Bibliography

A CLASSICAL BOOK ON THE THEORY OF NONLINEAR VIBRATIONS

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A. A. Andronov, A. A. Vitt and **S. É. Khaĭkin,** "Theory of Oscillations," second edition, revised and supplemented by N. A. Zheleztsov, Fizmatgiz, Moscow, 1959. 915 pages, 31.50 rub.

The history of this book is an unusual one. Immediately after the appearance of the first edition* in 1937, the entire issue of 3000 copies, which was not small for the time, was sold out, and by 1938 the book became a bibliographical rarity. In spite of this fact, interest in the book and knowledge of it developed rapidly, copies of the book passed from hand to hand, library copies were always in use — the book quickly became a fundamental requirement for everyone using the theory of oscillations in his own specialty. An entire generation of physicists and engineers, working in the general theory of oscillation and in its various applications (radiophysics, vibration, spring mounting, automatic control, etc.) was brought up on this book and grew up imbued with its ideas.

The most insistent demand for prompt publication of a second edition of the book, which was already developing in 1938-1939, became especially urgent in the years immediately after the war, when, owing to progress in radar and automation, the sphere of application of the theory of vibrations expanded with astonishing swiftness. In spite of the fact that only a small number of copies of this rare book penetrated beyond the border, references to it as the source of new directions in the theory of oscillation began to appear well before the war; after the war it gained world-wide fame. In 1947 a short exposition of the book, prepared by N. Minorsky, appeared in the U.S.A.,† and in 1949 a complete translation of the book‡ appeared under the editorship of S. Lepshetz. This translation encountered the fate of the Russian publication. The book was quickly sold out. Nevertheless, from that time on a Russian reader could obtain the English translation more readily than the Russian original.

In spite of all this, the second edition did not appear. Opposition to this came not from the publisher — proposals to reissue the book were made repeatedly — but from the authors. One of them — A. A. Vitt — was no longer alive at the time of appearance of the first edition. The second of the authors — S. É. Khaĭkin — no longer worked in the field of the general theory of nonlinear vibrations. And only one of the three authors — A. A. Andronov — headed the large group of Gorkiĭ physicists and mathematicians who continued to develop the theory of nonlinear vibrations in the directions pointed out in the book. It is natural that the reissue of the book was possible only with agreement of A. A. Andronov, and he persisted in declining to give it. He had a number of important reasons for this. The theory of nonlinear vibrations developed rapidly in those years, each year brought new methods, new facts, new interesting, previously unknown regions of application — it suffices to recall that the theory of automatic control, as an immense sphere of application of the theory of nonlinear vibrations, was "opened up" by physicists only after the war. A. A. Andronov, more than anyone else, saw that much in the book was obsolete or was becoming obsolete before one's eyes, and that in years of rapid progress of the theory of nonlinear vibrations republication of the book would mean essentially rewriting of it from the beginning. In his attitude toward the printed work, which was characteristic of A. A. Andronov, and which he cultivated among his students and co-workers, such a task would require several years. The last eight years of Andronov's life were clouded by heavy illness and he well knew that his days were numbered. Being at the center of scientific investigation of Gorkiĭ physicists, and giving all of himself to it, was it right for him to give up a significant part of his remaining years to the writing of a monograph? Could he, in this method of collective scientific collaboration, which the circle of Gorkiĭ physicists had cultivated, divert some of his coworkers in his remaining years to this work on the development of the book?

A. A. Andronov answered these questions with an unconditional negative.

In light of this unusual history of the book, it is understood how much of a departure the students and coworkers of A. A. Andronov* took on themselves in deciding to prepare a second edition of "The Theory of

*A. A. Andronov and S. É. Khaĭkin, Theory of Oscillations, first edition, ONTI NKTP SSSR, 1937.

†N. Minorsky, Introduction to Nonlinear Mechanics, J. W. Edwards, Ann Arbor, Michigan, 1947.

‡A. Andronov and S. Khaikin, The Theory of Oscillations, Princeton Univ. Press, 1949.

*Although the general decision to publish the new edition, its plan, and its basic ideas were discussed collectively in Gor'kiĭ, the principal part of the preparation of the second edition was carried out by N. A. Zheleztsov and especially by E. A. Leontovich-Andronova.

Oscillations" after his death. Here they adopted the only correct solution: not to omit any of the fundamental text of the first edition and, by only slightly correcting and changing it, supplement the book with a large number of new sections, written in the style of the book, but expressing new ideas, results, and examples. This plan of the revision of the book inevitably led to an increase in its size (the first edition had 32.5 and the second 57.25 printing signatures). On the other hand, they did not destroy the author's design of the book and retained "the concentrated" manner of presentation of the material, familiar to all of us who grew up on this book, by which the same ideas are gradually developed by considering ever more complicated cases, and examples play no less a role than the general theory.

The second edition of the book contains ten chapters, three appendices, and the preface to the first edition written by Academician L. Mandel'shtam.

Chapter 1 is devoted to linear systems with one degree of freedom. An analysis of the simplest systems of this type is used to acquaint the reader with the basic ideas by a qualitative consideration of all sets of possible motions in phase space.

Chapter 2 is devoted to conservative nonlinear systems with one degree of freedom. In it, an extraordinarily clear relationship is established between the curve characterizing the change in the potential energy and the course of the phase trajectory. In the last section of this chapter, the elementary foundations of the theory of integral invariants and other general properties of conservative systems are given, and also, as an interesting and unusual example of a conservative system, the example of Volterra is considered of the existence of two types of fish in a single reservoir.

In Chapter 3 the reader makes his first acquaintance with the basic properties and phase portraits* of nonconservative systems.

After a brief consideration of the properties of dissipative systems, the reader is acquainted by means of two classical examples (vacuum tube with Z characteristic and clock) with the possibility of the excitation in nonlinear systems of undamped oscillations which are not maintained by an external periodic action (i.e., autonomous systems). The term self-oscillation is introduced for the first time at this point.

Chapter 4 is devoted to degenerate dynamic systems of first order (or, as is sometimes said, systems with 1/2 degree of freedom). The concept is introduced of the "phase line" and it is shown that periodic motions are possible even in the simplest nonlinear systems of this type. In this chapter, the reader first meets with the problems of the determination of the dependence of the character of possible motions (i.e., the phase portrait) on parameters. As a fundamental example, a multivibrator with a single RC loop is considered.

All these basic concepts are developed and extended in Chapter 5 to a system with a single degree of freedom. In this chapter the concept of point transformations is first introduced and some general methods are put forth for the determination of the topological structure of the phase portrait (Poincaré indices, indications of the absence of closed trajectories, investigation of the behavior of phase trajectories at infinity, etc.) and for estimating the location of the limiting cycle.

Chapter 6 has basically a mathematical character. Its purpose is to introduce the reader to the foundations of Poincaré's qualitative theory of differential equations and to the development of this theory, principally by the Gorkiĭ mathematicians. Of greatest importance to the vibration physicist is the concept of rough systems, which is introduced in this chapter, i.e., systems for which the topological structure of the phase portrait is not changed by small changes in the parameters.

Chapter 7 is devoted to a special class of dynamical systems corresponding to phase trajectories not on a plane but on cylindrical surfaces.

In Chapter 8, piecewise linear dynamical systems are considered in detail. The method of point transformations serves as the principal apparatus for their investigation, while principal examples are taken from problems of the theory of automatic control. Chapter 9 is devoted to quantitative methods of study of nonlinear systems with a single degree of freedom. The well-known methods of Van der Pol and Poincaré are developed in it for the analysis of systems of the Thomson type.

The final chapter 10 is given over to a general theory of intermittent oscillations. Brief data on intermittent oscillations, given already in Chapter 4, are considerably generalized and developed. A large number of mechanical and electrical examples of intermittent systems are considered.

The appendices contain the principal theorems of the theory of differential equations, elementary information on the use of the cathode-ray oscillograph for the observation and recording of phase trajectories, and trigonometric formulas.

If we collect together all the differences between the first and second editions, then the following characteristics are clearly marked out: the theory of intermittent oscillations is deepened analytically, the physical facts connected with the discontinuity hypothesis and with vibrations of degenerate dynamic systems are made more precise; the vibrations of piecewise linear systems are now considered from the viewpoint and in terms of the theory of point transformations, developed in successive papers of the Gorkiĭ school; the exposition of the qualitative theory of differential equations has been broadened and improved, as has

*The term "phase portrait," which at first sounds almost like jargon, came about simply as a colorful description for the entire set of phase trajectories.

been the study of the qualitative picture of possible mutual locations of singular points, limit cycles, and the separatrices; analysis of phase portraits on a cylinder has been expanded and consideration of phase portraits on many-sheeted surfaces has been introduced; the number of examples has been increased appreciably and examples in the field of automatic regulation and control have been introduced in addition to electronic and ordinary mechanical examples; the exposition of quantitative methods of investigation of nonlinear systems of the Thomson type has been greatly developed (systems close to linear harmonic) as have also the methods of Van der Pol and Poincaré.

We shall now consider in detail each of these sections.

The problem of discontinuities arises every time the differential equations of a problem are simplified by neglecting terms containing higher derivatives. In the first edition of the book this circle of problems was considered only in passing. Later researches, especially those of the Gorkiĭ school, have advanced our understanding of both the physics of the phenomena and of the mathematical problems associated with it. These results are included in the second edition. They have been cast in the mold of a rigorous theory, which is set forth in the newly-written final chapter of the book. In connection with this chapter it is appropriate to make the following methodological observation. The entire book is devoted to systems with one degree of freedom, i.e., to differential equations of not higher than second order. In the last chapter, a number of facts are put forth which are applicable to an equation of arbitrary order, i.e., they are treated not in terms of two-dimensional, but of n-dimensional phase space.

The most outstanding results of the Gorkiĭ school in the field of the theory of oscillations have been associated in recent years with the application of the apparatus of the theory of point transformation to the solution of oscillatory problems and with an understanding of the fitting process as a point transformation of a surface into itself. Up to the present time no paper has been published specially devoted to the exposition of this apparatus as applied to the needs of oscillation theory. In the second edition of the book, a first attempt of this sort is made. Systems of second order do not give sufficient material for a demonstration of the power of the new ideas, but everything that can be done to explain the ideas within the scope of second-order systems has been done. The idea of point transformations is worked out not only in the chapter specially devoted to it, but in all the remaining chapters.

Even in the first edition of the book there is a section devoted to the general study of possible topological structures of the phase portrait, separate from the other material; reading of it demanded of the reader deep knowledge of the qualitative theory of differential equations and a thorough general preparation. In the second edition this section has been appreciably supplemented by E. A. Leontovich-Andronova. It now contains a rather complete picture of the possible decompositions of the phase plane and the possible coexistence of cycles, singular points and the separatrices. The concept of "the rough system" which is fundamental in applications, is now laid out with proper fullness. But a large price is paid for this fullness: reading of these sections requires great preparation. Perhaps a great part of this section should have been set in small type.

The first edition of this book, in addition to a consideration of the phase plane, contained only a very short consideration of trajectories on a cylindrical surface. In the second edition, the consideration of the cylindrical case has been expanded, and, what is most important, consideration of a trajectory on a multi-sheeted phase space is introduced. The idea that has arisen in recent years, in connection with the theory of clocks and relay systems, of considering multi-sheeted phase portraits, has proved very useful for many technical problems.

The problems of automatic control are the most prominent problems of this type. It is precisely in automation that the theory of oscillations has in recent years found a wide field for application of these methods. Perhaps one can say without exaggeration that automation has become for the theory of oscillations today what radio engineering was for it in earlier times. In this connection, the inclusion in the second edition of "Theory of Oscillations" of detailed examples from the field of automatic control is one of its most interesting characteristics.

Finally, a word on the quantitative methods, that is, the small-parameter methods of the type of Van der Pol, Poincaré, etc. Although A. A. Andronov himself and his colleagues made considerable contributions to the development of these methods, in general these methods were clearly not in the spirit of the Gorkiĭ school. Therefore, they were briefly treated in the first edition of the book. In the second edition, the material of the corresponding section, has been greatly augmented, but there still remains the general impression that this whole section lies outside of the plan of the book. The fundamental idea of the book — the deep penetration into the physics of oscillations, the ability to extract the maximum from a qualitative consideration of a problem and the exact solution of problems when they are correctly understood and applied — this basic idea does not permeate Chapter 9, which is devoted to the study of a system of the Thomson type by small-parameter methods.

Finally, a last word. In the first edition of the book the title was followed by the words "First Part." This was a promise to produce a book devoted to non-automatic systems and systems with many degrees of freedom. In the second edition the words "First Part" are omitted. Why? Did not A. A. Andronov leave behind

him a strong school, of which we are rightly proud? Does not the promise, given by the authors, of whom two are no longer alive and the third works in a different field, bind many of his students and co-workers?

The publication has not been completed; to complete it, not hurriedly but thoroughly, is a matter of honor for the Gorkiĭ physicists.

Translated by R. T. Beyer