NEW EXPERIMENTAL DATA ON π- AND μ-MESON DECAYS

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CONTENTS

1. INTRODUCTION

THIS is a survey of the experimental studies of the properties of "free" π and μ mesons that have been made in the last two years. This means that we shall scarcely touch at all on questions of the interaction of these mesons with matter, and shall consider only experiments in which those properties of π and μ mesons that are manifest in their decay have been studied. These are for the most part experiments in which parity nonconservation in $\pi \rightarrow \mu \rightarrow e$ decay was studied, and experiments that have become possible owing to this phenomenon. In the first paper of Lee and Yang on parity nonconservation in weak interactions[1] the two main possibilities for an experimental test of this hypothesis were pointed out: the decay of oriented nuclei and the decay of mesons and hyperons, in particular the $\pi \rightarrow \mu \rightarrow e$ decay. At the present time intense and well collimated beams of π and μ mesons are available in many laboratories, and the ideas suggested by Lee and Yang have been tested rapidly and in many ways. At the beginning of 1957 parity nonconservation in the $\pi \rightarrow \mu \rightarrow e$ decay was already a solidly confirmed fact.[2] These experiments, like the experiment with polarized Co^{60} nuclei in the case of nuclear β decay,[3] extended the domain of phenomena accessible to experimental study and involving parity nonconservation, and confirmed Lee and Yang's hypothesis that parity conservation fails in all processes in which the neutrino takes part.

Among β-decay phenomena, however, the decay of the μ meson into an electron and two neutrinos occupies a special place, owing to the fact that only weakly-interacting particles take part in this decay, whereas the examination of nuclear β decay is complicated by the presence of the strongly interacting nucleons. It is for this reason that there is no renormalization of the interaction constants in the $\mu \rightarrow e$ decay. Thus from the point of view of the theory, the decay of the μ meson into an electron and two neutrinos is the purest case of the interaction of four fermions. With regard to the π and μ decays the theory of the two-component neutrino[4-6] and the theory of the universal Fermi interaction proposed by Feynman and Gell-Mann[7] have made a number of clear predictions. Such experimental problems as the shape of the electron spectrum in the $\mu \rightarrow e$ decay, the existence of $\pi \rightarrow e$ decay, the value of the asymmetry parameter in the $\mu \rightarrow e$ decay, and the polarizations and helicities of the particles produced in $\pi \rightarrow \mu \rightarrow e$ decays have been of critical importance for the new theory of β decay. In a short time new experimental methods have been developed and a large number of experimental papers devoted to these problems have appeared. This work has gone on in parallel with the study of analogous problems in nuclear β decay and has reached fruition in the establishment of interaction types that confirm the validity of the universal Fermi interaction with vector and axial-vector coupling.

New possibilities for experiments have arisen as a result of these studies. For example, it has been found that the beams of μ mesons obtained from accelerators (and even those in cosmic radiation, though to a smaller degree) are polarized. This has made it possible to carry out an experiment of fundamental importance for understanding the nature of the μ meson — the determination of the magnetic moment of the μ meson. The very first experiments of this kind showed[2,78] that the g factor of the μ meson is close to 2, i.e., that the meson is indeed a Dirac particle. The accuracy of the measurements is steadily increasing, and we can expect very soon an experimental solution of the question of the radiative correction to the magnetic moment of the μ meson.

The phenomenon of asymmetry in the $\mu \rightarrow e$ decay has finally made it possible in principle to observe the existence of a new atomic system, muonium — a hydrogenlike atom with a μ meson as its nucleus.

The present survey is devoted to an examination of the experimental data on these phenomena. It is assumed that the reader is acquainted with the main theoretical ideas presented in the papers and lectures of

Lee and Yang,[1,4,8] of Landau,[5] of Feynman and Gell-Mann,[7] and in review papers.[9–12]

Our survey begins with an examination of the values of the masses and lifetimes of π and μ mesons that have been obtained up to the present time. The problem of the mass of the μ meson has taken on special significance in connection with the measurements of its magnetic moment: with the methods of measurement used so far it is just the error in the mass of the μ meson that limits the accuracy of the measurement of the magnetic moment.

We then consider the experimental data on the shape of the electron spectrum in $\mu \rightarrow e$ decay and on the energy dependence of the asymmetry parameter of the spatial distribution of the electrons.

After this we consider the data on the existence of decay of the π meson into an electron and a neutrino: $\pi \rightarrow e + \nu$. The existence of this decay follows from the Feynman-Gell-Mann theory which restricts β-decay interactions to the vector and axial-vector types, and these experiments are crucial for the present β-decay theory. In the same chapter we consider the search for the various radiative decay processes. This problem is a more concrete one since the establishment of the weak-interaction types.

In the last chapter of the survey we consider the question of the value of the asymmetry parameter in various substances. This problem is of significance in at least two ways. First, the theory predicts that the asymmetry parameter has the value $a = -0.33$. Therefore it is important to find out whether there are substances or experimental conditions for which the quantity a actually has this value. Second, the values of the asymmetry coefficient measured in various substances show a large spread, which is due to the depolarization of the μ mesons, and this provides possibilities for finding out about the mechanism of depolarization.

2. THE MASSES AND LIFETIMES OF π AND μ MESONS

We present here the values of the masses and lifetimes of the light mesons as known at the present time. An analysis of the data obtained up to the end of 1956 has been made in references 13 and 14. Since that time there has been practically no change in these data, and in Table I we give the values of the masses from the paper of Crowe.[14]

The mass of the π^0 meson is

$$M_{\pi^0} = (264.37 \pm 0.60).$$

The most precise measurements of the masses of light charged mesons are those described in a paper by Barkas and others; they were made by deflecting mesons in a magnetic field and then measuring their ranges in an emulsion calibrated with protons. The mass of the π^- meson has also been measured by means of determinations of the energy of the monochromatic γ radiation that arises in the capture of

TABLE I

Charge	π meson	μ meson	Method of measurement of mass
+	273.34 ± 0.33	206.93 ± 0.35	Magnetic field and range in emulsion[15]
−	272.8 ± 0.45	206.3 ± 0.47	The same
−	272.74 ± 0.40	206.33 ± 0.41	Spectrum of γ rays[16]
−	273.34 ± 0.13	206.93 ± 0.13	x-radiation of mesic atoms,[17,18]
Weighted average values	273.27 ± 0.11	206.86 ± 0.11	

π^- mesons slowed down in hydrogen,[16] $\pi^- + p \rightarrow n + \gamma$. Another method for accurate mass measurement that can be used for negatively charged mesons is the study of the absorption of the x-radiation of mesic atoms. The energy of this radiation can be determined to an accuracy of a few electron volts if one uses the phenomenon of the sharp decrease of the absorption coefficient that occurs when the energy of the x-ray quantum is smaller than the energy required to remove an electron from the K shell of an atom in the absorber. (The K edge of the absorption curve.) Such measurements of the absorption of the x-radiation of mesic atoms have been made by Koslov and others[17] for μ-mesic atoms and by Stearns and others[18] for π-mesic atoms. In these experiments the mesic-atom x-radiation emitted by the target in which the mesons were stopped was registered by a scintillation spectrometer. The observers placed various absorbers with monotonically varying values of Z between the target and the spectrometer, and found a sharp change of the intensity of a given line on passage through the K edge close to the energy of the given transition. At an energy of about 50 kev the K levels of neighboring elements differ by about 1.7 kev, so that by this method one cannot measure the x-ray energy precisely, but in certain favorable cases, by choosing suitable transitions in suitable mesic atoms, one can fix very precise values of upper and lower limits on such energies, and consequently get limits on the masses of the light mesons.

The passage from the energies of the x-ray quanta to the meson masses requires the taking into account of such factors as the interaction of the meson with the nucleus, the finite size of the nucleus, the electronic screening, and so on. These corrections are extremely small for transitions between far-out orbits in light elements. In such cases the corrections that have been mentioned can be neglected, and the most important correction is that associated with vacuum polarization.[19]

In their work[17] Koslov and others studied the 2p-1s transition in sulphur, the 3d-2p transition in phosphorus, and the 4f-3d transition in silicon, and fixed the following limits on the mass of the μ^- meson:

$$(206.77 \pm 0.04)\, m_e \leqslant M_{\mu^-} \leqslant (208.95 \pm 0.04)\, m_e.$$

The K edge of the absorption curve is known to an accuracy of about a tenth of an electron-volt, and the main error indicated here is due to the theoretical calculation of the vacuum-polarization effects.

By a similar method the following limits on the mass of the π^- meson have been obtained[18]

$$(272.2 \pm 0.30)\, m_e \leqslant M_{\pi^-} \leqslant (273.51 \pm 0.04)\, m_e.$$

The mean lifetimes of the light mesons have been measured repeatedly. The most exact value of the mean lifetime of μ^+ mesons was found by Bell and Hincks[20] by the method of delayed coincidences for the μ mesons of cosmic rays:

$$\tau_{\mu^+} = (2.22 \pm 0.02)\, 10^{-6} \text{ sec.}$$

Recently Dudziak and others[22] have made analogous measurements with an accelerator and obtained

$$\tau_{\mu^+} = (2.21 \pm 0.02)\, 10^{-6} \text{ sec.}$$

The lifetime of μ^- mesons depends on the substance in which they are stopped. Measurements in substances with small Z (< 10), in which the effect of capture can be completely neglected, have shown[21] that the lifetime of the free μ^- meson agrees with that of the μ^+ meson within the 4 percent error of the measurements.

An average of all the data on the lifetime of π mesons, obtained by different methods which do not agree with each other, gives[14]

$$\tau_{\pi^\pm} = (2.56 \pm 0.05)\, 10^{-8} \text{ sec.}$$

The existing data do not give any indication of a difference between the lifetimes of π^+ and π^- mesons.

The most reliable estimate of the lifetime of the π^0 meson, obtained in a paper of Anand,[23] is

$$\tau_{\pi^0} = \left(5.0 \begin{array}{l} +5 \\ -2 \end{array}\right) 10^{-15} \text{ sec.}$$

3. THE SPECTRUM OF THE ELECTRONS FROM $\pi \to \mu \to e$ DECAY AND THE ASYMMETRY OF THE SPATIAL DISTRIBUTION OF THE ELECTRONS

3.1. The Theoretical Spectrum of the Electrons and their Angular Distribution

In this chapter we shall examine the experimental data on the shape of the energy spectrum of the electrons emitted in $\mu \to e$ decay and on the dependence of the asymmetry of the spatial distribution of the electrons on their energy. Before proceeding to the experimental data, let us look at the main conclusions of the theory of β decay as applied to the decay of the μ meson.[4–6,24–29] We consider the decay of a μ meson into an electron, a neutrino, and an antineutrino

$$\mu \to e + \nu + \bar{\nu}.$$

As is well known from the theory of β decay, the requirement of relativistic invariance of the interaction has the result that the Hamiltonian describing the interaction of the four fermions that take part in a decay (μ, e, ν, $\bar{\nu}$ in the case of μ decay, and n, p, e, and ν in nuclear β decay) is the sum of five terms, corresponding to the scalar, pseudoscalar, tensor, vector, and axial-vector interaction types

$$H_{\text{int.}} = \sum_j \psi_e O_j \psi_\mu \{C_j (\bar{\varphi}_\nu O_j \varphi_\nu)$$
$$+ C'_j (\bar{\varphi}_\nu O_j \gamma_5 \varphi_\nu)\} + \text{Herm. adj. terms} \qquad (1)$$

where φ_ν is the field operator that describes the annihilation of a neutrino or the creation of an antineutrino, and $\bar{\varphi}_\nu$ is the operator that describes the annihilation of an antineutrino or the creation of a neutrino; the electron and meson field operators ψ_e and ψ_μ have analogous meanings. Each such operator is a four-component Dirac spinor, and between each pair of spinors there is an operator O_j (or $O_j\gamma_5$ in the terms that do not conserve parity). The operator O_j has the following values:

$$O_j = \begin{cases} 1 & \text{— scalar interaction} \\ \gamma_\mu & \text{— vector} \\ \frac{1}{2i}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu) & \text{— tensor} \\ i\gamma_\mu\gamma_5 & \text{— axial vector} \\ \gamma_5 & \text{— pseudoscalar,} \end{cases}$$

where γ_ν are the Dirac matrices, and $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$.

In the expression (1) C_j and C'_j are the interaction constants corresponding to the five interaction types we have listed. The constants C_j correspond to the parts of the interaction that do not involve parity *nonconservation, and* C'_j *to the parts that do involve* it. Thus in its most general form the β decay of the μ meson is determined by a set of ten complex interaction constants. If we neglect the mass of the electron ($m_e = 0$), the Hamiltonian (1) gives the following spectrum shape and angular distribution for the electrons from $\pi \to \mu \to e$ decay:

$$R(x, \Omega)\, dx\, d\Omega = \frac{1}{\tau}\, 12x^2 dx\, \frac{d\Omega}{4\pi} \Big\{(1-x) + \frac{2}{9}\varrho(4x-3)$$
$$\mp \cos\vartheta \Big[\alpha(1-x) + \frac{2}{9}\xi(4x-3)\Big]\Big\}. \qquad (2)$$

In this formula, which gives the decay probability of the μ meson referred to unit time, x is the energy of the electron expressed as a fraction of the maximum energy that it can receive in the decay ($0 < x < 1$); ϱ (the Michel parameter), α (the asymmetry parameter), and ξ (the Michel asymmetry parameter) are three parameters that depend in a definite way on the five pairs of coupling constants that appear in the Hamiltonian (1).

The first term of the formula (2), which does not contain $\cos\vartheta$, gives the spectrum of the decay electrons

$$R(x)\, dx = \frac{1}{\tau}\, 12\, x^2 dx\, \Big\{(1-x) + \frac{2}{9}\varrho(4x-3)\Big\}.$$

The second term, which occurs because of parity nonconservation, is proportional to

$$\mp \cos\vartheta \left[\alpha(1-x) + \frac{2}{9}\zeta(4x-3) \right]$$

and gives the asymmetry of the spatial distribution of the electrons. Here ϑ is the angle between the direction of the spin of the μ meson occurring in the $\pi \rightarrow \mu \rightarrow e$ decay and the direction of the momentum of the electron at the instant of the $\mu \rightarrow e$ decay; the signs $-$ and $+$ are for μ^- and μ^+ mesons, respectively.

The family of curves in Fig. 1 shows the spectrum of the decay electrons for various values of the Michel parameter ϱ. These curves intersect at $x = 3/4$ and give nearly equal values of the average energy of the decay electrons; these values lie in the range $x = 0.6 - 0.7$. It can be seen from Fig. 1 that the parameter ϱ characterizes the hardness of the spectrum: the smaller the value of ϱ, the more slow electrons there are in the spectrum. By measuring the spectrum of the decay electrons experimentally and comparing it with the spectra in Fig. 1 one can determine the value of the parameter ϱ. We have seen, however, that this parameter represents only one relation imposed on the ten unknown coupling constants that describe the $\mu \rightarrow e$ decay, and therefore a measurement of the value of ρ gives very little toward the determination of the constants C. For example, in the general case, if all five interaction types are involved in decay by the scheme $\mu \rightarrow e + \nu + \bar{\nu}$, the value of ϱ lies between 0 and 1, and if two identical neutrinos are emitted ϱ can lie between 0 and $3/4$.

The situation is much simpler in the two-component theory of the neutrino, in which, for $\mu \rightarrow e + \nu + \bar{\nu}$ decay, the ten parameters reduce to the two constants C_V and C_A. In fact,[4,6,9] this theory can be regarded as a special case of the four-component theory, if we set $C_S = C'_S = C_P = C'_P = C_T = C'_T = 0$, $C'_V = -C_V$, $C'_A = -C_A$. In the two-component theory of the neutrino the Michel parameter must have the value $\varrho = 3/4$ if there are neutrinos and antineutrinos, and $\varrho = 0$ if these particles are indistinguishable, i.e., for decay by the scheme $\mu \rightarrow e + 2\nu$ or $\mu \rightarrow e + 2\bar{\nu}$.

Thus the study of the shape of the spectrum is a critical experiment for the two-component theory: if

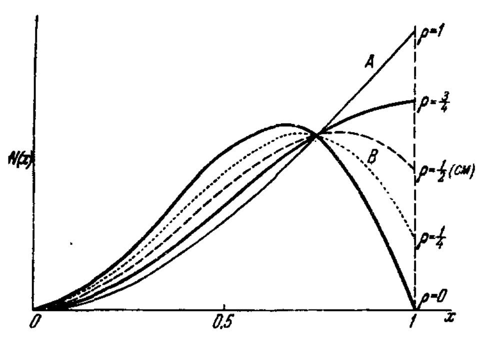


FIG. 1. Spectrum of the electrons from $\mu \rightarrow e$ decay for various values of the Michel parameter ρ. The abscissa is the energy x, and the ordinate is the decay probability.

this theory is correct, we must have either $\varrho = 0$ ($\nu = \bar{\nu}$) or $\varrho = 3/4$ ($\nu \neq \bar{\nu}$). Of course, if it is found experimentally that $\varrho = 0.75$ or $\varrho = 0$, this is by no means sufficient to establish the correctness of the two-component theory and fix the choice between the two possibilities $\mu \rightarrow e + \nu + \bar{\nu}$ and $\mu \rightarrow e + \nu + \nu$. In fact, these values of ϱ can be obtained by infinitely many combinations of the coupling constants C in the four-component theory also.

We shall now consider the experiments in which the spectrum of the decay electrons has been measured and the parameter ϱ has been determined.

3.2. The Spectrum of the Electrons from $\mu \rightarrow e$ Decay. Determination of the Parameter ϱ

The first measurement of the spectrum of positrons from $\mu \rightarrow e$ decay was made by Leighton, Anderson, and Seriff.[30] These authors worked with μ^+ and μ^- mesons in cosmic radiation and used the method of the "falling" Wilson chamber in a magnetic field. It was shown in this experiment that the energies of the decay electrons form a continuous spectrum extending up to 50 – 60 Mev. A similar conclusion was reached by G. B. Zhdanov and A. A. Khaĭdarov[31] and by Steinberger[32] on the basis of analyses of absorption curves of the electrons from decays of cosmic-ray μ mesons. It could be concluded from the continuous character of the spectrum that more than two particles are produced in the decay. The energy carried away by all the decay electrons was close to $1/3$ of the total energy released in the decays, so that it was highly probable that the decay leads to three light and weakly interacting particles.

Experiments by Hincks and Pontecorvo[33] and by Sard and Althaus[34] showed that the uncharged particles produced in $\mu \rightarrow e$ decay are not γ-ray quanta.

In subsequent years the shape of the spectrum of decay positrons was studied by many authors. Most of the early papers, however, are characterized by insufficient statistical accuracy and poor energy resolution. Moreover, in these papers there was a sizable systematic error in the determination of the parameter ϱ, which was due to the fact that the mass of the μ^- meson, and consequently also the limit of the spectrum, were not accurately enough known in those years. It is convenient to consider the measurements of the shape of the spectrum under the headings of the methods used.

The Emulsion Method. In this method the energies of the electrons from $\mu \rightarrow e$ decay are found from their multiple scattering in the Coulomb fields of the nuclei in the emulsion. Under the conditions usually chosen for the measurements (layers of emulsion of thickness 400 to 1000μ) the tracks of the electrons have minimum length 1 – 1.5 mm, fixed by the possibility of dividing a track into 15 to 20 independent segments of 50 – 100μ each, and the maximum track length, limited by emergence of the electrons from the thin layer of emulsion because of the scattering,

TABLE II. Summary of data on measurements of the Michel parameter ρ

Authors	Year	Source of mesons	Method	Sign of charge	Number of cases	Result, reduced to $m_\mu = 209.6$
Leighton et al.[30]	1949	Cosmic rays	Wilson chamber in magnetic field	+ and −	75	0.4
Bramson et al.[35,36]	1952	Accelerator	Emulsion	+	301	0.48 ± 0.13
Bonetti et al.[37]	1956	Cosmic rays	Emulsion	+ and −	506	0.57 ± 0.14
Besson et al.[38]	1958	Accelerator	Emulsion	+	915	0.67 ± 0.14
Vaĭsenberg[142]	1959	Accelerator	Emulsion	+ and −		0.72 ± 0.10
Vilain et al.[40]	1954	Accelerator	Wilson chamber in magnetic field	+	280	0.50 ± 0.13
Rosenson[39]	1958	Accelerator	Diffusion chamber in magnetic field	+	1300	0.67 ± 0.05
Sargent et al.[41]	1955	Accelerator	The same	−	415	0.64 ± 0.10
Crowe et al.[42]	1956	Accelerator	Double-focusing spectrometer	+		0.62 ± 0.03
Dudziak et al[44]	1959	Accelerator	Spiral-orbit spectrometer	+		0.74 ± 0.03
Anderson et al.[43]	1959	Accelerator	Spectrometer	+		0.74 ± 0.03

is rarely larger than 5 mm. Under these conditions the statistical accuracy of an energy measurement by the multiple-scattering method lies in the range from 8 to 20 percent, depending on the track length and the electron energy. Besides this dispersion in the measurements, in comparing the measured spectrum with theory one must remember the radiation losses of the electrons in the emulsion. The radiation length in the emulsion is close to 3 cm, and the fluctuation of these losses begins to have an effect comparable with the dispersion of the scattering measurements for track lengths of a few millimeters.

The values of the parameter ϱ obtained by the photographic-emulsion method are shown at the beginning of Table II (lines 2 to 5).

It must be noted that the parameter ϱ can also be determined not from the whole spectrum but from the behavior of the spectrum in the low-energy region. In the emulsion method this way of determining ϱ has a decided advantage over the ordinary analysis of the whole spectrum. Indeed, in the first place the low-energy region is most sensitive to the value of this parameter. From the spectrum of the decay positrons it is easily found [Eq. (2), Fig. 1] that if ϱ changes from 0 to 0.75 the relative number of low-energy particles ($x < 0.3 - 0.4$) changes by approximately a factor two, whereas the characteristics of the spectrum that depend on the larger energies change much more weakly with ϱ. For example, for the same limits of variation of ϱ the average energy of the decay electrons changes only from 0.6 to 0.7. A second circumstance that favors the determination of the parameter ϱ from the behavior of the spectrum in the low-energy region is that this energy range is subject to the least distortion by the dispersion of the scattering measurements and by bremsstrahlung in the emulsion itself, so that the corrections required are small. Finally, measurements in the low-energy region have an advantage in principle, though one that is far from having been realized because of the small statistics, in that the radiative corrections to the spectrum manifest themselves most strongly in the low-energy region, and since these corrections are extremely sensitive to the type of interaction, measurements of the beginning of the spectrum could give additional information about the coupling constants in $\mu \rightarrow e$ decay. In reference 142 the spectrum was measured for 1099 decay electrons, and all the data on the behavior of the spectrum in the low-energy region ($x < 0.3 - 0.4$) by the emulsion method were examined. These data are in good agreement with each other. After the introduction of small corrections for the "flat-chamber effect," which means that because of scattering in the medium the observed number of low-energy positron tracks is too small relative to the number of tracks of positrons of higher energies, and for the smearing out of the spectrum by errors of measurement, the value obtained for the Michel parameter was $\varrho = 0.72 \pm 0.10$.

Chambers in Magnetic Fields. The accuracy of measurements of the energies of decay electrons can be much improved as compared with the emulsion method if one uses a Wilson cloud chamber or a bubble chamber in a magnetic field of several kilogauss. By selecting decay-electron tracks several centimeters long one can measure momenta with a standard error of 3 to 5 percent. One of the researches of this type is that of Rosenson.[39] A diffusion chamber filled with hydrogen at pressure 20 atm and placed in a magnetic field of 7000 gauss was exposed to a beam of slow μ mesons. On the average the chamber registered about two stoppings of μ mesons in a pulse containing 10 to 20 mesons. The most essential criterion for the selection of positron tracks was that their length should exceed 5 cm. On the average there was one positron track suitable for measurement in every 13 exposures. From 25,000 pictures there were selected 1300 positrons, which make up the spectrum shown in Fig. 2. The abscissas are values of the momentum x, expressed as a fraction of the maximum momentum $P_{max} = 52.8$ Mev/c, and the ordinates are numbers of particles in intervals $\Delta x = 0.025$. A statistical analysis shows that the value of ϱ that fits best with these data is $\varrho = 0.67 \pm 0.05$.

The spectrum of decay electrons has also been measured in a hydrogen bubble chamber in a field of 8000 gauss, in a search for electronic decay of the π

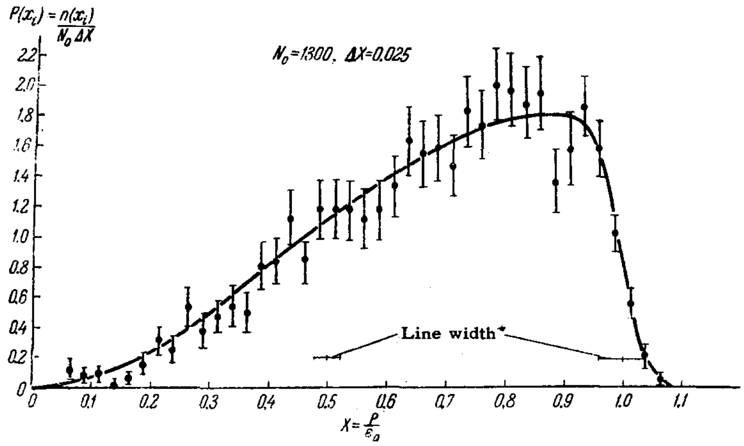


FIG. 2. Spectrum of electrons from $\mu \to e$ decay, according to measurements by Rosenson with a Wilson chamber in a magnetic field. Abscissas are energies x, ordinates are numbers of positrons in energy intervals $\Delta x = 0.025$.

meson.[92] This spectrum is shown in Fig. 27, and a description of the apparatus is given on page 214. It is seen from Fig. 27 that this spectrum is in good agreement with a value of ϱ close to 0.75.

Magnetic Spectrometers. Finally we must consider the data obtained by means of magnetic spectrometers.[42–44] At the Sixth Rochester Conference in 1956 work by Crowe and others was reported.[42] These authors worked with a double-focusing magnetic spectrometer, which had first been tested and calibrated in measurements on the scattering of electrons. They obtained $\varrho = 0.62 \pm 0.05$. In a search for $\pi \to e$ decay Anderson and others[43] carefully measured the spectrum of $\mu \to e$ positrons. Their apparatus is described in Chapter 6, and Fig. 30 shows the spectrum they obtained, which agrees with the value $\varrho = 0.74 \pm 0.03$. Dudziak, Sagane, and Vedder,[44] working with a spectrometer with a spiral orbit, obtained the value $\varrho = 0.741 \pm 0.027$.

Examining the data of Table I, we arrive at the conclusion that with the exception of the first papers the values of ϱ are in fairly good agreement with each other; and the small values of the parameter ϱ obtained in the early work are to a considerable extent explained by the unreliable value of the mass of the μ meson, which was not well known at that time. Averages of the values of ϱ obtained by the three methods we have discussed are shown in Table III.

We see that the average values of ϱ obtained by the three methods lie in the range 0.65 – 0.70. At any rate it follows from these data that the probability that ϱ is larger than $^3/_4$ is negligibly small.

Some reduction of the experimentally observed parameter ϱ is due to the radiation effects. They increase considerably the intensity of the spectrum in the low-energy region, and decrease it in the high-energy region. This softening of the spectrum manifests itself in a decrease of the effective value of ϱ calculated from a comparison of Eq. (2) with the experimental data. Calculations by Behrends et al.,[45] V. Kuznetsov,[46] Kinoshita and Sirlin,[47] and Berman[48] show that radiation effects can diminish $\varrho = 0.75$ by 5 to 6 percent, i.e., reduce it to 0.71. It must further be remembered that in the majority of experimental methods there is always some tendency to diminish the measured energy. Therefore, despite the difference between $\varrho = 0.75$ and the measured values of ϱ, at the present time it can be supposed that the experimental data we have examined do not contradict the demand of the two-component theory that ϱ have the value 0.75 for decay into an electron, a neutrino, and an antineutrino.

TABLE III. Values of the Michel parameter ρ

Photographic emulsion	Chambers in magnetic fields	Magnetic spectrometers
0.67 ± 0.07	0.65 ± 0.04	0.70 ± 0.02

3.3. The Asymmetry of the Spatial Distribution of the Electrons from $\pi \to \mu \to e$ Decay

In this section we consider the data on the asymmetry of the spatial distribution of the electrons from $\pi \to \mu \to e$ decay. Together with the experiments on the β decay of Co^{60}, the discovery of this asymmetry was the basic experiment that demonstrated parity nonconservation in the weak interactions. Here we shall consider the question of the dependence of the asymmetry on the energy. The problem of the value of the asymmetry coefficient, which is connected with the problem of depolarization, will be considered in Chapter 7.

As we have seen earlier, the nonconservation of spatial parity has the result that the decay probability depends on the cosine of the angle ϑ between the directions of the spin of the μ meson and the momentum of the decay electron. In the theory of the two-component neutrino[4–6] the expression (2) for the angular dependence of the decay probability is simplified and takes the following form:

$$R(x, \Omega)\, dx\, d\Omega \doteq \frac{1}{2\pi} x^2 [(3 - 2x) \mp \xi (1 - 2x) \cos \vartheta]\, dx\, d\Omega. \tag{3}$$

In this formula ξ is a parameter that depends on the coupling constants C_V and C_A in the following way:

$$\xi = \frac{C_V C_A^* + C_A C_V^*}{|C_V|^2 + |C_A|^2}. \qquad (4)$$

We note that for the μ^- meson with the same spin direction the coefficient ξ in Eq. (3) is taken with the other sign. Since the direction of the spin of the μ^- meson is opposite to that of the spin of the μ^+ meson, this means that if we observe experimentally a definite spatial asymmetry for the μ^+ decay, the sign of this asymmetry does not change if we go over from μ^+ mesons to μ^- mesons: if the positrons from $\mu^+ \to e^-$ decay emerge mainly backward, the same will be true for the electrons from $\mu^- \to e^-$ decay.

It follows from Eq. (4) that the possible values of ξ lie between -1 and $+1$. In particular, if $|C_V| = |C_A|$, i.e., if the decay of the μ meson is caused by only the vector and axial-vector types of interaction with equal magnitudes of the constants, as is the case in the theory of Feynman and Gell-Mann, then $|\xi| = 1$.

Let us examine some consequences of Eq. (3) that are convenient for comparison of experimental data with the theory.

1. The asymmetry coefficient for electrons of a given energy is given by

$$a(x) = \xi \frac{2x-1}{3-2x}. \qquad (5)$$

For $x = 1$, we have $a(1) = \xi$; as x decreases, $a(x)$ decreases to 0 in the center of the spectrum (at $x = 1/2$), and with further decrease of the energy it changes sign, and at low energies reaches the value

$$a(0) \cong -\frac{1}{3} \text{ (sic) (cf. Fig. 3)}$$

2. The asymmetry coefficient for the electrons with energies less than a given energy is

$$a(<x) = \frac{\xi}{2} \frac{(-2x^2+3x)}{2x^3-x^4}. \qquad (6)$$

From this it follows that in the Feynman–Gell-Mann theory the asymmetry coefficient averaged over the entire spectrum has the value: $a(<1) = -1/3$.

3. The asymmetry coefficient for the electrons with energies greater than a given energy is

$$a(>x) = \frac{\xi}{3}\left(\frac{1+x+x^2+3x^3}{1+x+x^2-x^3}\right).$$

Graphs of these functions are shown in Fig. 3.

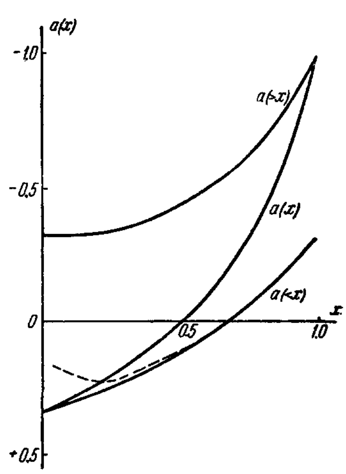


FIG. 3. Asymmetry coefficients for $\pi \to \mu \to e$ decay in the two-component theory of the neutrino. a(x) is the asymmetry coefficient for the positrons with energy x; a(<x), for those with energies from 0 to x; and a(>x), for those with energies from x to 1.

The values quoted for the coefficients $a(x)$, $a(>x)$, and $a(<x)$ are those obtained in the theory that does not take into account radiative corrections. The appropriate corrections have been calculated in papers by Kinoshita and Sirlin[47,49] and by V. Kuznetsov.[46] On the graph of $a(<x)$ the dashed part shows the quantity $a(<x)$ calculated with the radiative corrections included.[46,47] We see that the radiative corrections are important in the low-energy region, where they decrease the asymmetry coefficient by 30 to 40 percent, but do not affect the asymmetry coefficient at high energies.

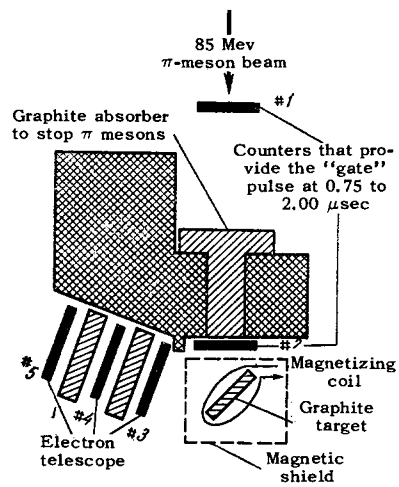


FIG. 4. Scheme of the experiment of Garwin, Lederman, and Weinrich,[2] made to study parity conservation in $\pi \to \mu \to e$ decay.

The first experiment in which the spatial asymmetry in $\pi \to \mu \to e$ decay was observed was that of Garwin, Lederman, and Weinrich.[2] The scheme of this experiment is shown in Fig. 4. The beam of monochromatic π^+ mesons, brought out from the accelerator in the usual way, contained about 10 percent μ^+ mesons, which mainly came from the decay of the π mesons in flight, near the cyclotron target and in the whole course of their passage through the deflecting and collimating system. The separation of the π and μ mesons was made by the usual method of absorption in a graphite block, which stopped the π^+ mesons but let through μ^+ mesons with the same initial momentum. The μ mesons were finally stopped in a graphite target. The incidence of a μ meson on the target was registered by a coincidence pulse from the scintillation counters 1 and 2. The subsequent $\mu \to e$ decay in the graphite target was detected by an electron telescope of two scintillation counters (3 and 4). The diagram shows a telescope of three counters 3, 4, and 5; this corresponds to a later variation of the experiment, in which the dependence of the asymmetry on the energy of the decay positrons was measured.

The front of the coincidence pulse from counters 1 and 2 was shifted by 0.75 μsec and opened an electronic circuit for 1.25 μsec. Thus the system counted electrons that emerged from the graphite target in the time interval from 0.75 to 2.00 μsec after the stopping of a meson in the target. Let us assume that the μ mesons are polarized in the direction of their motion. The problem in the experiment is to look for an asymmetry of the spatial distribution of the electrons relative to this direction, characterized [cf. Eq. (1)] by the angle ϑ between the directions of motion of the μ meson and emergence of the decay electron. As can be seen from the diagram, in the absence of any magnetic field at the carbon target the electron telescope registers electrons for which ϑ is close to 100°. If a current flows through the coil shown in Fig. 4, then in the constant magnetic field H perpendicular to the plane of the drawing the spin of the μ^+ meson will precess at the frequency $\omega = geH/(2m_\mu c)$ rad/sec (g is the gyromagnetic ratio for the μ meson and m_μ is its mass), and the angular distribution will be turned along with the spin around the direction of the μ-meson beam. Thus the angular distribution of the decay electrons can be obtained with unchanged geometry of the experiment by varying the current in the winding; to each value of the current there corresponds a certain mean angle of rotation of the μ-meson spin. The experimental curve obtained with a graphite target is shown in Fig. 5. The current in the magnetizing coil is plotted as abscissa (the maximum field intensity in the region of the target was 50 gauss), and the ordinate is the ratio of the number of counts N to the number of counts in the same time interval without the magnetic field. The solid curve corresponds to the theoretical distribution of the type $1 + a\cos\vartheta$ with $a = -1/3$ and with the gyromagnetic ratio of the μ^+ meson taken equal to $+2.00$. The theoretical curve takes into account the solid angle of the telescope that registered the electrons and the exponential decrease of the number of μ^+ mesons during the time the gate is open. We see that the experimental data are in excellent agreement with this curve; this led the authors to a number of conclusions that are now well known:

1. There is parity nonconservation in both the $\pi \to \mu$ and the $\mu \to e$ decays.
2. There is a very strong asymmetry in the distribution for mesons decaying in graphite, close to the maximum asymmetry $a = -1/3$ predicted by the theory.
3. The gyromagnetic ratio for the μ meson equals the value of g for a particle obeying the Dirac equation.

Subsequent experiments made in many laboratories greatly extended these results and improved their accuracy. These experiments included studies of the dependence of the asymmetry coefficient on the substance in which the μ mesons are stopped and of its variation with the positron energy, and measurements of the magnetic moment of the μ meson.

We shall return to these experiments in the appropriate parts of this survey. Before further discussion let us examine the qualitative explanation of the Lederman experiment from the point of view of the theory of the two-component neutrino.

As is well known, in the theory of the two-component neutrino the spin of the neutrino must be directed parallel (or else antiparallel) to its momentum. In accordance with this we introduce a new quantum number for the neutrino, its helicity H, which has the value $H = +1$ or $H = -1$, depending on which of these two possibilities exists in reality. By means of the idea of the helicity of the neutrino the phenomenon of polarization in $\pi \to \mu$ decay and of asymmetry in the subsequent $\mu \to e$ decay can be intuitively illustrated. Since the spin of the π meson is zero, $l = 0$, and the spin of the neutrino is parallel or antiparallel to its momentum, it follows from the law of conservation of spin that the μ meson emitted in the direction opposite to the neutrino must have the same helicity as the neutrino, $H_\mu = H_\nu = 1$ or else $H_\mu = H_\nu = -1$ (Fig. 6).

In the subsequent decay of the μ meson

$$\mu \to e + \nu + \bar{\nu}$$

we confine ourselves to a discussion of the extreme case in which the electron receives the maximum energy possible in the decay. Then the neutrino and antineutrino have equal momenta, and the momentum of the electron is opposite to them. The total angular momentum carried away by the neutrino and antineutrino emitted in the same direction is zero (the helicity of the antineutrino is opposite to that of the neutrino), and there remains for the electron the whole spin of the μ meson. But the electron, just like the neutrino, has a longitudinal polarization, i.e., a definite helicity. Therefore the direction of its emission can only be colinear with the direction of emission of the μ meson in the $\pi \to \mu$ decay: it will be in this direction if the helicities of the μ meson and the electron are the same, and will be in the opposite direction

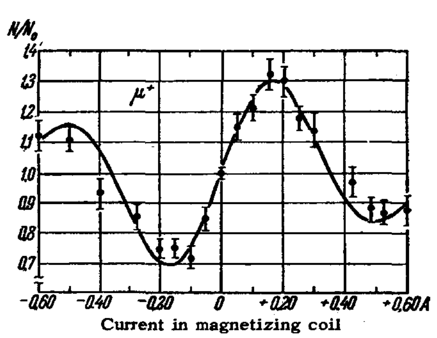


FIG. 5. Asymmetry of the spatial distribution of the electrons from $\pi \to \mu \to e$ decay in graphite. The abscissa is the current in the magnetizing coil, and the ordinate is the number of counts in the electron telescope.

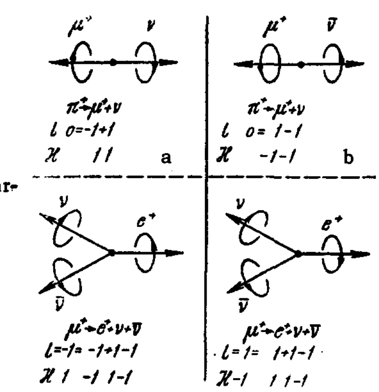


FIG. 6. Helicities of particles in $\pi \to \mu \to e$ decay.

if the electron has the helicity opposite to that of the μ meson.

Thus the helicities of the light particles produced in the decay give an intuitive explanation of the appearance of an asymmetry.

The experiment of Garwin et al.[2] showed that the positron from the $\mu \to e$ decay is emitted in the direction opposite to the direction of emission of the μ meson in the preceding $\pi \to \mu$ decay. From this it at once follows that its helicity is opposite to that of the μ meson, and thus also to that of the neutrino that is produced in the $\pi \to \mu$ decay.

The experiment of Garwin et al. does not, however, enable us to make a choice between the possibilities

$$\pi^+ \to \mu^+ + \nu \text{ and } \pi^+ \to \mu^+ + \bar{\nu},$$

shown in Fig. 6, and to solve the problem of what particle is emitted in the $\pi \to \mu$ decay, a neutrino or an antineutrino we need a direct measurement of the helicities of the particles produced in β decay. We shall return to this problem in the chapter on the measurement of the helicities of the light particles.

Now let us examine the investigations of the energy dependence of the asymmetry coefficient.

Experiments Made by Electronic Methods Using Counters and Magnetic Spectrometers. It is well known that the first experiment of Garwin et al. did not confirm the theoretically predicted sharp increase of the asymmetry coefficient with increasing energy. For example, when they placed between the counters 3, 4 of the electron telescope (Fig. 4) absorbers corresponding to electron ranges of 8 g/cm^2 (electrons of energy $E_e > 25$ Mev) and 16 g/cm^2 ($E_e > 35$ Mev), the authors found practically no increase of the ratio of maximum to minimum; in the former case it was 1.86 ± 0.20, and in the latter, 1.92 ± 0.19, instead of the expected 2.5. The reason for such a weak increase of the asymmetry coefficient in these measurements is

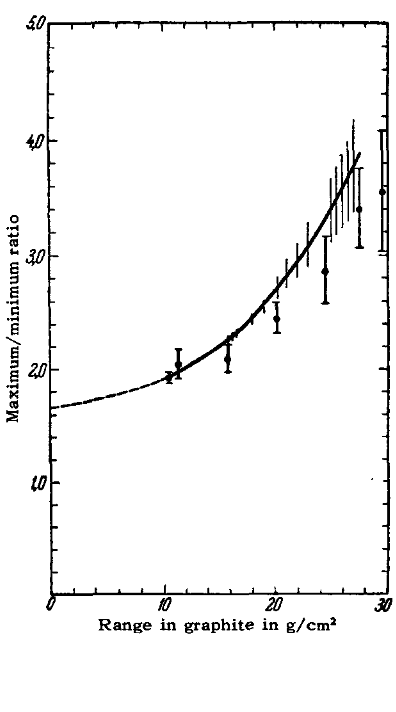


FIG. 7. Energy dependence of the asymmetry in $\pi \to \mu \to e$ decay according to the data of Weinrich.[50,143]

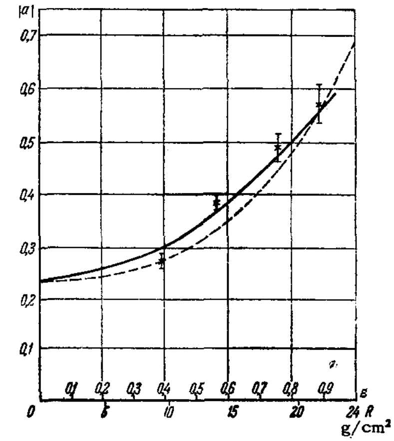


FIG. 8. Energy dependence of the asymmetry in $\pi \to \mu \to e$ decay according to the data of Mukhin, Ozerov, and Pontecorvo.[51]

that the electrons emit γ-ray quanta in the absorber. The γ-rays so produced are registered by the counters through secondary effects, mainly the Compton effect. This deceptively increases the apparent range of the slow electrons and smears out the growth of the asymmetry coefficient. In a second paper by the Columbia group[50] measures were taken against the registration of electrons through their bremsstrahlung: the number of scintillation counters in the telescope was increased to three (see Fig. 4) and the graphite absorber was divided into two parts. Mukhin, Ozerov, and Pontecorvo[51] have gone still further in this direction, using an apparatus in which the electron telescope consisted of 5 rows of scintillation counters, between which there were 4 layers of polyethylene absorbers. The results of these authors are shown in Figs. 7 and 8, where the abscissas are absorber thicknesses in g/cm^2 and the minimum electron energies corresponding to these thicknesses, and the ordinates are the measured values of the asymmetry; in the first case these values are just the ratios of maximum to minimum intensities on curves like that of Fig. 5, and in the second case the ordinates are actual values of the asymmetry coefficient. The solid curves on the two figures give the theoretical behavior of the asymmetry according to the theory of the two-component neutrino, corrected for the variations of the efficiency of the detecting system with angle and energy. As can be seen from these curves, the experimental data agree with the theory of the two-component neutrino.

Kruger and Crowe[52] have studied the energy dependence of the asymmetry coefficient by a magnetic-spectrometer method. The scheme of their experiment is shown in Fig. 9. The beam of μ mesons, previously freed from π mesons by absorption, was stopped in a target which was inclosed together with Helmholz coils in a magnetic shield. When there was no current in the coils the μ mesons retained their polarization and the electrons reaching the magnetic spectrometer corresponded to "forward" emission, i.e., to the minimum intensity I(0). When the Helmholtz coils were turned on there was a vertical field of 55 gauss, which made the μ mesons precess and produced complete depolarization. The intensity I(H) with the field H turned on corresponded to the symmetrical term of

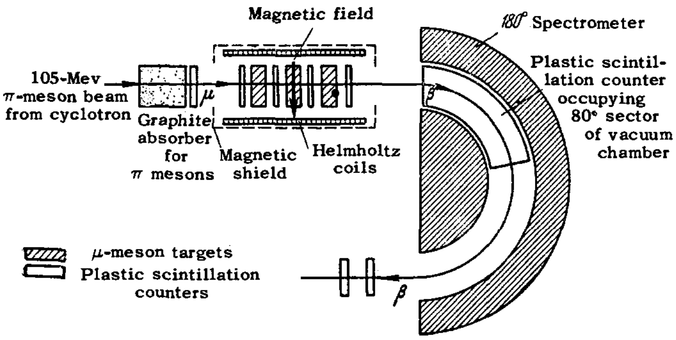


FIG. 9. Experiment of Kruger and Crowe for measuring the energy dependence of the asymmetry in $\pi \to \mu \to e$ decay by means of a magnetic spectrometer.

Eq. (1). Knowing I(0) and I(H) and the energy resolution of the apparatus, one can calculate the asymmetry for a given energy registered by the spectrometer. The energy dependence a(x) obtained in this work is shown in Fig. 10. It shows good agreement with the theory of the two-component neutrino.

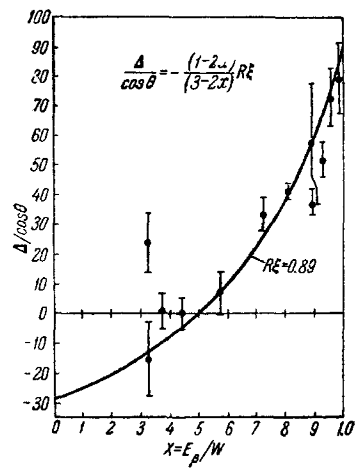


FIG. 10. Dependence a(x) of the asymmetry coefficient on the energy, according to the data of Kruger and Crowe. Solid line is a(x) according to the theory of the two-component neutrino.

The energy dependence of the asymmetry coefficient has also been measured in a paper by Cassels et al.[53] (Liverpool group). To determine the energy of the decay electrons these authors used a large NaI crystal of diameter 15 cm and length 12.5 cm, in which there was practically complete absorption of the electrons (Fig. 11) or their secondary radiation. This scintillation spectrometer distinguishes this apparatus from another apparatus used by the Liverpool group of investigators, in which measurements were made on the asymmetry of the decay in various substances and on the magnetic moment of the μ meson (cf. Fig. 35).

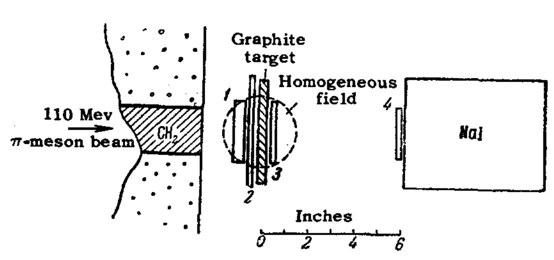


FIG. 11. The experiment of Cassels et al.[53] to measure the energy dependence of the asymmetry in $\pi \to \mu \to e$ decay.

The ratio of the asymmetry coefficients measured in this experiment for the cases in which the scintillation spectrometer registers electrons with energy larger than 48 Mev and for all the incident decay electrons is 2.79 ± 0.39, whereas after the introduction of corrections for the conditions of the experiment the theory of the two-component neutrino calls for this ratio to be 2.32 ± 0.06. Thus also for the results obtained by this method we can assert that there is agreement with the two-component theory.

Experiments Made with the Emulsion Method. In the study of the energy dependence of the spatial asymmetry of the electrons the emulsion method has a number of advantages over other methods. First, it has extraordinarily good angular resolution: the angle between the directions of the μ meson and the decay electron can easily be measured with an accuracy of about 1 percent. Second, the track of the electron in the emulsion can be seen from the point of the decay to the point where it passes out of the emulsion. This means that there is no "dead" space, as there is in experiments with targets in which the μ mesons are absorbed, and there is no lower limit on the energy that can be observed; the emulsion method allows the registration of the entire spectrum of the decay electrons, beginning at a few Mev, with practically the same efficiency. Third, the energy of a decay positron in the emulsion is determined by a direct method, based on its Coulomb scattering in the fields of the nuclei, whereas in experiments by electronic methods (without magnetic analysis) the energy is determined from the absorption, by a comparatively indirect method. We may add to this that the experiments we have considered hitherto, made by electronic methods, have required many tens and hundreds of hours of operation of the accelerator, whereas the corresponding exposures of emulsions can be made in hours. The first measurements of the energy dependence of the asymmetry coefficient were made in a research by Vaĭsenberg and Smirnit-skiĭ.[54] Figure 12 shows the data obtained in this work. In this diagram the abscissas are values of the energy x, and the ordinates are values of the asymmetry coefficient determined from mean values of $\cos\vartheta$, where ϑ is the angle between the projections on the plane of the emulsion of the directions of emission of the μ meson and the electron. The solid curve (1) gives the theoretical energy de-

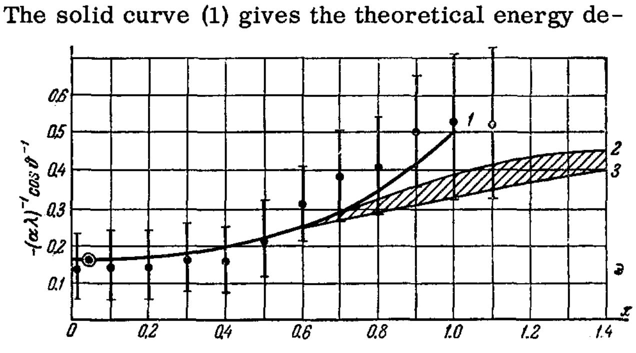


FIG. 12. The integrated asymmetry coefficient a(>x) as obtained by emulsion measurements.[54]

pendence of the asymmetry coefficient; the shaded region (2 and 3) corresponds to the theoretical curve as smeared out by the dispersion of the observations (the upper curve is for the smallest dispersion, the lower for the largest). This diagram shows a rapid rise of the integrated coefficient with increasing energy, which is in agreement with the predictions of the two-component theory. Similar results have been obtained by Babayan and others,[55] by Castagnoli and others,[56,57] and by Besson and Brisson-Fouche.[38]

It has been possible to obtain much more accurate data by placing the emulsions in a strong magnetic field, which removes the depolarization and thus increases the observed asymmetry coefficient (see Chapter 7). Figure 13 shows the differential spectrum of values of the asymmetry coefficient, measured in an emulsion placed in a magnetic field of 17,000 gauss.[58] Here the asymmetry coefficient was found to rise from $a \cong -0.08 \pm 0.01$ to $a = 0.28 \pm 0.02$ (see Chapter 7). In Fig. 13 the abscissas are positron energies and the ordinates are measured values of the asymmetry coefficient in the respective energy intervals. The dashed curves show the theoretical energy dependence predicted by the two-component theory, as smeared out by the extreme conditions of the measurements; the experimental data are in agreement with the theory of the two-component neutrino.

The energy dependence of the asymmetry coefficient has also been studied in bubble chambers, by measurements of the multiple scattering of the positrons.[59-61] The data obtained by this method are also in qualitative agreement with the theory of the two-component neutrino.

3.4. The Asymmetry of the Spatial Distribution of the Positrons in the Low-energy Region

The theory of the two-component neutrino calls for a very special behavior of the asymmetry coefficient in the low-energy region (cf. Fig. 3): at energies close to zero the asymmetry coefficient $a(x)$ is equal in magnitude and opposite in sign (i.e., it is positive) to the mean asymmetry coefficient a (< 1), and de-

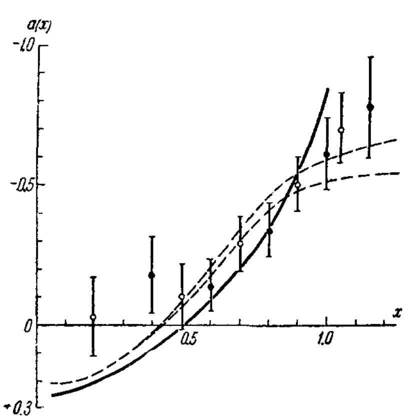


FIG. 13. The energy dependence of the asymmetry a(x) as found from measurements in an emulsion placed in a strong magnetic field.[58]

creases as the energy increases. In the center of the spectrum, at $x = 1/2$, the asymmetry coefficient vanishes, $a(x) = 0$, and the integrated coefficient $a(< x)$ vanishes at $x = 2/3$. A test of the theory in the low-energy region is difficult, because low-energy electrons are scantily represented in the spectrum: at low energies the number of particles with energies *smaller than* x *is proportional to* x^3. From the data given above on the energy dependence of the asymmetry coefficient it indeed follows that the asymmetry vanishes at the center of the spectrum. In order to observe the expected change of sign of the asymmetry at the very lowest energies it is necessary to examine measurements made by the emulsion method or with bubble chambers, where the initial parts of the electron tracks are accessible to measurement.

In reference 62 such an examination was made of all known results obtained from determinations of the energies of slow positrons from measurements of the multiple scattering in emulsions or bubble chambers. In Table IV we present the conclusions of this paper, in which data from references 63, 38, 55, 56, and 64 are used.

TABLE IV

	Energy range Δx					
	0—0.3			0—0.6		
	total number of particles N	emitted forward N_f	emitted backward N_b	N	N_f	N_b
	600	320	280	1779	879	900
Forward excess $\delta = \frac{N_f - N_b}{N}$	+0.07±0.04			−0.01±0.02		
Forward excess expected from theory	+0.04			0		

In the energy range $0 - 0.3$ the total number of particles observed in these researches was 600 (320 were emitted forward, and 280 backward). This corresponds to a forward excess of $\delta = (N_f - N_b)/(N_f + N_b) = 0.07 \pm 0.04$. In the range $0 - 0.6$ the observed value of the forward excess was $\delta = (879 - 900)/1779 = -0.01 \pm 0.02$.

If we take into account the smearing out of the theoretical spectrum caused by the conditions of the measurements, the differences in the mean values of the asymmetry coefficient and in the experimental geometry in the researches described in references 63, 38, 55, 56, and 64, and also the decrease of the asymmetry coefficient in the low-energy region on account of radiative corrections, then the expected values of the forward excess are about 0.04 in the energy range $0 - 0.3$ and about 0 in the range $0 - 0.6$. Thus the data that have been obtained in the low-energy region do not contradict the predictions of the two-component theory.

4. THE HELICITIES OF THE LEPTONS AND THE LAW OF CONSERVATION OF LEPTONIC CHARGE

Here we shall examine the experimental data that confirm the hypothesis of the conservation of leptonic charge. We shall begin with the phenomena of nuclear β decay. We use the name neutrino for the neutral particle that is produced in the capture of an electron by a proton (K capture)

$$p + e^- \to n + \nu. \tag{7}$$

Then it is obvious that the neutral particle produced in the forced decay of the proton,

$$p \to e^+ + n + \nu, \tag{8}$$

is also a neutrino. We shall suppose that the leptonic charge of the neutrino is +1, i.e., that the neutrino is a lepton. Then if the law of conservation of leptons is valid, it follows from Eqs. (7) and (8) that the positron is an antilepton and the electron is a lepton.

From the decay of the neutron

$$n \to p + e^- + \bar{\nu}$$

it follows that the neutral particle that is produced in this decay is an antineutrino (an antilepton).

Let us consider the decay of the μ^+ meson

$$\mu^+ \to e^+ + \nu + \bar{\nu}.$$

If the leptonic charge is conserved, then the leptonic charges of the μ meson and the positron are equal. Consequently the μ^+ meson is an antilepton. It follows from this that the neutrino produced in the decay of the π^+ meson

$$\pi^+ \to \mu^+ + \nu,$$

is a lepton (it has negative helicity, just like the neutrino produced in nuclear K capture), and an antineutrino is produced in the decay of the π^- meson.

Thus, calling the neutrino from the reaction (7) a lepton and assuming the conservation of leptonic charge, we must ascribe to the particles the following leptonic charges:

$$\begin{array}{ll} \quad\text{Lepton} & \quad\text{Antilepton} \\ \quad(+1) & \quad(-1) \\ \nu \begin{pmatrix} p + e^- \to n + \nu \\ \quad p \to e^+ + n + \nu \\ \pi^+ \to \mu^+ + \nu \end{pmatrix} & \bar{\nu} \begin{pmatrix} n \to p + e^- + \bar{\nu} \\ \pi^- \to \mu^- + \bar{\nu} \\ \end{pmatrix} \\ e^- & e^+ \\ \mu^- & \mu^+ \end{array}$$

The question of the helicities of these particles can be settled by experiment. It is well known that in nuclear β decay the theory of the two-component neutrino predicts two possibilities:

1. The V and A interactions occur in β decay and a neutrino of negative helicity is emitted.

2. The S, T, and P interactions exist, and the neutrino has positive helicity.

The most direct determinations of the helicity of the neutrino in β decay have been made by Goldhaber and others,[65] who used the method of resonance scattering of γ rays to measure the polarization of the neutrinos emitted in K-electron capture. From these experiments it follows that the helicity of the neutrino is negative (left-handed), which, together with experiments on the electron-neutrino angular correlation in β decay[66] confirms the first of the possibilities provided by the theory.

Knowing the helicity of the neutrino from nuclear β decay and using the law of conservation of leptonic charge, we can predict the helicities of the particles involved in the sequence of $\pi \to \mu \to e$ decays. In fact, an analysis of the experiment of Garwin and others[2] from the point of view of the ideas of the two-component theory about the helicities of the light particles (see Fig. 6) shows that if the law of conservation of leptons is true, then the electron from the μ decay and the neutrino from the π decay must have opposite helicities. But the neutrino from the π decay is a lepton, for which the experiments of Goldhaber and others[66] have established negative helicity. This means that the positron from μ^+ decay must have the opposite helicity, i.e., positive helicity (and the electron from μ^- decay must have negative helicity).

If the helicity of the electrons from $\mu \to e$ decay is measured, its agreement with the predicted sign will be a confirmation that the same neutrino is involved in light-meson decay and in β decay, and that the law of conservation of leptonic charge holds for these decays.

Let us now consider the experiments in which the helicity of the electrons from μ decay has been measured.[67,68] These experiments are based on the fact that the bremsstrahlung emitted by longitudinally polarized electrons preserves the helicity of the electrons, i.e., it has a circular polarization whose direction is unambiguously related to the helicity of the electrons. Now, the direction of the circular polarization of the bremsstrahlung γ-ray quanta can be measured by the Compton scattering of the γ rays in magnetized iron:[69] if the γ rays are propagated through the iron in the direction of the applied field, then a change of the direction of the field increases or decreases the transparency of the iron absorber for the γ rays, depending on the direction of their circular polarization.

The degree of longitudinal polarization of the electrons from $\mu \to e$ decay has been calculated by many authors.[24,29] We present the result in the form obtained by Überall[29] for the theory of the two-component neutrino

$$P = \frac{\{[\xi(3-2x) + (1-2x)\cos\vartheta]^2 + (1-\xi^2)\sin^2\vartheta\}^{\frac{1}{2}}}{3-2x+\xi(1-2x)\cos\vartheta}.$$

Here ξ is the asymmetry parameter and ϑ is the angle between the spin of the μ meson and the direction of emission of the electron. It follows from this formula that if the asymmetry parameter has the value

$\xi = 1$, the electrons have 100 percent polarization, which does not depend on the angle ϑ. This means that even unpolarized μ mesons are a source of completely polarized electrons.

The helicity of the electrons produced in $\mu \rightarrow e$ decay has been measured by the indicated method in a research by Culligan and others[67] performed at the Liverpool accelerator, and by Macq and others[68] in Berkeley. Figure 14 is the diagram of the apparatus from reference 67. The π^+ mesons were stopped in the carbon target C, where the sequence of $\pi \rightarrow \mu \rightarrow e$ decays occurred. (In the measurement of the helicity of electrons the μ^- mesons arising from the decay of π^- mesons in flight were stopped in the target.) The electrons emerging from the target were stopped in a lead plate. The polarization of the γ rays so produced was studied with an analyzer consisting of a block of iron magnetized by a coil and a scintillation γ-ray counter (NaI), which measured the transparency of the iron absorber for two opposite directions of magnetization — parallel and antiparallel to the direction of propagation of the γ rays. One of the main difficulties in this experiment was the avoidance of effects of the magnetic field on the scintillation counters. For this purpose all the multipliers were carefully shielded with soft iron and μ-metal, and the stray field of the magnet near the γ-ray counter was compensated by special windings shown in Fig. 14. A test showed that the effect of the stray field on the operation of the photomultiplier can be neglected.

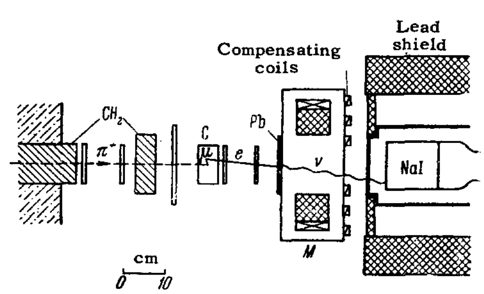


FIG. 14. Measurement of the helicity of the electrons from $\mu \rightarrow e$ decay (experiment of Liverpool group).[67]

To calibrate the system the Liverpool group[67] used bremsstrahlung from the β particles of Y^{90}, which are known to be highly polarized. In the calibration a 100 mC source of Y^{90} was inserted instead of the carbon target and the spectrum of the pulses in the γ-ray counter was measured for the two directions of the field. The result so obtained is shown in Fig. 15, where the pulse size in the γ-ray counter (in Mev) is plotted as abscissa, and the ordinate is the difference between the counts with the two opposite directions of the magnetic field. The curve clearly shows both the existence of the effect and its increase as the degree of polarization of the γ rays increases with increasing energy. The experimental results for μ^+ and μ^- mesons are shown in Fig. 16. Here the quantities plotted are the same as in Fig. 15. We see, first, that the effect of the magnetic field exists, and that it has opposite signs for μ^+ and μ^- mesons. The integrated effect for γ-ray energies larger than 12 Mev is $+4.7 \pm 1.2$ percent for μ^+ mesons and -5.6 ± 2.3 percent for μ^- mesons. The values found for this quantity in reference 68, for γ rays of energies larger than 8 Mev, were $+6.1 \pm 0.7$ and -4.9 ± 1.5 percent, respectively, which agrees with the theortically expected value of the asymmetry for 100 percent polarization of the electrons. In the case of μ^+ mesons the transparency of the magnet is larger in the case in which its north pole is turned toward the source of γ rays. Since the cross section for Compton scattering is smaller if the spin directions of the γ-ray photon and the electron coincide, this result means that in this case the photons, and thus also the positrons that produced them, have positive helicity. The correspond ing result for the helicity of the electrons from μ decay is that it is negative.

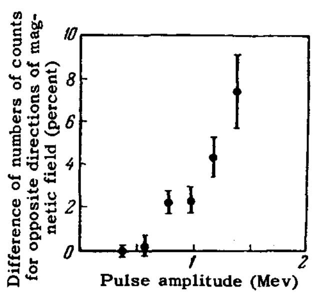


FIG. 15. Calibration of the "helicity" analyzer with γ rays from the bremsstrahlung of Y^{90} β rays. The abscissa is the pulse amplitude in the γ-ray counter (Mev), and the ordinate is the difference of the numbers of counts with opposite directions of the magnetic field (in percent).

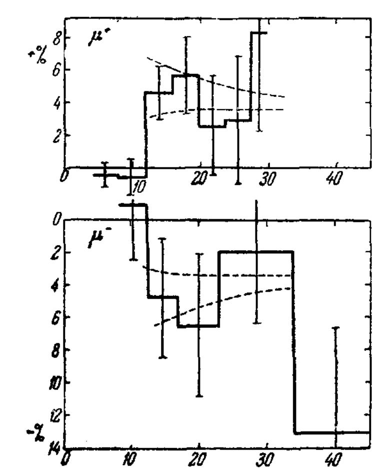


FIG. 16. Experimental data for measurements of helicities of electrons; the coordinates are the same as in Fig. 15.

These results are thus in agreement with the following assumptions:

1) the two-component theory holds, with the neutrino having negative helicity;

2) the law of conservation of leptons is valid;

3) there exists universal Fermi interaction containing the V and A interaction types;

4) e^+ and e^- are completely polarized in $\mu \rightarrow e$ decay. The experiments we have considered show that the neutral particle produced in the decay of the π^+ meson has negative helicity, so that from the two possibilities indicated on page 206 we can choose the first

$$\pi^+ \rightarrow \mu^+ + \text{neutrino}.$$

5. THE MAGNETIC MOMENT OF THE μ MESON

The measurement of the magnetic moment of the μ meson is of fundamental significance for an understanding of the nature of this particle. In the weak interactions the μ meson takes part as a lepton; that is, with respect to the weak interactions it has all the properties of the electron except that its mass is 207 times that of the electron. This is the most surprising paradox in elementary-particle physics, and theory evidently possesses no means of solving it. If the μ meson is actually a "heavy electron" its magnetic moment must be described by the Dirac equation, just as is that of the electron.

The magnetic moment of a Dirac particle is

$$\mu_0 = \frac{eh}{2mc}.$$

Owing to its interaction with the zero-point vibrations of the electromagnetic field a Dirac particle with this magnetic moment acquires an additional magnetic moment. Calculations of this additional magnetic moment, caused by radiative corrections, have been carried out in papers by Karplus and Kroll,[71] Sommerfield,[72] and Petermann[73] to fourth order in the charge. The theoretical values of the magnetic moments of the electron and the μ meson accepted at present are[72,73]

$$\mu = \mu_0\left(1 + \frac{\alpha}{2\pi} - 0.328\frac{\alpha^2}{\pi^2}\right) = 1.0011596\ \mu_0$$

$$(g_e = 2 \cdot 1.0011596)$$

for the electron and

$$\mu = \mu_0\left(1 + \frac{\alpha}{2\pi} + 0.75\frac{\alpha^2}{\pi^2}\right) = 1.0011654\ \mu_0$$

$$(g_\mu = 2 \cdot 1.0011654)$$

for the μ meson. Here the fine-structure constant α is taken to be given by $\alpha^{-1} = 137.039$,[74] and the gyromagnetic ratio g is expressed in its usual units e/2mc.

The most precise direct measurement of the magnetic moment of the electron is that made recently by Schupp and others.[75] To determine the magnetic moment of the electron these authors used an observation of the difference between the spin-precession frequency and the cyclotron frequency. The polarization of the electrons was obtained by Mott scattering, and this same method was used to analyze the degree of polarization after the electron passed through the magnetic field. The value obtained

$$g_e = 2 \cdot (1.0011612 \pm 0.0000024)$$

is in good agreement with the predictions of the theory, and the precision of these measurements is about five times that of the well known determinations of the magnetic moment of the electron[76,77] in which this magnetic moment was compared with that of the proton.

After these preliminary remarks we turn to the researches in which the magnetic moment of the μ meson has been measured.

As we have seen, the very first work of Garwin, Lederman, and Weinrich,[2] in which parity nonconservation in $\pi \to \mu \to e$ decay was brought to light, also gave a measurement of the magnetic moment of the μ meson (cf. page 201). This measurement reduces to a measurement of the precession frequency ω of the spin of the μ meson in a magnetic field H. The frequency of precession is given by

$$\omega = g\frac{eH}{2m_\mu c}\ \text{rad/sec}$$

from which relation, knowing the mass m_μ of the μ meson and measuring the field H, one can determine the value of the gyromagnetic ratio g_{μ^+}. In reference 2 the value found for the quantity g_{μ^+} was

$$g_{\mu^+} = 2.00 \pm 0.02.$$

By this same method of the spin precession in a constant magnetic field Cassels and others,[78] working at the Liverpool cyclotron, have obtained a more exact value of the gyromagnetic ratio of the μ meson. We have shown a diagram of their experiment in the chapter on the depolarization of μ^+ mesons; the experimental data from which the magnetic moment of the μ meson was found, i.e., the distribution in time of the pulses in the electron counter, corrected for the exponential decay of the μ mesons during the time of the observations, are shown in Fig. 17. An analysis of this curve by the method of least squares shows that if we try to fit these data with a relation of the form

$$A + B\cos(2\pi f t + \delta),$$

the best agreement is obtained with

$$f = \frac{\omega}{2\pi} = (1.382 \pm 0.006)\ \text{Mcs/sec}$$

For the field strength that was used, H = 101.9 ± 0.3 gauss and the meson mass 206.84± 0.12, this frequency corresponds to the gyromagnetic ratio

$$g_\mu = 2(1.002 \pm 0.007).$$

The error shown includes both the statistical fluctuations (0.4 percent) and the error in the value of the magnetic field, the uncertainty associated with the drift of the electronic system, and the error from the uncertainty in the mass of the μ meson. Thus the values of the magnetic moment of the μ meson measured from the frequency of its precession in a constant field confirm, to accuracy about 0.7 percent, that the magnetic moment of the μ meson agrees with that of a Dirac particle. This accuracy, however, is still in-

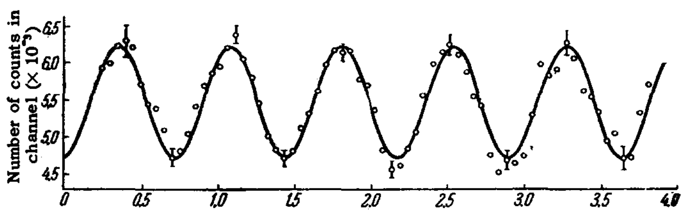


FIG. 17. The precession of the spin of the μ meson in a magnetic field.[78] The abscissa is the time; the ordinates are the numbers of decay electrons.

sufficient to detect the radiative correction to the magnetic moment, which has a value close to 0.1 percent.

It must be noted that in all methods that determine the magnetic moment of the μ meson from the spin precession in a magnetic field the accuracy of the measurement of the magnetic moment is limited by the accuracy with which the mass of the μ meson is known. We have seen (page 196) that at present the best value of the mass is 206.86 ± 0.11. Even if all of the errors in the measurements were eliminated or sharply diminished, this uncertainty would still give an error close to 0.05 percent in the value of the magnetic moment. This accuracy has indeed been attained in a research by Lundy and others,[79] who used an extremely clever and effective method of "stroboscopic coincidences" suggested by Telegdi.

The idea of the method is as follows. Suppose a beam of μ mesons polarized parallel to their motion is stopped at the time $t = 0$ in a target placed in a magnetic field perpendicular to the spins of the mesons (cf. Fig. 18).

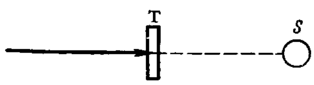


FIG. 18. Idea of the method of stroboscopic coincidences.

Then because of the spin precession in the magnetic field the intensity of the electrons emitted from the target in the given direction TS varies according to the law

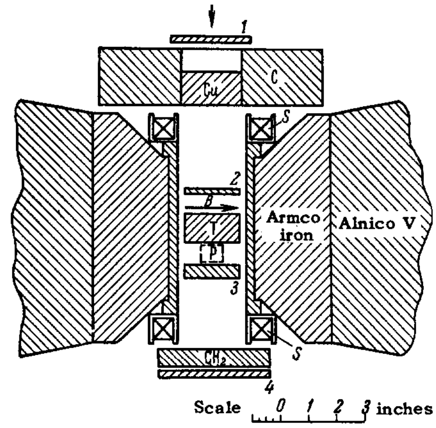
$$\exp\left(-\frac{t}{\tau}\right)(1 - a\cos\omega_H t),$$

where τ is the lifetime of the μ meson, a is the asymmetry coefficient, and

$$\omega_H = g\frac{eH}{2m_\mu c}\ \text{rad/sec}.$$

We place at S a counter for electrons. Then $\omega/2\pi$ times per second it will intersect the beam of electrons emerging from the target in the direction TS, at the instants when the spins of the μ mesons have the opposite direction. If by means of an electronic system we open the counter for the first time at the instant $t = 0$, and then open it periodically with the frequency $\omega/2\pi$, then the maximum number of counts of the counter, which registers high-energy electrons, will be observed when the frequency at which it is opened agrees with the frequency of the spin precession of the μ meson. A diagram of the apparatus is shown in Fig. 19. The beam of μ mesons (indicated by the arrow) was stopped in the target (graphite or bromoform, 8 g/cm^2). The magnetic field, produced by a large permanent magnet, was perpendicular to the direction of the meson beam, and thus also perpendicular to the spins of the μ mesons. The stopping of the μ mesons in the target was registered by coincidences of counters 1, 2, 3, and the decay electron was registered by coincidences of 2, 3, 4. The electronic part of the apparatus was constructed so that the pulse of the 1, 2, 3 coincidence turned on an oscillator of frequency

FIG. 19. Measurement of the magnetic moment of the μ meson by the method of stroboscopic coincidences.

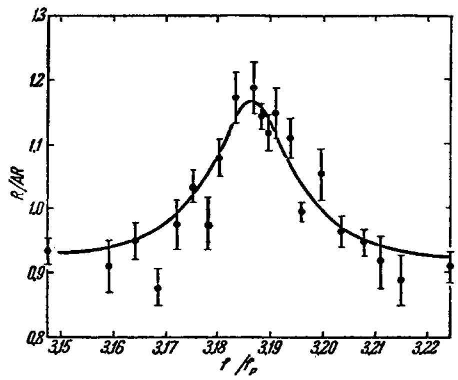


48.63 Mcs, which then stayed on for 6 μsec. The train of high-frequency oscillations was fed to the parallel inputs of two coincidence circuits R and AR, which respond to negative inputs only. The other two inputs of the coincidence circuits received the pulses from the electron counter 3, shifted by a line in such a way that a pulse that was physically simultaneous with that in counter 2 reached the coincidence circuit R at a maximum of the high-frequency oscillation, and reached AR a half period later. With this arrangement of the circuit, if the precession frequency agrees with the generator frequency, $\omega_H = \omega$, then the pulse from the decay electron in 3 will cause a coincidence in R and not cause one in AR. Thus by measuring the ratio of the coincidences R/AR as the frequency of the generator was varied, one could measure the resonance frequency by locating the maximum of this ratio. The measurements were made with a constant field of 3700 gauss, and the resonance was obtained by varying the frequency f_{osc} of the high-frequency field. The resonance curve is shown in Fig. 20; the abscissa is the ratio of f_{osc} to the proton resonance frequency f_p. The value found for this ratio at the maximum of the curve was

$$\frac{f_{\mu^+}}{f_p} = 3.1830 \pm 0.0011.$$

To get from this ratio the value of the gyromagnetic ratio g_{μ^+} for the μ^+ meson, we must know its mass, and also the mass m_p and magnetic moment μ_p of the proton:

FIG. 20. Resonance curve obtained by the method of stroboscopic coincidences.



$$g_{\mu^+} = 2\frac{m_{\mu^+}}{m_p}\frac{f_{\mu^+}}{f_p}\cdot\mu_p.$$

Taking $m_\mu = 206.86 \pm 0.11\ m_e$, $m_p = 1836.1\ m_e$, and $\mu_p = 2.7927$ nuclear magneton,[80] we get

$$g_\mu = 2\,(1.0015 \pm 0.0006).$$

In a paper by Garwin and others[81] it is stated that the Chicago group has obtained a somewhat better value of f_μ/f_p for a target of Al and bromoform. With the new value

$$\frac{f_\mu}{f_p} = 3.1838 \pm 0.0008$$

the corresponding gyromagnetic ratio is

$$g_\mu = 2\,(1.0017 \pm 0.0006),$$

which is to be compared with the theoretical value 2×1.00116.* Thus by this method it has been possible to attain an accuracy close to 0.06 percent, and almost all of this error is due to the uncertainty in the measurement of the mass of the μ meson. The accuracy of the method of "stroboscopic coincidences" has been considerably improved in work done by Garwin and others.[81] These authors increased the precession frequency and decidedly improved the accuracy with which the time interval between the μ meson's reaching the target and the emission of the electron was registered. Owing to this the accuracy of the determination of the resonance frequency was improved to 0.007 percent, so that at present the entire error in the determination of the magnetic moment of the μ meson is due to the uncertainty in its mass.

The data obtained by these authors are as follows:

Target	Measured ratio f_{μ^+}/f_p
Aluminum	3.1847 ± 0.0003 3.1848 ± 0.0003
Bromoform	3.1846 ± 0.0002

With these results, if we use the values of the masses of the μ meson and proton given above, and the proton magnetic moment $\mu_p = 2.7927$ nuclear magneton, we get

$$g_{\mu^+} = 2\,(1.0020 \pm 0.0005),$$

which exceeds the gyromagnetic ratio predicted by the theory by 1.6 times the standard error. If we use the value given on page 197 for the lower limit on the mass of the μ meson, which is known with great certainty,

$$m_{\mu^+} > (206.77 \pm 0.04),$$

we get

$$g_{\mu^+} \geqq 2\,(1{,}00154 \pm 0.00022),$$

which also does not agree with the theoretical value. Coffin, Garwin, and others[82] have measured the magnetic moment of the μ^+ meson by a magnetic-resonance method. In these measurements the signal indicating the fact that reorientation of the spin of the μ meson had occurred was the change of the asymmetry of the emission of positrons in the $\mu^+ \to e^+$ decay. The apparatus is shown in Fig. 21. The target, placed in a solenoid in which a high-frequency field was excited,

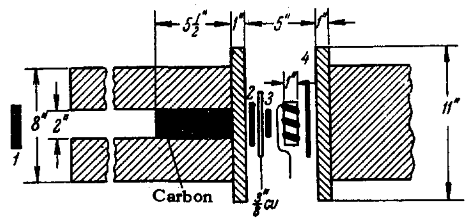


FIG. 21. Measurement of magnetic moment of the μ meson by magnetic resonance method.

and the scintillators, were between the poles of an electromagnet which produced a constant magnetic field parallel to the spins of the μ^+ mesons. After first being slowed down in a graphite absorber placed in a channel through one pole of the electromagnet, the mesons were stopped in the target. The stopping of μ^+ mesons was registered by coincidences 1, 2, 3, $\bar{4}$. Besides this, the pulse from 1, 2, 3, $\bar{4}$ started the excitation of the high-frequency coil and opened the circuit for the registration of decay electrons. This circuit was opened for 3 μsec, beginning 2 μsec after the 1, 2, 3, $\bar{4}$ coincidence. The circuit registered decay positrons that were emitted backward during the 3-μsec time interval, which were counted by 2, 3, $\bar{1}$, $\bar{4}$ coincidences. The pulsed radio-frequency field H_1 excited in the solenoid surrounding the target caused reorientation of the meson spins. At resonance, when the frequency of the field in the solenoid is $\omega = g\,(eH_1/2mc)$ rad/sec, the angle through which the spin is turned during the time t is $\vartheta = \frac{1}{2}\int_0^t g\left(\frac{eH_1}{2mc}\right)dt$. A rotation through 180°, which from the quantum-mechanical point of view corresponds to 100 percent probability for transition from the state $m_s = \mp\frac{1}{2}$ to the state $\pm\frac{1}{2}$, requires an impulse from the high-frequency field of $\int_0^t H\,dt = 70$ gauss μsec. The apparatus registered decay electrons emitted in the backward direction. Therefore when there were no transitions the registered intensity was a maximum; a decrease of the intensity indicated the occurrence of magnetic transitions.

The resonance curve obtained in this experiment is shown in Fig. 22, where the abscissa is the ratio of the frequency f_r of the oscillating field in the solenoid to the proton-resonance frequency in the same constant field, and the ordinate is the number of counts in the electron telescope 2, 3, $\bar{1}$, $\bar{4}$ for 64,000 stoppings of μ

*Similar measurements, but with much less accuracy, have been made with μ^- mesons (in graphite) in a field of 300 gauss (f_{osc} = 3.945 Mes) produced by Helmholtz coils. The value of the gyromagnetic ratio so obtained is

$$g_{\mu^-} = 2\,(0.9993 \pm 0.0042).$$

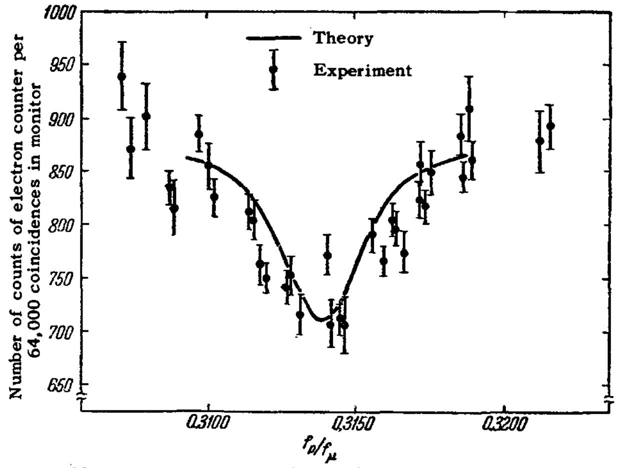


FIG. 22. Resonance curve obtained by the magnetic-resonance method. The abscissa is the ratio of the generator frequency to the proton resonance frequency in the same magnetic field, and the ordinate is the number of counts in the electron counters.

mesons in the target. The solid curve shows the expected form of the resonance curve. The center of the experimental distribution of points, from which the resonance frequency was determined, was found by the method of least squares on the assumption that the distribution of experimental points must be symmetrical with respect to this center. For the value of the resonance frequency the result $f_\mu/f_p = 3.1865 \pm 0.0022$ was obtained, from which we get, for $m_\mu = 206.86 \pm 0.11$,

$$g_{\mu^+} = 2\,(1.0026 \pm 0{,}0009).$$

A summary of all measurements that have been made on the magnetic moment of the μ meson is shown in Table V; from this table it can be seen that in the most precise measurements there is a systematic shift of the measured values of g_{μ^+} toward values larger than $g_{\mu^+} = 2\,(1.0012)$, which is predicted by theory for a Dirac particle. The solution of this problem, which is of primary importance, requires either a great improvement in the accuracy with which the mass of the μ^+ meson is known, or else a resort to other methods for determining g_{μ^+} that dispense with the use of this mass. One such method is the comparison of the spin precession frequency in a magnetic field

$$\omega = g\,\frac{eH}{2mc}$$

with the "cyclotron" frequency of the orbital motion of the μ meson in the same constant field

$$\omega = \frac{eH}{m_\mu c}$$

as has been done for the electron.[75]

The ratio of these frequencies does not depend on the mass of the μ^+ meson. At present preparations for such experiments are being made in several laboratories.

6. ELECTRONIC DECAY OF THE π MESON. SEARCHES FOR RADIATIVE DECAYS OF π AND μ MESONS

6.1. The $\pi \to e + \nu$ Decay

The main type of decay for the π meson is decay into a μ meson and a neutrino

$$\pi \to \mu + \nu.$$

We have seen that according to all of its properties so far measured the μ meson is a "heavy electron:" thus like the electron it exists in two charge states, with the mass of the positively charged μ meson equal to that of the negative μ meson; the magnetic moment of the μ meson, like that of the electron, agrees within the limits of accuracy of the measurements with that of a Dirac particle; and also in the weak interactions the properties of the μ meson are analogous to those of the electron. This follows, for example, from the existence of the two analogous weak interactions that involve four fermions:

$$e^- + p \to n + \nu,$$
$$\mu^- + p \to n + \nu.$$

The first of these is K capture, which is well known in β decay. The second phenomenon is the capture of a μ meson by a proton. As early as 1948-49 attention was called to the fact that the coupling constants that appear in the interactions describing these two reactions are very nearly equal.[83,84] This fact is the basis of the idea of the "universal interaction" between the four fermions μ, e, ν, and $\bar{\nu}$. From the existence of

TABLE V. Measurements of the magnetic moment of the μ^+ meson

Authors	Method	Mass value used (in units m_e)	g_{μ^+}
Garwin et al.[2]	Precession in constant magnetic field	206.84 ± 0.12	2.00 ± 0.02
Cassels et al[78]	The same	± 0.12	2.044 ± 0.014
Lundy et al.[79]	Precession in constant magnetic field (stroboscopic coincidences)	206.86 ± 0.11	2(1.0015 ± 0.0006)
Garwin et al.[81]	The same	206.86 ± 0.11	2(1.0020 ± 0.0005)
		206.77 ± 0.04	2(1.00154 ± 0.00022)
Coffin et al.[82]	Magnetic resonance	206.86 ± 0.11	2(1.0026 ± 0.0009)

such a universal interaction it follows that along with the usual decay of the π meson into a μ meson and a neutrino there must also occur a decay into an electron and a neutrino:

$$\pi \rightarrow e + \nu.$$

The theoretical estimates of the probability of such a decay are based on the treatment of the π meson as an intermediate state of a nucleon-antinucleon pair. This pair annihilates into a μ meson and a neutrino:

$$\pi \rightarrow \mathrm{N} + \overline{\mathrm{N}} \rightarrow \mu + \nu. \tag{9}$$

The universality of the four-fermion interaction makes possible the electronic decay of the π meson

$$\pi \rightarrow \mathrm{N} + \overline{\mathrm{N}} \rightarrow \mathrm{e} + \nu. \tag{10}$$

Since the π meson is a pseudoscalar particle, such a transition is possible only in the theory of weak interaction with pseudoscalar or axial-vector coupling.

The corresponding calculations have also been made in papers by Ruderman and Finkelstein[85] and by Steinberger,[86] where it has been shown that for pseudoscalar coupling the ratio of the probabilities of decays of the types (9) and (10) is

$$w = \frac{\pi \rightarrow \mathrm{e} + \nu}{\pi \rightarrow \mu + \nu} \cong \frac{M_\pi^2 - M_e^2}{M_\pi^2 - M_\mu^2} = 5.4,$$

so that this type of interaction can be rejected at once, whereas for the axial-vector interaction

$$w = \frac{\pi \rightarrow \mathrm{e} + \nu}{\pi \rightarrow \mu + \nu} \cong \frac{M_\pi^2 - M_e^2}{M_\pi^2 - M_\mu^2} \cdot \frac{M_e^2}{M_\mu^2} = 1.36 \cdot 10^{-4}.$$

This result took on especial importance quite recently, when in connection with the discovery of parity nonconservation there appeared the theory of Feynman and Gell-Mann, which restricts the possible interactions in β decay to the vector and axial-vector types.[7] In the final analysis almost all of the experimental data on β decays were in agreement with this theory, and only the absence of the $\pi \rightarrow e + \nu$ decay could still be a crucial experiment for the theory.

Several difficult experimental investigations have been devoted to the search for $\pi \rightarrow e$ decay. In the first papers (Friedman and Rainwater,[87] Lokanatan and Steinberger,[88] Anderson and Lattes[89]) the $\pi \rightarrow e$ decay was not detected, since the sensitivity and reliability of the experimental methods were insufficient, or only bordered on what was necessary. Only in researches carried out in 1958[90,92,93] was the existence of the $\pi \rightarrow e$ decay shown quite convincingly.

The work of Friedman and Rainwater,[87] which was evidently the first search for $\pi \rightarrow e$ decay, was done in the initial period of the operation of accelerators, when there were still no intense and well collimated external π-meson beams. In this experiment Ilford G-5 plates were placed near the internal target of the Columbia 164-inch accelerator, which was bombarded by a proton beam of energy 385 Mev. The stray magnetic field of the accelerator focused slow π^+ mesons on the plate. To each point of entry of a π^+ meson into a plate there corresponded a definite range of the π^+ meson in the emulsion. Among 1419 mesons selected in this way according to their ranges and momenta and giving $\pi^+ \rightarrow \mu^+ + \nu$ decays, there was found only one particle that stopped in the emulsion and emitted a decay electron instead of a μ^+ meson. This particle, however, could also have been a μ^+ meson arising from the decay of a π meson in flight.

Thus this experiment, in which the sensitivity was small, showed that the ratio of the probabilities of the $\pi \rightarrow e + \nu$ and $\pi \rightarrow \mu + \nu$ decays is smaller than 10^{-3}.

In later work the sensitivity of the methods was greatly increased. The experimental problem is to single out a monochromatic line of electrons with energy close to 70 Mev that arise from $\pi \rightarrow e$ decays, against the background of a much larger number of electrons from $\mu \rightarrow e$ decays, which have a continuous energy spectrum that breaks off at about 53 Mev.

The arrangement of the experiment of Lokanatan and Steinberger[88] is shown in Fig. 23. After first being slowed down in graphite, the beam of 60-Mev π^+ mesons from the Columbia University accelerator was stopped in a polyethylene target. The electrons emerging from this target were registered by an electron telescope of four counters separated by layers of graphite absorber. Coincidence of the pulses in counters 1, 2 (pulse M) served to monitor the beam. The passage of particles through the electron telescope was registered by coincidences of 3, 4, 5, 6 (pulse D), which were discriminated with a resolving power $\tau \sim 10^{-6}$ sec. Moreover, the electronic circuit made it possible to distinguish the "fast" coincidences $\mathrm{MD_f}$ (pulse D delayed relative to pulse M by a time less than than 10^{-7} sec) from the "slow" coincidences (time interval between pulses D and M less than 1.8×10^{-6} sec).

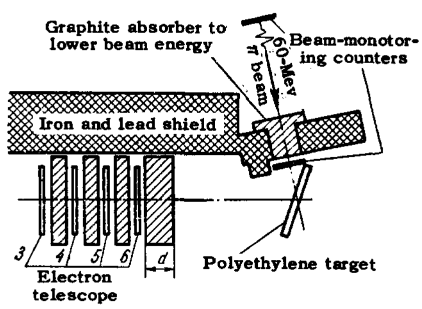


FIG. 23. Experiment of Lokanatan and Steinberger:[88] search for $\pi \rightarrow e + \nu$ decay.

The separation of the monochromatic electrons of $\pi \rightarrow e$ decay from the electrons of the continuous spectrum was made by the absorption method, for which purpose the thicknesses of the filters in the electron telescope were suitably chosen. A supplementary criterion for the selection was the separation of the electron pulses into pulses $\mathrm{MD_f}$ and $\mathrm{MD_s}$: Since $\pi \rightarrow e$ decay occurs with the lifetime of the π meson, cases of $\pi \rightarrow e$ decay must be registered mainly by $\mathrm{MD_f}$ coincidences, whereas the $\mathrm{MD_s}$ coincidences mostly registered the μ^--e electrons.

In this work no $\pi \rightarrow e$ decays were found and, analyzing the sensitivity of their apparatus, the authors concluded that the probability of $\pi \rightarrow e$ decay is not larger than $w = 1:17{,}000$. Allowing for the fact that many estimates in their work were rather rough, and in particular the estimate of the efficiency of the electron telescope might be considerably too high, it must be said that their result is not in sharp contradiction with the theory, which requires $w \approx 1:9000$.

In their work, Fazzini and others[90] used the main ideas of the method of Lokanatan and Steinberger. Improvements they made enabled them to increase greatly the sensitivity of the apparatus to $\pi \rightarrow e$ decays. The main improvement of the method consisted of a large increase in the efficiency of the electron telescope and an improvement of the time resolution of the pulses from the $\pi \rightarrow \mu \rightarrow e$ and $\pi \rightarrow e$ decays. The arrangement of the experiment is shown in Fig. 24. The electron telescope consisted of eight scintillation counters $(5-12)$, separated by seven layers of graphite absorber. Coincidences 1, 2, 3, $\overline{4}$ (counter 3 was the target in which the π mesons were stopped) registered the stopping of the π mesons. Coincidences $5-12$ registered the decay electrons. The fast-coincidence circuit recorded all coincidences of $(1, 2, 3, \overline{4})$ with $(5-12)$ that were within the limits of from 60 to 160 mμsec after the stopping of a π meson $(1, 2, 3, \overline{4})$. When there was such a coincidence the pulses from counters 3 and 12 were fed to a "fast" oscilloscope, whose screen was photographed. This oscilloscope also received the pulse from a large scintillation counter (NaI) with which the electron telescope ended.

A typical oscillogram of ordinary $\pi \rightarrow \mu \rightarrow e$ decay is shown on the same diagram with the apparatus. The pulses π and μ arose in counter 3 as the result of the stopping of a π meson and its decay to a μ meson. Pulse e(3) in counter 3 is from the decay electron. This electron caused the $5-12$ coincidence and the pulse in counter 12 which can also be seen on the oscillogram.

The second oscillogram, on which the pulse from the μ meson is absent, can correspond to the looked-for $\pi \rightarrow e$ decay, but can also be ordinary $\pi \rightarrow \mu \rightarrow e$

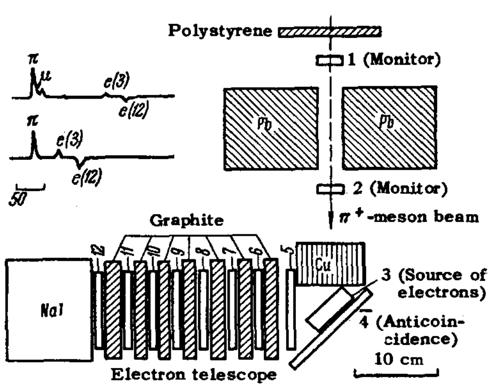


FIG. 24. Experiment of Fazzini and others[90] on the measurement of $\pi \rightarrow e$ decay.

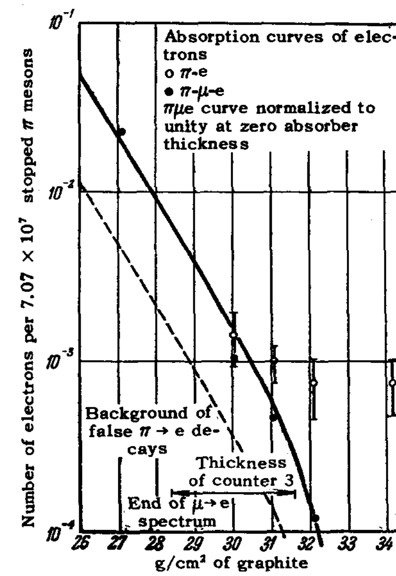


FIG. 25. Absorption of $\pi \rightarrow \mu \rightarrow e$ and $\pi \rightarrow e$ decays in the experiment of Fazzini et al. The abscissa is the thickness of graphite absorber in g/cm², and the ordinate is the number of electrons per 7.07×10^7 stopped π mesons.

decay in which the pulse from the μ meson has not been resolved by the scintillation counter, or else an accidental coincidence of pulses $(1, 2, 3, \overline{4})$ and $(5-12)$. To separate the actual $\pi \rightarrow e$ decays from the background of unresolved $\pi \rightarrow \mu \rightarrow e$ decays and accidental coincidences the authors studied the dependence of the number of $\pi \rightarrow \mu \rightarrow e$ and $\pi \rightarrow e$ decays on the thickness of absorber in the electron telescope, and the distribution of these decays in time. Figure 25 shows "absorption curves" for the $\pi \rightarrow e$ and $\pi \rightarrow \mu \rightarrow e$ oscillograms. The ordinate is the number of cases of $\pi \rightarrow \mu \rightarrow e$ or $\pi \rightarrow e$ decay per 4×10^3 stopped π^+ mesons, and the abscissa is the total thickness of absorber between the counters $5-12$. Complete absorption of the electrons from $\mu \rightarrow e$ decay corresponds to 30 g/cm² of carbon. Meanwhile, with thicknesses of about 30, 31, 32, and 34 g/cm² there were observed 40 $\pi \rightarrow e$ decays in which the pulses from the π meson and the electron were separated by time intervals longer than 8.3 mμsec. The distribution in time of these 40 decays agrees with the lifetime of the π^+ meson, $\tau_{\pi^+} = (22 \pm 4) \times 10^{-8}$ sec. Thus these data are a very convincing proof of the existence of $\pi \rightarrow e$ decay. The total number of π^+ mesons stopped in counter 3 during the experiment was 1.71×10^8. The solid angle of the electron telescope is about 0.8 percent of 4π. Let us assume that it had 100 percent efficiency for the registration of $\pi \rightarrow e$ electrons. We must reject from the 40 $\pi \rightarrow e$ decays four false $\pi \rightarrow e$ decays, and add about 13 decays that had occurred during the time of 8.3 mμsec. We get:

$$\omega = \frac{R(\pi \rightarrow e+\nu)}{R(\pi \rightarrow \mu+\nu)} = \frac{(40-4+13)}{1.71 \cdot 10^8 \cdot 8 \cdot 10^3} \cong 4 \cdot 10^{-5}.$$

This quantity must be regarded as a lower limit on this probability, mainly because the efficiency of the electron telescope is of course not 100 percent. In particular, it has been remarked in a note by Tolhoek[91] that the last result of this group, obtained after a determination of the efficiency of the electron telescope by the Monte Carlo method, is

$$\omega = (1.4 \pm 0.2)\,10^{-5}.$$

The electron decay of the π meson has also been observed by means of a bubble chamber, in work done by a group at Columbia University.[92] A liquid-hydrogen bubble chamber of diameter 30 cm and depth 15 cm was placed in a field of 8800 gauss and exposed to a beam of slow π^+ mesons. On the average the chamber showed about 10 stoppings of π^+ mesons per picture. In the overwhelming majority of cases the observed decay electrons arose from the decay of μ mesons, which in turn came from $\pi^+ \rightarrow \mu^+$ decay, with the two successive steps of the $\pi \rightarrow \mu \rightarrow e$ decay clearly visible in the picture. In approximately 1/40 of the cases of appearance of a decay electron no intermediate slow μ meson was observed. Such electrons come either from the decay of μ mesons that have stopped in the chamber after being produced by $\pi \rightarrow \mu$ decays in flight, or else from the much less probable event of $\pi \rightarrow e + \nu$ decay. To separate the extremely rare cases of $\pi \rightarrow e + \nu$ decay from the $\mu \rightarrow e$ decays, the momenta of the decay electrons were measured from their deflections in the magnetic field; the maximum momentum of an electron from $\mu \rightarrow e$ decay is 53 Mev/c, while $\pi \rightarrow e$ decay gives monochromatic electrons of momentum ~ 70 Mev/c. Among 65,000 $\pi \rightarrow \mu \rightarrow e$ decays the authors observed 1766 decays in which no intermediate μ meson could be seen. In addition they made special measurements of the spectrum for 2893 $\pi \rightarrow \mu \rightarrow e$ decays. The two spectra are shown in Figs. 26 and 27. We see that out of the 2983 $\pi \rightarrow \mu \rightarrow e$ decays there is no case in which the measured electron energy exceeds 62 Mev, whereas in the spectrum of the $\mu \rightarrow e$ decays there are 6 energy values that are grouped near 70 Mev. They must therefore be ascribed to $\pi \rightarrow e$ decays. The average energy for these 6 cases is 72.9 ± 1.5 Mev, which is higher than the expected value of 69.8 Mev. This is to be explained by systematic errors in the calibration.

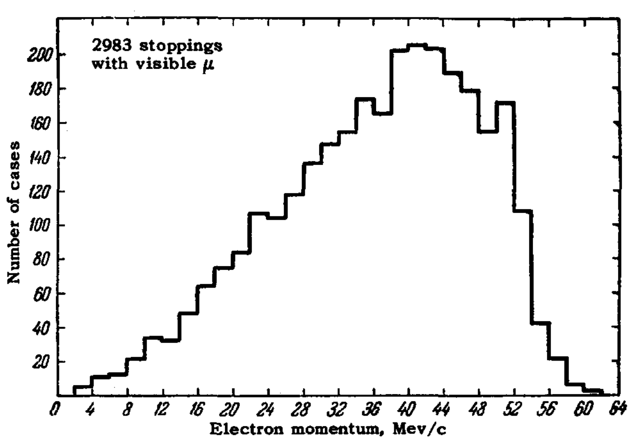


FIG. 26. Spectrum of electrons from $\pi \rightarrow \mu \rightarrow e$ decay; $\pi \rightarrow \mu$ decays visible in chamber.

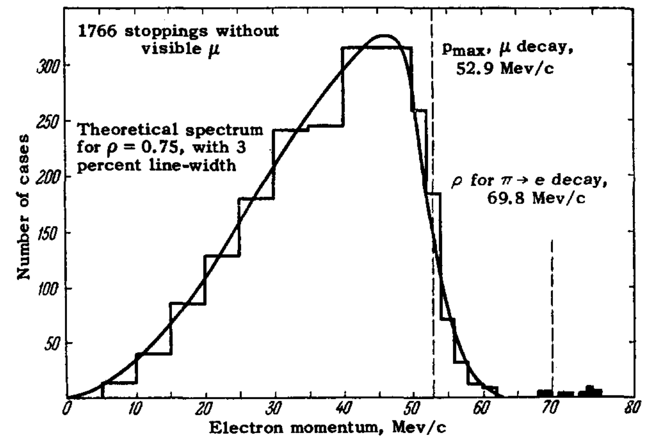


FIG. 27. Spectrum of electrons from $\mu \rightarrow e$ decay; $\pi \rightarrow \mu$ decays not visible in chamber.

The relative probability of $\pi \rightarrow e$ decay found in this experiment is 6/65,000 = 1/10,800 ± 40%, which is in qualitative agreement with the probability w ~ 1/9000 expected from theory.

Let us now examine the experiment of Anderson and Lattes[89] and the recent experiment of Anderson and others,[83] which is a continuation of the former experiment.

In this work the magnetic-spectrometer method was used to separate the 70 Mev $\pi \rightarrow e$ electrons from the $\mu \rightarrow e$ electrons. In the first work[89] no electronic π decay was found, and from an estimate of the sensitivity of the apparatus for $\pi \rightarrow e$ electrons it was concluded that the relative probability of $\pi \rightarrow e$ decay could not be larger than 2×10^{-5}. After this decay had been found by the methods already considered, Anderson and others[93] repeated their experiment in greatly improved form. A diagram of the apparatus is shown in Fig. 28. The same magnetic spectrometer as in reference 89 was used to separate the $\pi \rightarrow e$ electrons from the $\mu \rightarrow e$ electrons. The way the method was improved was that instead of a single detector for the decay electrons three scintillators 4a, 4b, and 4c were placed side by side at the output of the spectrometer, so as to register the $\pi \rightarrow e$ decay electrons in three adjacent energy intervals. The block

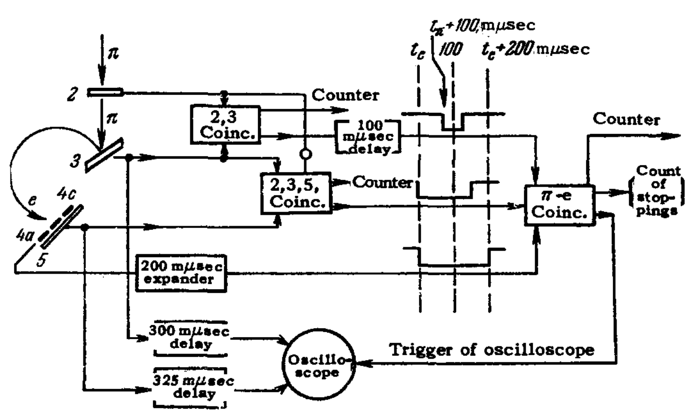


FIG. 28. Experiment of Anderson et al.[93]: search for $\pi \rightarrow e + \nu$ decay.

scheme of the electronics is clear from Fig. 28. The (2, 3) coincidence from the passage of a π^+ meson through counters 2, 3 generated a 50 mμsec pulse, which was shifted 100 mμsec by a delay line. A pulse of duration 150 mμsec, generated by a (2, 3, 5) coincidence, accompanied the electron passing through the spectrometer. The block $\pi \to e$ (a), and the blocks $\pi \to e$ (b) and $\pi \to e$ (c) not shown in the diagram, registered coincidences between the pulses (2,3) and (2, 3, 5) and pulses from the electron counters 4a, 4b, and 4c; the output pulse of these circuits triggered the sweep of a fast oscilloscope, whose deflecting plates were connected through delay lines to the photomultiplier anodes of scintillators 3 and 5. Thus a $\pi \to e$ decay was shown on the screen of the oscilloscope by a large positive pulse from the stopping of the π meson in counter 3, a subsequent smaller positive pulse from the electron emerging from 3, and, finally, a small negative pulse from the electron reaching 5. The system registered decay times from 24 to 150 mμsec. Figure 29 shows the distribution in energy (measured by the current in the spectrometer magnet windings) of the pulses from $\pi \to e$ decays. It is constructed from 159 pulses, of which, in the opinion of the authors, 39 ± 15 are background.

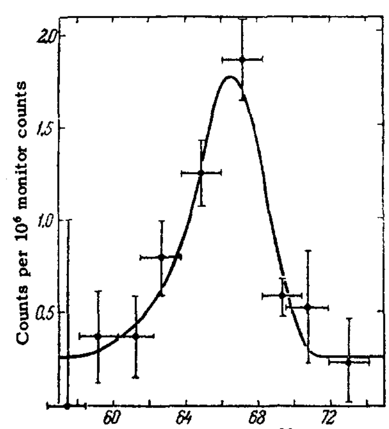


FIG. 29. Energy distribution of electrons from $\pi \to e$ decays in the experiment of Anderson et al.[93] The abscissa is the electron energy (from current in spectrometer winding), and the ordinate is the number of $\pi \to e$ decays.

We remark that in this same work with the spectrometer the energy spectrum was measured for the electrons from $\mu \to e$ decay; this spectrum is shown in Fig. 30. The value of the parameter ρ found from this spectrum is (cf. Table I)

$$\rho = 0.74 \pm 0.03.$$

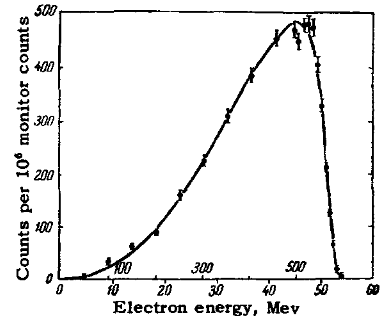
These authors obtained for the relative probability of $\pi \to e$ decay the value

$$\omega = (1.03 \pm 0.20) \cdot 10^{-4},$$

which agrees with the value 1.14×10^{-4} predicted by the theory when radiative corrections are included.[97]

Thus these researches have shown both the existence of $\pi \to e$ decay and agreement of the probability

FIG. 30. Spectrum of electrons from $\mu \to e$ decays in the experiment of Anderson et al.[93]



of such decay with the predictions of the theory of the universal four-fermion interaction with vector and axial-vector terms.

6.2. The Radiative β Decay of the π Meson

Besides the electron decay of the π meson that we have just considered, $\pi \to e + \nu$, there must also exist so-called radiative $\pi \to e$ decay, i.e., decay in which a γ-ray quantum is emitted in addition to the electron and neutrino

$$\pi \to e + \nu + \gamma.$$

The relative probability of radiative $\pi \to e$ decay

$$\rho = \frac{w(\pi \to e + \nu + \gamma)}{w(\pi \to \mu + \nu)}$$

has been calculated in references 95, 96, and 97. This process is forbidden for the scalar interaction but allowed for the tensor interaction, for which its probability ρ is of the order of 10^{-3}. In the paper by V. G. Vaks and B. L. Ioffe the quantity ρ has been calculated on the basis of the universal weak interaction of the form proposed by Feynman and Gell-Mann, i.e., on the assumption of the vector and axial-vector interaction terms with equal and opposite coefficients. In this case the quantity ρ must be of the order of 5×10^{-6}. An experimental search for this process has been made by Cassels and others,[98] working at the Liverpool cyclotron. A diagram of the apparatus is shown in Fig. 31.

After being slowed down, a π^+ meson beam was stopped in a polyethylene target. Radiative $\pi \to e + \nu + \gamma$ decays were registered by counter coincidences 1, 2, 3, $\bar{4}$. The pulse in counter 1 was produced by the

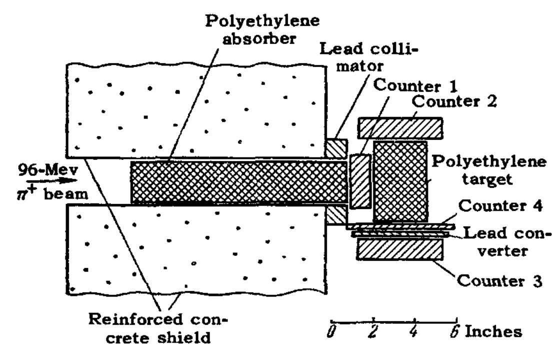


FIG. 31. Experiment of Cassels et al.[98]; search for $\pi \to e + \nu + \gamma$ decay.

π^+ meson passing through this counter. Counter 2 served to register the decay electron. The system of counters 3, 4 with the lead converter placed between them was designed to register γ-ray quanta, which would not cause a flash of light in counter 4 (anticoincidence $\overline{4}$) but would produce an electron in the converter, after which the electron would make counter 3 operate. The coincidence of pulse 1 with 2, 3, $\overline{4}$ occurred in the time interval from 11 to 62 mμsec after the operation of counter 1. After about 75 hours of operation of the apparatus about 110 pulses 1, 2, 3, $\overline{4}$ had been registered, but most of them must be attributed to the background of accidental coincidences. The direct result of the experiment is the number of coincidences 1, 2, 3, $\overline{4}$ per pulse in counter 1, as computed after the background was subtracted. This quantity, in turn, is equal to the product of the desired probability ϱ by the efficiency η of the system for the registration of $\pi \to e + \nu + \gamma$ decays. The measured value was

$$\eta\varrho = (0.16 \pm 0.29)\cdot 10^{-7}.$$

The value of η as estimated by the authors was

$$\eta = 6\cdot 10^{-3},$$

from which we have for the value of ϱ

$$\varrho = (3 \pm 5)\cdot 10^{-6}.$$

This result means that with a sensitivity of the apparatus of the order of 5×10^{-6} radiative decay was not observed; that is, the upper limit on the quantity ϱ is close to 10^{-5}. This result does not agree with the expected value of ϱ for the T interaction term, and gives evidence in favor of the V-A type of interaction; thus it is yet another confirmation, albeit an indirect one, for the universal-interaction theory of Feynman and Gell-Mann.

6.3. Search for $\mu \to e + \gamma$ Decays

Several researches, with increasing sensitivities, have been devoted to the search for $\mu \to e + \gamma$ decays.[101,100,102,103] The existence of this branch in the decay of the μ meson follows from theoretical considerations associated with the possible existence of an intermediate particle — a charged boson of spin 1 (a "vector" boson). It is well known that the weak interaction of four fermions is described in the theory as the interaction of two currents corresponding to the two pairs of fermions; Feynman and Gell-Mann[7] have surmised that the V-A interaction may occur not locally, but by transfer of these currents by a heavy charged boson. The existence of such an intermediate particle leads to a number of consequences for $\mu \to e$ decay. For example, the Michel parameter is increased from the value $\rho = 0.75$ to $\rho \cong 0.75 + \frac{1}{3}(m_\mu/m_B)^2$, where m_μ and m_B are the masses of the μ meson and the charged boson; the lifetime of the μ meson is shortened in the ratio $1:[1 + \frac{3}{5}(m_\mu/m_B)^2]$ in comparison with that predicted by the V-A interaction theory of Feynman and Gell-Mann, and there is a possible decay $\mu \to e + \gamma$, with a relative probability given by the theory as about

$$w = \frac{\mu \to e + \gamma}{\mu \to e + \nu + \bar{\nu}} \sim 10^{-4}.$$

In work by Meshkovskiĭ and others[100] a search for such a decay was made by means of a bubble chamber filled with freon. With 10,000 stoppings of μ^+ mesons in the chamber, and an efficiency for detecting the γ ray from $\mu \to e + \gamma$ decay of about 25 percent, they did not find a single decay that could be reliably interpreted as a $\mu \to e + \gamma$ decay, so that it follows that the probability of such a decay has the upper limit $w < 4 \times 10^{-4}$.

In work by Lokanatan and Steinberger,[101] which has been only briefly mentioned in the literature, there was also a failure to find $\mu \to e + \gamma$ decay, and an analysis of the sensitivity of the apparatus and the experimental errors allowed the authors to fix as an upper limit on w the value $w \leq 2 \times 10^{-5}$.

The general arrangement of the apparatus of Davis and others,[102] who also looked for $\mu \to e + \gamma$ decay, is shown in Fig. 32. It consists of a carbon target S, in which 32-Mev π^+ mesons are stopped, and two counters: a counter for positrons and a counter for γ-ray quanta; coincidences in these counters can register $\mu \to e + \gamma$ decays.

The positron counter consists of two scintillation counters e_1 and e_2 and a water Cerenkov counter e_3, and registers electrons with energies $E \geq 35$ Mev. Its efficiency was determined by placing it in a beam of fast electrons, and was found to be 75 percent for 55-Mev electrons.

The γ-ray counter consists of three scintillation counters, a radiator (a lead plate of thickness 6 mm), and a water Cerenkov counter γ_4, connected for coincidences $\bar{\gamma}_1$, γ_2, γ_3, γ_4. Calibration of the γ-ray counter was carried out in a beam of electrons and

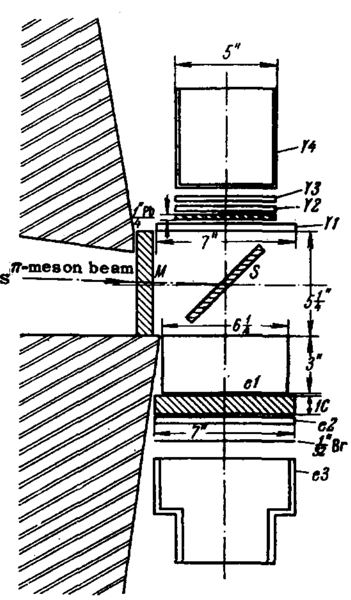


FIG. 32. Experiment of Davis and others:[102] search for $\mu \to e + \gamma$ decay.

also in a beam of γ rays coming from a liquid-hydrogen target in which π^- mesons were absorbed. The electronic circuit registered simultaneously the total count of e-γ coincidences and that of accidental e-γ coincidences, distinguished from the total count by a time delay.

The final result: $w = (1.7 \pm 0.9) \times 10^{-5}$ in one series of measurements and $w = (0.9 \pm 7) \times 10^{-6}$ in another, shows that practically no evidence of $\mu \rightarrow e + \gamma$ decay was found, and that the probability of such decay has the upper limit $w \leq 10^{-5}$.

Work done by Berley and others[103] shifted the upper limit on w considerably further into the domain of small values. The method of this work was analogous to that of Davis and others; two telescopes (one for electrons and one for photons), placed on opposite sides of the target in which the π^+ mesons were stopped, registered coincidences between the electron and the photon produced in $\mu \rightarrow e + \gamma$ decay. In addition to selecting $\mu \rightarrow e + \gamma$ decays, the coincidence circuit was used to obtain photographs of the pulses from the counters of the two telescopes on the screens of two oscilloscope tubes. A particular method used in testing the sensitivity of the apparatus was to check its ability to register ordinary radiative decays $\mu \rightarrow e + \nu + \bar{\nu} + \gamma$, for which purpose the sensitivity threshold of the γ-ray telescope was lowered.

In this work from among 2.6×10^7 decay electrons there were obtained 4 photographs that satisfied the criteria imposed for $\mu \rightarrow e + \gamma$ decay. The expected background of accidental coincidences could give about two such photographs. Thus this work also did not give definite evidence of the existence of $\mu \rightarrow e + \gamma$ decay, but provided a much more sensitive determination of an upper limit on the probability of $\mu \rightarrow e + \gamma$ decay, $w = \dfrac{R(\mu \rightarrow e + \gamma)}{R(\mu \rightarrow e + \nu + \bar{\nu})}$, which was set at 2×10^{-6} with a 90 percent confidence level.

Thus the probability of $\mu \rightarrow e + \gamma$ decay turns out to be much smaller than the value predicted by the weak-interaction theory of Feynman and Gell-Mann as expressed in terms of coupling through an intermediate vector boson; together with the data on the value of the Michel parameter ρ this evidently renders such a possibility doubtful.

7. THE DEPOLARIZATION OF μ MESONS

7.1. Measurements of the Asymmetry Coefficient in Various Substances

The very first experiments on the spatial asymmetry[2] revealed that, despite the expected complete polarization of the μ^+ mesons at the instant of decay, the asymmetry coefficient a in the formula for the angular distribution of the decay positrons, $dN = (1 + a \cos \vartheta)\, d\Omega$, depends on the substance in which the stopping of the mesons occurs. This is a result of the depolarization which the μ^+ meson experiences in the

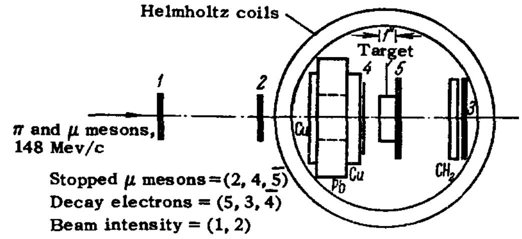


FIG. 33. Arrangement of the experiment of Swanson:[104] measurement of the asymmetry coefficient in various substances.

substance; this has the consequence that in Eq. (1) or Eq. (4) the coefficient a must be replaced by $a\xi$, where ξ is the average value of the polarization of the μ^+ mesons at the instant of decay. For complete depolarization $\xi = 0$. In experiments on the angular correlation in $\pi \rightarrow \mu \rightarrow e$ decay, which have been carried out with accelerators and electronic registering systems, tests have been made on a large number of different substances. The most accurate data are those of Swanson (Chicago group).[104] Just as in the experiment of Garwin and others, the spatial asymmetry of $\mu \rightarrow e$ decays was studied with a system of counters fixed in space, and a magnetic field caused precession of the spin of the μ meson, and along with it, of the axis of the $\mu \rightarrow e$ correlation. A virtue of this method is the constancy of the geometrical conditions for the electrons leaving the target and the counter telescope used to register them. Unlike the Columbia group, Swanson worked with a constant magnetic field, and measured for each $\mu \rightarrow e$ decay in the target the time interval between the entrance of the μ meson and the emergence of the decay electron. Since the spin of the μ meson precesses with constant frequency in the constant magnetic field, each value of the time corresponds to a definite and known orientation of the spin in space. This method uses all of the decays in obtaining the asymmetry coefficient, and therefore secures a much greater statistical accuracy of the measurements than was obtained in the work of the Columbia group. The geometry of the experiment and a block diagram of the electronic apparatus are shown in Figs. 33 and 34. The π^+ mesons of the beam were absorbed in copper filters and the μ^+ mesons

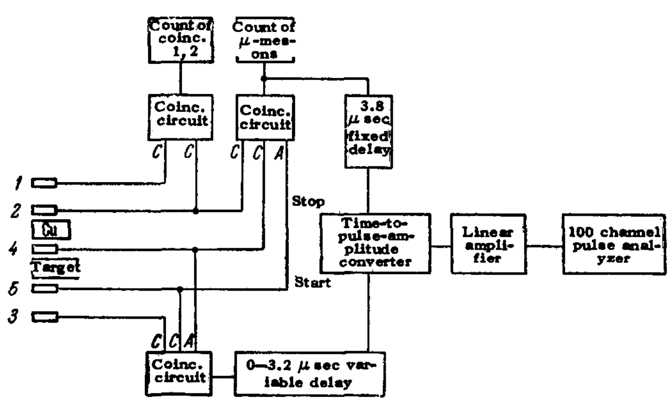


FIG. 34. Block diagram of electronics in Swanson's experiment.[104]

were stopped in the target (area 60 cm^2, thickness 5 g/cm^2), which was in a constant magnetic field produced by Helmholtz coils, where they decayed. Coincidences 1, 2 served for continuous intensity registration ("monitoring") of the meson beam. Registration of the μ mesons stopped in the target was obtained from the coincidences 2, 4, 5. Coincidences 3, 4, 5 registered the decay electrons emerging from the target in the direction of counters 3, 5. The corresponding connections of the counters are shown in the block diagram, where C and A denote coincidence and anticoincidence, respectively. The pulses from the 5, 3, $\bar{4}$ coincidences (positron counts) and the 2, 4, $\bar{5}$ coincidences (μ-meson counts) were used to start and stop the circuit of the time converter, which gave pulses with amplitudes proportional to the time intervals between the pulses 5, 3, $\bar{4}$ and 2, 4, $\bar{5}$.

After linear amplification the pulses from the time converter went to a 100 channel pulse analyzer, which sorted them by amplitude, i.e., by the time of precession, which is uniquely connected with the spatial orientation of the spin of the μ meson.

A similar method for studying the asymmetry has been developed also by the Liverpool group,[78] who worked with μ^+ mesons of energy 110 Mev. Their apparatus, with the arrangement shown in Fig. 35, is very much like that of the Chicago group; it uses the same principle of precession in a constant magnetic field, conversion of the time interval between the entrance of the μ^+ meson and the emergence of the electron into the amplitude of a pulse, and sorting of the amplitudes by a 100 channel amplitude analyzer. The effectiveness of this method for increasing the statistical accuracy of the results can be seen from the following figures. During an hour's operation of the apparatus used by the Chicago group the monitor counters registered 1.5×10^7 coincidences, which corresponded to 10^6 stoppings of μ mesons and 4×10^4 decay positrons. This gave a measurement of the asymmetry coefficient a with an accuracy of ± 0.01.

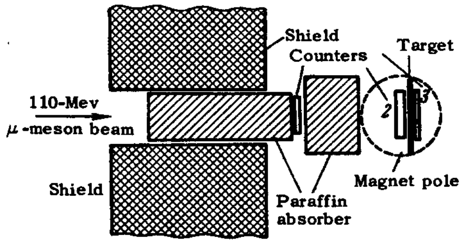


FIG. 35. Arrangement of the experiment of Cassels et al.[78] Measurement of the asymmetry coefficient in various substances and of the magnetic moment of the μ meson.

Table VI shows the values of the asymmetry coefficient a for the substances studied in the experiments we have described. The values of the asymmetry coefficient given in the table have been corrected for the exponential decay of the μ^+ mesons, for the geometrical conditions of the experiments, and for the energy loss and scattering of the decay positrons in the target where the μ^+ mesons were stopped.

The data in the table show that for certain substances — mainly metals and graphite — the asymmetry coefficient has its largest values, in the range 0.22 — 0.28; in other substances the values of the coefficient

TABLE VI. Asymmetry coefficients for μ^+ mesons

Substance	Assymmetry coefficient	Error	Substance	Assymmetry coefficient	Error
Graphite	0.229	0.008	HCl, P_H 1.2	0.158	0.011
"	0.29*	0.05	P_H 11.2	0.141	0.011
"	0.24**	0.04	Polyethylene	0.146	0.012
Lampblack	0.253	0.021	"	0.20*	0.03
Bromoform	0.34*	0.06	Polystyrene	0.070	0.010
Diamond	0.045	0.008	Polystyrene + 2%		
"	0.209	0.010	p-terphenyl	0.06**	0.04
"	0.25*	0.02	Propane	0.170	0.020
"	0.222	0.012	Benzol	0.046	0.012
"	0.201	0.014	Phenylcyclohexane	0.084	0.011
"	0.254	0.013	Chloroform	0.184	0.015
"	0.20*	0.05			
"	0.253	0.012	P	0.025	0.017
"	0.21*	0.06	S	0.014	0.011
"	0.24**	0.02		0.03*	0.06
"	0.213	0.011	CsI	0.031	0.013
"	0.23	0.02	NaCl	0.041	0.009
"	0.022	0.009	MgF_2	0.136	0.009
"	0.03*	0.03	MgO	0.079	0.012
Fused quartz	0.038	0.009	Photographic emulsion	0.087	0.009
Crystalline quartz	0.01	0.02	" "	0.13*	0.02
Water, P_H 6.5	0.141	0.011	AgBr	0.02*	0.02
Distilled water	0.15*	0.03	Gelatin	0.12*	0.03
" "	0.15**	0.05	CCl_4 (liquid)	0.26**	0.05
NaOH, P_H 12	0.131	0.012	Methyl alcohol	0.16**	0.05

Note: Data marked * are from reference 50; those marked ** are from reference 78; the rest of the data are from reference 104.

The values given in the table are subject to uncertainties owing to the kinematic depolarization of the μ^+ mesons, which can be different for beams of different energies and different conditions of collimation.

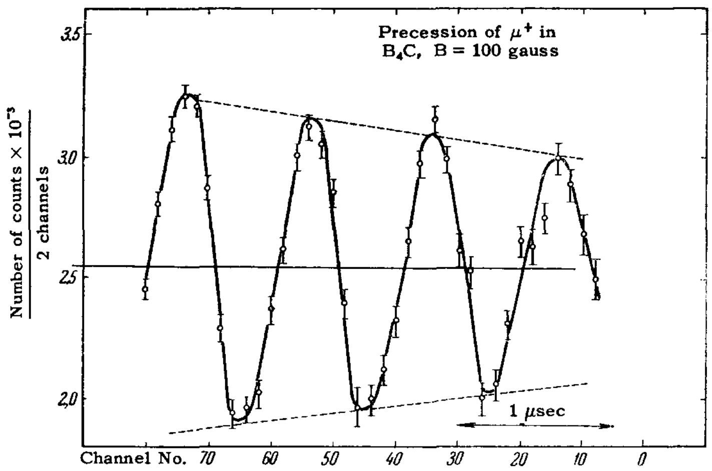


FIG. 36. Asymmetry of $\mu \to e$ decay in boron carbide.

vary over a wide range, and fall to values close to zero in aluminum oxide, silver bromide, and fused quartz, which shows that the μ^+ mesons are almost completely depolarized in these substances.

7.2. The Relaxation Time of the Spin of the μ^+ Meson

In the measurements discussed above estimates were also made of the relaxation time of the spin of the μ^+ meson in the various substances. These estimates showed that the mechanism of depolarization of the μ^+ mesons acts very quickly in all of these substances. This can be seen, for example, from curves like that of Fig. 36. This curve was obtained from measurements in graphite (sic), for which the asymmetry coefficient is large ($a \sim 0.22 - 0.29$). The curve is corrected for decay, and we see that the amplitude of the precession is constant (sic) for a few microseconds of registration of the decay positrons.

Similar curves with constant modulation amplitude were also obtained for the other substances, independently of the measured value of the asymmetry coefficient. The constancy of the amplitude for a few microseconds of observation indicates that the depolarization occurred before the beginning of the registration of decay electrons, i.e., that the relaxation time of the μ^+-meson spin is smaller than $10^{-7} - 10^{-6}$ sec. A similar result has been obtained by the Columbia group.[50] To get an estimate of the relaxation time these authors doubled the magnitude of the magnetic field that rotated the spin of the meson, and at the same time made the delay time and gating pulse only half as large. When this was done there was no change of the amplitude of the intensity modulation, which indicated rapid depolarization. The rapid depolarization in all substances speaks in favor of the existence of some universal mechanism of depolarization that is the same for all substances. This mechanism may possibly be depolarization through the formation of muonium.

The only exception to this general behavior is boron carbide (cf. Fig. 36), for which the asymmetry is close to the maximum possible at the beginning of the cycle ($a = 0.23$) and then decreases in accordance with a relaxation time of 6.5 μsec, which indicates that there is a different mechanism of depolarization in this substance.

7.3. The Asymmetry Coefficient in Photographic Emulsions

The value of the asymmetry coefficient for $\pi^+ \to \mu^+ \to e^+$ decays in photographic emulsions has been the object of detailed study. Almost simultaneously with the work of Garwin and others,[2] Friedman and Telegdi[105] showed the existence of an angular asymmetry in $\pi \to \mu \to e$ decays in emulsion. Thereupon many authors made similar measurements. A summary of all known data on the emulsion Ilford-G5, with which almost all foreign authors have worked, is shown in Table VII. This same table contains the data of Vaĭsenberg and others,[113] Gurevich and others,[114] and Ivanov and others,[115] who have worked with the emulsion NIKFI-R.

The data shown in the table are very hard to reconcile with a single mean value. This conclusion, which seems probable on inspection, is confirmed by an analysis of Table VII by the usual χ^2 test for a binomial distribution with 11 degrees of freedom; the value of χ^2 is 25 or 30, if we treat the values of a in the Ilford-G5 and NIKFI-R emulsions all together. The agreement is more acceptable ($\chi^2 \sim 12$) if we confine ourselves to a separate treatment of the data for

TABLE VII. The asymmetry coefficient a in emulsions

Authors	Source of mesons	Emulsion	
Castagnoli[106,107]	Cosmic rays	Ilford-G5	0.20 ± 0.04
Fowler[108]	” ”	”	0.03 ± 0.04
Bøggild[109]	” ”	”	0.21 ± 0.04
Bhowmik[110]	” ”	”	0.081 ± 0.044
Babayan[55]	” ”	”	0.135 ± 0.040
Biswas[111]	Accelerator	”	0.095 ± 0.045
Davis[112]	”	”	0.190 ± 0.060
Friedman[105]	”	”	0.174 ± 0.045
Chadwick[116]	”	”	0.149 ± 0.033
Gurevich[114]	”	NIKFI-R	0.092 ± 0.018
Vaisenberg[113]	”	”	0.066 ± 0.018
Ivanov[115]	”	”	0.065 ± 0.041

TABLE VIII

NIKFI-R accelerators	Ilford-G5		
	all data	cosmic rays	accelerators
–0.077 ± 0.012	–0.139 ± 0.014	–0.133 ± 0.018	–0.148 ± 0.021

the Ilford-G5 emulsion. The data for the NIKFI-R emulsion also agree fairly well among themselves. Table VIII shows weighted average values of the asymmetry coefficient for the two emulsions.

Thus it is very probable that μ^+ mesons experience a much greater (sic) depolarization in NIKFI-R emulsion than in Ilford-G5 emulsion; the ratio of the depolarizing properties of these two emulsions is close to

$$\frac{0.139 \pm 0.014}{0.077 \pm 0.012} = 1.84 \pm 0.33.$$

It follows from the data of Table VI that silver bromide almost completely depolarizes the μ mesons that are stopped in it. Therefore the difference that is observed between the asymmetry coefficients for the two types of emulsion must be ascribed either to the different properties of the gelatins in the Ilford-G5 and NIKFI-R emulsions, or to a different distribution of the stoppings of μ^+ mesons between the gelatin and the AgBr in the two emulsions. The latter possibility is very improbable, since the degree of dispersion of the AgBr crystals and the chemical composition is almost exactly the same in the two emulsions.[117,118]

7.4. The Asymmetry Coefficient in Diluted Emulsion

The first mention of the fact that an increase of the gelatin content in an emulsion leads to an increase of the asymmetry coefficient is due to Chadwick and others,[116] who got for Ilford-G5 emulsion diluted twofold the value $a = -0.190 \pm 0.033$, which is to be compared with the average value $a = -0.139 \pm 0.014$ for Ilford-G5 emulsion.

Similar measurements for NIKFI-R emulsion, made in references 113, 114, give

$$a_{\text{NIKFI-R}\cdot 2} = -0.127 \pm 0.028.$$

Thus twofold dilution increases the asymmetry coefficient in NIKFI-R emulsion by a factor

$$\frac{0.127 \pm 0.028}{0.077 \pm 0.012} = 1.65 \pm 0.40.$$

Experiments with diluted emulsion confirm the idea that the component of the emulsion in which the polarization of μ^+ mesons is to a large extent preserved is the gelatin.

We have considered all the known experimental data on the values of the asymmetry coefficient in various substances and in photographic emulsions. We now go on to the question of the mechanism of depolarization of μ^+ mesons.

7.5. The Kinematic Depolarization of μ^+ Mesons

In the majority of experiments made by electronic methods the μ^+ mesons are present as a 10 to 15 percent impurity which arises from decays in flight in a beam of π mesons of the same momentum, selected by magnetic analysis. In all these experiments there is an appreciable kinematic depolarization of the μ^+ mesons, which occurs because the μ^+ mesons that are selected by the magnetic field as having the proper momentum and that emerge from the collimator with their directions of motion in a definite and relatively narrow range of angles have come from decays which, in the rest system of the π^+ mesons, lay in a considerably larger range of angles $\Delta\vartheta$. Since the component of the polarization vector along the axis of the collimator decreases with increase of the angle ϑ, it is obvious that the polarization of the μ-meson beam emerging from the collimator will be less than unity. The quantitative analysis of this problem made in references 119 — 122 leads to the following results.

Let us denote by $\mathbf{u}$ and η the velocity and energy of the π meson in the laboratory system (in units c and $m_\pi c^2$), and by $\mathbf{v}$, $\mathbf{v}'$ and ϵ, ϵ' the velocity and energy of the μ meson (in units c and $m_\mu c^2$) in the π-meson system and the laboratory system, and let us denote the corresponding unit vectors by $\mathbf{v}_1$, $\mathbf{v}_1'$, etc. Then if in the π-meson system the μ meson was completely polarized along its momentum, in the laboratory system its polarization will be

$$\xi' = \frac{\eta v \varepsilon}{v'\varepsilon'}\left(1 + \frac{\mathbf{v}_1\mathbf{u}}{v}\right),$$

where $\mathbf{v}_1\mathbf{u} = u\cos\vartheta$ and ϑ is the angle between the direction of motion of the π meson and the direction of emission of the μ meson in the laboratory system. After some simple kinematical transformations this same quantity can be put in the form

$$\xi' = \frac{v + \mathbf{v}_1\mathbf{u}}{|\mathbf{u} + \mathbf{v}\cdot\mathbf{u}_1 u + \eta^{-1}(\mathbf{v} - \mathbf{v}\cdot\mathbf{u}_1\mathbf{u}_1)|}$$

and also in the form

$$\xi' = \frac{1}{v'v}\left(1 - \frac{\eta}{\varepsilon'\varepsilon}\right),$$

where the speed and energy of the μ meson in the π-meson system are $v = 0.27$ and $\epsilon = 1.04$.

Let us consider the following special cases: 1) Let $\mathbf{v}_1\mathbf{u} = u$, i.e., the μ meson is emitted in the direction of motion of the π meson ($\cos\vartheta = 1$); then $\mathbf{v}\mathbf{u}_1 = v$, and $\xi' = v + u\,|\mathbf{u} + \mathbf{v}|^{-1} = 1$, so that a μ meson emitted forward completely retains its polarization.

2) In just the same way, if the μ meson is emitted backward, then

$$\xi' = \frac{v - u}{|\mathbf{u} - \mathbf{v}|} = -1,$$

so that a μ meson emitted backward ($\cos\vartheta = -1$) has polarization of the opposite sign. These two limiting cases correspond to the minimum and maximum energies of the π meson

$$\eta_{\min} = \varepsilon'\varepsilon(1 - v'v),$$
$$\eta_{\max} = \varepsilon'\varepsilon(1 + v'v).$$

3) For what follows it is interesting to determine the value of the angle ϑ in the π-meson system for

which $\xi' = 0$, i.e., the angle of emission which corresponds to transverse polarization of the μ meson in the laboratory system.

The formula for ξ' shows that $\xi' = 0$ for $\cos\vartheta = -v/u$. To this angle ϑ there corresponds in the laboratory system the angle ϑ' given by the relation

$$\tan\vartheta' = \eta^{-1}\frac{\sin\vartheta}{\cos\vartheta + \frac{u}{v}} = \eta^{-1}\frac{\sqrt{1-\frac{v^2}{u^2}}}{\frac{u}{v}-\frac{v}{u}}.$$

We note that for given u this is the maximum possible angle of emission in the laboratory system (the "threshold" angle). The μ mesons emitted at this angle are transversely polarized.

We now assume that we register μ mesons with prescribed values ϵ' and v' of the energy and speed. Such μ mesons can arise from π mesons with energies in the range from η_{min} to η_{max}, and the average polarization $\langle\xi'\rangle$ of the μ mesons registered on stopping can be obtained by integrating the product of the expression for ξ' and the π-meson spectrum $N(\eta)$ between the limits η_{min} and η_{max}. Here two limiting cases are of interest. In the first case, which occurs with cosmic rays, the geometry of the experiment is such that it allows the registration of μ mesons that are emitted over the range of angles from 0 to π in the rest system of the π meson, which corresponds to the energy range from η_{min} to η_{max}. In this case, assuming that the cosmic-ray π-meson spectrum is described by a power law

$$N(\eta)\,d\eta \doteqdot \eta^{-\gamma}\,d\eta,$$

and averaging the expression for ξ' over this spectrum over the range from η_{min} to η_{max}, we get for the average value of the μ-meson polarization the expression

$$\langle\xi'\rangle = \left\{\frac{1}{3}\gamma(v'v) + \text{terms}\sim(v'v)^3 + \dots\right\}.$$

Neglecting the terms of third order in vv' and setting $\gamma = 2.5$, $v' = 0.27$, and $v \sim 1$, we get $\langle\xi'\rangle = 0.23$, from which it follows that the cosmic-ray μ mesons retain just about $1/4$ of their polarization.

Experimental studies of the polarization of cosmic-ray μ mesons have been made in references 121, 123, 124, by examining the asymmetry of the spatial distribution of the decay electrons that emerge from an absorber in which the μ mesons are stopped. These experiments agreed with the cosmic-ray μ-meson polarization expected on the basis of the considerations we have presented.

The other extreme case occurs in experiments with accelerators. Magnetic analysis and careful collimation of the μ mesons have the result that the μ mesons stopped in the target of the apparatus originate from π mesons with energies in a relatively small energy range from η_{min} to $\eta_{min} + \Delta\eta$. This means that one is using not all angles from 0 to π in the rest system of the π meson, but only a narrow range of angles around $\vartheta = 0$. In this case the change of intensity in the range $\Delta\eta$ can be neglected, and integration of the expression for ξ' over this range gives

$$\langle\xi'\rangle = 1 - \frac{\Delta\eta}{2\varepsilon'\varepsilon v'v}.$$

We have $\epsilon v = 1.04 \times 0.27 = 0.28$. Furthermore, in the experiments on the asymmetry that we have considered the energy of the μ mesons was close to $\epsilon' = 2.1$, and their speed was close to $v' = 0.9$.

Thus

$$\xi' = 1 - \frac{\Delta\eta}{0.28\cdot 2.1\cdot 0.9} \simeq 1 - 2\Delta\eta,$$

and from this we get, for example for $\Delta\eta = 0.1$, 80 percent polarization instead of 100 percent, i.e., a 20 percent loss of polarization.

An upper limit on the experimentally observed kinematic depolarization can be obtained from the data of Table VI if we assume: 1) that there is no depolarization in metals, and 2) that the maximum asymmetry coefficient is $a_{max} = -1/3$. Then the value of the polarization ξ' will be given by

$$\xi' = \frac{a}{-\frac{1}{3}},$$

where a is the average value of the asymmetry coefficient for metals and graphite. To get this quantity we form from the data of the table a weighted average $\langle a\rangle$ for graphite, lampblack, Al, Be, Li, Mg, and Si from the measurements of the Chicago group.[104] We get $\langle a\rangle = -0.230 \pm 0.005$, from which we get for the average polarization of the μ-meson beam (lower limit) $\xi' = 0.23/0.33 = 0.70$, with accuracy about 2 percent. Thus in experiments with accelerators the kinematic depolarization can amount to some tens of percent.

7.6. The Depolarization of μ^+ Mesons in Matter

The depolarization of a μ^+ meson in a medium in which it is stopped can be divided into two stages: the depolarization in flight, which occurs during the process of slowing down of the μ^+ meson, and the depolarization that occurs after the meson has been very much slowed down. The depolarization in flight, caused by scattering of the mesons by the Coulomb fields of nuclei and electrons, has been calculated in several papers,[125-127] whose authors come to the conclusion that this type of depolarization can be neglected. For example, Wentzel[126] got the following expression for the mean square angle of rotation of the spin caused by Coulomb scattering by atomic nuclei during the slowing down of the particle until it is at rest:

$$\langle\vartheta^2\rangle = \frac{7}{32}\frac{m_e}{m_\mu}\left(\frac{v}{c}\right)^4.$$

For μ^+ mesons arising from $\pi\to\mu$ decays in emulsion ($v/c = 0.27$) this quantity is negligibly small, and even for $v/c \sim 0.8 - 0.9$, which corresponds approximately to the speeds of mesons in counter experiments with accelerators, $\langle\vartheta^2\rangle$ is very small.

The depolarization of μ^+ mesons after they have been slowed down is due to two causes: the action of magnetic fields present in the substance on the magnetic moment of the free μ^+ meson, and the formation of atomic systems, in particular of muonium. Let us consider the first process. A magnetic field perpendicular to the spin of a meson makes the spin precess with the Larmor frequency

$$\omega = g\frac{eH}{2m_\mu c} \cong 10^5 H \,\text{rad/sec}.$$

from which it follows that during the mean lifetime of a μ^+ meson, which is about 2×10^{-6} sec, a constant magnetic field of 5 gauss produces complete depolarization ($\omega\tau \sim 1$). It is obvious that internal magnetic fields that may be strong but that last extremely short times and change direction chaotically will have much weaker effects. For example, let us consider the depolarization of a μ^+ meson in a gas that is at a pressure of 100 atm. Let us estimate the depolarization of a μ^+ meson in collisions with atoms of the molecules of the gas, after it is at thermal velocities. During such a collision the μ^+ meson is acted on by the magnetic fields of the electrons, and the angle of precession is $\vartheta_0 = 10^5 H\tau$. For the collision time τ we take the time of flight of the μ^+ meson through a Bohr radius, $\tau \cong 10^{-8}/10^6 = 10^{-14}$ sec, and for H we take the field of an electron at the distance of the Bohr radius ($H \sim 10^5$ gauss). We get for the angle of precession during one collision $\vartheta_0 = 6 \times 10^{-5}$ rad. The average precession angle during the lifetime of the μ^+ meson will be $\sim \vartheta_0 n^{1/2}$, where n is the number of such collisions. In a gas at 100 atm pressure there are $10^{11} - 10^{12}$ collisions per second, which gives 10^6 collisions during the lifetime. Thus $\vartheta \sim 10^{-4}$ rad (sic). We see that the local fields of the electrons in the gas cannot produce an appreciable depolarization. Let us now consider the other mechanism of depolarization, which consists of the formation of muonium — a hydrogen-like neutral atom made up of a μ^+ meson and an electron. It is well known that when the speed of a charged particle passing through matter falls to a value close to the speed of the orbital electrons the particle can capture and then lose electrons. This phenomenon of charge transfer was studied in early experiments by Rutherford and P. L. Kapitza with α particles. Let us consider a μ^+ meson being slowed down in a gas. When its speed becomes comparable with that of the electrons in the gas molecules, i.e., at an energy

$$E \sim \frac{e^2}{a_0}\frac{m_\mu}{m_e} \cong 6 \cdot 10^3 \text{ ev},$$

it can capture an electron and form muonium. The capture cross section is of the order of πa_0^2, and the cross section for ionization of muonium (loss of the electron) is of the same order of magnitude. Thus during the slowing down of a μ meson repeated charge transfer occurs, through successive captures and losses of electrons, just as in the slowing down of protons, α particles, or multiply charged ions. The final result is that during their slowing down μ^+ mesons get out of the range of energies in which the formation of muonium is possible in a very short time, and reach thermal speeds, where they "await" decay either in the free state or in muonium atoms. From this obviously highly simplified picture it follows that the depolarization of the μ^+ meson can be related in an important way to the formation of muonium. Therefore let us consider the properties of this system. It is obvious that as long as we neglect the fine-structure and hyperfine-structure interactions in muonium and the difference in mass between the meson and proton (infinitely heavy nucleus) the properties of this atom are like those of the hydrogen atom. Let us suppose that the muonium is formed only in the 1S state, in which the spin-orbit interaction that leads to the fine structure of the terms is absent, and consider the effect of the interaction of the spin of the nucleus (the μ^+ meson) with that of the orbital electron (which leads to the hyperfine structure of the terms) on the initial polarization of the μ^+ meson. Owing to this interaction the 1S state of muonium splits up into $^1S_0$ and $^3S_1$ states, whose energies differ by the hyperfine-structure interaction energy, given by the well known formula of Fermi:

$$\Delta W = -\frac{32\mu_e\mu_\mu}{3a_{e\mu}^3} = 1{,}84 \cdot 10^{-5} \text{ ev } = h \cdot 4.44 \cdot 10^3 \text{ Mcs}.$$

Here the μ's are the magnetic moments of the electron and the μ meson and $a_{e\mu}$ is the radius of the Bohr orbit of muonium, which is practically equal to the radius of the Bohr orbit of the hydrogen atom. Suppose that, as is almost always the case in the experiments we have considered, the μ^+ meson is completely polarized, and that the corresponding spin function is α_μ. For the unpolarized electrons of the medium there are functions α_e and β_e that correspond to spin directions parallel and antiparallel to that of the spin of the μ^+ meson. Then the muonium atom can be formed with equal probabilities in states with parallel spins ($\alpha_\mu\alpha_e$) and with antiparallel spins ($\alpha_\mu\beta_e$). The first of these states is a pure triplet state of muonium ($^3S_1$, m = 1); the second state is twofold degenerate and is a superposition of the states $^1S_0$ and $^3S_1$ with m = 0. It is obvious that the state $^3S_1$ (triplet muonium) preserves the polarization of the μ^+ meson, provided that the action of external fields does not lead to a spatial reorientation of the triplet atom, which is very sensitive to such action, since it has a large magnetic moment, equal to that of the electron (more exactly to the sum of the magnetic moments of the electron and the μ meson). In the second state of the muonium atom, described by a superposition of the states $^1S_0$ and $^3S_1$, the meson will be completely depolarized in a very short time. In fact, such a system will make periodic transitions between the states $^1S_0$, m = 0 and $^3S_1$, m = 0. The frequency of these transitions is determined by the hyperfine-structure interaction energy, $\omega = \Delta W/\hbar$ rad/sec. If at the initial time the muonium is in the state $^3S_1$, $m = 0$, then after a time $\sim 10^{-10}$ sec it goes over into the state $^1S_0$, $m = 0$, and the mean polarization of the meson will oscillate in time according to the law $\langle S(t) \rangle = \frac{1}{4}(1 + \cos \omega t)$. Thus the state $\alpha_\mu \beta_e$ will be completely depolarized in a time $\sim 10^{-10}$sec. From the point of view of these ideas half of the μ^+ mesons that have formed muonium are completely depolarized, and the other half keeps its polarization. If f is the fraction of the μ^+ mesons that has reached thermal speeds in the form of muonium, the asymmetry coefficient must be diminished from its maximum value a_0 to the value $a_0(1 - f/2)$. Therefore even if all of the μ^+ mesons form muonium $(f = 1)$, the asymmetry coefficient can be diminished only to half of its maximum value. Even a cursory inspection of the data of Table V shows, however, that a can be much smaller than half of the maximum value. There are many causes which, even if the picture we have examined of the depolarization through the formation of muonium is correct, can still reduce the asymmetry coefficient to very small values. First of all, in dense media, where the mobility of the muonium is small, the triplet muonium, which is responsible for the half of the polarization that is preserved, can be depolarized by local fields, or else can get converted in the singlet state, as happens with triplet positronium.[128] Another mechanism causing additional depolarization can be exchange of the electron from triplet muonium with an electron from the surrounding medium, which contains unpolarized electrons. At each such exchange there is a probability $\frac{1}{2}$ for preservation of the polarization of the μ^+ meson, and an equal probability for depolarization. A sequence of a small number of such exchanges can almost destroy the polarization of the triplet muonium and bring the asymmetry coefficient down to very small values. We note that in metals, where such exchanges occur at a high frequency, much exceeding the frequency of the hyperfine-structure interaction, there must be no depolarization. In fact, if the time interval between two successive exchanges is smaller than the time for transition between the $^1S_0$ and $^3S_1$ states, depolarization does not have time to happen. This means for practical purposes that in metals muonium is not formed as a stable system.

7.7. Attempts to Observe the Triplet State of Muonium

In papers by Swanson and by Cassels and others[104,144] attempts have been made to observe the existence of the triplet state of muonium in substances in which μ^+ mesons have been stopped. The idea of the experiments is as follows. In the triplet state the muonium atom has a magnetic moment equal to that of the electron (if we neglect the magnetic moment of the μ meson). This means that in a field that is about one one-hundredth of that used for the precession of the free μ meson triplet muonium will precess with a frequency close to that of the precession observed with free μ mesons. After reducing the field to 0.4 gauss, Swanson[104] looked for the precession of muonium in fused quartz, liquid silicon, and teflon. The Liverpool group made studies in a field of 0.7 gauss on polystyrene + 2% p-terphenyl, liquid carbon tetrachloride, methyl alcohol, and water. In these experiments liquids were used as targets in the hope that in liquids the action of local magnetic fields capable of depolarizing triplet muonium would be less prominent than in solids. In none of these substances could a modulation of the number of decay electrons be observed; Swanson's experiments showed that the amplitude of the modulation corresponding to triplet muonium was 0 ± 8 percent of that corresponding to the precession of the free μ^+ meson in graphite. The experiments of the Liverpool group gave for this quantity the value 0 ± 2 percent.

An experimental test of the question of the existence of triplet muonium can also be based on somewhat different considerations. Let us consider the asymmetry coefficients in a photographic emulsion as measured by two methods: 1) following individual trajectories in the emulsion, and 2) electronic methods, with the emulsion as the target. Let us suppose that the triplet state of muonium exists, that the probability for the formation of muonium after stopping is f, and that the value of the asymmetry coefficient, not affected by depolarization, is a. Then in case 1) the asymmetry coefficient measured in the emulsion is given by $a_1 = a_0(1 - f/2)$. In case 2), when the electronic system is set for the detection of the precession of the free μ^+ meson, the triplet muonium eludes observation, and the asymmetry coefficient measured in such an experiment is given by $a_2 = a_0(1 - f)\xi$, where ξ is the mean depolarization of the μ^+ mesons incident on the target. On the assumption that the maximum value is $a = -\frac{1}{3}$, we have found for ξ the value $\xi = 0.70 \pm 0.014$. From Table VI we have for the quantity a_2 the value $a_2 = 0.087 \pm 0.009$, and after correction for the kinematic depolarization this gives as an upper limit the result

$$a(1 - f) = 0.126 \pm 0.013.$$

Although this is indeed somewhat smaller than the quantity $a_1 = -0.14$ for Ilford-G5 emulsion, as obtained by the method of trajectories, because of insufficient statistical accuracy of the two quantities it does not seem possible to draw definite conclusions. Finally, let us note one more fact, which evidently speaks against the existence of triplet muonium. We have seen above that a field of several gauss, acting during the lifetime of the μ^+ meson, causes depolarization. Obviously for triplet muonium, with a magnetic moment 200 times that of the meson, depolarization will be caused by a field as small as several hundredths of a gauss. For just this reason Friedman and Telegdi[105] and some other investigators have observed μ^+ decays in emulsions irradiated under conditions of careful magnetic screening $(H < 10^{-3}$ gauss). No significant difference has been found between the values of the asymmetry coefficient measured under these conditions and under conditions of irradiation without screening, with stray fields of some tenths of a gauss, or in cosmic rays affected by the earth's field (see in particular Table VII).

Thus the data that have been presented indicate that there is no appreciably intense triplet state of muonium. This could be so either if the general idea of depolarization through the formation of muonium does not correspond to reality and muonium is not formed, or if some mechanism acts that leads to rapid depolarization of the triplet state.

7.8. Preservation of Polarization by Means of a Magnetic Field

Here we shall examine to what extent available experimental data agree with the hypothesis that the fundamental mechanism responsible for depolarization is the formation of muonium. The first serious argument which, if it does not directly support just this hypothesis, at any rate supports the existence of some mechanism of depolarization that is universal for all substances, is found in the small time of depolarization ($\sim 10^{-6} - 10^{-7}$ sec) of the free μ^+ meson that has been established from experiments that measure the asymmetry. Depolarization because of the hyperfine-structure interaction in muonium occurs in a time of about 10^{-10} sec, and thus the experimental data on the depolarization time agree with the hypothesis of depolarization through the formation of muonium. A more direct confirmation of this hypothesis has been obtained in experiments on the restoration of polarization by means of a magnetic field. The idea of these experiments is to produce the Paschen-Back effect in muonium. If the substance in which the muonium is formed is in a magnetic field that is so strong that the energy of the electron in the field is much larger than the energy of the interaction between the magnetic moments of the μ meson and electron in muonium, then, as is well known, the coupling between the two spins is broken and they precess separately around the field. The interaction energy for the hyperfine structure can be written in the form

$$W = 2(\mu_e - \mu_\mu) H_0,$$

where μ_e and μ_μ are the magnetic moments of the electron and the μ meson and H_0 is a magnetic field characteristic for muonium, of the order of the magnetic field that the μ^+ meson produces in the region of the first Bohr orbit, where the electron is. Since for muonium the energy W is 1.85×10^{-5} ev, knowing the magnetic moments of the electron and μ meson, we find $H_0 = 1580$ gauss. When the external magnetic field is much larger than this quantity the coupling between the spins is broken and there will be no depolarization because of the hyperfine-structure interaction.

In the intermediate case of not too large fields the state of the muonium atom in a magnetic field is a mixture of the field-free singlet and triplet states. This means that in the state $m = 0$, which in the absence of a magnetic field is characterized by complete depolarization, there is now, in the magnetic field, a certain polarization, which owes its origin to the admixture of the triplet state. The quantum-mechanical calculation based on perturbation theory has been done in a paper by Ferrell[128] and shown that the mean value of the polarization of the μ^+ meson increases in the following way with the strength of the applied magnetic field:

$$\Delta p = \frac{f}{2} \frac{x^2}{1+x^2}.$$

In this formula f is the fraction of mesons that have formed muonium atoms in the ground state, and $x = H/H_0$ is the ratio of the applied magnetic field to the magnetic field $H_0 = 1580$ gauss that characterizes the hyperfine-structure interaction in muonium. In the strong-field limit $H \gg H_0$ the polarization is completely restored. This corresponds to the Paschen-Back effect. On the basis of this result it is not hard to find how the asymmetry coefficient varies as the magnetic field is increased. It is obvious that

$$a(H) = a_0 \left\{ \left(1 - \frac{f}{2}\right) + \frac{f}{2} \frac{x^2}{1+x^2} \right\}$$
$$= a_0 \left\{ 1 - f + f\left(\frac{1+2x^2}{2(1+x^2)}\right) \right\}; \qquad (11)$$

here a_0 is the maximum value of the asymmetry coefficient, which it would have in the absence of depolarization through the formation of muonium; $1 - f/2 = 1 - f + f/2$ is the part of the polarization that comes from μ mesons that did not form muonium $(1 - f)$ and from muonium in the triplet state $(f/2)$; and $(f/2)x^2/(1 + x^2)$ is the part of the polarization restored by the magnetic field. From this formula it follows in particular that even if all of the μ^+ mesons stopped in the substance form muonium $(f = 1)$ the asymmetry coefficient cannot be smaller than $a/2$. We have already seen that this does not correspond to reality, and that one possible cause of the discrepancy is exchange of electrons of triplet muonium with unpolarized electrons of the medium. Let the number of such exchanges be n and let the time between two successive exchanges be larger than the time characteristic of the hyperfine-structure interaction but much smaller than the mean lifetime of the μ^+ mesons. For this case Sens and others[130] have suggested a simple generalization of Eq. (11),

$$a = a_0 \left\{ 1 - f + f\left(\frac{1+2x^2}{2(1+x^2)}\right)^n \right\}, \qquad (12)$$

which explains the decrease of a to values much smaller than $a_0/2$.

Studies of the effect of a magnetic field on the polarization of μ^+ mesons have been made both by electronic methods and with photographic emulsions. Sens, Swanson, and others studied the effect of a magnetic field on the polarization of μ^+ mesons by placing emul-

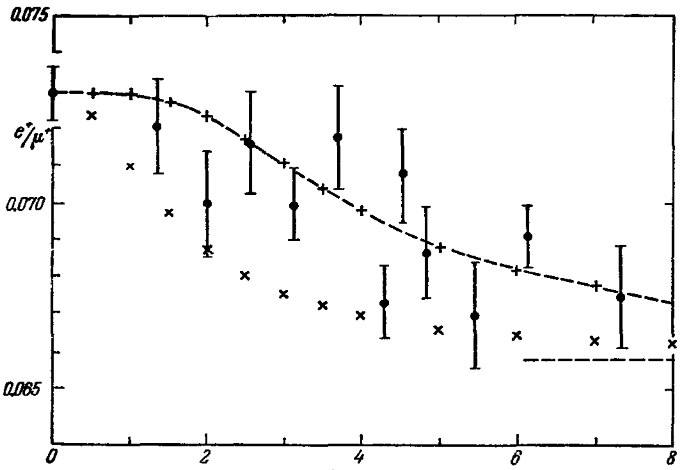


FIG. 37. Results of an experiment on the preservation of the polarization of μ mesons by means of a magnetic field.

sion or vitreosil (fused quartz) in the longitudinal magnetic field of a solenoid and measuring the asymmetry of the electron emission with counters. Their results are shown in Fig. 37. Here the abscissa is the field strength and the ordinate is the number of positrons emitted forward per stopped μ^+ meson. We see an increase of the asymmetry (a decrease of the number of positrons emitted forward) with increase of the magnetic field. The crosses show the behavior of the asymmetry according to Eq. (11), and the dashed line the behavior according to Eq. (12) on the assumption of 11 to 13 exchanges of electrons. As can be seen from the diagram, the best agreement with the experimental data is obtained on the assumption that the muonium atoms have time to exchange electrons with the surrounding medium several times. The results obtained with photographic emulsions placed in magnetic fields are shown in Table IX. These results are to be compared with the asymmetry coefficients without magnetic field, $a = -0.077 \pm 0.012$ for NIKFI-R emulsion and $a = -0.140 \pm 0.014$ for Ilford-G5 emulsion.

7.9. The True Value of the Asymmetry Coefficient

An important deduction from the mechanism of depolarization through muonium formation which we have been discussing is the expectation that the polarization will be completely preserved in magnetic fields that are much larger than H_0. This gives a possibility for finding the value of the asymmetry coefficient in the absence of depolarization. We have seen that the theory of Feynman and Gell-Mann calls for the value $a = -1/3$. Measurements have been made in strong mag-

TABLE IX. Effect of magnetic field on asymmetry coefficient in emulsion

Authors	Field (kilogauss)	Asymmetry coefficient	Emulsion
Heughebaert[131]	0.1	0.22 ± 0.12	Ilford-G5
Gurevich[114]	1.1	0.16 ± 0.04	NIKFI-R
Vaisenberg[113]	2.5	0.20 ± 0.02	"
Orear[132]	9.0	0.249 ± 0.036	Ilford-G5
Barkas[133]	14.25	0.23 ± 0.05	"
Vaisenberg[113]	17.0	0.28 ± 0.02	NIKFI-R
Lynch[134]	25.0	0.296 ± 0.02	Ilford-G5
Gurevich[135]	27.0	0.32 ± 0.02	NIKFI-R

Note: Unlike all the data of Table VIII, the result of Orear and others ($a = -0.249 \pm 0.036$) was obtained by using not $\pi \to \mu \to e$ decays, but $\mu \to e$ decays; therefore to get the "true" value of a one would have to take into account the kinematic depolarization.

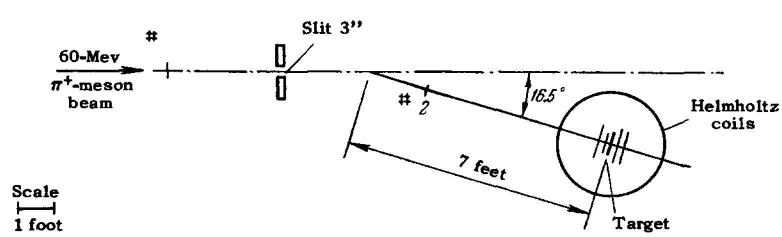
netic fields of 17 to 25 kilogauss.[134,113,133,135] The data of Table IX show that in such fields the asymmetry coefficient is very close to the value $a = -0.33$.

Another experiment for the determination of the limiting value of the asymmetry coefficient has been made by Bardon and others,[136] who used μ^+ mesons emitted near the threshold angle in the decay of 60 Mev π^+ mesons. We have seen that the μ^+ mesons emitted at this angle are transversely polarized. The arrangement of the experiment is shown in Fig. 38. Two counters 2 and 3 (this counter is placed in front of the target) pick out the μ^+ mesons emitted in the angular range $16.5° \pm 0.75°$ relative to the primary beam of π^+ mesons (16.5° is the threshold angle for 60-Mev π mesons). The circle indicates Helmholtz coils which compensate the stray magnetic field. The solenoid used to produce the rotation of the spins of the μ^+ mesons was wound directly on the bromoform target in which the μ^+ mesons were stopped. Scintillation counters registered the decay electrons emitted backward (3, 6, 5) and forward (4, 5, 3) in the time interval 1.5 — 4.5 μsec after the stopping of a μ^+ meson in the bromoform target. The pulse from the stopping of the μ^+ in the target (1, 2, 3, 5) produced in the solenoid a magnetic field of duration 1 μsec, which turned the spin of the meson through $-20°$, so that its spin was directed at one of the electron telescopes. The value obtained for the asymmetry coefficient in this experiment was

$$a = 0.275 \pm 0.011.$$

After the introduction of corrections (which are easy to make in this case) for the small kinematic de-

FIG. 38. Arrangement of the experiment of Bardon and others[136] for determining the asymmetry coefficient in the absence of kinematic depolarization.



polarization arising from the finite aperture of the telescope that registers the μ^+ mesons, for the angular resolution of the electron telescope, and for its sensitivity to the electron spectrum, the writers get for the true value

$$(a) \geqslant 0.325 \pm 0.015,$$

which is very close to the limiting value of a predicted by the two-component theory. An examination of the results of experiments with emulsions in magnetic fields together with the results of this experiment shows that the asymmetry coefficient is actually very close to the limiting value $a = -1/3$ predicated by the theory of the universal V-A interaction, although of course the accuracy of the experiments is insufficient for the assertion that there is complete agreement of the measured values with the theoretical predictions.

It must be remarked that an examination of Table IX reveals a monotonic increase of the asymmetry coefficient as we go from a field of 17,000 gauss to a field of 27,000 gauss. If the Paschen-Back effect occurs, then for muonium the effect of a field of 17,000 gauss should be practically no different from that of a field of 27,000 gauss. Therefore it is important to get statistically more accurate data in this range of field strengths.

7.10. The Depolarization of μ^- Mesons

In the first work of Garwin and others[2] it was shown that the asymmetry coefficient for μ^- mesons stopped in graphite is of the same sign as that for μ^+ mesons but has the much smaller numerical value $a \cong -1/20$, which is about 15 percent of the asymmetry coefficient for μ^+ mesons.

Ignatenko and others[107] have made similar measurements, using targets of liquid hydrogen, graphite, oxygen (H_2O), magnesium, sulfur, zinc, cadmium, and lead to stop the μ^- mesons. Their results are shown in Table X.

We see that hydrogen completely depolarizes the μ^- mesons stopped in it, and the other substances depolarize μ^- mesons about 5 to 6 times as strongly as they depolarize μ^+ mesons. As is well known, the decay of a μ^- meson in matter takes place while it is in a mesic atom, and therefore the mechanism of its depolarization must be very different from that for μ^+ mesons. The time spent in matter by a μ^- meson can be considered in terms of the following sequence:

1. Slowing down to speeds at which capture into an orbit of a mesic atom is possible.
2. Transitions from the region of the continuous spectrum into a bound state in one of the distant orbits of a mesic atom.
3. Cascade transitions to lower orbits, ending with a transition into the ground state, from which most of the decays of μ^- mesons occur.

In the first stage the process of depolarization is the same for μ^+ and μ^- mesons, but with the difference that the μ^- meson cannot form muonium. The second stage occurs in a very short time. Therefore we can assume that these two stages do not give rise to any appreciable depolarization. Owing to the large mass of the μ^- meson its capture into a distant orbit of a mesic atom occurs at a large quantum number of the order of $(m_\mu/m_e)^{1/2} \cong 14$. The main depolarization of the mesons indeed occurs in the process of cascade transitions from distant orbits to the K orbit of the mesic atom. This depolarization is caused by the spin-orbit interaction of the μ^- meson and occurs because the time the μ^- meson spends in a given level is much longer than the period of precession of the meson spin. This depolarization has been calculated in references 138 — 140, and the main result of this work is that a completely polarized μ^- meson retains about 17 percent of the polarization when it has reached the K shell. This result holds for all spinless nuclei, for which there is no hyperfine-structure interaction, which would increase the depolarization. Let us examine the data of Table X, which were obtained for substances with nuclei without spin (except H). We see that the average absolute value of the asymmetry coefficient for these nuclei is close to 0.05, whereas for μ^+ mesons the magnitude of the asymmetry coefficient in graphite, where all the depolarization is of the kinematic type, is close to 0.25. Assuming that the kinematic depolarization is the same for μ^- and μ^+ mesons, we find that at the time of its decay a μ^- meson has a polarization close to $(0.05/0.25) = 20\%$ of the initial value, which agrees with the estimate cited. An exception is that μ^- mesons are completely depolarized when they are stopped in liquid hydrogen. The mechanism of this depolarization has been explained in a paper by Zel'dovich and Gershteĭn,[141] who showed that the main contribution to this process comes from scattering of neutral mesic hydrogen atoms by protons,

$$(\mu^-H) + H \to (\mu^-H) + H.$$

The scattering occurs with a large effective cross section for transfer of the μ^- meson to the other proton, with simultaneous transition of the mesic hydrogen atom to a lower state of the hyperfine structure. Owing to the fact that the probability of such transitions in liquid hydrogen (10^9 sec^{-1}) exceeds by three orders of magnitude the probability for decay of the μ^- meson (0.5×10^6 sec^{-1}) in the mesic hydrogen

TABLE X

Substance	$-a$
H_2	0.01 ± 0.01
C	0.040 ± 0.005
O	0.043 ± 0.005
Mg	0.058 ± 0.008
S	0.042 ± 0.006
Zn	0.056 ± 0.011
Cd	0.055 ± 0.012
Pb	0.054 ± 0.013

atom, during the lifetime of the meson these atoms go over entirely into the ground state of the hyperfine structure, and the result is complete depolarization.

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