theory of radiation and absorption to plasma waves (plasmons) can be both convenient and effective. As an example we can point to the calculation of reabsorption of plasma waves and the determination of the stability criteria for a beam of particles moving in a plasma given in reference 23.

A beam instability arises if there is a growth of the perturbations which arise in the beam (waves). From the quantum-mechanical point of view, the absorption coefficient for waves in the beam must be negative ($\mu < 0$;cf. Sec. 2). This situation obtains if the particles in the beam can radiate or if the velocity distribution of the particles in the beam is such that the induced emission exceeds the absorption. A particle of sufficiently high velocity ($v \gg \sqrt{\kappa T/m}$) moving in an isotropic plasma can, as we have indicated, radiate Cerenkov plasma waves. The absorption coefficient μ is negative (there is more emission than absorption) if there are more particles in the beam in the upper states than in the lower states [cf. Eqs. (10)--(11)]. If the particles do not possess internal degrees of freedom (i.e., free electrons, protons, etc.) or if the change in the internal state is neglected, the upper state corresponds to a higher velocity. Whence it follows immediately that a stream will be unstable if there are more fast particles than slow particles for a given velocity range, i.e., if the velocity distribution $f_s(v)$ has a positive derivative.* The same instability criterion, $df_s/dv > 0$, can be obtained classically,[36, 36b] but a special analysis is needed. In many ways (depending on the complexity of the problem) the quantum method is more effective for obtaining the stability criterion in the case already mentioned in Sec. 2, i.e. motion of a beam of charged particles in a magnetoactive plasma (here we must take account of the change in the component of the particle velocity perpendicular to the magnetic field or, in quantum-mechanical language, the transitions between the energy levels associated with motion perpendicular to the field, which is quantized[24]).

The possibility of emission of Cerenkov radiation implies the possibility of absorption of this radiation by particles, regardless of its origin. It follows that in a plasma, in addition to absorption due to collisions** there must be absorption due to a Cerenkov mechanism. In an isotropic plasma there is no such absorption mechanism for transverse waves because there is no Cerenkov radiation.***

But plasma waves are absorbed, even in the absence of collisions. The mechanism responsible for absorption was analyzed a long time ago by a completely different approach.[35] In this approach we consider the linearized kinetic equation for the plasma electrons (cf. for example, references 34, 35 and 38):

$$\frac{\partial f_1}{\partial t} + \mathbf{v}\nabla_{\mathbf{r}} f_1 + \frac{e}{m}\mathbf{E}\nabla_{\mathbf{v}} f_0 = 0, \quad f = f_0 + f_1, \quad |f_0| \gg |f_1|. \quad (37)$$

(Here we have neglected collisions and $f_0(v)$ is the zeroth approximation to the distribution function, i.e., at equilibrium, the Maxwellian distribution). Then, Fourier analysis i.e. substitution of $f_1(\mathbf{v}, \mathbf{r}, t) = g(\mathbf{v})\, e^{i(\omega t - \mathbf{k}\cdot\mathbf{r})}$ yields the expression

$$i(\omega - \mathbf{kv}) f_1 = \frac{e}{m}\mathbf{E}\nabla_{\mathbf{v}} f_0. \quad (38)$$

If $\omega \neq \mathbf{k} \cdot \mathbf{v}$ division by $\omega - \mathbf{k} \cdot \mathbf{v}$ yields an expression for f_1 ; substituting f in the field equation,

$$\text{curl curl } \mathbf{E} + \frac{\partial^2 \mathbf{E}}{c^2\, \partial t^2} = -\frac{4\pi}{c^2}\frac{\partial \mathbf{j}_t}{\partial t}, \quad \mathbf{j}_t = e\int \mathbf{v} f_1\, d\mathbf{v},$$

we then obtain a dispersion equation that relates ω and $\mathbf{k}$. This equation can be written in the

*For simplicity we consider radiation in the direction of motion of a one-dimensional stream, $\mathbf{v}$ (in the general case $f_s(v)$ is the function $f_s(v_k)$, where $v_k = v\cos\theta$ is the projection of the flow velocity in the direction of the radiated waves $\mathbf{k}$; cf. reference 23). When $f_s(v_k)$ is given by $f_s(v_k) = \text{const} \cdot \exp\{-\frac{M}{2\kappa T_s}(v_k - v_0\cos\theta)^2\}$, (Fig. 3), $\mu > 0$ in region II, where $\frac{df_s(v_k)^s}{dv_k} > 0$. Because of the Cerenkov condition (1) $v\cos\theta = v_k = c/n_3(\omega)$ the radiation emitted by particles with different v_k (in particular, radiated by particles from regions I and II in Fig. 3), is at different frequencies; thus there is no interference between these waves and stability obtains even if $\mu < 0$ for only a small range of values of v_k. We may note that for any isotropic three-dimensional electron velocity distribution function $f = f(v^2)$, $f(v_k) = \int f(v^2)\, d\mathbf{v}_\perp$ ($\mathbf{v}_\perp$ is the projection of the velocity perpendicular to $\mathbf{k}$) cannot have a positive derivative and, in accordance with reference 36a, the distribution is stable.

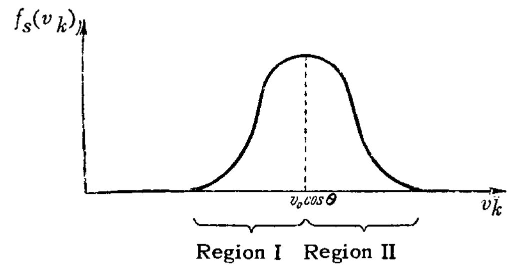

FIG. 3.

**Bremsstrahlung is produced in collision between particles; hence the inverse process is the absorption which arises as a result of collisions.

***If the usual Maxwelliam velocity distribution is used, formally it is found that there is (although extremely small) a non-vanishing absorption of transverse waves even in the absence of collisions (cf. for example reference 37). This result is erroneous however and is due to the fact that a nonrelativistic Maxwelliam distribution does not necessarily preclude the presence of particles with velocities $v > c$. If the problem is treated relativistically, or if particles with velocities $v > c$ are disregarded, the non-collisional absorption of transverse waves in an isotropic plasma is found to be identically zero (this result also follows from the absence of Cerenkov radiation in this case). R. Z. Sagdeev has called the attention of the author to the error concerning this point in reference 37.

form $\frac{c^2k^2}{\omega^2} = n^2_{1,2,3}$ where $n_{1,2,3}$ is the refractive index for a given wave: transverse ($(n_{1,2})$) or longitudinal (n_3). However, if $\omega = \mathbf{k}\cdot\mathbf{v}$ Eq. (38) cannot be divided by $\omega - \mathbf{k}\cdot\mathbf{v}$ and it can be shown [35] that the longitudinal wave which propagates in the plasma is attenuated. But the condition

$$\omega = \mathbf{k}\cdot\mathbf{v} = \frac{\omega n v}{c}\cos\theta \qquad \left(k^2 = \frac{\omega^2 n^2}{c^2}\right) \tag{1a}$$

is precisely the Cerenkov condition (1). In an isotropic plasma this condition can be satisfied only for plasma waves whose absorption actually represents the inverse Cerenkov effect (the wave is attenuated and the plasma electrons which satisfy the Cerenkov condition gain energy).*

If there is an external magnetic field, radiation is produced in the magnetoactive plasma as a result of collisions (bremsstrahlung), by virtue of the Cerenkov effect, and as a consequence of acceleration of particles in the magnetic field ("magnetic bremsstrahlung" or synchrotron radiation). Correspondingly, there are three absorption mechanisms. It should be noted, however, that the classification of radiation and absorption in terms of Cerenkov and magnetic bremsstrahlung effects is somewhat arbitrary for the following reason. A particle (electron) in a magnetic field moves along a helical line, rotating with a frequency $\omega_H^* = \omega_H \frac{mc^2}{E} = \frac{eH_0}{mc}\frac{mc^2}{E}$ (E is the total energy). In vacuum, motion of this kind leads to radiation at frequencies $s\omega_H^*$ ($s = 1, 2, \ldots\ldots$; the Doppler of the frequency shift is neglected).** In a plasma, however, the nature of the radiation (intensity, direction, and polarization) is changed and, in addition to the frequencies $s\omega_H^*$, there is a continuous radiation spectrum; this radiation is the Cerenkov radiation (if the particles move strictly along the field lines, in general there is no magnetic bremsstrahlung radiation). On the other hand, if the particles move in a circle in the plane perpendicular to the field H_0, only discrete frequencies are radiated $s\omega_H^*$, i.e., in the terminology used here, only magnetic bremsstrahlung is radiated. Physically, however, it is apparent that if the radius of curvature is sufficiently large and if $E/mc^2 \gg 1$ the radiation spectrum becomes essentially continuous; thus in the corresponding frequency region it is similar to Cerenkov radiation.[13] It follows from these considerations that in the general case it is only logical to use a unified analysis of the magnetic bremsstrahlung and Cerenkov radiation[13] and absorption.

We now consider in somewhat greater detail the frequency of the radiation which is emitted (and absorbed) in a magnetoactive plasma. For this purpose we write the equation for the field amplitudes used in Sec. 3 [cf. Eq. (16)]:

$$\frac{e}{n_{\lambda_j}}(\mathbf{v}\;\mathbf{a}^*_{\lambda_j})\,e^{-i\mathbf{k}_\lambda\mathbf{R}} \equiv f(t), \tag{39}$$

where $\omega^2_{\lambda j} = \frac{c^2k_\lambda^2}{n^2_{\lambda j}}$, $\mathbf{R}(t)$ and $\mathbf{v} = d\mathbf{R}/dt$ are respectively the radius vector and the velocity of the radiating particle.

Equation (39) is obtained by a substitution of the expansion in (16) into the equation for the vector potential (18), multiplication by $\mathbf{a}^*_{\lambda j}e^{-i\mathbf{k}_\lambda\mathbf{r}}$ and integration over space. If we neglect constant factors, the form of the "force" term f(t) in Eq. (39) is clear immediately because when $\mathbf{j}_e = e\mathbf{v}\,\delta(\mathbf{r}-\mathbf{R})$

$$\int (\mathbf{j}_e\mathbf{a}^*_{\lambda j})\,e^{-i\mathbf{k}_\lambda\mathbf{r}}\,d\mathbf{r} = e\,(\mathbf{v}\mathbf{a}^*_{\lambda j})\,e^{-i\mathbf{k}_\lambda\mathbf{R}}$$

[cf. Eq. (18)].

Equation (39) has solutions for $q_{\lambda j}$ which increase in time, corresponding to radiation, only at frequencies $\omega_{\lambda j}$ contained in the spectrum of the force term f(t). For example, if the electron moves uniformly $\mathbf{R} = \mathbf{v}t$ and the force spectrum contains the single frequency $\omega = \mathbf{k}\cdot\mathbf{v}$. Hence the radiation condition assumes the form $\omega_{\lambda j} = \omega = \mathbf{k}\cdot\mathbf{v}$ i.e. the Cerenkov condition (1a). (This method of deriving the radiation condition[43] is no less graphic than the use of the interference analysis or the conservation laws.)[15]

For the case of an electron in a magnetic field H_0 which is parallel to the z axis:

$$\mathbf{R} = \{R_0\cos\omega_H^*t,\ R_0\sin\omega_H^*t,\ v_zt\},\quad \mathbf{v} = \{-v_\perp\sin\omega_H^*t,\ v_\perp\cos\omega_H^*t,\ v_z\},$$
$$v_\perp = R_0\omega_H^*,\ f(t) = \mathrm{const}\,(-a_x^*v_\perp\sin\omega_H^*t + a_y^*v_\perp\cos\omega_H^*t + a_z^*v_z)$$
$$\times\exp\{-i\,[kR_0\sin\theta\sin\omega_H^*t + kv_z\cos\theta t]\}, \tag{40}$$

where, for simplicity we have taken $k_x = 0$ and θ is the angle between k and H_0 (z axis). Expanding the plane wave in terms of Bessel function

$$\exp\{-ik_\lambda R_0\sin\theta\sin\omega_H^*t\} = \sum_{s=-\infty}^{\infty} J_s(k_\lambda R_0\sin\theta)\,e^{-is\omega_H^*t},$$

*The physical interpretation of the non-collisional attenuation of plasma waves was essentially given in reference 36 but no explicit references were made to Cerenkov radiation. We would not care to say when such references first appeared but, as is well known, this matter has been discussed, for example, in references 41 and 42.

**This effect is considered below.

we obtain the resonance condition [cf. Eqs. (39) and (40)] :

$$\omega = s\omega_H^* + kv_z \cos\theta; \quad s = 0, \pm 1, \pm 2, \pm 3, \ldots \quad (41)$$

For s = 0 this condition is identical with the Cerenkov condition (1) -- (1a) with $v = v_z$; at the same time, all the $s \neq 0$ terms in Eq. (39) vanish only when the motion is strictly along the field, in which case $R_0 = 0$. When $s \neq 0$, in place of Eq. (41) we can write

$$s > 0: \ \omega = \frac{s\omega_H^*}{1 - \frac{v_z}{c} n \cos\theta}; \quad s < 0: \ \omega = \frac{s\omega_H^*}{\frac{v_z}{c} n \cos\theta - 1}. \quad (42)$$

where the frequency, as before, is positive.

If $v_\perp \ll v_z$, the electron in the magnetic field radiates in the same way as two appropriately chosen dipoles moving along the field with velocity $v_z \cong v$; this case corresponds to values $s = \pm 1$ (more precisely, the intensity of the higher harmonics is small if k $R_0 \sin\theta = (\omega/c) n v_\perp \omega_H \sin\theta \ll 1$. Equation (42) with $s = \pm 1$ is in complete agreement with Eq. (3) for the Doppler effect in a medium (obviously for motion in a magnetic field $\omega_i \sqrt{1-\beta^2} = \omega_i \frac{mc^2}{E} = \omega_H \frac{mc^2}{E}$, since ω_i is the frequency in the system in which the center of gravity of the radiator is at rest).

Turning from radiation to absorption, we see that in a magnetoactive plasma there must be absorption of waves with frequencies given by Eq. (41), corresponding to magnetic bremsstrahlung and Cerenkov radiation (taking account of the Doppler effect). We may note that this same result can be obtained[44] by analyzing (for motion of an electron in a magnetic field) the frequency spectrum of the force which acts on this electron in the field associated with the wave (the frequency of the force term is not equal to the frequency of the field E since the electron is displaced and is in a field of different intensity at different moments of time).

Above we have discussed only the radiation and absorption conditions. The actual calculation of the radiation intensity and absorption coefficient is a separate problem, which can be extremely complicated. This problem has been solved by the kinetic-equation method[45, 46] and by other methods[41, 47] and a summary of the pertinent results for a nonrelativistic plasma is given in reference 38. It may not be out of place here to note that the non-collisional absorption of waves in a magnetoactive plasma is of great importance, not only at ultra-high temperatures (in thermonuclear systems), but also, for example, in the solar corona (temperature $T \sim 10^6$ degrees; cf. references 26, 38, 47, and 48).

5. CERENKOV RADIATION OF DIPOLE MOMENTS IN A CONTINUOUS MEDIUM AND IN CHANNELS AND SLITS

Usually, only the Cerenkov radiation due to point charges or charge bunches is considered. At the same time it is clear that Cerenkov radiation must be emitted by any radiator which moves with a velocity v which is greater than the phase velocity of light in a medium c/n. In other words, the radiation condition (1) also applies for any multipole; in particular, for electric and magnetic dipoles,[6, 10, 15, 49--57] The dipole radiation is modified fundamentally (we neglect the higher multipoles) and is usually appreciably smaller than the radiation of a single charge. Thus, in order-of-magnitude terms, when $v \sim c$ and $n \sim 1$ the radiation intensity of an electric dipole p = ed is smaller than that of a charge e by a factor $p^2\omega^2/e^2c^2 \sim (d/\lambda)^2$; in the case of a magnetic dipole m this ratio is of order m^2w^2/e^2c^2 (the appearance of the factor $(d/\lambda)^2$ is easily understood if we consider a dipole as two charges -e and +e separated by a distance d; cf. reference 49).

The magnetic dipole Cerenkov radiation of elementary particles (electron, neutron, etc.) or nuclei is extremely weak and is not of interest. The situation is changed when we consider particle bunches which, under certain conditions, can radiate as point charges, with charge and multipole moment corresponding to the total bunch. It is precisely this case which may arise when particle bunches or current loops move in a magnetoactive plasma or when such objects move along the axis of a channel or slit or close to a slow-wave system etc. The calculation of the Cerenkov radiation of dipole moments is also of interest for methodological reasons; in particular, using this technique it is possible to obtain information concerning the magnetic moment of particles with various spin values.[15, 51, 52, 57] Furthermore, for a long time there have been certain unresolved questions (cf. references 6, 10, 15, 49, 50, 53, 54) in the problem of the Cerenkov radiation of a magnetic moment. Finally, it was only recently that a definitive answer was given to the problem of how the Cerenkov radiation of a dipole moment is affected when the dipole moves in a channel or slit.[10, 11] For these reasons we shall consider the Cerenkov radiation of dipole moments in some detail.

Consider a particle with charge e, electric dipole moment $\mathbf{p}$, and magnetic moment $\mathbf{m}$, which moves with velocity $\mathbf{v}$. The current density associated with the particle is (ρ_e is the charge density, $\mathbf{M}$ is the magnetization, $\mathbf{p}$ is the polarization):

$$\mathbf{j} = \varrho_e \mathbf{v} + c\,\mathbf{curl}\,\mathbf{M} + \frac{\partial \mathbf{P}}{\partial t} = e\mathbf{v}\delta(\mathbf{r}-\mathbf{v}t) + c\,\mathbf{curl}\{\mathbf{m}\,\delta(\mathbf{r}-\mathbf{v}t)\} + \frac{\partial}{\partial t}\{\mathbf{p}\delta(\mathbf{r}-\mathbf{v}t)\}. \quad (43)$$

We assume for simplicity that the medium is isotropic and nonmagnetic (the magnetic permeability $\mu = 1$). Then, assuming that the vector potential $\mathbf{A}$ satisfies the condition div $\mathbf{A} = 0$, we obtain the following equation (cf. for example, Eq. (18) for $\epsilon_{\alpha\beta} = \epsilon\delta_{\alpha\beta}$ or reference 58):

$$\Delta\mathbf{A} - \frac{\varepsilon}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\frac{4\pi}{c}\mathbf{j} + \frac{\varepsilon}{c}\frac{\partial}{\partial t}\nabla\varphi, \quad \Delta\varphi = -\frac{4\pi\varrho}{\varepsilon}, \quad (44)$$

$$\left.\begin{array}{c} \mathbf{A} = \sum\limits_{\lambda,\,j}(q_{\lambda j}\mathbf{A}_{\lambda j} + q^*_{\lambda j}\mathbf{A}^*_{\lambda j}), \quad \mathbf{A}_{\lambda j} = c\sqrt{\frac{4\pi}{\varepsilon}}\,\mathbf{a}_{\lambda j}\exp\{i\mathbf{k}_\lambda\mathbf{r}\}' \\ \mathbf{a}_{\lambda j}\mathbf{a}_{\lambda i} = \delta_{ij}, \quad \mathbf{k}_\lambda\mathbf{a}_{\lambda j} = 0, \quad j = 1,\ 2, \\ \mathscr{H} = \int \frac{\varepsilon E^2_{tr} + H^2}{8\pi}\,d\mathbf{r} = \sum\limits_{\lambda,\,j}(p_{\lambda j}p^*_{\lambda j} + \omega^2_{\lambda j}q_{\lambda j}q^*_{\lambda j}), \\ \mathbf{E}_{tr} = -\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}, \ \mathbf{H} = \mathbf{curl}\ \mathbf{A}, \quad p_{\lambda j} = \frac{dq_{\lambda j}}{dt} \equiv \dot q_{\lambda j}, \quad \omega^2_{\lambda j} = \omega^2_\lambda = \frac{c^2k^2_\lambda}{\varepsilon}. \end{array}\right\} \quad (45)$$

Here, ϕ is the scalar potential, $\mathscr{H}$ is the energy of the transverse field and $\mathbf{a}_{\lambda j}$ is the polarization vector. Substituting the current (43) in Eq. (44) and integrating with respect to $d\mathbf{r}$, we have after multiplication by $A^*_{\lambda j}$:

$$\ddot q_{\lambda j} + \omega^2_\lambda q_{\lambda j} = \frac{1}{c}\int(\mathbf{j}\mathbf{A}^*_{\lambda j})\,d\mathbf{r} = \sqrt{\frac{4\pi}{\varepsilon}}\{e(\mathbf{a}_{\lambda j}\mathbf{v}) + ic\ \mathbf{m}\cdot[\mathbf{k}_e \times a_{\lambda j}] + i(\mathbf{a}^*_{\lambda j}\mathbf{p})(\mathbf{k}^*_\lambda\mathbf{v})\}\exp\{-i\mathbf{k}^*_\lambda\mathbf{v}t\} \quad (46)$$

Integrating Eq. (46), for example with the initial conditions $q_{\lambda j}(0) = p_{\lambda j}(0) = 0$, we can find the energy $\mathscr{H}$. This energy contains a part which increases with time and is due to a resonance when the Cerenkov condition $\omega_\lambda = \mathbf{k}\cdot\mathbf{v}$ (cf. Sec. 4) is satisfied. The part of $\mathscr{H}$ which increases in time, to be discussed below, does not depend on the initial conditions and can be easily computed by introducing the density of states $dZ_i(\omega) = \frac{\varepsilon^{\frac{3}{2}}\,\omega^2\,d\omega\,d\Omega}{(2\pi c)^3}$ and integrating over the angle θ (between $\mathbf{k}$ and $\mathbf{v}$) where $d\Omega = \sin\theta\,d\theta d\phi$. It is clear from Eq. (46) that the radiation of the charge e and the radiation of the moments $\mathbf{p}$ and $\mathbf{m}$ are shifted in phase by $\pi/2$ so that there is no interference between the radiation from the charge and the moments. In other words, the energy radiated per unit time is equal to the sum of the radiation due to the charge [Eq. (2)] and the energy due to the Cerenkov radiation of the moments

$$\mathscr{H} = \frac{dW}{dt} = \frac{1}{2\pi vc^2}\sum_{j=1,\,2}\int d\omega\int_0^{2\pi} n^2\omega^3\left\{\mathbf{m}[\mathbf{s}\times\mathbf{a}_j] + \frac{1}{n}(\mathbf{a}_j\mathbf{p})\right\}^2 d\varphi, \quad (47)$$

where $n^2(\omega) = \epsilon(\omega)$ is the dielectric permittivity of the medium, $\cos\theta = \cos\theta = \frac{c}{n(\omega)\,v}$, $\mathbf{s} = \frac{\mathbf{k}}{k}$ and θ and ϕ are the polar and azimuthal angles in the coordinate system in which the z axis is parallel to the velocity $\mathbf{v}$. The integration over frequency in Eq. (47) is carried out over the region for which $c/n(\omega)\,v \ll 1$. At first glance it might appear that dispersion, i.e., the frequency dependence of n, has not been considered in this calculation. It can easily be shown, however, that dispersion is taken into account in Eq. (47) (cf., for example, reference 16).

For the case in which the magnetic moment $\mathbf{m}$ is parallel to the velocity, we have from Eq. (47):[15]

$$\frac{dW}{dt} = \frac{m^2}{vc^2}\int n^2\left(1 - \frac{c^2}{v^2n^2}\right)\omega^3\,d\omega. \quad (48)$$

For an electric dipole, using Eq. (47), we can also immediately obtain the familiar expressions.[6, 49] The question indicated in the introduction of this section concerns the case in which the magnetic moment is perpendicular to the velocity. If this moment is $\mathbf{m}_0$ and $\mathbf{p}_0 = 0$ in the system in which the particle is at rest, then, as is well known, in the laboratory system $\mathbf{m} = \mathbf{m}_0$ and $\mathbf{p} = (1/c)\,\mathbf{v}\times\mathbf{m}$ In this case (for $\mathbf{v}\perp\mathbf{m}$)

$$\frac{dW}{dt} = \frac{m^2}{2vc^2}\int n^2\omega^3\left\{2\left(1 - \frac{1}{n^2}\right)^2 - \left(1 - \frac{v^2}{n^2c^2}\right)\left(1 - \frac{c^2}{n^2v^2}\right)\right\}d\omega. \quad (49)$$

This expression agrees with that obtained in reference 6 but differs from the results obtained by other authors.[49, 53, 54] For instance, in reference 49 the following expression was obtained instead of Eq. (49):

The origin of this discrepancy is the following: in references 49, 53, and 54 a "true" magnetic dipole is used (dipole formed from magnetic poles). However, a moving "true" magnetic dipole is equivalent to a current moment only in vacuum. If one introduces magnetic poles with a density given by $\rho_m(\mathbf{r})$, the field equations have the form (we set $\wp = 0$, $\mathbf{j} = 0$, $\mathbf{B} = \mu\mathbf{H}$; cf. for example reference 59):

$$\operatorname{curl}\mathbf{H} = \frac{1}{c}\frac{\partial\varepsilon\mathbf{E}}{\partial t}, \quad \operatorname{div}\varepsilon\mathbf{E} = 0,$$
$$\operatorname{curl}\mathbf{E} = -\frac{1}{c}\frac{\partial\mu\mathbf{H}}{\partial t} - \frac{4\pi}{c}\varrho_m\mathbf{v}, \quad \operatorname{div}\mu\mathbf{H} = 4\pi\rho_m,$$

whence

$$\operatorname{curl}\operatorname{curl}\mathbf{H} + \frac{\varepsilon\mu}{c^2}\frac{\partial^2\mathbf{H}}{\partial t^2} = -\frac{4\pi}{c}\varepsilon\frac{\partial(\varrho_m\mathbf{v})}{\partial t},$$
$$\operatorname{curl}\operatorname{curl}\mathbf{E} + \frac{\varepsilon\mu}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = -\frac{4\pi}{c}\operatorname{curl}(\varrho_m\mathbf{v}). \tag{51}$$

However, when there are electric charges and currents (for $\wp_m = 0$)

$$\left.\begin{aligned}\operatorname{curl}\operatorname{curl}\mathbf{H} + \frac{\varepsilon\mu}{c^2}\frac{\partial^2\mathbf{H}}{\partial t^2} &= \frac{4\pi}{c}\operatorname{curl}(\varrho\mathbf{v}),\\ \operatorname{curl}\operatorname{curl}\mathbf{E} + \frac{\varepsilon\mu}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} &= -\frac{4\pi}{c}\mu\frac{\partial(\varrho\mathbf{v})}{\partial t}.\end{aligned}\right\} \tag{52}$$

The magnetic-pole equations are obtained from the charge equations by making the substitutions

$$\mathbf{E}\to\mathbf{H}, \quad \mathbf{H}\to-\mathbf{E}, \quad \varrho\to\varrho_m, \quad \mu\to\varepsilon. \tag{53}$$

Thus, the actual current moment for $\mu = 1$ is equivalent to a "true" magnetic moment only in vacuum, in which case $\epsilon = 1$. In a medium, however, where $\epsilon \neq 1$, the moving "true" magnetic moment m has an electric moment given by $(\epsilon/c)(\mathbf{v}\times\mathbf{m})$ and not $(1/c)(\mathbf{v}\times\mathbf{m})$. This substitution is equivalent to taking account of the electrical polarization of the medium which is carried along by the dipole itself.[50] In other words, the "true" magnetic dipole is equivalent to a current moment "made" from a material with permittivity ϵ, which is therefore polarizable. It is interesting that this case can be realized for bunches; in this case (at the frequency considered) ϵ, the dielectric permittivity in the bunch itself, must be equal to the permittivity of the surrounding medium (for example, a plasma in a magnetic field).

Using a quantum-mechanical calculation,[15,51,53,55–57] which starts with the Pauli or Dirac equations (or equations for particles with spin 1, 3/2 and 2), we obtain an expression such as Eq. (48) or (49). If the spin is parallel or anti-parallel to the velocity $\mathbf{v}$ an expression such as (49) is obtained only for transitions in which there is a spin flip, since it is only in this case that there are components of the spin operator perpendicular to $\mathbf{v}$. In essence, however, the Cerenkov radiation of a magnetic moment has no quantum-mechanical specification.

In contrast with Eqs. (2) and (48), a characteristic feature of Eq. (49) is the fact that the integrand does not vanish at threshold ($\cos\theta = c/nv = 1$):

$$\frac{dW}{dt} = \frac{m^2}{vc^2}\int n^2\omega^3\left(1 - \frac{1}{n^2}\right)^2 d\omega$$

(we may note that in reference 57 precisely this expression was obtained by a quantum-mechanical calculation for a particle with magnetic moment. However, this result need not be considered paradoxical since the energy dW/dt does vanish at threshold and increase smoothly above the threshold. Actually, if dispersion is taken into account, as the velocity increases radiation appears only at frequencies corresponding to the maximum value of $n(\omega)$. Furthermore, if recoil is considered, automatically, in the quantum-mechanical calculation (cf. Sec. 2) Eq. (7) is obtained; in this equation the mass m is the mass of the entire bunch. By virtue of Eq. (7), even with n = const, as the velocity v increases radiation appears at only one frequency, in this case the frequency $\omega = 0$; thus, the region of integration and the quantity in Eq. (49) increase gradually with increasing v.

We may note that in the quantum-mechanical calculation it is also possible to obtain an expression such as Eq. (50); in this case, to the Dirac equation for the charged particle it is necessary to add an appropriate term, proportional to $\gamma_j\gamma_k G_{ik}$ (for a particle with non-kinematic magnetic moment we must replace $\gamma_i\gamma_k F_{ik}$ by $\gamma_i\gamma_k H_{ik}$); here $F_{ik} = \{\mathbf{H}, i\mathbf{E}\}$, $H_{ik} = \{\mathbf{H}, i\mathbf{D}\}$ and $G_{ik} = F_{ik} - H_{ik}$ and the γ_i are the Dirac matrices. However, there is no basis for making these changes for an individual particle and the use of such a quantum-mechanical calculation would be meaningless in the case of a bunch.

In conclusion, we consider the Cerenkov radiation of dipole moments which move in voids -- channels and slits (for simplicity we assume that in the void $\epsilon = 1$ and $\mu = 1$). In the case of a charge, it is well known[8–10] that as the radius of the channel or the width of the slit approaches zero the Cerenkov radiation becomes that which obtains for motion in a continuous medium (when $\cos\theta \sim 1$ this is the case when $a/\lambda \ll 1$, where a is the radius of the channel or the width of the slit and $\lambda = \lambda_0/n$ is the wavelength in the medium). At first glance it might appear that this result would apply for dipoles and other multipoles; in general, however, such is not the case.

In order to compute the effect of thin channels (slits) on Cerenkov radiation it is convenient to use the reciprocity theorem

$$\int_{(1)} \mathbf{j}_\omega^{(1)} \mathbf{E}_\omega^{(2)}\, d\mathbf{r} = \int_{(2)} \mathbf{j}_\omega^{(2)} \mathbf{E}_\omega^{(1)}\, d\mathbf{r}.$$

where $\mathbf{j}_\omega^{(1,2)}$ is the Fourier component of the density of the "transverse" current in regions 1 and 2; the field $\mathbf{E}_\omega^{(2)}$ is produced by the current 2 in region 1 and the field $\mathbf{E}_\omega^{(1)}$ by current 1 in region 2 (cf. for example references 38 and 59*). Writing the current in the form $\mathbf{j} = \varrho_e \mathbf{v} + \frac{\partial \mathbf{P}}{\partial t} + c \operatorname{curl} \mathbf{M}$, we have

$$\int_{(1)} [(\varrho_e \mathbf{v})_\omega^{(1)} \mathbf{E}_\omega^{(2)} + i\omega (\mathbf{P}_\omega^{(1)} \mathbf{E}_\omega^{(2)} - \mu \mathbf{M}_\omega^{(1)} \mathbf{H}_\omega^{(2)})]\, d\mathbf{r} = \int_{(2)} [(\varrho_e \mathbf{v})_\omega^{(2)} \mathbf{E}_\omega^{(1)} + i\omega (\mathbf{P}_\omega^{(2)} \mathbf{E}_\omega^{(1)} - \mu \mathbf{M}_\omega^{(2)} \mathbf{H}_\omega^{(1)})]\, d\mathbf{r}, \quad (54)$$

where μ is the magnetic permeability of the medium at points 1 and 2. In the case of Cerenkov radiation of a point charge which moves along the z axis

$$(\varrho_e \mathbf{v})_\omega^{(1)} = \frac{e}{2\pi} \mathbf{v}\, e^{-i\frac{\omega z}{v}} \delta(x)\, \delta(y)$$

and, placing an electric dipol $\mathbf{p}^{(2)} = \int \mathbf{P}_\omega^{(2)} d\mathbf{r}$ at a point removed from the trajectory, 2, we have

$$\frac{e}{2\pi} \int \mathbf{v}\mathbf{E}^{(2)}(0,\ 0,\ z)\, e^{-i\frac{\omega z}{v}}\, dz = i\omega\, \mathbf{p}^{(2)} \mathbf{E}(2), \quad (55)$$

where $\mathbf{E}(2) \equiv \mathbf{E}^{(1)}(2)$ is the radiation field at 2, which is of interest to us (the subscript ω is omitted). If the charge moves in a thin channel or narrow slit (i.e. if $a/\lambda \ll 1$) the quantity $\mathbf{v} \cdot \mathbf{E}^{(2)}(0, 0, z)$ remains the same as for a continuous medium since the tangential components of the field $\mathbf{E}^{(2)}$ are continuous. Hence, as is clear from Eq. (55), the radiation field $\mathbf{E}$ is the same as for a continuous medium.

For a radiating electric dipole, $\mathbf{P}^{(1)} = \mathbf{p}\, \delta(z - vt)\, \delta(x) \delta(y)$, , we have

$$\frac{1}{2\pi} \int \mathbf{p}\mathbf{E}^{(2)}(0,\ 0,\ z)\, e^{-i\frac{\omega z}{v}}\, dz = \mathbf{p}^{(2)} \mathbf{E}(2). \quad (56)$$

if the dipole $\mathbf{p} \equiv \mathbf{p}^{(1)}$ is parallel to the axis of the channel or if it lies in the plane of the slit, when $a/\lambda \ll 1$ the radiation field again is the same as for a continuous medium. For a dipole which is perpendicular to the plane of the slit, because of the fact that the component normal to the boundary is continuous $\mathbf{D} = \epsilon \mathbf{E}$:

$$\mathbf{p}\mathbf{E}^{(2)}(0,\ 0,\ z) = \varepsilon(\omega)\, \mathbf{p}\mathbf{E}_0^{(2)}(0,\ 0,\ z), \quad (57)$$

where $\mathbf{E}_0^{(2)}$ is the field produced by dipole 2 in the continuous medium (Fig. 4). If the Cerenkov radiation field of dipole 1 (with moment $\mathbf{p}^{(1)} \equiv \mathbf{p}$) in the continuous medium is denoted by $\mathbf{E}_0$, using the reciprocity theorem we have

$$\frac{1}{2\pi} \int \mathbf{p}\mathbf{E}_0^{(2)} e^{-i\frac{\omega z}{v}}\, dz = \mathbf{p}^{(2)} \mathbf{E}_0(2). \quad (58)$$

If there is a slit, using Eq. (57) and the reciprocity theorem, we have

$$\frac{1}{2\pi} \int \mathbf{p}\mathbf{E}^{(2)} e^{-i\frac{\omega z}{v}}\, dz = \frac{1}{2\pi} \varepsilon \int \mathbf{p}\mathbf{E}_0^{(2)} e^{-i\frac{\omega z}{v}}\, dz = \mathbf{p}^{(2)} \mathbf{E}(2). \quad (59)$$

It is apparent from Eqs. (58) and (59) that the Cerenkov radiation field is $\mathbf{E} = \epsilon\, \mathbf{E}_0$, that is to say ϵ times larger than for a dipole which moves in a continuous medium. For a dipole which is perpendicular to the axis of a narrow cylindrical channel (circular) we find $\mathbf{E} = \frac{2\varepsilon}{\varepsilon + 1} \mathbf{E}_0$. . Since the magnetic field in the wave zone is proportional to the electric field, the radiated energy in these cases (slit and channel) is larger by factors of ϵ^2 and $[2\epsilon/(\epsilon + 1)]^2$ respectively.** A dipole which

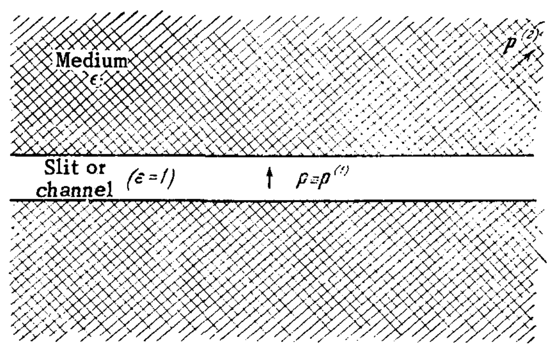

FIG. 4.

*In the form given here, the reciprocity theorem applies for any fixed linear medium, but only in the absence of an external magnetic field. In the presence of a magnetic field, in which case the tensors ϵ_{ik}' and μ_{ik}' are not symmetric, the generalized reciprocity theorem must be used (cf. reference 38, Sec. 29).

**To compute the radiated energy by means of Eq. (47), when there is a channel or slit, it is necessary to replace $\mathbf{p}$ by the appropriate expression obtained from Eq. (56); for example, for a dipole perpendicular to the axis of a circular channel we use $2\epsilon \mathbf{p} / (\epsilon + 1)\, \mathbf{p}$ in place of $\mathbf{p}$.

is oriented in an arbitrary direction may be regarded as made up of dipoles parallel and perpendicular to the axis of the channel (slit) and, using the superposition theorem, the problem can be reduced to a combination of the preceding problems. It is clear from Eq. (54) that when $\mu = 1$ the existence of a narrow channel has no effect on the radiation of a magnetic dipole **m**. If there are both electric and magnetic dipoles the radiated fields can be combined (obviously not the energy), i.e., the problem can again be solved easily.

A moving current moment and a "true" magnetic moment placed in a hollow void must obviously give the same radiation. This conclusion has been verified by direct calculation of the radiation of different kinds of dipoles moving in a circular channel;[11] in the particular case of a thin channel, it is found, as is to be expected, that the preceding result applies--the field of the electric dipole is increased by a factor $2\epsilon/(\epsilon + 1)$.

Because the Cerenkov radiation of a moving electric dipole (when $\mu \neq 1$ also a magnetic dipole) depends on the shape of a void which may be made as narrow as desired, one might question the validity of Eqs. (47) or (48--49) for motion of dipoles in a continuous medium. From the reciprocity theorem it is clear that here we are concerned with whether the actual field which acts on the dipole, E_{eff}, is the average macroscopic field E. For dipoles fixed in a medium this approximation does not hold in general (i.e. $E_{eff} = E$). However, when a charge or dipole moment moves along some trajectory, the average field acting on a "physically infinitesimal" portion of the path is precisely the macroscopic field. This conclusion concerning the validity of the original expressions (44) -- (46) for motion of a particle in a continuous medium can be verified by obtaining this expression (or the equivalent wave equation) by averaging the equations of microscopic electrodynamics. Thus, in our opinion there is no question as to the validity of Eqs. (47) -- (49) for Cerenkov radiation of point dipoles in a continuous medium.

We have indicated the effectiveness of a technique based on the reciprocity theorem for calculating Cerenkov radiation in narrow channels. This technique has also been used in analyzing transition radiation;[60] it should also be useful for analyzing a number of other problems in the theory of Cerenkov radiation and transition radiation in the presence of boundaries.[61]

In this review we have considered only certain aspects of the theory of superlight radiation --the radiation produced by superlight motion of particles in a medium. Nonetheless, the results indicate that there are a number of new and interesting points in this field which will be valuable in the development of concepts and methods for the investigation of plasmas and the motion of particle beams in plasmas and close to arbitrary media, etc.[2,3,61,62] Thus, the theory of radiation for superlight motion in a medium, which arises basically in connection with Cerenkov radiation in isotropic bodies, has rather wide application and is a subject of continuing development.

[1] I. E. Tamm and I. M. Frank, Dokl. Akad. Nauk SSSR **14**, 109 (1937).

[2] B. M. Bolotovskiĭ, Usp. Fiz. Nauk **62**, 201 (1957).

[3] J. V. Jelley, Cerenkov Radiation, Pergamon Press, New York, 1958.

[3a] I. M. Frank, Usp. Fiz. Nauk **68**, 397 (1959).

[4] Millimeter and Submillimeter Waves, Coll. of Papers, I.L. 1959. (Russ. Transl.).

[5] V. L. Ginzburg, JETP **10**, 608 (1940); J. Phys. (U.S.S.R.) **3**, 101 (1940).

[6] I. M. Frank, Izv. Akad. Nauk SSSR, Ser. Fiz. **6**, 3 (1942).

[7] V. L. Ginzburg, Usp. Fiz. Nauk **51**, 343 (1953); Fortschr. Phys. **1**, 659 (1954).

[8] V. L. Ginzburg and I. M. Frank, Dokl. Akad. Nauk SSSR **56**, 699 (1947).

[9] L. S. Bogdankevich and B. M. Bolotovskiĭ, JETP **32**, 1421 (1957), Soviet Phys. JETP **5**, 1157 (1957).

[10] V. L. Ginzburg and V. Ya. Éĭdman, JETP **35**, 1508 (1958), Soviet Phys. JETP **8**, 1055 (1959).

[11] L. S. Bogdankevich, J. Tech. Phys. (U.S.S.R.) **29**, 1086 (1959), Soviet Phys. -- Tech. Phys. **4**, in press.

[12] A. A. Kolomenskiĭ, Dokl. Akad. Nauk SSSR **106**, 982 (1956), Soviet Phys. --Doklady **1**, 133 (1956).

[13] V. Ya. Éĭdman, JETP **34**, 131 (1958), Soviet Phys. JETP **7**, 91 (1958); **36**, 1335 (1959), Soviet Phys. JETP **9**, 957 (1959).

[14] K. A. Barsukov, JETP **36**, 1485 (1959), Soviet Phys. JETP **9**, 1052 (1959); See also J. Tech. Phys. (U.S.S.R) **29**, 954 (1959), Soviet Phys. -- Tech. Phys. **4**, 868 (1960).

[14a] L. S. Bogdankevich, JETP **36**, 835 (1959), Soviet Phys. JETP **9**, 589 (1959).

[15] V. L. Ginzburg, JETP 10 589 (1940); J. Phys. (U.S.S.R.) 2, 441 (1940).
[16] V. L. Ginzburg, JETP 10, 601 (1940); J. Phys. (U.S.S.R.) 3, 95 (1940).
[17] A. A. Kolomenskiĭ, JETP 24, 167 (1953).
[17a] Yu. A. Ryzhov, Радиофизика (Известия вузов) (News of the Universities, Radiophysics) 2 (1959).
[18] G. Marx and G. Gyorgyi, Ann. Physik. 16, 241 (1955).
[19] G. Gyorgyi, Magyar Fizikai Folyoirat, 3, 369 (1955).
[20] V. L. Ginzburg and I. M. Frank, Dokl. Akad Nauk SSSR 56, 583 (1947).
[21] V. L. Ginzburg and V. M. Faĭn, JETP 35, 817 (1958), Soviet Phys. JETP 8, 567 (1959).
[22] V. L. Ginzburg and V. Ya. Éĭdman, JETP 36, 1823 (1959), Soviet Phys. JETP 9, 1300 (1959).
[23] V. L. Ginzburg and V. V. Zhelenznyakov, Астроном. журнал 35, 694 (1958), Soviet Astronomy 2, 653 (1959).
[24] V. V. Zheleznyakov, Радиофизика (Известия вузов) (News of the Universities, Radiophysics) 2, 14 (1959).
[25] V. L. Ginzburg, Dokl. Akad. Nauk SSSR 56, 145, 253 (1947); Izv. Akad. Nauk SSSR, Ser. Fiz. 11, 165 (1947).
[26] V. L. Ginzburg and V. V. Zheleznyakov, Радиофизика (Известия вузов) (News of the Universities, Radiophysics) 1, 9 (1958); Paris Symposium on Radio Astronomy, Stanford Univ. Press, 1959, p. 574; Астроном. журнал (Soviet Astronomy) in press.
[26a] I. E. Tamm, Usp. Fiz. Nauk 68, 387 (1959), Science 131, 206 (1960).
[27] L. I. Mandel'shtam, Собрание трудов (Coll. Works) Vol. II, p. 334, 1947; JETP 15, 475 (1945).
[28] V. L. Ginzburg, JETP 34, 1593 (1958), Soviet Phys. JETP 7, 1096 (1958).
[29] V. E. Pafomov, JETP 32, 366 (1957), Soviet Phys. JETP 5, 307 (1957); JETP 36, 1853 (1959), Soviet Phys. JETP 9, 1321 (1959).
[30] V. L. Ginzburg and V. V. Zheleznyakov, Радиофизика (Известия вузов) (News of the Universities, Radiophysics) 1, No. 2, 59 (1958).
[31] A. P. Belousov, JETP 9, 658 (1939).
[32] V. L. Ginzburg, Труды ФИАН. (Proc. Inst. Phys. Acad. Sci. USSR) 3, 195 (1946).
[33] V. Ya. Éĭdman, Радиофизика (Известия вузов) (News of the Universities, Radiophysics) 2, 1959.
[34] A. A. Vlasov, JETP 8, 291 (1938).
[35] L. D. Landau, JETP 16, 574 (1946).
[36] D. Bohm and E. P. Gross, Phys. Rev. 75, 1851 (1949).
[36a] L. M. Kavrizhnykh and A. A. Rukhadze, JETP (in press).
[36b] A. I. Akhiezer and Ya. B. Faĭnberg, Usp. Fiz. Nauk 44, 321 (1951).
[37] Gershman, Ginzburg, and Denisov, Usp. Fiz. Nauk 61, 561 (1957).
[38] V. L. Ginzburg, Распространение электромагнитных волн в плазме (Propagation of Electromagnetic Waves in Plasma) Fizmatgiz (in press).
[39] D. Pines and D. Bohm, Phys. Rev. 85, 338 (1952).
[40] D. Pines, Revs. Modern Phys. 28, 184 (1956); Usp. Fiz. Nauk 62, 399 (1957).
[41] V. D. Shafranov, JETP 34, 1475 (1958), Soviet Phys. JETP 7, 1019 (1958).
[42] Proceedings of the Second International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1959, Atomizdat, 1959.
[42a] Bernstein, Greene, and Kruskal, Phys. Rev. 108, 546 (1957).
[43] V. L. Ginzburg, Dokl. Akad. Nauk SSSR 24, 130 (1939).
[44] M. E. Gertsenshteĭn, JETP 27, 180 (1954).
[45] B. N. Gershman, JETP 24, 659 (1953); 37, 695 (1959), Soviet Phys. JETP 10, 497 (1960).
[46] A. G. Sitenko and K. N. Stepanov, JETP 31, 642 (1956), Soviet Phys. JETP 4, 512 (1957).
[47] V. V. Zheleznyakov, Радиофизика (Известия вузов) (News of the Universities, Radiophysics) 3 (1959).
[48] V. L. Ginzburg and V. V. Zheleznyakov, Астроном. журнал. 36, 233 (1959), Soviet Astronomy 3, 235 (1959).
[49] I. M. Frank, Сборник памяти С. И. Вавилова (Collection in Memory of S. I. Vavilov), U.S.S.R. Acad. Sci. 1952, p. 173.
[50] V. L. Ginzburg, ibid., p. 193.
[51] V. L. Ginzburg, JETP 11, 620 (1941); 12, 425 (1942); J. Phys. USSR 5, 47 (1941); 7, 115 (1943).
[52] M. Ya. Shirokobokov, JETP 19, 481 (1949).
[53] V. Ya. Éĭdman, Ученые записки Горьковского гос. университета Sci. Notes, Gorkii State University) 30, 229 (1956).
[54] N. L. Balazs, Phys. Rev. 104, 1220 (1956).
[55] A. A. Sokolov and Yu. M. Loskutov, JETP 32, 630 (1957), Soviet Phys. JETP 5, 523 (1957).
[56] M. I. Ryazanov, JETP 32, 1244 (1957), Soviet Phys. JETP 5, 1013 (1957).
[57] Yu. M. Loskutov and A. B. Kukanov, JETP 34, 477 (1958), Soviet Phys. JETP 7, 328 (1958).
[58] V. L. Ginzburg, JETP 9, 981 (1939).

[59] L. A. Vaĭnshteĭn, Электромагнитные волны (Electromagnetic Waves), Soviet Radio, 1957.
[60] V. L. Ginzburg and I. M. Frank, JETP **16**, 15 (1946); J. Phys. (U.S.S.R.) **9**, 353 (1945).
[61] B. M. Bolotovskiĭ and V. E. Pafomov, Usp. Fiz. Nauk (in press).
[62] G. M. Garibyan, JETP **37**, 527 (1959), Soviet Phys. JETP **10**, 372 (1960).

Translated by H. Lashinsky