STRONG MAGNETIC FIELDS

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I. STRONG MAGNETIC FIELDS AND THEIR APPLICATION

THE use of strong magnetic fields is necessary for many physical and technical investigations. A pioneer in this region has been P. L. Kapitza, who in 1924 produced a magnetic field of 500,000 oersteds by making use of the discharge of a set of storage batteries through a solenoid of low resistance, and in 1927 obtained a field of 300,000 oersteds by short-circuiting a powerful current generator with a solenoid of low resistance (electro-mechanical method of obtaining strong fields).

At the present time, field intensities up to 1.6×10^6 oersteds have been obtained under laboratory conditions, and such fields do not represent the possible limit.

A short list of researches on obtaining strong magnetic fields is given in Table I.

The achievement of strong magnetic fields (in what follows, we shall call a magnetic field strong if its intensity exceeds 20 or 30 kilooersteds) is possible by several means. Constant magnetic fields with intensities up to 50 or 70 kilooersteds can be obtained either by means of electromagnets with iron cores or by means of iron-free solenoids. F. Bitter in 1936 obtained a field intensity of 10^5 oersteds in a volume of 25 cm^3 by this method; the required power in this case amounted to 1700 kilowatts. Pulsed magnetic fields have been obtained by using the discharge of a capacitor bank or a storage battery through a low resistance coil. A field of a million oersteds with a time duration of 10 to 10,000 μsec can be obtained by this method.

Further increase of the field intensity is made difficult because the great mechanical forces which arise and the heat rise destroy the solenoid. A very interesting idea, which will perhaps find application in obtaining strong magnetic fields, is the possibility of creating a solenoid in which mechanical stresses are absent. For this purpose it is necessary that the vector current density be parallel to the magnetic field vector, i.e.,

$$\mathbf{i} = \alpha \mathbf{H}.$$

The possibility of the creation of such a construction has been considered by a number of authors.[81,82,83,108] Thus it was shown by Furth et al.[108] that the use of a

TABLE I

Author	Year	Field, kilo-oersteds	Method	Volume cm^3	Reference
P. L. Kapitza	1924	500	Discharge of batteries	0.005	2
Wall	1926	450	Discharge of capacitor bank	0.5	3
P. L. Kapitza	1927	320	Electro-mechanical method	2	4
Bitter	1936	100	Constant current	25	19
Haas and Westerdijk	1946	200	Discharge of battery	1	28
D. Schoenberg	1950	90	Discharge of capacitor bank	3	46
G. M. Strakhovskiĭ	1952	150	Discharge of dummy line. Obtained pulses of rectangular shape	1	45,86
Olsen	1953	150	Discharge of capacitor bank	0.15	49
Myers	1953	250	"	0.5	50
V. S. Komel'kov, N. G. Aretov	1956	700	"	—	74
Furth and Waniek	1956	600	"	0.1	64
Levin and Furth	1956	1500	"	—	63
Foner and Kolm	1956	750		—	65
G. I. Budker	1956	—	Relativistically stabilized electron beam	—	76
Levin, Furth, and Waniek	1957	1600	Discharge of capacitor bank	—	108
Chandrasekhar and Levin	1956	—	Magnetic fields without mechanical forces acting on the coil	—	81
Furth and Waniek	1957	—			108

toroidal construction would, if not completely eliminate, at least significantly decrease the action of mechanical forces. This was also realized in the plasmoids obtained by Bostick.[83,108]

Recently, new branches of physics have been intensively developed where strong magnetic fields are created and used, namely plasma physics and accelerators with iron-free electromagnets.[75,79,80,82,83,112-114] Strong magnetic fields can be created by means of a relativistically stabilized beam of electrons possessing strong self-focusing, while the magnetic field of the beam itself is much larger than the external magnetic field. This problem was first studied in detail by G. I. Budker.[76,84] Calculation has shown that with a current of 1000 amp, the field on the surface of the beam amounts to 50×10^3 oersteds, while the field necessary to keep this beam in an orbit of 1 m radius has an intensity of no more than 1350 oersteds.

Somewhat aside from the mentioned possibilities of obtaining strong magnetic fields is the question of the use of intra-atomic magnetic fields, which have a very high intensity, reaching a half million oersteds.[52]

Kapitza first employed his strong magnetic fields in the study of magnetostriction and galvanomagnetic effects.[4,5,7-12] Subsequently, such fields obtained application in different branches of physics: in nuclear physics, optics, solid state physics, etc. In nuclear physics, in the study of elementary particles of high energy, use is made of Wilson chambers, photoplates, and bubble chambers immersed in strong constant or pulsed magnetic fields.[41,45,55,58,59,60,64,86,90,91,97,111,115] This permits us to determine the sign of the charge and the momentum of the relativistic particles, especially short-lived ones such as mesons and hyperons. In this connection, the problem of the synchronization of the action of an accelerator producing a pulsed beam of fast particles with a pulsed magnetic field is of great importance. Contemporary technology permits us to produce such synchronization with an accuracy up to a fraction of a microsecond.[45,56,86] Unfortunately, pulsed magnetic fields are quite inapplicable in the study of cosmic rays. Strong magnetic fields can also be used for the polarization of atomic nuclei, which is necessary in many experiments.[52,103] The degree of polarization is determined by the following relation

$$P = \frac{1}{3}\frac{I+1}{I}\frac{\mu H}{kT}.$$

Calculation shows, for example, that for 50% polarization of nuclei at a temperature 0.01° K, the necessary field is about 75 kilooersteds. The interesting possibility of the application of strong magnetic fields to the polarization of an electron beam injected into an accelerator has been pointed out by L. N. Rozentsveĭg.[71,95]

Strong magnetic fields find widespread use in the study of the properties of matter. In particular, great interest attaches to the investigation of the Cotton-Mouton effect, the study of magneto-optical and galvanomagnetic effects, and also magnetostriction and magnetization in strong pulsed fields. A great number of researches have been devoted to the study of the de Haas effect.[38,46,47,64,70,93] Use of strong magnetic fields has permitted workers to carry out an investigation of resonance phenomena (cyclotron, ferro- and antiferromagnetic resonance) in the region of submillimeter radio waves,[35,66,68,96] which opens up great possibilities for new studies in radio spectroscopy.

II. ELECTROMAGNETS WITH IRON CORES

Electromagnets with iron cores are usually employed to obtain magnetic fields up to 50×10^3 oersteds. There exists a large number of constructions of laboratory electromagnets for 30,000 to 50,000 oersteds,[14,20,22,44,94,109] the designs of which are the result of compromises to satisfy what are often contradictory requirements, such as: 1) the necessity of obtaining fields of high magnetization, 2) the necessary homogeneity of the field in the working space, 3) accessibility of the working space, 4) effectiveness of cooling of the magnet, 5) power consumption and the efficiency of operation, etc. Figure 1 illustrates the most frequently used scheme for an electromagnet.

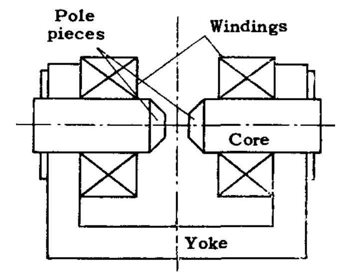

FIG. 1. Typical arrangement of a laboratory electromagnet.

The parts of the magnetic circuit where the induction reaches the highest values — the pole pieces are usually made of iron. cobalt alloys, while the yoke and cores are soft iron. The maximum fields obtained in such electromagnets is limited by the saturation of the magnetic circuit, which sets in, in the case of ordinary iron, at B = 17,000 — 20,000 gauss and in the case of special alloys at B = 24,000 — 26,000 gauss.

However, correct choice of the shape of the pole pieces permits one to obtain fields up to 70 kilooersteds. i.e., fields which considerably exceed saturation density. Choice of the optimal shape of the pole faces is quite complicated, in view of the absence of working methods of calculation of pole pieces with account of saturation, and in most cases their configuration is determined experimentally.

Let us consider the pole pieces shown in Fig. 2. If the magnetization at each point is parallel to the OO′ axis, and if the induction at the pole pieces reaches

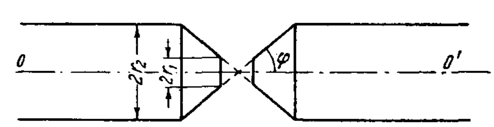

FIG. 2. Shape of pole pieces used to obtain a large magnetic field.

saturation, then the field at the point P can be calculated by the equation:[20]

$$H = 2.3B_s \sin^2 \varphi \ln \frac{r_2}{r_1}. \quad (2.1)$$

It is not difficult to find the optimal value of the angle φ, which is equal to 54°44′; substituting this value in Eq. (2.1), we obtain:

$$H = 0.88B_s \ln \frac{r_2}{r_1}. \quad (2.2)$$

The shape of pole pieces usually employed for obtaining large fields is close to that pictured in Fig. 2. However, the fields obtainable in practice are generally somewhat smaller than those computed by Eq. (2.2); this is explained by the fact that the magnetization of the poles is not parallel to their axes. In the general case, the pole of the electromagnet can be considered as a field created by a large number of magnetic dipoles, of which the electromagnet is composed. The field of each dipole can be expressed by the formula

$$\mathbf{H} = \frac{3(\boldsymbol{\mu}\mathbf{r})\,\mathbf{r}}{r^5} - \frac{\boldsymbol{\mu}}{r^3}.$$

The component of this field in the direction of the x axis has the form

$$H_x = \frac{M}{r^3}(2\cos\vartheta\cos\varphi + \sin\vartheta\sin\varphi)$$

(see Fig. 3).

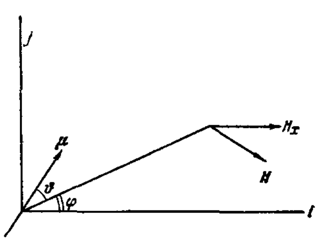

FIG. 3

It is not difficult to find for what value of the angle ϑ the field intensity H_x reaches a maximum, for fixed values of r and φ. Setting

$$\frac{\partial H_x}{\partial \vartheta} = 0,$$

we get

$$(H_x)_{max} = \frac{2\mu}{r^3} \frac{1+\cos^2\varphi}{(1+3\cos^2\varphi)^{1/2}}. \quad (2.3)$$

This equation yields the field created by N magnetic dipoles:

$$H_x = \frac{B_s}{4\pi} \int_v \frac{1}{r^3} \frac{1+\cos^2\varphi}{(1+3\cos^2\varphi)^{1/2}} dv. \quad (2.4)$$

Equation (2.4) does not always reduce to elementary functions, and in a number of practically important cases, it is necessary to carry out numerical integration. Calculations of electromagnets of simple forms, which begin with this equation, were made by Bitter.[20] For many experiments, it is necessary to obtain a sufficiently homogeneous field over a significant volume. The homogeneity of the field in the gap of an electromagnet depends on the ratio $2r_1/l$, and increases with this ratio. References 18 and 88 are devoted to a calculation of the degree of homogeneity of the magnetic field and to problems of its improvement by the use of pole pieces of more complicated shape. However, these calculations are rather involved and inaccurate for magnets when strong saturation is present.

As an example, let us consider a laboratory magnet (Fig. 4), with which fields up to 50 kilooersteds were obtained.[94] The magnet consisted of a yoke of diameter 630 mm, cores of diameter 630 mm with a winding and set of pole pieces (cores). The yoke and the cores were cast from carbon-free steel; the pole pieces were prepared from the best carbon-free iron; the magnet weighed 3.5 tons. The winding consisted of a conductor of cross section 25 mm^2 and had 170,000 ampere turns. Air cooling was employed. The dependence of the field on the distance between the poles of the electromagnet is shown in Fig. 5.

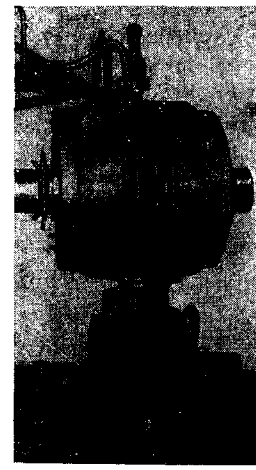

FIG. 4. Laboratory electromagnet.

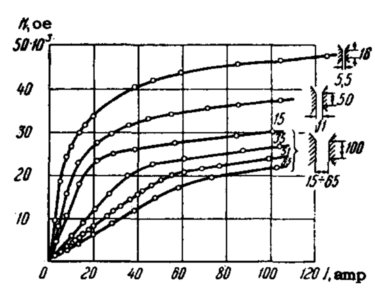

FIG. 5. Dependence of the field intensity on the current in the windings for different gaps and diameters of the pole pieces of the electromagnet (see Fig. 4).

III. IRON-FREE CONSTANT ELECTROMAGNETS

Much larger magnetic fields can be obtained from solenoids. The equations given in this section for the calculation of magnetic fields are applicable for solenoids fed either by direct or pulsed currents. We consider the fundamental types of solenoids employed in obtaining strong magnetic fields. If i is the current density in amp/cm^2, then the field for a circular current-bearing loop of radius r on the axis at a distance x from the plane of the loop and created by an element of current is equal to

$$dH = \frac{2\pi i r^2}{10} \frac{dr\,dx}{(r^2+x^2)^{3/2}}. \quad (3.1)$$

The power necessary to maintain this current is

$$dw = 2\pi \varrho i^2\, dr\, dx, \tag{3.2}$$

where ρ is in ohm/cm^2.

These equations are fundamental to calculations for any types of solenoids. If only part of the volume of the solenoid is filled with conductors (as is actually the case), then the result must be multiplied by a filling factor λ.

A. Cylindrical coil with constant current density. We introduce the notation (Fig. 6)

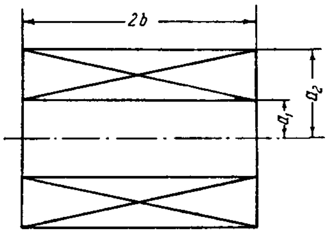

FIG. 6. Cylindrical solenoid.

$$\frac{a_2}{a_1} = \alpha, \qquad \frac{b}{a_1} = \beta.$$

In accord with (3.1) and (3.2), we obtain

$$H = \frac{\pi i\, \lambda}{5} \int_{a_1}^{a_2} r^2\, dr \int_{-b}^{+b} \frac{dx}{(r^2+x^2)^{3/2}} = \frac{2\pi}{5} b\lambda i \ln \frac{a_2+(b^2+a_2^2)^{1/2}}{a_1+(b^2+a_1^2)^{1/2}}, \tag{3.3}$$

$$w = 2\pi \varrho i^2 \lambda b\,(a_2^2 - a_1^2). \tag{3.4}$$

Eliminating i, we obtain an expression for the field intensity at the center of the solenoid:

$$H = G_1 \left(\frac{w\lambda}{\varrho a_1}\right)^{1/2}, \tag{3.5}$$

where

$$G_1 = \frac{(2\pi)^{1/2}}{5} \left(\frac{\beta}{\alpha^2 - 1}\right) \ln \frac{\alpha+(\beta^2+\alpha^2)^{1/2}}{1+(\beta^2+1)^{1/2}}. \tag{3.6}$$

The quantity G_1 reaches a maximum, equal to 0.179, at $\alpha = 3$ and $\beta = 2$.

B. Cylindrical coil with optimal radial distribution of the current. Let the dependence of the current density on the radius of the coil be given as some function $i = f(r)$; then

$$H = \frac{\pi\lambda}{10} \int_{a_1}^{a_2} r f(r)\, dr \int_{-b}^{+b} \frac{dx}{(r^2+x^2)^{3/2}}, \qquad w = 4\pi\varrho\lambda \int_{a_1}^{a_2} r f^2(r)\, dr.$$

It can be shown[19] that, in the optimal case, the field in such a solenoid can be written in the form

$$H = G_2 \left(\frac{w\lambda}{\varrho a_1}\right)^{1/2}. \tag{3.7}$$

The quantity

$$G_2 = 0{,}1 \left\{\frac{2\pi}{\beta} \ln \alpha^2 \frac{(1+\beta^2)}{(\alpha^2+\beta^2)}\right\}^{1/2} \tag{3.8}$$

reaches a maximum, equal to 0.225, at $\beta \approx 2$ and $\alpha = \infty$.

If a sufficiently strong current of several thousand amperes passes through the coil of the solenoid winding, one can obtain a field of the order of 100,000 oersteds. In this case the problem of cooling the solenoid takes on a transcending importance. When low powers are used, the cooling only of the outer surface of the solenoid is possible, which becomes inadequate when the dissipated power reaches tens of kilowatts. Water cooling has obtained the widest use, although cooling by means of certain organic liquids is also possible.[77,102] Cooling by liquid nitrogen or helium is also possible.[46,49,53] Calculation of heating and forced cooling does not present any great difficulties.[15,118]

In working with solenoids, it is important to know the degree of homogeneity of the magnetic field in the working region, which depends on many parameters and cannot be represented in the form of an expression that is suitable for computation. Let us make several remarks relative to the calculation of the homogeneity of the field. The magnitude of the field on the axis of the solenoid at a distance x from the center O is expressed by the following formula:

$$H_x(x, 0) = \frac{2\pi n I}{20 b\,(a_2 - a_1)} \Big[(b+x) \ln \frac{a_2+[(b+x)^2+a_2^2]^{1/2}}{a_1+[(b+x)^2+a_1^2]^{1/2}} + (b-x) \ln \frac{a_2+[(b-x)^2+a_2^2]^{1/2}}{a_1+[(b-x)^2+a_1^2]^{1/2}} \Big]. \tag{3.9}$$

The field in a solenoid of finite length has both an axial and a radial component, which can be represented as follows:

$$H_x(x, r) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n!)^2} \left(\frac{r}{2}\right)^{2n} H^{(2n)}(x, 0), \tag{3.10}$$

$$H_r(x, r) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,(n-1)!} \left(\frac{r}{2}\right)^{2n-1} H_x^{(2n-1)}(x, 0). \tag{3.11}$$

Here $H_x(x, r)$ is the axial component of the field at the point (x, r); $H_r(x, r)$ is the radial component at the same point;

$H_x(x, 0)$ is the field intensity on the axis at a distance x from O. The values of the derivatives can be found by differentiation of Eq. (3.9). The calculation is usually limited to a finite number of terms in Eqs. (3.10) and (3.11).

Figure 7 shows the design of a solenoid prepared for obtaining fields up to 10^5 oersteds in an appreciable volume.[23] The field distribution in the solenoid is shown in Fig. 8, from which it is seen that it is possible to obtain a homogeneous field with accuracy to within 1 per cent over a volume of 25 cm^3. The solenoid is fed by direct current up to 10,000 amp at 170 volts. Water cooling was at a flow rate of 500 liters/sec; the required power was 1700 kw. The graphs shown in Fig. 9 illustrate the relationship between the maximum field which can be obtained by means of a water-cooled solenoid, the applied power, and the internal diameter of the solenoid.

IV. PULSED MAGNETIC FIELDS

Magnetic fields with intensity higher than 100,000 oersteds can be obtained by means of solenoids fed by pulsed current. Inasmuch as the current density in

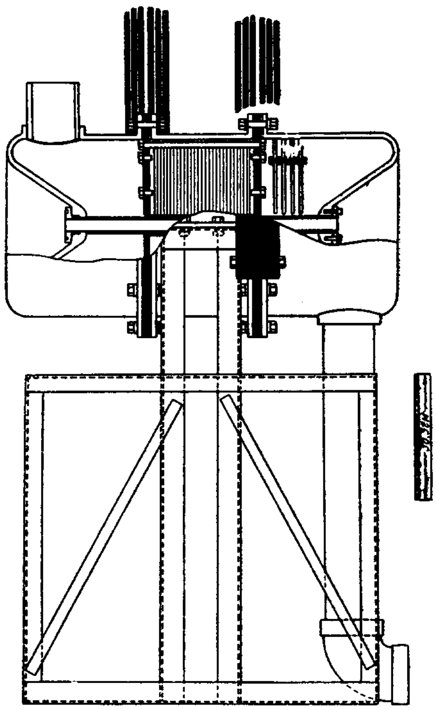

FIG. 7. Design of a solenoid for obtaining a constant magnetic field of intensity up to 100,000 oersteds.

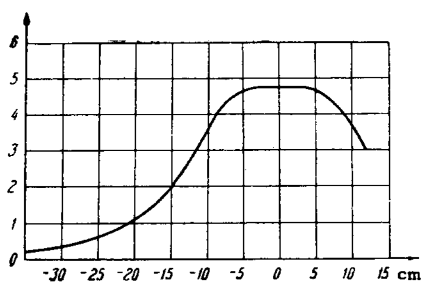

FIG. 8. Distribution of the field intensity along the axis of the solenoid (represented in Fig. 6).

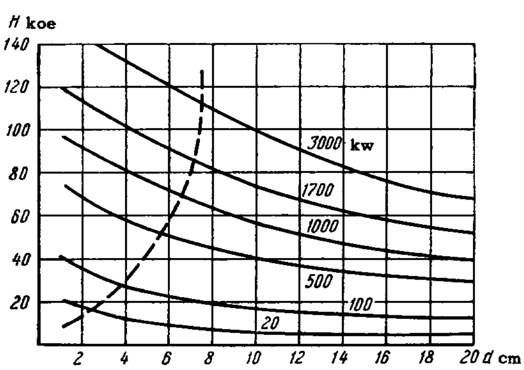

FIG. 9. Dependence of the field intensity on the diameter of the solenoid for different power consumptions.

this case is very large, use of direct currents is not possible because of cooling difficulties. Pulsed current of sufficient density can be obtained by various means: by a discharge of a bank of capacitors through a solenoid of low resistance, discharge of a chemical battery, or by electro-mechanical means.[2,4,16,17,21,22,24,30,45,74,105]

Figure 10 shows the principal circuit arrangement for obtaining pulsed magnetic fields. The energy

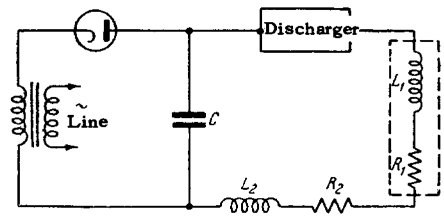

FIG. 10. Circuit for obtaining pulsed magnetic fields.

stored in the capacitors is almost completely transformed into the energy of the magnetic field:

$$W = \frac{1}{2} CV^2 = \frac{1}{2} LI^2 + W_{\text{loss}} = \frac{1}{8\pi} \int H^2 \, dv + W_{\text{loss}}.$$

Here C is the capacitance of the bank, V is the operating voltage, and L is the inductance of the coil and leads. The current in such a circuit is given by

$$I = \frac{V}{L\omega} e^{-\mu t} \sin \omega t.$$

Hence the maximum current, which determines the maximum field, is equal to

$$I_{\max} = \frac{V}{\left(\frac{L}{C}\right)^{1/2}} \exp\left[-\frac{\mu}{\omega} \tan^{-1} \frac{\omega}{\mu}\right]. \quad (4.1)$$

where the following notation has been introduced:

$$\omega = \left[\frac{1}{LC} + \frac{R^2}{4L^2}\right]^{1/2}, \quad \mu = \frac{R}{2L}, \quad L = L_1 + L_2, \quad R = R_1 + R_2,$$

L_1 and R_1 are the inductance and resistance of the solenoid, while L_2 and R_2 are the corresponding values of the conducting leads and the discharge. To obtain maximum field, it is necessary that the external inductance L_2 and the external resistance R_2 have the smallest possible value, and also that the condition

$$\frac{R}{2L} \ll \frac{1}{(LC)^{1/2}}$$

be satisfied. These requirements lead to the necessity of the use of special capacitors which have a minimum inductance, to a proper choice of current leads, and to an optimal geometry of the coil. In obtaining fields of the order of a million oersteds, the current reaches 100,000 and even 1,000,000 amperes at an operating voltage of several ten thousands of volts. Thus, for example, experiments are described in reference 74 with a current of 2.1×10^6 amp at an operating voltage of 50 volts, while the rate of change of the current reached 2×10^{12} amp/sec. In this experiment the possibility of obtaining currents up to 12×10^6 amp was pointed out.

At the present time the discharge of a capacitor bank is used in the majority of cases for obtaining large pulsed currents. In the construction of such current generators, as has already been pointed out above, special attention is paid to minimizing the inductance of the current leads and of the other connectors. This is necessary not only for obtaining maximum fields, but also for obtaining sufficiently steep fronts, inasmuch as the duration of the front of the pulse is proportional to $\sqrt{LC}$.

Another method of obtaining pulsed currents, which has not lost its importance, is the electromechanical method, which allows one to obtain sufficiently long pulses. This method, for example, was used by M. Oliphant for obtaining a pulsed current feeding an iron-free electromagnet of an accelerator in which protons are accelerated to 10 Bev.[72] The discharge of a storage battery is employed, which also has certain advantages, since it makes it possible to obtain current pulses of duration of the order of 0.01 sec.

In the production of pulsed currents, an important part of the apparatus is the discharge gap, which determines to a significant degree the rate of increase of the current in the coil and the accuracy of the synchronization of the pulsed fields with the radiation pulses of the accelerator (for the latter purpose, for example, mechanical discharge is completely unsuitable). Use of the following types of dischargers is possible: ignitrons,[45,64,86] gas-discharge tubes,[56] dischargers with breakdown of a solid dielectric,[74] vacuum,[110] and mechanical[2,108] dischargers.

Use of pulse techniques thus permits one to obtain significantly larger magnetic fields; in this case, the mean power required is much less than in the case of direct current and much less stringent requirements are necessary for the cooling system. However, obtaining of pulsed fields also brings on a number of difficulties, the principal among which is the problem of the mechanical stability of the solenoid. Actually, fields of the megagauss range contain colossal energy densities, and great mechanical stresses are produced in the coil. These can be expressed in terms of two components: axial, which compresses the coil along the axis, and radial, which tends to rupture it. The effect of these forces can be reduced somewhat if a sufficiently rigid shell is placed around the coil. However, this does not solve the problem completely, since limitations appear that are connected with the yield of the material of the coil under the action of the large stresses. A detailed analysis of the stresses arising in the coil has been carried out in a number of researches.[4,6,108] These are quite cumbersome, and an approximate estimate of the stress can be made by means of the following formula:

$$p = \frac{H^2}{8\pi} \cdot 10^{-6},$$

where the pressure p is expressed in kg/cm^2 and H in oersteds. The values of the maximum fields that can be obtained without changing the properties or destroying the material of which the coil is composed are shown in Table II. When fields of a half-million oersteds are produced in a coil, stresses of the order of 7×10^3 kg/cm^2 appear. We note that the elastic limit of phosphor bronze is $(3 \text{ to } 4) \times 10^3$ kg/cm^2,

TABLE II

H oersteds	
1 000 000	
800 000	W
	Steel
600 000	2% Be+Cu
	1% Be+Cu
	1% Cd+Cu
400 000	Cu
200 000	

while that of beryllium is $(7 \text{ to } 8) \times 10^3$ kg/cm^2. However, for short pulses, it is possible to obtain even larger fields. Fields up to 1.6×10^6 oersteds were obtained in reference 108 in just such a fashion. From the viewpoint of increasing the dynamic stability of the solenoid, it is advantageous to generate very short pulses of the magnetic field of the order of 1 microsecond; in this case, the action of the stress on the coil can be regarded as a ballistic impact (mechanical and thermal). Experiments show that if the duration of the pulse is short, then the stability of the coil will be appreciably increased in this case.

In addition to mechanical limitations in the production of pulsed magnetic fields, there are also temperature limitations. The temperature rise in a pulse time τ_i can be estimated by starting from the following relation

$$\Delta T = \int_0^{\tau_i} \frac{RI^2\,dt}{v\sigma d}.$$

Here v is the volume occupied by the conductor, σ is the specific heat, and d is the density. For short pulses, one can neglect the thermal conductivity of the coil. For simplicity, it is assumed that

$$I^2 = \frac{1}{2} I_m^2,$$

I_m is the current amplitude. Then

$$\Delta T = \frac{RI_m^2 \tau_i}{2v\sigma d}.$$

The value of I_m can be found from Eq. (4.1). It is evident that the heating of the coil is determined both by the specific heat and the thermal conductivity of the coil itself and by the duty cycle and duration of the pulses. Use of one form of cooling or another permits a certain reduction in the heating, which is particularly important in the generation of long pulses. If the coil is cooled by liquid nitrogen or helium,[28,46,49,53,62] change in the conductivity at low temperatures must be taken into account in the calculation of the field. By using helium temperatures, it is possible to obtain magnetic fields of intensities of 2.5×10^5 oersteds of duration of 0.1 sec.[46] The obtaining of strong pulsed fields is also complicated by the presence of skin effect. Actually, in the case of current pulses of short duration, the current flows over the surface of the conductor and overheats it greatly.

Let us consider typical designs of coils used to obtain pulsed magnetic fields. The simplest design is the coil considered in Sec. III.

In the case of pulsed fields, it is appropriate to express Eq. (3.5) in the following form:

$$H=\left(\frac{20C}{b}\right)^{1/2}\frac{V}{a}SK. \tag{4.2}$$

Here H is expressed in oersteds, C is the capacitance of the bank in microfarads, V is the operating voltage in volts, S and K are constants smaller than unity; S depends only on the geometry of the coil, while K represents the decrease in the field due to the Joule heat. The expression for S can be represented in the form

$$S=\frac{\pi(\beta)^{1/2}}{10\sqrt{5}(\alpha-1)}\ln\frac{\alpha+(\beta^2+\alpha^2)^{1/2}}{1+(\beta^2+1)^{1/2}}.$$

The dependence of S on the parameters of the coil is shown graphically in Fig. 11. As is to be expected, S ≈ 1 if the length of the coil is much greater than the diameter. The value of K as a function of RT/L

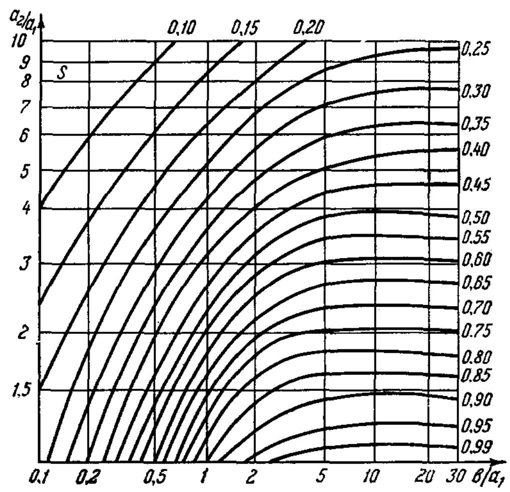

FIG. 11. Dependence of the coefficient S on the parameters of the coil (for notation, see Fig. 6).

(where T is the period) can be found from Table III. The inductance of the coil, $L = \lambda a_1 n^2$, depends on its parameters and can be determined by means of the graphs shown in Fig. 12, plotted under the assumption that the inductance is independent of the filling factor; this holds only for λ = 1. However, deviation from the computed data does not exceed several thousandths.

TABLE III

$\frac{RT}{L}$	K	$\frac{RT}{L}$	K
0	1	3	0.638
0.01	0,999	3,5	0.586
0.05	0.994	4	0.537
0.1	0.987	5	0,451
0.2	0.975	6	0,379
0.3	0.962	7	0.319
0.4	0.950	8	0.270
0,5	0.937	10	0.195
0.6	0.925	12	0.143
0.8	0.900	15	0.0923
1.0	0.875	20	0.0463
1,2	0.850	25	0.0239
1,6	0.800	30	0.0125
2.0	0.751	40	0.0035
2.5	0.693	50	0.00099

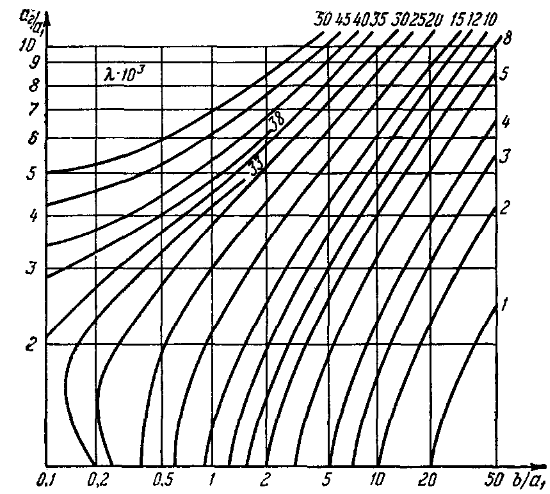

FIG. 12. Dependence of $\lambda = \frac{L}{n^2 \cdot a_1}$ on the parameters of the coil (L = inductance of the coil in microhenries, b = number of turns, and a_1 = internal diameter of the coil).

All of the calculations were carried out under the assumption of a quasistationary mode, in which the skin effect could be neglected. The tables given permit one to compute the field generated by such a coil, the length of the pulse, and the dissipated power.

We shall now consider other designs of solenoids for obtaining pulsed fields.

C. Cylindrical coil with trapezoidal cross section and homogeneous distribution of current density (Fig. 13). For such a coil,

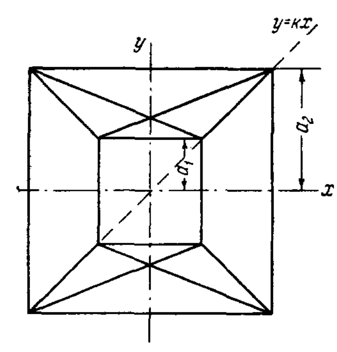

FIG. 13. Coil with trapezoidal winding cross section.

$$H=\frac{2\pi i}{5}\frac{\lambda K}{(1+K^2)^{1/2}}(a_2-a_1), \qquad w=\frac{4\pi}{3}\varrho K i^2\lambda(a_2^3-a_1^3),$$

where, by eliminating i, we obtain:

$$H=G_3\left(\frac{w\lambda}{\varrho a_1}\right)^{1/2}, \tag{4.3}$$

$$G_3=\frac{(3\pi)^{1/2}}{5}\left(\frac{K}{K^2+1}\right)^{1/2}\frac{\alpha-1}{(\alpha^3-1)^{1/2}}. \tag{4.4}$$

The dependence of G_3 on the parameters $\alpha = a_2/a_1$ and K is shown in the following graphs (Figs. 14 and 15).

D. Cylindrical coil with trapezoidal cross section,

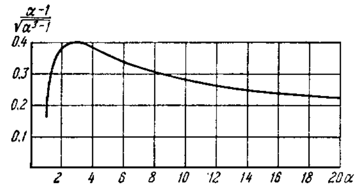

FIG. 14.

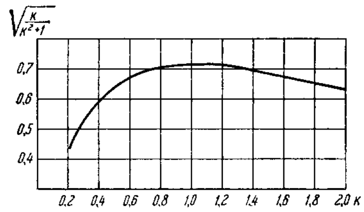

FIG. 15

in which the current density is inversely proportional to the radius. This case can easily be realized in practice by use of a strip of variable thickness. By using Eqs. (3.1) and (3.2) we get:

$$H=\frac{2\pi i_0}{5}\frac{K}{K^2+1}\ln\frac{a_2}{a_1}, \qquad w=4\pi\varrho\lambda i_0^2 K\left(a_2-a_1\right).$$

By assuming the following dependence of the current on the radius,

$$i=\frac{i_0}{r},$$

we get (after eliminating i):

$$H=G_4\left(\frac{w\lambda}{\varrho a_1}\right)^{1/2}, \qquad G_4=\left(\frac{K}{K^2+1}\right)^{1/2}\frac{\ln\alpha}{(\alpha-1)^{1/2}}. \tag{4.5}$$

The dependence of G_4 on K is the same as in the case C, while the dependence on α is shown in Fig. 16.

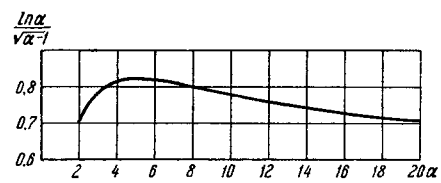

FIG. 16

E. Cylindrical coil of rectangular cross section with a current density inversely proportional to the radius of the coil. This type of coil is frequently used today in the production of pulsed fields, inasmuch as such a coil possesses great rigidity. The design of such a coil is shown at the end of this section.

In accord with Eqs. (3.1) and (3.2), we get

$$H=\frac{2\pi i_0\lambda}{5}\ln\left[\alpha\,\frac{\beta+(1+\beta^2)^{1/2}}{\beta+(\alpha^2+\beta^2)^{1/2}}\right], \qquad w=4\pi\varrho i_0^2\lambda\ln\alpha. \tag{4.6}$$

Here $\alpha = a_2/a_1$, $\beta = b/a_1$, a_1 = internal radius, a_2 = external radius of the coil, 2b = its length.

After eliminating i_0, we get

$$H=G_5\left(\frac{w\lambda}{\varrho a_1}\right)^{1/2}, \tag{4.7}$$

$$G_5=\frac{\pi^{1/2}\ln\left[\alpha\,\frac{\beta+(1+\beta^2)^{1/2}}{\beta+(\alpha^2+\beta^2)^{1/2}}\right]}{5\,(\beta\ln\alpha)^{1/2}}. \tag{4.8}$$

The dependence of G_5 on α and β is shown in Fig. 17.

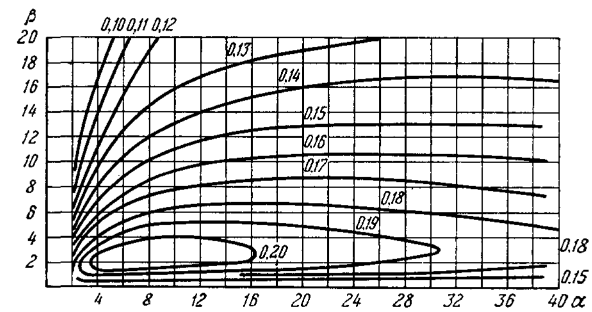

FIG. 17. Value of the quantity G_5 for various values of α and β.

F. Coil for which i = f (r, x). It can be shown that in the optimal case, the expression for the current has the following form:

$$i=\frac{1}{20}\cdot\frac{a}{\mu\varrho\,(a^2+x^2)^{1/2}}.$$

Here μ is some constant which can be represented in terms of w. The field in this case also can be represented in the form

$$H=G_6\left(\frac{w\lambda}{\varrho a_1}\right)^{1/2}. \tag{4.9}$$

The maximum value of G_6 is equal to 0.272, and is achieved at $\alpha = \infty$ and $\beta = \infty$.

Table IV gives characteristics that make it possible to compare coils of different types from the viewpoint of obtaining maximum field.

The practical designs of coils for obtaining pulsed magnetic fields described below by no means exhausts all the possibilities; however, they are the most widely used. The choice of the type of coil is determined by the purpose for which it is destined and by the parameters of the current generator. For estimating the maximum field intensity, it is convenient to use the formula

$$H\simeq 10V\left(\frac{C}{v}\right)^{1/2},$$

where H is expressed in oersteds, V in volts, C is the capacitance of the capacitor bank in microfarads, and v is the working volume of the solenoid in cm^3. The design of a solenoid destined for obtaining record-breaking fields[64,99,117] is shown in Fig. 18; it consists of a plane spiral of beryllium bronze with mica insu-

FIG. 18. Details for the construction of a a solenoid for obtaining a pulsed magnetic field.

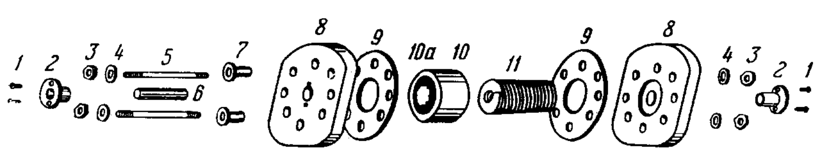

TABLE IV

Type of coil	Distribution of the current density	G	G_{max}	Values of the parameters at which H_{max} is achieved		
				α	β	K
A		$\frac{(2\pi)^{1/2}}{5}\left(\frac{\beta}{\alpha^2-1}\right)\ln\frac{\alpha+(\beta^2+\alpha^2)^{1/2}}{1+(\beta^2+1)^{1/2}}$	0.179	3	2	—
B	Optimal	$\frac{(2\pi)^{1/2}}{10}\left\{\frac{1}{\beta}\ln\alpha^2\frac{(1+\beta^2)}{(\alpha^2+\beta^2)}\right\}^{1/2}$	0.225		2	—
C		$\frac{(3\pi)^{1/2}}{5}\left(\frac{K}{K^2+1}\right)^{1/2}\frac{\alpha-1}{(\alpha^3-1)^{1/2}}$	0.172	2.7	—	1
D	$i=\frac{i_0}{r}$	$\frac{\pi^{1/2}}{5}\left(\frac{K}{K^2+1}\right)^{1/2}\frac{\ln\alpha}{(\alpha-1)^{1/2}}$	0.201	4.5	—	1
E	$i=\frac{i_0}{r}$	$\frac{\pi^{1/2}}{5}\cdot\frac{\ln\left[\alpha\frac{\beta+(1+\beta^2)^{1/2}}{\beta+(\alpha^2+\beta^2)^{1/2}}\right]}{(\beta\ln\alpha)^{1/2}}$	0.209	6	2	—
F	Optimal	—	0.272	∞	∞	—

lators. The entire coil is enclosed in a ceramic shell which somewhat increases its mechanical strength. Usually, the coils are made out of mechanically strong alloys which possess high electrical conductivity. The most suitable from this point of view are the beryllium-copper alloys. Parameters are given in Table V for several coils of such a type, along with their operating characteristics.

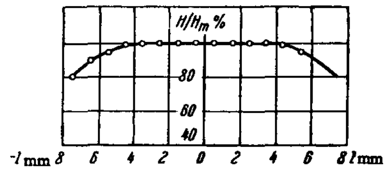

FIG. 19. Field distribution along the axis of the solenoid.

TABLE V

Internal diameter in cm	External diameter in cm	Thickness of the turn in cm	Number of turns	T μsec	H_{max} kilo-oersteds
2.54	5.08	0.127	32	480	145
2.54	5.08	0.127	16	260	175
2.54	5.08	0.127	6	135	110
1.80	5.08	0.127	26	325	250
1.80	5.08	0.127	13	190	270
1.71	3.80	0.127	25	280	315
1.71	3.80	0.127	11	160	360
0.95	2.54	0.127	12	135	575
0.95	2.54	0.127	18	175	575
0.95	3.80	0.127	17	170	540
0.95	3.80	0.127	8	85	580
0.95	3.80	0.127	5	74	430
0.63	2.54	0.076	19	160	700
0.47	2.54	0.076	13	120	750
0.47	2.54	0.076	15	140	900

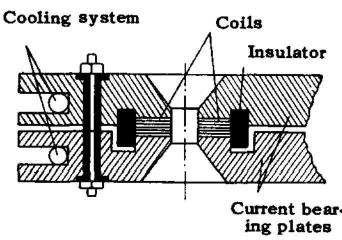

FIG. 20. Design of a solenoid for obtaining a pulsed field of intensity up to 500,000 oersteds.

Similar solenoids enable one to obtain sufficiently high homogeneity of the field in the working space. The field distribution along the axis of the solenoid[117] is shown in Fig. 19.

The practical design of a pulsed solenoid for obtaining fields up to 5×10^5 oersteds is shown in Fig. 20; heavy plates conducting the current ensure minimum external resistance and inductance. The coil has an internal diameter of 4.8 mm and length 9.5 mm which assures homogeneity of the field to within 5 per cent in the working space of diameter 2.3 mm and length 3.2 mm.

As a second example, we consider the "ribbon type" of coil (type D in accord with Table IV). The arrangement of such a coil is shown in Fig. 21. The external and internal loops are connected with heavy wires which serve as conducting leads. By means of such

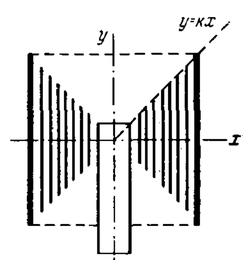

FIG. 21. Coil with windings of ribbon type.

a coil, a field of 150 kilooersteds was obtained with a half-period of 230 μsec in the discharge of a 2000-microfarad capacitor bank charged to 1 kilovolt.[99] The advantage of such a coil, as was pointed out above, lies in its comparatively simple structure. Use of ordinary coils (type A) is also possible. For example, in the work of Piekara,[73] coils are described that consist of 100 — 200 turns of copper wire of diameter 0.7 — 1.0

mm, by means of which fields were obtained up to 350 $\times 10^3$ oersteds in a volume of 1 cm^3. Sometimes pairs of coils are used (Helmholtz coils),[99] whose construction is pictured in Fig. 22, while the field distribution is shown in Fig. 23.

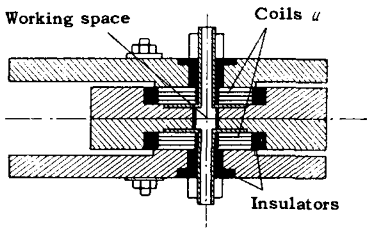

FIG. 22. Helmholtz coil.

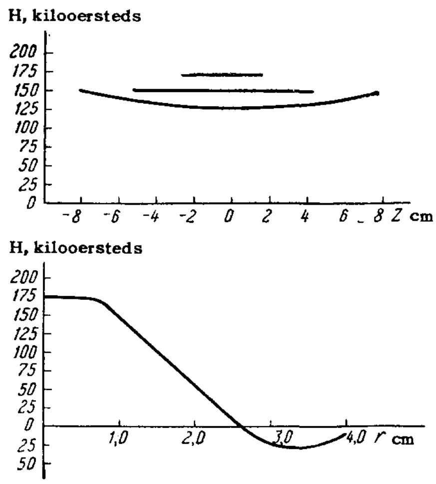

FIG. 23. Distribution of the magnetic field in a Helmholtz coil.

V. SHAPE AND LENGTH OF THE FIELD PULSE

Physical experiments in pulsed magnetic fields impose an additional restriction on the shape and length of the pulsed field. Thus, for example, carrying out of experiments with resonance phenomena in the infrared and submillimeter region requires a sufficiently flat peak of the pulse, and in other experiments pulses of sufficient duration are required. In the simplest circuits for obtaining pulsed magnetic fields, the length and shape of the pulse are determined by the period of the characteristic oscillations and the periodic or aperiodic character of the oscillations, which depends on the relation between the resistance and reactance of the circuit. By choosing a particular ratio between L and R, one can increase or decrease the damping in the circuit as desired, and consequently change the shape of the pulse. Usually, for obtaining sufficiently long pulses, it is necessary to sacrifice the intensity of the field somewhat. The record in this relation is the result obtained by P. L. Kapitza in 1924, when he obtained a field of 0.5 $\times 10^6$ oersteds over a time of 0.003 sec.[2]

Use of pulse transformers[64,108] permits one to obtain a pulse length of the order of hundredths of a second. Figure 24 shows a block diagram for employing a pulse transformer. Such an arrangement permits us to obtain pulses of the field of duration 50 — 100 μsec by means of the magnet A and a length of 500 — 10,000 μsec by means of the pulse transformer and magnet B.

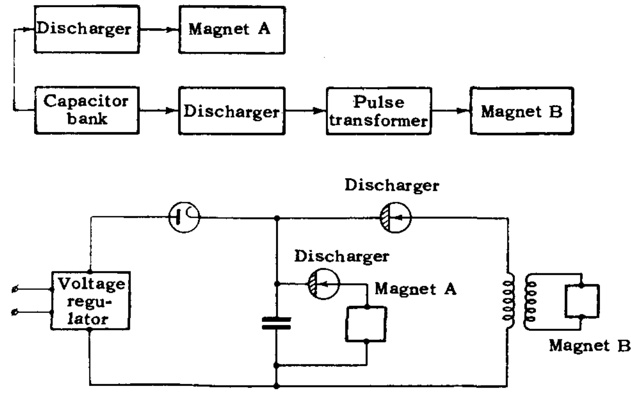

FIG. 24. Circuit with pulse transformer for obtaining pulses of field of duration up to 10,000 μsec.

De Haas and Westerdijk thus obtained a field of 200,000 oersteds over a time of 0.1 sec by cooling the solenoid with liquid hydrogen; their researches demonstrated the possibility of increasing the time interval up to 0.5 sec. It is possible to obtain pulses of almost rectangular shape and the increase in length of the pulse by application of an artificial circuit for shaping the pulse.[45,65,101] For this purpose the capacitors are connected in separate groups, and are connected among themselves by corresponding inductances. This arrangement makes it possible to obtain pulses of sufficient length without appreciable loss of intensity. The maximum length obtained and the amplitude of the field in this case are also limited by the disrupting mechanical forces and by heating of the coil. For obtaining long pulses it is appropriate to make use of electromechanical and chemical current generators.[4]

In a number of scientific and technical investigations, the obtaining of unidirectional fields is required. For this purpose it is necessary to use discharge gaps which possess unidirectional conductivity. The dischargers usually employed do not possess such properties. This difficulty can be avoided if use is made of special ignition devices which, by exploding, cut off the oscillatory discharge in the coil. Finally, for a number of studies, application of magnetic fields in the form of a unidirectional pulse of rectangular shape is necessary. The arrangements previously considered of obtaining pulsed magnetic fields by means of a discharge of a capacitor bank possess two disadvantages from this viewpoint: 1) the discharge has an oscillatory character; 2) the pulse has no flat peak. This problem is solved by the use of a dummy line. In this case, it is possible to obtain rectangular pulses and their synchronization with the pulse under study (for example, with the operation of an accelerator) with accuracy up to fractions of microseconds.

Such an arrangement was first proposed by Strakhovskiĭ in 1951.[45,86] The circuit of the generator of rectangular pulses is shown in Fig. 25; $L_1C_1 - L_4C_4$ form the dummy line computed for the required length of the pulse, R is the wave resistance, I is the discharger (ignitron). The discharge is produced by a

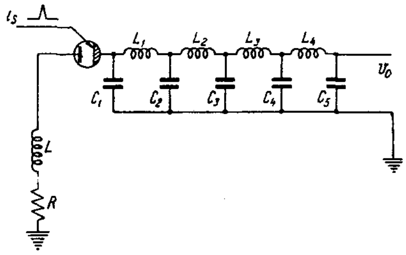

FIG. 25. Circuit for obtaining a strong magnetic field in the form of pulses of rectangular shape.

starting pulse which enters from the synchronizing transmitter. The inductance of the working coil is approximately twice that of the remaining ones. The pulsed field obtained has a form close to rectangular (Fig. 26) and its length is determined by the parameters of the dummy line while the amplitude depends on L, R and the voltage V to which the dummy line is connected. In certain cases it is advantageous to employ a dummy line in the form of a series of resonant circuits (Fig. 27). Processes taking place in

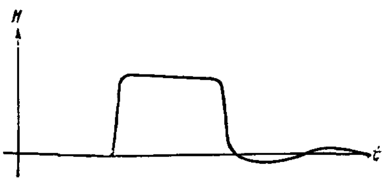

FIG. 26. Shape of the pulsed magnetic field.

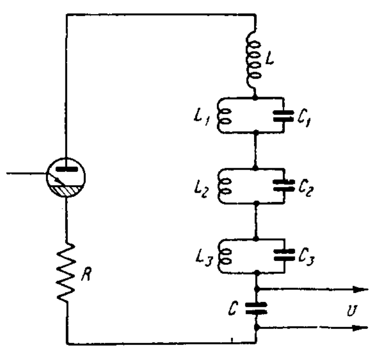

FIG. 27. Circuit of a dummy line in the form of a series of parallel circuits. L – solenoid in which the pulsed magnetic field of rectangular shape is generated.

dummy lines are described at length in books on pulse techniques.[37] We note that in such circuits it is important to make a correct choice of the value of the resistance R, inasmuch as the latter determines the pulse shape. To obtain strong currents, the wave resistance of the line must have a sufficiently small value. The pulse length can be computed from the equation,

$$\tau_i = 2n\sqrt{LC},$$

where n is the number of units in the dummy line, and L and C are the inductance and capacitance of each unit.

MEASUREMENT OF PULSED MAGNETIC FIELDS

A strong magnetic field can be measured by the usual ballistic method if a small test coil is placed in the magnetic field to be measured. This arrangement makes it possible to measure the field intensity with an accuracy to within one percent both in the case of constant fields and in the case of unidirectional pulsed fields.

The magnitude of the magnetic field can also be found by measuring the current strength in the solenoid. In the measurement of strong pulsed currents, a Rogovskiĭ loop[74] or non-inductive measuring shunts with pulsed oscillators are usually employed.

For measurement of the intensity of the pulsed field, one can use the Faraday effect. By measuring the rotation of the plane of polarization, and by knowing the Verdet constant, one can obtain the value of the magnetic field in the region under study; carbon bisulfide, carbon tetrachloride, and water can be used as the optical media. This method permits measurement of the field within one percent.[73,108] Finally, strong magnetic fields can be measured by other magneto-optic (Cotton-Mouton effect, Paschen-Back effect, etc) or galvanomagnetic effects. However, these methods do not have widespread application in view of the absence of sufficiently suitable and laboratory developed designs.[38,72]

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