ON MULTIPLE PRODUCTION IN COLLISIONS OF ULTRA-HIGH ENERGY PARTICLES*

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Usp. Fiz. Nauk **70**, 333-350 (February, 1960)

1. INTRODUCTION

Theories of multiple production of particles at ultra-high energies are of considerable interest largely because in the process of their experimental verification we also test our fundamental space-time and field-operator concepts "in the small", i.e., on intervals considerably smaller than $1/\mu$, where μ is the meson mass;** on distances less than 10^{-13} cm, and on time intervals smaller than $\sim 10^{-23}$ sec.

Of course, there exists no true and complete theory in this case. One may speak only of attempts making use of "phenomonlogical" postulates in one respect or another. From the point of view noted above these attempts, or theories, may be divided into two groups.

On the one hand, among the theories one sometimes includes formulations which avoid the critical and fundamental problems mentioned previously. These formulations are adapted to the interpretation of the available experiments; in making these formulations attempts are made to restrict oneself to spatial dimensionsof the order of $1/\mu$ or greater, and also to the most general conservation laws for the system as a whole, and for its parts which have already attained a separation exceeding $1/\mu$. The principal argument in favor of these theories is that one is dealing with collisions of so called geometrical cross section $\sigma \sim 1/\mu^2$, i.e., with impact parameters of the order of $1/\mu$, and that therefore, allegedly, it is not necessary to "penetrate more deeply." However, this turns out to be in a large measure illusory.

Often such theories reduce simply to outlines and to kinematical models whose purpose is to describe in a convenient manner certain aspects of the phenomenon. Without any doubt they are very useful, since a successful outline of such a type formulates certain requirements to be satisfied by any future theory. Among such theories we may include the fireball model, and also the better developed theories of this class, such as the so called "shaking-off theories." According to these theories, the colliding nucleons undergo a non-adiabatic shock (which is perhaps the more significant the smaller is the impact parameter) and as a result of it lose a part of their meson cloud (the theory of Lewis, Oppenheimer and Wouthuysen (L.O.W.), the Hu Ning theory, and their modifications). However the problem of the mechanism of the shaking off of the meson cloud or, more simply, the problem of the decay of the system excited by the collision, either remains completely uninvestigated, or (in the older theories) is solved in an unsatisfactory manner, for example, by means of perturbation theory. Moreover, the interaction of the created particles among themselves is here completely ignored, although, as we shall see later, this is a very important effect. Such theories have existed since 1949. During recent years interest in them has been revived, on the one hand, in connection with experimental indications of the existence of "double humped" stars; and, on the other hand, because a conviction has become widespread that the meson theory, with its concept of a single meson exchange between nucleons, is applicable to large impact parameters. Such theories are obviously intended for grazing particle collisions, when the impact parameter is of order $1/\mu$, although some authors consider that they are applicable to arbitrary impact parameters.

The theories belonging to the other class - the hydrodynamical theories (Heisenberg-Landau) strictly speaking, refer to head-on collisions of particles. The essential point of these theories is the concept that at the instant of collision practically the whole energy of the colliding particles is liberated within a small volume within which there is formed a region of highly heated vacuum containing a large number of interacting particles undergoing transformations into one another. The process of expansion and cooling of this condensed bit of matter is of the greatest significance. The hydrodynamical theories start from quite definite hypotheses or postulates:

1) our space-time concepts hold on an arbitrarily small scale;

2) the production of a large number of particles means that many degrees of freedom are excited and that, therefore, the system is a quasi-classical one;

3) the interaction is sufficiently strong, so that the collision between the particles is accompanied by the transfer of all, or practically all, the momentum.

In the most highly developed form of the theory all subsequent arguments are strictly consistent, but may lead to different results depending on the choice of

*Revised text of reports at the Moscow Cosmic Ray Conference and the Kiev High Energy Physics Conference (July, 1959).

**We everywhere take $\hbar = c = 1$.

one or another additional postulate, for example, of the equation of state for highly excited vacuum. During recent years a considerable degree of progress has been made both in the formal development of the theories, and also in the clarification of the field-theoretic foundations of the basic assumptions. Nevertheless, their validity cannot be considered to be to any extent established, in spite of the agreement between many of the conclusions with experiment. To a large extent this uncertainty is due to an insufficiency of the experimental data.

We note that a common error in the experimental estimates of the correctness of these theories consists of the fact that they are applied without any discrimination to *arbitrary* collisions between nucleons and nuclei. But the hydrodynamical theory may be applied to grazing collisions only after certain modification, as a result of which a separate branch of the theory is obtained. The point is that this theory first of all, deals with the decay, the decomposition, of the excited system, but not with its process of formation. It is assumed that the optimum conditions for the production of a condensed bit of nuclear matter at a high temperature occur in the case of head-on collisions, for example, in the collision of a particle with a column of nuclear matter carved out by the particle if its collision with the nucleus is to a sufficient degree a central one. However, insofar as, for example, a grazing collision between two nucleons is concerned, one should first evaluate on the basis of some additional theory or outline-the nature of the excitation, the initial parameters (energy, momentum) of the condensations being produced (this may, for example, be carried out on the basis of the model of single meson exchange), and only then should the hydrodynamical theory be applied to the decay of the system.

There exist now excellent systematic reviews of all these theories, the recent review by Koba and Takagi[1] extending up to 1958. Therefore, in the present article we shall not present all the theoretical work in detail. We shall restrict ourselves to a discussion of certain basic questions of principle, and of the latest papers which have appeared since the review by Koba and Takagi.

2. THE FOUNDATIONS OF THE HYDRODYNAMICAL THEORY OF HEAD-ON COLLISIONS

We shall first of all consider the hydrodynamical theory of head-on collisions, for example, of the collisions between two nucleons of very high energy, when the energy of the incident nucleon in the laboratory system, E_L, exceeds, let us say, 10^{12} ev.

The first of the postulates lying at the basis of the theory (the validity of space-time concepts in the small) is simply a bold hypothesis which requires verification. Of course, we must still determine down to how small distances is it actually necessary to extend these concepts in order to derive this or that theoretical conclusion which is confirmed by experiment. The Lorentz-contracted nucleon, which at rest has a radius of the order of $1/\mu$, has in the center of mass system in which its energy is equal to

$$E_C \sim \sqrt{\tfrac{1}{2} M E_L} \qquad (1)$$

(M is the nucleon mass), the form of a very thin disk of thickness.

$$\Delta \sim \frac{1}{\mu}\frac{M}{E_C} \sim \frac{1}{\mu}\sqrt{\frac{M}{E_L}}. \qquad (2)$$

When $E_L \sim 10^{12} - 10^{13}$ ev (a region which can be well studied in the case of stars) this corresponds to dimensions of the order of $10^{-2}(1/\mu) \sim 10^{-15}$ cm, while in the case of collisions giving rise to extensive atmospheric showers the corresponding dimensions are smaller still by one or two orders of magnitude. However, by no means all the results of the theory are sensitive to the conditions existing in the initial stages of expansion of the small bit of excited vacuum.

The second assumption is that the system is quasi-classical. An attempt may be made to evaluate it on the basis of the change in the action during the collision process.[26] If the colliding Lorentz-contracted nucleons exchange a fraction κ of their energy E_C, then for a collision time Δ (the velocity is almost equal to $c=1$) the increment δS of the action S is equal to

$$\delta S \sim \varkappa E_C \cdot \Delta \sim \frac{M}{\mu}\varkappa \sim 7\varkappa. \qquad (3)$$

The system may be regarded as quasi-classical if $\delta S \gg 1$. Consequently, in any case we must have $\kappa \sim 1$ if we want to regard the system as quasi-classical in all its stages. Thus, it is necessary to assume that during the collision a very strong interaction exists which guarantees an instantaneous transfer of the whole energy. Therefore the third postulate is simply equivalent to the second one, and it is not necessary to formulate it separately. But even in this case the classical margin is not very great.

One might put forward the requirement that not only the system as a whole, but each element of the system separately, should be quasi-classical. As has been noted by Blokhintsev[2] such a requirement is poorly satisfied in the initial stages. One may look for an escape from this difficulty by beginning the hydrodynamical discussion at a later time, when the system has already expanded by a certain factor, and by treating this initial volume as a parameter (Landau). Only the logarithm of this parameter appears in the formulas and, therefore, its exact value is not important.

It is of interest to note[26] that in the collision of two π mesons the condition of being quasi-classical is not satisfied even for the system as a whole. We have here instead of formulas (2) and (3)

$$\Delta_\pi \sim \frac{1}{\mu}\cdot\frac{\mu}{E_C}, \tag{2a}$$

$$\delta S_{\pi\pi} \sim E_C\cdot\frac{1}{\mu}\cdot\frac{\mu}{E_C} = 1. \tag{3a}$$

Therefore, the possibility is not excluded that this process will take place in a somewhat different manner. However, by choosing a later stage as our initial stage we may get rid of doubts with respect to this point as well. But it is just in such an approach that it is necessary to introduce as a separate, third, postulate the requirement of the complete transfer of energy during a very small interaction time. Therefore, in listing the fundamental assumptions, we have included it as a separate assertion, although perhaps it is in fact already contained in the second postulate.

The postulates enumerated above lead naturally to a theory which has the hydrodynamical form and which utilizes the hydrodynamical terminology. Such an unexpected conclusion often scares physicists off by its paradoxical nature: hydrodynamics "inside the nucleon"! However, it should be emphasized that if one accepts the postulates given above then this conclusion is logically inevitable.

As has been shown in a series of papers, largely by Japanese theoreticians (Ezawa, Tomozawa and Umezawa,[3] Iso and Namiki,[4] Ito and Tanaka[5]) quantum field theory may be naturally expressed in hydrodynamical terminology if we start with the same assumptions-the quasi-classical nature of the system and the unrestricted validity of the field operator and space-time concepts. It is, of course, true that this quantum field-theoretical foundation, which ought to lead both to a definite equation of state for the heated vacuum and to an explicit expression for its other hydrodynamical (viscosity and thermodynamical properties, has not yet yielded such results. The reason for this is that here the usual difficulties of the quantum theory of strongly interacting particles become apparent. However, a few things may be obtained.

The point is that in the case of a system of interacting fields we can write the energy-momentum tensor operator $\hat{T}_{ik}$, $i, k = 1,\ldots,4$, expressed in terms of the operator wavefunctions of the interacting fields. For simplicity we shall assume that we have only nucleons ($\hat{\psi}$ functions) and π mesons ($\hat{\phi}$). All the physical quantities characterizing the system are expressed in terms of the expectation values of the corresponding operators in terms of these functions in the usual manner. If the system is sufficiently large, and if we are interested in one of its local properties within a much smaller region, then at a temperature T we must also carry out a thermodynamic averaging. Such a small part of the system is as is well known, no longer described, by a wave function, but by a density matrix $\hat{\mathscr{P}}_T$ corresponding to the given temperature. The thermodynamic average of the operator $\hat{T}_{ik}$ is obtained in the usual manner by taking the trace of the product of $\hat{\mathscr{P}}_T$ and $\hat{T}_{ik}$. However, in addition, it is customary to subtract from it the value of the same quantity at $T = 0$.

$$\langle \hat{T}_{ik}\rangle \equiv Sp\,(\hat{\varrho}_T\hat{T}_{ik}) - Sp\,(\hat{\varrho}_{T=0}\hat{T}_{ik}). \tag{4}$$

Indeed, we shall be interested only in quantities which arise as a result of T being different from zero.

Just as in the case of any energy-momentum tensor the T_{44} component can here also be interpreted as the energy density ϵ (with opposite sign), while the T_{ii} components with $i = 1, 2, 3$ are equal to the pressure p. Thus, from the quantum field-theoretical description follows the hydrodynamical description of the system. Such a description is, of course, always possible. The essence of the problem, however, consists of being able to confine ourselves to the averaged characteristics of the hydrodynamical type, i.e., of being able to neglect the quantum fluctuations. But this is evidently possible if the system is a quasi-classical one, if the particle number density is everywhere sufficiently great. The role of the second postulate reduces just to this point.

If we have the expression for $\hat{T}_{ik}$ in terms of $\hat{\psi}$ and $\hat{\phi}$ we can obtain the relation between ϵ and p. Indeed, $\hat{\psi}$ and $\hat{\phi}$ are connected by a definite equation of motion. If we make use of it, we can show that

$$3p - \varepsilon = -M\,\langle\hat{\bar{\psi}}\hat{\psi}\rangle - \mu^2\,\langle\hat{\varphi}^+\hat{\varphi}^-\rangle - F_{\text{int}}, \tag{5}$$

where $\hat{\bar{\psi}}$, $\hat{\psi}$, $\hat{\phi}^+$, and $\hat{\phi}^-$ are certain creation and annihilation operators, while F_{int} is the term due to the interaction between the meson and the nucleon fields. Thus, we would obtain the equation of state—the relation between ϵ and p — if we could evaluate the right hand side of expression (5). But, in general, this cannot be done. If we neglect the interaction, $F_{int} \to 0$, then we can assert that at a sufficiently high temperature, when ϵ and p are also large, we can neglect the remaining terms on the right hand side, and therefore have

$$\varepsilon = 3p. \tag{6}$$

This equation of state holds for the electromagnetic field in a black body cavity and, in general, for a relativistic gas in the case of small interaction. In his hydrodynamical theory Landau assumed this from the outset as an additional postulate. However, if the interaction is arbitrary, its validity is not evident *a priori*. In the papers of the Japanese theoreticians it is shown that in certain cases F_{int} vanishes even in the case of a strong interaction. Thus, it vanishes in the case of those interactions which are known to be renormalizable. But in the case of a nonrenormalizable interaction the situation is quite unclear.

(for accidental reasons it coincides with the results of Fermi's statistical theory in which the interaction between the created particles is not taken into account at all). It, apparently, is in reasonably good agreement with experiment. The particle spectrum also turns out to have a different form (cf. below).

The reason for this discrepancy has remained unclear for a long time (cf. reference 1). It was sometimes ascribed to the fact that Landau had neglected the viscosity (which is permissible, since the Reynolds' number turns out to be very great in this case), etc.

However, the situation has become clarified as a result of Milekhin's work[11] who examined the expansion of a finite highly heated volume by the two methods in parallel: on the one hand, in accordance with the equations of relativistic hydrodynamics "a-la-Landau", but using an equation of state of a more general type:

$$p = c^2\varepsilon, \quad 0 < c^2 < 1, \tag{14}$$

where c^2 is not necessarily equal to 1/3 (c^2 has the meaning of the square of the velocity of sound in this medium, and can hardly exceed 1/3); and, on the other hand, "a la Heisenberg," i.e., in the case of the wave field ϕ, initially confined to a small volume, but obeying the wave equation with an interaction Lagrangian of a type more general than in Heisenberg's case:

$$L = L_0 + L_{int},$$
$$L_{int} = L_{int}\left(\sum_{i=1}^{4}\left(\frac{\partial\varphi}{\partial x_i}\right)^2\right). \tag{15}$$

Explicit calculations were carried out to the end in the case of

$$L_{int} = \lambda\left\{\sum_{i=1}^{4}\left(\frac{\partial\varphi}{\partial x_i}\right)^2\right\}^{\nu}, \tag{16}$$

where λ is the interaction constant, ν is an arbitrary positive integer. When $\nu = 1$ the equation practically does not differ from the equation for the free field. For $\nu = 2,3,\ldots$ we have an essentially nonlinear equation.

It turns out that both treatments lead to results identical in all detail, if c^2 in the equation of state in the hydrodynamical treatment is related in a definite way to the index ν (which characterizes the nonlinearity of the interaction) in the quasi-classical field theory. Specifically, we must take

$$c^2 = \frac{1}{2\nu - 1}. \tag{17}$$

In particular, Landau's theory, $c^2 = 1/3$, corresponds to the field theory with $\nu = 2$. And indeed Khalatnikov,[12] Koba,[13] and Taniuti[14] had already shown that the relativistic hydrodynamics utilized by Landau may be put into Lagrangian form with the Lagrangian containing a term of just such a type

$$L_{int}^{(\mathrm{Land})} \sim \left\{\sum_{i=1}^{4}\left(\frac{\partial\varphi}{\partial x_i}\right)^2\right\}^2, \tag{18}$$

where ϕ is the velocity quasi-potential. At the same time, the results of Heisenberg's field theory may be obtained from Landau's hydrodynamic theory if we take an equation of state for the substance which differs from equation (6) utilized by Landau, specifically, if we take $c^2 = 0$. Indeed, if the Born-Infeld Langrangian (8) is expanded into a series, then this series will contain all values of ν up to $\nu = \infty$, which in accordance with (17) implies $c^2 = 0$. The agreement between the two theories is so complete that it is even possible to construct in field theory an expression which has the meaning of entropy, and which is conserved in the process of the dispersal of the particles (in Landau's case, because of the neglect of viscosity and heat conductivity, the dispersal of the particle bunch, after the passage of the shock waves has been completed, is isentropic).

Thus, the concrete consequences of the theory depend on the equation of state or on the form of the interaction Lagrangian. Apparently, the multiplicity actually observed corresponds better to $c^2 = 1/3$, than to $c^2 = 0$. Consequently, whether in the field or in the hydrodynamical picture, and as is now clear, this is essentially one and the same thing (in both cases we have simply a quasi-classical approximation to quantum field theory), the problem may be formulated as follows: we assume that our space-time concepts are valid within arbitrarily small four-dimensional volumes, and that the concept of *second-quantized fields is itself valid.* We shall investigate as to what follows from this, and we shall check this theory experimentally.

The simplest consequences of the theory pass such a test successfully. However, this of course, can by no means be regarded as a confirmation of the correctness of the special postulates. Unfortunately, experiments are still very meager, and many consequences of the theory are insensitive to the initial assumptions, and may also be explained in quite a different manner. It is, therefore, all the more important to continue the detailed development of the theory and its comparison with experiment.

3. COMPARISON BETWEEN THEORY AND EXPERIMENT

Let us examine the principal conclusions of the hydrodynamical theory.

a) *The composition of the particles produced* is one of the elements most characteristic of the theory. We know that the interactions between π and K-mesons, and also between nucleons and hyperons are approximately equally strong. Thus, for example, the cross section for the interaction between K mesons and nucleons at relativistic energies (hundreds of Mev) is

The possibility is not excluded that progress in this problem will be possible as a result of the application of the new methods of quantum statistics.[6,7] As has been noted in a recent paper by Fradkin,[8] the situation is more favorable here than in the usual field theory problems because in this case a new large parameter appears in the problem – the temperature T. In particular, there is hope that the temperature dependence of physical quantities might be determined. In any case, the quantum field-theoretical aspect of the problem of a highly excited vacuum, with which one has to deal in the theory of multiple production of particles at high energies, has now been raised to the level of a general problem of field theory, and one might expect a further application of the methods of this theory. Among the present problems, in addition to the derivation of the equation of state, we should here list: the determination of the coefficient α in the relation between ϵ and T, $\epsilon = \alpha T^4$, the determination of the viscosity and of the heat conductivity of the medium, the determination of the correlation functions for the current, etc. It should be noted that the operator expressions for the viscosity and for the correlation functions have been obtained already (cf., for example, references 4 and 9). However, they can be evaluated only in the case of weak coupling, which, unfortunately, is completely inadequate for this problem.

All the foregoing refers to the quantum field-theoretic foundation of the hydrodynamical description of the system and of the equation of motion. A question can also arise with respect to the basis of the hydrodynamical equations of motion. The answer in this case is obvious: both in hydrodynamics and in field theory the equations of motion may be written in the following form

$$\frac{\partial T_{ik}}{\partial x_k} = 0 \tag{7}$$

(in the case of quantum field theory we interpret T_{ik} as the operator $\hat{T}_{ik}$). To the extent to which the relationship between T_{ik} and $\hat{T}_{ik}$ has been established we can say that the hydrodynamic equations are simply the averaged quantum field-theoretical equations.

However, is such a hydrodynamic theory unique? Until recently a strange contradiction existed with respect to this point.

Heisenberg, who as far back as 1936 had pointed out the necessity for the quasi-classical treatment of the region of the vacuum excited by the collision and stopping of fast particles, has proposed in the course of many years different semi-qualitative approaches of a hydrodynamical nature. The last, the most highly developed and consistent form of his theory[10] reduces to a discussion of the spread of a wavepacket initially confined within a small region, with the assumption being made that this (classical) field satisfies a wave equation with an interaction Langrangian of a definite type. After having tried different Langrangians, Heisenberg chose the expression for the total Langrangian which had been used by Born and Infeld in a quite different problem:

$$L = l^{-4}\left(1 - \sqrt{1 + l^4 \sum_{i=1}^{4} \left(\frac{\partial \varphi}{\partial x_i}\right)^2}\right). \tag{8}$$

Here l is a constant having the dimension of length. If the field ϕ is confined to a volume smaller than l, then its gradients are large, and the expression for L is essentially nonlinear. After the field has spread and the following inequality holds $l^4 \left(\frac{\partial \phi}{\partial x_i}\right)^2 << 1$, the square root may be expanded in a power series and we obtain

$$L \approx -\frac{1}{2} \sum_{i=1}^{4} \left(\frac{\partial \varphi}{\partial x_i}\right)^2. \tag{9}$$

This is the Lagrangian of the free wave field (with zero rest mass) and the corresponding field can be written as a superposition of plane waves each of which describes one of the particles observed at the end of the process. Such a discussion (the initial volume in this case was regarded as infinitely thin) led Heisenberg to the conclusion that the total number of created particles is equal to

$$N_H \sim \frac{E_C}{M} \sim \left(\frac{E_L}{M}\right)^{1/2}. \tag{10}$$

At the same time the spectrum of their energies in the center of mass system has the form

$$dN_H \sim \frac{d\omega}{\omega^2}, \tag{11}$$

where ω is the energy of a single created particle.

This multiplicity N_H is very large. It implies an almost complete dissipation of energy, its almost complete transformation into the rest energy of the particles. Indeed, in the limiting case, if all the mesons were created at rest we should have obtained

$$N_{\max} = \frac{2E_C}{\mu}. \tag{12}$$

It is true, that in Heisenberg's case additional factors appear as a result of a more precise discussion which somewhat alter the result, $\sim [1/\ln(E_C/M)]\ (E_C/M)$. However, this difference is not great, and apparently this conclusion does not agree with experiment.

On the other hand, Landau has examined the expansion of a compressed heated volume in accordance with relativistic hydrodynamics, making use of the equation of state (6), and has obtained a considerably lower multiplicity

$$N_{\text{Land}} \sim \left(\frac{E_C}{M}\right)^{1/2} \sim \left(\frac{E_L}{M}\right)^{1/4} \tag{13}$$

less than the cross section for the interaction between nucleons by a factor of only 2 or 3. One might think that in multiple production the fraction of each kind of particles is determined by their statistical weight (thus, in Fermi's statistical theory, in which the interaction between the created particles wasn't taken into account at all, the ratio of the numbers of π mesons and of nucleons-antinucleons turned out to be 3: 8, three kinds of π mesons and eight kinds of nucleons of different spins). At the same time experiment, apparently, shows that in the range $E_L \sim 10^{12}$-10^{13} Mev, 80 to 90% of particles in a star are π mesons.[1,19] On the other hand, this result had been long ago predicted by the hydrodynamical theory and is characteristic of it: in the process of expansion and cooling of the condensation the strong interaction between all the fields leads to a gradual disappearance of all the heavier particles, and when the temperature drops to the critical temperature T_c, at which the interaction decreases sharply, $T_c \sim \mu$, the proportion of particles of mass $M_i > \mu$ will be determined by their Boltzmann factor $\exp(-M_i/T_c) \ll 1$. Thus, the experimentally observed composition of showers confirms the decisive role played by the interaction between fields during the process of dispersal, and therefore requires an analysis of the expansion and cooling of the small bit of excited vacuum. At the same time it follows from the foregoing that the composition of particles is determined by the later (final) stage of the dispersal and cannot say anything about the validity of space-time concepts in the small.

b) *The multiplicity* -- the total number of particles produced-is determined by the dissipation of energy, by the change in its entropy. In the hydrodynamical theory, since viscosity and heat conductivity were not taken into account, the change in entropy takes place only in the very early period, in the process of formation of the bit of excited vacuum. Therefore, in contrast to what was said about the composition, the decisive role is here played by the behavior of the system during the period when its dimensions are exceptionally small.

Let us examine the nucleon-nucleus collision (under the condition that we are not dealing with the periphery of the nucleus). If the energy of the particle if sufficiently great, then both the nucleus and the nucleon are contracted so strongly that the passage through the nucleus lasts only a very short time: in the center of mass system the time is of the order $(n+1/\mu)(M/E_C)$ where n is the number of nucleons in the nucleus which are in the path of the incident particle. Therefore, in the transverse direction the interaction does not have time to extend beyond a distance $1/\mu$, and only the incident particle and the column of nuclear matter forced out by its participate in the reaction. In this case Landau's theory gives the following value[15] for the total number of created particles N_0:

$$\left.\begin{aligned} N_0 &= k(n+1)\left(\frac{E_L}{M}\right)^{1/4} && \text{for} \quad n < 3{,}7, \\ N_0 &= 1{,}85k(n-1)^{3/4}\left(\frac{E_L}{M}\right)^{1/4} && \text{for} \quad n > 3{,}7; \end{aligned}\right\} \tag{19}$$

k is a coefficient of the order of unity. If an average is taken over all possible traversals through a nucleus of atomic mass number A, we have

$$\overline{N}_0 \approx 2k\left(\frac{E_L}{M}\right)^{1/4} A^{0.2}. \tag{20}$$

The dispersal is approximately symmetric forward and backward in the coordinate system which differs very little from the center of mass system of the nucleon and of the column of nuclear matter.[16]

Thus, the number of particles in the column affects N_0 much more strongly than the energy of the primary particle. Just for this reason alone one might expect a considerable scatter in the multiplicities for a given initial particle energy in interactions with heavy nuclei. As a rule, the observed stars with $N_0 \sim 100$ result from the collision of particles of not very high energy ($E_L \sim 10^{12} - 10^{13}$ ev) with the column of nuclear matter in a fairly heavy nucleus.

Apparently the experimental data may be put into reasonable agreement with these predictions. However, in a large number of cases there exists a discrepancy which may be ascribed to the fact that here we have to deal with grazing collisions or, more precisely, with those cases when the excited state is formed with an incomplete transfer of energy, for example in the case when the inelasticity coefficient differs from unity.

c) *The energy distribution* of the created particles has a rather peculiar character in Landau's theory. One must distinguish between the main group of particles arising from the region of the so-called "non-trivial solution" in hydrodynamics, and the so-called progressive wave.[17] Two such waves are propagated in both directions ahead of the main mass of the particles. Each comprises only about one particle, but contains a significant fraction (severals tens of percent) of the total energy of the process. Such a particle may be a nucleon, but in the overwhelming fraction of cases it is a π meson, and in one third of the cases it is a π^0 meson. The existence of such a distinctive particle may play an essential role in the development of a nuclear cascade process in the atmosphere, in particular, of an extensive atmospheric shower. With respect to the great majority of the particles, if ω is the energy of the particle, we obtain in this case in the center-of-mass system approximately the following distribution in energy:

$$\frac{dN}{N_0} \approx \frac{1}{\sqrt{2\pi L}} \frac{d\omega}{\omega} e^{-\left[\frac{1}{\sqrt{2L}} \ln \frac{\omega}{\mu}\right]^2} \quad (\omega \gg \mu), \tag{21}$$

be determined by the quantum field theory of which we spoke earlier (cf. Sec. 2). However, this problem has not yet been solved at the present time. On the other hand, the relativistic nucleon-meson plasma with which we are, in fact, dealing is characterized by a strong interaction, and it is definitely incorrect to evaluate its electrical characteristics on the basis of the gas model. Therefore, at the present time we can only put forward one hypothesis or another in the expectation that a comparison with experiment will enable us to conclude whether it is justified or not, and in this way to find out something about the properties of the radiating medium.

The simplest hypothesis is based on the fact that we are interested in very high temperatures, much higher than the masses of all the particles. Therefore, these masses must drop out from the characteristics of the medium, and the only quantity of the dimensions of length and time is $1/T$. A similar situation occurs, for example, in a black body cavity where the correlation lengths and times are of the order of $1/T$. This hypothesis finds support in the fact that the commonly accepted equation of state (6) at these temperatures no longer contains any masses [cf. (5) and (6)]. On the other hand, the arguments given above are not very convincing, since the electromagnetic radiation is associated with the dissipative and relaxation characteristics of the medium, and not directly with the equation of state.

If we retain the hypothesis indicated earlier (and remember that it has no secure foundation) we conclude that, just as in the case of a black body, the intensity of radiation from an element of volume dV at rest during a time element dt is equal to

$$dI = AT^5\, dV\, dt, \tag{23}$$

where A is a dimensionless constant. The spectrum of the radiation has a maximum at $\omega \sim T$. On the basis of this we can show that the number of quanta radiated during the whole dispersal time is given by

$$N_\gamma \sim 9Ae^2 N_0^{\frac{4}{3}}, \tag{24}$$

where N_0 is the total number of π mesons produced. This radiation is emitted during the later stages of the dispersal, in particular, when the temperature falls below a certain threshold temperature T_1:

$$T_1 \sim \sqrt{T_0 \mu} \ll T_0, \tag{25}$$

where T_0 is the initial temperature of the condensation (until that time the condensation has existed for too short a time to emit radiation of frequency ω, corresponding to its temperature T).

In such a treatment it is considered that the electromagnetic radiation carries away the energy E_γ which comprises a small fraction of the total energy $2E_C$; in particular, it turns out that

$$\frac{E_\gamma}{E_C} \sim 5Be^2 \left(\frac{N_0}{n+1}\right)^{2/3}, \tag{26}$$

where B is a dimensionless constant of order of magnitude unity, $e^2 = 4\pi/137 \approx 1/10$. We see that for a sufficiently great initial energy E_L [we must remember that $T_0 \sim \mu N_0/(n+1) \sim \mu(E_L/M)^{1/4}$] the role of the electromagnetic radiation becomes more important, and at sufficiently high initial energies a new situation arises: the γ radiation may become equal to the π-meson radiation, and may come into equilibrium with it. Then it will carry off a fraction of energy corresponding to its number of degrees of freedom. There are two such degrees of freedom (two polarizations), and in the case of π mesons there are three degrees of freedom. Consequently, at energies which exceed a certain threshold energy and which lie, possibly, somewhere in the interval between 10^{14} and 10^{16} ev, all the π mesons carry off 3/5 of the energy, while the γ quanta carry off 2/5 of the energy. Since π mesons also decay into γ quanta, $(3/5)E_L$ of the energy will be accounted for by electromagnetic radiation.

In exactly the same way the electrical fluctuations may in principle give rise to electron and μ-meson pairs. Since this process is of higher order in e^2 (the probability will contain an extra factor e^2) it may become significant only at still higher energies.

Of course, this conclusion is based on the hypothesis expressed by (23). If it does not hold, for example, if the right-hand side of this formula contains a factor of the type μ/T, then the temperature dependence in (26) will be correspondingly lower and the relative role of γ-radiation will either increase with energy more slowly, or will not increase at all. Therefore, the study of the electromagnetic component of stars may, in principle, lead to definite conclusions with respect to the electrical characteristics of hot vacuum.

4. "GRAZING COLLISIONS"

Experiment clearly indicates that the nature of a star is determined not only by the initial energyof the incident particle, and by the nature of colliding particles, but also by some other additional parameter. Thus, the multiplicity may assume quite different values at one and the same energy (cf., for example, Fig. 1.1 in reference 1). It has been shown that such a scatter is

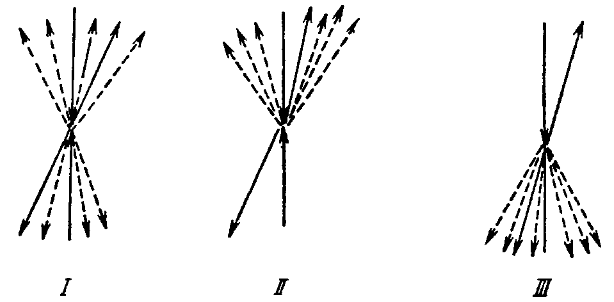

Figure 1.

where L is the characteristic parameter of Landau's theory which in accordance with more exact calculations[18] is given by:

$$L = 0.56 \ln \frac{E_L}{M} + 1.6 \ln \frac{2}{n+1} + 1.6. \qquad (22)$$

This value is somewhat higher than L_{Land}, which was given by Landau. The form of the spectrum given here depends on the chosen equation of state. We can say roughly that for the great majority of the particles the spectrum has the character of $d\omega/\omega$, if $\mu << \omega < \mu \exp\sqrt{2L}$. The spectrum breaks off sharply at the indicated upper limit. In a nucleon-nucleon collision this upper limit lies at $\omega \sim 40\ \mu$ if $E_L \sim 10^{13}$ ev, or at $\omega \sim 90\ \mu$, if $E_L \sim 10^{16}$ ev. Thus, the spectrum falls off slowly up to quite high meson energies. It is of interest to note that in the paper by Schein and coworkers,[19] in which the meson energies were studied in a large number of stars, a special note is made of the presence of mesons of energies (in the center of mass system) of several Bev.

d) *The angular distribution* of the particles, is as is well known from hydrodynamic theory, very peaked. This is associated with the fact that the expansion of the condensation which initially has the form of a very thin disk proceeds for a long time as a one-dimensional process. When expansion in the perpendicular direction also begins to occur, i.e., when the so-called three-dimensional stage begins, the cooling becomes very rapid and the process ends (the critical temperature is attained) long before the particles have time to acquire an appreciable transverse velocity component.[18] Therefore, the transverse component of the momentum $p_\perp$ of the created particles is determined not so much by the "macroscopic" velocity of motion of the fluid, as by the thermal motion of the particles in the fluid.[20] In view of the fact that the critical temperature is constant, $p_\perp$ depends weakly on the initial energy of the process, and must be of the order of 2 or 3 μ. Only very large values of E_L can give rise to a dragging out of the three-dimensional stage and contribute to an increase in the hydrodynamical part of $p_\perp$.

The three-dimensional stage was evaluated by Landau very roughly. Recently it has been studied more rigorously.[18] As a result an even greater concentration of particles in the two cones forward and backward was obtained. However, one must keep in mind the fact that in the course of the derivation certain points were again neglected as a consequence of which the result cannot be regarded as completely definitive.

The fact that $p_\perp$ is constant in the case of mesons is now an experimentally established fact.[1] However, one should keep in mind that such a value of p is not specific for the hydrodynamical theory. Practically in any theory we would have $p_\perp \sim \mu$. Thus, the spreading of the wavepacket which initially had the transverse dimension of the order of $1/\mu$, would have given the same result (only Fermi's theory would have led to a sharply different conclusion).

Since the heavier particles (of mass M_i) are nonrelativistic at the temperature T_c, their transverse momentum $p_{\perp i}$ will be considerably larger than in the case of π mesons, $p_{\perp i} \sim \sqrt{2M_i T_c} >> \mu$. This experimentally verified conclusion is also not specific for Landau's theory: if the heavier particles in the nucleon are concentrated within a region smaller than $1/\mu$ the spreading of their wavepacket will also yield large $p_{\perp i}$.

A comparison of the distribution curves for the quantity $p_\perp$ with the theoretical values is one of the methods of determining the critical temperature of the dispersal. At present it is customary to take $T_c = \mu$.

The fact that the particles are sharply peaked forward and backward is by itself an insufficient characteristic for comparison with theory. The depth of the dip at $\vartheta_C = \pi/2$, where ϑ_C is the angle between the direction of motion of the newly created particle and the direction of motion of the initial nucleon, is a significant parameter. If the particle distribution is plotted not with respect to ϑ_C, but with respect to $\eta = \ln \tan(\vartheta_c/2)$, then Landau's theory leads to a Gaussian distribution with respect to η, with the maximum lying at $\vartheta_C = \pi/2$ (in this scale, the wider the curve the more strongly peaked is the forward and backward distribution). At the same time stars were recently found which give two maxima in this scale. They clearly indicate a superposition of two independent processes of dispersal. Within the framework of the hydrodynamic theory these two processes could be the dispersal of two condensations arising in the collision of nucleons which exchange their energy and momenta only partially, for example in the case of grazing collisions. We shall return to this point later. However, it should be noted that such stars with two maxima comprise only a small fraction of the stars that have been studied experimentally so far.[21]

e) *The electromagnetic radiation* that accompanies multiple production originates first in the decay of the π° mesons born in this process. However, the dispersal of the condensed bit of excited vacuum may lead to another mechanism of production of γ quanta (and also of electron and μ meson pairs) which is associated with the specific features of this phenomenon and which in principle may yield additional characteristics of the dispersal process. We are speaking here not about the "stopping" radiation of the colliding protons, which is insignificant, but, in simple terms, about the black-body radiation from a hot body. At high temperatures with which we are dealing, strong fluctuations of the charge density must occur even if the original collision is between two neutrons. The fundamental question in this case is the problem of the size and of the pulsation frequency of these fluctuations. Of course, this radiation could be evaluated if we knew the complex dielectric permittivity of the medium. In principle it should to a great extent due to the difference in the lengths of the columns n carved out of the nucleus (cf. See 3, in particular, Eq. (19).[22] However, in such an analysis n sometimes turns out to be considerably smaller than unity. On the other hand, the analysis of a number of stars, for which sufficient data are available, showed that it is not N_0 which is a single valued function of E_L, but the quantity N_0/η, where η is the coefficient of inelasticity (cf. reference 1, Sec. 3.2.3).[23] Finally, it was shown[24] by means of a Wilson cloud chamber on the basis of a small number of extensively studied stars, that the general character of the angular distribution of the products also has a sharply different character in the center of mass system. The three types of angular distributions obtained are roughly shown schematically in Fig. 1.

All this suggests that we should investigate theoretically those collisions which are characterized by an incomplete exchange of energy between the colliding particles. The meson theory of nucleon interaction definitely indicates that such collisions should exist. Indeed, it is possible that the colliding nucleons exchange only one (or two) mesons, as is the case in the region of quite low energies and at large distances between the particles. As is well known, this is the origin of the potential of the Yukawa type nuclear forces. At the present time it is not possible to assert that such processes are not important at small impact parameters, i.e., that head-on collisions are always accompanied by a complete exchange of energy and lead to the process discussed earlier in Secs. 2 and 3. However, the distant collisions are in any case "few meson" ones. Therefore, we shall now discuss such collisions, and we shall refer to them as grazing collisions. Since the possibility is not excluded that similar collisions also occur at small impact parameters the term "grazing" should be understood as a conventional one.

Thus, we go over to the opposite limiting case of impact parameters greater than or of order of $1/\mu$. The fact that the interaction has a finite range or, what is the same thing, that it is due to a meson field, i.e., to particles of rest mass μ, gives the process definite distinctive features.

First of all, the classical concept of the field is inapplicable here. The collision time is so short that the uncertainty in the energy is great, and for impact parameters $\gtrsim 1/\mu$ we must take into account the quantum structure of the field. Actually this reduces to the fact that we must take into account the finiteness of the amounts of transferred momentum and transferred energy.

In particular, as may be easily shown, even if the impact parameter is of order $\gtrsim 1/\mu$ the colliding particles may in exchanging a meson transfer to one another a very large fraction of their energy, for example of order $(\mu/M)E_C$ while the square of the transferred four-momentum remains of order μ^2, and even the square of the three-dimensional transferred momentum is of the same order. For this reason the result of the interaction may be almost as catastrophic as in the case of a head-on collision. But the frequency of such events drops off sharply as the impact parameter increases. In this connection it is not possible to accept as correct the frequently appearing statements that the excitation of the nucleons continues to fall off indefinitely as the impact parameter increases. Generally speaking the magnitude of the perturbation is determined by the parameter μ/M. Correspondingly, the coefficients of inelasticity cannot be smaller than this quantity in any significant number of cases.

The general consideration that a strong perturbation of the nucleon state may arise as a result of a transfer of a small perpendicular momentum lay at the basis of many papers which did, as well as of those which did not, utilize the hydrodynamical theory of dispersal. However, firstly, the question as to what happens to the nucleons which are excited was solved in different ways. Secondly, the authors of certain such theories thought it possible to ascribe to these concepts an exhaustive significance, and to assume that they are characteristic of all collisions, including head-on ones.

It is clear that a perturbation of the state of one or two nucleons which, moreover, retain their individuality will lead, generally speaking, to the appearance of two processes, although macroscopically, for example, in a photographic plate, they will appear as one. Its characteristics will, of course, not be the same as for a head-on collision, and if they are confused then this in itself is equivalent to the introduction of a far reaching hypothesis concerning the identity of these processes. Neither in analyzing an experiment nor in a theoretical discussion should this be done *a priori*.

The transition between the two limiting cases, the head-on collision and the "few-meson" grazing collision, is not very distinct. We can only expect that when we go from the region of a diffuse single-meson cloud to the region where this concept becomes inapplicable ("core", "Kern"), the number of mesons participating virtually will increase sharply, and it will no longer be rational to make a distinction between such a case and a head-on collision.

From a theoretical point of view the process may be represented by the Feynman diagram of Fig. 2A. In exchanging a meson of mass μ and of four-momentum q the colliding particles are excited into states which may be characterized by total energies E_1 and E_2 *and* by momenta $\mathbf{p}_1$ and $\mathbf{p}_2$. They correspond to "rest masses" $\mathfrak{M}_1$ and $\mathfrak{M}_2$.

$$\mathfrak{M}_i^2 = E_i^2 - \mathbf{p}_i^2. \tag{27}$$

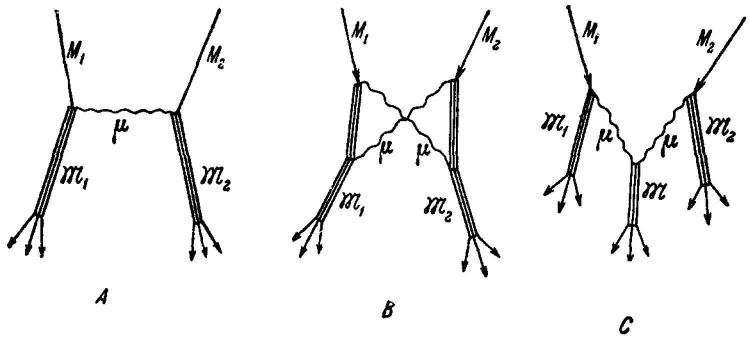

Figure 2.

The transferred momentum q may be expressed in terms of $\mathfrak{M}$.

This by no means determines the true nature of the process: here we can have both a true formation of isobars with their subsequent decay (which is shown in the diagram), or a direct production of the particles which are finally observed, and also a sequential emission of mesons. Among the simplest diagrams we can also have the two-meson diagrams of the type of Figs. 2B and 2C. The diagram of Fig. 2C where an interaction of virtual mesons takes place, is of particular interest. It corresponds to a relatively small perturbation of the state of the primary particles and, consequently, to stars of type I in Fig. 1. The problem of the relative role played by diagrams A and B, on the one hand, and by C on the other hand is theoretically very important.

At the present time we do not have a complete theory; in particular, we do not know the expression for the operator of the vertex part (if we adopt perturbation theory for the pseudoscalar meson, then it is given by γ_5). Of course, it is possible to utilize the experimental values of the cross section for the interaction between a π meson and a nucleon, and on making use of certain approximations to express the vertex part in terms of this quantity. This method was recently successfully applied to the analysis of nucleon collisions in the energy range $E_L \sim 9$ Bev.[25] However, in the range of ultra high energies an analysis can at present be made only of the propagator, and on the assumption that it defines the region of significant values of $q(q^2 \lesssim \mu^2)$ we can obtain from this the possible values of $\mathfrak{M}$. Essentially we are concerned with finding the kinematically possible masses of the excited system. If we know them, we can apply the hydrodynamical theory in full measure to the decay of every such system. The results will differ from the results presented in Sec. 3 because, firstly, generally speaking, there will be two decaying condensations; secondly, the rest energy of each one of them will be significantly smaller than the total energy of the process; thirdly, they will each decay in its own rest system which differs from the centre of mass coordinates of the whole system.

Such an analysis[26] shows that the excitations may be divided, generally speaking, into two groups: into *symmetric* ones attaining the maximum value

$$\mathfrak{M}_1 \sim \mathfrak{M}_2 \sim \sqrt{2\mu E_C}, \qquad (28)$$

and *asymmetric* ones, in which case a still greater excitation is possible

$$\mathfrak{M}_1 - M \leqslant \mu, \quad \mathfrak{M}_2 \leqslant 2\sqrt{\frac{\mu}{M}}E_C. \qquad (29)$$

We see that in the case of asymmetric excitation we can speak only of the decay of a system which has in its rest system an energy which is smaller by only a small factor than the total energy of the process E_C.

This result has a simple obvious meaning, and has been obtained long ago[27] within the framework of the Weizsäcker-Williams method. Indeed, when the nucleons pass at an appreciable distance from each other the field of each one may be decomposed into a flux of mesons each carrying on the average a momentum $\mathbf{q} \sim (\mu/M)\mathbf{p}$ and an energy $\omega \sim (\mu/M)E_C$, where $\mathbf{p}$ and E_C are the momentum and the energy of each nucleon in the center-of-mass system. The other nucleon can collide with such a meson, and from this the excited system will originate of exactly the same kind as in the head-on collision of two particles. Therefore, we can here apply the usual hydrodynamical theory of head-on collisions (Secs. 2 and 3). The energy of this system in its rest system is given in accordance with the laws of conservation of energy and momentum (we must remember, that the nucleon and meson energies are added, while their momenta are subtracted) by the following expression:

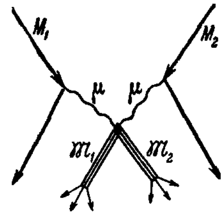

Figure 3.

$$\mathfrak{M} = \sqrt{\left(E_C + \frac{\mu}{M}E_C\right)^2 - \left(\mathbf{p} - \frac{\mu}{M}\mathbf{p}\right)^2} = \\ = \sqrt{M^2 + \mu^2 + 2\frac{\mu}{M}(E_C^2 + \mathbf{p}^2)} \approx 2\sqrt{\frac{\mu}{M}}E_C. \quad (30)$$

From this we can see that, even when only a single meson is exchanged, according to the hydrodynamic theory a number of particles may be produced which may attain the maximum value of

$$N_0 \sim \left(\frac{2\sqrt{\frac{\mu}{M}}E_C}{\mu}\right)^{1/2} \sim \left(\frac{E_L}{\mu}\right)^{1/4}. \quad (31)$$

which almost corresponds to the average number of particles in a head-on collision of nucleons. On this method we can base a classification of different types of stars[26].

Here it is also appropriate to recall the "stars with two maxima" (cf. Sec. 3, and also reference 1, Sec. 2.2.1). One might have thought that the analysis presented above would lead to their natural explanation. However, it turns out that only a fraction of the cases leads to quantitative agreement. In other cases new approaches are required. Of course, the number of stars of this type that have been studied is still not very great, and statistically the results are not sufficiently secure. However, an impression is ever more strongly formed, that here the phenomenon proceeds according to a different scheme. In this connection the explanation of such stars given by Bubelev and Zatsepin[28] is of interest. In terms of Feynman diagrams their scheme may be represented as collisions of virtual π mesons which sometimes lead to the diagram of Fig. 2C. (even without nucleon excitation), and sometimes to the mutual scattering of π mesons accompanied by their excitation and subsequent decay (in accordance with this idea they constitute the "fireball"), as shown in Fig. 3. It is assumed that the "merging" of π meson occurs only if they exchange a momentum exceeding a certain critical value. The published cases of stars with two maxima may be understood, as it turns out, if we assume that the critical value of the transverse momentum is of order of magnitude M.

CONCLUSION

The theory of multiple production at ultra high energies has attained a considerable degree of consistency and orderliness. The quantum field-theoretical approach to it has been developed sufficiently to allow the application of the usual methods of the theory of elementary particles. Due to the existence of a new large parameter (the ratio of the temperature to the mass of the particles) it may be regarded as quite possible that these methods will in this case lead to results sooner than in the usual problems of the theory of elementary particles. However, the difficulties which are yet to be overcome are exceptionally great. A special difficulty is presented by the problem of the formation of the initial state of the expanding condensation. For the time being this point is based on a specially introduced postulate.

One might think that the theory should be extended by taking weak interactions into account. Already in Heisenberg's first paper[29] the remarkable fact was noted that the β interaction has a special nonlinearity, as a result of which at a sufficiently high energy (an estimate yields $E_L \sim 10^{14}$ - 10^{15} ev) perturbation theory ceases to be valid: processes of higher order in the coupling constant (and in the number of particles produced) become, if they are computed in accordance with perturbation theory, of equal probability with processes of lower order. On this basis the opinion has been frequently expressed that at sufficiently high energies the weak interactions become strong. In such a case they should be taken into account on the same footing as the meson forces.

It should be emphasized, that for the time being such an approach appears to be excessively simplified. Indeed, at a sufficiently high energy higher order processes become equally probable with processes of lower order. However, the lower order processes themselves become of negligibly low probability. Thus, for example, the process $\mu^+ + e^- \to \nu + \tilde{\nu}$ has, according to perturbation theory, the following cross section

$$\sigma_1 \sim \frac{G^2}{M^2}\left(\frac{E_C}{M}\right)^2.$$

where $G^2 \sim 10^{-10}$ is the weak-interaction constant. The higher order processes differ from each other roughly speaking by factors $G^2 (E_C/M)^4$. Perturbation theory becomes invalid when such a factor approaches unity, i.e., when we have

$$G^2\left(\frac{E_C}{M}\right)^2 \sim \left(\frac{M}{E_C}\right)^2.$$

But in such a case the value of σ, itself becomes negligibly small, viz., of order

$$\sigma_1 \sim \frac{1}{E_C^2} \sim \frac{1}{\mu^2}\cdot 10^{-5}\left(\frac{\mu}{M}\right)^2 \sim 10^{-7}\frac{1}{\mu^2}.$$

It is difficult to say as to what will happen in the region in which perturbation theory is, inapplicable but at the present time we do not see any special reasons for taking these effects into account in the range of cosmic ray energies open to investigation.

A special problem arises resulting from attempts of applying nonlocal theories. But this goes beyond the framework of the formulation of the problem adopted in the present review.

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Translated by G. Volkoff