THE SPECTRA OF SYSTEMS OF INTERACTING PARTICLES AND COLLECTIVE ENERGY LOSSES DURING PASSAGE OF CHARGED PARTICLES THROUGH MATTER

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INTRODUCTION

This article is fundamentally devoted to a study of two closely connected problems. These are, first, the problem of collective-excitation spectra in systems of interacting particles, and second, the problem of energy losses due to excitation of collective oscillations upon passage of particles through matter.

In systems of strongly-interacting particles, such as liquids and solids, plasmas and nuclear matter, we may speak only of energy levels and states of the system as a whole. The study of the whole spectrum of levels of such a system is very complex. It is considerably easier to study the weakly-excited states, i.e., the states which do not differ much from the equilibrium state. Thus, the vibrations of ions in a solid with respect to the crystalline lattice points are states differing little from the equilibrium state, and may be spoken of as weakly-excited states. In this case, the excitation levels of the system are phonons, or quanta of sound. Another example is that of plasma oscillations in a plasma or in metals. In the quantum treatment, we may speak here of quasi-particles (phonons, plasmons, etc.) with a definite energy and momentum, whereas in the classical treatment we speak of waves with a definite frequency and wave vector. The relation of the energy of the quasi-particles to the momentum, or respectively, of the frequency to the wave number, is referred to below as the excitation spectrum.

The excitation states, naturally, may obey the Fermi-Dirac or the Bose-Einstein statistics. Out treatment of collective oscillations is actually devoted to the study of excitation states obeying the Bose-Einstein statistics. Such excitation states include sound waves in solids, phonon-roton excitations in superfluid helium, and spin waves. The latter are an example of a Bose-Einstein excitation manifested in a system of particles obeying Fermi-Dirac statistics.

Wave processes are the analog in classical physics of the Bose-Einstein excitation states. One such process is the propagation of longitudinal plasma waves. The question of the study of excitation spectra in systems of charged particles is taken up in Secs. 3--5 of this article. Here the study of excitation spectra is based on the use of equations for the quantum distribution function (density matrix), a number of properties of which are studied in Sec. 1.

In Sec. 6 is studied the problem of energy losses on passage of a fast charged particle through matter, due to excitation of collective oscillations. In essence, the losses are associated with the excitation in the medium of electromagnetic oscillations (both transverse and longitudinal). The spectra of these oscillations are actually determined by the dielectric constant of the medium. The latter, if spatial dispersion is taken into account, is a function both of the frequency and of the wave vector of the corresponding oscillations. Hence, the energy losses of a fast particle are also completely determined by the dielectric constant.

However, there are cases in which the formulas given in Sec. 6 for the energy losses cannot explain the experimental results. The well-known Langmuir paradox serves as an example; this consists in the fact that the experimentally-observed distances at which electrons entering a plasma will transmit their energy to it turn out to be much smaller than the relaxation distances which may be obtained from the formulas of Sec. 6.

In order to analyze this case, another approach has been studied in Sec. 2 to the problem of energy losses of charged particles on passing through a plasma. This approach is based on the use of kinetic equations, by means of which we may also describe the process of transfer of energy from the charged particles in the excitation of collective oscillations. In the case in which particles entering a plasma do not change the properties of the medium appreciably, the expressions obtained for the retarding force agree with the expressions which may be obtained on the basis of the formulas of Sec. 6.

When an electron beam of sufficient intensity passes through a plasma, these conditions are no longer fulfilled. That is, we may not consider that the properties of the plasma in the presence of the beam are characterized by the same excitation spectrum or the same dielectric constant as in the absence of the beam. In this case, one must solve a system of simultaneous non-linear equations for the electrons of the beam and the electrons of the plasma. Section 7 is devoted to one of the particular problems of the type in which a sufficiently intense electron beam passes through a plasma. The results obtained permit us to explain in basic outline the rapid change in energy of electrons passing through a plasma, as was first observed by Langmuir.

In order to make it easier for the reader to become acquainted with our article, we have tried to arrange the presentation so that each section may, on the one hand,

be read practically independently of the others, and on the other hand, so that it may successively cover all the questions stated above. The authors do not consider that the bibliography given in the article is exhaustive. In articles devoted to a study of such a relatively broad set of problems, such oversights are apparently unavoidable.

Finally, we shall point out that Sec. 3 contains the results of the joint work of the authors published in 1952,[3] in which some fundamental problems of the quantum theory of plasma oscillations were presented. We shall note that analogous problems in the quantum theory of the plasma have been treated by other methods also by other authors, not in connection with the cited reference. However, the method which we used in reference 3 seems to us to be the simplest. In the course of several years since the time that reference 3 was written, the theory of the quantum plasma has been developed along various lines by the authors of this article. The results obtained along one of these lines, together with corresponding results obtained by other authors, are presented in Secs. 2 and 7, written by Yu. L. Klimontovich, while those obtained along another line are given in Secs. 4, 5, and 6, written by V. P. Silin.*

1. THE EQUATION FOR THE QUANTUM DISTRIBUTION FUNCTION

For the statistical description of the quantum properties of macroscopic systems, we may use the equation for the density matrix. The most complete analogy with the classical method involving the distribution function in coordinate momentum phase space is found in using the mixed representation for the density matrix. Here, the matrix is a function of the coordinates and momenta, just as is the classical distribution function. In this article, we shall use the density matrix in the form proposed by Wigner, and shall designate it as the "quantum distribution function." [1-5]

The quantum distribution function is related to the ordinary density matrix in the coordinate representation by the following transformation:

$$f_N(\mathbf{q}, \mathbf{p}, t) = \frac{1}{(2\pi)^{3N}} \int e^{-i\tau\mathbf{p}} \varrho\left(\mathbf{q} - \frac{1}{2}\hbar\tau,\ \mathbf{q} + \frac{1}{2}\hbar\tau,\ t\right) d\tau. \tag{1,1}$$

In the case of a pure ensemble, the distribution function is expressed directly in terms of the wave function of the system

$$f_N(\mathbf{q}, \mathbf{p}, t) = \frac{1}{(2\pi)^{3N}} \int e^{-i\tau\mathbf{p}} \Psi^*\left(\mathbf{q} - \frac{1}{2}\hbar\tau, t\right) \Psi\left(\mathbf{q} + \frac{1}{2}\hbar\tau, t\right) d\tau. \tag{1,2}$$

In Eqs. (1,1) and (1,2), N is the number of particles in the system being studied, and the vectors $\mathbf{q}$ and $\mathbf{p}$ denote the sets of all coordinates and momenta of the particles. The use of the quantum distribution function permits us in a unitary way to describe both classical and quantum systems. The transition from the quantum description to the classical ($\hbar \to 0$) is very graphic.

We shall note some properties of the quantum distribution function.

1. $f_N(\mathbf{q}, \mathbf{p}, t)$ is a real function, but can take on negative values. Hence, only the distributions obtained from the function $f_N(\mathbf{q}, \mathbf{p}, t)$ by integration over the coordinates or the momenta characterize the probability of finding the system in a state of given q or p, respectively:

$$\int f_N(\mathbf{q}, \mathbf{p}, t)\, dp = \overline{\Psi^*(\mathbf{q}, t)\, \Psi(\mathbf{q}, t)} = \varrho_N(\mathbf{q}, t),$$

$$\int f_N(\mathbf{q}, \mathbf{p}, t)\, dq = \overline{\varphi^*(\mathbf{p}, t)\, \varphi(\mathbf{p}, t)} = F_N(\mathbf{p}, t).$$

2. The mean values of a function $A(q, p)$ of the coordinates and momenta are determined, as in classical statistical mechanics, by:

$$\bar{A} = \int A(\mathbf{q}, \mathbf{p})\, f_N(\mathbf{q}, \mathbf{p}, t)\, d\mathbf{q}\, d\mathbf{p}.$$

Thus, in order to obtain the mean values of physical quantities, we may use the quantities themselves, rather than the operators corresponding to them.

3. The symmetry properties of the quantum distribution function are not so graphic as is the case with the density matrix in the coordinate representation. Hence, in order to study the symmetry properties of the function $f_N(\mathbf{q}, \mathbf{p}, t)$, we must refer to the density matrix in the coordinate representation $\rho(\mathbf{q}', \mathbf{q}, t)$, for which the following relations hold true:

$$P\varrho(\mathbf{q}', \mathbf{q}, t) = \varrho(\mathbf{q}', \mathbf{q}, t) = \varrho(\mathbf{q}', \mathbf{q}, t)\, P$$

*In addition, the development along this line in reference 3 is covered in Sec. 3.

for systems of particles obeying Bose-Einstein statistics, and

$$P\varrho(\mathbf{q}', \mathbf{q}, t) = (-1)^P \varrho(\mathbf{q}', \mathbf{q}, t) = \varrho(\mathbf{q}', \mathbf{q}, t) P$$

for the Fermi-Dirac case. In these expressions, P is the transformation operator. The equation determining the variation in the quantum distribution function may be found by a Fourier transformation of the equation for the density matrix in the coordinate representation. It may be written in the form

$$\frac{\partial f_N}{\partial t} = \frac{i}{\hbar (2\pi)^{6N}} \int \dots \int \left[H\left(\eta + \frac{1}{2}\hbar\mathbf{k}, \mathbf{r} - \frac{1}{2}h\tau\right) - H\left(\eta - \frac{1}{2}\hbar\mathbf{k}, \mathbf{r} + \frac{1}{2}\hbar\tau\right)\right] \times$$
$$\times f_N(\mathbf{r}, \eta, t)\, e^{i\tau(\eta - \mathbf{p}) + i\mathbf{k}(\mathbf{r} - \mathbf{q})}\, d\tau\, d\mathbf{k}\, d\eta\, d\mathbf{r}. \qquad (1,3)$$

In this equation, H is the Hamiltonian of the system being studied. If, for example, $H = \mathbf{p}^2/2m + U(\mathbf{q})$, Eq. (1, 3) takes the form

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{m}\frac{\partial f}{\partial \mathbf{q}} = \frac{i}{(2\pi)^3 \hbar} \int \dots \int \left[U\left(\mathbf{q} - \frac{1}{2}\hbar\tau\right) - U\left(\mathbf{q} + \frac{1}{2}\hbar\tau\right)\right] \times$$
$$\times f(\eta, \mathbf{q})\, e^{i\tau(\eta - \mathbf{p})}\, d\tau\, d\eta.$$

In the classical case ($\hbar \to 0$), Eq. (1, 3) goes over into the equation for the classical distribution function

$$\frac{\partial f_N}{\partial t} = \sum_{1 \leq i \leq N} \left\{ \frac{\partial H}{\partial \mathbf{q}_i}\frac{\partial f_N}{\partial p_i} - \frac{\partial H}{\partial p_i}\frac{\partial f_N}{\partial \mathbf{q}_i} \right\} = [H f_N], \qquad (1,4)$$

where [ ] is the classical Poisson bracket.

In establishing the form of the equation for the quantum distribution function, it is possible to avoid use of the concept of the density matrix, and rather, to proceed directly from Eq. (1, 4), transforming it in such a way as to take into account the finite volume of the cell in the phase space being studied.[3]

2. THE KINETIC EQUATION FOR THE QUANTUM DISTRIBUTION FUNCTION

The direct application of Eq. (1, 3) for the function f_N is very difficult in most cases, since the desired function depends on a vast number of variables $(6N + 1)$. However, for practical purposes, e. g., in calculating the mean values of physical quantities, in deriving the equation of state, in calculating the fluctuations of physical quantities, etc., it is sufficient to know the distribution functions depending on the coordinates and momenta of one and of two particles, i. e., the first-order and second-order distribution functions:

$$f_1(\mathbf{q}_1, \mathbf{p}_1, t) = \int f_N(\mathbf{q}, \mathbf{p}, t)\, d\mathbf{q}_2, \dots, d\mathbf{q}_N, d\mathbf{p}_2, \dots, d\mathbf{p}_N,$$
$$f_2(\mathbf{q}_1, \mathbf{q}_2, \mathbf{p}_1, \mathbf{p}_2, t) = \int f_N(\mathbf{q}, \mathbf{p}, t)\, d\mathbf{q}_3, \dots, d\mathbf{q}_N, d\mathbf{p}_3, \dots, d\mathbf{p}_N.$$

Hence, it is natural to try to obtain equations which would contain only the simplest distribution functions f_1 and f_2. However, the equation for the first-order distribution function contains the second-order function, while the equation for the second-order distribution function contains the third-order function, etc. Thus, we obtain a chain of equations in which the equation defining the function f_s contains the function f_{s+1}.

Such a chain of equations for the classical distribution function was first studied in the papers of N. N. Bogolyubov[6] and of Bron and Green. Using the method developed in these references, we may obtain from Eq. (1, 3) for the function f_N a chain of equations for the quantum distribution function,[3] relating the functions f_s and f_{s+1}. The chain of these equations, of course, is completely analogous to the chain of equations for the density matrix, which was studied in the paper of N. N. Bogolyubov and K. P. Gurov.[7] Thus, for example, we may consider systems of interacting particles in which the Hamiltonian may be written as the sum of kinetic energies of individual particles and the potential energy of their interaction in pairs

$$H = \sum_{1 \leq i \leq N} \frac{\mathbf{p}_i^2}{2m} + \sum_{1 \leq i < j \leq N} U(|\mathbf{q}_i - \mathbf{q}_j|), \qquad (2.1)$$

Here, the first equation of the chain, relating the functions f_1 and f_2, has the following form:

$$\frac{\partial f_1}{\partial t}+\frac{\mathbf{p}}{m}\frac{\partial f_1}{\partial \mathbf{q}}-\frac{i}{(2\pi)^3\hbar}\int\cdots\int\left[U\left(\left|\mathbf{q}_1-\mathbf{q}_2-\frac{1}{2}\hbar\boldsymbol{\tau}\right|\right)-U\left(\left|\mathbf{q}_1-\mathbf{q}_2+\frac{1}{2}\hbar\boldsymbol{\tau}\right|\right)\right]\times$$
$$\times f_2(\mathbf{q}_1,\ \mathbf{q}_2,\ \boldsymbol{\eta},\ \mathbf{p}_2,\ t)\,e^{i\boldsymbol{\tau}(\boldsymbol{\eta}-\mathbf{p}_1)}\,d\mathbf{q}_2\,d\mathbf{p}_2\,d\boldsymbol{\tau}\,d\boldsymbol{\eta}. \qquad (2.2)$$

As $\hbar\to 0$, this equation goes over into the corresponding classical equation. Of course, a rigorous solution of the chain of equations considered here would be as complex as the solution of the original equation, since in the final analysis, it requires a knowledge of the function f_N.

However, as is well known, the so-called kinetic equations are of fundamental significance for physical applications. These are closed equations for the first-order distribution function f_1:

$$\frac{\partial f_1}{\partial t}=S(\mathbf{q},\ \mathbf{p};\quad f_1(\mathbf{q},\ \mathbf{p},\ t)),$$

in which the rate of change of the distribution function at time t is completely determined by assigning the distribution function f_1 at the same instant of time t. Among the kinetic equations, for example, belong the Boltzmann equation, the Fokker-Planck equation in phase space and the quantum kinetic equation corresponding to it, and the self-consistent equation for the distribution function. The fruitfulness of the latter in solving problems concerning plasma oscillations was first shown by A. A. Vlasov.[8]

In the kinetic equation, the initial distribution f_1 is completely determined by the value of this function at later moments of time. On the other hand, we may see from the chain of equations for the distribution functions that the form of the distribution function f_1 is determined not only by its own initial value, but also by the initial values of all the higher distribution functions f_2, ..., f_N, or by the initial values of all the correlation functions. Hence it follows that the solutions of the kinetic equations are particular solutions of the chain of equations for the distribution functions, for which the form of the correlation functions is completely determined by the values of the first-order distribution function f_1 alone. It is precisely the study of such solutions which permits us to arrive at the kinetic equations, starting with the chain of equations. Such an approach to the solution of this problem was developed in the well-known monograph of N. N. Bogolyubov. Even if we limit ourselves to the indicated class of solutions of the chain for the distribution functions, the problem of deriving the kinetic equations still remains too complex. Hence, the chain of equations is commonly terminated by approximation of the higher distribution functions by the lower ones. Of course, the nature of the approximation to be made is determined by the set of problems which the kinetic equations are intended to describe and by the properties of the system being studied.

We shall assume that the system being studied may be divided into two sub-systems, one of which is near to the equilibrium state, while the other is in a non-equilibrium state. As a result of the interaction of these sub-systems over a certain period of time τ, the relaxation time, an equilibrium state is established in the system as a whole. The causes leading to the establishment of equilibrium are different in different cases.

In a plasma, the processes establishing the equilibrium state have several causes. If the concentration of neutral molecules is small, the relaxation time is determined by the collisions of charged particles and by the process of excitation of random plasma oscillations. In this case, the corresponding kinetic equation for the plasma is the Fokker-Planck equation in momentum space.[9-11] Such an equation has been derived by L. D. Landau from the Boltzmann equation, taking into account only collisions. Because of the slow decline of Coulomb forces with distance, this equation contains divergent integrals. In order to obtain finite expressions, the limits of integration are terminated at large and at small distances. In order to derive the kinetic equation in the case of the spatially-inhomogeneous distribution of non-equilibrium particles, one may use the method of N. N. Bogolyubov.[6]

In reference 9, the Fokker-Planck equation for a plasma was derived by an approximate solution of the chain of equations for the distribution functions. Due to the fact that the Debye correlation was taken into account in the coefficients, the divergence at large distances in equation derived in this way was eliminated.

A more general kinetic equation was derived in reference 11, taking into account also the process of excitation of longitudinal oscillations. For an inhomogeneous distribution of charged particles, the equation for the electrons of the plasma may be written in the following form:

$$\frac{\partial f_1}{\partial t}(\mathbf{p},\ t)=\sum_{\alpha,\beta}\frac{\partial}{\partial p_\alpha}B_{\alpha\beta}\frac{\partial}{\partial p_\beta}f_1+\frac{\partial}{\partial \mathbf{p}}(\mathbf{A}f_1),\qquad \alpha,\beta=1,\ 2,\ 3. \qquad (2.3)$$

Here, $B_{\alpha\beta}$ and $\mathbf{A}$ are, respectively, the diffusion coefficients and the systematic friction. Each of these may be resolved into two parts, corresponding to the processes of collision and excitation of longitudinal oscillations

$$B_{\alpha\beta} = B^{(col)}_{\alpha\beta} + B^{(osc)}_{\alpha\beta}; \quad \mathbf{A} = \mathbf{A}^{(col)} + \mathbf{A}^{(osc)}.$$

The expressions for $B^{(col)}$ and $\mathbf{A}^{(col)}$ were derived in references 10 and 9 for large and small values of the energies of the charged particles. The expressions for $B^{(osc)}$ and $\mathbf{A}^{(osc)}$ have the following form: [11]

$$B^{(osc)}_{\alpha\beta} = \frac{e^2\varkappa T}{2\pi}\int \delta\left(\omega_L - \frac{\mathbf{kp}}{m}\right) a_{\mathbf{k}\alpha}\, a_{\mathbf{k}\beta}\, d\mathbf{k}, \quad (2.4)$$

$$\mathbf{A}^{(osc)} = \frac{e^2}{2\pi m}\int \delta\left(\omega_L - \frac{\mathbf{kp}}{m}\right) \mathbf{a}_{\mathbf{k}}(\mathbf{a}_{\mathbf{k}}\mathbf{p})\, d\mathbf{k}. \quad (2.5)$$

Here T is the temperature of the electrons, K is the Boltzmann constant, $\omega_L = \sqrt{4\pi e^2 n/m}$ is the Langmuir frequency for the electrons of the plasma, and $a_{\mathbf{k}\alpha}$ is the component of a unit vector in the direction of the wave vector. It follows from the expressions (2, 4) and (2, 5) that the coefficients $B^{(osc)}_{\alpha\beta}$ and $\mathbf{A}^{(osc)}$ differ from zero only when the condition is fulfilled for Cerenkov radiation of longitudinal plasma waves, $p/m > \omega_L/k$. Hence, as the result of the deceleration of an electron with momentum $\mathbf{p}$, only plasma waves with wave number $k > \omega_L m/p$ may be excited. Since the maximum value of the wave number is determined by the value of the Debye radius $r_d = \sqrt{KT/4\pi e^2 n}$, the deceleration of electrons due to radiation of longitudinal waves is possible only under the condition that their velocity is greater than the thermal velocity.

We may integrate over the wave number in Eqs. (2, 4) and (2, 5). In Eq. (2, 4), only the terms with $\alpha = \beta$ differ from zero. If, in calculating the integrals the polar axis is taken parallel to the direction of motion of the particles, then we arrive at the following expressions [11] for the diffusion coefficients:

$$B^{(osc)}_{33} = \frac{e^2\varkappa T}{v^3}\omega_L^2 \ln\frac{v}{v_T}; \qquad B^{(col)}_{11} = B^{(osc)}_{21} = \frac{me^2\omega_L^2}{4v} \quad \text{for} \quad v \gg v_T. \quad (2.6)$$

Here v_T is the mean thermal velocity; $v = p/m$. Equation (2, 5) determines the decelerating force F acting on a charged particle due to the radiation of longitudinal waves. By integration, we arrive at the following expression:

$$F^{(osc)} = \frac{e^2\omega_L^2}{v^2}\ln\frac{v}{v_T}. \quad (2.7)$$

Expressions (2, 6) and (2, 7) agree with the corresponding expressions obtained by a number of authors by different methods. The decelerating force $F^{(osc)}$ is of the same order of magnitude as $F^{(col)}$ 10, 9

$$F^{(col)} = \frac{e^2\omega_L^2}{v^2}\ln\frac{r_d}{a}; \qquad a = \frac{e^2}{mv^2}. \quad (2.8)$$

The kinetic equation (2, 3) was derived in reference 11 by approximate solution of the chain of equations for the classical distribution functions of the electrons and the plasma oscillations. Such a chain of equations contains two equations for the first-order distribution functions $f_1(\mathbf{q}, \mathbf{p}, t)$ and $F_1(Q_{\mathbf{k}}, P_{\mathbf{k}}, t)$, the former determining the probabilities of various states of the electron, and the latter determining the probabilities of various values of the coordinate and the momentum of the plasma oscillation having the wave vector $\mathbf{k}$. The second-order distribution function $\phi_2(\mathbf{q}, \mathbf{p}, Q_{\mathbf{k}}, P_{\mathbf{k}}, t)$ enters into the equations for these functions. The third-order distribution function enters into the equation for F_2, etc. The approximation of the higher functions by the lower ones consists in assuming that the third-order distribution functions are equal to the product of the first-order distribution functions, while the second-order functions are expressed in the forms

$$f_2 = f_1 f_1 + G(\mathbf{q}, \mathbf{p}, \mathbf{q}', \mathbf{p}', t), \quad (2.9)$$

$$\Phi_2 = f_1 F_1 + g(\mathbf{q}, \mathbf{p}, Q_{\mathbf{k}}, P_{\mathbf{k}}, t). \quad (2.10)$$

Thus, only pair correlations are taken into account. Eq. (2, 3) describes the process of establishment of the equilibrium state in momentum space (the spatial distribution being uniform); in deriving this equation from the system of equations for the functions f_1 and F_1, the first term in Eq. (2, 9) is not essential. Thus, the phenomenon of establishment of equilibrium in this case is due to the correlation between the values of the coordinates and momenta of the electron and those of the plasma oscillations. In addition, in deriving Eq. (2, 3) from the system of equations for the functions f_1, F_1, it is essential to assume that the plasma oscillations and the plasma particles surrounding the injected particle occur at the initial moment in a state of thermal equilibrium.

In another limiting case, we may find the kinetic equation for the distribution function F_1 describing the process of establishment of the equilibrium state of the plasma oscillations. [11] This equation is also a Fokker-Planck equation in the phase space of the coordinates and momenta of the plasma oscillations. We shall give here an equation derived from it for the mean value of the coordinate of a plasma oscillator with the wave vector $\mathbf{k}$:

$$\left.\begin{aligned}&\ddot{\overline{Q}}_{\mathbf{k}}+2\gamma_{\mathbf{k}}\dot{\overline{Q}}_{\mathbf{k}}+\omega_{\mathbf{k}}^2\overline{Q}_{\mathbf{k}}=0,\\&\overline{Q}_{\mathbf{k}}=\int Q_{\mathbf{k}}F_1\,dQ_{\mathbf{k}}\,dP_{\mathbf{k}}.\end{aligned}\right\}\qquad(2.11)$$

Here, $\omega_{\mathbf{k}}^2 = \omega_L^2 + (3\kappa T/m)\,k^2$ is the square of the frequency of the plasma oscillation with the wave vector k, and $\gamma_k = \sqrt{\pi/8}\,(\omega_L/r_d^3k^3)\,\exp\{-\tfrac{1}{2}r_d^2k^2\}$ is the damping factor of the plasma oscillations, the expression for which was first derived in a paper by L. D. Landau.[20] By using the corresponding chain of equations for the quantum distribution functions of the electrons and the plasma oscillations, we may derive the quantum kinetic equations describing the process of establishment of the equilibrium state in the plasma with quantum effects taken into account. Such an equation has been studied in reference 13, with collisions between charged particles alone being taken into account. The approximation of the higher distribution functions by the lower ones applied in this paper is analogous to that given in the paper of N. N. Bogolyubov and K. P. Gurov.[7]

If the system of charged particles being studied is located in the vicinity of a dielectric or in a decelerating system, the kinetic equation will contain terms describing the processes of deceleration and diffusion due to the radiation of electromagnetic waves.[14] The state of the system being studied is characterized in this case by the distribution function of the coordinates and momenta of the charged particles and the coordinates and momenta of the electromagnetic-field oscillators with various wave numbers. If the initial distribution of the coordinates and momenta of the oscillators is an equilibrium distribution, while the state of the electrons is near to the equilibrium state, then a kinetic equation of the Fokker-Planck type may also be derived for the distribution function of the electrons. However, now terms $B_{\alpha\beta}^{(\mathrm{rad})}$ and $A^{(\mathrm{rad})}$ appear in the diffusion coefficients and the systematic friction, due to the radiation of electromagnetic waves. The expressions for these coincide with the expressions (8) and (9), the only difference being that the plasma frequency ω_L is replaced by $ck/\sqrt{\epsilon} = \omega_k$, and now $a_k \perp k$. The coefficient $A^{(\mathrm{rad})}$ determines the decelerating force in the direction of the field exerted on a particle with momentum p. The expression for the decelerating force has the form

$$F^{(\mathrm{rad})}=\left(\frac{e}{c}\right)^2\int\left(1-\frac{c^2}{\varepsilon v^2}\right)\omega_k\,d\omega_k$$

and coincides with the well-known expression from the theory of Cerenkov radiation. With the converse assumptions, in which the electrons are in an equilibrium state at the initial moment, while the state of the field oscillators is near to the equilibrium state, a kinetic equation may be derived[16] for the distribution function of the field oscillators. The mean values obtained by means of this equation for the coordinates of the electromagnetic oscillators have the form:

$$\ddot{\overline{Q}}_{\mathbf{k}}+2\gamma_{\mathbf{k}}\dot{\overline{Q}}_{\mathbf{k}}+\Omega_{\mathbf{k}}^2\overline{Q}_{\mathbf{k}}=0,\qquad(2.12)$$

$$\Omega_k^2=\omega_L^2+\frac{c^2k^2}{\varepsilon};\qquad \gamma_k=\left(\frac{\pi m}{8\kappa T}\right)^{1/2}\frac{\omega_L^2}{k}\exp\left(-\frac{mc^2}{2\varepsilon\kappa T}\right).\qquad(2.13)$$

The expression given for γ_k is valid for $\Omega_k/k < c$. In the contrary case, $\gamma_k = 0$.

Thus, by an approximate solution of the chain of equations for the distribution functions, we may derive the kinetic equations describing the processes of establishment of statistical equilibrium in the plasma.

It follows from what has been stated that, when the spatial distribution is uniform, the kinetic equation for the electrons is a Fokker-Planck equation describing the process of establishment of the equilibrium distribution in momentum space. In order to derive this equation, we must at least take into account the correlation of pairs of charged particles. The first term on the right-hand side of Eq. (2, 9) turns out in this case to be inessential. The opposite situation may arise when the distribution is inhomogeneous. Namely, if the dimensions of the inhomogeneities are much smaller than the relaxation distance of the processes discussed above, then in a number of cases it may turn out not to be essential to take these processes into account. Hence, under these conditions, the first term in Eq. (2, 9) will now be the fundamental one, and we may terminate the chain of equations for the distribution functions of the electrons with the first equation by assuming

$$f_2(\mathbf{q},\mathbf{p},\mathbf{q}',\mathbf{p}',t)=f_1(\mathbf{q},\mathbf{p},t)\,f_1(\mathbf{q}',\mathbf{p}',t),\qquad(2.14)$$

thus arriving at the kinetic equation for the self-consistent field. Thus, if Eq. (2, 14) is substituted into the first equation of the chain for the quantum distribution function (2, 2), we obtain the quantum kinetic equation for the self-consistent field. This equation goes over, as $\hbar \to 0$, into the well-known classical equation for the self-consistent field. The self-consistent equa-

tion for the quantum distribution function may be made more accurate if, in the approximation of the second-order distribution function in terms of the first-order function, we take into account the correlation due to the identity of the particles. In order to do this, we shall use the following expression in place of Eq.(2, 14):

$$f_2(\mathbf{q}_1, \mathbf{q}_2, \mathbf{p}_1, \mathbf{p}_2, t) = f_1(\mathbf{q}_1, \mathbf{p}_1, t) f_1(\mathbf{q}_2, \mathbf{p}_2, t) \mp \int \varrho\left(\mathbf{q}_1 + \frac{1}{2}\hbar\tau_1, \mathbf{q}_2 - \frac{1}{2}\hbar\tau_2\right) \times \varrho\left(\mathbf{q}_2 + \frac{1}{2}\hbar\tau_2, \mathbf{q}_1 - \frac{1}{2}\hbar\tau_1\right) e^{-i\tau_1\mathbf{p}_1 - i\tau_2\mathbf{p}_2}\, d\tau_1\, d\tau_2, \quad (2, 15)$$

which may be derived from the corresponding expression for the density matrix in the coordinate representation. In Eq.(2, 15), the minus sign refers to systems obeying Fermi-Dirac statistics, while the plus sign refers to those obeying Bose-Einstein statistics. In the intermediate case, in which the dimensions of the spatial inhomogeneities are comparable with the relaxation distance, a more complex kinetic equation is obtained, which is a generalization of the Fokker-Planck equation taking the self-consistent interaction into account. If the relaxation distance or time τ is known, then in many cases we may use a simplified kinetic equation, in which the collision integral or the terms describing diffusion and systematic friction are replaced by the term

$$-\frac{1}{\tau}(f - f_0), \quad (2, 16)$$

where f_0 is the equilibrium distribution function.

The kinetic equations for the plasma given here may be used to derive hydrodynamic equations which take relaxation processes into account.

3. SPECTRA OF COLLECTIVE OSCILLATIONS IN THE SELF-CONSISTENT-FIELD APPROXIMATION

The method of the self-consistent field[8] has been widely applied in the theory of the classical plasma. By means of this method, as is known, it is especially simple to study problems associated with collective effects in systems of charged particles. We might say that the application of the self-consistent-field approximation in the classical theory of the plasma was a transfer of the quantum method of the self-consistent field of Hartree[15] into the classical field. Hence, the expediency of using such a method in the theory of the quantum plasma is quite obvious. We shall present some results with which we are familiar in this field.*

In the self-consistent field approximation, the wave function is assumed to have the form of the product of the corresponding functions of the separate particles. Hence, likewise, the density matrix and, correspondingly, the quantum distribution function are also so assumed. In this approximation, the correlation of the separate particles is neglected. On the other hand, one may directly derive thereby from the Schrödinger equation for a system of many particles the following equation for the single-particle distribution function $f(\mathbf{q}, \mathbf{p}, t)$:[3]

Here it is assumed that the particles interact by a central force law, and $U(|\mathbf{q}|)$ is the energy of pair interaction,

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{m}\frac{\partial f}{\partial \mathbf{q}} + \frac{i}{(2\pi)^3\hbar}\int\left[U\left(\left|\mathbf{q} - \mathbf{q}' + \frac{1}{2}\hbar\tau\right|\right) - U\left(\left|\mathbf{q} - \mathbf{q}' - \frac{1}{2}\hbar\tau\right|\right)\right] \times f(\mathbf{q}, \eta, t) f(\mathbf{q}', \mathbf{p}', t)\, e^{i\tau(\eta - \mathbf{p})}\, d\tau\, d\eta\, d\mathbf{q}'\, d\mathbf{p}' = 0. \quad (3, 1)$$

which is equal in the case of electrons to $e^2/|\mathbf{q}|$ (here, in general, we must take into account the presence of the compensating charge of the ions).

In the case in which $\hbar$ may be considered to be a small quantity, i. e., when the inhomogeneities are macroscopic, or in the case of oscillations not of microscopic wavelength, Eq. (3, 1) takes the form:

This equation is the well-known kinetic equation of the self-consistent approximation, which has been widely applied in the description of the plasma, and was ap-

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{m}\frac{\partial f}{\partial \mathbf{q}} - \frac{\partial f}{\partial \mathbf{p}}\frac{\partial}{\partial \mathbf{q}}\int U(|q - q'|) f(\mathbf{q}', \mathbf{p}', t)\, d\mathbf{q}'\, d\mathbf{p}' = 0. \quad (3, 2)$$

parently first proposed by Vlasov.[8]

We shall base the discussion of collective oscillations in this section on Eqs. (3, 1) and (3, 2). Considering the collective oscillations as weakly-excited states, we shall assume that the distribution function f is only

*The results presented below are essentially connected with the taking into account of the influence of the motion of particles on their self-consistent field. We must note that these effects, at least as to order of magnitude, may also be studied in the hydrodynamic approximation of Bloch.[16, 17]

slightly different from the equilibrium homogeneous spatial distribution $f_0(\mathbf{p})$** (we shall neglect surface effects, considering the system to be infinite, as is possible in case that the wavelength of the vibrations is small in comparison with the dimensions of the body).

Then, we assume a dependence on the coordinates of the form $e^{i\mathbf{kq}}$, as is natural in the study of wave motion in an infinite medium. Thus we derive from (3, 1) an equation for the small non-equilibrium correction to the distribution function $\phi_{\mathbf{k}}(\mathbf{p})\exp\{i\mathbf{qk}\}$ (here terms in ϕ^2 are omitted)

$$\frac{\partial\varphi}{\partial t}+i\frac{\mathbf{kp}}{m}\varphi+\frac{i}{\hbar}\nu(\mathbf{k})\left[f_0\left(\mathbf{p}+\frac{\hbar\mathbf{k}}{2}\right)-f_0\left(\mathbf{p}-\frac{\hbar\mathbf{k}}{2}\right)\right]\int\varphi_{\mathbf{k}}(\mathbf{p}')\,d\mathbf{p}'=0, \tag{3.3}$$

where

$$\nu(\mathbf{k})=\int U(\mathbf{q})\,e^{i\,\mathbf{kq}}\,d\mathbf{q}.$$

In view of the fact that the term in (3, 3) due to the interaction of particles is proportional to the number density of particles $\int\phi\,d\mathbf{p}'$, it is clear that the spectrum of the collective oscillations derived below from (3, 3) will correspond to the density oscillations, or longitudinal waves.

It is not nearly always that the collective oscillations turn out to be undamped. Naturally, this means that we may speak of the collective oscillations as new degrees of freedom of the system only in case that the damping is sufficiently small. In connection with this, it is convenient[20] in the solution of Eq. (3, 3) to consider the initial solution and to study the possibility of obtaining solutions which have the asymptotic form $\exp\{-\gamma t-i\omega t\}$ for long periods of time. Using the Laplace-Mellin transformation, we derive from (3, 3)

$$\varrho_k(t)=\frac{\frac{1}{(2\pi i)}\int\limits_{-i\infty+\sigma}^{i\infty+\sigma}ds e^{st}\int\frac{\varphi_{\mathbf{k}}(0,\,\mathbf{p})}{s+i\mathbf{kp}/m}\,dp}{\left[1-\frac{\nu(\mathbf{k})}{\hbar}\int\frac{f_0\left(\mathbf{p}+\frac{\hbar\mathbf{k}}{2}\right)-f_0\left(\mathbf{p}-\frac{\hbar\mathbf{k}}{2}\right)}{(\mathbf{kp}/m)+s/i}\,d\mathbf{p}\right]}. \tag{3.4}$$

Here, $\phi_{\mathbf{k}}(0,\,\mathbf{p})$ is the value of the non-equilibrium correction to the distribution function at the initial instant of time.

We may easily convince ourselves that the value of the frequency ($\omega = -\operatorname{Im} s$) and the damping factor $\gamma = -\operatorname{Re} s$ are determined by the poles of the integrand of (3, 4), i. e., they may be found from the following dispersion equation:[3]

$$1=\frac{\nu(\mathbf{k})}{\hbar}\int\limits_{-U-}\frac{f_0\left(\mathbf{p}+\frac{\hbar\mathbf{k}}{2}\right)-f_0\left(\mathbf{p}-\frac{\hbar\mathbf{k}}{2}\right)}{(\mathbf{kp}/m)-\omega+i\gamma}\,d\mathbf{p}. \tag{3.5}$$

We note that, in integrating (3, 5) over the component of the momentum parallel to $\mathbf{k}$, we must pass below the pole $(\mathbf{kp}/m)=\omega-i\gamma$.

In the case of electrons, for which $\gamma(\mathbf{k})=4\pi e^2/k^2$, Eq. (3, 5) has been derived by the method given here in reference 3 and has been studied in reference 21 and 22.

We note that, for electrons, Eq. (3, 5) may be rewritten in the form

$$1=\frac{4\pi e^2}{m}\int\frac{f_0(\mathbf{p})\,d\mathbf{p}}{(\omega-\mathbf{kp}/m)^2-(\hbar k^2/2m)^2}. \tag{3.5}$$

Such a dispersion equation was later derived by another method by Bohm and Pines.[23] An analogous method of collective variables has been used by Zubarev[24] as applied to an electron gas. Finally, Eq. (3, 5) has also been derived by the method of the self-consistent field by Zyryanov and Eleonskiĭ[25] and by Ferrell.[26]

In the case in which the momentum of the collective excitation state is small in comparison with the root-mean-square momentum of the equilibrium state, we may go over from Eq. (3, 5) to the classical limit. Here we have

$$1=\nu(k)\int\limits_{-U-}\frac{\mathbf{k}\frac{\partial f_0}{\partial\mathbf{p}}}{(\mathbf{kp}/m)-\omega+i\gamma}\,d\mathbf{p}. \tag{3.6}$$

Such an oscillation spectrum in the classical theory of the electron plasma has been studied in the references 8, 20, and 27. Here, it turned out[20] that in the region of wavelengths small or comparable with the Debye screening distance $r_d=\sqrt{\kappa T/4\pi e^2 n}$ (n being the number of electrons per cm^3, T the temperature, and κ the Boltzmann constant), the plasma oscillations of charge density, or as is obviously the same, the oscillations of the longitudinal electromagnetic

**Self-consistent oscillations of electrons occurring in the periodic field of a lattice have been studied in the papers of Zyryanov[18] and Feĭnberg.[19]

field, are strongly damped. On the contrary, in the region of long wavelengths ($\lambda >> r_d$), the damping is small, and

$$\omega^2 = \omega_L^2 + \frac{3\varkappa T}{m} k^2; \quad \gamma = \omega \sqrt{\frac{\pi}{8}} \frac{\exp\left(-\frac{1}{2} r_d^2 k^2\right)}{r_d^3 k^3},$$

where the so-called plasma frequency $\omega_L = \sqrt{4\pi e^2 n/m}$.

There is another case in which it is also possible to go over to the classical limit; this occurs, for example, in the case of the degenerate Bose-Einstein gas. This case is possible under the condition that the energy $\hbar\omega$ of the collective oscillation is large in comparison with the corresponding energy $\hbar^2 k^2/2m$ of the separate particle. In particular, this may be seen from the spectrum of density oscillations of a degenerate Bose-Einstein gas [$f_0 = n\delta(p)$] derived from (3, 5), and having the form[3]

$$\omega^2 = \frac{\nu(k)\, n}{m} k^2 + \frac{\hbar^2 k^4}{4m^2}.$$

Such a spectrum was first derived by Bogolyubov[28] in the theory of superfluidity (see also reference 29).

We shall now take up the study of the collective oscillations of a degenerate electron gas. A problem of this type for long wavelengths, for which $\hbar k$ is much smaller than the Fermi limiting momentum $p_0 = (3\pi)^{1/3} n^{1/3} \hbar$, was first studied by Gol'dman.[30] In our study, not assuming $\hbar k$ to be small in comparison with p_0, we have derived from (3, 5) the following:[21, 22]

$$\left.\begin{aligned} 1 - \frac{3}{2}\frac{m^2\omega_L^2}{p_0^2 k^2}\Bigg(-1 + \frac{1}{2\hbar k p_0}\Bigg\{\Bigg[\left(m\frac{\omega - i\gamma}{k} + \frac{\hbar k}{2}\right)^2 - \\ - p_0^2\Bigg]\ln\frac{\omega - i\gamma + \hbar k^2/2m + k p_0/m}{\omega - i\gamma + \hbar k^2/2m - k p_0/m} - \\ - \Bigg[\left(m\frac{\omega - i\gamma}{k} - \frac{\hbar k}{2}\right)^2 - p_0^2\Bigg]\ln\frac{\omega - i\gamma - \hbar k^2/2m + k p_0/m}{\omega - i\gamma - \hbar k^2/2m - k p_0/m}\Bigg\} + \\ + 2\pi i \frac{m}{k p_0} F\Bigg) = 0, \\ F = 0, \quad \left(\omega \mp \frac{\hbar k^2}{2m}\right)^2 - \gamma^2 > \left(\frac{p_0 k}{m}\right)^2, \\ F = (-\omega + i\gamma), \quad \left(\omega \mp \frac{\hbar k^2}{2m}\right)^2 - \gamma^2 < \left(\frac{p_0 k}{m}\right)^2, \\ F = \pm \frac{(p_0 k/m)^2 - (\omega - i\gamma + \hbar k^2/2m)^2}{2\hbar k^2/m}, \\ \left(\omega \mp \frac{\hbar k^2}{2m}\right)^2 - \gamma^2 < \left(\frac{p_0 k}{m}\right)^2 < \left(\omega \mp \frac{\hbar k^2}{2m}\right)^2 - \gamma^2. \end{aligned}\right\} \quad (3.7)$$

This dispersion equation has a rather unwieldy appearance. Before we proceed to the study of some simple limiting cases, we shall make the following remarks. Oscillations of an electron gas will be undamped only in case that $F = 0$. As may easily be seen, this takes place in the region of sufficiently large ω, or of sufficiently long wavelengths. Hence, it is only in this region that we may speak of longitudinal oscillations of an electron gas as being a new degree of freedom. In the long-wavelength region, we obtain from Eq. (3, 7)[3]

$$\omega^2 = \omega_L^2 + \frac{3}{5}\frac{p_0^2 k^2}{m^2} + \left(\frac{\hbar k^2}{2m}\right)^2. \quad (3.8)$$

Here we consider that $k << k_c = m\omega_L/p_0$ and that $\omega_L >> \hbar k^2/2m$. Thus, in this region the spectrum of longitudinal oscillations is similar to the oscillation spectrum of a high-temperature plasma, with the distinction that in the present case the chaotic motions of the electrons are due to the Fermi energy, rather than to the temperature. In addition, Eq. (3, 8) takes quantum effects into account. As the wavelength decreases, the spectrum of longitudinal oscillations goes over into the single-particle spectrum. Thus, under the condition that $\hbar^2 k^2/2m << 4\pi e^2 n/k^2 << p_0^2/2m$, we have

$$\hbar\omega = \frac{(p_0 + \hbar k)^2 - p_0^2}{2m}\left\{1 + \exp\left[-\frac{2}{3}\frac{p_0^2 k^2}{\omega_L^2} - 2\right]\right\} \quad (3.9)$$

Within the accuracy of an exponentially small term, this expression coincides with the excitation energy of free electrons raised upon excitation above the Fermi surface.

We note that, for forces with a finite radius of action [$\nu(0)$ is finite], we have in place of Eq. (3, 8):[3]

$$\omega^2 = \left(\frac{\nu(0)\, n}{m} + \frac{3}{5}\frac{p_0^2}{m}\right)k^2 + \left(\frac{\hbar k^2}{2m}\right)^2. \quad (3.10)$$

Such a spectrum is similar to the spectrum of a Bose-Einstein gas, and corresponds to phonon oscillations. We must emphasize the distinction of these oscillations*, as well as of plasma oscillations, from ordinary sound in a gas. In the latter case, the particles interact only at the moment of collision. Hence, as the mean free path of the particles increases, the particles practically do not interact within the period of the oscillation, and consequently the damping becomes very great, and the oscillations cannot be propagated. This is precisely the picture which must hold true in a completely degenerate system of particles obeying Fermi-Dirac statistics and interacting only at the moment of collision, since at a temperature of absolute zero the mean free path in such a system becomes infinite. On the contrary, in the present case, the oscillations of self-consistent sound are propagated under conditions in which collisions are inessential, and the excitations which result here are due to the presence of self-consistent interactions between the particles. As a result, the particles by no means interact at the moment of collision alone, but on the contrary, are continually moving in the general self-consistent field. We must note that, as may be seen from the material to be presented in the next section, it is important to take into account exchange interaction in the spectrum of zero sound. While the latter does not change the physical picture, it alters the numerical magnitude of the velocity of self-consistent zero sound.

In a system of charged particles, the Hartree approximation also permits us to derive the spectrum of the transverse oscillations. The classical approximation** leads to a spectrum of the transverse oscillations having the form

$$\omega^2 = \omega_L^2 + [c^2 + \overline{(pk)^2}/m^2k^2]\, k^2, \qquad (3, 11)$$

where c is the velocity of light, and the bar denotes averaging over the equilibrium state. For a degenerate electron gas, $(\overline{\mathbf{pk}})^2 = (1/5)\, p_0^2 k^2$.

In our nonrelativistic treatment, the velocity of the particles is small in comparison with the speed of light. Hence, the influence of the motion of the particles in Eq. (3, 11) may be neglected.

The spectrum of the transverse oscillations permits us to write immediately an expression for the dielectric constant of the electron gas, taking into account the definition of the dielectric constant, $\epsilon = c^2k^2/\omega^2$. We shall call such a dielectric constant the transverse dielectric constant[32] (see Sec. 6). The point is that in taking into account the motion of the particles, as may be seen from (3, 11), ϵ^{tr} depends not only on ω, but also on k. The role of the longitudinal dielectric constant ϵ^l is played by the expression on the left-hand side of Eq. (3, 7), which is the condition for existence of longitudinal waves, which are permitted only when the dielectric constant vanishes. We may easily convince ourselves of this by taking into account the fact that the energy of interaction of the charged particles entering into Eq. (3, 1) is determined by the scalar potential of the electric field. Hence, on expressing the charge density in terms of this interaction energy by means of (3, 1), we may easily find $\epsilon^l(\omega, k)$, which relates the longitudinal components of the electric induction and electric field intensity.

The results formulated above for the electron gas remain valid for the case of a system of electrons and ions. However, here there arises an additional excitation branch, that of acoustic excitations. The velocity of these excitation states may easily be determined by calculating the compressibility. Actually, for example, in

*Following Landau,[31] we refer to these vibrations as zero sound, or as self-consistent sound.

**In the classical limit,[8] the self-consistent-field approximation for charged particles, as is known, corresponds to a system of Maxwell equations in which the current density and the charge density are determined by means of a distribution function obeying the equation (see, e. g., reference 33.)

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{m}\frac{\partial f}{\partial \mathbf{q}} + e\left(\mathbf{E} + \frac{1}{c}[\mathbf{vH}]\right)\frac{\partial f}{\partial \mathbf{p}} = 0. \qquad (3, 2')$$

The quantum treatment of the transverse field oscillations leads to a dispersion equation of the oscillations having the following form:[3]

$$1 = \frac{4\pi e^2(\omega - i\gamma)}{m[(\omega - i\gamma)^2 - c^2k^2]} \int \frac{d\mathbf{p}\,(\mathbf{pe}_k)}{\omega - i\gamma - \mathbf{pk}/m}\left\{\mathbf{e}_k\frac{\partial f_0}{\partial \mathbf{p}} - \frac{(\mathbf{pe}_k)}{m\omega}\left[\mathbf{k}\frac{\partial f_0}{\partial \mathbf{p}} - \frac{1}{\hbar}\left(f_0\left(\mathbf{p} + \frac{\hbar\mathbf{k}}{2}\right) - f_0\left(\mathbf{p} - \frac{\hbar\mathbf{k}}{2}\right)\right)\right]\right\},$$

where $\mathbf{e}_k$ is a unit vector perpendicular to the wave vector $\mathbf{k}$. Here, the dielectric constant, which determines the propagation of the transverse waves, has the form

$$\varepsilon^{tr} = 1 + \frac{4\pi e^2}{m(\omega - i\gamma)} \int \frac{d\mathbf{p}\,(\mathbf{pe}_k)}{\omega - i\gamma - \mathbf{pk}/m}\left\{\mathbf{e}_k\frac{\partial f_0}{\partial \mathbf{p}} - \frac{(\mathbf{pe}_k)}{m\omega}\left[\mathbf{k}\frac{\partial f_0}{\partial \mathbf{p}} - \frac{1}{\hbar}\left(f_0\left(\mathbf{p} + \frac{\hbar\mathbf{k}}{2}\right) - f_0\left(\mathbf{p} - \frac{\hbar\mathbf{k}}{2}\right)\right)\right]\right\}$$

the case of a degenerate gas, the pressure will be determined by the electrons, and $P \approx [(3\pi^2)^{2/3}/5]\, h^2 n_{0e}^{5/3}/m$, while the density will be determined by the ions: $\rho \approx m_i n_{0i} = m_i n_{0e}/Z$. Then

$$U^2 = \frac{\partial P}{\partial \varrho} = \frac{Z\hbar^2 (3\pi)^{2/3} n_{0e}^{2/3}}{3 m_i m_e} = \frac{Z p_{0e}^2}{3 m_i m_e}. \qquad (3, 12)$$

Here, U is the velocity of sound, p_{0e} is the limiting momentum for electrons, m_i and m_e are the masses of an ion and an electron, and Z is the number of electrons per ion. The velocity of sound (3, 12) has been derived in reference 34 (see also reference 35) and in reference 36 (see also reference 22) from an analysis of the collective movements of the plasma. Here, the problem was also studied in reference 36 of the damping of sound waves in an electron-ion plasma. We note that the concepts applied above by us to Eq. (3, 12) are often also applied to the high-temperature plasma (see reference 37), whereby a value of the velocity of sound waves of the order of magnitude of $\sqrt{\kappa T/m_i}$ is obtained. However, such waves are actually not observed in a high-temperature plasma. This may be associated with the strong damping of these waves observed in reference 36. We shall give below some results of this work. We shall consider a system of two equations of the type of (3, 2) [for example, in the form (3, 2′)] for electrons and ions, respectively, and shall solve the problem for small deviations from equilibrium, as was done above in the study of Eq. (3, 2). It is not difficult to find thus the dispersion equation for the oscillations in the system of electrons and ions

$$1 = \frac{4\pi e^2}{k^2} \int \frac{\mathbf{k}\frac{\partial f_{0e}}{\partial \mathbf{p}}}{\mathbf{pk}/m - \omega + i\gamma}\, d\mathbf{p} + \frac{4\pi e^2 Z^2}{k^2} \int \frac{\mathbf{k}\frac{\partial f_{0i}}{\partial \mathbf{p}}}{\mathbf{pk}/m - \omega + i\gamma}\, d\mathbf{p}. \qquad (3, 13)$$

Here, Z is the number of electrons per ion, and f_{0e} and f_{0i} are, respectively, the equilibrium distribution functions of the electrons and the ions.

In the region of long wavelengths, large in comparison with the screening distance of the Coulomb field (r_d or r_c, respectively, for the cases of high temperatures and of a degenerate electron gas), we may neglect the term unity on the left-hand side of Eq. (3, 13). This leads to the condition that ω and γ become proportional to the wave number k. In the case of the degenerate electron gas, the analysis of Eq. (3, 13) is essentially simplified. This is so, first, because the velocity of sound is small in comparison with the velocity of the electrons (this is also true at high temperatures), and second, because the velocity of sound turns out to be large in comparison with the velocity of the ions. As a result, we obtain

$$\omega = \sqrt{\frac{Z}{3}\frac{m_e}{m_i}}\frac{P_{0e}}{m_e} k; \qquad \gamma = \frac{\pi Z}{12}\frac{P_{0e} k}{m}. \qquad (3, 14)$$

The expression derived for the velocity of sound, naturally, agrees with that determined from the compressibility [see (3, 12)]. The latter, as we must note, gives the correct order of magnitude for the velocity of sound in metals. The damping factor γ turns out to be small in comparison with ω; this permits the propagation of sound to great distances. The damping, as determined by Eq. (3, 14) corresponds to the fact that the sonic oscillations of a degenerate plasma are states showing a natural width of the levels. We must indicate that in the study of the absorption of sound in metals, such a damping has recently been observed experimentally, and has been interpreted theoretically in references 38 and 39.

Finally, we point out analogous results relating to the high-temperature case, in which we can use the Maxwell distribution for the electrons and the ions. In this case, the velocity of sound is near to the thermal velocity of the ions. If we designate

$$\omega = xk\sqrt{\kappa T/m_i}; \qquad \gamma = yk\sqrt{\kappa T_i/m_i},$$

we obtain the following equation from (3, 13) for the determination of x and y:*

$$\frac{T_i}{ZT_e} + 1 + \frac{x - iy}{\sqrt{2\pi}} \int_{-U}^{U} \frac{d\xi}{\xi - x + iy} e^{-\xi^2/2} = 0. \qquad (3, 15)$$

Here, T_e and T_i are the temperatures of the electrons and the ions. We may easily convince ourselves that

*With regard to the integral in (3, 15), see reference 40.

solutions of this equation corresponding to the case of small damping exist only under the condition $T_i \ll ZT_e$. Otherwise, the damping is large.

4. THE INFLUENCE OF THE CORRELATION OF PARTICLES ON THE SPECTRA OF COLLECTIVE OSCILLATIONS (MICRO-TREATMENT)

There is apparently at present no microscopic theory that considers in any degree of completeness the influence of the correlation of particles on the spectrum of the collective excitations. Hence, we shall present below only certain special results which, as we consider, permit us to see the general regularities due to correlation, and may facilitate the understanding of the phenomenological approach presented in the next section.

The fundamental assumption, which essentially simplifies the taking into account of correlation, must be considered to be the statement: in the region of excitation wavelengths that are long in comparison with the correlation distance, the correlation functions of pairs of particles are practically indistinguishable from their equilibrium values. It is just this situation which occurs in the Hartree-Fock[41] approximation, and which permits us to take into account the exchange correlation of particles. We shall now begin to discuss this case.

Following Dirac,[42] we shall represent the density matrix in the form of the determinant of the density matmatrices of the individual particles:

$$\varrho_n = \begin{vmatrix} (s_1', \mathbf{r}_1' | \varrho | s_1, \mathbf{r}_1), \ldots, (s_1', \mathbf{r}_1' | \varrho | s_n, \mathbf{r}_n) \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ (s_n', \mathbf{r}_n' | \varrho | s_1, \mathbf{r}_1), \ldots, (s_n', \mathbf{r}_n' | \varrho | s_n, \mathbf{r}_n) \end{vmatrix} \cdot \quad (4,1)$$

Here, ($|\rho|$) is the density matrix of an individual particle, the state of which is characterized by its coordinate r and its spin index s. Here ρobeys the following equation of the Hartree-Fock approximation:

$$\left\{ i\hbar \frac{\partial}{\partial t} + \frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial r^2} - \frac{\partial^2}{\partial r'^2} \right) \right\} (s', \mathbf{r}' | \varrho | s, \mathbf{r}) =$$
$$= \sum_{s''} \int d\mathbf{r}'' \left[U(|\mathbf{r} - \mathbf{r}''|) - U(|\mathbf{r}' - \mathbf{r}''|) \right] \{ (s', \mathbf{r}' | \varrho | s, \mathbf{r}) (s'', \mathbf{r}'' | \varrho | s'', \mathbf{r}'') - \quad (4,2)$$
$$- (s', \mathbf{r}' | \varrho | s'', \mathbf{r}'') (s'', \mathbf{r}'' | \varrho | s, \mathbf{r}) \}.$$

It is convenient now to go over to the Wigner representation

$$f_{s's}(\mathbf{q}, \mathbf{p}) = \frac{1}{(2\pi)^3} \int \left(s', \mathbf{q} - \frac{\hbar\tau}{2} | \varrho | s, \mathbf{q} + \frac{\hbar\tau}{2} \right) e^{-i\tau\mathbf{p}} d\tau. \quad (4,3)$$

Here we may immediately write the equation for the case of small deviations from the equilibrium state

$$f_{s's}(\mathbf{q}, \mathbf{p}) = \frac{1}{2} \delta_{s's} f_0(\mathbf{p}) + \delta f_{ss'}(\mathbf{q}, \mathbf{p}) \quad (4,4)$$

(here $f_0 = [2/(2\pi\hbar)^3] \left[\exp\left(\frac{\varepsilon_0(p) - \mu}{\varkappa T} \right) + 1 \right]^{-1}$, $\varepsilon_0(p)$ is the energy of the particle, and μ is the chemical potential). We limit ourselves to the case in which the wavelength of the excited state is much greater than the wavelength of particles on the Fermi surface, agreeing in order of magnitude with the mean distance between particles. Then we have[43,44]

$$\frac{\partial \delta f}{\partial t} + \left\{ \frac{\mathbf{p}}{m} - \frac{1}{2} \mathrm{Sp}_{s'} \frac{\partial}{\partial \mathbf{p}} \int \hat{\Phi}^0(\mathbf{p}, \mathbf{p}') f_0(\mathbf{p}') \, d\mathbf{p}' \right\} \frac{\partial \delta f}{\partial \mathbf{q}} -$$
$$- \frac{1}{2} \frac{\partial f_0}{\partial \mathbf{p}} \frac{\partial}{\partial \mathbf{q}} \mathrm{Sp}_{s'} \left\{ U(|\mathbf{q} - \mathbf{q}'|) \delta f(\mathbf{q}', \mathbf{p}') \, d\mathbf{q}' \, d\mathbf{p}' - \int \hat{\Phi}^0(\mathbf{p}, \mathbf{p}') \delta f(\mathbf{q}', \mathbf{p}') \right\} = 0, \quad (4,5)$$

where

$$\hat{\Phi}^0(\mathbf{p}, \mathbf{p}') = \nu\left(\left| \frac{\mathbf{p} - \mathbf{p}'}{\hbar} \right| \right) \frac{1 + \hat{\sigma}\hat{\sigma}^1}{2},$$

$\hat{\sigma}$ being the spin operator, and $\nu(\mathbf{k}) = \int U(\mathbf{q}) e^{i\mathbf{k}\mathbf{q}} d\mathbf{q}$.

The quantity $\hat{\Phi}^0$ is the exchange component of the amplitude of forward scattering for the collision of two particles, as calculated by the Born approximation. For

forces with a small radius of action, very small in comparison with the characteristic distance of the variation δf, the presence of $U(|\mathbf{q} - \mathbf{q}'|)$ in Eq. (4, 4) leads to the appearance of the complete Born forward-scattering amplitude for two particles of spin one-half

$$-\nu(0)+\nu\left(\left|\frac{\mathbf{p}-\mathbf{p}'}{\hbar}\right|\right)\frac{1+\hat{\sigma}\hat{\sigma}'}{2}.$$

The approximate character of Eq. (4, 4) is obvious even from the fact that it gives the Born approximation for the scattering amplitude. Hence, the Hartree-Fock approximation is not accurate in the case of strong interaction of particles. Further, the derived scattering amplitude does not depend on the presence of particles other than the pair colliding. Generally speaking, it is highly essential to take into account the other particles*.[45]

We note that the second term in the first curly bracket of Eq. (4, 5) corresponds to the change in mass of the particle. In view of the fact that δf differs from zero only at the Fermi surface, we may speak of the appearance of an effective mass of the particle. As is known, the correction to $(1/m)$ in the Hartree-Fock approximation becomes infinite in the case of the Coulomb interaction of particles. However, in Eq. (4, 5), there is still another term containing $\hat{\Phi}^0$ and showing the same singularity, besides the contribution to the effective mass. If we assume here that the energy of the particle, which is the argument of the function f_0, corresponds to the energy of a free particle, then the singular terms in Eq. (4, 5) compensate each other. In such an approximation, we may easily estimate the influence of exchange effects on the spectrum of plasma oscillations of an electron gas. In the long-wavelength region, as we may easily convince ourselves, we have**[43]

$$\omega^2=\omega_L^2+\frac{3}{5}\frac{p_0^2k^2}{m^2}-\omega_L^2\frac{3}{20}\left(\frac{\hbar k}{p_0}\right)^2.$$

Hence it is clear that the influence of exchange correlation on the relation of the frequency to the wave vector may be neglected only in the case of an electron gas of high density, for which the Fermi energy $(p_0^2/2m)$ is considerably greater than the energy of the plasma oscillations $\hbar\omega_L$ (see also references 26 and 50).

In the case of forces with a finite radius of action, we take as the energy $\epsilon_0(p)$ of the particle the corresponding expression from the Hartree-Fock approximation

$$\varepsilon_0(p)=\frac{p^2}{2m}+\int\left\{\nu(0)-\frac{1}{2}\nu\left(\left|\frac{\mathbf{p}-\mathbf{p}'}{\hbar}\right|\right)\right\}f_0(\mathbf{p}')\,d\mathbf{p}'.$$

Further, in this case the possibility arises of the existtence of other excitation states besides the density oscillations.[43,48,49] An analysis of all of the discussion here is made difficult by the necessity of making definite specifications on the form of $\nu(|p - p'|/\hbar)$. If, however, we make no specifications about the form of the forward scattering amplitude, then Eq. (4, 5) will differ in no way from the corresponding equation in the theory of the Fermi liquid. We shall proceed to the study of the results of the latter theory in the next section.

5. THE INFLUENCE OF CORRELATION OF PARTICLES ON THE SPECTRA OF COLLECTIVE EXCITED STATES. A PHENOMENOLOGICAL THEORY OF THE DEGENERATE ELECTRON FERMI FLUID

Landau's theory of the Fermi fluid[51] is a direct generalization of the Hartree-Fock approximation, also neglecting the difference of the correlations from their equilibrium values. We shall give below some results obtained on the basis of such a theory. Here it is convenient to make some preliminary remarks about the use of the model of independent particles in the description of the quantum plasma which is formed by the electrons in a metal.

The electron theory of metals successfully uses for the description of many of the properties of metals the conception of electrons as being independent particles, thus considering the electrons in a metal essentially as a gas. In view of the fact that the electrons are situated in a lattice field, their properties differ greatly from those of free electrons, as characterized, in particular, by the dispersion law, i. e., the relation of the energy of the electron to its quasi-momentum*. Another reason for the difference of the electrons from a gas of free particles is associated with the interelectronic interaction, which is by no means small. This may be seen immediately from the fact that the mean energy of Coulomb interaction of the electrons in a metal agrees

*However, as we see it the exclusion law, which is based on the Pauli principle and which is realized because of the filling of the levels below the Fermi surface, and which is taken into account, e. g., in the Bethe-Goldstone equation[46] in the theory of nuclear matter, does not change the expression for $\hat{\Phi}^0$. This is because, in our treatment, the scattering of particles at the Fermi surface is essential, the exclusion law not being valid for these particles.

**The same expression has subsequently been derived in reference 47.

*We shall not discuss the effects due to the periodic lattice field (just as in the previous sections), but shall analyze only the collective effects.

in order of magnitude with their mean kinetic energy. Hence, we may naturally expect a significant influence of the correlation of electrons on a number of properties of metals.

In fact, in the various approximate methods of taking into account the correlation of electrons, an appreciable influence of correlation on a number of properties of metals has been found (the determination of para-magnetic susceptibility,[52,53] the determination of the heat capacity and the correlation energy of an electron gas of high density,[54] for the influence of correlation on the vibration spectrum of an electron plasma, see Sec. 4). However, all such treatments are microscopic theories, whose applicability to real metals is highly problematical. This is due to the absence in real metals of a small parameter, resulting from the practical coincidence in order of magnitude of the kinetic and potential energies of the electrons. This situation compels us to pay especial attention to a phenomenological theory, which takes into account the correlation of particles rather fully.

We must note that the correlation of particles is ordinarily not taken into account in the kinetic theory of electrons in a metal. Actually, the kinetic equation of Boltzmann is commonly used here. In such a treatment, a partial account of the correlation of electrons, as arising at small distances and resulting in collisions, may be made with the aid of the collision integral of electrons with electrons. As is known,[55] such collisions play a small role*, and the correlation thus taken into account does not reproduce the essential effect.

On the other hand, as was shown above, taking into account the correlations due to the identity of electrons essentially changes the kinetic equations, even when collisions are neglected.[43] A phenomenological way of taking precisely these effects into account is contained in the theory of the Fermi fluid.

In a quantum fluid, due to the significant self-consistent interaction of the particles, the energy of an individual particle depends on the state of the surrounding particles. In this regard, naturally, it turns out that the energy of the system of particles is no longer equal to the sum of the energies of the individual particles, but is some function of the distribution function. Under these conditions, it is more natural to speak not of particles (e. g., atoms) of which the fluid consists, but of quasi particles, inasmuch as the latter are essentially different from free particles.

For an infinitesimally small variation in the distribution function $\delta f_{ss'}(\mathbf{q}, \mathbf{p})$, the variation in the energy density of the system has the form:

$$\delta E(\mathbf{q}) = \sum_{ss'} \int \varepsilon_{ss'}(\mathbf{q}, \mathbf{p})\, \delta f_{ss'}(\mathbf{q}, \mathbf{p})\, d\mathbf{p}. \qquad (5.1)$$

This formula is essentially a definition of the energy (the Hamiltonian) of a quasi particle, which differs from the energy of a free particle due to the interaction with surrounding particles.

Further, upon variation of the distribution function, the energy of the quasi particle also varies[44]

$$\delta\varepsilon_{ss'}(\mathbf{q}, \mathbf{p}) = \sum_{s''} \int F_{ss''}(\mathbf{q}, \mathbf{q}', \mathbf{p}, \mathbf{p}')\, \delta f_{s''s'}(\mathbf{q}', \mathbf{p}')\, d\mathbf{q}'\, d\mathbf{p}'. \qquad (5.2)$$

The function F is the second variational derivative of the energy density, and plays an essential role in determining a number of properties of the fluids. This quantity depends, in general, on the momenta $\mathbf{p}$ and $\mathbf{p}'$ and the spins, and is also a function of the coordinates.

In the theory of He^3, the dependence on the coordinates was actually taken to have the form $\delta(\mathbf{q} - \mathbf{q}')$. In the present case of an electron fluid, this is no longer valid, because of the Coulomb force law. In fact, if we should use the self-consistent Hartree approximation for particles interacting according to a force law with a potential energy $U(|\mathbf{q} - \mathbf{q}'|)$, the function would have the following form in this case:

$$F^{(H)}_{ss''}(\mathbf{q}, \mathbf{q}', \mathbf{p}, \mathbf{p}') = \delta_{ss''} U(|\mathbf{q} - \mathbf{q}'|). \qquad (5.3)$$

It is clear that for short-range forces, the approximation using the δ-function on the right-hand side of this formula is permissible for all those states in which the characteristic dimension of the inhomogeneities which arise is large in comparison with the radius of action of the forces.

In the Hartree approximation correlation is completely neglected, since the distribution function of the system is taken in the form of the product of the distribution functions of the individual particles. Hence it is clear that the difference $(F - F^{(H)})$ must be due to the effect of the correlations of the particles. In the general case, we may say only rather little about this quantity. However, the case is of fundamental interest, in which the radius of correlation is much smaller than the distances at which the distribution function varies appreciably. In such a case, we may assume that

$$F(\mathbf{q}, \mathbf{q}', \mathbf{p}, \mathbf{p}') - F^{(H)} \simeq \delta(\mathbf{q} - \mathbf{q}')\, \hat{\Phi}(\mathbf{p}, \mathbf{p}'). \qquad (5.4)$$

* An exception to this might be the contribution of inter-electronic collisions on the absorptivity of a metal in the frequency region in which $\hbar\omega >> \kappa T$ (see references 31, 56, and 57).

Correspondingly Eq. (5, 2) takes the form

$$\delta\varepsilon(\mathbf{q}, \mathbf{p}) = \int U(|\mathbf{q}-\mathbf{q}'|)\,\delta f(\mathbf{q}', \mathbf{p}')\,d\mathbf{q}'\,d\mathbf{p}' + \int \hat{\Phi}(\mathbf{p}, \mathbf{p}')\,\delta f(\mathbf{q}, \mathbf{p}')\,d\mathbf{p}'. \tag{5.5}$$

If the forces have a small radius of action, then formula (5, 5) agrees with that applied in the theory of He^3. [51] Due to the fact that, in the He^3 case, both the radius of action of the forces and the radius of correlation of the particles are quantities of the order of interatomic distances, it is clear from what has been stated above that a localized relation between $\delta\epsilon$ and δf is completely correct.

Another situation occurs in the case of a system of electrons. The long-range character of the Coulomb forces compels us to use a non-localized relation between $\delta\epsilon$ and δf, similar to (5, 5). However, we must make some remarks here. In using Eq. (5, 5), we must satisfy the condition (5, 4). Hence, we shall deal with those correlations which may arise in a system of electrons. In particular, the correlation of electrons occurring in a degenerate state arises as a result of the identity of the particles. An estimate of this correlation may be obtained in the Hartree-Fock approximation. However, we may say immediately that the distances at which this type of correlation is substantial are of the order of magnitude of the interelectronic distances. Further, for electrons, for example those of a metal, the correlations are also substantial, which are due to interactions (the Coulomb correlation, which leads to the so-called correlation energy of the degenerate electron gas). This correlation is similar at large distances to the Debye correlation of particles, as in electrolytes, while at short distances it is associated with the strong mutual repulsion of the electrons. An important peculiarity of these correlations at the electron densities which occur in real metals is the fact that the characteristic distances at which the correlations are substantial turn out to be not much larger than the interatomic distances. [53,58] In line with this, we can say that relation (5, 4) may be used for the electrons in a metal. Thus, in Eq. (5, 5), the nearest-neighbor correlation will be given correctly by the second term containing the function $\hat{\Phi}(\mathbf{p}, \mathbf{p}')$.

The first term in Eq. (5, 5) takes into account the influence of long-range Coulomb forces, and in the present case of electrons, $U(|\mathbf{q}-\mathbf{q}'|) = e^2/|\mathbf{q}-\mathbf{q}'|$. We note that, for spatially-homogeneous distributions, the first term in this formula in its literal sense gives a divergent expression. This situation is due to the fact that, in the study of a system of electrons, it is necessary to introduce a background of positive charges compensating for the charge of the electrons, and corresponding to the electronic charge of the ions. Hence, in the term in Eq. (5, 5) under discussion, we must take δf to mean the deviation of the distribution function from a spatially-homogeneous distribution preserving the neutrality of the system of particles.

The result of what has been presented above may be the following. The nearest-neighbor correlation may be taken into account in a way similar to that taken in the case of He^3 with the aid of the Landau theory. The long-range Coulomb correlation may be taken into account by the introduction of a self-consistent field. Obviously, we may thus take into account not only the Coulomb (longitudinal) force, which we discussed above, but also the transverse electromagnetic field. The latter is analogous to what is commonly done when the Boltzmann equation is used in the kinetic theory of metals, the Lorentz force being determined by the self-consistent electromagnetic field. [8,33] Hence, the field equations may be written in the form

$$\left.\begin{aligned} &\operatorname{curl}\mathbf{E} + \frac{1}{c}\frac{\partial\mathbf{H}}{\partial t} = 0, \quad \operatorname{div}\mathbf{E} = 4\pi e\,\mathrm{Sp}\int f\,d\mathbf{p}' - 4\pi\varrho_+, \\ &\operatorname{curl}\mathbf{H} - \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} = \frac{4\pi e}{c}\,\mathrm{Sp}\int\frac{\partial\varepsilon}{\partial\mathbf{p}'}f\,d\mathbf{p}' + 4\pi\beta\operatorname{curl}\mathrm{Sp}\int\hat{\sigma}f\,d\mathbf{p}'; \quad \operatorname{div}\mathbf{H} = 0, \end{aligned}\right\} \tag{5.6}$$

where $\hat{\sigma}$ is the Pauli spin matrix, β is the magnetic moment of an electron, and ρ_+ is the compensating charge of the positive background of ions. In these equations, $f_{ss'}$ is a distribution function determined by the following kinetic equation: [44]

$$\begin{aligned} &\frac{\partial f}{\partial t} + \frac{1}{2}\left(\frac{\partial\varepsilon_1}{\partial\mathbf{p}}\frac{\partial f}{\partial\mathbf{q}} + \frac{\partial f}{\partial\mathbf{q}}\frac{\partial\varepsilon_1}{\partial\mathbf{p}}\right) - \frac{1}{2}\left(\frac{\partial\varepsilon_1}{\partial\mathbf{q}}\frac{\partial f}{\partial\mathbf{p}} + \frac{\partial f}{\partial\mathbf{p}}\frac{\partial\varepsilon_1}{\partial\mathbf{q}}\right) + e\mathbf{E}\frac{\partial f}{\partial\mathbf{p}} + \\ &+ \frac{e}{2c}\left\{\left[\frac{\partial\varepsilon_1}{\partial\mathbf{p}}\times\mathbf{H}\right]\frac{\partial f}{\partial\mathbf{p}} + \frac{\partial f}{\partial\mathbf{p}}\left[\frac{\partial\varepsilon_1}{\partial\mathbf{p}}\times\mathbf{H}\right]\right\} - \frac{i}{\hbar}[\varepsilon_1, f] = \hat{I}(f). \end{aligned} \tag{5.7}$$

Here, $\hat{I}$ is the operator corresponding to the collision integral. For equilibrium distributions, $\partial\epsilon_1/\partial\mathbf{p} = \mathbf{v}$ is the velocity of the electrons, while in the case of small deviations from equilibrium

$$\delta\varepsilon_1(\mathbf{q}, \mathbf{p}) = -\beta\hat{\sigma}\mathbf{H} + \int \hat{\Phi}(\mathbf{p}, \mathbf{p}')\,\delta f(\mathbf{q}, \mathbf{p}')\,d\mathbf{p}'. \quad (5.8)$$

Here, the possible spin-orbit interactions have been neglected. We may easily convince ourselves* that the following equality holds

$$\mathrm{Sp}\int \mathbf{p} f\,d\mathbf{p} = \mathrm{Sp}\int m\frac{\partial\varepsilon_1}{\partial\mathbf{p}} f\,d\mathbf{p}. \quad (5.9)$$

From relation (5,9) it follows that

$$\frac{\mathbf{p}}{m} = \frac{\partial\varepsilon_1}{\partial\mathbf{p}} - \int \hat{\Phi}\frac{\partial f}{\partial\mathbf{p}'}\,d\mathbf{p}'. \quad (5.10)$$

If this relation, which was found by Landau,[51] is substituted into (5,9), we obtain the following condition:

$$\mathrm{Sp}\int \hat{\Phi}(\mathbf{p}, \mathbf{p}')\frac{\partial f}{\partial\mathbf{p}'} f(\mathbf{p})\,d\mathbf{p}\,d\mathbf{p}' = 0. \quad (5.11)$$

which may be imposed on Φ in view of the arbitrary character of the distribution function. We note that in the case in which Φ corresponds to the Hartree-Fock approximation and depends only on the difference $\mathbf{p} - \mathbf{p}'$, relation (5,11) is automatically satisfied. This relation is also satisfied for equilibrium distributions.

In particular, Eq. (5,7) permits us to study the plasma oscillations and other oscillations of a degenerate electron fluid. References 58 and 59 are devoted to this problem. The oscillations of an uncharged Fermi fluid have been discussed in references 31 and 60.

First of all, we shall consider oscillations accompanied by density oscillations, and therefore leading to the appearance of a longitudinal electromagnetic field. These are the plasma waves. If the condition $\omega_L >> kv_0$ is satisfied, we have in the long-wavelength region the following expression for the spectrum of these oscillations:[58]

$$\omega^2 = \omega_L^2 + \frac{v_0 p_0}{m}\left(\frac{3}{5} + \frac{1}{3}\left[A_0 + \frac{4}{25}A_2\right]\right)k^2. \quad (5.12)$$

Here A_0 and A_2 are the coefficients in the Legendre polynomial expansion of the function

$$\frac{8\pi p_0^2}{(2\pi\hbar)^3 v_0^2}\frac{1}{2}\mathrm{Sp}\,\hat{\Phi}(\mathbf{p}, \mathbf{p}') = \sum_n A_n P_n(\cos\chi).$$

where χ is the angle between $\mathbf{p}$ and $\mathbf{p}'$, $\mathbf{p}_0$ is the Fermi limiting momentum, and v_0 is the velocity of electrons at the Fermi surface, which, according to (5,10) is related to $\mathbf{p}_0$ by the relation

$$\frac{p_0}{m} = v_0\left(1 + \frac{A_1}{3}\right). \quad (5.14)$$

The limiting frequency ω_L which appears in Eq. (5,12) coincides with the corresponding value obtained by the Hartree approximation. However, the correlation of particles plays a substantial role in the small correction term in (5,12) determining the dispersion of the plasma waves. An estimate of the quantities A_0 and A_2 may be made under the assumption that the function Φ is determined by the amplitude of forward scattering, as calculated in the Born approximation, for a screened Coulomb interaction using the actual parameters characteristic of the majority of metals. This estimate shows that these quantities are not large, and in such an approximation, at least, they do not change the order of magnitude of the term proportional to k^2. However, in real metals, the situation may turn out to be more complex. In particular, the coefficients A_n of the expansion may turn out to be significantly greater than those obtained in the estimate stated above.

If we make no assumptions about the form of the function Φ, then the expressions for the excitation spectra of an electron fluid, generally speaking, will be determined by highly complex relations, and will correspond to the roots of the following equation for various polarizations l ($|l| < n$):

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* The momentum density and the current density may be determined by the following equations:

$$P(q) = \int \mathbf{p}_1 f_n(\mathbf{p}_1\mathbf{q}_1 \ldots \mathbf{p}_n\mathbf{q}_n)\,d\mathbf{p}_1\,d\mathbf{p}_2\,d\mathbf{q}_2 \ldots d\mathbf{q}_n \equiv \int \mathbf{p} f(p, q)\,d\mathbf{p},$$

$$j(q) = e\int \frac{\mathbf{p}_1}{m} f_n(\mathbf{p}_1\mathbf{q}_1 \ldots \mathbf{p}_n\mathbf{q}_n)\,d\mathbf{p}_1, d\mathbf{p}_2, d\mathbf{q}_2 \ldots d\mathbf{q}_n \equiv e\int \frac{\partial\varepsilon}{\partial\mathbf{p}} f(\mathbf{q}, \mathbf{p})\,d\mathbf{p}.$$

The latter takes into account the fact that

$$p_1/m = \frac{\partial}{\partial p_1}\left(\sum_i p_i^2/2m + U_i + \sum_{i,j} U_{ij}\right).$$

Equation (5,9) follows from a comparison of the expressions for the momentum density and the current density defined in terms of f_n and f.

$$\left|\delta_{nn'}\delta_{ll'}+ikv_0\frac{A_{n'}}{2n'+1}\left[\sqrt{\frac{(n'+1)^2-l'^2}{4(n'+1)^2-1}}N^{l,l'}_{n,n'+1}+\sqrt{\frac{n'^2-l'^2}{4n'^2-1}}N^{l,l'}_{n,n'-1}\right]+\right.$$
$$+\frac{4\pi ie^2c^2}{\omega^2-c^2k^2}\frac{8\pi p_0^2}{(2\pi\hbar)^3}\left[-\frac{k}{\sqrt{3}}\delta_{n'0}\delta_{l'0}N^{l,0}_{n,1}+\right.$$
$$\left.\left.+\frac{\omega v_0}{3c^2}\left(1+\frac{A_1}{3}\right)\delta_{n'1}(\delta_{l',0}+\delta_{l',1}+\delta_{l',-1})N^{l,l'}_{n,n'}\right]\right|=0. \quad (5.15)$$

where

$$iN^{l,l'}_{n,n'}=\delta_{ll'}\int\frac{do\,Y^{l*}_n(\vartheta,\varphi)\,Y^l_{n'}(\vartheta,\varphi)}{kv_0\cos\vartheta-\omega}. \quad (5.16)$$

and the $Y^l_n(\vartheta,\Phi)$ are normalized spherical harmonics.

If we assume for simplicity that only A_0 and A_1 differ from zero, then for longitudinal waves ($l = 0$) we obtain from (5, 15)

$$\left(1+\frac{A_1}{3}\right)[1-A_0\eta(s)]-\left\{A_1s^2+\frac{3\omega_L^2}{k^2v_0^2}\right\}\eta(s)=0. \quad (5.17)$$

where

$$\eta(s)=\frac{s}{2}\ln\frac{s+1}{s-1}-1\quad(s=\omega/kv_0). \quad (5.18)$$

In the long-wavelength region, $kv_0 \ll \omega_L$, relation (5, 12) follows from this for the case $A_2 = 0$. Under the assumption that only A_0 and $A_1 \neq 0$, the spectrum of the transverse oscillations ($l = 1$) is determined by the relation

$$1-\frac{1}{2}A_1\left\{\frac{1}{3}-(s^2-1)\eta(s)\right\}-\frac{3}{2}\frac{\omega_L^2}{\omega^2-c^2k^2}\left\{1-(s^2-1)\eta(s)\right\}=0. \quad (5.19)$$

If A_1 is neglected, this relation coincides with the corresponding dispersion relation in the self-consistent-field approximation, describing undamped oscillations.[3, 21, 22, 32] The oscillations become damped in the case in which $|s| < 1$. Under these conditions, the logarithm in formula (5, 18) should be taken to mean the principal value of the logarithm minus $2\pi i$. Taking the quantity A_1 into account in the region $s \gg 1$ leads to the following relation between ω and k [see (5, 14)]:

$$\omega^2=\omega_L^2+k^2\left(c^2+\frac{1}{5}\frac{p_0}{m}v_0\right),\quad \omega\gg kv_0. \quad (5.20)$$

In other words, in this frequency region, the taking into account of correlation is manifested only in a small term. We note that if we neglect A_1, i. e., in the self-consistent-field approximation, Eq. (5, 19) permits no undamped solutions in the frequency region $\omega \sim kv_0$.

We shall ascertain what changes are brought about in this relation by taking correlation of particles into account. It is easy to see that when the following inequality is satisfied

$$\omega_L\gg\omega\gg(v_0/c)\,\omega_LA_1^{-1/2}. \quad (5.21)$$

as takes place for the majority of metals in the infra-red region, we obtain from (5, 19)

$$(s^2-1)\eta(s)=\frac{1}{3}-\frac{2}{A_1}. \quad (5.22)$$

This relation corresponds to the spectrum of the transverse oscillations of an uncharged Fermi fluid and has been studied in reference 60. Solutions of (5, 22) are possible only when $A_1 > 6$. Estimates for a number of metals carried out in reference 61 gave $A_1 \lesssim 3$. Hence, we may suppose that for such metals, transverse zero sound is hardly possible. In general, however, the question of the existence of metals in which the propagation of zero sound is possible may apparently be solved only experimentally, until we obtain more information about the correlation of electrons in metals.

We note that, as may be seen from (5, 15), the presence of the charge of the electrons is not essential for polarizations with $l > 1$. Hence, in this case, the theory of zero sound in an electron fluid is analogous to that for He^3.[31, 60] However, in order that this sound be possible, it is necessary that the coefficients of the expansion in (5, 13) with $n > 1$ should also differ from zero. In the special case in which all A_n with $n > 2$ are zero, the dispersion equation for zero sound with $l = 2$ has the form

$$\frac{1}{5}-\frac{4}{A_2}=\frac{3}{2}(s^2-1)\left\{\frac{1}{3}+(s^2-1)\left[1-\frac{s}{2}\ln\frac{s+1}{s-1}\right]\right\}. \quad (5.23)$$

Speaking of the nature of zero sound, there is little that may be added to what has been said above in the discussion of collective oscillations in the Hartree self-consistent-field approximation. We note that, for zero sound to exist, it is necessary that the oscillation period be small in comparison with the time for estab-

lishment of equilibrium in the electron fluid. For example, as we know, this condition is satisfied in the case of the inequality (5, 21). At first glance, it might seem that the permitted frequency region for zero sound might be broadened substantially on lowering the temperature. However, this is actually not the case. The fact is that in the frequency region $\hbar\omega \gtrsim \varkappa T$, the electrons participating in collisions are distributed in an energy range $\sim \hbar\omega$ from the Fermi surface.[62, 31, 63, 56, 57] This naturally leads to an increase in the frequency of collisions, making the mean free path of an electron practically independent of the temperature for sufficiently high frequencies ω.

We shall take up now some peculiarities of collective oscillations which are possible in the case in which the electron fluid is situated in a constant magnetic field (H_0). Electromagnetic oscillations (longitudinal-plasma oscillations and transverse oscillations) are modified here in a way analogous to that which takes place in an ordinary plasma. The oscillations corresponding to zero sound are also modified. In the long-wavelength region, in which the condition $kv_0 \ll \Omega = ev_0H_0/cp_0$ is satisfied, we have the following simple expression for the spectrum of these oscillations

$$\omega_{n,l} = l\Omega\left(1+\frac{A_n}{2n+1}\right)+O\left(\left(\frac{kv_0}{\Omega}\right)^2\right), \quad (1<|l|\leqslant n). \tag{5.24}$$

In the case of a gas of free electrons, (5,24) corresponds to the spectrum of equidistant electron levels in a magnetic field. As we see, taking into account the correlation of particles makes these levels no longer equidistant. However, we should note that the excitation states of (5, 24) correspond to only a fraction of the possible states of the electrons in the magnetic field. In particular, these excitation states are similar to plasma waves, and hence, they obey Bose-Einstein statistics.

In addition to these effects, which are associated with the orbital motion of the electrons, excited states associated with the spin moment are also possible in a system of electrons. In order to study these excited states, it is convenient to use the function $\sigma(\mathbf{q}, \mathbf{p}) = \sum_{ss'}(\hat{\sigma})_{ss'} f_{s's}(\mathbf{q}, \mathbf{p})$. Then, from (5, 7) we have, in particular, the following equation describing the oscillations of the component of σ normal to the constant magnetic field ($\sigma^{(\pm)} = \sigma_x \pm i\sigma_y$):

$$\frac{\partial\delta\sigma^{(\pm)}}{\partial t}+\left\{\mathbf{v}\frac{\partial}{\partial\mathbf{q}}+\frac{e}{c}\left([\mathbf{v}\times\mathbf{H}]\frac{\partial}{\partial\mathbf{p}}\right)\pm i\frac{2\gamma(p)}{\hbar}\mathbf{H}_0\right\}\left(\delta\sigma^{(\pm)}-\frac{\partial f_0}{\partial\varepsilon}\delta\varepsilon_2^{(\pm)}\right)=0. \tag{5.25}$$

Here

$$\delta\varepsilon_2(\mathbf{q}, \mathbf{p}) = -\beta\mathbf{H}+\frac{1}{12}\mathrm{Sp}\int(\hat{\sigma}\Phi(\mathbf{p}, \mathbf{p}')\hat{\sigma}')\,\delta\sigma(\mathbf{q}, \mathbf{p}')\,d\mathbf{p}', \tag{5.26}$$

where H is the variable magnetic field, and the function $\gamma(p)$ determines the paramagnetism of the electron fluid, and is equal to [51, 59]

$$\gamma(p_0) = \beta\left\{1+\frac{2}{(2\pi\hbar)^3}\frac{p_0^2}{v_0}\int \mathrm{Sp}_{s'}\mathrm{Sp}_s\frac{1}{12}\Phi(p_0, p_0')\,\hat{\sigma}\,\hat{\sigma}'\,d\theta'\right\}^{-1}. \tag{5.27}$$

In the long-wavelength region, (5, 25) gives

$$\omega_{l,n} = (\pm\Omega_0 + l\Omega)\left(1+\frac{B_n}{2n+1}\right)+O(k^2), \tag{5.28}$$

where $\Omega_0 = 2\gamma(p_0)H_0/\hbar$, and B_n are the coefficients in the Legendre polynomial expansion

$$\frac{8\pi p_0^2}{(2\pi\hbar)^3 v_0}\frac{1}{12}\mathrm{Sp}_s\mathrm{Sp}_{s'}\hat{\sigma}\hat{\sigma}'\Phi(\mathbf{p}, \mathbf{p}') = \sum_n B_n P_n(\cos\chi). \tag{5.29}$$

If $l = 0$, then (5, 28) gives $\omega_{0,0} = \pm 2\beta H_0/\hbar$, which corresponds to the ordinary Bloch frequency. In the short-wavelength region the spectrum of spin waves, generally speaking, is very complex. In the special case in which we may consider that only B_0 differs from zero, and that the wave vector $\mathbf{k}$ is parallel to the direction of the constant magnetic field H_0, the dispersion equation may be written in the form

$$-\frac{1}{B_0} = 1-\frac{s}{2}\ln\left|\frac{s\pm s_0+1}{s\mp s_0-1}\right|, \tag{5.30}$$

where $s = \omega/kv_0$, and $s_0 = \Omega/kv_0$. For longer wavelengths, (5, 30) gives

$$\omega = kv_0 s = \pm \Omega_0 (1 + B_0)\{1 + k^2 v_0^2 / 3 B_0 \Omega_0^2\}. \quad (5, 31)$$

If B_0 is positive, then the frequency increases with increase in the absolute magnitude of the wave vector. Here, in the region in which the inequality $kv_0 >> \Omega_0$ is satisfied, the dispersion equation (5, 30) takes the form $(1/B_0) = \eta(s)$, and possesses solutions only when B_0 is positive.[31] Hence it is clear that in this wavelength region spin waves are impossible if $B_0 < 0$. For longer wavelengths, according to Eq. (5, 31), the frequency decreases in this case with increase in the wave vector. The spin waves indicated here are due to the paramagnetism of the electrons. Here, the dispersion, the relation of the frequency of the waves to the wave vector, is determined by function (5, 29), which results, in particular, from the exchange correlation of the electrons. The exchange correlation is extremely essential in the case of ferromagnetic metals, in which it determines a spectrum of spin waves in the electron fluid[64] having the form $\omega \sim k^2$.

6. LOSSES OF A CHARGED PARTICLE ON PASSING THROUGH MATTER, ASSOCIATED WITH THE EXCITATION OF COLLECTIVE OSCILLATIONS IN THE MEDIUM

As we have seen in the preceding sections, both transverse and longitudinal oscillations of the electromagnetic field are possible in a medium, considered to be a system of many particles. These oscillations depend essentially on the properties of the medium, as may be seen, in particular, from the possibility of propagation of longitudinal waves, which in general cannot exist in a vacuum. Such oscillations may be excited by a charged particle, and correspondingly may cause a definite fraction of the energy losses of the charged particle in passing through matter. We must note that these collective oscillations (of the electromagnetic field) may be propagated in the medium only when the absorption is small. In absorbing media, they will not propagate, but will be rather quickly absorbed by the medium. This essentially corresponds to the transfer of energy from the charged particle to the medium by means of collective oscillations.

An example of the excitation of collective oscillations of the electromagnetic field in a medium is the Cerenkov radiation of a charged particle*.[65, 66] Here, in an isotropic medium oscillations of the transverse electromagnetic field are excited. The problem of the excitation of both transverse and longitudinal oscillations of the electromagnetic field in the medium, and of the energy losses of the charged particle passing through the medium, corresponding to these excitations, has been studied in essence by Fermi[68] in a rather general manner (see also references 69--75). In such studies, the dielectric constant $\epsilon(\omega)$, which is a function of the frequency of the alternating field, has been used as the characteristic of the electromagnetic properties of the medium. Here, the following expression is valid for the value of the energy loss of a fast particle per unit path length, in the Tamm-Frank-Fermi theory:

$$W(q_0) = \frac{ie^2Z^2}{\pi} \int_{-\infty}^{\infty} \omega\, d\omega \int_0^{q_0} q\, dq \frac{1}{q^2v^2 + \omega^2} \left\{ \frac{1}{\varepsilon(\omega)} - \frac{v^2}{c^2} \frac{q^2}{q^2 + \omega^2 \left(\frac{1}{v^2} - \frac{\varepsilon(\omega)}{c^2} \right)} \right\}. \quad (6, 1)$$

Here eZ is the charge of the particle, v is its velocity, and hq_0 is the maximum momentum transferred by the charged particle during ionization. We shall not go into any detailed analysis of Eq. (6, 1), inasmuch as it is given in the references given above. We shall only note that the first term in the integrand of (6, 1) corresponds to longitudinal losses, while the second term corresponds to transverse losses, that is, they are due to the excitation of vibrations of the longitudinal and transverse fields, respectively. It is expedient to consider the case of transparent media, in which $\epsilon(\omega)$ is a real quantity. In this case, the formula for the energy losses may be written in the form[67, 70, 75]

$$W(q_0) = \frac{e^2Z^2}{v^2} \sum_s \frac{\omega_s}{|d\varepsilon(\omega_s)/d\omega_s|} \ln \frac{vq_0}{\omega_s} + \frac{e^2Z^2}{c^2} \int_{\varepsilon(\omega)v^2/c^2 > 1} \left(1 - \frac{c^2}{v^2\varepsilon(\omega)} \right) \omega\, d\omega. \quad (6, 2)$$

where the ω_s are the frequencies of the longitudinal vibrations of the electromagnetic field in the medium, and are determined by the condition

$$\varepsilon(\omega_s) = 0. \quad (6, 3)$$

and it is assumed that $vq_0 >> \omega_s$.

Thus, in addition to the Cerenkov losses, the expression for which is given by the integral in Eq. (6, 2), there

*See also the review reference 67.

occur losses associated with the excitation of longitudinal, or as they are also often called, polarization or Bohr waves*.

A special case, but apparently one of the most interesting cases of oscillations of the longitudinal field, is the so-called plasma oscillations in an electron gas with a frequency equal to

$$\omega_L = \sqrt{4\pi e^2 n/m}. \qquad (6,4)$$

where m is the mass of an electron, and n is the number of electrons per unit volume. The dielectric constant of the electron gas, dissipation and random motion being neglected, is ordinarily written in the form[76]

$$\varepsilon(\omega) = 1 - \omega_L^2/\omega^2. \qquad (6,5)$$

which, naturally, agrees with (6, 3) and gives Eq. (6, 4). The problem of the excitation of plasma oscillations by a moving charge is often studied with the aid of some model (see, e. g., references 8, 12, and 77). This permits us to concretize the derived formulas greatly. However, such concretization often does not permit us to see the general physical regularities; in connection with this, we shall try as much as possible to postpone the moment when we have to use model representations, and in particular, to use the contents of the preceding sections.

In recent years, the question of polarization losses of charged particles has been widely discussed in the literature, and we might say that it has experienced its second birth in connection with the problem of discrete losses of electrons passing through thin films.[79] There is every basis for assuming that such discrete losses are associated with the excitation of longitudinal vibrations determined by relation (6, 3). Here we may speak in quantum-mechanical language of the emission of a quantum of the electromagnetic field with longitudinal polarization and with a frequency ω_s, often referred to as a plasmon.[80] Such an interpretation of discrete losses has been developed in references 81 and 23. A corresponding theory of losses has been developed in the papers of many authors.[81, 82, 32, 83, 75, 84, 38, 5, 86, 80, 87, 88]

One of the peculiarities of the recently-developed theory of energy losses of charged particles is that of taking into account the spatial dispersion of the dielectric constant*. In other words, as the wave depending on the time and the coordinates as $e^{-i\omega t + i\mathbf{kr}}$ propagates in the medium, the dielectric constant is a function not only of the frequency ω, but of the wave vector k. In the case of an isotropic medium, in distinction from the usual treatment, the dielectric constant is no longer a scalar, but in general has the form[32]

$$\varepsilon_{ij}(\omega, \mathbf{k}) = \varepsilon^{tr}(\omega, k)\{\delta_{ij} - k_i k_j/k^2\} + (k_i k_j/k^2)\,\varepsilon^{l}(\omega, k). \qquad (6,6)$$

Thus, the propagation of longitudinal and transverse waves in a medium is determined, generally speaking, by differing dielectric constants.

The formula for the total losses of a fast charged particle per unit path on passing through a medium with spatial dispersion has the form

$$W(q_0) = \frac{ie^2Z^2}{\pi}\int_{-\infty}^{\infty}\omega\,d\omega\int_0^{q_0}\frac{q}{q^2v^2+\omega^2}\left\{\frac{1}{\varepsilon^l(\omega, \sqrt{q^2+\omega^2/v^2}}\right.$$
$$\left. -\frac{v^2}{c^2}\frac{q^2}{q^2+\omega^2\left[\frac{1}{v^2}-\frac{1}{c^2}\varepsilon^{tr}(\omega, \sqrt{q^2+\omega^2/v^2})\right]}\right\}dq. \qquad (6,7)$$

In studying discrete losses, fast but non-relativistic particles have been considered, for which only the longitudinal losses are significant. These losses are characterized by the first term on the right-hand side of Eq. (6, 7). Hence, we shall proceed now to a more detailed study of the longitudinal losses.

We may derive without difficulty from Eq. (6, 7) an expression for the probability per unit path of scattering of the particle by an angle between θ and $\theta + d\theta$, with emission of a longitudinal quantum (plasmon) in the frequency interval ω to $\omega + d\omega$:

$$\frac{dW^l(\theta, \omega)}{\hbar\omega\,d\theta\,d\omega} = \frac{2e^2Z^2}{\hbar v^2}\frac{\theta}{\theta^2+\left(\frac{\hbar\omega}{vp}\right)^2}\operatorname{Im}\frac{1}{\varepsilon^l\left(\omega, \sqrt{\frac{p^2\theta^2}{\hbar^2}+\frac{\omega^2}{v^2}}\right)} \qquad (6,8)$$

*The possibility of emergence from matter of radiation associated with the excitation in the medium of longitudinal plasma waves has been indicated in reference 78.

*Optical phenomena in a medium having spatial dispersion have been studied in references 89, 90, and 91. A well-studied phenomenon in which spatial dispersion is manifested is the anomalous skin-effect.[92]

Here the fact is taken into account that, in the emission of the longitudinal photon, the momentum of the fast particle is changed only slightly ($p >> \hbar k$), and that the particle is scattered at a small angle $\theta >> 1$.

In the case in which we may neglect spatial dispersion, the probability of scattering may easily be related to the optical constants of the medium, the index of refraction n and the absorption coefficient κ, according to the relation[75]

$$\operatorname{Im}\frac{1}{\varepsilon} = \frac{2n\varkappa}{(n^2+\varkappa^2)^2}. \qquad (6,9)$$

Since the absorption in a given medium may not be small, Eq. (6, 3), strictly speaking, may not be satisfied, since the simultaneous vanishing of the real and imaginary parts may be due to pure coincidence. Hence, in order that we may speak of discrete losses, it is necessary that Eq. (6, 9) should have a sharp maximum in a definite frequency region. The determination of the optical properties of metals permits us in some cases to determine the value of (6, 9), and for the alkali metals and silver it actually possesses a rather sharp maximum (see references 75, 93, and 94). It is obvious that the determination of n and κ for metals over a wide range of frequencies would be highly useful in the further elucidation of the nature of discrete losses.

It is pertinent here to proceed to the next problem in the interpretation of the spectra of characteristic losses of fast electrons. The question is often discussed as to whether at least part of the discrete losses are associated with transitions of individual electrons between different bands. First of all, we note in this regard that the use of Eq. (6, 1) to describe the losses is correct for impact parameters $(1/q_0)$ much larger than the interatomic distances, i. e., for solids in the case in which $q_0 << 10^8 \text{cm}^{-1}$ Hence, for scattering at an angle smaller than $< v > / v$ (where $< v > = 10^8$ cm/sec), the distribution of fast electrons is determined by the dielectric constant of the medium alone. In this sense, the discrete losses in small-angle scattering are always due to collective effects. Further, the existence of a band structure in the spectrum of the electrons in the solid is reflected, of course, in the form of $\epsilon(\omega)$ as a function of the frequency. Hence, in particular, in the vicinity of certain frequencies, for which $\hbar\omega$ is near to the distance between bands, a sharp increase in the quantity (6, 9) is possible in principle. This increase, naturally, would lead also to a line in the spectrum of characteristic losses.

This case, however, is very complex. It is essential to analyze in a simpler way the spectrum of the discrete losses in the case in which the width of the forbidden band is large in comparison with the plasma frequency of the conduction electrons. Here the dielectric constant may be represented in the form:

$$\varepsilon(\omega) = \varepsilon_0 - \omega_L^2/\omega^2. \qquad (6,10)$$

where ϵ_0 is the frequency-independent contribution to the dielectric constant from the electrons of the filled bands. Here the quantity n may be determined from measurements of the optical constants of the metal in the frequency region for which $\omega^2 << \omega_L^2$. Corresponding experiments[94] for Na give a value for the number of conduction electrons agreeing with high accuracy with the number of atoms per unit volume; experiments on the discrete losses give the same value, and thus confirm the correctness of the optical experiments. On the other hand, such an agreement completely reveals the mechanism of the discrete losses, which in this case is completely due to the conduction electrons.

On the contrary, the measurement of the number of conduction electrons for a number of other elements[95, 96, 61] (for example Cu, Ag, Au, Al) gives for these elements an energy of the plasma oscillations much smaller than the observed value of the characteristic losses. In connection with this, we note that Eq. (6, 10) may be applicable to the case in which the spectrum of electronic levels is separated by a band whose width is large in comparison with the frequency of the plasmon (the case of valence electrons), while the characteristic binding energies of the electrons lying below this band are small in comparison with the plasma frequency*.[97] However, in this case the determination of the discrete losses may in essence only determine n.

In order to understand the field of applicability of relation (6, 9), we note the following. If the dissipation is small, for naturally only in this case will the losses be in any degree discrete, the characteristic parameter determining the role of spatial dispersion can only be the quantity

$$\Pi = \sqrt{< v^2 >}\ (k/\omega). \qquad (6,11)$$

where $\sqrt{< v^2 >}$ is the root-mean-square velocity of the particles of the medium. (For the conduction electrons in a metal, $\sqrt{< v^2 >}$ is of the order of magnitude of the velocity at the Fermi surface.) Here the spatial dispersion becomes the determining factor only when the parameter of (6, 11) is not small in comparison with unity. As applied to Eq. (6, 8), this means that the quantity which is not to be small must be

$$\Pi^2 = \frac{< v^2 > p^2}{\hbar^2\omega^2}\theta^2 + \frac{< v^2 >}{v^2}. \qquad (6,12)$$

* Cited in reference 80.

Hence, it is clear that the role of spatial dispersion becomes important only in scattering at angles which are not small in comparison with

$$\theta_1 = \hbar\omega/(p\sqrt{<v^2>}). \qquad (6,13)$$

Here the fact has been taken into account that the velocity of the fast particle is large in comparison with the root-mean-square velocity of the particles of the medium ($v^2 >> <v^2>$). The angular distribution of the scattered electrons, according to (6,8), has a sharp maximum in the region of angles considerably smaller than θ_1 and of the order of magnitude of:

$$\theta_0 = \hbar\omega/pv. \qquad (6,14)$$

In this angular region and in the vicinity of the line in the spectrum of losses, Π is a small quantity.

We shall proceed now to the study of effects in which spatial dispersion is essential. In the region in which Π is small in comparison with unity, ϵ^l may be represented in the form**:[89]

$$\varepsilon^l(\omega, k) = \varepsilon(\omega) - \delta\Pi^2. \qquad (6,15)$$

For a gas of free electrons, δ is positive.

A calculation based on the theory of the Fermi liquid gives the following expression for $\epsilon^l(\omega, k)$ for small values of k (cf. (5,12)):[58]

$$\varepsilon^l(\omega, k) = 1 - \frac{\omega_L^2}{\omega^2} - \frac{v_0 p_0}{m\omega^2}\left(\frac{3}{5} + \frac{1}{3}\left[A_0 + \frac{4}{25}A_2\right]\right)k^2. \qquad (6,16)$$

The quantity $\epsilon(\omega) = \epsilon'(\omega) + i\epsilon''(\omega)$ in (6,15) is the ordinary dielectric constant. Assuming δ to be real, we may write the following expression for the differential probability of scattering:

$$\frac{dW^l(\theta, \omega)}{\hbar\omega\, d\theta\, d\omega} = \frac{2e^2Z^2}{\hbar v^2}\frac{\theta}{\theta^2+\theta_0^2}\frac{\varepsilon''(\omega)}{[\varepsilon'(\omega) - \delta(\theta^2+\theta_0^2)/\theta_1^2] + [\varepsilon''(\omega)]^2}. \qquad (6,17)$$

Experiments studying the dependence of the energy losses of an electron on the scattering angle, in an angular range not too small in comparison with θ_1, permit us to determine δ (see references 98 and 80). In the general case, the full interpretation of such experimental results is not trivial. In essence, only in the model of free electrons may we make a prior calculation of δ. Even in such a relatively simple model as that of the Fermi fluid, δ is determined by parameters which may vary from metal to metal, and which depend on the exchange and Coulomb correlations of the electrons. Hence, the experiments on determination of the angular scattering distribution of electrons actually give new characteristics of metals. As we have seen from the example of the Fermi fluid, these characteristics cannot be reduced to any of the known characteristics of the metals.

We shall proceed now directly to an analysis of some peculiarities of the losses of charged particles, which are specific for the model of a degenerate electronic Fermi fluid. From Eq. (5,17), and expression for $\epsilon^l(\omega, k)$ follows, which we shall write in the form

$$\varepsilon^l(\omega, k) = 1 - \frac{3\omega_L^2}{k^2v_0^2}\frac{\eta(s)}{(1+A_1/3)[1-A_0\eta(s)] - A_1 s^2\eta(s)}. \qquad (6,18)$$

Dissipation in the electron fluid, as associated, for example, with collisions of the electrons with the lattice or with impurities, may be taken into account by introducing the time of free flight τ. This leads to the situation that in Eq. (6,18), $s = (\omega + i\tau)kv_0$. Equation (6,18) assumes that $\hbar k$ is small in comparison with the limiting Fermi momentum. An essential peculiarity of the given expression for ϵ^l, distinguishing it from that derived from the Hartree approximation,[32,21] is the singularity which appears under the condition

$$(1 + A_1/3)[1 - A_0\eta(s)] - A_1 s^2\eta(s) = 0 \qquad (6,19)$$

and which is associated with the excitation of zero sound.[31]

In order to clarify the qualitative picture associated with the possibility of exciting zero sound, we shall discuss a case in which $A_1 = 0$, while A_0 is large in comparison with unity. Then the solution of Eq. (6,19) acquires the form

$$s = \sqrt{\frac{A_0}{3}} \gg 1. \qquad (6,20)$$

Here ϵ^l may be represented in the form

$$\varepsilon^l(\omega, k) = 1 - \frac{\omega_L^2}{\omega^2 - (A_0/3)k^2v_0^2}. \qquad (6,21)$$

Correspondingly, such a value of the dielectric constant will give a dependence of the differential scattering

** In the Hartree approximation for a degenerate electron gas, we have in this region

$$\varepsilon^l(\omega, k) = 1 - \frac{\omega_L^2}{\omega^2} - \frac{3}{5}\frac{v_0^2k^2}{\omega^2}; \quad <v^2> = \frac{3}{5}v_0^2; \quad \delta = +1.$$

probability on θ and ω in the form

$$\frac{\theta}{\theta^2+\theta_0^2}\frac{\omega_L^2\,\delta\left(\omega-\sqrt{\omega_L^2+(A_0/3)(v_0^2p^2/\hbar^2)(\theta^2+\theta_0^2)}\right)}{\sqrt{\omega_L^2+(A_0/3)(v_0^2p^2/\hbar^2)(\theta^2+\theta_0^2)}}. \quad (6,22)$$

This formula agrees with that which may be derived from (6, 16) and (6, 17), but differs in that it is valid also when the latter term under the radical sign in (6, 22) is not small in comparison with ω_L^2. Hence, in the region of angles larger than $\theta_2=\sqrt{(3/A_0)(\hbar\omega_L/v_0p)}$, the energy lost may turn out to be proportional to the scattering angle, being equal to $\hbar\omega=\sqrt{(A_0/3)}\,v_0p\theta$. This corresponds to the excitation of a quantum of zero sound in (6, 20). Here, in view of the fact that s is larger than unity, ϵ^l is real. Hence, the maximum scattering angle is determined by the smallness of $p\theta$ in comparison with the limiting momentum p_0 of the Fermi distribution, for which our expression for ϵ^l becomes invalid. We note that, for an ideal electron gas, ϵ^l becomes complex for $\theta>(\hbar\omega_0/v_0p)$, and consequently, at such angles the line in the spectrum of losses in the gas becomes diffuse.

In addition to the variation in the value of the energy lost, the angular distribution also varies. Namely, if the angular distribution has the same form in the region of angles smaller than θ_2 that it has in the case of the electron gas, being proportional to $[\theta/(\theta^2+\theta_0^2)]$, then for $\theta>\theta_2$, the angular distribution is proportional to $[1/(\theta^2+\theta_0^2)]$.

The expression for the spectrum of the zero sound has a simple form even in the case in which $A_0\ll 1$ and is positive. Then

$$s=1-e^{-(2/A_0)}. \quad (6,23)$$

Hence, the excitation of zero sound is manifested in this case only in a narrow region near the angle $\theta=(\hbar\omega/pv_0)$.

We shall proceed now to the study of frequencies which are, on the one hand, much smaller than ω_L, but on the other hand, are much larger than the frequency of collisions $1/\tau$. Then, for $k<\omega/v_0$, or correspondingly for small angles, for which $\theta^2+\theta_0^2<\theta_1^2=(\hbar\omega/pv_0)^2$, ϵ^l is real and does not vanish. Hence, in this region there are no losses. For larger angles ($\theta>\theta_1$), the longitudinal dielectric constant becomes complex. Hence, in this angular region losses must occur. However, we cannot rely on the obtaining of any sort of distinct line in the spectrum of losses in this case.

We shall give some attention to a factor which might lead to a relation of the value of the discrete losses to the scattering angle, and which is associated with optical anisotropy. For example, in the case of a uniaxial crystal, when a fast electron moves along the symmetry axis, the expression for the differential scattering probability has the form:

$$\frac{dW^l(\theta,\omega)}{\hbar\omega\,d\theta\,d\omega}=\frac{2e^2Z^2}{\hbar v^2}\,\mathrm{Im}\,\frac{\theta}{\varepsilon_\perp(\omega)\,\theta^2+\varepsilon_\parallel(\omega)\,\theta_0^2}.$$

It is obvious that in this case, when the absorption is small, i. e., when the imaginary part of ϵ is small, the line of discrete losses will be determined by the nodes of the expression

$$\varepsilon_\perp(\omega)\,\theta^2+\varepsilon_\parallel(\omega)\,\theta_0^2.$$

In view of the fact that $\epsilon_\perp$ and $\epsilon_\parallel$ can vanish simultaneously only by pure chance, the nodes of the given expression will depend essentially on the scattering angle θ. In particular, for $\theta\ll\theta_0$, the position of the line is determined by the point at which $\epsilon_\perp(\omega)$ vanishes. Since the fundamental contribution to the total losses due to excitation of plasmons is that of angles $\lesssim\theta_1$, a broadening of the line takes place when we measure the total losses.[58, 59, 85]

7. NON-LINEAR EFFECTS DURING PASSAGE OF CHARGED PARTICLES THROUGH A PLASMA

The kinetic equations describing relaxation processes in a plasma were discussed in the second section of this article.

As early as 1928, Langmuir found that electron beams passing through a plasma transferred their energy to the electrons of the plasma in considerably shorter distances than the relaxation distance found on the basis of the formulas of Secs. 2 and 6. Both methods of analysis of energy losses discussed in these sections are based in essence on the assumption that the particles entering the plasma do not change the properties of the plasma. Thus, for electrons, it is assumed in the derivation of the Fokker-Planck equation given in Sec. 2 that the state of the electrons of the plasma surrounding the injected particle and the state of the plasma vibrations are equilibrium states.

In the method presented in Sec. 6, this is manifested in the fact that, while particles are passing through the plasma, the state of the latter is characterized by the same complex dielectric constant as in the absence of the particles. This implies that the charged particles entering the plasma do not change its properties.

The existence of the Langmuir effect indicates that such a treatment is not adequate, i. e., the assumptions on which the results of Secs. 2 and 6 are based are not fulfilled in this case.

In the solution of the more general problem, in which neither of the subsystems (the electrons of the beam

and the plasma oscillations) is in a state of statistical equilibrium, it is necessary to solve a system of non-linear simultaneous equations for the electrons of the beam and the plasma.

In experiments in which the Langmuir phenomenon is manifested, the relaxation distance calculated on the basis of the formulas of Secs. 2 and 6 is 10^3 to 10^6 times greater than the distance at which the electrons of the beam transfer their energy to the plasma electrons. Thus it follows that in the inhomogeneous case, which occurs when ordered oscillations appear in the plasma at the expense of the energy of the electrons of the beam, we need not consider those terms in the approximation of the second distribution function which lead to the relaxation terms in the kinetic equation. Under these conditions, the second distribution function may be represented in the form of the product of the first distribution functions. As a result, we arrive at the kinetic equation with a self-consistent field.

In this section will be discussed the solution of the indicated problem for a classical system of beam and plasma only. The generalization of the results given below to the quantum case may be carried out by a method analogous to that given below, but using the quantum distribution function.

Thus, let the electron beam enter the plasma along the x axis at the point $x = 0$. We shall consider the one-dimensional case. Then the system of self-consistent equations for the distribution function of the electrons of the beam and the plasma and the electric potential may be written in the following form:

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{e}{m}\frac{\partial \varphi}{\partial x}\frac{\partial f}{\partial v} = 0. \qquad (7,1)$$

$$\frac{\partial^2 \varphi(x,\ t)}{\partial x^2} = 4\pi e\left\{\int_{-\infty}^{\infty} f(x,\ v,\ t)\,dv - n_+\right\}. \qquad (7,2)$$

We assume that the charge of the electrons is compensated by a positively-charged background of ions.

In order to solve the problem of the deceleration of the electron beam due to the excitation of plasma waves, we must find a wave solution of Eqs. (7, 1) and (7, 2) satisfying the given boundary conditions at $x = 0$. Here, if we consider that ϕ is a known function, then by solving Eq. (7, 1) with respect to ϕ and eliminating f from Eq. (7, 2), we may derive a single non-linear equation for the electric potential.

From the solution of the linearized system (7, 1) and (7, 2),[12, 27, 99, 100] it follows that longitudinal waves appear in the plasma, increasing along the x axis, and with phase velocities less than the mean velocity $\bar{v}$ of the electrons of the beam. The rate of growth of the plasma waves which appear depends on the velocity of the electrons of the beam and on their concentration. With sufficiently small concentrations of electrons in the beam, the growth of the plasma waves may be arbitrarily small.

Thus, if the concentration of electrons in the beam is small in comparison with that in the plasma, the solution of the non-linear equation for the potential may be sought in the form of a slowly-growing wave

$$\varphi(x,\ t) = \varphi_0(x)\sin(\omega t - kx + \Psi(x)). \qquad (7,3)$$

where $\phi_0(x)$ and $\Psi(x)$ are the slowly varying amplitude and phase.

We shall first consider the case in which a steady-state in the wave has been established, i. e., its amplitude and phase do not depend on x. In such a state, the potential is a function only of $x - v_p t$. The solution of Eq. (7, 1) for ϕ in this case has the following form:

$$f(x,\ v,\ t) = \Phi\left(\pm\sqrt{(v-v_p)^2 - \frac{2e}{m}\varphi(x-v_p t)} + v_p\right)$$

$$+ \text{ for } v > v_p$$

$$- \text{ for } v < v_p$$

where Φ is an arbitrary function. If we eliminate the function f from Eq. (7, 2) with the aid of expression (7, 4), we obtain the equation for ϕ

$$\frac{\partial^2 \varphi}{\partial x^2} = 4\pi e\left\{\int \frac{\Phi(v)}{\sqrt{1+\frac{2e\varphi(x-v_p t)}{m(v-v_p)^2}}}\,dv - n_+\right\}. \qquad (7,5)$$

If we take the function Φ to be the Maxwell function, Eq. (7, 5) agrees with the equation given in the paper of Bohm and Gross.[27]

A solution of Eq. (7, 5) in the case in which the temperature of the electrons of the plasma and the beam is zero is given in a paper by Akhiezer and Lyubarskii.[101] In another paper by Akhiezer, Lyubarskii, and Feinberg,[102] a solution for this equation has been found for the more general case in which the temperature of the electrons of the plasma is not zero. The expression for the function Φ in this case may be given in the form

$$\Phi = \Phi_0\left(\frac{mv^2}{2}\right) + n_1\delta(v-\bar{v}). \qquad (7,6)$$

In this expression, n_1 is the concentration of electrons in the beam, $\bar{v}$ is their velocity, and Φ_0 is an arbitrary function of the energy.

These solutions cannot be used directly for the problem under consideration of the deceleration of the elec-

tron beam, since when the thermal losses are small, the transfer of energy from the beam to the wave takes place precisely in the region of growth of the wave. Hence, in order to solve this problem, we must study the process of establishment of the steady-state in the wave. In addition, the problem is still not resolved as to whether a growing-wave solution will in general approach a solution satisfying Eq. (7, 5).

However, even from a special solution of this sort, we may conclude that the conditions of applicability of the linear approximation for the electrons of the plasma and the electrons of the beam are not identical. For example, let us assume the function Φ on the right-hand side of Eq. (7, 5) to be expression (7, 6) or a more general expression taking into account the thermal fluctuations in the velocities of the beam electrons. Then we may easily see that for $v_p >> \sqrt{\kappa T/m}$, the linear approximation for the plasma electrons is valid under the condition $e\phi << mv_p^2/2$, while it is valid for the beam electrons under the condition $e\phi << m(\overline{v} - v_p)^2/2$. If $m(\overline{v} - v_p)^2 << mv_p^2$, non-linear effects for the beam electrons will have an effect at considerably lower potentials than for the plasma electrons. The use of the non-linear approximation for the beam electrons alone may be justified, of course, only in case that the steady-state value of the amplitude obtained from such a solution is such that

$$e\varphi_{ss} << mv_p^2/2.$$

Thus, when the concentration of electrons in the beam is small (but, of course, still large enough to sustain the plasma oscillations at the expense of the energy of the beam), instead of the system of equations (7, 1) and (7, 2), we may consider a system of equations for the distribution function f_1 of the beam electrons alone, while the equation for ϕ will be the wave equation for the plasma waves. The values of the phase velocity and the damping factor of the plasma wave may be taken from the linear theory of plasma oscillations.

As a result, we arrive at the following system of equations:

$$\frac{\partial f_1}{\partial t} + v\frac{\partial f_1}{\partial x} + \frac{e}{m}\frac{\partial \varphi}{\partial x}\frac{\partial f_1}{\partial v} = 0, \qquad (7,7)$$

$$\frac{\partial^2 \varphi}{\partial x^2} + 2\frac{\gamma}{v_p}\frac{\partial \varphi}{\partial x} - \frac{1}{v_p^2}\frac{\partial^2 \varphi}{\partial t^2} = 4\pi e\left\{\int f_1\, dv - n_{+1}\right\}, \qquad (7,8)$$

$$\varphi = \varphi_0(x)\sin(\omega_k t - kx + \Psi(x)). \qquad (7,9)$$

In these equations, $\omega_k^2 = \omega_L^2 + (3\kappa T/m)k^2$, where γ is the damping factor including both the damping factor of the plasma waves, as found by Landau, and the possible damping due to collisions. The functions $\phi_0(x)$ and $\Psi(x)$, as before, are the slowly varying amplitude and phase.

Taking expression (7, 9) for the potential, we can find a solution of Eq. (7, 7) for a given value of the distribution function of the beam electrons at $x = 0$. We shall denote the known function f_1 at $x = 0$ as $f_1^{(0)}(v^{(0)})$; then $\int f_1^{(0)}\, dv^{(0)} = n_{+1}$. Here and later on, the superscript (0) will denote that the function or variable refers to the point $x = 0$.

Since Eq.(7, 7) is a first-order linear differential equation for a given function ϕ, its solution is determined by the solution of its characteristic equation. Considering the fact that the amplitude and the phase vary slowly, this equation may be written in the following form:

$$\frac{d^2X}{dt^2} \approx -\frac{ek}{m}\varphi_0(x)\cos(\omega_k t - kx + \Psi(x)). \qquad (7,10)$$

If we find from Eq. (7, 10) the functions $v^{(0)} = v^{(0)}(x, v, t)$ and $t^{(0)} = t^{(0)}(x, v, t)$, the solution of Eq. (7, 7) may be written in the form

$$f_1(x, v, t) = f_1^{(0)}(v^{(0)}(x, v, t)).$$

Using this solution, we may find expressions for the current and the density

$$j = -e\int v f_1^{(0)}(v^{(0)}(x, v, t))\, dv;$$

$$\varrho = -e\int f_1^{(0)}(v^{(0)}(x, v, t))\, dv.$$

If we substitute the expression thus found for ρ into the right-hand side of Eq. (7, 8), we obtain the non-linear equation for the potential. At low electron-beam intensities, the right-hand side of this equation is small, and hence we may use a well-known method in the theory of non-linear oscillations to find simpler equations for the amplitude and the phase of the wave. In order to do this, we must find the Fourier components of the density ϱ considering the amplitude and the phase of the beam to be constant during the integration. Let $\rho = \rho^{(1)}\cos(\omega_k t - kx) + \rho^{(2)}\sin(\omega_k t - kx)$. Then

$$\varrho^{(i)} = -\frac{e}{\pi}\int_0^{2\pi}\int \begin{matrix}\cos\\ \sin\end{matrix}[\omega_k t - kx] \times f_1^{(0)}(v^{(0)}(x, v, t))\, dv\, d(kx), \qquad i = 1, 2. \qquad (7,11)$$

Eq. (7, 11) may be simplified by using the Liouville theorem $dx\, dv = dx^{(0)}\, dv^{(0)}$; or since $dx = v\, dt$, and $dx^{(0)} = v^{(0)}\, dt^{(0)}$, $dx\, dv = v^{(0)}\, dt^{(0)}\, dv^{(0)}$. Replacing

the variables t and v in Eq. (7, 11) by $t^{(0)}$ and $v^{(0)}$, we obtain the following expression for the Fourier components of the density:

$$\rho^{(i)} = -\frac{e}{\pi v_p}\int_0^{2\pi}\int \begin{matrix}\cos\\ \sin\end{matrix}[\omega_k t(t^{(0)}, v^{(0)}, x) - kx]\, f_1^{(0)}(v^{(0)})\, v^{(0)}\, dv^{(0)}\, d(\omega_k t^{(0)}). \tag{7, 12}$$

Eq. (7, 12) permits us to find $\rho^{(i)}$ when the function $t = t(t^{(0)}, v^{(0)}, x)$ is known. In order to determine this function, we refer to the equation of motion (7, 10). If we consider that the variable t on the right-hand side of this equation is a known function of the coordinates $v^{(0)}$ and $t^{(0)}$, then the integral in Eq. (7, 10) may be represented in the form

$$v = v^{(0)}\left\{1 - 2\frac{ek}{mv^{(0)2}}\int_0^x \varphi_0(x')\cos[\omega_k t^{(0)} + \omega_k(t - t^{(0)}) - kx' + \Psi(x')]\,dx'\right\}^{1/2},$$

and we thus obtain the integral equation for the function $t = t(t^{(0)}, v^{(0)}, x)$:

$$t - t^{(0)} = \frac{1}{v^{(0)}}\int_0^x \left\{1 - \frac{2ek}{mv^{(0)2}}\int_0^{x'} \varphi_0(x'') \times \cos[\omega_k t^{(0)} + \omega_k(t - t^{(0)}) - kx'' + \Psi(x'')]\,dx''\right\}^{-1/2} dx'. \tag{7, 13}$$

Due to the complexity of Eq. (7, 13), the solution of the problem without the application of numerical methods may be carried out approximately only in certain special cases; these cases are primarily of theoretical significance. We shall discuss some of them.

Let μ denote a small parameter. We shall consider the case in which the steady-state value of the amplitude of vibration is such that $e\phi_0/mv^{(0)2} \sim \mu^2$, and the maximum excitation is found for a wave for which $(v^{(0)} - v_p)/v^{(0)} \sim \mu$. Under these conditions, and under the condition that the amplitude and phase of $\phi_0(x)$ change slowly, $\Psi(x)$ in Eq. (7, 13) may be simplified by series expansion of the square root, retaining only the first and second terms of the series. As a result, Eq. (7, 13) acquires the form

$$t - t^{(0)} = \frac{x}{v^{(0)}} + \frac{ek}{mv^{(0)3}}\int_0^x (x - x')\,\varphi_0(x')\cos[\omega_k t^{(0)} + \omega_k(t - t^{(0)}) - kx' + \Psi(x')]\,dx'. \tag{7, 14}$$

Eq. (7, 14) contains two length parameters. One of these, $A = k(v^{(0)} - v_p)/v^{(0)}$, is determined by the difference between the velocity of the electron beam and the phase velocity of the most rapidly growing plasma wave at $x = 0$. The other, α, characterizes the rate of change of the amplitude and phase of the wave. We shall now discuss the case in which $\alpha/A \sim \mu$.

We shall designate the energy ratio $e\phi_0/m(v^{(0)} - v_p)^2$ by X and find an approximate solution of Eq. (7, 14) as a power series in X, assuming that $x \lesssim 1$. Retaining the terms up to X^3 inclusively, we obtain the following expression for $t - t^{(0)}$:

$$\omega(t - t^{(0)}) = \frac{\omega x}{v^{(0)}} - \left(X - \frac{5}{16}X^3\right)\cos[\omega t^{(0)} - Ax + \Psi] - \frac{X^2}{8}\sin 2[\omega t^{(0)} - Ax + \Psi].$$

We now substitute this expression into the integrand of Eq. (7, 11), and integrate over $t^{(0)}$, and retain again the terms in X up to X^3, making use of the formula

$$\int \frac{f_1(v - \bar{v})}{(v - v_p)^n}\,dv = \int \frac{f_1(v - v)}{(v - v_p)^n}\,dv + i\pi\frac{1}{(n-1)!}\left(\frac{\partial^n f_1}{\partial v^n}\right)_{v = v_p}, \tag{7, 15}$$

in which the symbol $\int$ indicates that the integral is taken in the sense of its principal value. Keeping in mind the fact that $\rho = \rho_1 \cos(\omega_k t - kx) + \rho_2 \sin(\omega_k t - kx)$, we obtain as a result the following expression for the density of the beam:

$$\varrho = -\frac{e^2}{m}\int\left(1-\frac{3}{8}\left(\frac{e\varphi}{m(v-v_p)^2}\right)^2\left(\frac{v_\Phi}{v}\right)^2\right)\frac{f_1(v-\bar{v})}{(v-v_p)^2}\,dv\varphi + \\ + \frac{\pi e^2}{mk}\left[\left(\frac{\partial f_1}{\partial v}\right)_{v=v_p} - \frac{3}{8\cdot 4!}\left(\frac{e\varphi_0}{m}\right)^2\left(\frac{\partial^5 f_1}{\partial v^5}\right)_{v=v_p}\right]\frac{\partial\varphi}{\partial x}. \qquad (7,16)$$

Substituting this expression into the right-hand side of Eq. (7,8), we obtain the non-linear equation for the potential under the condition that the solution of this equation is sought in the form (7,9).

If the ratio of the concentrations of electrons in the beam and the plasma is such that the parameter of the slowness of change of the amplitude and phase of the wave is of the same order of smallness as the parameter characterizing the smallness of the right-hand side of the equation for ϕ, then, on equating terms of the same order of smallness, we obtain the following equations for the amplitude and phase of the wave:

$$\frac{d\varphi_0}{dx} = \alpha\varphi_0 - \beta\varphi_0^3, \qquad (7,17)$$

$$\frac{d\Psi}{dx} = -\frac{2\pi e^2}{mk}\int\left[1-\frac{3}{8}\left(\frac{e\varphi_0}{m(v-v_p)^2}\right)\left(\frac{v_\Phi}{v}\right)^2\right]\frac{f_1(v-\bar{v})}{(v-v_p)^2}\,dv. \qquad (7,18)$$

In Eq. (7,17), the notation has been introduced

$$\alpha = \frac{2\pi^2 e^2}{mk}\left(\frac{\partial f_1}{\partial v}\right)_{v=v_p} - \frac{\gamma}{v_p}; \qquad \beta = \frac{3}{8\cdot 4!}\frac{2\pi e^2}{mk}\left(\frac{e}{m}\right)^2\left(\frac{\partial^5 f_1}{\partial v^5}\right)_{v=v_p}.$$

For self-excitation of the oscillations to occur, it is necessary that the coefficient α should be positive. If the value of the damping coefficient is basically determined by the damping factor of the plasma oscillations, i. e., collisions play only a small role, the condition for self-excitation may be written in the form $(\partial f^{(0)}/\partial v)_{v=v_p} > 0$, where $f^{(0)}$ is the distribution function for all the electrons (plasma and beam) at $x = 0$. This condition of self-excitation corresponds to that derived in references 12, 27, 99, and 100.

We shall now concretize the form of the distribution function $f_1^{(0)}$. With a sufficient degree of accuracy, it may be apparently be given in the form

$$f_1^{(0)} = n_1\left(\frac{m}{2\pi\varkappa T_1}\right)^{1/2}\exp\left[-\frac{m(v-\bar{v})^2}{2\varkappa T_1}\right].$$

In this expression n_1, T_1, and $\bar{v}$ are the concentration, temperature, and velocity of the beam electrons. With such a choice of the boundary distribution function, the condition of self-excitation is satisfied in the region of phase velocities $\Delta v_p \sim \sqrt{\varkappa T/m}$. The coefficient α is at a maximum for a wave with a phase velocity such that $\bar{v} - v_p = \sqrt{\varkappa T_1/m}$. Since $\bar{v}$ and $v_p >> \sqrt{\varkappa T_1/m}$, and $v_p = \omega_k/k \approx \omega_L/k$, we find that the value of the wave number of the most rapidly growing wave is $k \approx \omega_L/\bar{v}$. For this wave, the coefficients α and β have the form

$$\alpha = k\left[\sqrt{\frac{\pi}{22}}\frac{n_1}{n}\frac{\bar{v}^2 m}{\varkappa T_1} - \frac{\gamma}{\omega_L}\right]; \quad \beta = \frac{3}{32}\sqrt{\frac{\pi}{22}}\frac{n_1}{n}\bar{v}^2\frac{me^2}{(\varkappa T_1)^3}.$$

It follows from the expression for α that, for the assigned values of the parameters $\bar{v}$; T_1, and n, there always exists a lower limit for the values of the concentration of the electrons in the beam for which self-excitation of the oscillations is possible. With lower values of the concentration, the transition to the equilibrium velocity distribution during motion of the electrons through the plasma takes place by relaxation processes alone, as described in the first part of this article.

The solution of Eq. (7,17) for the amplitude may be written in the form

$$\varphi_0(x) = \frac{\varphi^{(0)}e^{\alpha x}}{\sqrt{1+\frac{\alpha}{\beta}\varphi^{(0)2}(e^{2\alpha x}-1)}}. \qquad (7,19)$$

In this expression, $\phi^{(0)}$ is the value of the amplitude at $x = 0$. We shall denote by the symbol ϕ_{ss} the steady-state value of the amplitude. For small values of x ($x << 1/\alpha$), the solution of Eq. (7,19) increases exponentially with increase in x. For large values of x, the amplitude approaches the value

$$\varphi_{ss} = \sqrt{\frac{\alpha}{\beta}}. \qquad (7,20)$$

If the value of γ is so small that the second term in the expression for α may be neglected, then $e\phi_{ss} \approx 3\kappa T_1$. We may see from the derived formulas that the steady-state value of the amplitude of the wave approaches zero as $T_1 \to 0$, i. e., in the case of a beam of uniform velocity. It follows from the expression for α that in this case the condition of self-excitation of the waves is not satisfied. Of course, this does not mean that, when a beam of uniform velocity passes through a plasma, plasma waves do not appear. The fact here is that the solution discussed here was obtained under the condition $\alpha \ll A$, $X \sim 1$. Under this condition, plasma waves in the hydrodynamic approximation actually do not appear. In order to describe the appearance of waves in this case, we must study the solution under the condition that $\alpha \sim A$.

We shall now discuss Eq. (7,18) for the change in phase of the wave. This equation determines the change in the wave number of the plasma wave with increasing x ($d\Psi/dx = -\Delta k$). It follows from Eq. (7,18) that Δk is determined by two terms, one of which depends on the amplitude. They are both of the order of $\mu^2 k$, and thus are small in comparison with the initial difference between the velocity of the beam electrons and the velocity of the wave, which is of the order of μ. We shall estimate now the distance at which the energy of the beam electrons is transformed into the energy of plasma oscillations. In order to do this, we shall find the value of the ratio of the flux of electrical energy of the plasma waves in the region in which a steady-state of the wave has already been established to the value of the energy of the electron beam at $x = 0$. This ratio is equal to

$$\frac{s_{\text{wave}}}{s_{\text{el}}} = \frac{n}{n_1}\left(\frac{e\varphi_0}{m\bar{v}^2}\right)^2. \qquad (7,21)$$

We shall estimate the order of magnitude of this ratio. Since $e\phi_0/mv_0^2 \sim \mu^2$, and $\alpha/k \sim \mu^2$, then it follows from the expression for α that the ratio of concentrations of electrons of the beam and the plasma $n_1/n \sim \mu^4$. From Eq. (7,21) we find that, with these values of the parameters, the ratio of energy fluxes is the order of unity. Thus, the beam transfers its energy into the excitation of plasma oscillations at a distance l, of the same order of magnitude as the distance at which the steady-state is established. If we designate the wavelength of the plasma wave as λ, then $\lambda \ll l \gtrsim 1/\alpha$. For currents of 20–50 ma, $\kappa T_1 \sim 1$ ev, $m\bar{v}^2/2 \sim 20$ ev, $n_1 \sim 10^8$, and $n \sim 10^{10}$, the distance l is of the order of a centimeter. At the same time, the relaxation distance calculated from the same data by Eq. (1,2) is equal to about 10^5 cm.

In a paper of Looney and Brown,[104] standing waves were found in a plasma upon passage of an electron beam through it. The standing waves appear in the presence of a reflecting electrode.

In order to find the conditions for the appearance of standing waves upon passage of an electron beam through a plasma, a solution for the electric field may be sought, for example, in the form

$$\varphi_s = \varphi_0(t)\sin(\omega t + \Psi(t))\sin\frac{s\pi x}{L}, \qquad s = 1, 2, \ldots,$$

where $\phi_0(t)$ and $\Psi(t)$ are the amplitude and phase of the wave, slowly varying with time, and L is the length of the region along the direction of motion of the beam. Calculation shows that, for $n_1/n \ll 1$, the condition of self-excitation is best satisfied for the frequencies and wavelengths determined by the formulas

$$\omega_L \approx \frac{s\pi\bar{v}}{L}; \qquad \lambda_s \approx \frac{2L}{s}, \qquad s = 1, 2\ldots$$

It follows from these formulas that the transition from the fundamental mode of oscillation to a higher one may take place either on increasing the concentration of electrons in the plasma or on decreasing the mean velocity of the beam electrons.

Under the experimental conditions of Looney and Brown, the basic factor is the concentration of electrons in the beam, i. e., $n_1/n \gg 1$. This case may also be analyzed. Under these conditions, the square of the frequency of the oscillations for a given mean velocity will be proportional to the concentrations of electrons in the beam, or the current.

In a linear approximation, the theory of the excitation of standing waves in a plasma for any ratio n_1/n has been discussed in references 105 and 106.

We shall note that another sort of spatial periodicity has been observed in the well-known paper of Merrill and Webb[103] (see also the new references 107, 108, and 109). There is no opportunity to spend any time here on this question. We only note that the value of the spatial periodicity in this case is determined by the wavelength of the most rapidly growing plasma wave, and is equal to

$$\lambda = 2\pi\frac{\bar{v}}{\omega_L}\left(1 - \sqrt{\frac{\kappa T_1}{m}}\Big/\bar{v}\right).$$

The results of the experiments of Merrill and Webb also indicate the presence of growing waves in the plasma. The value of λ obtained from the formula given above agrees well with the value of the spatial periodicity found in this paper.

As is known, plasma waves may also appear due to the energy of the relative motion of the electrons and ions. This effect may give an appreciable contribution to the temperature increase in a plasma when a strong pulse of electric current is passed through it. With a pulse duration of $\sim 10^{-3}$ sec, establishment of the plasma oscillations will be possible. The rate of damping of these oscillations will determine the value of the energy transformed directly into heat.

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Translated by M. V. King

Translator's Comments

This article is not a complete review of the subject of energy losses of charged particles passing through matter, but rather, is a rather thorough summary of the development of one particular theoretical treatment of this subject, in which the authors have made numerous contributions. This treatment uses the concept of the quantum distribution function, as developed by Wigner[1] and Moyal[2], together with the concept of collective excited states, as developed by Bohm et al.[3, 4, 5] These concepts are applied to the problem of the energy losses incurred by charged particles as a result of excitation of plasma oscillations, both in plasmas and in metals, and are used to explain such phenomena as the Langmuir paradox. The phenomena are treated in the self-consistent-field approximation, and then, considerable discussion is devoted to the influences of exchange and Coulomb correlations on the interacting systems.

The treatment of losses in thin metallic films is not exhaustive, but covers primarily the subject of discrete losses, in view of the interest of the authors in the theoretical application of the concept of collective oscillations to this phenomenon. One section covers the phenomenological theory of the Fermi fluid, including such predicted phenomena as that of zero sound. The final section discusses the case in which the interaction of an electron beam with a plasma is far stronger than that calculated for the interaction of single charged particles with the plasma, by virtue of the excitation of sustained plasma oscillations, such that the plasma can no longer be assumed to have its original properties, e. g., dielectric constant. The non-linear theory applicable to this case is outlined.

Thus, the article is a valuable summary of the theoretical work of the authors on the subject of energy losses, in view of the fact that much of this work is not readily available in many American libraries, especially since their earlier papers were published before the period in which translated Russian journals became available. In addition, the authors have drawn on a wide variety of other work pertinent to their main subject, including both theoretical and experimental papers, recognizing the implications of their treatment over a wide range of topics. Thus, the 109 cited references form an extensive, although by no means exhaustive bibliography of the subject, being chosen, in general, for their pertinence with regard to the theories being developed, rather than to the entire subject of energy losses of charged particles.

An idea of how much of the field has been covered in the article, and how much omitted, may be gained by comparison with a recently-published collected volume,[6] since some of the sections of this volume cover very similar topics to those covered by the article, although not with the same theoretical treatment.

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