NONLINEAR PHENOMENA IN A PLASMA LOCATED IN AN ALTERNATING ELECTROMAGNETIC FIELD

V. L. Ginzburg and A. V. Gurevich

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INTRODUCTION

One of the characteristic features of a plasma (ionized gas) is the appearance of nonlinear effects even at relatively small and readily attainable electric fields.

This is caused by two factors: first, the mean free path of the electrons in the plasma is quite considerable, and therefore the electron may acquire from the field a large energy; secondly, the energy transfer from the electrons to the atoms, molecules, and ions is made difficult by the small ratio of the electron mass to the mass of these heavy particles. As a result the plasma electrons in an electric field become heated, and the complex dielectric permittivity of the plasma begins to depend on the field intensity. In other words, the polarization and the conduction current are no longer proportional to the field E, and consequently

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the electrodynamic processes in the plasma, and in particular, the propagation of electromagnetic waves, acquire a nonlinear character (violation of the principle of superposition, etc.).

The effect of the field on the properties of a plasma can be neglected in the first approximation if

$$E \ll E_p = \sqrt{3kT \frac{m}{e^2} \delta (\omega^2 + \nu_0^2)} = 4.2 \cdot 10^{-10} \sqrt{\delta T (\omega^2 + \nu_0^2)}\ v/cm \quad (0.1)$$

Here E_p is the characteristic "plasma field" (see Sec. 1.2), e, m, k are the electron charge and mass and Boltzmann's constant, T is the absolute temperature of the plasma in the absence of a field, δ is the average relative fraction of energy transferred upon collision of an electron with a heavy particle (in elastic collisions $\delta = 2m/M$, where M is the mass of the heavy particle), ω is the radian frequency of the field, and ν_0 $\nu^{(0)}_{eff}$ is the effective number of collisions between the electron and the heavy particles in the absence of a field. We note that in criterion (0.1) the plasma is considered, for simplicity, to be isotropic, which it is in the absence of an external magnetic field.

It is natural to call an electric field that satisfies condition (0.1) a weak field. Under the influence of a strong field ($E \gtrsim E_p$), and especially a very strong field ($E >> E_p$), the properties of the plasma are substantially changed.

Estimates show that the values of the "plasma field" E_p are in many cases quite small. In fact, for example, in the E layer of the ionosphere, $\nu_0 \sim 10^5$, $T \simeq 300$, and $\delta \simeq 10^{-3}$, while in the F layer $\nu_0 \sim 10^3$, $T \simeq 10^3$, and $\delta \simeq 10^{-4}$. Therefore, in the ionosphere, at low frequencies

$$\omega^2 \ll \nu_0^2 \quad (0.2)$$

the field $E_p \sim 10^{-5}$ -- 10^{-7} v/cm. In the sun's corona

$$\left(\nu_0 \sim 10,\ T \sim 10^6,\ \delta = \frac{2m}{M_p} = \frac{1}{918}\right)$$

subject to the same condition (0.2) $E_p \sim 10^{-7}$ v/cm. For a denser plasma, or at higher frequencies

$$\omega^2 \gg \nu_0^2 \quad (0.3)$$

the "plasma field" is, naturally, considerably greater. For example, in the ionosphere at $\omega = 2\times 10^6$ ($\lambda = 2\pi c/\omega \simeq 1$ km), $E_p \approx 5 \times 10^{-4}$ v/cm, and when $\omega = 2\times 10^7$ ($\lambda = 2\pi c/\omega \simeq 100$ m) $E_p \approx 5 \times 10^{-3}$ v/cm. In the solar corona, in the meter band investigated by radio astronomic methods, $E_p \sim 10$ v/cm; when $\lambda = 1$ cm we already get in the corona $E_p \sim 10^4$ v/cm. Finally, in laboratory installations ($\nu_0 \sim 10^6$ to 10^9, $T \sim 10^4$, $\delta \sim 10^{-1}$ -- 10^{-3}) one obtains $E_p \sim 10^{-3}$ to 10 v/cm for a low-frequency electric field [subject to condition (0.2)] and $E_p \sim (10^{-11}$ to $10^{-10}) \sqrt{T} \times \omega$ v/cm at high frequencies.

Thus, nonlinearity becomes substantial in plasma in fields which are not too large, at least from the point of view of values customary under laboratory conditions or high power radio transmitters. In non-conducting pure liquids and solids (with the exception of ferroelectrics) the situation is different. Here the effect of the field on the properties of the medium can usually be neglected up to fields on the order of 10^6 to 10^7 v/cm, which approach electric fields of atomic scale, $E_q \sim e/d^2 \sim 10^8$ v/cm (d is the dimension of the molecule or the lattice constant). In metals and in semiconductors the conduction electrons can be likened up to a point to electrons in a plasma. However, the range of nonlinearity in metals is in practice nearly unattainable, since the high conductivity prevents the production of a sufficiently strong field in a metal (in addition, the nonlinearity is decreased by electron degeneracy[1-3]). In semiconductors, nonlinearity is observed without particular difficulty and qualitatively many deductions obtained in the investigation of nonlinear phenomena in gaseous plasma apply here. Yet, nonlinear effects in semiconductors are in general less clearly pronounced than in a gas; the quantitative theory in the two cases is also different. We shall therefore consider only gaseous plasma (certain results pertaining to semiconductors are given in references 4--7).

The present article is devoted to the theory of nonlinear phenomena in a plasma. The equations of plasma dynamics (in the general formulation -- the field equations and the kinetic equations for the plasma particles) are themselves nonlinear, and thus the theory of nonlinear phenomena covers in broad outlines a considerable portion of plasma physics. We plan here to throw light on a considerably narrower but clearly outlined group of problems. In the first part of the article (Secs. 1 & 2) we shall consider the effect of a homogeneous electric field $\mathbf{E} = \mathbf{E}_0 \exp(i\omega t)$ on a nonrelativistic and nondegenerate (classical) plasma (the frequency ω may be equal to zero, corresponding to a dc field). The plasma can in this case be in a homogeneous and permanent ("external") magnetic field $\mathbf{H}_0$. Macroscopic (hydrodynamic) motion of the plasma is neglected.

The effect of the field on the plasma reduces in this formulation of the problem to the variation of the distribution function of the plasma electron velocities. This distribution function must be determined as a function of the field intensity $\mathbf{E}_0$, the frequency ω, the magnetic field $\mathbf{H}_0$, and the plasma parameters. The distribution function of heavy particles (molecules or ions) will remain Maxwellian with temperature T; in the stationary mode, the only one considered here, this assumption is usually justified.

Knowing the electron velocity distribution function, one can determine the average kinetic energy (or the effective electron temperature T_e) and the total electric current density $\mathbf{j}_t$. In the particular case of a weak field, the electrons and the heavy particles have the same temperatures, and the current density $\mathbf{j}_t$ is proportional to the field $\mathbf{E}$.

The determination of the properties of a plasma in a homogeneous field of any intensity is of interest in the analysis of many problems in gas-discharge physics, the problem of plasma heating, etc. Calculation of the current $\mathbf{j}_t$ is, in addition, a necessary preliminary stage in the solution of electrodynamic problems, particularly problems connected with the propagation of electromagnetic waves in a plasma. The second part of the article (Sec. 3) is indeed devoted to the theory of nonlinear effects that arise when radio waves propagate in a plasma, specifically in the ionosphere. As to gas discharge[8, 9] (particularly at high and microwave frequencies[10], plasma heating in an inhomogeneous field,[11] the theory of nonstationary processes in a homogeneous plasma[12, 13](including the problem of the "runaway electrons"[14], etc, these problems will not be considered.

1. PLASMA IN A HOMOGENEOUS ELECTRIC FIELD (ELEMENTARY THEORY)

Under the influence of an electric field, the electron velocity distribution in a plasma ceases to be in equilibrium: the electrons acquire an accelerated motion in the direction of the field. This motion of the electrons along the field is slowed down by collisions with the heavy plasma particles -- molecules, atoms, or ions. As a result of these two processes -- acceleration by the field and retardation by collisions -- a certain non-equilibrium velocity distribution of the electrons is established in the stationary state; this distribution must be determined, in particular, in order to find the electric current in a field of arbitrary intensity.

In the general case, the kinetic equation for the distribution function must be used to solve this problem. However, to disclose the physical picture, and frequently also to obtain sufficiently accurate quantitative formulas, it is convenient and useful to use a simpler although approximate theory, which we call "elementary".

In the elementary theory the state of the plasma is characterized by two quantities: the average velocity of directed motion of the electrons, $\mathbf{u}$, and the electron temperature T_e. From its definition, the velocity $\mathbf{u}$ is related to the total electric current density $\mathbf{j}_t$ by

$$\mathbf{j}_t = \frac{\partial \mathbf{P}}{\partial t} + \mathbf{j} = eN\mathbf{u}, \qquad (1.1)$$

where $\mathbf{P}$ is the polarization of the plasma, $\mathbf{j}$ the conduction current density, and N the electron concentration.[2)] The quantities $\mathbf{P}$ and $\mathbf{j}$ are introduced here to establish a correspondence with the usual concepts of macroscopic electrodynamics.

The electron temperature T_e is determined in the elementary theory by the relation

$$\frac{3}{2} kT_e = \overline{K} = \frac{\overline{mv^2}}{2}, \qquad (1.2)$$

where $\overline{K}$ is the average kinetic energy of the plasma electrons; since the electron velocity distribution in the plasma is by far not always Maxwellian (see Sec. 2), it would be more correct, naturally, to call the temperature T_e the effective electron temperature.

The principal task of the elementary theory is obviously the derivation of equations for $\mathbf{u}$ and T_e. The next step is to find the values of $\mathbf{u}$ and T_e themselves as functions of $\mathbf{E}$, ω, $\mathbf{H}_0$, and the plasma parameters. The accuracy of the elementary theory and the character of the approximations involved in it can be explained in a logical manner only on the basis of a kinetic consideration (see Sec. 2).

1.1 Electron Current. Dielectric Constant and Conductivity of Plasma

The equation for the average electron velocity $\mathbf{u}$ can be derived from the following considerations. In the absence of collisions each electron moves independently of the others; its velocity $\mathbf{v}$ should obviously satisfy the equation $m\, d\mathbf{v}/dt = e\mathbf{E} + (e/c)\,[\mathbf{v} \times \mathbf{H}_0]$, where $\mathbf{H}_0$ is the constant magnetic field.[3)] We now represent the velocity $\mathbf{v}$ in the form $\mathbf{v} = \mathbf{v}_0 + \mathbf{u}$ and average the equation for $\mathbf{v}$ over all the electrons at a given instant of time, considering here that $\overline{\mathbf{v}_0} = 0$. The equation for the average velocity ($\mathbf{u} = \overline{\mathbf{v}}$) will consequently be the same as the equation for the total electron velocity $\mathbf{v}$:

$$m \frac{d\mathbf{u}}{dt} = e\mathbf{E} + \frac{e}{c}\,[\mathbf{u} \times \mathbf{H}_0].$$

This averaging over all electrons is at the same time tantamount to using an average electric field; in a plasma this field is equal to the average macroscopic field

[2)] We neglect the ion motion here and elsewhere. In the absence of a magnetic field the contribution of the ions is determined by the ratio mN_i/MN (N_i is the ion concentration) and is always small if the concentration of negative ions is not very high. In the presence of an external magnetic field, the role of the ions can be neglected if the ion gyro frequency $\Omega_H = |e|\,H_0/Mc$ is considerably lower than the frequency ω of the electric field (this condition is not necessary when $\Omega_H \ll \nu^{(i)}$, where $\nu^{(i)}$ is the ion collision frequency).

of phenomenological electrodynamics (see reference 15, Sec. 57).

Under the influence of the collisions, the velocity $\mathbf{u}$ should obviously decrease; the time required for the momentum to decrease by $m\mathbf{u}$ will be denoted by $\tau_{eff} = 1/\nu_{eff}$. Then the friction force due to the collision is $-m\nu_{eff}\mathbf{u}$ and the equation for $\mathbf{u}$ becomes

$$m\frac{d\mathbf{u}}{dt} = e\mathbf{E} + \frac{e}{c}[\mathbf{u} \times \mathbf{H}_0] - m\nu_{eff}\mathbf{u}. \quad (1.3)$$

It must be emphasized that different collisions between the electron and heavy particles produce different changes in the momentum, in view of the velocity distribution of the electrons and the different impact parameters. The time τ_{eff} is therefore a certain average effective time during which each electron experiences several collisions, the average net result of which is a change of the average momentum by $m\mathbf{u}$. In the same sense, $\nu_{eff} = 1/\tau_{eff}$ is the effective number of electron collisions per unit time. The calculation of ν_{eff} is a problem in kinetic theory and necessitates knowledge of the corresponding effective cross section. It is clear, however, even without these calculations, that the number of collisions depends on the electron velocity and, for example, in the case of a velocity-independent effective cross section q we have $\nu_{eff} = qN_m v$, where N_m is the concentration of molecules (we neglect here collisions with ions) and v is the average electron velocity. Since the total electron velocity in any field is close to its random velocity (see below), we can consider the velocity v to be proportional to $\sqrt{T_e}$. Thus, for collisions with molecules, when in the first approximation the cross section q is independent of the velocity, we can put

$$\nu_{eff\,m} = \nu^{(0)}_{eff\,m}\sqrt{\frac{T_e}{T}}, \quad (1.4)$$

where $\nu^{(0)}_{eff\,m}$ is the number of collisions in the absence of a field (or in a weak field), i. e., when $T_e = T$.

For collisions between electrons and ions, when q is inversely proportional to v^4 (Rutherford scattering), we have in the first approximation

$$\nu_{eff\,i} = \nu^{(0)}_{eff\,i}\left(\frac{T}{T_e}\right)^{3/2}. \quad (1.5)$$

We shall treat the expressions for ν_{eff} in greater detail in Sec. 2. It is important to emphasize here only that within the framework of the elementary theory there are grounds for assuming that the number of collisions ν_{eff} depends only on the electron temperature T_e, but not on the velocity $\mathbf{u}$ (see Sec. 1.2).

In the absence of an external magnetic field (in an isotropic plasma) we obtain for an electric field $\mathbf{E} = \mathbf{E}_0\exp(i\omega t)$ from (1.3).

$$\mathbf{u} = \frac{e\mathbf{E}}{m(i\omega + \nu_{eff})} = \frac{e\mathbf{E}}{m}\left(\frac{\nu_{eff}}{\omega^2+\nu^2_{eff}} - i\frac{\omega}{\omega^2+\nu^2_{eff}}\right). \quad (1.6)$$

We consider here only the steady-state solution, and assume also that T_e, meaning also ν_{eff}, is independent of the time (although they may depend on the field amplitude E_0).

In macroscopic electrodynamics, in the linear approximation, one usually introduces the dielectric permittivity ϵ and the conductivity σ, defined by the relations $\mathbf{P} = (\epsilon - 1)\mathbf{E}/4\pi$, $\mathbf{j} = \sigma\mathbf{E}$. Expressed in these terms, Eq. (1.1) becomes

$$\left.\begin{aligned} \mathbf{j}_t &= i\omega\mathbf{P} + \mathbf{j} = \left(i\omega\frac{\varepsilon-1}{4\pi} + \sigma\right)\mathbf{E} \\ \varepsilon' &= \varepsilon - i\frac{4\pi\sigma}{\omega}\ . \\ &= \frac{i\omega}{4\pi}(\varepsilon'-1) = eN\mathbf{u}, \end{aligned}\right\} \quad (1.7)$$

Comparing (1.6) and (1.7) we get

$$\varepsilon = 1 - \frac{4\pi e^2 N}{m(\omega^2+\nu^2_{eff})}$$
$$\sigma = \frac{e^2 N\nu_{eff}}{m(\omega^2+\nu^2_{eff})}\ . \quad (1.8)$$

The forms of the quantities ϵ and σ remain obviously the same if ν_{eff} depends on the field intensity (because ν_{eff} is a function of T_e and T_e is a function of E_0) and the medium becomes nonlinear. It is therefore convenient to use the concept of dielectric permittivity and conductivity even in the nonlinear theory [at first under conditions when the expressions of type (1.6) are valid with a time-independent number of collisions ν_{eff}]. If the introduction of ϵ and σ is inadvisable, the expression for $\mathbf{j}$ must be used directly.

In the presence of a magnetic field $\mathbf{H}_0$ the plasma be-

3) We neglect here and later the action of the alternating magnetic field of the radio wave; this is usually valid when $u/c \ll 1$.

comes anisotropic (magnetoactive) and one must write instead of (1.7)

$$\left.\begin{aligned} \mathbf{j}_{t,\,i} &= (i\omega\mathbf{P}+\mathbf{j})_i = \left(i\omega\,\frac{\varepsilon_{ik}-\delta_{ik}}{4\pi}+\sigma_{ik}\right)E_k = \frac{i\omega}{4\pi}\,(\varepsilon'_{ik}-\delta_{ik})\,E_k = eNu_i, \\ \varepsilon'_{ik} &= \varepsilon_{ik} - i\,\frac{4\pi\sigma_{ik}}{\omega}\,,\quad \delta_{ik}=1 \text{ when } i=k,\ \delta_{ik}=0 \text{ when } i\neq k, \end{aligned}\right\} \tag{1.9}$$

where repeated indices imply summation (one must not confuse the imaginary number i with the index i !) and the velocity $\mathbf{u}$ must be determined from (1.3). If the field $\mathbf{H}_0$ is aligned with the z axis, then it is readily shown that (see reference 15, Sec. 62):

$$\left.\begin{aligned} \varepsilon_{xx} &= \varepsilon_{yy} = 1-\frac{4\pi e^2N}{m}\cdot\frac{1}{2\omega}\left\{\frac{\omega-\omega_H}{(\omega-\omega_H)^2+\nu^2}+\frac{\omega+\omega_H}{(\omega+\omega_H)^2+\nu^2}\right\}, \\ \varepsilon_{yx} &= -\varepsilon_{xy} = i\,\frac{4\pi e^2N}{m}\cdot\frac{1}{2\omega}\left\{\frac{\omega-\omega_H}{(\omega-\omega_H)^2+\nu^2}-\frac{\omega+\omega_H}{(\omega+\omega_H)^2+\nu^2}\right\}, \\ \varepsilon_{zz} &= 1-\frac{4\pi e^2N}{m}\cdot\frac{1}{\omega^2+\nu^2},\quad \varepsilon_{yz}=\varepsilon_{xz}=0; \\ \sigma_{xx} &= \sigma_{yy} = \frac{e^2N}{m}\cdot\frac{\nu}{2}\left\{\frac{1}{(\omega-\omega_H)^2+\nu^2}+\frac{1}{(\omega+\omega_H)^2+\nu^2}\right\}, \\ \sigma_{yx} &= -\sigma_{xy} = i\,\frac{e^2N}{m}\cdot\frac{\nu}{2}\left\{\frac{1}{(\omega-\omega_H)^2+\nu^2}-\frac{1}{(\omega+\omega_H)^2+\nu^2}\right\}, \\ \sigma_{zz} &= \frac{e^2N\nu}{m\,(\omega^2+\nu^2)},\quad \sigma_{yz}=\sigma_{xz}=0. \end{aligned}\right\} \tag{1.10}$$

Here $\omega_H = |e|H_0/mc$ is the gyromagnetic frequency and $\nu \equiv \nu_{eff}$. It is seen from (1.10) that in a high-frequency electric field ($\omega^2 \gg \nu^2$), in the case when the frequency ω approaches the gyro frequency ω_H, there is a resonant increase in the conductivity, usually called gyromagnetic (or cyclotron) resonance. The reason for this phenomenon is quite understandable. An alternating electric field of frequency ω, directly perpendicular to $\mathbf{H}_0$, can be thought of as consisting of two circularly polarized fields rotating with frequency ω in opposite directions. In addition, the electrons themselves rotate about the direction of the magnetic field with frequency $\omega_H = |e|H_0/mc$. Consequently, from the point of view of the response of the electrons to the component of E that rotates in the same direction as the electron in the magnetic field, the presence of a magnetic field is equivalent to a reduction of frequency by ω_H. Therefore, when ω is close to ω_H, the effect of this component of the alternating electric field on the electron is equivalent to the effect of a constant electric field, which indeed causes of the resonant increase in electron velocity and consequently in the conductivity of the plasma at $\omega \simeq \omega_H$.

1.2. Electron Temperature

The equation for the effective electron temperature, $T_e = (2/3k)\,(\overline{mv^2}/2)$, is obtained from the law of conservation of energy. The electric field performs on the plasma the work $\mathbf{j}\cdot\mathbf{E} = eN\mathbf{u}\cdot\mathbf{E}$ per unit time, or the work $e\mathbf{u}\cdot\mathbf{E} = \mathbf{j}_t\cdot\mathbf{E}/N$ per electron. On the other hand, the electrons lose energy by collision with the heavy particles. This energy has an average value $\delta_{eff}\nu_{eff}(3/2)k\,(T_e - T)$ per unit time, where δ_{eff} is a certain coefficient, which has the physical meaning of the average relative fraction of energy lost in a single collision (at the same time δ_{eff} is the ratio of the time of relaxation τ_{eff} of the average momentum to the time of relaxation $\tau_{eff}^{(k)}$ of the average energy). The definition of δ_{eff} will be made more precise in Sec. 2. Here we shall merely identify the effective and average values with the true values and assume that in each collision (the number of collisions being ν_{eff}) the fast electron transmits to the heavy particle an energy $\delta_{eff}mv^2/2$ (the electron is considered fast if its energy $mv^2/2 = 3kT_e/2$ is considerably greater than the energy $3kT/2$ of the heavy particles; under such conditions the heavy particles can be considered immobile). Then the fast electron will give up per unit time an energy

$$\delta_{eff}\cdot\nu_{eff}\,\frac{mv^2}{2} = \delta_{eff}\cdot\nu_{eff}\cdot\frac{3}{2}\,kT_e.$$

However, as the electron energy is decreased, this value of transmitted energy cannot be the actual energy, since when $T_e = T$ (thermal equilibrium), it is obvious that the average electron energy is not changed at all by collision. The physical situation is that as T_e approaches T, the heavy particles can no longer be considered immobile and energy is not only transmitted from the electron to these particles, but in certain collisions, to the contrary, the electrons acquire energy. To take this into account, therefore, the energy transferred to the heavy particles is written in the form

$$\delta_{eff}\cdot\nu_{eff}\cdot\frac{3}{2}k(T_e-T).$$

We can now write the energy balance for the electrons in a plasma in the form

$$\frac{d}{dt}\left(\frac{3}{2}NkT_e\right)=\mathbf{jE}-\frac{3}{2}\delta_{eff}\cdot\nu_{eff}\cdot Nk(T_e-T),$$

or

$$\frac{dT_e}{dt}=\frac{2}{3kN}\cdot\mathbf{j}\cdot\mathbf{E}-\delta_{eff}\cdot\nu_{eff}(T_e-T). \quad (1.11)$$

A very important fact is that under stationary conditions in a plasma δ_{eff} is always less than unity.[4)]

Because of this, the stationary random velocity of the electron v is always much greater than its directed velocity u even in a strong electric field. To prove this and to clarify in general the character of the solutions of Eq. (1.11), let us draw on a few particular cases.

In the absence of the electric field, if $\delta_{eff}\nu_{eff}$ = const (i. e., it is independent of T_e) we have

$$T_e=T+(T_e-T)_{t=0}\cdot e^{-\delta_{eff}\cdot\nu_{eff}\cdot t}, \quad (1.12)$$

i. e., the temperature relaxation time is actually equal to $\tau_{eff}^{(k)}=(\delta_{eff}\nu_{eff})^{-1}$. In accounting for the temperature dependence of $\delta_{eff}\nu_{eff}$, the situation becomes more complicated, but remains usually qualitatively approximately the same as when $\delta_{eff}\nu_{eff}$ = const. We note that that Eq. (1.3) for $\mathbf{u}$ in the absence of a field and ν_{eff} = const has a solution $\mathbf{u}=\mathbf{u}_{t=0}\exp(-\nu_{eff}t)$; thus, the momentum relaxation time, $\tau_{eff}=1/\nu_{eff}$ is considerably less than $\tau_{eff}^{(k)}=(\delta_{eff}\nu_{eff})$ when $\delta_{eff}\ll 1$. We shall disregard from now on the relaxation terms in the expressions for $\mathbf{u}$ and T_e, since only steady-state processes are considered.

To find $\mathbf{u}$ and T_e in the electric field, it is necessary to solve simultaneously (1.3) and (1.11), which is in general a complicated matter. The situation becomes simplified if ν_{eff} = const and δ_{eff} = const. Then in a field $\mathbf{E}=\mathbf{E}_0\exp(i\omega t)$ the solution (1.6) is satisfied, or, on going to real quantities

$$\mathbf{E}=\mathbf{E}_0\cos\omega t,\ \mathbf{j}_t=eN\mathbf{u}=\frac{e^2N\mathbf{E}_0}{m(\omega^2+\nu^2)}(\nu\cos\omega t+\omega\sin\omega t).$$

Furthermore,

$$\frac{dT_e}{dt}=\frac{e^2E_0^2}{3km(\omega^2+\nu^2)}(\nu+\nu\cos 2\omega t+\omega\sin 2\omega t)-\delta\nu(T_e-T),$$

$$T_e-T=\frac{e^2E_0^2}{3km\delta(\omega^2+\nu^2)}\left\{1+\frac{(\delta\nu^2-2\omega^2)\delta}{4\omega^2+\delta^2\nu^2}\cos 2\omega t+\frac{\omega\nu(2+\delta)\delta}{4\omega^2+\delta^2\nu^2}\sin 2\omega t\right\}. \quad (1.13)$$

Here and in many cases below we shall omit the index "eff".

At very low frequencies, when

$$\omega\ll\delta\nu, \quad (1.14)$$

we have, accurate to a small term of order $\omega/\delta\nu$

$$T_e-T=\frac{2e^2E_0^2}{3km\delta\nu^2}\cos^2\omega t=\frac{2e^2E^2(t)}{3km\delta\nu^2}, \quad (1.15)$$

where we take into consideration the fact that in (1.14) ω is certainly much less than ν, since $\delta\ll 1$. In the other limiting case

$$\omega\gg\delta\nu, \quad (1.16)$$

we have accurate to terms of order $\delta\nu/\omega$ and δ

$$T_e-T=\frac{e^2E_0^2}{3km\delta(\omega^2+\nu^2)}=\frac{2\overline{E^2}}{3km\delta(\omega^2+\nu^2)} \quad (1.17)$$

($\overline{E}^2$ is the time average of E^2). Thus, in the case of (1.16) the temperature T_e is constant in the first approximation;

4) In a weakly-ionized plasma in monatomic gases at low electron temperatures (less than of the order of 1 ev), $\delta_{eff}=2m/M\sim 10^{-4}$ to 10^{-5} (M is the mass of the atom). In molecular gases under the same conditions $\delta_{eff}\sim 10^{-3}$. With increase of the electron temperature the role of inelastic collisions becomes greater and δ_{eff} increases; this increases the degree of ionization and causes breakdown, after which the ionization increases sharply. Simultaneously electron-ion collisions begin to play a substantial role, causing δ_{eff} again to decrease. The maximum value of δ_{eff} in stationary conditions until breakdown occurs is apparently on the order of 10^{-1}. In a completely ionized plasma δ_{eff} is equal to $2m/M$, as before. For details see Sec. 2.

the ac component of T_e (with frequency 2ω) has a low amplitude, $\delta\nu/\omega$ or δ times smaller than the dc component of T_e. The fact that the electron temperature is approximately constant in an alternating electric field when $\omega >> \delta\nu$ is quite understandable. In fact, as shown eariler, the relaxation time for the electron temperature in a plasma is on the order of $\tau^{(k)}_{eff} = (\delta_{eff} \cdot \nu_{eff})^{-1}$ and therefore the electron temperature cannot change substantially within the time $1/\omega << 1/\delta\nu$, during which the electric field changes. As a result, the temperature is established at a certain average time-independent level, and the deviations from this level are small.

If the dependence of $\delta_{eff}\nu_{eff}$ on T_e is taken into account, (1.11) can be solved subject to conditions (1.16) by series expansion in powers of $(\delta_{eff}\nu_{eff})/\omega$ and δ_{eff}. In the first approximation the electron temperature T_e is constant; it is determined by the relation

$$T_e - T = \frac{e^2E_0^2}{3km\delta(T_e)(\omega^2+\nu^2(T_e))}. \quad (1.18)$$

It is clear therefore that even in a very strong field (when $T_e >> T$) the average electron velocity $\bar{v}$ is close to the random velocity, since

$$\bar{v} \sim \sqrt{\frac{kT_e}{m}} \sim \frac{eE_0}{m\sqrt{\delta}\sqrt{\omega^2+\nu^2}}, \quad (1.19)$$

whereas the ordered velocity [see (1.6)] is

$$|\mathbf{u}| = \frac{eE_0}{m\sqrt{\omega^2+\nu^2}} \sim \sqrt{\delta}\,\bar{v}. \quad (1.20)$$

In a constant electric field, E = const, obviously $\mathbf{u} = e\mathbf{E}/m\nu$ and according to (1.11) we have in the stationary state, for arbitrary dependence of $\delta_{eff}\nu_{eff}$ on T_e,

$$T_e - T = \frac{2E^2}{3km\delta\nu^2}. \quad (1.21)$$

Equation (1.21) can be derived from (1.18) by putting in it $\omega = 0$ and replacing the amplitude E_0 by $\sqrt{2}E$, where E is the intensity of the constant electric field. Thus the heating of an electron gas in a constant field is the same as in an alternating field of low frequency $\omega^2 << \nu^2$, which is quite understandable, since when $\omega^2 << \nu^2$ the alternating field acts on the electron on the average like a constant field $E = E_{eff} = E_0/\sqrt{2}$. It follows from (1.21) that the condition $|\mathbf{u}| \lesssim \sqrt{\delta}\bar{v}$ is satisfied also in a constant field.

Thus, for any frequency ω, as stated above, $u << \bar{v}$ by virtue of the condition $\delta_{eff} << 1$.

The expressions (1.18) and (1.21) are implicit solutions of Eq. (1.11) for T_e, since the collision frequency ν_{eff} depends under real conditions on T_e [see (1.4) and (1.5)]; the dependence of δ_{eff} on T_e will not be considered here within the framework of elementary theory, although it can be derived in principle.

Before we find an explicit expression for T_e, let us write the solution (1.18) in the form

$$\frac{T_e}{T} = 1 + \left(\frac{E_0}{E_p}\right)^2 \frac{\omega^2+\nu_0^2}{\omega^2+\nu^2(T_e)}, \quad (1.22)$$

where $\nu_0 \equiv \nu^{(0)}_{eff}(T)$ is the number of collisions at $T_e = T$ and E_p is the characteristic "plasma field"

$$E_p = \sqrt{3kT\frac{m}{e^2}\delta(\omega^2+\nu_0^2)}. \quad (1.23)$$

It is seen from (1.22) that if the amplitude of the electric field intensity is lower than the "plasma field" ($E_0 << E_p$), then the electron temperature changes only slightly under the influence of the field. The changes in the electron collision frequency (1.4) and (1.5) are equally insignificant here, as are consequently the conductivity and the dielectric permittivity of the plasma. Thus, an electric field $E_0 << E_p$ barely affects the plasma, and will be called "weak."

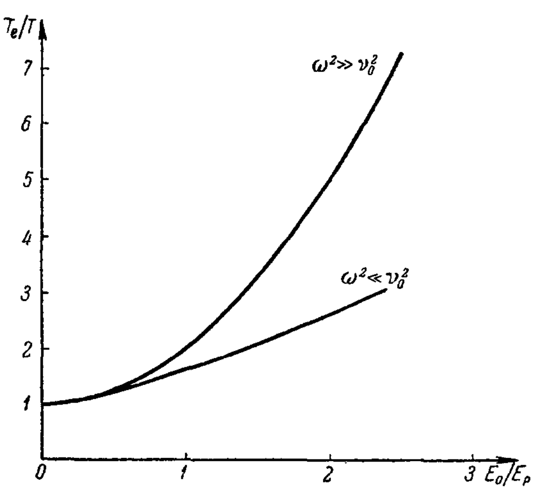

Figure 1

If, however, $E_0 \gtrsim E_p$, then the electron temperature, and hence the other parameters of the plasma ($\nu_{eff}, \epsilon, \sigma$) change significantly under the influence of an electric field. As already mentioned in the introduction, such fields will be called strong, and fields with $E_0 >> E_p$ will be called very strong.

Solving the algebraic equation (1.22) for the case when collisions between electrons and molecules play the principal role, i. e., when $\nu_{\mathrm{eff}}(T_e) = \nu_0 \sqrt{T_e/T}$ [see (1.4)], we get

$$T_e = T\left[1 + \frac{\omega^2+\nu_0^2}{2\nu_0^2}\left(\sqrt{1 + \frac{4\nu_0^2}{\omega^2+\nu_0^2}\left(\frac{E_0}{E_p}\right)^2} - 1\right)\right]. \tag{1.24}$$

The dependence of T_e on E_0/E_p when $\omega^2 \gg \nu_0^2$ and $\omega^2 \ll \nu_0^2$ is shown in Fig. 1. It is seen from the diagram that in this case the electron temperature increases monotonically with increasing E_0.

At high frequencies $\omega^2 \gg \nu^2$, as is clear from (1.18)

$$\frac{T_e}{T} = 1 + \frac{e^2E_0^2}{3kTm\delta\omega^2}. \tag{1.25}$$

This expression for T_e is independent of ν or ν_0 and consequently is valid not only for collisions between electrons and molecules, but also for collisions with ions.

An interesting peculiarity arises[13] at low frequencies $\omega^2 \ll \nu_0^2$ in the case of collisions with ions. It is easy to see that the connection between T_e and E_0/E_p, defined by Eq. (1.22), is single valued at low frequencies only when the collision frequency does not decrease with increasing T_e (as takes place for collisions with molecules) or else diminishes not faster than $T_e^{-1/2}$. In collisions with ions, this condition is naturally not satisfied ($\nu \sim T_e^{-3/2}$). As a result, when $\omega^2 < \nu_0^2$, in a definite range of values of field amplitude $E_k^{\mathrm{II}} < E_0 \leqslant E_k^{\mathrm{I}}$ one value of E_0 corresponds not to one, as usual, but to three stationary states with different electron temperatures (Fig. 2). However, only two of these, corresponding to the lowest and to the highest curves of Fig. 2 are stable; the state corresponding to the middle curve is unstable. The critical field for the lower curve is $E_k^{\mathrm{I}} = 0.28\, E_p(0)$, where $E_p(0)$ is the "plasma field" (1.23) for $\omega = 0$. The absence of a "low temperature" stationary state for $E_0 \geqslant E_k^{\mathrm{I}}$ is due to the fact that the energy imparted to the electrons by the low-frequency electric field increases sharply with increasing electric temperature ($\mathbf{E}\cdot\mathbf{j} \sim 1/\nu \sim T_e^{3/2}$), whereas the energy transferred to the electrons by the ions diminishes [$\delta\nu(T_e - T) \sim T_e^{-1/2}$]. Therefore in a sufficiently strong electric field ($E_0 \gg E_k^{\mathrm{I}}$) the electrons can no longer transfer to the ions all the energy they absorb, and the electron temperature begins to increase. As the temperature increases, however, the collision frequency decreases and when it drops below the field frequency the low-frequency condition is violated. This makes possible a second ("high-temperature") stable stationary state (1.25) for a strongly-heated electron gas, when $\nu^2(T_e) \ll \omega^2$. The transition from the low temperature state to the high temperature one is shown by the arrow

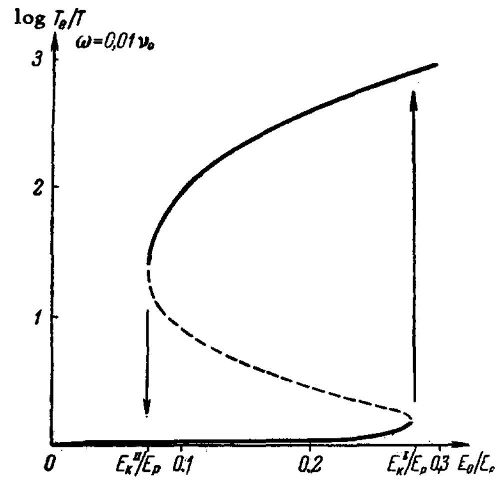

Figure 2.

in Fig. 2. The reverse transition occurs at a field $E_k^{\mathrm{II}} \approx 1.7(\omega/\nu_0)^{3/2} E_p(0)$; naturally, E_k^{II} is weaker than E_k^{I}. This gives rise to a unique hysteresis in the dependence of the electron temperature on the amplitude of the electric field.

In the case of collisions with ions the low-temperature state of the electron gas becomes unstable also in a constant electric field; the corresponding critical field is $E_k = E_k^{\mathrm{I}}/\sqrt{2} \simeq 0.2\, E_p(0)$. In this case, unlike the alternating field, there is no second stationary state [since the case (1.25) cannot be realized, naturally, when $\omega = 0$]; therefore the electron temperature increases continuously with time when $E > E_k$ (see reference 13).

We note, furthermore, that in this case of collisions with ions, in a very strong constant electric field,

$$E > E_c \simeq \sqrt{kT_e m}\cdot\frac{\nu(T_e)}{e}$$

the average directed electron velocity also becomes unstable. This is connected with the fact that under nonstationary conditions one can no longer assume the average directed velocity of the electrons, u, to be much less than the random velocity [as is always the case under stationary conditions; see (1.20)]. As a result, the number of collisions between the electrons and the ions begins to depend substantially on the velocity u, diminishing as $1/u^3$ [cf. (1.5)]. Therefore in a field $E > E_c$ the average electron velocity can increase so strongly, that the role of the collisions becomes negligibly small and the electrons are uniformly accelerated by the field. For the same reason the fastest electrons of the plasma, namely the electrons belonging to the "tail" of the distribution function ($v \gg \sqrt{kT_e/m}$), are not in a stable state even when $E < E_c$, for they are uniformly accelerated by the field; such electrons are customarily called run-away electrons.[14] Thus, for a pure electron-ion plasma in the "tail" of the distribution function the stationary state is not realized at all in a constant field. However, under conditions when $E \ll E_c$, and particularly under stationary conditions (as regards temperature) $E < E_k = E_k^I/\sqrt{2}$, the number of run-away electrons is small. In an alternating electric field, the electron run-away effect is missing. A detailed examination of these nonstationary phenomena in low-frequency and constant electric fields is outside the scope of this article.

The electron temperature is found to be constant, in first approximation (in an alternating electric field with frequency $\omega \gg \delta\nu$), also when the plasma is subject to an external magnetic field $\mathbf{H}_0$. In this case we put in (1.11) $dT_e/dt = 0$ and $j_i = \sigma_{ik}E_k$, σ_{ik} being defined by expression (1.10). As a result we obtain for T_e the equation

$$\frac{T_e}{T} = 1 + \left(\frac{E_0}{E_p}\right)^2 (\omega^2 + \nu_0^2) \left\{ \frac{\cos^2\beta}{\omega^2 + \nu^2(T_e)} + \frac{\sin^2\beta}{2[(\omega - \omega_H)^2 + \nu^2(T_e)]} + \frac{\sin^2\beta}{2[(\omega + \omega_H)^2 + \nu^2(T_e)]} \right\} \tag{1.26}$$

Here $\omega_H = |e|H_0/mc$ is the gyromagnetic frequency and β is the angle between the field $\mathbf{E}$ and $\mathbf{H}_0$. It is seen from (1.26) that in the case of a high-frequency electric field ($\omega^2 \gg \nu^2$), if the frequency ω is close to the gyrofrequency ω_H, a resonant increase of the electron temperature takes place. This temperature increase is the consequence of the resonant increase in conductivity, noted above.

In a constant electric field Eq. (1.26) also applies, if we put $\omega = 0$ and replace the amplitude E_0 by $\sqrt{2}\ E$, where E is the intensity of the constant electric field. It must be recalled, however, that in this case the role of the ion current can be neglected, as was done everywhere above, only if $\Omega_H \lesssim \nu^{(i)}$, where $\nu^{(i)}$ is the collision frequency for ions.

Within the framework of elementary theory, Eqs. (1.3) and (1.11) for $\mathbf{u}$ and T_e are the starting point for the analysis of the behavior of a plasma in an arbitrary field, including one of arbitrary frequency ω or one with a more complicated dependence on the time (for example, if the alternating electric field is amplitude modulated at a low frequency Ω). It is precisely these equations that are frequently used in the theory of nonlinear effects that arise in the propagation of radio waves in the ionosphere (see Sec. 3), and also in many other cases.

We note that the elementary theory is exact only when the collision frequency ν and the fraction of energy δ transferred in a single collision are the same for all electrons, i. e., are independent of the electron velocity. In a plasma, however, ν and δ are actually functions of v. The substitution of average or effective values ν_{eff} and δ_{eff} for $\nu(v)$ and $\delta(v)$, as is done in the elementary theory, is not a quite rational operation, and its accuracy must be checked by kinetic calculations, as will be done later (see Sec. 2.5b). Naturally, when ν and δ depend weakly on v the results of the elementary and kinetic analysis should be close to each other, as is indeed the case.

2. PLASMA IN A HOMOGENEOUS ELECTRIC FIELD (KINETIC THEORY)

In the kinetic theory, the state of the electron gas in a plasma located in an electric and magnetic field is described by a distribution function $f(\mathbf{v}, \mathbf{r}, t)$. Here, by definition, the average number of electrons in a volume $d\mathbf{v}\cdot d\mathbf{r} = dv_x dv_y dv_z dx\, dy\, dz$ is equal to $f\ d\mathbf{v}\cdot d\mathbf{r}$, where $\mathbf{v}$ is the electron velocity and $\mathbf{r}$ the corresponding radius vector. It follows, therefore, that the electron density N of interest to us, the average electron energy $\overline{K}$, and the electron current $\mathbf{j}$ at the point $\mathbf{r}$ and at the instant t can be expressed with the aid of the function f in the following manner

$$\left.\begin{aligned} N &= \int f(\mathbf{v}, \mathbf{r}, t)\, d\mathbf{v}, \\ \overline{K} &= \frac{1}{N}\int \frac{mv^2}{2} f(\mathbf{v}, \mathbf{r}, t)\, d\mathbf{v}, \\ \mathbf{j} &= \int e\mathbf{v} f(\mathbf{v}, \mathbf{r}, t)\, d\mathbf{v}. \end{aligned}\right\} \tag{2.1}$$

2.1 The Kinetic Equation

The Boltzmann kinetic equation, from which the function f should be determined, has the following form[5)]

$$\frac{\partial f}{\partial t} + \mathbf{v}\,\mathrm{grad}_{\mathbf{r}} f + \frac{e}{m}\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}\times\mathbf{H}_0]\right)\mathrm{grad}_{\mathbf{v}} f + S = 0. \tag{2.2}$$

Here S is the so called collision integral, which describes the variation of the function f when electrons collide with

[5)]The applicability of the Boltzmann equation to a plasma is limited by the conditions $e^2N^{1/3} \ll kT$ (the interaction energy per particle should be much less than the particle kinetic energy). In addition, we assume that $kT \gg \hbar^2N^{2/3}/m$ (condition of nondegenerate plasma). The derivation of the kinetic equation can be found, for example, in reference 16.

each other or with all other particles of the plasma

$$S = \iint d\mathbf{v}_1\, d\Omega q(u, \theta)\, u\{f(\mathbf{v})F(\mathbf{v}_1) - f(\mathbf{v}')F(\mathbf{v}_1')\}, \tag{2.3}$$

where $\mathbf{v}_1$ is the velocity of the particle with which the electron collides (we call it particle 1), $u = |\mathbf{v} - \mathbf{v}_1|$, $q(u, \theta)$ is the differential effective scattering cross section, $\mathbf{v}'$ and $\mathbf{v}_1'$ are the velocity of the electron and of particle 1 prior to the collision (after the collision their velocities are $\mathbf{v}$ and $\mathbf{v}_1$ respectively), F is the distribution function of the particles 1. The integration in (2.3) is over the velocities of particle 1 ($d\mathbf{v}_1$) and over the scattering angles $d\Omega = \sin\theta d\theta d\phi$, where θ is the angle between $\mathbf{v} - \mathbf{v}_1$ and $\mathbf{v}' - \mathbf{v}_1'$.

Electrons in plasma are significantly affected by elastic and inelastic collisions with molecules [S_m^{el} and S_m^{inel}], collisions with ions (S_i) and collisions with each other (S_e), i.e., in general $S = S_m^{el} + S_m^{inel} + S_i + S_e$. In collisions with heavy particles (molecules or ions) their distribution functions F will be considered Maxwellian. If collisions between electrons are considered, then $F = f$. The interelectron collisions make Eq. (2.2) nonlinear.

Let us see now how to simplify Eq. (2.2) by using the principal features of the behavior of electrons in a plasma. We take into account first that, as was shown earlier in the elementary analysis, the random (thermal) velocity of the electron in the stationary state is always much greater than its directed velocity. Accordingly, one can expect under the same conditions the distribution function to depend essentially only on the absolute value of the velocity, and not on its direction. It is therefore convenient to separate from the distribution function its principal part $f_0(v, \mathbf{r}, t)$ which depends only on the absolute value of the velocity (and which is symmetrical) from the directional part f_1, i. e., to expand the angular portion of the distribution function in a series of spherical functions in velocity space.

Let us consider first, for the sake of simplicity, the isotropic case ($H_0 = 0$) and assume that the spatial gradient of the distribution function is directed along the z axis, parallel to the field $\mathbf{E}$. Then there is only one separated direction $\mathbf{E}$ (i. e., the z axis) and the distribution can be expanded in zero-order spherical functions, i. e., in Legendre polynomials $P_k(\cos\theta_1)$, where θ_1 is the angle between $\mathbf{E}$ and $\mathbf{v}$

$$f(\mathbf{v}, \mathbf{r}, t) = \sum_{k=0}^{\infty} P_k(\cos\theta_1)\, f_k(v, \mathbf{r}, t). \tag{2.4}$$

Let us now substitute the expansion (2.4) in Eq. (2.2), multiply it by the polynomials $P_k(\cos\theta_1)$, and integrate it over the angles, using the orthogonality and other properties of the Legendre polynomials (see reference 17, p. 394); we also take into account the fact that

$$\mathbf{E}\,\mathrm{grad}_{\mathbf{r}} f = E\cos\theta_1 \frac{\partial f}{\partial v} + \frac{E\sin^2\theta_1}{v}\frac{\partial f}{\partial(\cos\theta_1)}.$$

We then obtain instead of (2.2) the following system of linked functions $f_0, f_1, f_2, \ldots$

$$\left.\begin{aligned}
&\frac{\partial f_0}{\partial t} + \frac{v}{3}\frac{\partial f_1}{\partial z} + \frac{eE}{3mv^2}\frac{\partial}{\partial v}(v^2 f_1) + S_0 = 0,\\
&\frac{\partial f_1}{\partial t} + v\left(\frac{\partial f_0}{\partial z} + \frac{2}{5}\frac{\partial f_2}{\partial z}\right) + \frac{eE}{m}\left[\frac{\partial f_0}{\partial v} + \frac{2}{5v^3}\frac{\partial}{\partial v}(v^3 f_2)\right] + S_1 = 0,\\
&\frac{\partial f_2}{\partial t} + v\left(\frac{2}{3}\frac{\partial f_1}{\partial z} + \frac{3}{7}\frac{\partial f_3}{\partial z}\right) + \frac{eE}{m}\left[\frac{2}{3}v\frac{\partial}{\partial v}\left(\frac{1}{v}f_1\right) + \frac{3}{7v^4}\frac{\partial}{\partial v}(v^4 f_3)\right] + S_2 = 0,\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
&\text{where}\\
&\qquad S_k = \frac{2k+1}{4\pi}\int P_k(\cos\theta_1)\, S\, d\Omega_1.
\end{aligned}\right\} \tag{2.5}$$

We note that in examining collisions with heavy particles, when the collision integral is linear with respect to the electron distribution function f, the integral S_k depends only on the function f_k. In fact, substituting in the collision integral the expansion (2.4), multiplying it by $P_k(\cos\theta_1)$, and integrating over $d\Omega_1$, we have

$$\begin{aligned}
S_k &= \frac{2k+1}{4\pi}\int d\mathbf{v}_1\, d\Omega \int q(u, \theta)\, uP_k(\cos\theta_1)\left\{F\sum_k P_k(\cos\theta)\, f_k - \right.\\
&\left. - F'P_k(\cos\theta_1')\, f_k'\right\} d\Omega_1 = \int d\mathbf{v}_1\, d\Omega q(u, \theta)\, u\{f_k\cdot F - F'\cdot f_k'\cdot P_k(\cos\theta)\}.
\end{aligned} \tag{2.5a}$$

Here, in the integration over $d\Omega_1$, it is taken into consideration that $\cos\theta_1' = \cos\theta\cos\theta_1 + \sin\theta\sin\theta_1\cos\phi_1$ (θ_1 is the angle between $\mathbf{v}'$ and $\mathbf{E}$), and use is made of the theorem for the addition of Legendre polynomials (for more details see references 18 -- 20).

It is seen from (2.5) that the resultant chain of equations can be terminated with the first two, if the function f_2 can be neglected compared with the fundamental function f_0, or more accurately if $\frac{\partial f_0}{\partial v} \gg \frac{1}{v^3}\frac{\partial}{\partial v}(v^3 f_2)$ and $\frac{\partial f_0}{\partial z} \gg \frac{\partial f_2}{\partial z}$.

Taking into account the fact that $S_1 = \nu f_1$ with sufficient accuracy (this will be proved in Sec. 2.2) and that correspondingly $S_2 \simeq \nu f_2$, we can express the function f_2 approximately in terms of f_0. For example, in a spatially-homogeneous plasma ($\partial f/\partial z = 0$) under stationary conditions $\partial f_1/\partial t = i\omega f_1$, $\partial f_2/\partial t \approx i\omega f_2$; therefore, as is clear from (2.5), $f_1 = \frac{eE}{m(i\omega+\nu)}\frac{\partial f_0}{\partial v}$ and consequently

$$|f_2| \sim \left|\frac{eE}{m(i\omega+\nu)}\, v\frac{\partial}{\partial v}\left(\frac{1}{v}f_1\right)\right| \simeq \frac{e^2E^2}{m^2(\omega^2+\nu^2)}\, v\left|\frac{\partial}{\partial v}\left(\frac{1}{v}\frac{\partial f_0}{\partial v}\right)\right|.$$

Then the condition $\frac{\partial f_0}{\partial v} \gg \frac{1}{v^3}\frac{\partial}{\partial v}(v^3 f_2)$ can be rewritten in the form

$$\frac{e^2E^2}{m^2(\omega^2+\nu^2)}\frac{1}{v^2}\left|\frac{\partial}{\partial v}\left(v^2\frac{\partial f_0}{\partial v}\right)\right| \ll f_0. \qquad (2.6a)$$

If the inhomogeneity of the plasma is taken into account along with its nonstationary nature, it is necessary to stipulate also that the following conditions be satisfied

$$\left|\frac{\partial f_0}{\partial t}\right| \ll \sqrt{\omega^2+\nu^2}\, f_0, \qquad (2.6b)$$

$$\frac{v}{\sqrt{\omega^2+\nu^2}}\left|\frac{\partial^2 f_1}{\partial z^2}\right| \ll \left|\frac{\partial f_0}{\partial z}\right|. \qquad (2.6c)$$

Here $\nu = \nu(v)$ is the collision frequency for an electron having a velocity v (see Sec 2.2), while ω is defined in (2.6) as $\omega \sim |\partial f_2/\partial t|/f_2$.

In the case of interest to us, of a stationary distribution in a spatially homogeneous plasma, conditions (2.6b) and (2.6c) are always satisfied. The decisive condition is therefore (2.6a). If we confine ourselves to the use of this condition for an average velocity $\bar{v} \sim \sqrt{kT_e/m}$ and put $\partial f_0/\partial v \sim f_0/\bar{v}$, we arrive at the requirement

$$\frac{e^2E_0^2}{kT_e(\omega^2+\nu^2(T_e))} \simeq \delta \ll 1,$$

where expression (1.18) is used for T_e. Thus, condition (2.6a) applied to the average values is found to be identical with the initial requirement $\delta_{eff} \ll 1$, which, as already indicated in Sec. 1, is always satisfied. Let us see now how condition (2.6a) is satisfied at different velocities v. For small v, condition (2.6a) may not be satisfied only at electron velocities that are $\sqrt{\delta}$ smaller than its average velocity; this region of velocity is usually of little importance. Condition (2.6a) may also be violated at high electron velocities. Here in the case of high frequencies ($\omega^2 \gg \nu^2$) this condition is not satisfied only in the insignificant region of velocities when v exceeds the average velocity by a factor $1/\sqrt{\delta}$. At lower frequencies, and in particular in a constant field, the region of velocities for which condition (2.6a) is not satisfied is in general more significant. For example, in the case of collisions with ions, when $\nu \sim v^{-3}$ for a Maxwellian distribution function f_0, condition (2.6a) is not satisfied if $(mv^2/kT_e - 3)v^6 \gtrsim \delta(kT_e/m)^3$ i. e., even if $v \gtrsim 3\sqrt{kT_e/m}$. In this case it is necessary to carry out a special investigation of the electron distribution function at high velocities.

Thus, condition (2.6a) -- the condition that only the first two of the chain of equations (2.5) need be retained -- is usually fairly well satisfied in a stationary spatially-homogeneous plasma.[6] Conditions (2.6b) and (2.6c) indicate the permissible degree of nonstationarity and inhomogeneity in the plasma. By virtue of these conditions, the energy and the density of the electrons should not change considerably over a time $1/\sqrt{\omega^2+\nu^2}$, and the electron current should not change over the effective electron mean free path $l_{eff} = v/\sqrt{\omega^2+\nu^2}$.

A completely analogous expansion of the distribution function can be made also in the presence of a constant magnetic field $\mathbf{H}_0$ in the plasma, and also for an arbitrary direction of the spatial gradient of the distribution function. In this case, separating again the symmetrical (dependent only on the absolute value of the velocity) portion of the distribution function $f_0(v, \mathbf{r}, t)$ from its directed portion $\mathbf{v}\cdot\mathbf{f}_1(v, \mathbf{r}, t)/v$, and neglecting the remaining terms (i. e., putting $f = f_0 + \mathbf{v}\cdot\mathbf{f}_1/v$), we can reduce Eq. (2.2) to the following systems of equations for the functions f_0 and f_1:[4]

$$\frac{\partial f_0}{\partial t} + \frac{v}{3}\operatorname{div}_r \mathbf{f}_1 + \frac{e}{3mv^2}\frac{\partial}{\partial v}(v^2\mathbf{E}\mathbf{f}_1) + S_0 = 0, \qquad (2.7a)$$

$$\frac{\partial \mathbf{f}_1}{\partial t} + v\,\operatorname{grad}_r f_0 + \frac{e\mathbf{E}}{m}\frac{\partial f_0}{\partial v} + \frac{e}{mc}[\mathbf{H}_0 \times \mathbf{f}_1] + \mathbf{S}_1 = 0. \qquad (2.7b)$$

In the absence of a magnetic field and when $\operatorname{grad}_r f$ is parallel to $\mathbf{E}$, Eqs. (2.7) coincide with the first two equations of the system (2.5), as they should. The con-

[6] The foregoing pertains to the calculation of the principal terms. Naturally, when one calculates small correction terms of order δ, the function f_2 must be taken into account. This is important, for example, in the calculation of the small variable terms on the order of $\delta\nu/\omega$ (see Secs. 3.1 and 3.5), provided only that the condition $\delta\nu/\omega \gg \delta$ is not satisfied.

ditions of the applicability of Eqs. (2.7) are the same as the conditions of the applicability of the first two equations of (2.5).

2.2 Transformation of the Collision Integral

Before we proceed to an analysis of the different types of electron collisions in a plasma, we note the most important feature of these collisions: in a majority of cases the principal role is played by collisions that change only slightly the electron energy and sometimes its momentum.[7] In such cases, the change in the distribution function due to the collisions, i. e., the change in the electron density in velocity space, can be naturally represented in the form

$$S=\left[\frac{\partial f}{\partial t}\right]_{\text{CT}}=-\operatorname{div}\mathbf{j}_{\mathbf{v}}, \tag{2.8}$$

where $\mathbf{j}_{\mathbf{v}}$ is the density of the particle flux due to the collisions at the point $\mathbf{v}$ in velocity space. Relation (2.8) is the usual continuity equation in velocity space. The flux density $\mathbf{j}_{\mathbf{v}}$ is naturally given, for small momentum changes, by the following expression

$$\mathbf{j}_{\mathbf{v}}=\frac{1}{2}\int\int d\mathbf{v}_1\,d\Omega q(u,\theta)\,u\,\Delta\mathbf{v}\,\{f(\mathbf{v})\,F(\mathbf{v}_1)-f(\mathbf{v})\,F(\mathbf{v}_1')\}. \tag{2.9}$$

Here $\Delta\mathbf{v}=\mathbf{v}'-\mathbf{v}$ is the change in the electron velocity after one collision; the remaining quantities have the same meaning as in the ordinary collision integral. It can be shown that expressions (2.8) and (2.9) for the collision integral when $\Delta\mathbf{v} \ll \mathbf{v}$ are identical with the ordinary expression (2.3) (see reference 21).

If the distribution function depends only on the modulus of the velocity [i. e., $f=f_0(v,\mathbf{r},t)$], then expressions (2.8) and (2.9) assume a particularly simple form

$$\left.\begin{aligned} S_0 &= -\frac{1}{v^2}\frac{\partial}{\partial v}(v^2 j_{v_0}),\\ j_{v_0} &= \frac{1}{2}\int\int d\mathbf{v}_1\,d\Omega q(u,\theta)\,u\,(v'-v)\{f_0(v)\,F(\mathbf{v}_1)-f_0(v')\,F(\mathbf{v}_1')\}. \end{aligned}\right\} \tag{2.10}$$

We note also that in this case, when not only the electron velocity but also the state of particle 1 (and its velocity v_1) change only slightly in one collision, the expression in the curly brackets of Eqs. (2.9) and (2.10) can be simplified to the form

$$f(\mathbf{v})\,F(\mathbf{v}_1)-f(\mathbf{v})\,F(\mathbf{v}_1')=(\Delta\mathbf{v}\,\text{grad}_{\mathbf{v}} f)\,F(\mathbf{v}_1)-(\Delta\mathbf{v}_1\,\text{grad}_{\mathbf{v}_1}F)\cdot f(\mathbf{v}), \tag{2.9a}$$

where $\Delta\mathbf{v}$, as before, is equal to $\mathbf{v}'-\mathbf{v}$; $\Delta\mathbf{v}_1=\mathbf{v}_1'-\mathbf{v}_1$. In calculating the integrals in (2.9) and (2.10), $\Delta\mathbf{v}$ and $\Delta\mathbf{v}_1$ must be expressed in terms of the velocities $\mathbf{v}$ and $\mathbf{v}_1$ before integrating over $d\mathbf{v}_1$ and $d\Omega$.

a) Elastic Collisions with Neutral Particles (Molecules). When a light particle (electron) strikes a heavy one, the energy or, the modulus of the velocity of the light particle changes only slightly. Using this fact, we can assume in the first approximation $v'=v$ and $v_1'=v_1$. Considering also[8] that the velocity of the electron is much greater than the velocity of the heavy particle v_1, we obtain from (2.5a)

$$\begin{aligned} S_{m_1}^{\text{el}} &= \int\int d\mathbf{v}_1\,d\Omega q(u,\theta)\,u\{\mathbf{f}_1(v)\,F(\mathbf{v}_1)-P_1(\cos\theta)\,\mathbf{f}_1(v')\,F(\mathbf{v}_1')\}=\\ &= \mathbf{f}_1(v)\cdot\int\int d\mathbf{v}_1\,d\Omega q(u,\theta)\,vF(\mathbf{v}_1)(1-\cos\theta)=\nu_m(v)\cdot\mathbf{f}_1(v),\\ \nu_m(v) &= N_m\cdot v\int q(v,\theta)(1-\cos\theta)\,d\Omega. \end{aligned} \tag{2.11}$$

[7] In elastic, and sometimes also in inelastic collisions between electrons and neutral particles, there is a slight change in the electron energy (because of the small electron mass). In collision with ions and collisions between electrons, in most cases there is a slight change not only in the energy by also in the momentum of the electron (owing to the peculiarity of the Coulomb interaction).

[8] In the present article we actually always assume that the average energy of heavy particles does not greatly exceed the average electron energy. Naturally, other conditions are also possible; for example, in solar corpuscular streams it is not the average energies of the electrons and ions that are equal, but their average velocities.

Here $\nu_m(v)$ is the number of electron collisions and $q(v, \theta)$ is the differential effective elastic scattering cross section of the electron.[9] In collisions with a heavy sphere of radius a (with which one can approximate elastic collisions between electrons and neutral particles at low velocities), we know that $q(v, \theta) = \pi a^2/4$, and consequently

$$\nu_m(v) = \pi a^2 N_m v, \quad (2.12)$$

where N_m is the molecule concentration.

In the same approximation a value of zero is obtained for S^{el}_{m0} as should be if energy exchange is completely neglected. We can calculate S^{el}_{m0} from the expressions (2.9a) and (2.10), since the energy or the velocity modulus of the electron, as noted above, changes but little in one collision. Let us find now the change in the absolute value of the velocity after one collision. As is known from the laws of elastic impact (see, for example, reference 22, Sec. 17), the electron velocity after collision is

$$\mathbf{v} = \frac{m\mathbf{v}' + M\mathbf{v}'_1}{m+M} + \mathbf{n}\frac{M}{m+M}|\mathbf{v}' - \mathbf{v}'_1| \simeq \mathbf{n}v'\left(1 - \frac{v'_1}{v'}\cos\psi\right) + \mathbf{v}'_1$$

(the last expression takes into account the fact that the velocity of the molecule $v_1 \ll v$; ψ is the angle between $\mathbf{v}$ and $\mathbf{v}_1$, $\mathbf{n}$ is a unit vector directed parallel to $\mathbf{v}' - \mathbf{v}'_1 \simeq \mathbf{v}'$). It follows therefore that $v \simeq v' - v'_1(\cos\theta_1 - \cos\psi)$ and $\Delta v = v' - v \simeq v'_1(\cos\theta_1 - \cos\psi) \simeq v_1(\cos\theta_1 - \cos\psi) = v_1(\cos\theta\cdot\cos\psi + \sin\theta\sin\psi \times \cos\varphi - \cos\psi)$. Here θ_1 is the angle between $\mathbf{v}_1$ and $\mathbf{n}$, and θ and ϕ are the scattering angles (i. e., the angles between $\mathbf{n}$ and $\mathbf{v} - \mathbf{v}' \simeq \mathbf{v}$). Finally, we obtain directly from the law of conservation of energy in the collisions that $\Delta v_1 = -mv\Delta v/Mv_1$.

Substituting these equations in the integral (2.10) [taking (2.9a) into account] and integrating over $d\Omega = \sin\theta\, d\theta\, d\phi$ and $d\mathbf{v}_1 = v_1^2 \sin\psi\, dv_1\, d\psi\, d\psi_1$, and assuming furthermore that the heavy particles have a Maxwellian distribution with temperature T, i. e.,

$$F(\mathbf{v}_1) = \left(2\pi\frac{kT}{M}\right)^{-\frac{3}{2}} N_m \exp\left[\frac{-Mv_1^2}{2kT}\right].$$

We obtain

$$j_{v_0} = \frac{1}{2}\int_0^\infty v_1^2\, dv_1 \int_0^\pi \sin\theta\, d\theta \int_0^{2\pi} d\varphi \int_0^\pi \sin\psi\, d\psi \int_0^{2\pi} d\varphi_1\cdot v\cdot q(v, \theta)\left\{v_1^2(\cos\theta\cdot\cos\psi + \sin\theta\sin\psi\cos\varphi - \cos\psi)^2\frac{\partial f_0}{\partial v}F + \frac{m}{M}vv_1(\cos\theta\cdot\cos\psi + \sin\theta\sin\psi\cos\varphi - \cos\psi)^2\frac{\partial F}{\partial v_1}\cdot f_0\right\} = \nu_m(v)\frac{kT}{M}\frac{\partial f_0}{\partial v} + \nu_m(v)\frac{m}{M}vf_0.$$

Here $\nu_m(v)$ is the electron collision frequency, determined from (2.11). The expression obtained in velocity space for the flux due to the collisions between the electrons and the heavy particles has a clear physical meaning: the flux j_{v_0} consists, firstly, of the "diffusion flux" $-\nu\frac{kT}{M}\frac{\partial f_0}{\partial v} = \frac{d}{dt}\overline{v^2}\frac{\partial f_0}{\partial v}$, which occurs in the presence of a gradient in the velocity distribution of the electrons and is due to the fact that the particles with which the electron collides have a non-zero velocity; secondly, a contribution is made to j_{v_0} by the "transport flux" $\nu mvf_0/M = \overline{dvf_0/dt}$, which represents the losses in the random velocity (or energy) of the electron upon collision.

The expression sought for the collision integral S^{el}_{m0} has, consequently, the following form

$$S^{el}_{m0} = -\frac{1}{2v^2}\frac{\partial}{\partial v}\left\{v^2\delta_{el}\nu_m\left[\frac{kT}{m}\frac{\partial f_0}{\partial v} + vf_0\right]\right\}. \quad (2.13)$$

[9] In the same approximation we find that $S_2 = \nu_2(v)f_2$, where

$$\nu_2(v) = N_m v\int q(v, \theta)(1 - P_2(\cos\theta))\, d\Omega.$$

From this it is clear that the quantities ν and ν_2 are of the same order of magnitude, as we agreed upon in the derivation of the conditions for the validity of Eqs. (2.5) and (2.7).

Here $\delta_{el} = 2m/M$ is the average fraction of the energy lost by the electrons in one elastic collision.[10] Comparing (2.13) and (2.11) we verify that the expansion of the collision integral is in powers of the parameter δ_{el}, so that the succeeding terms can always be neglected.

b) Inelastic Collisions with Neutral Particles. Inelastic collisions between electrons and neutral particles are accompanied by the excitation of rotational, vibrational, or optical levels, and also by ionization. In addition, so called second-order impacts are possible, in which the energy of the excited state of the molecule is transferred to the incoming electron. An exact calculation of all these inelastic processes is quite complicated; in addition, their cross sections are known only in a few cases (see references 23 and 24). Therefore there is no complete theory of inelastic collisions, in which the problem is solved as accurately as in the case of elastic collisions. In spite of this fact, it is possible to analyze relatively simply two important limiting cases; specifically, we consider cases when the electron energy is considerably greater than the energy of the excited level, or the ionization energy ($K \gg \hbar\omega$), and when, to the contrary, the electron energy is only slightly higher than the excitation energy ($K - \hbar\omega \ll K$).

In the former case the expression for the integral of inelastic collisions is found in the same manner as for elastic collisions. It is merely necessary to consider that the energy lost by the electron in inelastic impact is consumed essentially in excitation of the molecule, and this is connected with a transfer of energy $\hbar\omega$ (thus, $v' - v = \hbar\omega/mv$); in this case the neutral particle simply goes from the ground state into the excited state. We then have[25]

$$\left.\begin{aligned} S^{inel}_{m0} &= -\frac{1}{2v^2}\frac{\partial}{\partial v}\left\{v^2 r_\omega\left[\frac{kT_\omega}{m}\frac{\partial f_0}{\partial v}+vf_0\right]\right\}, \\ S^{inel}_{m1} &= \nu_\omega \mathbf{f}_1. \end{aligned}\right\} \quad (2.14)$$

[10] By definition $U = \delta K\nu$ is the average energy transferred by electrons of velocity v to heavy particles when $K \gg 3kT/2$ (see Sec. 1.2). On the other hand $U = N_m v\int m^2(\Delta\mathbf{v})^2/M\, q(v,\theta)\,d\Omega$, since the heavy particle, which can be considered stationary, acquires after the collision a momentum $m\Delta\mathbf{v} - m(\mathbf{v}' - \mathbf{v})$ and an energy $[m(\Delta v)]^2/2M$ (the impact is considered elastic, and the term linear in $\Delta\mathbf{v}$ in the expression for the energy vanishes when averaged over the directions of the molecule velocities). Choosing as the axis the initial direction of the electron velocity $\mathbf{v}'$, we have $(\Delta v_z)^2 = v^2(1-\cos\theta)^2$

and $(\Delta v_x)^2+(\Delta v_y)^2 = v^2\sin^2\theta$.

Consequently $U = \dfrac{m^2}{M}v^3 N_m\int q(v,\theta)(1-\cos\theta)\,d\Omega$

where $K = mv^2/2$, and ν is defined by (2.11). Obviously therefore $\delta_{el} = 2m/M$, and this result is independent of the cross section $q(v,\theta)$.

Here ν_ω is the number of inelastic collision, accompanied by the excitation of a quantum $\hbar\omega$ (as we shall call the transfer to the molecule of an energy $\hbar\omega$, consumed in excitation of some level)

$$\nu_\omega(v) = v\,(N^0_m + N^{ex}_m)\int q_\omega(v,\theta)(1-\cos\theta)\,d\Omega,$$

where $q_\omega(v,\theta)$ is the differential effective scattering cross section in inelastic collision, N^0_m and N^{ex}_m are the number of molecules in the ground and excited states respectively. Furthermore, $r_\omega(v)$ is the fraction of the energy lost per unit time by the electron to excitation of a quantum $\hbar\omega$

$$r_\omega(v) = \frac{2\hbar\omega}{mv^2}(N^0_m - N^{ex}_m)v\int q_\omega(v,\theta)\,d\Omega,$$

and T_ω is the effective temperature

$$T_\omega = \frac{\hbar\omega}{2k}\,\frac{N^0_m + N^{ex}_m}{N^0_m - N^{ex}_m}.$$

It is important to emphasize that in the case when the quantum $\hbar\omega$ is small not only compared with the electron energy, but also compared with the energy of neutral particles ($\hbar\omega \ll kT$), and if the neutral particles have a Boltzmann distribution $N^{ex}_m/N^0_m = \exp(-\hbar\omega/kT)$ (i. e., if the collisions with the electrons do not substantially change the number of excited molecules), then the effective temperature T_ω is equal to the molecule temperature T.

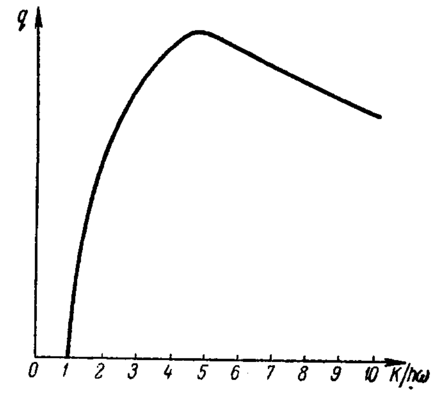

Figure 3.

In the second limiting case, when the electron energy exceeds but little the excitation energy, the colliding electron merely goes from the region of large energies into the region of small energies ($K \sim 0$).[26,27] Therefore at large energies

$$\left.\begin{aligned} S^{inel}_{m0} &= \nu_\omega f_0 = N_m v\int q_\omega(v,\theta)\,d\Omega, \\ S^{inel}_{m0} &= \nu_\omega \mathbf{f}_1, \end{aligned}\right\} \quad (2.15)$$

where ν_ω is the total frequency of the excitations of the level $\hbar\omega$ (it is assumed that $N_m^{ex} \ll N_m^0$). The fact that the electron cannot simply vanish but goes into the region of small energies ($K \sim 0$) is taken into account here by adding to the equation for f_0 a δ-function source of electrons, $-Q\,\delta(0)/4\pi v^2$, where $Q = dN/dt = 4\pi \int_{\hbar\omega}^{\infty} \nu_\omega f_0 v^2\,dv$.

We note in conclusion that in the general case the characteristic dependence of the total cross section of the inelastic collision on the electron energy has the form shown in Fig. 3: the total cross section $q(v)$ vanishes when $K < \hbar\omega$, then it increases, reaching a maximum $K \sim (3 \text{ to } 5)\hbar\omega$, and then starts to diminish slowly. Since in one impact the electron loses an energy $\hbar\omega$, it is clear from the diagram that, generally speaking, the most probable elastic collisions are those for which the fraction of the energy lost by the electron is small.

We note also that in those cases when not one but several levels $\hbar\omega_i$ can be excited, we have $S = \sum_i S_{\omega_i}$. It must be taken into account also that some of the inelastic collisions -- ionization and the effective recombination (recombination, capture of electron by a molecule, etc) -- are accompanied by a change in the number of electrons in the plasma. It is therefore necessary to add in the collision integral for the function f_0 the terms

$$-\nu_r(v)\,f_0 + \int_{\sqrt{\frac{2\hbar\omega_i}{m}}}^{\infty} \nu_{ion}(v', v)\,f_0(v')\,v'^2\,dv',$$

the first of which describes the effective recombination, and the second describes ionization. Here $\nu_r(v)$ is the total recombination frequency

and $\qquad \nu_{ion}(v', v) = N_m v' \int q_{ion}(v', v, \theta)\,d\Omega$—

is the ionization frequency, i. e., the number of ionizations produced per second by electrons of velocity v', which lead to the appearance of a new electron of velocity v, where $\hbar\omega_i$ is the ionization energy. These terms usually do not exert a noticeable influence on the form of the distribution function (see reference 9, Sec. 47 and reference 15, Sec. 56); they do determine, however, the concentration of the electrons in the plasma.

c) Collisions with Ions. To describe elastic collisions between electrons and ions one can employ the foregoing general expressions for the integral of elastic collisions between an electron and neutral particles, without modification, since the only assumption made in their derivation was $m \ll M$. It is necessary only to calculate the number of collisions between the electrons and the ions, $\nu_i(v)$. For this purpose one substitutes in (2.11) the Rutherford formula for the differential effective cross section for the scattering of an electron by an ion. We then have

$$\nu_i(v) = 2\pi N_i v \left(\frac{e^2}{2mv^2}\right)^2 \int_{\theta_{min}}^{\pi} \frac{1-\cos\theta}{\sin^4\frac{\theta}{2}} \sin\theta\,d\theta =$$
$$2\pi N_i \frac{e^4}{m^2v^3} \ln\left(1 + \cot^2\frac{\theta_{min}}{2}\right),$$

where N_i is the concentration of the ions, which for simplicity are assumed to be singly-charged.

If we consider the scattering of an electron on a free ion, then integration should be carried out from 0 to π (i. e., $\theta_{min} = 0$) and the collision frequency diverges logarithmically at small θ. In a plasma, however, the ions are not entirely free: as a result of interaction between the ions and the electrons, the field of each ion, under equilibrium conditions, has a Coulomb character only to distances on the order of the Debye radius D, where[11]

$$D = \left[\frac{kTkT_e}{4\pi e^2 N\,(kT + kT_e)}\right]^{1/2}.$$

At distances greater than D, the Coulomb field of the ion drops off rapidly (exponentially) as a result of screening (see, for example, reference 15, Sec. 56). Consequently, D is the maximum distance at which a substantial interaction between the electron and the ions still takes place, i. e., the maximum impact parameter. It can be used to express the minimum scattering angle (see, for example, reference 22, Sec. 19): $\theta_{min} = 2\tan^{-1}(e^2/mv^2D) \simeq 2e^2/mv^2D$ Therefore

$$\nu_i(v) = 2\pi N_i \frac{e^4}{m^2v^3} \ln\left(1 + \frac{D^2m^2v^4}{e^4}\right). \quad (2.16)$$

It is important that $D^2(kT_e)^2 e^{-4} \sim k^3T_e^3 e^{-6} N^{-1}$ always be a large quantity in the cases of interest to us [see footnote[5]]. This means that the second term in the logarithm is always the principal term. Consequently, the principal contribution to the number of collisions between the electron and the ions is made by the weak scattering -- scattering by small angles. In one such collision, the change in either the energy or in the electron momentum is insignificant. In fact, the fraction of the energy lost by the electron when scattered by an angle θ is $\delta_k = 2m(1 - \cos\theta)/M$. Considering that the principal role is played by collisions that lead to the scattering by a small angle, on the order of θ_{min}, we find

[11] This expression is valid for $N_i = N$, as in equilibrium in the absence of negative ions; in general, however, N must be replaced by the concentration of positive ions, N_+.

$$\delta_k \sim \delta_{k\,\min} = \theta^2_{\min} \frac{m}{M} = \frac{m}{M} \left(\frac{e^2 N^{\frac{1}{3}}}{kT_e}\right)^3 \ll 1.$$

Analogously, the change in the momentum is

$$\delta_p = |\mathbf{p} - \mathbf{p}_1| \,/\, |\mathbf{p}| \sim \delta_{p\,\min} = \left(\frac{e^2 N^{\frac{1}{3}}}{kT_e}\right)^3 \ll 1.$$

It must be emphasized that although the change in the momentum in one impact is small, the change in the energy is considerably smaller: $\delta_k / \delta_p \sim m/M$.

Scattering by large angles adds to the number of collisions only a term of order unity, which is small compared with the main logarithmic term. A similar correction in equilibrium plasma results from an exact solution of the problem of scattering in a Debye field, and also the interaction greater than the Debye radius (so called collective effects) which are not taken into account in formula (2.16) (see, for example, references 28 or 29).

It should also be noted that expression (2.16) is obtained under the assumption that the classical theory is correct, i. e., subject to the condition $e^2 zZ/\hbar v \gg 1$; when $z = 1$ this means that $v < 3 \times 10^8$ cm/sec, or $T_e = mv^2/3k \ll 3 \times 10^5$ °K. A quantum calculation, however, leads only to a change in the logarithmic term, for example, for $e^2/\hbar v \ll 1$ (i. e., when $T_e \gg 3 \cdot 10^5$ °K) in (2.16) it is necessary to replace the term $D^2 m^2 v^4/e^4$ under the logarithm sign by $\frac{D^2 m^2 v^4}{e^4} \cdot \frac{4e^4}{\hbar^2 v^2}$. The expression under the logarithm changes also slightly in an alternating electric field, of frequency higher than $\omega_0 = (4\pi e^2 N/m)^{1/2}$ for in this case the average collision time $\Delta\tau \sim D(kT_e/m)^{-1/2} \sim 1/\omega_0$ is greater than the time $1/\omega$, in which the field changes (see reference 15, Secs. 59, 81, and 82). Finally, it is necessary to introduce analogously in formula (2.16) changes if $\omega_H > \omega_0$; in this case the average radius of curvature $r_H \sim (kT/m)^{1/2}/\omega_H$ is less than the Debye radius $D \sim (kT_e/m)^{1/2} \omega_0^{-1}$. All these changes, disregarding the limiting cases ($\omega \gg \omega_0$ or $\omega_H \gg \omega_0$) are in practice of little importance, since they change the effective number of collisions only by a few percent and only sometimes by as much as 10 -- 20%.

Inelastic collisions between electrons and ions, which lead to their excitation and multiple ionization, do not differ at all from inelastic collisions with neutral particles, considered above. However, owing to the large values of the maximum elastic impact parameter (D), the role of inelastic collisions is greatly reduced. Collisions accompanied by bremsstrahlung of electrons, which are of importance at high electron energies, will not be considered here (see, for example, reference 11).

d) Interelectron Collisions. The principal role in a collision between an electron and ions, as seen above, is played by long-range collisions, which lead to weak scattering. Both the energy and the momentum of the electrons are changed only slightly by one such collision. This is the consequence of the singularity of the Coulomb interaction and therefore pertains not only to collisions between electrons and ions, but also to collisions between electrons and electrons. The difference lies only in the fact that the fraction of the energy and the fraction of the momentum, lost by the electron when colliding with another electron is of the same order $\delta_k/\delta_p \sim 1$, whereas in the collision with the ion $\delta_k/\delta_p \sim m/M$ (see Sec. 2.2c).

Thus, in considering the integral of interelectron collisions, one can use the differential expressions derived earlier for S. In addition, we can integrate in this expression over the scattering angles $d\Omega$ [using the fact that $q(\theta, u)$ has a sharp maximum at $\theta \sim 0$]. We then find that the integral of the collisions between the electrons are satisfied by the expression (2.8), where[21]

$$\mathbf{j}_\mathbf{v} = \frac{1}{2N} \int d\mathbf{v}_1 \nu(u) \{u^2 (f(\mathbf{v}) \operatorname{grad}_{\mathbf{v}_1} f(\mathbf{v}_1) - f(\mathbf{v}_1) \operatorname{grad}_\mathbf{v} f(\mathbf{v})) - \mathbf{u}[f(\mathbf{v})(\mathbf{u} \operatorname{grad}_{\mathbf{v}_1} f(\mathbf{v}_1)) - f(\mathbf{v}_1)(\mathbf{u} \operatorname{grad}_\mathbf{v} f(\mathbf{v})]\}. \tag{2.17}$$

Here $\mathbf{u} = \mathbf{v} - \mathbf{v}_1$, $\nu(u)$ is the number of collisions (2.16), where v must be replaced by u and N_i by $N_e \equiv N$; account is also taken of the fact that the scattering particles are electrons, i. e., that $F(\mathbf{v}_1) \equiv f(\mathbf{v}_1)$.

Let us consider now S_{0e} -- the integral of interelectron collisions for the function f_0. We note here that as a result of the nonlinearity of the integral of interelectron collisions, the integrals S_{ke} depend, generally speaking, not on the function f_k alone. However, the integral S_{0e} depends only on f_0, since the terms of type $f_0 \mathbf{f}_1$ drop out upon integration over the angles, and terms of type $\mathbf{f}_1^2$ can be neglected compared with f_0^2 [since, as can be seen from (2.5), $\mathbf{f}_1^2 \lesssim \delta f_0^2$, and the entire system of equations (2.7) is correct only accurate to terms $0(\delta)$]. Therefore, putting $f = \mathrm{f}_0(v)$ in (2.17), we can readily perform the integration over the angles. Then

$$S_{0e} = -\frac{1}{v^2} \frac{\partial}{\partial v} \left\{ v^2 \left[A_1(f_0) v f_0 + A_2(f_0) \frac{\partial f_0}{\partial v} \right] \right\}, \tag{2.18}$$

where

$$\left.\begin{aligned} A_1 &= -\frac{1}{N}\int d\mathbf{v}_1 v_1 \frac{\partial f_0}{\partial v_1}(1-\cos^2\theta_1)\,\nu(u) = \\ &= \frac{4\pi\nu(v)}{N}\int_0^v v_1^2 f_0(v_1)\,dv_1, \\ A_2 &= \frac{1}{N}\int v_1^2 f_0(1-\cos^2\theta_1)\,\nu(u)\,d\mathbf{v}_1 = \\ &= \frac{4\pi\nu(v)}{3N}\left\{\int_0^v v_1^4 f_0(v_1)\,dv_1 + v^3\int_v^\infty v_1 f_0(v_1)\,dv_1\right\}. \end{aligned}\right\} \quad (2.18a)$$

Here θ_1 is the angle between $\mathbf{v}$ and $\mathbf{v}_1$, $u = |\mathbf{v}-\mathbf{v}_1|$; when integrating over the angles we neglected the variation of the logarithmic term in $\nu(u)$ (compared with the variation of the principal term $\sim 1/u^3$).[30,12,31] For fast electrons, whose velocity v is much greater than the average velocity of the plasma electrons, the coefficients A_1 and A_2 assume a simple form: $A_1 = \nu(v)$, and $A_2 = 2\overline{K}\nu(v)/3m$, where $\overline{K}$ is the average energy of the scattered electrons (in the case of a Maxwellian electron velocity distribution, $2\overline{K}/3m = kT_e/m$).

We note that if expression (2.18) for S_{0e} is multiplied by v^2 or by v^4 and integrated over all the velocities v (from 0 to ∞), then the corresponding integral vanishes identically, regardless of the type of the function $f_0(v)$:

$$\int_0^\infty S_{0e}\cdot v^2\,dv = 0, \qquad \int_0^\infty S_{0e}v^4\,dv = 0. \qquad (2.18b)$$

These relations reflect the conservation of the number of particles and of the energy in collisions between electrons. For a Maxwellian distribution, naturally, $S_{0e} = 0$.

The integral of electron collisions for the function $\mathbf{f}_1$ depends both on $\mathbf{f}_1$ and on f_0. The expression for S_{1e}, as seen directly from (2.3), has the form

$$S_{1e} = \frac{3}{4\pi}\int q_e(u,\theta)\,u\frac{\mathbf{v}}{v}\left\{\frac{\mathbf{v}\mathbf{f}_1(v)}{v}f_0(v_1) + \frac{\mathbf{v}_1\mathbf{f}_1(v_1)}{v_1}f_0(v) - \frac{\mathbf{v}'\mathbf{f}_1(v')}{v'}f_0(\mathbf{v}_1) - \frac{\mathbf{v}_1'\mathbf{f}_1(v_1')}{v_1'}f_0(v')\right\}d\mathbf{v}_1\,d\Omega\,d\Omega_1. \qquad (2.19)$$

We note that the expression for the integral S_{1e} can be obtained also from (2.17) (where the weak scattering of the electron in each collision is made use of). In this case S_{1e} is a rather complex integro-differential expression, linear in $\mathbf{f}_1$ and containing a large number of terms. We shall therefore not cite it here, referring the reader to corresponding papers.[32–34]

2.3 Solution of the Kinetic Equation. Strongly Ionized Plasma.

The final form of the system of equations for the electron distribution function $f(\mathbf{v},\mathbf{r},t) = f_0(v,\mathbf{r},t) + \mathbf{v}\cdot\mathbf{f}_1(v,\mathbf{r},t)/v$, i.e., for the functions f_0 and $\mathbf{f}_1$, can be represented in the form

$$\frac{\partial f_0}{\partial t} + \frac{v}{3}\operatorname{div}_r\mathbf{f}_1 + \frac{e}{3mv^2}\frac{\partial}{\partial v}(v^2\mathbf{E}\mathbf{f}_1) = = \frac{1}{2v^2}\frac{\partial}{\partial v}\left\{v^2\delta_{el}(\nu_m^{el}+\nu_i)\left[\frac{kT}{m}\frac{\partial f_0}{\partial v}+vf_0\right]\right\} + S_{mo}^{inel}(f_0) + \frac{1}{v^2}\frac{\partial}{\partial v}\left\{v^2\left[A_1(f_0)vf_0 + A_2(f_0)\frac{\partial f_0}{\partial v}\right]\right\}, \qquad (2.20a)$$

$$\frac{\partial\mathbf{f}_1}{\partial t} + v\operatorname{grad}_r f_0 + \frac{e\mathbf{E}}{m}\frac{\partial f_0}{\partial v} + \frac{e}{mc}[\mathbf{H}_0\times\mathbf{f}_1] = -\nu(v)\mathbf{f}_1 - S_{1e}(\mathbf{f}_1). \qquad (2.20b)$$

Here $\delta_{el} = 2m/M$, ν_m^{el} and ν_i are the numbers of elastic collisions of the electron with molecules (2.12) and ions (2.16), $\nu(v) = \nu_m^{el}(v) + \nu_i(v) + \nu_m^{inel}(v)$ is the total number of electron collisions, where

$$\nu_m^{inel}(v) = \sum v\,(N_{mi}^0 + N_{mi}^{ex})\int q_{\omega_i}(v,\theta)(1-\cos\theta)\,d\Omega \qquad (2.14a)$$

is the number of inelastic collisions between an electron with molecules.[12] Next, $S^{inel}_{m0}(f_0)$ is the collision integral for the function f_0, which describes inelastic collisions between electrons and molecules (the expressions for $S^{inel}_{m0}(f_0)$ for two limiting cases are given Sec. 2.2b), $A_1(f_0)$, $A_2(f_0)$ and $S_{1e}(\mathbf{f}_1)$ are the integral expression (2.18a) and (2.19), which describe the variation of the functions f_0 and $\mathbf{f}_1$ due to collisions between electrons.

Proceeding now to solve Eqs. (2.20) let us dwell first on one of their singularities, which will be of essential use later on. As noted in Sec. 1, the time of relaxation of the electron energy, $\tau^{(k)} = 1/\delta_{eff}\nu_{eff}$, is always much greater than the momentum relaxation time $\tau = 1/\nu_{eff}$. In this connection, the relaxation time is much greater for the function f_0 than for the function $\mathbf{f}_1$. As a result, the function f_0 always changes more slowly than function $\mathbf{f}_1$; consequently, when integrating Eq. (2.20b) for the function $\mathbf{f}_1$, the function f_0 can be considered in first approximation as constant and independent of the time. This facilitates greatly the integration of Eq. (2.20b). The simple approximate expression obtained thereby for $\mathbf{f}_1$, as shown in reference 35, is accurate up to terms less than or of the order of δ_{eff}, i. e., to the same degree of accuracy with which Eqs. (2.7) and (2.20) are in themselves accurate. The problem reduces therefore to an integration of only one equation for the function f_0.

In the equation for the function f_0, the last term in the right part of the equation, due to collisions between electrons, has an order of $\nu_e f_0$, where ν_e is the frequency of the interelectron collisions. The remaining terms, which describe the collisions between electrons and heavy particles, have an order of $\delta\nu f_0$, where $\nu = \nu_m + \nu_i$. It is clear that, depending on the relation between ν_e and $\delta\nu$, the form of the function f_0 is determined either by the interelectron collisions, or by collisions between electrons with heavy particles. We shall therefore first consider separately these two important limiting cases: the case of "strongly ionized plasma" when $\nu_e \gg \delta\nu$, and the case of "weakly ionized plasma," when $\nu_e \ll \delta\nu$ (in a completely ionized plasma we always have $\nu_e \gg \delta\nu = \delta\nu_i$, since $\delta \ll 1$ and $\nu_e \sim \nu_i$; on the other hand, at a very low degree of ionization, when the concentration of the electrons is sufficiently small, $\nu_e \ll \delta\nu = \delta\nu_m$; the terms "strongly ionized" and "weakly ionized" are of course arbitrary). The solution of the problem for any electron concentration, i. e., for any relation between ν_e and $\delta\nu$, is the subject of Sec. 2.5a. There we give criteria for the applicability of the formulas obtained in each of the limiting cases indicated above.

a) Distribution Function (Maxwellian Distribution). In a strongly ionized plasma, when $\nu_e \gg \delta\nu$, the form of the function f_0 is determined by the interelectron collisions. The solution of Eq. (2.20a) must be sought in this case by the method of successive approximations $f_0 = f_{00} + f_{01} + \ldots$, considering in the zero approximation, naturally, only collisions between electrons. In a homogeneous plasma we then obtain from (2.20a) the following chain of equations

$$S_{0e}(f_{00}) = -\frac{1}{v^2}\frac{\partial}{\partial v}\left\{v^2\left[A_1(f_{00})\,vf_{00} + A_2(f_{00})\frac{\partial f_{00}}{\partial v}\right]\right\} = 0, \tag{2.21a}$$

$$\frac{1}{v^2}\frac{\partial}{\partial v}\left\{v^2\left[A_1(f_{01})\,vf_{00} + A_1(f_{00})\,vf_{01} + A_2(f_{01})\frac{\partial f_{00}}{\partial v} + A_2(f_{00})\frac{\partial f_{01}}{\partial v}\right]\right\} =$$
$$= \frac{\partial f_{00}}{\partial t} + \frac{e}{3mv^2}\frac{\partial}{\partial v}(v^2\mathbf{E}\mathbf{f}_{10}) - \frac{1}{2v^2}\frac{\partial}{\partial v}\left\{v^2\delta_{el}(\nu^{el}_m + \nu_i)\left[\frac{kT}{m}\frac{\partial f_{00}}{\partial v} + vf_{00}\right]\right\} + S^{inel}_{m0}(f_{00}), \tag{2.21b}$$

. . . .

We see directly from (2.21a) that the zero-approximation function f_{00} is Maxwellian

$$f_{00} = N\left(\frac{m}{2\pi kT_e}\right)^{3/2}\exp\left\{-\frac{mv^2}{2kT_e}\right\}, \tag{2.22}$$

since it is precisely for a Maxwellian distribution that the integral of interelectron collisions (2.18) vanishes. Physically this result is quite understandable: owing to the interelectron collisions, the Maxwellian distribution should become established within a time $1/\nu_e$; when $\nu_e \gg \delta\nu$ this process is much faster than the process of transfer of energy to heavy particles, meaning the function f_0 should be close to Maxwellian. With this the

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[12] The expression for S^{inel}_{m1}, strictly speaking, should be written in the form

$$S^{inel}_{m1} = v\sum_i (N^0_{mi} + N^{ex}_{mi})\int q_{\omega_i}(v,\theta)\,d\Omega\cdot\mathbf{f}_1(v) -$$
$$- v\sum_i N^0_{mi}\int q_{\omega_i}(v,\theta)\cos\theta\,d\Omega\,\mathbf{f}_1\left(\sqrt{v^2 + \frac{2\hbar\omega_i}{m}}\right) -$$
$$- v\sum_i N^b_{mi}\int q_{\omega_i}(v,\theta)\cos\theta\,d\Omega\,\mathbf{f}_1\left(\sqrt{v^2 - \frac{2\hbar\omega_i}{m}}\right).$$

It is easy to see, however, that by virtue of the condition $\delta_{eff} \ll 1$, which must be satisfied for Eqs. (2.20) to be in themselves correct, the expression for S^{inel}_{m1} can be approximately represented in the form (2.14a). On the average, the error introduced thereby does not exceed δ_{eff}.

electron density N and the electron temperature T_e in (2.22) need not necessarily be constant, and are certain functions of the time. They are determined from the condition of solvability of Eq. (2.21b) for the next (first-order) approximation. In fact, as indicated in Sec. 2.2d (see 2.18b), if we multiply the left half of (2.21b) by v^2 or v^4 and integrate it over the velocities, then the corresponding integral vanishes identically (independent of the form of the function f_{01}). Consequently, the right half of (2.21b) should also vanish at the same time. This leads to equations for the density and temperature of the electrons. In fact, multiplying (2.11b) by $4\pi v^2$ and integrating it over v, we obtain

$$\frac{d}{dt}\left(4\pi \int_0^\infty v^2 f_{00}\, dv\right) + \int_0^\infty 4\pi v^2 S_{m0}^{inel}(f_{00})\, dv = 0,$$

or, taking (2.22) into account,

$$\frac{dN_e}{dt} + (\nu_{rec} - \nu_{ion}) N_e = 0, \tag{2.23}$$

where,

$$\nu_{ion} = \sqrt{\frac{2}{\pi}}\left(\frac{m}{kT_e}\right)^{3/2} \times \int_0^\infty \int_{\sqrt{\frac{2\hbar\omega_i}{m}}}^\infty v^2 v'^2 \nu_{ion}(v, v') \exp\left\{\frac{-mv'^2}{2kT_e}\right\} dv\, dv'$$

is the total ionization frequency, and

$$\nu_{rec} = \sqrt{\frac{2}{\pi}}\left(\frac{m}{kT_e}\right)^{3/2} \int_0^\infty v^2 \exp\left\{\frac{-mv^2}{2kT_e}\right\} \nu_r(v)\, dv$$

is the total frequency of effective recombination (see Sec. 2.2b). Equation (2.23) is usually called the ionization-balance equation.[13]

[13] If there exist also substantial external ionizing factors (such as photo-ionization by solar ultraviolet in the ionosphere) these must, naturally, also be taken into account in Eq. (2.23).

Quite analogously, multiplying (2.21b) by $(1/2)\, mv^2 \cdot 4\pi v^2$ and integrating over v, we obtain

$$\frac{d}{dt}\left(2\pi m \int_0^\infty v^4 f_{00}\, dv\right) - \frac{4\pi e}{3} \mathbf{E} \int_0^\infty v^3 \mathbf{f}_{10}\, dv + \delta_{el} 2\pi m \int_0^\infty v^3 (\nu_m^{el} + \nu_i) \times$$

$$\times \left(v f_{00} + \frac{kT}{m}\frac{\partial f_{00}}{\partial v}\right) dv + 2\pi m \int_0^\infty v^4 S_{m0}^{inel}(f_{00})\, dv = 0.$$

Considering (2.22) and (2.23), we rewrite this equation in the form

$$\frac{dT_e}{dt} + \delta_{\text{eff}}(T_e)\, \nu_{\text{eff}}(T_e)\,(T_e - T) = \frac{2e\mathbf{E}}{3kN_e} \mathbf{j}_t(T_e). \tag{2.24}$$

Here ν_{eff} denotes a parameter, determined by the relation

$$\nu_{\text{eff}}(T_e) = \frac{4\pi m}{3N_e kT_e} \int_0^\infty v^4 \nu(v) f_{00}\, dv =$$

$$= \frac{\sqrt{2}}{3\sqrt{\pi}}\left(\frac{m}{kT_e}\right)^{5/2} \int_0^\infty \nu(v)\, v^4 \exp\left\{-\frac{mv^2}{2kT_e}\right\} dv \tag{2.25}$$

where $\nu(v) = \nu_m^{el}(v) + \nu_i(v) + \nu_m^{inel}(v)$ is the number of collisions between the electron and the heavy particles; ν_{eff} it is natural to call the effective number or the effective frequency of electron collisions. Furthermore, δ_{eff} is another characteristic parameter, having the meaning of an average relative fraction of the energy transfer to the electron by the heavy particles within a time $1/\nu_{eff}$ (see Sec. 1):

Here $\delta_{el} = 2m/M$, ν_{eff}^{el} are the effective number of elastic collisions [calculated also from formula (2.25), but provided that $\nu(v)$ takes into account only elastic

$$\delta_{\text{eff}}(T_e) = \delta_{el} \frac{\nu^{el}_{\text{eff}}}{\nu_{\text{eff}}} + + \frac{1}{\nu_{\text{eff}}(T_e - T)} \left[\frac{4\pi m}{3N_e k} \int_0^\infty v^4 S^{inel}_{m0}(f_{00})\, dv + (\nu_{ion} - \nu_{rec})\, T_e \right]. \qquad (2.26)$$

collisions with molecules, $\nu_m^{el}(v)$, and with ions $\nu_i(v)$]. $S^{inel}_{m0}(f_{00})$ is part of the collision integral, describing the inelastic collisions of the electrons having a Maxwellian distribution with molecules [the expressions for $S^{inel}_{m0}(f_{00})$ for the two limiting cases are given in Sec. 2.2b]. Finally, j_t is the density of the total electron current, determined from Eq. (2.1).

An estimate of the first-approximation function shows that $f_{01} \sim (\delta\nu/\nu_e) f_{00}$. Consequently, in a strongly ionized plasma, for the symmetrical part of the distribution function f_0, accurate to terms of order $\delta\nu/\nu_e$, we can confine ourselves to the zeroth (Maxwellian) approximation (for more details see Sec. 2.5a).

b) Effective Number of Collisions. In the case of collisions with molecules, as already indicated, one can assume for low electron energies that the electron free path is independent of its velocity, i. e., that $\nu_m(v)$ is determined by (2.12). Inserting this equation into (2.25) we get

$$\nu_{\text{eff}} = \frac{8\sqrt{2}}{3\sqrt{\pi}} \sqrt{\frac{kT_e}{m}}\, \pi a^2 N_m = \nu^{(0)}_{\text{eff}} \sqrt{\frac{T_e}{T}}, \qquad (2.27)$$

where $\nu^{(0)}_{\text{eff}\, m}$ is the effective number of collisions between electrons and molecules in a weak field, when $T_e = T$.

In case of collisions with ions, we obtain by substituting expression (2.16) for $\nu_i(v)$ into (2.25), (for more details see reference 15, Sec. 61)

$$\nu_{\text{eff}\, i} = \frac{2}{3}\sqrt{8\pi}\, \frac{e^4 N_i}{\sqrt{m}\,(kT_e)^{3/2}} \ln\left(\frac{kT_e D}{e^2}\right) \approx \frac{5.5 N_i}{T_e^{3/2}} \ln\left(\frac{2\cdot 10^2 T_e}{N^{1/3}}\right), \qquad (2.28)$$

where N_i is the ion concentration and $D = kTkT_e/[4\pi e^2 N(kT + kT_e)]$ is the Debye radius. The quantity kT_eD/e^2, under the logarithm sign in (2.28), is always much greater than unity.[14] As a result, even at relatively large variations of electron temperature, the logarithm changes only slightly; we can, therefore, always assume

$$\nu_{\text{eff}\, i} = \nu^{(0)}_{\text{eff}\, i} \left(\frac{T}{T_e}\right)^{3/2}, \qquad (2.29)$$

where $\nu^{(0)}_{\text{eff}\, i}$ is the effective number of collisions between electrons and ions in a weak field, when $T_e = T$.

Equations (2.27) and (2.29) coincide with expressions (1.4) and (1.5), used in Sec. 1.

In the case when the plasma contains heavy particles of different kinds, the effective number of electron collisions thus determined is merely equal to the sum of the effective number of collisions between the electron and the particles of each kind.

c) Relative Fraction of the Energy Transferred. In the case of elastic collisions between electrons and heavy particles, such as molecules or ions, it is clear from (2.26) that $\delta_{\text{eff}} = \delta_{el} = 2m/M$.

To be able to calculate δ_{eff} also in the presence of inelastic collisions, it is necessary to know the effective cross sections of all the inelastic processes [see formula (2.26)]. They are known sufficiently well at the present time only for monatomic inert gases. Suitable calculations show that the relative fraction of the energy transferred δ_{eff} is equal in these cases to δ_{el} up to temperatures on the order of 1 ev, and then increases rapidly (exponentially) with increasing electron temperature (see Table I).

In diatomic gases (hydrogen, oxygen, nitrogen) both vibrational and rotational levels can be excited. Little is still known about the effective cross sections of these processes (see reference 23 and 24) so that δ_{eff} cannot be calculated. Experimental investigations of δ_{eff} and of its temperature dependence are reported in many papers.[23,36–41] The results of these measurements in hydrogen, oxygen, nitrogen, and air are listed in Table I.[15] As can be seen from the table, all these

[14] This circumstance was already used when making the transition to the last equation (2.28), where no attention was paid to a factor of order unity over the logarithm. For the same reason one can always assume in (2.28) that

$$D = \sqrt{\frac{kT}{4\pi e^2 N}} \quad \left(\text{when } T_e = T \right.$$

$$D = \sqrt{\frac{kT}{8\pi e^2 N}}, \text{ and when } T_e \gg T \quad \left. D = \sqrt{\frac{kT_e}{4\pi e^2 N}}\right).$$

[15] The values of δ_{eff} are given here as obtained from the latest papers.[37–39] The dependence of δ_{eff} on T_e agrees in these data with that obtained by earlier authors,[23] although there is a considerable discrepancy in the absolute values. The plasma temperature in the experiment is $T \sim 290°$, and a special verification at lower T disclosed no variation of δ_{eff} at all.[40]

Table 1

Values of $\delta_{eff} \times 10^3$

T_e	He $\delta_{el} = 2.7 \cdot 10^{-4}$	H_2 $\delta_{el} = 5.4 \cdot 10^{-4}$	O_2 $\delta_{el} = 3.4 \cdot 10^{-5}$	N_2 $\delta_{el} = 3.9 \cdot 10^{-5}$	Air $\delta_{el} = 3.7 \cdot 10^{-5}$	Ionosphere		
						100 km	200 km	300 km
500°	—	2.3	—	—	—	—	—	—
1000°	0,27	2.5	3.7	0,47	0.89	0.86	0.08	0,06
2000°	0.27	2.2	6.7	0.36	1.2	1.2	0,12	0.06
3000°	0.27	2.2	8.6	0.33	1.6	1.5	0.16	0.06
4000°	0.27	2,5	9.0	0.32	1,7	1.6	0,18	0,06
5000°	0.27	3,0	8.7	0,34	1.7	1.6	0.22	0,06
6000°	0.27	3.4	8.2	0.38	1.7	1.6	0.26	0,07
7000°	0.27	3.9	7.7	0.45	1.7	1.6	0.32	0,07
8000°	0,27	4.4	7.2	0.60	1.7	1,6	0.43	0,08
9000°	0.27	4.8	6.8	0.82	1.8	1.7	0.60	0.09
10000°	0.27	5.3	6.6	1.15	2.0	2.0	0,85	0,11
12000°	0.27	6,1	7.7	2.40	3.2	3.1	1.8	0.23
15000°	0.27	7.2	21	9.8	11	10.6	7.7	1.13

gases are characterized by a δ_{eff} that varies little with T_e from room temperatures to temperatures on the order of 1 or 2 ev; at higher temperatures, δ_{eff} increases sharply.

If the gas is a mixture of several gases, the value of δ_{eff} can be readily obtained from the formula

$$\delta_{eff} = \frac{\sum_k \delta_{eff\,k} \cdot \nu_{eff\,k}}{\sum_k \nu_{eff\,k}}, \tag{2.26a}$$

where $\nu_{eff\,k}$ and $\delta_{eff\,k}$ is the effective number of collisions and fraction of energy transferred for the gas of kind "k" as determined from (2.25) and (2.26). A corresponding calculation of δ_{eff} for air (from data obtained for nitrogen and oxygen) is in good agreement with the directly measured δ_{eff} (see reference 25). The values of δ_{eff} in the ionosphere are listed in Table I and were also calculated with the aid of (2.26a).

d) Electron Current. Dielectric Permittivity and Conductivity of Plasma. To obtain the value of the electron current $\mathbf{j}_t$, it is necessary to determine the function $\mathbf{f}_1$, since

$$\mathbf{j}_t = e \int \mathbf{v} f \, d\mathbf{v} = \frac{4\pi e}{3} \int_0^\infty v^3 \mathbf{f}_1 \, dv. \tag{2.30}$$

It is consequently necessary to solve Eq. (2.20b).

Inserting in this equation f_{00} instead of f_0, we find that the dependence of f_0 on the time t can be neglected here (see beginning of Sec. 2.3). If, furthermore, the interelectron collisions are insignificant in the equation for $\mathbf{f}_1$, Eq. (2.20b) in the homogeneous case becomes in fact algebraic. In this approximation its solution, which can be verified by direct substitution, is

$$\mathbf{f}_{10} = -\mathbf{u} \frac{\partial f_{00}}{\partial v},$$

where $\mathbf{u}$ is the velocity of the directed motion of the electron, determined by the equation

$$\frac{\partial \mathbf{u}}{\partial t} + \nu(v)\,\mathbf{u} = \frac{e\mathbf{E}}{m} + \frac{e}{mc}[\mathbf{u} \times \mathbf{H}_0] \tag{2.32}$$

We note that the equation for $\mathbf{u}$ is quite analogous to Eq. (1.3) for the directed velocity in elementary theory, the only difference being that ν in (2.32) depends, generally speaking, on the velocity of the random motion v, and consequently also $\mathbf{u} = \mathbf{u}(v, t)$.

Inserting the resultant function f_{10} in (2.30) and integrating over the velocity v, we obtain an expression for the current $\mathbf{j}_t$, and consequently also for the conductivity and the dielectric permittivity of the plasma, since $\mathbf{j}_t = [\sigma + i\omega(\epsilon - 1)/4\pi]\,\mathbf{E}$ (see Sec. 1.1).

The formulas obtained for ϵ and σ can be written in the form

$$\left.\begin{aligned}\varepsilon &= 1-\frac{4\pi e^2 N}{m\,(\omega^2+\nu^2_{eff})}\,\mathscr{K}_\varepsilon\left(\frac{\omega}{\nu_{eff}}\right),\\ \sigma &= \frac{e^2 N\nu_{eff}}{m\,(\omega^2+\nu^2_{eff})}\,\mathscr{K}_\sigma\left(\frac{\omega}{\nu_{eff}}\right).\end{aligned}\right\}\quad (2.33)$$

Here ν_{eff} is the effective collision frequency, determined from (2.25), and $\mathscr{K}_\epsilon(z)$ and $\mathscr{K}_\sigma(z)$ are certain functions, the numerical values of which, for collisions with either molecules or ions, are listed in Table II and in Fig. 4.5 (the analytical expressions for the functions $\mathscr{K}_\epsilon$ and $\mathscr{K}_\sigma$ are quite complicated (see references 20 and 43).[16] The coefficients $\mathscr{K}_\epsilon$ and $\mathscr{K}_\sigma$ represent the dispersion of the electron collision frequency; they show the extent to which the values of σ and ϵ calculated in the kinetic theory differ from the corresponding values obtained with the aid of elementary formulas (1.8). It is seen from Table II and from Fig. 4.5 that in the case of collisions with molecules the coefficients $\mathscr{K}_\epsilon$ and $\mathscr{K}_\sigma$ are close to unity. To the contrary, in collisions with ions $\mathscr{K}_\epsilon$ and $\mathscr{K}_\sigma$ may differ considerably from unity, particularly at low frequencies $\omega \lesssim \nu_{eff}$.

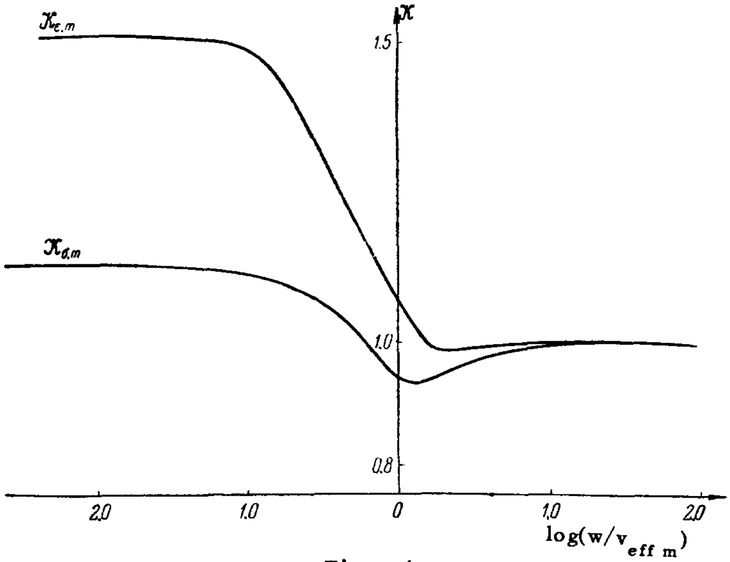

Figure 4.

Earlier, in the determination of the function f_1, we disregarded the collisions between electrons. This is true for collisions with molecules (when $\nu_{effm} \gg \nu_{effi}$) and also for collisions with ions ($\nu_{effi} \gg \nu_{effm}$), provided the plasma is multiply ionized or contains a large number of negative ions (when $N_i \gg N_e$). If the plasma is singly ionized and there are no negative ions, the collisions between electrons can play a substantial role. To determine f_1 in this case, it is necessary to include the integral term in (2.20b) and to find a solution of this integral equation. Such a solution was obtained in reference 44 (see also references 45 and 46) for a constant electric field by expanding the function f_1 in Laguerre polynomials; in reference 13 this solution is generalized to include the case of an alternating electric field. The same problems were solved also in references 32, 33, 47, and 48, where the authors used integro-differential ("diffusion") expressions for the collision integral (see Sec. 2.2d); the results of these investigations agree with those obtained in reference 44, as they should.

The calculation of ϵ and σ with allowance for inter-electron collisions shows that these quantities can, as before, be represented in the form (2.33).[13] All that changes here are the functions $\mathscr{K}_\epsilon$ and $\mathscr{K}_\sigma$; these are also represented in Table II and in Fig. 5 (solid curves). It is seen from Fig. 5 that allowance for the collisions between electrons reduces the values of the functions $\mathscr{K}_\epsilon$ and $\mathscr{K}_\sigma$, but they still remain sufficiently different from unity. We note also that at high frequencies ($\omega^2 \gg \nu^2_{eff}$) the functions $\mathscr{K}_\epsilon$ and $\mathscr{K}_\sigma$, with allowance for collisions between electrons, are close to unity, i. e., the influence of the collisions between electrons is insignificant at higher frequencies.[49] The problem of the influence of electron collisions is solved also for doubly- and triply-ionized plasma.[44]

[16] We note that the functions $\mathscr{K}_\epsilon$ and $\mathscr{K}_\sigma$ depend on one variable ω/ν_{eff} only in the case of a power-law dependence of ν on v (i. e., when $\nu \sim v^\alpha$). If the dependence of ν on v is more complicated, the coefficients $\mathscr{K}_\epsilon$ and $\mathscr{K}_\sigma$ depend on two variables, ω/ν_{eff} and T_e (see reference 42). If collisions between electrons and both molecules and ions are significant simultaneously, the values of the functions $\mathscr{K}_\epsilon$ and $\mathscr{K}_\sigma$ lie between those for ions and those for molecules. To determine these values, it is necessary to perform suitable computations (an exception is the case of high frequencies $\omega^2 \gg \nu^2_{eff}$, when $\mathscr{K}_\epsilon = \mathscr{K}_\sigma = 1$.

Table 2

ω/ν_{eff}	Collisions with molecules		Collisions with ions			
	$\mathcal{K}_{\varepsilon,m}$	$\mathcal{K}_{\sigma,m}$	$\mathcal{K}_{\epsilon,i}$ with allowance for inter-electron collisions	$\mathcal{K}_{\epsilon,i}$ without allowance for inter-electron collisions	$\mathcal{K}_{\sigma,i}$ with allowance for inter-electron collisions	$\mathcal{K}_{\sigma,i}$ without allowance for inter-electron collisions
0	1,51	1.13	4.59	19,8	1.95	3.39
0.01	1.51	1,13	4.59	19.5	1,95	3,38
0.05	1.50	1.13	4.51	15.8	1.92	2.76
0.1	1.48	1.12	4.34	11.1	1,86	2.12
0.2	1.40	1.09	3.79	5.47	1.65	1.40
0.5	1.19	1.02	2.30	2.44	1.07	0.90
1.0	1,07	0.94	1,41	1.52	0.72	0.68
2,0	0.985	0.95	1,05	1.15	0.62	0.59
4.0	1.0	0.98	0,97	1.01	0,73	0.67
6.0	1.0	0.99	0,98	0.97	0.82	0.72
10.0	1.0	1,0	0,99	0.98	0,92	0.78
35,0	1.0	1.0	1,00	0.99	0.99	0.91
∞	1	1	1	1	1	1

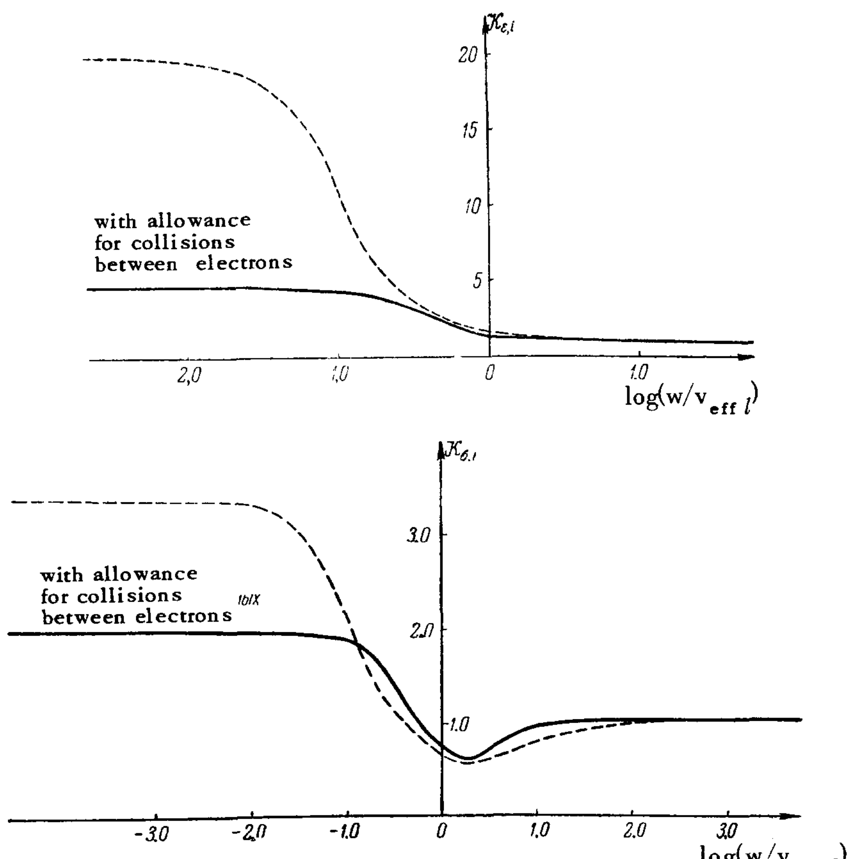

Figure 5.

With the aid of the same functions $\mathscr{K}_\epsilon$ and $\mathscr{K}_\sigma$ it becomes possible to express the components of the tensors ϵ_{ik} and σ_{ik} in an anisotropic plasma, i. e., in the presence of a constant magnetic field $\mathbf{H}_0$. Here the tensor components ϵ_{ik} and σ_{ik} in the direction parallel to the magnetic field (ϵ_{zz}, σ_{zz}) satisfy, as before, expressions (2.33); in a plane perpendicular to $\mathbf{H}_0$ (xy plane), we have:[13]

$$\varepsilon_{xx}=\varepsilon_{yy}=1-\frac{4\pi e^2N}{m}\frac{1}{2\omega}\left\{\frac{(\omega-\omega_H)\,\mathscr{K}_\varepsilon\left(\frac{|\omega-\omega_H|}{\nu_{\mathrm{eff}}}\right)}{(\omega-\omega_H)^2+\nu^2_{\mathrm{eff}}}+\frac{(\omega+\omega_H)\,\mathscr{K}_\varepsilon\left(\frac{|\omega+\omega_H|}{\nu_{\mathrm{eff}}}\right)}{(\omega+\omega_H)^2+\nu^2_{\mathrm{eff}}}\right\},$$

$$\varepsilon_{yx}=-\varepsilon_{xy}=i\frac{4\pi e^2N}{m}\frac{1}{2\omega}\left\{\frac{(\omega-\omega_H)\,\mathscr{K}_\varepsilon\left(\frac{|\omega-\omega_H|}{\nu_{\mathrm{eff}}}\right)}{(\omega-\omega_H)^2+\nu^2_{\mathrm{eff}}}-\frac{(\omega+\omega_H)\,\mathscr{K}_\varepsilon\left(\frac{|\omega+\omega_H|}{\nu_{\mathrm{eff}}}\right)}{(\omega+\omega_H)^2+\nu^2_{\mathrm{eff}}}\right\},$$

$$\sigma_{xx}=\sigma_{yy}=\frac{e^2N}{m}\frac{\nu_{\mathrm{eff}}}{2}\left\{\frac{\mathscr{K}_\sigma\left(\frac{|\omega-\omega_H|}{\nu_{\mathrm{eff}}}\right)}{(\omega-\omega_H)^2+\nu^2_{\mathrm{eff}}}+\frac{\mathscr{K}_\sigma\left(\frac{|\omega+\omega_H|}{\nu_{\mathrm{eff}}}\right)}{(\omega+\omega_H)^2+\nu^2_{\mathrm{eff}}}\right\},$$

$$-\sigma_{xy}=\sigma_{yx}=i\frac{e^2N}{m}\frac{\nu_{\mathrm{eff}}}{2}\left\{\frac{\mathscr{K}_\sigma\left(\frac{|\omega-\omega_H|}{\nu_{\mathrm{eff}}}\right)}{(\omega-\omega_H)^2+\nu^2_{\mathrm{eff}}}-\frac{\mathscr{K}_\sigma\left(\frac{|\omega+\omega_H|}{\nu_{\text{эфф}}}\right)}{(\omega+\omega_H)^2+\nu^2_{\mathrm{eff}}}\right\},$$

$$\omega_H=\frac{|e|H_0}{mc}. \qquad (2.34)$$

These expressions for ϵ_{ik} and σ_{ik} differ from the corresponding expressions (1.10), obtained in the elementary theory, only in the presence of the factors $\mathscr{K}_\epsilon$ and $\mathscr{K}_\sigma$. Therefore, in particular, a resonant increase in conductivity σ_{ik} can occur, as before, near the gyro frequency (at $\omega \simeq \omega_H$). The value of $\mathscr{K}_\sigma$ affects in this case the height of the resonant curve; in particular, the collisions between electrons lower the height of the resonance, by reducing $\mathscr{K}_\sigma$ and $\mathscr{K}_\epsilon$ (see Table II).[47]

The formulas given here are valid, naturally, not only in a strong field but also in a weak electric field. Furthermore, in a weak field the distribution function f_0 is usually Maxwellian with $T_e = T$, regardless of the degree of plasma ionization. The expressions obtained for ϵ_{ik} and σ_{ik} can consequently be used to calculate the conductivity and the dielectric permittivity of the plasma in a weak alternating electric field of any frequency ω (this is of significance, for example, in problems connected with propagation of radio waves[15]).

e) Electron Temperature. Substituting the expressions obtained for the effective collision frequency ν_{eff}, of the relative fraction of the energy transferred δ_{eff}, and the current $\mathbf{j}_t$ into (2.24) and solving this equation, we can determine the electron temperature. It is significant that the equation obtained here for T_e is close to the equation of the elementary theory (1.11). Therefore its solution is completely analogous to the solution of Eq. (1.11), considered in Sec. 1. For example, in a rapidly alternating electric field (when $\omega \gg \delta\nu_{\mathrm{eff}}$) the temperature of the electrons is constant, as previously; it is given by the equation

$$\frac{T_e}{T}=1+\left(\frac{E_0}{E_p}\right)^2\frac{\delta_{\mathrm{eff}}(T)}{\delta_{\mathrm{eff}}(T_e)}\frac{\omega^2+\nu^{(0)2}_{\mathrm{eff}}}{\omega^2+\nu^2_{\mathrm{eff}}}\mathscr{K}_\sigma\left(\frac{\omega}{\nu_{\mathrm{eff}}}\right). \qquad (2.35)$$

Here E_p is again the characteristic "plasma field":

$$E_p=\sqrt{3kT\frac{m}{e^2}\delta_{\mathrm{eff}}(T)(\omega^2+\nu^{(0)2}_{\mathrm{eff}})}.$$

It is seen therefore that Eq. (2.35) differs from the corresponding Eq. (1.16) of elementary theory only in the coefficient $\mathscr{K}_\sigma$, and also in the fact that the number of collisions ν_{eff}, which has remained somewhat indeterminate, is now accurately determined by Eq. (2.25); in addition, the quantity $\delta = \delta_{\mathrm{eff}}$ is assumed to be independent of T_e in (1.16). In the cases of collisions with molecules, the coefficient $\mathscr{K}_\sigma$ is close to unity; therefore the analysis of this case as given in Sec. 1 remains completely in force. The same pertains to collisions with ions at high frequencies ($\omega^2 \gg \nu_{eff}^2$). The factor $\mathscr{K}_\sigma$ can influence substantially the electron temperature only in the case of collisions with ions at low frequencies, and also in the region of gyro resonance.

2.4 Weakly Ionized Plasma

In a weakly ionized plasma the collisions between electrons are insignificant in the equation for the function f_0 (since $\nu_e \ll \delta\nu$) and these can be disregarded in first approximation. They are even less significant in the equation for the function $\mathbf{f}_1$, since $\nu_e \ll \delta\nu \ll \nu$. Therefore the function $\mathbf{f}_1$ in a homogeneous weakly-ionized plasma, accurate to terms of order δ, is always given by the expression (2.31), $\mathbf{f}_1 = -\mathbf{u}\,\partial f_0/\partial v$, where the velocity $\mathbf{u} = \mathbf{u}(v, t)$ is defined by (2.32). Substituting this value of $\mathbf{f}_1$ in (2.20a) we obtain finally the following equation for the function f_0

$$\frac{\partial f_0}{\partial t} - \frac{1}{2v^2}\frac{\partial}{\partial v}\left\{ v^2 \left[\left(\delta_{el}\,(\nu_m^{el} + \nu_i)\frac{kT}{m} + \frac{2e\mathbf{E}\mathbf{u}}{3m} \right)\frac{\partial f_0}{\partial v} + \delta_{el}\, v\, (\nu_m^{el} + \nu_i)\, f_0 \right] \right\} + S_{m0}^{inel}(f_0) = 0. \quad (2.36)$$

Depending on the relation between the time $1/\omega$, during which the electric field changes significantly, and the relaxation time for the function $f_0(\tau) \sim 1/\delta\nu$, we distinguish here cases of slowly varying field ($\omega \ll \delta\nu$, and rapid ones ($\omega \gg \delta\nu$) (the same as in the analysis of the electron temperature in elementary theory or in a strongly-ionized plasma). In the former case, which is quasi-stationary, the dependence of f_0 on the time in Eq. (2.36) can be disregarded; in particular, this takes place naturally in the case of a constant electric field. On the other hand, for a rapidly alternating electric field, $\omega \gg \delta\nu$ the function f_0 does not have a chance to change as rapidly as the field; it therefore settles at a certain average level, independent of the time, and the variable deviations from this level are small, of amplitude on the order of $\delta\nu/\omega$ (the same as the observations of the electron temperature in elementary theory). Consequently, in both cases we can neglect in first approximation the term $\partial f_0/\partial t$ in Eq. (2.36), and thereby get rid in fact of the time variable. This allows us to find an analytical solution for Eq. (2.36) for many important cases: for elastic collisions, in inert gases and in a molecular plasma. We now proceed to analyze these solutions.

a) Case of Elastic Collisions. If all the collisions are elastic, the $S_{m0}^{inel} = 0$ in (2.36). Therefore in a constant electric field $\mathbf{E}$ we have, according to (2.32), $\mathbf{u} = e\mathbf{E}/m\nu$ at $\mathbf{H}_0 = 0$ and Eq. (2.36) is written as

$$\frac{1}{v^2}\frac{\partial}{\partial v}\left\{ v^2 \left[\left(\delta_{el}\nu\frac{kT}{m} + \frac{2e^2E^2}{3m^2\nu} \right)\frac{\partial f_0}{\partial v} + \delta_{el}\cdot\nu v f_0 \right] \right\} = \frac{1}{v^2}\cdot\frac{\partial}{\partial v}\{v^2 j_v\} = 0. \quad (2.37)$$

(Here $\nu = \nu(v) = \nu_m^{el} + \nu_i$). Multiplying this equation by v^2 and integrating from 0 to v, we see that $j_v = 0$, since in the absence of an electron source $[v^2 j_v]_{v=0} = 0$. Integrating now the equation $j_v = 0$ over the velocities, we obtain

$$f_0 = C\exp\left\{ -\int_0^v \frac{mv\,dv}{kT + \dfrac{2e^2E^2}{3m\delta_{el}\nu^2}} \right\}. \quad (2.38)$$

We therefore obtain a Maxwellian distribution in a weak field, but in a strong field the distribution function f_0 may differ substantially from Maxwellian, since ν depends on v. For example, in a strong electric field upon collision with molecules -- hard spheres -- the function f_0 is determined by the well known Druyvestein formula[50]

$$f_0 = C\exp\left\{ -\frac{3m^2\delta_{el}}{8e^2E^2l^2} v^4 \right\},$$

where $l = v/\nu(v) = 1/\pi a^2 N_m$ is the mean free path of the electron, C a constant determined from the normalization condition (2.1a), and the term kT is neglected in (2.38), which is permissible for a strong field.

The Druyvestein distribution at large electron velocities differs greatly from Maxwellian: it drops off much more rapidly than a Maxwellian one. The calculation of the function f_0 with allowance for the exact dependence of the collision frequency and the velocity for different inert gases was made in references 51 and 63. The effect of a constant magnetic field is taken into account in reference 4 [the magnetic field changes the velocity $\mathbf{u}$ (v), and accordingly f_0 also changes].

We considered above only the case of a constant quasi-stationary electric field ($\omega \ll \delta\nu$). Quite analogously the problem is solved also in a rapidly-alternating ($\omega \gg \delta\nu$) electric field, for in this case we can neglect in first approximation the derivative $\partial f_0/\partial t$. The function f_0 now assumes the form [52–55]

$$f_0 = C \exp\left\{-\int_0^v \frac{mv\,dv}{kT+\left(\frac{e^2E_0^2}{3m\delta_{el}}\right)\varphi(v)}\right\}. \quad (2.40)$$

Here the function $\phi(v)$ without the magnetic field is equal to $[\omega^2 + \nu(v)^2]^{-1}$, and in the presence of a magnetic field

$$\varphi(v) = \frac{\cos^2\beta}{\omega^2+\nu^2(v)} + \frac{\sin^2\beta}{2[(\omega-\omega_H)^2+\nu^2(v)]} + \frac{\sin^2\beta}{2[(\omega+\omega_H)^2+\nu^2(v)]}, \quad (2.41)$$

where β is the angle between **E** and **H**, ω_H is the gyromagnetic frequency, E_0 is the amplitude, and ω is the frequency of the alternating electric field.

The distribution function (2.40) coincides at low frequencies ($\omega + \omega_H \ll \nu$) with the distribution (2.38) for a constant electric field except that the corresponding constant field is here, naturally, found to be equivalent to the effective field $E_{eff} = E_0/\sqrt{2}$).

Corrections to the function f_0 with periodic variation in time were calculated in references 56 and 57 devoted to nonlinear effects in the ionosphere.

Elastic collisions are produced in monoatomic (inert) gases at low medium electron energy (up to 1 ev).

b) Molecular Plasma. We define molecular plasma as one formed in diatomic or polyatomic gases. In such a plasma there can be excited not only optical but also rotational and vibrational levels, the energy of which is low ($\hbar\omega \sim 10^{-2}$ to 10^{-4} ev for rotational levels and $\hbar\omega \sim 0.1$ to 0.5 ev for vibrational ones). Therefore inelastic collisions in such a plasma become important even at electron energies on the order of 10^{-2} ev, i. e., at room temperatures.

In a plasma in a diatomic gas (hydrogen, oxygen, nitrogen, air) at low average electron energy (less than or on the order of 1 ev), the principal role is played by losses due to the excitation of rotational levels, the energy of which is naturally small compared with the average electron energy (as found both by computation[58,59] and experiment [39,60]). Consequently, the principal role is played in these cases by such inelastic electron collisions, at which only a small part of the energy is lost. The integral of inelastic collisions for the function f_0 can therefore be represented in the form

$$S_{mo}^{inel}(f_0) = -\frac{1}{2v^2}\frac{\partial}{\partial v}\left\{v^2 R_H(v)\left[\frac{kT}{m}\frac{\partial f_0}{\partial v} + v f_0\right]\right\}, \quad (2.14b)$$

where $R_H(v) = \Sigma_i r_{\omega i}$ is a summary function, describing the energy losses of the electron in inelastic collisions (see Sec. 2.2b).[17] Substituting this expression for S_{mo}^{inel} into Eq. (2.36), we can verify that it actually coincides with the equation considered above for the case of elastic electron collisions; it is merely necessary to replace $\delta_{el} = 2m/M$ by

$$\delta(v) = \frac{\delta_{el}(\nu_m^{el}+\nu_i)+R_H(v)}{\nu(v)} = \frac{R(v)}{\nu(v)}. \quad (2.42)$$

Accordingly, the solution of this equation coincides for molecular plasma also with the solutions considered above: it is again enough merely to replace δ_{el} by $\delta(v)$. For example, in a strong constant electric field we have instead of the Druyvestein distribution (2.39) in a molecular plasma

$$f_0 = C \exp\left\{-\frac{3m^2}{2e^2E^2l^2}\int_0^v v^3\delta(v)\,dv\right\}. \quad (2.43)$$

To obtain finally the form of the distribution function in molecular plasma, it is also necessary to calculate the function $\delta(v) = R(v)/\nu(v)$, which can be done by using for $R(v)$ the expression (2.14b). However, to perform this calculation it is necessary to know the cross sections of all the inelastic processes, which are still unknown (see references 23 and 24). Another method has therefore been proposed in reference 25 to determine the total loss function $R(v)$. In fact, the fraction of the energy lost by the electron δ_{eff}, in a strongly-ionized molecular plasma, as is clear from (2.26), is related with the function $R(v)$ by

$$\int_0^\infty R(v)v^4 \exp\left\{-\frac{mv^2}{2kT_e}\right\}dv = \frac{3\sqrt{\pi}}{\sqrt{2}}\left(\frac{kT_e}{m}\right)^{5/2}\delta_{eff}(T_e)\,\nu_{eff}(T_e). \quad (2.44)$$

This relation can be considered as an integral equation with respect to $R(v)$, since its right half is known from experiment. Thus, it is possible, in principle, to determine $R(v)$ from (2.44), and consequently to determine $\delta(v)$. *The* results of the corresponding calcula-

[17] It is assumed here that the temperature of the heavy particles is also higher than the average energy of the rotational quanta, as usually occurs (the energy of the rotational quanta $\hbar\omega \sim 2$ to $100°$).

tion for hydrogen, oxygen, nitrogen, and air are given in Fig. 6.

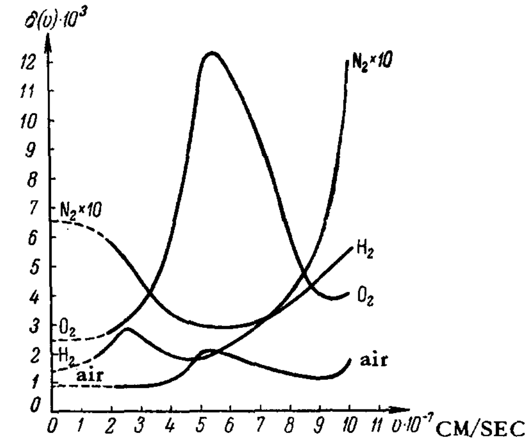

Figure 6.

Inserting the resultant function $\delta(v)$ into (2.38), (2.40), etc, we can calculate the distribution function of the electrons in a molecular plasma. The results of such a calculation for electrons in hydrogen in a high frequency electric field are shown in Fig. 7. The ordinates represent $-\ln f_0$, and the abscissas represent $-v^2/\bar{v}^2$ where $\bar{v}^2 = 2\bar{K}/m$ is the mean squar electron velocity. The dotted line corresponds to a Maxwellian distribution function [the distribution would be Maxwellian were δ independent of v, as occurs, for example, in the case of elastic collisions; see Eq. (2.40)]. It is seen from the figure that in this case the deviations of the distribution function from Maxwellian are not very large, they increase with increasing average electron energy.

c) Inert Gases. In inert gases at low average electron energy (up to 1 ev) the principal role is played by elastic collisions between electrons and atoms of the gas. At higher energies, the losses due to fast electrons, which are capable of exciting optical levels or of ionizing the atoms, become more significant. Here, if the average electron energy is low -- lower than the minimum excitation energy $\hbar\omega$ (on the order of 10 ev), the basic inelastic losses are obviously due to the electrons whose energy exceeds $\hbar\omega$ only slightly (since the number of electrons having a high energy and being consequently capable of inelastic collisions at $K > \omega\hbar$, diminishes rapidly with increasing K). In this case for the integral of inelastic collisions, the function f_0 can be considered to satisfy with good approximation the limiting formula $S_{m0}^{inel} = \nu_\omega(v) \cdot f_0$ [see Eq. (2.15)]. Therefore in a constant electric

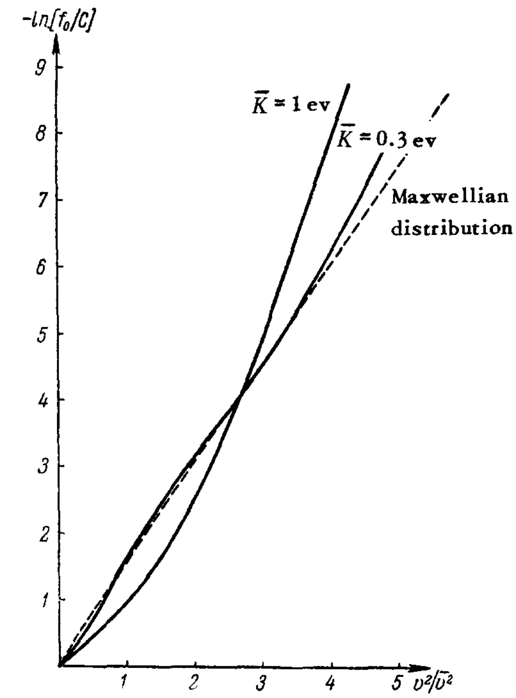

Figure 7.

field in inert gases the equations for the function f_0, with allowance for inelastic collisions of electrons, assume the form

$$-\frac{1}{2v^2}\frac{\partial}{\partial v}\left\{v^2\left[\left(\delta_{el}\nu_m^{el}\frac{kT}{m}+\frac{2e^2E^2}{3m^2(\nu_m^{el}+\nu_\omega)}\right)\frac{\partial f_0}{\partial v}+\nu_m^{el}vf_0\right]\right\}+\nu_\omega(v)f_0=0. \qquad (2.45).$$

In addition, it is necessary to add, at $v = 0$, the electron source $Q = dN/dt = 4\pi\int_{\hbar\omega}^{\infty}\nu_\omega(v)\,v^2 f_0\,dv$ (see Sec. 2.2b). The cross section for the inelastic collision vanishes at electron energies less than the excitation energy $\hbar\omega$, and when $K > \hbar\omega$ it is possible to assume approximately that it increases linearly with increasing electron energy, i. e., $\nu_\omega(v) = \nu\left(\frac{mv^2}{2\hbar\omega} - 1\right)/l_H$, where l_H is the effective mean free path of the electron between two inelastic collisions.

In solving (2.45) it is advisable to distinguish between the two regions, $mv^2/2 \leq \hbar\omega$ and $mv^2/2 > \hbar\omega$. In the former region the distribution function, as before, is determined by (2.37), since there are no inelastic collisions here. However, in solving it it is necessary to take into account the presence of the source Q when $v = 0$; consequently the flux j_v does not vanish in this region, $j_v = C_1/v^2$, where C_1 is the integration constant. The solution of Eq. (2.37) in the first region leads thus to a distribution function that differs from the Druyvestein function in the presence of an additional factor.[61] In the second region, as can be readily seen:[26]

$$f_0 = C\sqrt{\frac{mv^2}{2\hbar\omega}-1}\, iH_{1/3}\left\{i\sqrt{\frac{2}{3}}\frac{\hbar\omega}{eE\sqrt{ll_H}}\left(\frac{mv^2}{2\hbar\omega}-1\right)\right\}. \tag{2.46}$$

Here $H_{1/3}$ is the Hankel function of order 1/3 and $l = v/(\nu_m^{el} + \nu_\omega)$ is the electron free path, which is independent of the velocity. Both these distributions join at $K = \hbar\omega$, and from this one determines the constants C and C_1. The distribution function (2.46) diminishes with increasing electron velocity much more sharply than the Druyvestein distribution function, i. e., the "tail" of the distribution function in the region $K > \hbar\omega$ is so to speak cut off because of the inelastic collisions, as should be. Kovrizhnykh[61] investigated also the case of an arbitrary dependence of the mean free path l and the excitation cross section on the velocity v.

We note that usually not one but several levels may be excited, and therefore the dependence $\nu_\omega(v)$ has, generally speaking, a more complicated form. A corresponding calculation for helium and hydrogen, with allowance of all the excited levels, is given in references 27 and 27a. The problem is solved quite analogously also for a alternating electric field.[62]

d) Electron Current and Average Electron Energy. Using the expressions obtained above for the distribution function, it is easy to determine the electron current and the average electron energy in a weakly-ionized plasma

$$\begin{aligned} \varepsilon &= 1 + \frac{(4\pi e)^2}{3}\int_0^\infty \frac{v^3}{\omega^2+\nu^2}\frac{\partial f_0}{\partial v}\,dv,\\ \sigma &= -\frac{4\pi e^2}{3}\int_0^\infty \frac{\nu v^3}{\omega^2+\nu^2}\frac{\partial f_0}{\partial v}\,dv,\\ \overline{K} &= \frac{2\pi m}{N}\int_0^\infty v^4 f_0\,dv, \end{aligned} \tag{2.47}$$

where $\nu = \nu(v)$ is the total number of electron collisions.

These expressions for the case of elastic collisions were discussed in references 50 and 4 for a constant electric field, in references 52 and 43 for an alternating field, and in references 43 and 54 for the presence of a magnetic field, too; the case of a molecular plasma is considered in reference 25, and the calculations for a plasma in inert gases have been made in reference 27. For different limiting cases simple formulas were obtained; in general the formulas are, naturally, complicated; frequently the values of ϵ, σ, and $\overline{K}$ are obtained only by numerical integration.

It is important to emphasize that the results of the calculation of ϵ, σ, and $\overline{K}$ for weakly-ionized plasma, using formula (2.47), differ almost always only slightly (up to 10 — 15%) from the results of the calculation of the same quantities by means of the simpler formula given above for a strongly-ionized plasma (we have in mind, naturally, results that are comparable under the same field intensity, the same values of l and δ_{el}, etc). For example, in a strong constant electric field in the case of elastic collisions with molecules we have for a weakly-ionized plasma $\overline{K} \simeq 0.604\, eEl/\sqrt{\delta_{el}}$, and for a strongly-ionized plasma

$$\frac{3}{2}kT_e = \overline{K} \simeq 0{,}613\,\frac{eEl}{\sqrt{\delta_{el}}}.$$

2.5. Arbitrary Degree of Ionization. Concerning the Elementary Theory.

a) Transition from a Strongly Ionized Plasma to a Weakly Ionized Plasma. We considered above the limiting cases of a weakly-ionized plasma, when the collisions between electrons are insignificant, and a strongly-ionized plasma, when, to the contrary, the form of the function f_0 is determined precisely by the collision between electrons. We consider now an intermediate case, when the form of the function f_0 is substantially influenced both by collisions between electrons and collisions of electrons with heavy particles.[31] In the equation for the function $\mathbf{f}_1$ in this case we can neglect the collisions between electrons, since $\nu_e \sim \delta\nu \ll \nu$. Therefore the function $\mathbf{f}_1$ is written, as previously, in the form $\mathbf{f}_1 = -\mathbf{u}\,\partial f_0/\partial v$ [see Eq. (2.31)].

The problem reduces therefore to an analysis of one equation for the function f_0

$$\frac{\partial f_0}{\partial t} - \frac{1}{2v^2}\frac{\partial}{\partial v}\left\{v^2\left(\left[\delta_{el}(\nu_m^{el}+\nu_i)\frac{kT}{m} + \frac{2e\mathbf{Eu}}{3m} + 2A_2(f_0)\right]\frac{\partial f_0}{\partial v} + \right.\right.$$
$$\left.\left. + [\delta_{el}(\nu_m^{el}+\nu_i) + 2A_1(f_0)]\,vf_0\right)\right\} + S_{m0}^{inel}(f_0) = 0, \tag{2.48}$$

where the coefficients A_1 and A_2 are integrals that depend on the function f_0 (2.18).

Under stationary conditions (constant or rapidly-varying electric field) the first term in (2.48) can be neglected. The solution of the remaining nonlinear integro-differential equation can be obtained by the iteration method. This method gives good convergence, since the variation of the function f_0 when going from weakly-ionized plasma to a strongly-ionized one causes only a small change in the integral coefficients $A_1(f_0)$ and $A_2(f_0)$ (compare with Sec. 2.4d).

Choosing as the zero approximation $f_0^{(0)}$ a Maxwellian distribution function with an electron temperature, which must be determined from Eq. (2.24), we can verify that in the next approximation

$$f_0^{(1)} = C\exp\left\{-\int_0^v \frac{v\,dv\,[\delta_{el}\nu + 2A_1^{(0)}]}{\frac{kT}{m}\delta_{el}\nu + \frac{2e\mathbf{Eu}}{3m} + 2A_2^{(0)}}\right\}, \qquad (2.49)$$

Here we consider for simplicity only elastic collisions of electrons and

$$A_2^{(0)} = A_2(f_0^{(0)}) = \frac{kT_e}{m}A_1^{(0)} = \frac{kT_e\nu_e}{m}\left[\Phi(x) - \frac{2}{\sqrt{\pi}}\,x\exp\{-x^2\}\right],$$

where $\Phi(x) = 2(\pi)^{-1/2}\int_0^x \exp(-z^2)\,dz$ is the probability integral, $x = v(2kT_e/m)^{-1/2}$, and $\nu_e = \nu_e(v)$ is the collision frequency between the electrons [it is given by formula (2.16) if N_i is replaced by N]. In a strongly-ionized plasma the principal role is played in (2.49) by the coefficients $A_1^{(0)}$ and $A_2^{(0)}$, since the function $f_0^{(1)}$ is in this case a Maxwellian one. In a weakly-ionized plasma, to the contrary, the coefficients $A_1^{(0)}$ and $A_2^{(0)}$ can be neglected, and the function $f_0^{(1)}$ in a strong field is a Druyvestein function, as should be. Fig. 8 shows the transition from a Maxwellian distribution to

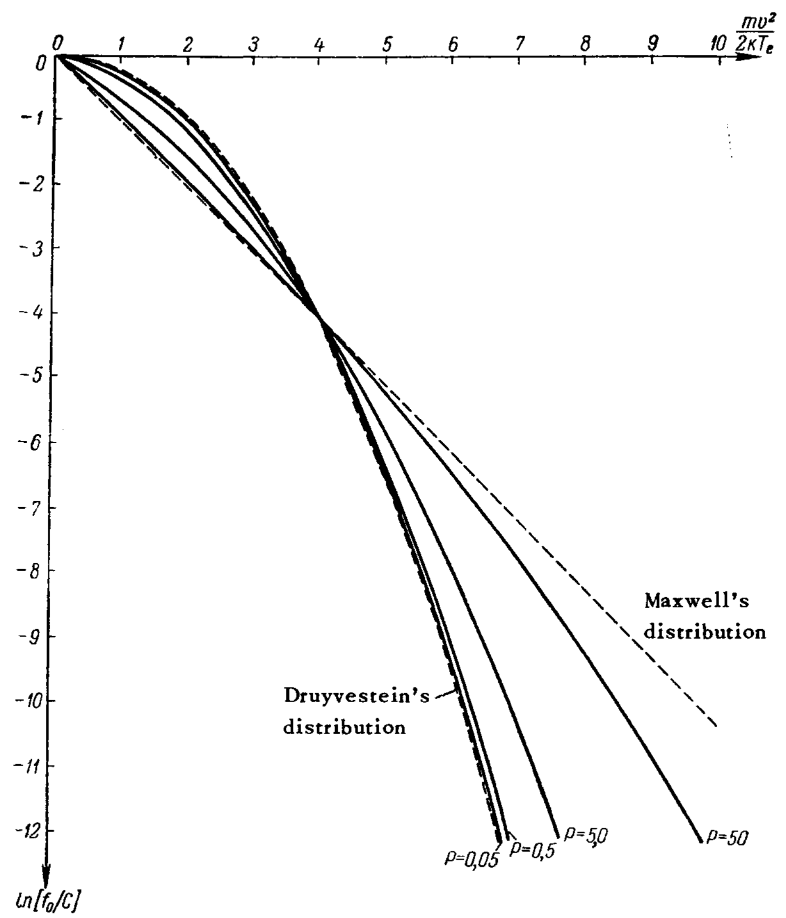

Figure 8.

the Druyvestein distribution as a function of the degree of ionization of the plasma, or more accurately, as a function of the parameter

$$p=\frac{\nu_e(v_0)}{\delta\nu_m(v_0)}=\frac{N_e}{N_m}\frac{\pi e^4\ln\left(\frac{kT_eD}{e^2}\right)}{\delta(kT_e)^2\pi a^2}\approx 6\cdot 10^7\frac{N_e}{N_m}\left(\frac{1\text{ ev}}{kT_e}\right)^2\left(\frac{10^{-16}\text{ cm}^2}{\pi a^2}\right)\left(\frac{10^{-4}}{\delta}\right). \qquad (2.50)$$

[Here $v_0=(2kT_e/m)^{1/2}$, $T_e=eEl/\sqrt{6\delta}$ is the electron temperature]. It is seen from the diagram that approximately halfway between the Maxwellian and the Druyvestein distributions lies a curve corresponding to a value $p=5$. Consequently, at $p\gg 5$ the distribution is Maxwellian and the plasma can be considered "strongly ionized," i. e., the formulas obtained in Sec. 2.3 can be used. When $p\ll 5$ the plasma is "weakly ionized." It must be noted that the region of the transition is stretched out, particularly strongly at high velocities, i.e., in the "tail" of the distribution function: we see from the diagram, for example, that in the "tail" the deviations from the Maxwell distribution are considerable even at $p=50$. In general the Maxwellian distribution in the "tail" (i. e., at large v) can be considered as taking place only when p is greater than $(mv^2/2kT_e)^2$.

b) Conditions of Applicability of Elementary Theory. We already noted above that the expressions for the dielectric permittivity, the conductivity, and the mean electron energy in a plasma are quite complicated in the general case. We have also seen that for the same quantities one obtains in the elementary theory very simple formulas convenient for computation. It is therefore important to ascertain when the mean quantities such as ϵ, σ, and $\overline{K}$ can be calculated by means of the elementary theory and when the use of kinetic theory is essential for this purpose.

An important qualitative difference between elementary and kinetic theories appears only in certain cases in the analysis of nonstationary effects.[14,42] For stationary effects, considered in the present article, the elementary and the kinetic theories always lead to a qualitatively identical result. Therefore in the analysis of the question of applicability of elementary theory, we can speak here only of the magnitude of the quantitative error, which can be tolerated in the elementary calculation.[18)]

Let us consider first the simplest case, when the electron collision frequency ν and the fraction of the energy δ are independent of the electron velocity.[64–68,25] The solution of the kinetic equation (2.20) has in this case the following form

$$f_0=N\left(\frac{m}{2\pi kT_e}\right)^{3/2}\exp\left\{-\frac{mv^2}{2kT_e}\right\},\quad \mathbf{f}_1=-\mathbf{u}\frac{\partial f_0}{\partial v}, \qquad (2.51)$$

where the temperature T_e and the average directed velocity of the electron are determined by

$$\left.\begin{aligned}&\frac{dT_e}{dt}+\delta\nu(T_e-T)=\frac{2}{3k}e\mathbf{E}\mathbf{u},\\&\frac{d\mathbf{u}}{dt}+\nu\mathbf{u}=\frac{e}{m}\left(\mathbf{E}+\frac{1}{c}[\mathbf{u}\times\mathbf{H}]\right).\end{aligned}\right\} \qquad (2.52)$$

These equations for $\mathbf{u}$ and T_e are identical with the equations of elementary theory (1.3) and (1.11) for constant $\delta_{eff}\equiv\delta$ and $\nu_{eff}\equiv\nu$. In other words, the elementary theory actually corresponds to the assumption that ν and δ are independent of v. It is therefore clear that in those cases when ν and δ do not depend too much on v, the error admitted in the elementary calculation should be small [instead of $\nu(v)$ and $\delta(v)$ in the elementary theory, i. e., in Eq. (2.52), it is natural to use here the values $\nu_{eff}(T_e)$ and $\delta_{eff}(T_e)$, determined in accordance with (2.25) and (2.26)].

The corresponding analysis, carried out in references 20 and 25, shows that in a strongly-ionized plasma (i. e., for a Maxwellian distribution), the discrepancy between the results of the elementary and kinetic calculations of $\mathbf{j}$ and $\overline{K}$ is insignificant, provided the following condition is satisfied

$$\frac{D_\nu}{\omega^2+\nu_{eff}^2}=\frac{1}{\omega^2+\nu_{eff}^2}\frac{\sqrt{2}}{3\sqrt{\pi}}\left(\frac{m}{kT_e}\right)^{5/2}\int_0^\infty(\nu(v)-\nu_{eff})^2v^4\exp\left\{-\frac{mv^2}{2kT_e}\right\}dv\ll 1. \qquad (2.53)$$

Here D_ν is a quantity that characterized the deviation of the number of collisions of the electron from its average (effective) value; in other words, D_ν characterizes the dependence of ν on v. If, for example,

[18)]The foregoing pertains, naturally, to the calculation of the average quantities ($\overline{K}$, $\mathbf{j}$). Obviously, to find the velocity distribution of the electrons one cannot avoid the use of kinetic theory in one form or another.

ν = const, then $D_\nu = 0$; when $\nu = A \cdot v$, when $\nu_{eff} \sim \sqrt{T_e}$, $D_\nu/\nu^2_{eff} \simeq 0.1$; when $\nu = A/v$ (when $\nu_{eff} \sim T_e^{-1/2}$), $D_\nu/\nu^2_{eff} = 0.1$; when $\nu = Av^2$ (when $\nu_{eff} \sim T_e$), $D_\nu/\nu^2_{eff} = 0.4$. In a real plasma in collisions with molecules, the number of collisions is usually proportional to T_e^α, $0 \lesssim \alpha \leq 0.8$ (see references 23 and 24); in this case the criterion (2.53) is always satisfied, so that the error in calculating σ, ϵ, and $\bar{K}$ by means of the formulas of elementary theory, is relatively small. For example, when $\nu_{eff} \sim T_e^{0.5}$, as can be seen from Table II and Fig. 4, the maximum error, which is obtained when $\omega = 0$, is 13% for σ and 51% for ϵ. At high frequencies $\omega^2 \gg \nu^2_{eff}$ the elementary calculation is found to be in general accurate, owing to the choice of the effective frequency of collisions in the elementary theory in the form of Eq. (2.25). The number of collisions with ions depends greatly on the electron velocity (2.16); the ratio $D_\nu/\nu^2_{eff\,i}$ is found in this case to be a rather large quantity

$$\frac{D_\nu}{\nu^2_{eff\,i}} \simeq 6\,\frac{\left(\frac{kT_e}{e^2N^{1/3}}\right)^{3/4}}{\ln^2\left(\frac{kT_e}{e^2N^{1/3}}\right)} \gg 1.$$

As a result, in collisions with ions condition (2.53) is satisfied only for a high frequency electric field at $\omega \gg \sqrt{D_\nu} \sim (10 \text{ to } 100)\,\nu_{eff\,i}$. If $\omega \lesssim \sqrt{D_\nu}$, then it is necessary, generally speaking, to use the results of the kinetic theory in calculation of σ and ϵ i. e., it is necessary to take into account the correction coefficients $\mathscr{K}_\sigma$ and $\mathscr{K}_\epsilon$, listed in Table II and in Fig. 5. The maximum difference between elementary formulas and the kinetic formulas occurs at a constant field ($\omega = 0$), when [19] $\mathscr{K}_\sigma = 1.95$ and $\mathscr{K}_\epsilon = 4.59$ (we recall that in elementary theory $\mathscr{K}_\sigma = \mathscr{K}_\epsilon = 1$).

In a weakly-ionized plasma the distribution function can deviate substantially from Maxwellian. In order for the elementary calculation to be accurate in this case it is essential that, in addition to (2.53), the following conditions also be satisfied

$$\left.\begin{aligned} \frac{1}{2}\frac{T_e}{\nu_{eff}}\frac{d\nu_{eff}}{dT_e}\,\frac{\gamma(T_e-T)}{\gamma(T_e-T)+T_e} &\ll 1,\\ \frac{1}{2}\frac{T_e}{\delta_{eff}}\frac{d\delta_{eff}}{dT_e}\,\frac{\gamma(T_e-T)}{\gamma(T_e-T)+T_e} &\ll 1, \end{aligned}\right\} \quad (2.54)$$

where

$$\gamma = \frac{T_e}{\delta_{eff}}\cdot\frac{d\delta_{eff}}{dT_e} + \frac{2\nu^2_{eff}}{\omega^2+\nu^2_{eff}}\,\frac{d\nu_{eff}}{dT_e}\,\frac{T_e}{\nu_{eff}}$$

[19] This pertains to a completely ionized plasma with singly-charged ions.

To satisfy conditions (2.54) it is essential that at low frequency $\omega^2 \lesssim \nu^2_{eff}$ and ν_{eff} and δ_{eff} depend weakly on T_e [if ν_{eff} and δ_{eff} are proportional to T_e^α, then criterion (2.54) is satisfied only if $-0.25 \leq \alpha \leq 1$]. At high frequency, $\omega^2 \gg \nu^2_{eff}$ it is necessary merely that δ_{eff} depend weakly on T_e. An important factor is that the error of the elementary theory increases rapidly in the region where ν_{eff} and δ_{eff} decrease with increasing T_e. In a plasma formed in either monoatomic or molecular gas, conditions (2.53 and 2.54) are usually satisfied (with the exception of the high-energy region $kT_e \gtrsim 2$ ev, where δ_{eff} increases vigorously with increasing T_e, and also in the region of the Ramsauer effect in heavy inert gases). Therefore the error admitted in the calculation of σ and $\bar{K}$ by the formulas of elementary theory and in the case of a weakly-ionized plasma is usually small (up to 40%); the error in the calculation of ϵ in a low-frequency electric field may be greater (up to 100%).

(The article and the cited literature will be concluded in the next issue).

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