PROPAGATION OF ACOUSTIC AND INFRARED WAVES IN NATURAL WAVEGUIDES OVER LONG DISTANCES*

L. Brekhovskikh

Usp. Fiz. Nauk **70**, 351–360 (February, 1960)

1. INTRODUCTION

It is well known that acoustic and infrared waves can be propagated in the ocean and in the atmosphere up to very large distances. A number of powerful explosions carried out in the atmosphere at the surface of the earth at different times and different occasions show that the sound of an explosion of a charge ranging in weight from several tons to several thousand tons can be registered at distances from several hundred up to several thousand kilometers (see, for example, refs. 1, 2, 3). It is also known that powerful nuclear explosions in the atmosphere can be recorded at virtually any point on the surface of the earth. In the ocean, the conditions for propagation of sound at long distances is still more favorable. Thus, the sound of an underwater explosion of several kilograms of TNT can be recorded at distances of 5 to 6 thousand kilometers.[4] In one of the experimental underwater nuclear explosions, the ability of the sound to propagate under water up to large distances, and also to be reflected from islands, underwater eminences, etc., was used for the study of the relief of the bed of a large region of the ocean.[5] In this same experiment, data were obtained on the damping of low frequency sounds in the ocean.

The ability of sound to propagate over long ranges in the ocean and in the atmosphere comes from the presence in these media of natural acoustical waveguides, also known as sound channels.

Natural waveguides arise as a consequence of the specific dependence of the sound velocity on the vertical coordinate. It is of interest that the relative changes in the sound velocity are small. They do not exceed 15 per cent in the ocean and 30 per cent in the atmosphere. However, they profoundly affect the propagation of sound waves to large distances.

These waveguide bands, both in the ocean and in the atmosphere, have an appreciable extension along the vertical. The most favorable conditions of propagation take place for a sound source located near the "axis" of the waveguide, which coincides with the level of minimum sound velocity.

In tropical oceans, the axis of the acoustical waveguide is located at depths of 1000–1500 m. As one goes to higher latitudes, its depth generally decreases. In northern latitudes, it can reach the surface of the ocean. In this case we have a surface waveguide.

In the atmosphere, the axis of the waveguide lies at altitudes of 15 – 30 km, while the waveguide itself has an extension of several tens of kilometers along the vertical. In spite of the large difference in the characteristics of atmospheric and oceanic waveguides, the fundamental physical properties of the phenomenon are the same in both cases. These general laws of behavior will be considered below.

It should be pointed out that the presence in the atmosphere of waveguides for electromagnetic waves is well known. Thus the low-frequency electromagnetic signal from a lightning flash or a nuclear explosion is propagated at great distances in the waveguide bounded below by the surface of the earth and above by the ionosphere.

The process of long-distance propagation of elastic waves in the earth's crust has a somewhat more specific character. However, even here, phenomena of the type of waveguide propagation are encountered in a number of cases.

2. ACOUSTIC WAVEGUIDE IN THE OCEAN

We shall begin this discussion with long-distance propagation of sound in the ocean. Here the theoretical analysis of the problem is seen to be somewhat simpler than in the case of the atmosphere.

The dependence of the sound velocity on temperature and salinity as well as depth (i.e., the hydrostatic pressure) is described by the formula of Del Grosso:

$$c = 1448.6 + 4.618t - 0.0523t^2 + 0.00023t^3 + 1.23\ (5 - 35) - 0.011\,(5 - 35)\,t + \\ + 0.0027 \cdot 10^{-5}\,(s - 35)\,t^4 - 2 \cdot 10^{-7}\,(s - 35)^4\,(1 + 0.577t - 0.0072t^2) + 0.018z.$$

* Paper read at the third International Congress on Acoustics, Stuttgart, West Germany, September 1959.

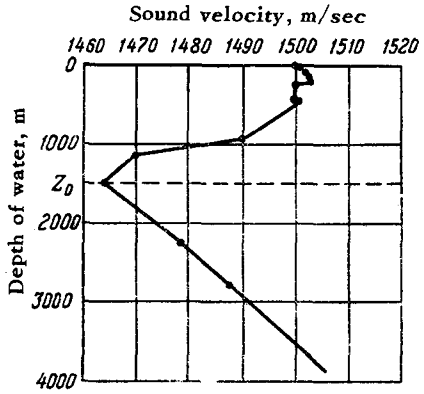

Figure 1a.

where c is the sound velocity in m/sec, t the temperature in °C, s the salinity in grams per liter, and z the depth in meters. A typical example of the dependence of sound velocity on depth in the ocean at medium latitudes is given in Fig. 1a. The curve plotted in the figure was computed on the basis of data obtained on February 15, 1932, in the Atlantic Ocean (35°56' S, 69°00' W). The sound velocity has a minimum at a depth $z_0 \sim 1500$ m. Above this minimum it increases because of the heating of the upper layers of the water, while below it, it increases because of the increase in hydrostatic pressure. The region of the minimum of the sound velocity is the central zone of the waveguide, and the level $z = z_0$ is its "axis". Sound waves have a tendency to concentrate in the region of the sound channel as they propagate. If we turn to the ray picture (Fig. 1b), we see that the rays starting out from the radiator at small angles to the horizontal (emission angles) are propagated in the waveguide without reflection from the bottom or surface of the ocean. This excludes the possibility of absorption of sound by reflection from the bottom and scattering of from roughnesses in the bottom or the surface.

The damping of sound waves of low frequency in water is very small. The attenuation coefficient, expressed in decibels per kilometer, is given[5] by the formula

$$\alpha = 0.036 f^{3/2},$$

where f is the frequency in kilocycles. For example, it follows from this that the intensity of the sound wave at $f = 50$ cps is reduced by absorption in the water by a factor of 10 only at a distance of 26 000 km.

However, the decrease in the strength of the sound with distance in the underwater waveguide is by no means monotonic. For example, let us consider the simplest form of asymmetric sound channel, in which the sound velocity is constant in a layer of thickness $2H$ and increases linearly on both sides of it (Fig. 2).

The set of rays emerging from a radiator placed on the axis of the waveguide is shown in Fig. 3. The density of the rays is strongly inhomogeneous in space. The intensity of the sound will have local maxima in regions of concentration of the rays, the so-called caustics, which are the envelopes of families of curves. The caustics are denoted in Fig. 3 by heavy lines.*

The sound intensity for each of the rays in the simplest case of a nondirectional source can be computed from the relation

$$I = \frac{W}{4\pi r^2} F,$$

where r is the horizontal distance, while F is the focusing factor. The latter is equal to the ratio of the area of the transverse cross section of the ray tubes in the case of a homogeneous medium and in the case under consideration at the same distance. As a rule, F is close to unity. The regions close to the caustic form an exception. According to ray theory, $F = \infty$ on the caustics themselves, i.e., the sound field in the neighborhood of a caustic cannot be computed in the ray approximation. The more exact theory (see reference 6, § 38), although close in nature to the ray theory, gives for the focusing factor on the caustic itself,

$$F = \frac{2^{5/3} \cdot r \cdot v(0)}{k \tan \chi_0 \sin \chi \, (d^2 r / d\xi_0^2)^{2/3}},$$

where $k = k(z)$ is the wave number at the receiver, χ_0 is the angle of inclination of the ray touching the caustic at its emergence from the source, χ is the angle of inclination of the same ray at the point of observation, $\xi_0 = k_0 \cos \chi_0$, and r is the horizontal distance between the source and the point of observation.** The form of the function $r(\xi_0)$ depends upon the shape of the waveguide [on the form of the function $c(z)$]; $v(0) = 0.62927$ *is* the value of the Airy function at zero argument. The change in the amplitude of the sound field in the region around a caustic is described by the Airy functions with their well-known monotonic decrease from the concave side of the caustic (whence the rays

* The drawing does not show those parts of the caustics which are formed from rays emanating from the source at very small angles of inclination with the horizontal.

** For simplicity we have set $R \sim r$, where R is the "inclined" distance between the point of reception and the point of observation.

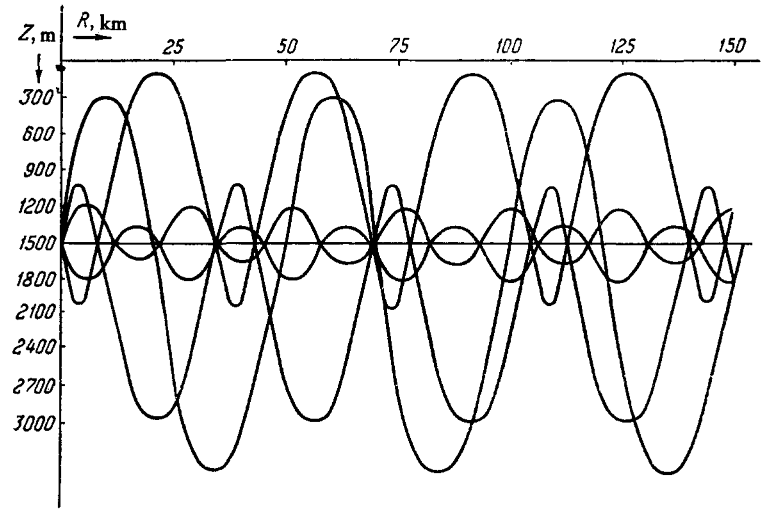

Figure 1b.

forming the caustic do not arise) and with the slower decrease (with simultaneous oscillations) on the convex side of the caustic (where two rays of the family forming the caustic intersect at each point).

Figure 4 shows a diagram of the rays for the case in which the source is displaced from the axis of the channel and is denoted in the picture by the continuous horizontal line. In this case the smaller fraction of the radiated energy is maintained in the channel. If the observer moves away from the source in the horizontal direction, then zones of audibility of the source will alternate with zones of "silence"(also called regions of acoustic shadow) where no one of the rays penetrates. Sound energy will frequently penetrate into the zone of silence because of diffraction effects.

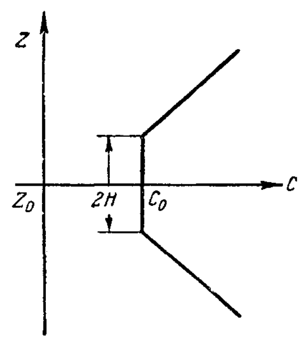

Figure 2

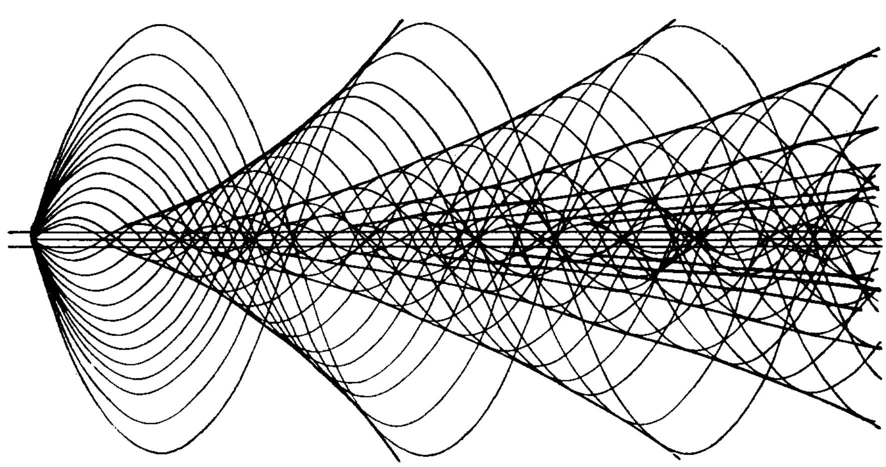

Figure 3

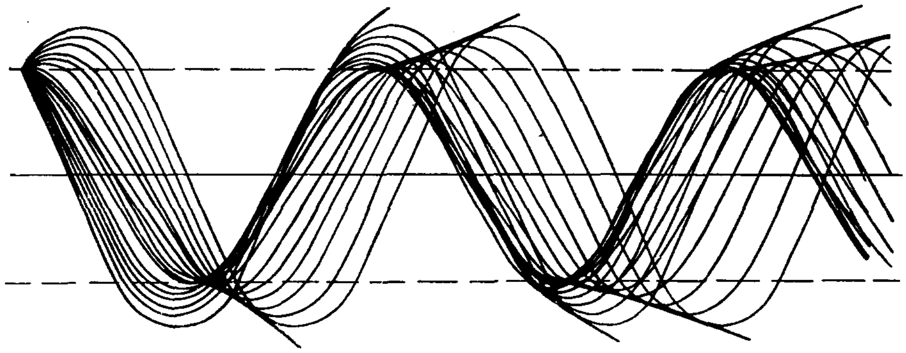

Figure 4

The description of the sound field in the region of zones of silence by any sort of improvement of or addition to ray theory, as was done in the case of the field near the caustics, is not possible. For this case it is necessary to construct a theory which is wave by its very nature. Such a theory must also describe the sound field in waveguides whose transverse dimensions are comparable with the wavelength of the sound or do not exceed it by very much. In this case the ray representations are also inapplicable in principle.

According to wave theory, the sound field at each point of space can be represented in the form of a superposition of a number of "normal modes." Each normal mode is propagated along the waveguide with a definite phase velocity and has a definite distribution of amplitudes along the vertical that is characteristic of it. A decrease of the sound intensity in the "zones of silence" is brought about by an appreciable attenuation of these waves as a result of their mutual interference.

The sound field in a waveguide characterized by the dependence given below for the sound velocity on the vertical coordinate z has been investigated in detail by Yu. L. Gazaryan:[6,7]

$$c(z) = c_1\left(1 + \frac{M}{\cosh^2\frac{z}{H}}\right)^{-1}, \quad M > 0.$$

The value of H can be called the effective width of the waveguide. The velocity of sound on the axis of the waveguide and at large distances from it is equal to $c_1(1+M)^{-1}$ and c_1, respectively.

In waveguide theory, an important role is played by the parameter

$$\nu = \frac{1}{2} + \sqrt{\frac{1}{4} + (k_1H)^2 M}, \quad k_1 \equiv \frac{\omega}{c_1}.$$

The waveguide can contain the wave only for $\nu > 2$. This gives the following expression for the maximum wavelength channeled in the guide:

$$\lambda_{max} = \pi H \sqrt{2M}.$$

The dependence of the amplitude of several normal modes on z/H for $\nu = 8.5$ is shown in Fig. 5b. A plot of $c(z/H)$ i.e. the shape of the waveguide in this case, is shown in Fig. 5a.

It should be noted that although the underwater sound channel was discovered comparatively recently (first in the U.S.A. and then, independently, in the U.S.S.R.) the fundamental laws of propagation of sound in such a waveguide are well understood at the present time (see references, 6, 8, 9, 10, etc.).

3. SOUND WAVEGUIDE IN THE ATMOSPHERE

The velocity of sound in the atmosphere is described with sufficient accuracy by the relation (see, for example, reference 1)

$$c(z) = 20.1\sqrt{T(z)} \text{ m/sec},$$

where $T(z)$ is the absolute temperature of the air at altitude z. The dependence of the air temperature on the altitude is well known in its general form at the present time. The temperature curve obtained in the U.S.S.R. on the basis of a large amount of data of experiments with meteorological rockets[12] is plotted in Fig. 6.* There are two temperature minima, located at altitudes of about

*The data obtained in the U.S.A. did not differ in any appreciable way from the data given here.

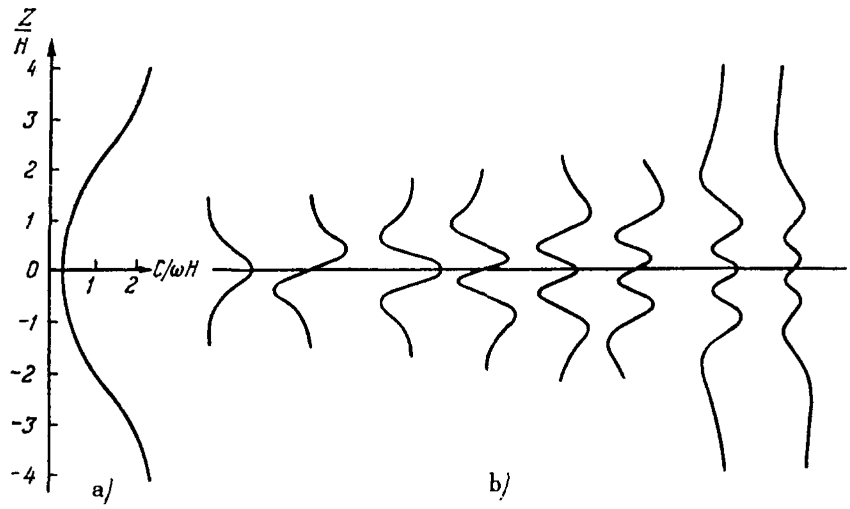

Figure 5

15 and 80 km. These give rise to two minima in the sound velocity, which leads to two acoustic waveguides at the appropriate altitudes. For a sound source (for example an explosion) located at altitudes of less than 50 km, and an observer located close to the earth's surface, the lower waveguide will play the more important role.

Although the general form of the curve, and in particular the presence of two minima, will remain constant there can be significant deviations of the real dependence of $T(z)$ from the given curve. In particular, at altitudes of less than 5 or 10 km, there will be appreciable seasonal temperature variations which can strongly affect the character of the propagation of sound waves. The solid curve in Fig. 6 from the viewpoint of its character at low altitudes, refers to summer conditions

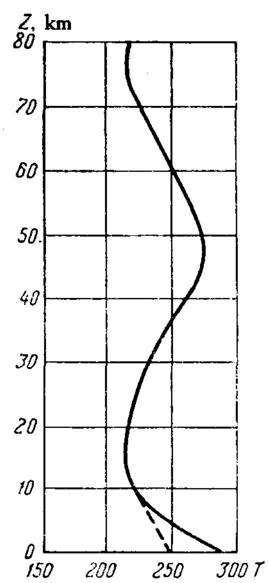

Figure 6

(temperature at the surface of the earth approximately +18 °C). Rays are drawn in Fig. 7 that start out from the radiator located at an altitude 8 km, for this particular case. We see that the rays have a tendency to concentrate in a waveguide that extends over the range of altitudes from 3 to 40 km. However, the rays trapped by the guide do not reach the surface of the earth.

We now assume that the temperature at the earth's surface is −23° C and the curve of the dependence of the temperature on the altitude extends to 10 km, as indicated by the broken line in Fig. 6. The ray picture in this case will have the form shown in Fig. 8. In this case, sound waves reach the surface of the earth, but at certain distances zones of silence appear.

However, one must keep in mind the highly variable applicability of ray representations for the analysis of the propagation of infrasonic waves in the atmosphere. The periods of pressure oscillations in waves excited by powerful explosions or eruptions of volcanoes at distances of several thousand kilometers and more are appreciable (from fractions up to tens of minutes). Two recordings (obtained by K. I. Bolishov) of nuclear explosions (equivalent to several megatons of TNT) obtained at distances of 5,000 and 11,500 km are shown in Figs. 9 and 10. The periods of the vibrations amounted to 5 to 8 minutes at the beginning of each recording and decreased to approximately 1 minute at the end. The wavelength for a period of 1 minute is equal to about 20 km. Thus over the extent of a single wavelength, the properties of the atmosphere change along the vertical by an appreciable amount and the fundamental condition of the applicability of ray theory remains unsatisfied.

Furthermore, use of the customary equation

$$\Delta\varphi + k^2(z)\varphi = 0, \quad k(z) = \frac{\omega}{c(z)}$$

for the analysis of the propagation of waves is seen to be possible only for waves whose periods satisfy the condition

$$\frac{T}{2\pi} \ll \frac{c}{g},$$

where g is the acceleration due to gravity. This equation can be assumed to be satisfied in sufficient degree for periods less than one minute. For longer periods, the effect of the force of gravity becomes significant and the equations become quite complicated.

It must also be remarked that the wind has an important effect on wave propagation in the upper layers of the atmosphere. Account of the wind significantly changes the curve of the dependence of the effective velocity of sound on height, in which the different cases are along the direction of the wind and against it. This leads to the result that in the direction of the "upper" wind, sounds of even small nuclear explosions (equiv-

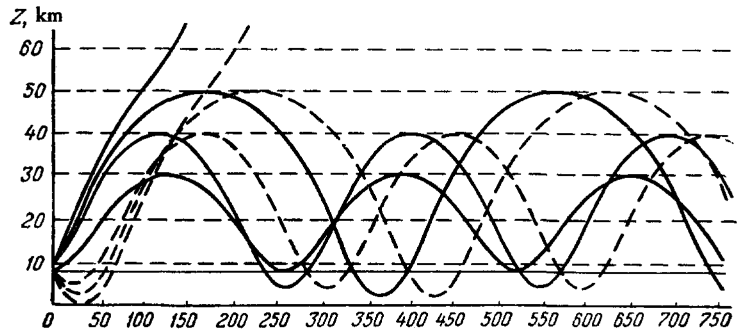

Figure 7

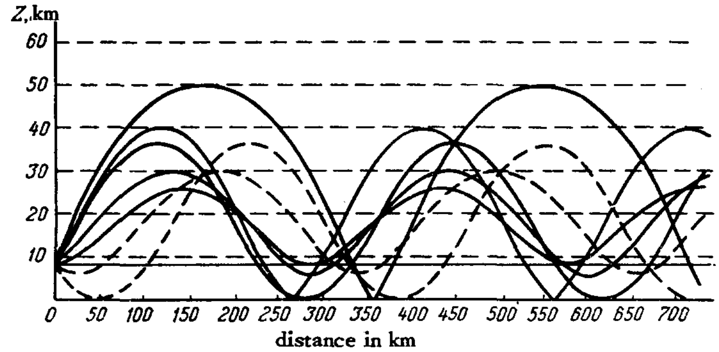

Figure 8

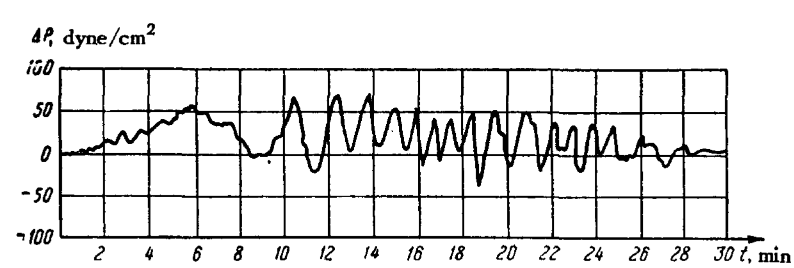

Figure 9

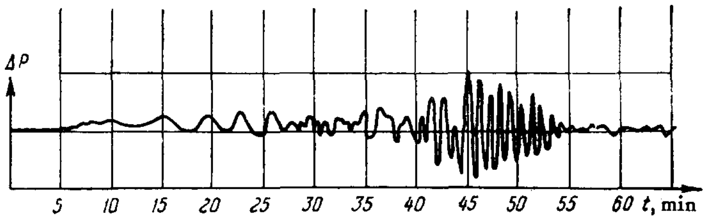

Figure 10

alent to the order of 1 kiloton) can be recorded at distances of the order of 2 or 3 thousand km. Against the wind, reliable recordings of these sources can not exceed 700 or 800 km.

The absorption of sound in the atmosphere has no essential effect on the frequencies of waves of interest to us. For a given frequency, the absorption increases with increase in height and becomes very large when the air becomes so rarefied that the length of the mean free path of the molecules is comparable with the order of magnitude of the wavelength. The absorption coefficient of the sound wave is equal to[1]

$$\alpha = 30.1 \frac{s}{\lambda^2},$$

where s = is the mean free path at a given altitude, and λ is the wavelength. At an altitude of 120 km, $s \approx 60$ cm. For a period of 0.5 minute, this yields a value $\alpha = 10^{-7}\ \mathrm{km}^{-1}$. Thus even at the altitude of the second waveguide, the absorption is not particularly strong at the very great distances in which we are interested. However, it must be kept in mind that the damping of sound waves brought about by their scattering on the inhomogeneities of a turbulent character and by diffusion of the sound energy through the boundaries of the guide can be of greater significance.

4. CHANGE OF SHAPE OF A SOUND PULSE IN PROPAGATION TO GREAT DISTANCES

A characteristic property of waveguide propagation is the fact that a sound pulse of short duration, which is obtained, for example, from an explosion, is converted at large distances to a long succession of comparatively regular vibrations. For example, the recordings in Figs. 9 and 10 have lengths of about 30 and 60 minutes, respectively.

In order to understand the reason for this interesting phenomenon and to make clear its quantitative behavior, it is necessary to consider the dispersive properties of the waveguide.

As was pointed out above, the sound field of a source radiating waves which are sinusoidal in time is a superposition of a definite number of normal modes. The higher the number of the normal mode, the greater is its attenuation with distance. Therefore, in many cases and, in particular, at large distances in an atmospheric waveguide, one need consider only one or the first few normal modes.

For simplicity in the following discussion, we shall consider only the first normal mode. We assume that the radiated sound pulse can be represented in the form of a Fourier integral in which a broad spectrum of frequencies is represented. The group velocity of the normal wave will be different for different positions of the elementary frequency interval $d\omega$. Therefore, the different frequencies will arrive at a distant point at different times and the total duration of the signal will increase in proportion to the distance.

If we assume that the radiated pulse is of very short duration and, consequently, all frequencies are contained in its spectrum with the same intensity, then the form of the sound signal at large distances will be determined exclusively by the shape of the dispersion curve, i.e., the dependence of the group velocity on frequency. The latter is in turn characteristic for the given waveguide. For example, let us consider the form of the sound pulse in the case of the dispersion curve shown in Fig. 11, where the frequency is plotted along the abscissa and the group velocity v along the ordinate. Such a dispersion curve exists, for example, in the case of sound propagation in a homogeneous liquid layer lying on a liquid half-space.[11] The sound signal at a given point distant R from the source will begin with vibrations of long period, since the low-frequency waves, as is seen from the curve, are propagated with maximum velocity. At the time $t = R/v_1$, high frequency vibrations are superimposed on the low frequency. At a subsequent time, for example at $t' = R/v_2$, the signal will be a superposition of the two frequencies ω_2' and ω_2'' arriving simultaneously. With passage of time the frequencies ω_2' and ω_2'' approach each other, and the signal concludes with vibrations of frequency ω_0, which are propagated with the minimum velocity v_0. It can be shown that as a consequence of absorption the frequencies $\omega > \omega_0$ will make a small contribution to the signal at large distances. Then it will be necessary to consider only the low frequency branch of the dispersion curve ($\omega < \omega_0$). The signal produced by it will have a period which increases continuously with passage of time. This we again see from Figs. 9 and 10, although in the case of an atmospheric waveguide, as a more detailed theoretical consideration shows, the process of propagation takes place in a much more complicated fashion.

It is also necessary to take it into account that some complications will be introduced in the picture of the received signal by the second and subsequent normal modes, which we have neglected in our discussions. Each of these has its own dispersion curve and the weight assigned to it in the sound signal can be estimated in the same manner as in the case of the first normal mode.

Thus, if we have sufficient information on the character of the guide and the form of the source, the form of the sound signal can be predicted at arbitrary distances even in very complicated cases. However, in practice, this can require very considerable calculations.

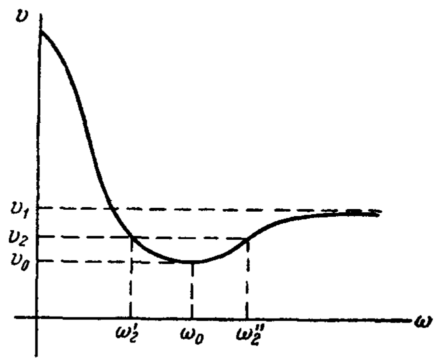

Figure 11

[1] S. K. Mitra, The Upper Atmosphere, U. of Calcutta, 1952.

[2] E. F. Cox, Abnormal Audibility Zones in Long Distance Propagation through the Atmosphere, J. Acoust. Soc. Am. **21**, 6 (1949)

[3] E. F. Cox, Microbarometric Pressures form Large High Explosive Blasts, J. Acoust. Soc. Am. **19**, 832 (1947)

[4] M. Ewing and D. Worzel, Long Range Sound Propagation (Russian translation in collection "Sound Propagation in the Ocean," IL, 1951.

[5] M. J. Sheehy and R. Halley, Measurement of the Attenuation of Low-Frequency Underwater Sound, J. Acoust. Soc. Am. **29**, 464 (1957)

[6] L. M. Brekhovskikh, Волны в слоистых средах, (Waves in layered media) (Acad. Sci. Press, 1957) (English translation, Academic Press, 1960)

[7] Yu. L. Gazaryan, The Problem of Waveguide Propagation of Sound in Inhomogeneous Media. Акуст. журн. **2**, 133 (1956); Soviet Phys. – Acoustics **2**, 134 (1956)

[8] L. M. Brekhovskikh, The Propagation of Sound in a Liquid Layer with Constant Propagation Velocity Gradient. Dokl. Akad. Nauk SSSR **62**, 469 (1948)

[9] L. M. Brekhovskikh, Propagation of Sound in an Underwater Sound Channel, Dokl. Akad. Nauk SSSR **69**, 157 (1949).

[10] L. D. Rozenberg, A new Phenomenon in Hydroacoustics, Dokl. Akad. Nauk SSSR **69**, 175 (1949)

[11] C. Pekeris, The Theory of Sound Propagation in Shallow Water. (Russian Translation in collection "Sound Propagation in the Ocean," IL, 1951.

[12] P. P. Alekseev, E. A. Besyadoskiĭ, et al, Rocket Studies of the Atmosphere, Метеорология и гидрология, (Meteorology and Hydrology) No. 8, 2 (1957).

Translated by R. T. Beyer.