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PROCESSES OF FRAGMENTATION AND FISSION IN INTERACTIONS BETWEEN HIGH ENERGY PARTICLES AND NUCLEI

N. A. Perfilov, O. V. Lozhkin, and V. P. Shamov

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INTRODUCTION

Nuclear reactions induced by $10^2 - 10^4$ Mev particles are best described at present by the "cascade-evaporation" model,[1] according to which a cascade of nucleon-nucleon collisions is initiated in the nucleus by an incoming fast particle. Since the de Broglie wavelength for a particle of energy of several hundred Mev is considerably smaller than the dimensions of the nucleus, one can visualize a particle trajectory within the nucleus and collisions between the particle and the individual nucleons of the nucleus. Since the time of collision of the incident particle with the nucleons is considerably shorter than the time between collisions of nucleons within the nucleus, the nuclear cascade has time to develop in the unexcited nucleus at $T = 0$. The existence of an aggregate of occupied states for the intranuclear nucleons leads to definite exclusion rules for the energy transferred in the collisions, and increases the mean free path of the fast particle in the nucleus.

The collision between an incident particle and the nuclear nucleons gives rise to an intranuclear cascade of fast protons and neutrons (and even pions if the incident particle has a high energy) and to the formation of a residual nucleus in an excited state; the excitation energy is determined by the nature of the development of the intranuclear cascade and can vary from very small values up to the total energy of the incident particle.

If the interaction between the incident particle and the nucleus is pictured in this manner, the disintegration products can be of two types: knock-on particles emitted from the nucleus during the development of the cascade, and evaporated particles, emitted upon de-excitation of the residual nucleus. If the excitation energy of the residual nucleus is larger than the threshold for fission at a given Z^2/A ratio, the fission reaction will also have a definite probability. Consequently, when a nucleus of mass A is bombarded by protons of several hundred Mev we obtain a large collection of new nuclei with masses ranging from $A + 1$ (disregarding secondary reactions) to unity, distributed in accordance with a definite law, depending on the energy of the acting particles and the mass number of the target nucleus.

The representation introduced by Serber[1] for the mechanism of the first stage of the nuclear reaction with fast particles allows us to predict, by using a Monte Carlo technique, the average number of particles of the nuclear cascade, their angular and energy distributions, and the distribution of the residual nuclei by excited states. The principal outlines of the computation methods were given by Goldberger[2] and used by him to calculate the nuclear cascade in the interaction between 90-Mev neutrons and heavy nuclei. Similar calculations were made later by others[3-8] for interactions between high-energy particles and nuclei. Recently nuclear-cascade calculations were made with the aid of computers for various incident-particle energies (up to 2 Bev) and different target nuclei, with allowance for meson production in the nucleus.[9]

The results of the latest calculations reduce briefly to the following:

1) The average number of cascade nucleons per inelastic interaction between a particle and the nucleus is independent of the target material (from Al to U), but increases rapidly with increasing incoming-particle energy, approximately from 3 at 0.4 Bev to 8 at 2 Bev.

2) Production of pions in nuclear interactions has approximately equal probability for all target elements at a given energy of incident protons, but increases by approximately one order of magnitude as the energy

increases from 0.4 to 2 Bev.

3) The average excitation energies of the residual nuclei increase with increasing energy of the incident particles and with increasing mass of the target nucleus.

The results obtained are in satisfactory agreement with the experimental estimates of the excitation energies of the residual nuclei, the number of cascade particles, and their angular distributions. The results of calculations for the evaporation from residual excited nuclei[10 – 11] are also in sensible agreement with the experiment. Calculations for the evaporation cascade were made recently with a computer,[12] and satisfactory agreement was obtained with experiment. In view of this, particular interest attaches to an examination of those features of high-energy nuclear reactions, which do not fit within the described "cascade-evaporation" model. These features include the production of multiply-charged particles with $Z \geqslant 3$, known for quite a long time but still unexplained. A study of these features shows at high energy of incident particles that the Serber "cascade-evaporation" model for nuclear reactions should be supplemented by additional processes. Thus, a special process, called fragmentation, was proposed as early as in 1953 – 1954; to interpret nuclear reactions that lead to the production of multiply-charged particles, there is no unified point of view concerning the mechanism of this process, let alone the fact that the very proposal of a new mechanism for nuclear disintegration requires additional experimental proof. We consider it therefore of interest to review the available data on the production of multiply-charged particles in nuclear disintegrations, and the first part of this article is indeed devoted to this topic. The second part is devoted to an examination of the mechanism of nuclear fission at high excitation energies – phenomena which occupy a unique place among the processes that occur in interactions between high-energy, particles and nuclei.

1. FRAGMENTATION

Since we cannot outline beforehand the conditions under which fragmentation, as a special source of nuclear disintegration products, takes place, we must examine fully all the problems connected with the production of multiply-charged particles in the disintegration of complex nuclei. For the sake of convenience we shall use the term "fragmentation" to cover all cases of production of multiply-charged particles with $Z \geqslant 3$ in nuclear disintegrations, regardless of the mechanism by which they are produced; we shall call these particles "fragments."

By way of an example of nuclear disintegrations accompanied by the emission of such fragments as will be discussed here, Fig. 1 shows a microphotograph of the disintegrations of heavy nuclei in an emulsion, induced by 660-Mev protons. In these disintegrations, multiply-charged particles – the fragments – are emitted in addition to the α particles and protons.

1. Cross Section of the Fragmentation Process

The first observations [13 – 26] of multiply-charged particles among products of nuclear reactions, obtained in investigations of nuclear disintegrations produced by cosmic rays in nuclear emulsions and then in the investigation of disintegrations produced by artificially accelerated particles,[27 – 30] have shown that the disintegrations with multiply-charged particles with $Z \geqslant 3$ are quite rare. The difficulty of working with small cross sections explains the fact that we still do not have enough experimental data on the cross sections for the production of various isotopes of light nuclei under different conditions. The factual material on the cross sections for the production of various fragments is listed in Table I. In its compilation we confined ourselves only to proton bombardment and only to targets in the

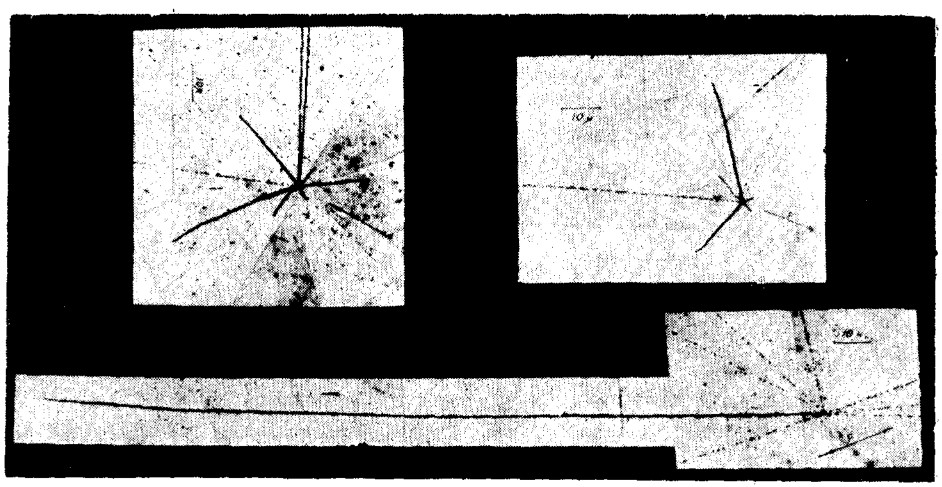

FIG. 1. Microphotographs of nuclear disintegrations, accompanied by emission of fragments.

range from aluminum to uranium, without touching upon the lighter nuclei, where the fragmentation is obscured by the process of formation of residual nuclei.

An examination of the experimental data shows that the process of fragment formation in nuclear disintegration depends greatly on the energies of the incident particles for all atomic numbers of the target nuclei. This process has low probability at incident-particle energies of several hundreds Mev, but begins to play a noticeable role at energies on the order of 1 Bev. The cross sections for the production of isotopes of light nuclei increase by a factor of approximately $10^2 - 10^3$ in the energy region under consideration. It is seen from Fig. 2, on which is shown the dependence of the cross section for the production of F^{18} and Na^{24} on the energy of the incident protons in the disintegration of Cu, Ag, and Pb, that in the energy region near 1 Bev the production of multiply-charged particles depends most strongly on the energy of the incident particles. At proton energies greater than 2 or 3 Bev, the increase in the cross section for production of multiply-charged particles slows down.

The dependence of the total cross section for the production of multiply-charged particles on the energy of the incident protons was obtained by the nuclear emulsion method.[51] Fig. 3 shows this dependence for fragments with $Z \geq 4$, which arise in the disintegration of silver or bromine nuclei. The strong increase in the number of disintegrations with fragments at proton energies greater than 400 or 500 Mev is seen from this figure.

The general increase in the yield of multiply-charged particles with increasing energy of bombarding particles is found to be connected with the increase in the total energy transferred to the nucleus in the collision with the proton. This connection can be characterized by a dependence of the probability of observing a disintegration with a fragment on the total number of all the particles produced in the disintegration. This was first shown by Perkins[52] in a study of disintegrations produced by cosmic rays, and subsequently confirmed by research on disintegrations produced by 300 – 660 Mev[53,80] and 6.2 Bev[55] protons. Figure 4 shows data on the dependence of the probability of disintegration accompanied by a multiply-charged particle on the number of charged particles in the disintegration, the latter including only the relatively low-energy products, i.e., protons with energies less than 30 Mev and α particles of all energies. The total number of such particles in the disintegration characterizes the energy transferred in the collision. When the energy transferred by collision becomes of the order of magnitude of the total binding energy of the nucleus, the probability of emitting a fragment becomes a quantity on the order of unity. Thus, the increase in the yield of multiply-charged particles with increasing energy of the incident particles may be due to the increase in the relative fraction of large energy transfers to the nucleus with increasing particle energy, a fact that follows both from data on the prong composition of the stars obtained by the nuclear emulsion method,[53], and from data on the yield of isotopes that have values of Z and A quite different from the initial nucleus and obtained by radiochemical means.[38,42,43,55] The same follows from Monte Carlo calculations of the nuclear-cascade process.[9]

However, the number of multiply-charged particles produced in the disintegrations is found to be connected not only with the total energy transferred, but also with the total number of cascade particles in the disintegration.[53,55] As the total number of cascade particles in the disintegration increases, the probability of observing disintegrations with fragments also increases. It has been found at the time that multiply-charged particles appear with equal probability in stars with many and few shower particles (i.e., fast pions) in the disintegrations produced by cosmic rays.[52]

The fact that a transfer of large energy is required to make possible the production of a multiply-charged particle in a disintegration is manifest also in the difference in the distribution of disintegrations with fragments over the total number of emitted particles, relative to the analogous distribution for ordinary disintegrations at a given incident-particle energy. Disintegrations with multiply-charged particles have a much larger average number of emitted charged particles than ordinary disintegrations. Fig. 5 shows distribution data for 660 Mev incident protons.[53,54,58] The average number of emitted charged particles in an ordinary disintegration is 3.5, and the average number of emitted particles in disintegrations with fragments is equal to six. When the incident-particle is 460 Mev, these values are 2.6 and 4.8 respectively.[54,53] From an examination of the data of Table I one can draw definite conclusions on the dependence of the cross section for the production of radioactive isotopes of light nuclei on the atomic number of the target nucleus. Figures 6 and 7 show the dependence of the cross sections for the production of light isotopes on the mass number of the target nucleus, as obtained in various investigations. As can be seen from Fig. 6, in the range of mass numbers up to 200, the cross section for the production of light isotopes drops significantly with increasing target mass number at proton energies less than 3 Bev, and the drop is more clearly pronounced at low incident-particle energies, both for the case of Be^7 and for F^{18}. When the energy of the bombarding protons is 2.2 Bev, as noted by Katkoff,[75] the cross section for the production of Li^8 increases with the atomic number of the target nucleus: the cross section rises from approximately 1×10^{-27} cm^2 to 10×10^{-26} cm^2 in the range of mass numbers from Al to U. At the same time, as can be seen from Fig. 6, where Wright's data are

TABLE I

Cross Section for the Production of isotopes of light nuclei

Target nucleus	Isotope produced	Proton						
		120	220	340	370—390	410—420	480	600
Al	Be^7			140 [36]				
	C^{11}			190 [36]		280 [49] 290 [47]		330 [47]
	N^{13}					97 [49] 75 [47]		86 [47]
	O^{15}							650 [47]
	F^{18}			550 [36]		700 [47]		690 [47]
Fe	Na^{22}			2 [48]				
	Na^{24}			2,6 [48]				
	Si^{31}			12 [48]				
	P^{32}			4.4 [48]				
Cu	Be^7			50 [36]				
	C^{11}				3.0 [32]		5.7 [38]	
	F^{18}				3.0 [32]	8.0 [37]		
	Na^{22}			3.7 [33–35]				
	Na^{24}	0.09 [31]	0.22 [31]	1.3 [31] 3.0 [33–35]	3.0 [32]		5.6 [31] 5.0 [38]	
	Mg^{28}							
	P^{32}	0.07 [31]	0.22 [31]	1.8 [31] 12 [33–35]			24 [31]	
	P^{33}							
Ag	Be^7			10 [36]				
	C^{11}			1.0 [46]			~3.0 [41]	
	F^{18}			1.0 [46]		1.6 [37]		
	Na^{24}			1.0 [46]			3.0 [41]	
	Mg^{28}			0.1 [46]				
	P^{32}						3.0 [41]	
La	Na^{24}	0.099 [31]	0.3 [31]	0.5 [31]			2 [31]	
	P^{32}			0.73 [31]			1.4 [31]	
Ta	Na^{24}			0.6 [46]				
	Mg^{28}			0.35 [46]				
Au	Be^7			1.4 [36] 1.0 [46]				
	C^{11}						0.67 [68]	
	F^{18}					0.44 [37]		
	Na^{24}		0.59 [31]	0.13 [31]			2.7 [68] 3.7 [31]	
	P^{32}			0.3 [31]			0.34 [68] 1.1 [31]	
Pb	F^{18}							0.5 [13]
	Na^{22}							
	Na^{24}				3.0 [43]			<2 [43]
	Mg^{28}				≤0.1 [43]			2.4 [43]
	P^{32}				≤1.0 [43]			~2 [43]
	P^{33}				≤1.0 [43]			

in interactions between protons and nuclei, $\sigma \times 10^{29} cm^2$

energy, Mev								
660—680	1000	1400	1600	2000	2200	3000	4500	5900
	760 [47]	830 [47]			1270 [47]	1160 [47]		
	540 [47]	560 [47]			610 [47]	660 [47]		
		160 [47]			180 [47]	120 [47]		
		760 [47]			650 [47]	760 [47]		
	760 [47]	670 [47]			710 [47]	700 [47]		
	500 [50]				1000 [42]	1190 [50]		
	370 [50]							
					⩾65 [42]			
	51 [44]			140 [44]	100 [42]	170 [44]	250 [44]	340 [44]
					180 [42]			
25 [31]	100 [44]			330 [44]	320 [42]	400 [44]	460 [44]	480 [44]
32 [38]								
					41 [42]			
95 [39]					640 [42]			
31 [31]								
					90 [42]			
	290 [50]				1130 [50]	1210 [50]		
	220 [50]							
	20 [44]			55 [44]		170 [44]	190 [44]	150 [44]
	30 [44]			140 [44]		270 [44]	410 [44]	330 [44]
21 [31]								
	20 [44]			85 [44]		230 [44]		870 [44]
	170 [50]				590 [50]	840 [50]		
	100 [50]							
	7 [44]			25 [44]		73 [44]	170 [44]	250 [44]
8.1 [31]	44 [44]			220 [44]		500 [44]	740 [44]	940 [44]
2.2 [31]								
	5.0 [44]		18 [43]	49 [44]	39 [43]	83 [44]	110 [44]	140 [44]
	3.9 [43]							
			<220 [43]		<150 [43]	<270 [43]		
	36 [43]		140 [43]	230 [44]	230 [43]	360 [43]	720 [44]	920 [44]
	36 [44]							
	7.5 [43]		27 [43]		48 [43]			
	9.0 [43]		31 [43]		80 [43]			
	~1 [43]		~10 [43]		~20 [43]			

Cross Section for the Production of isotopes of light nuclei

Target nucleus	Isotope produced	Proton						
		120	220	340	370—390	410—420	480	600
Th	Na^{24} P^{32}						18 [31] 12 [40] 3 [31] 2,8 [40]	
U	F^{18} Na^{24} P^{32}						4,8 [40] 0,75 [40]	

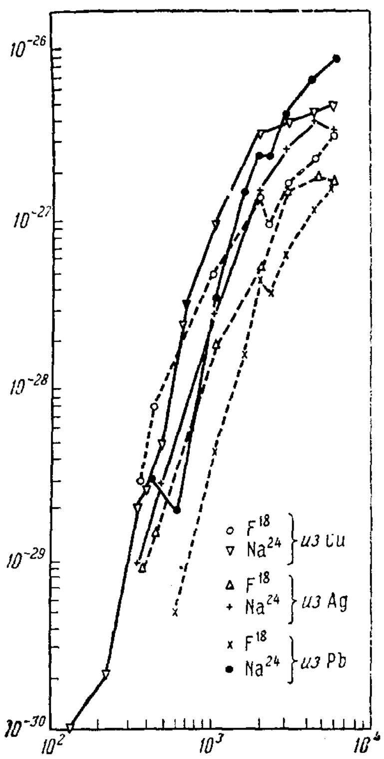

FIG. 2. Dependence of the cross section production of F^{18} and Na^{24} on the incident-proton energy in the disintegration of Cu, Ag and Pb.

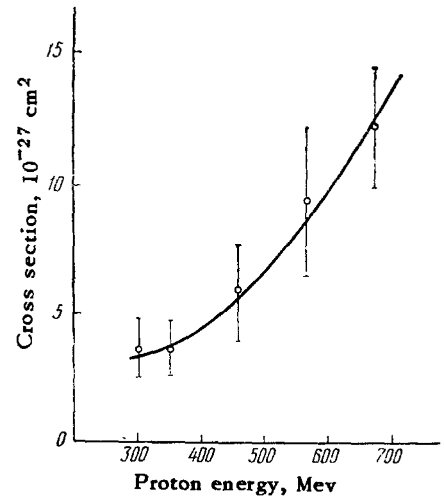

FIG. 3. Dependence of total cross section of production of fragments with $Z > 4$ on the energy of the incident protons.[51]

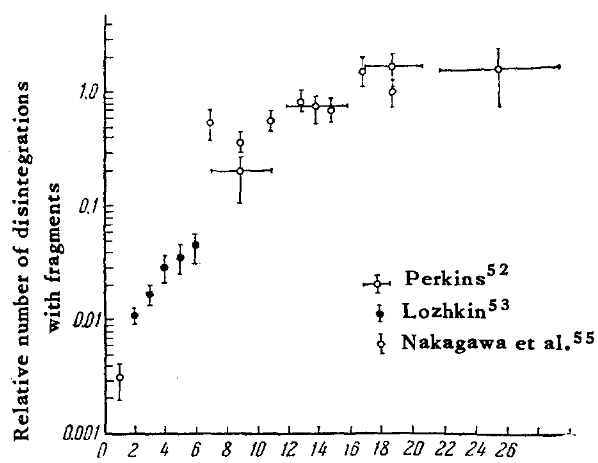

FIG. 4. Dependence of probability of production of disintegration with fragment on the number of charged particles emitted in the disintegration.

shown[27], the cross section for production of Li^8 by 340-Mev protons decreases rapidly with increasing target mass number. Thus, the example of Li^8, Be^7, and F^{18} discloses a clear cut connection between the bombarding-particle energy and the dependence of the fragment-production cross section on the target mass number.

As can be seen from Fig. 7, the drop in the cross section for production of Na^{24} in the range of target mass numbers from 64 to 180 is much less than for the lighter isotopes (Fig. 6). When the incident particles in interactions between protons and nuclei, $\sigma \times 10^{29} \mathrm{cm}^2$

energy, Mev								
660—680	1000	1400	1600	2000	2200	3000	4500	5900
	13[44] 63[44]			55[41] 290[44]		140[44] 590[44]	200[44] 1130[44]	320[44] 1200[44]

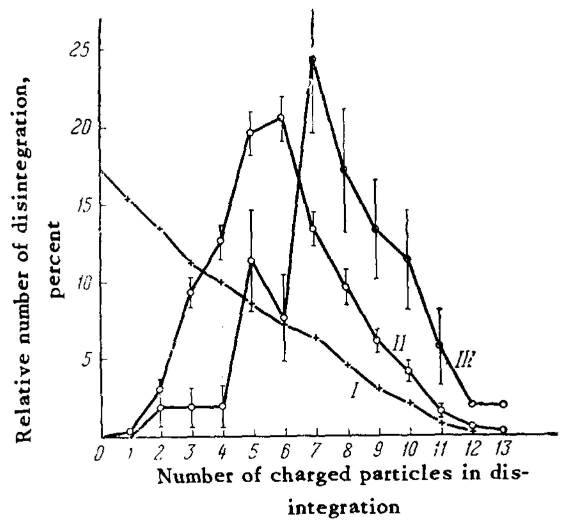

FIG. 5. Distribution of disintegrations of silver and bromine nuclei by 660-Mev proton by the number of emitted charged particles: I – disintegrations without fragments;[54] II – disintegrations with one fragment,[58] III – disintegrations with two fragments.[53]

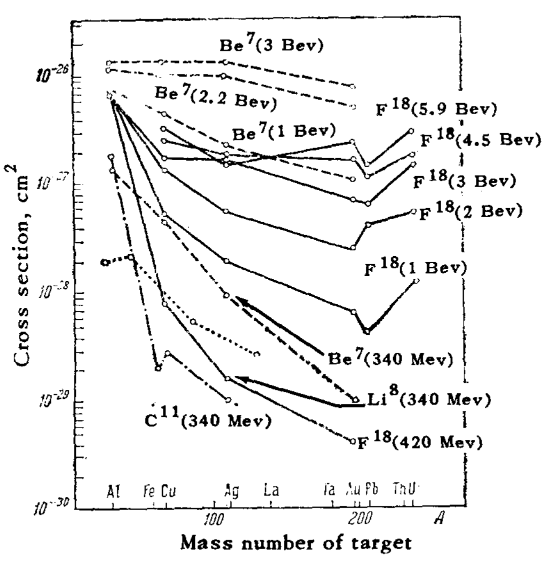

FIG. 6. Dependence of the cross section of production of light isotopes on the mass number of the target nucleus at different incident-proton energies.

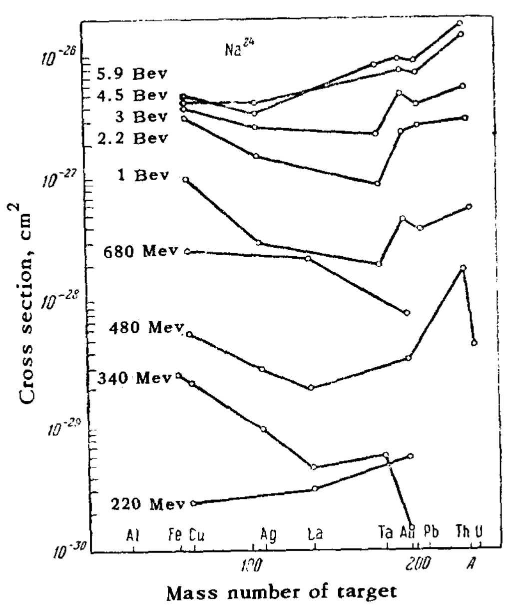

FIG. 7. Dependence of the cross section of production of Na^{24} on the mass number of the target at different incident-proton energies.

have energies of several Bev, to the contrary, a certain increase is seen in the Na^{24} cross section with increasing target mass number. A much stronger increase in the cross section for the production of the investigated light isotopes is observed in the region of target mass numbers from 200 to 238, over a broad range of incoming particle energies. In addition, it follows from the same drawings that the cross section for the production of relatively light nuclides (such as Be^7 or F^{18}) in nuclear disintegrations increases with energy of the incident particle much stronger in the region of heavier nuclei, than in the region of the lighter targets. This situation does not prevail in the case of Na^{24}, where the cross section increases just as much for medium target nuclei as for light ones. The foregoing dependence of the cross section for the production of several light isotopes in the disintegration of medium and heavy nuclei still needs experimental confirmation, since most of the data known at present pertain to F^{18} and Na^{24}. The cross sections for the production of lighter nuclei at various incident-particle energies and various targets have been investigated but little. There exists, furthermore, certain contradictions among the available experimental data; an example is the dependence of the cross section for Na^{24} production at proton energies of 220 and 340 Mev (Fig. 7).

The dependence of the total cross section for the production of multiply-charged particles on the atomic number of the target for 660 Mev incident particles was determined also by the nuclear-emulsion method.[51] In that reference, the increase in the total cross section for the production of fragments with $Z > 4$ with energies $> 2 - 3$ Mev nucleon was determined for the region of target mass numbers from 30 to 200. It is concluded in references 59 and 60, from a study of the secondary reactions produced by the Li in the bombardment of the targets by 480-Mev protons, that the cross section for the production of Li remains constant in the disintegration of various nuclei (from Cu to Pb). The investigation of the special cases of uranium fission with production of a light multiply-charged particle, cited in references 61 and 62, indicates that the ratio of the cross section for the production of multiply-charged particles to the total inelastic-interaction cross section is constant in a large region of nuclei from Ag to U, indicating that the cross section for fragment production increases with the atomic number of the target.

We see thus a definite discrepancy between the conclusions that can be drawn from radiochemical data on the cross section for the production of light isotopes, and the conclusions that follow from an examination of the total cross sections of production of all isotopes at a given Z of the fragment. Should further research confirm this discrepancy, it could be attributable to a change in the relative fraction of stable isotopes among the nuclides emitted in the disintegrations with change in the target mass number. Actually, if the relative fraction of the stable isotopes increases with increasing A of the target then (since the majority of the fragments are stable isotopes, as will be shown below), it may turn out that as the total cross section for fragment production increases, the cross section for the production of radioactive isotopes decreases. That the relative fraction of different neutron-deficient and neutron-rich isotopes changes with the mass number of the target is evidenced, for example, by the fact that the ratio of the cross section for the production of F^{18} and Na^{24} changes with A of the target (as found by Caretto, Hudis, and Friedlander[44,45]), the ratio $\sigma_{Na^{24}}/\sigma_{F^{18}}$ being three times smaller for copper and silver than it is for nuclei heavier than gold.

The data given above pertain in their entirety to the production of multiply-charged particles in disintegrations induced by fast protons. Naturally, multiply-charged particles are produced also in disintegrations induced by other fast particles – mesons, neutrons, deuterons, etc., if the particles have sufficient energy. The experimental data obtained for these cases are however much less plentiful. Most of the data obtained indicate that the production of multiply-charged particles proceeds similarly for different bombarding particles. Titterton,[28] in a study of the formation of Li^8 by bombardment of emulsions with 150-Mev neutrons and 170-Mev protons, found an approximately equal probability of their appearance in either case. For disintegrations induced by 190-Mev deuterons and 340-Mev protons, Wright[27] found nearly equal cross sections for the production of Li^8 on nuclei from Ne to Xe. The cross section for the production of fragments with $Z > 3$ in disintegration of photoemulsion nuclei induced by neutrons with a most probable energy of 395 Mev, was obtained by Sidorov and Grigor'ev.[63] This cross section was found to be $(2.8 \pm 1.4) \times 10^{-27}$ cm^2, which is close to the cross section obtained with protons (see Fig. 3), but a direct comparison is impossible, since the disintegrations of silver and bromines were not separated in this investigation from those of C, N and O.

Blau and Oliver[64] investigated disintegrations induced by 750-Mev pions and the resultant yield of multiply-charged particles with $Z \geqslant 3$. From among 249 stars with $n \geqslant 5$ prongs, 16 stars were found with fragments (fragment range $\geqslant 30\,\mu$ and dip angle in emulsion $\leq 30°$), amounting to approximately 6%. According to Perkins,[52] a value of approximately 10% is obtained, for fragments with the same ranges and the same dip angles in emulsion, in disintegrations produced by cosmic rays at $n = 8 - 13$ prongs. We see that there is no great difference in the relative probability in either case. The process of production of multiply-charged particles for 280-Mev positive pions was investigated by Ivanova.[65] The production cross section of multiply-charged particles with $Z > 4$ in the disintegration of Ag and Br nuclei by positive pions of this energy is $(0.62 \pm 0.2) \times 10^{-27}$ cm^2. It is found to be somewhat less than that obtained in disintegrations induced by 300-Mev protons (see Fig. 3). Other characteristics of disintegrations with multiply-charged particles, induced by pions, are found to be quite close to those obtained for proton-induced disintegrations. Thus, in disintegrations with fragments induced by 280-Mev pions, the average number of prongs is 4.9, whereas in disintegrations without fragments the number is approximately 3.3, considering, as before, only relatively slow disintegration products. We see here therefore the same preferred production of fragments in multiple-prong disintegrations, as in the case of disintegrations induced by fast protons.

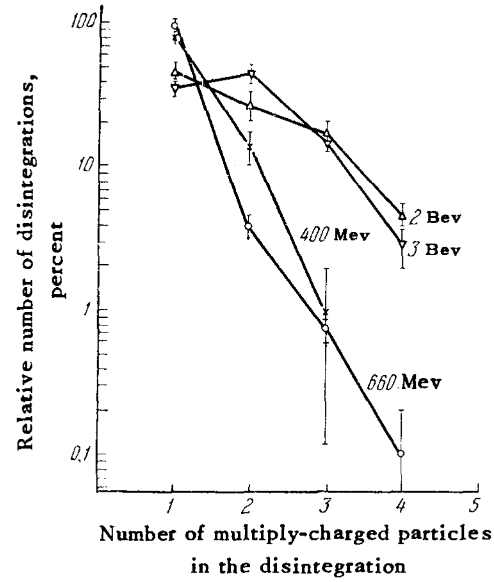

FIG. 8. Dependence of the yield of disintegrations with fragments on their number at different incident-proton energies: fragments with $Z \geqslant 4$ for 660-Mev protons[53] and fragments with $Z > 3$ for 0.4, 2, and 3 Bev protons.[66]

2. Multiplicity of the Fragmentation Process

A striking fact in the production of fragments by disintegration of complex nuclei by fast particles is that two and more multiply-charged particles appear in a single disintegration. The probability of production of such disintegrations increases greatly with increasing energy of the incident particles. In an investigation of nuclear disintegrations induced by 660-Mev protons, Lozhkin[53] showed that approximately 5% of the nuclear disintegrations with multiply-charged particles having $Z \geqslant 4$ is made up of disintegrations accompanied by emission of two or more multiply-charged particles with $Z \geqslant 4$. Perkins[52] obtained for disintegrations produced by cosmic rays in emulsions the following figures: 55 disintegrations with one multiply-charged particle of $Z \geqslant 3$ are accompanied by ten disintegrations with two such particles and one disintegration with three particles, amounting to approximately 17%. It is difficult to compare these data with the results of Nakagawa, Tamai, and Nomoto,[55] who investigated disintegrations induced by 6.2 Bev protons. The latter confined their attention only to fast multiply-charged particles and found six disintegrations with two and more multiply-charged particles among 73 disintegrations with multiply-charged particles of $Z \geqslant 3$, meaning 9%.

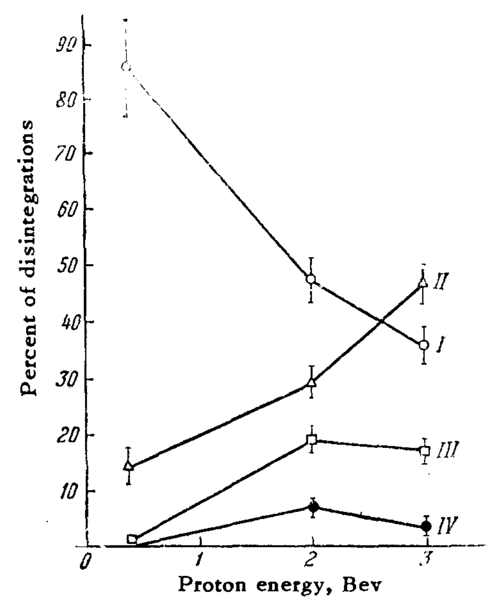

FIG. 9. Dependence of the yield of disintegrations with different number of fragments with $Z \geqslant 3$ (I – one fragment, II – two fragments, III – three fragments, IV – four fragments) on the energy of the incident protons.[66]

The number of multiply-charged particles in nuclear disintegrations was investigated in detail[52] at protons with energies 660 Mev[53] and energies of 0.4, 2, and 3 Bev.[66] Figure 8 shows the yield of disintegrations with different number of fragments, plotted from the data obtained in these references. We see that the yield of disintegrations diminishes rapidly with increasing number of multiply-charged particles in the disintegration. As the energies of the incident particles increases, the relative fraction of disintegrations with two and more fragments increases, and at 3 Bev the yield of disintegrations with two multiply-charged particles of $Z \geqslant 3$ becomes greater than the yield of disintegrations with a single multiply-charged particle. Figure 9 shows data on the dependence of the yield of disintegrations with different number of fragments on the energy of the incident particles, obtained by Baker and Katkoff.[66] The figure shows clearly the rapid increase in multiplicity of the production of multiply-charged particles with increasing incident-particle energy. A certain drop in the number of disintegrations with three and four multiply-charged particles in the range of incident particle energies above 2 Bev may be due, generally speaking, to the fact that at these energies complete pulverization of the silver and bromine nuclei is possible.

An investigation of the characteristics of nuclear disintegrations, in which two and more multiply-charged particles are produced, carried out for 660-Mev incident protons,[67] has lead to the following conclusions. The distribution of these disintegrations by number of emitted charged particles shifts noticeably towards the multiple-prong disintegrations (see Fig. 5). The average number of prongs in these disintegrations, 7.7, is greater than the average number of prongs in disintegrations with a single fragment, 6. Such a shift in the distribution by number of prongs towards the region of multiple-prong stars, for stars with two and more high-energy fragments (Including the nucles He_2^4 with a range greater than 700 μ) was noted earlier by Sorensen.[69] Thus, disintegrations with two and more fragments occur at even greater energy transfers than disintegrations with a single fragment. As before, the increase in multiplicity of the production of fragments with increasing incident-particle energy can be related to the increase in the relative fraction of large energy transfers. Other characteristics of disintegrations with two multiply-charged particles will be discussed further along with the features of a disintegration with a single multiply-charged particle.

3. Nature of Fragments Produced in Nuclear Disintegrations

The question of the mass and charge distribution of multiply-charged particles emitted in nuclear disintegrations is unfortunately one that has been studied the least. One of the reasons, in addition to the small cross section for the production of the investigated multiply-charged particles, is that among the known light-isotopes there are very few isotopes with half lives that can be measured conveniently (only six out of 41 among the isotopes of nuclei from Li to Na). In addition, the question of the nature of the fragments, produced in the disintegration of nuclei, becomes complicated by the fact that in principle a difference may exist between experimentally observed multiply-charged nuclides and those directly produced in the disintegration. This difference may be due either to a definite n/p ratio, or to the large excitation energy of the produced fragments, at which the fragment may have a great probability of decay with emission of nucleons. In particular, heretofore unknown isotopes of light nuclei can be expected to be produced in such nuclear disintegrations. However, before we consider the possible properties of the fragments produced in the disintegrations, let us examine the known properties of fragments observed in experiment.

Certain information concerning the relative probability of observing fragments with different Z and A in neutron-induced nuclear disintegrations can be gained from an examination of Table I. With the exception of the cross section for the production of Be^7, which is quite high in all cases, a rather strange uniformity is observed in the cross sections for the isotopes C^{11} and and F^{18}, and in some cases also for Na^{24}. However, there are still not enough experimental data, and the errors are too large to consider this ratio of isotope yields to be reliable. In addition, the available experimental data exhibit certain contradictions. For example, the dependence of the cross section on the fragment mass number of the disintegration of silver by 340-Mev protons is the opposite of that of lead by 600-Mev protons.

More definite data have been obtained at the present time by the method of nuclear emulsions. Figure 10 shows the available data on the dependence of the fragment yield on their charge. These distributions have been obtained at different times and at different incident-particle energies: 660 and 6200 Mev protons and cosmic rays.[52,53,55,58,70]

All the experimental data fit quite well an exponential curve of the type $P \sim \exp(-Z^n)$, where $n = 0.7$ to 1.0. It is very interesting that the differences between the distributions obtained at 660 and 6200 Mev proton energies are quite insignificant, considering that the fragments investigated in reference 35 have ranges greater than 25 μ and this leads to an underestimate of the number of fragments with large charges. This charge distribution of the fragments, determined with nuclear emulsions, differs significantly from the distributions of the yields of individual isotopes (see Table I). To establish the cause of this difference, it is necessary to analyze the cross section data for nuclides of definite charge, i.e., for the entire sum of isotopes of a given element, and the cross sections for definite isotopes at a given Z of the element.

It is known from radiochemical work that among the fragments observed in the disintegration of nuclei by fast particles include well known β-active isotopes of light nuclei, both neutron deficient (Be^7, C^{11}, F^{18}) and neutron rich (Li^8, Ni^{24}, P^{33}), and the cross sections for their production are known. Investigations with nuclear emulsions have yielded information regarding the total cross sections for the production of fragments with definite charges. A comparison of the various data for given target nuclei and a given energy of incident particles leads to definite conclusions on the nature (i.e., the ratio Z/A) of most of multiply-charged particles. Table II lists the cross sections for fragments with definite Z in proton bombardment of silver and bromine. These cross sections were obtained from

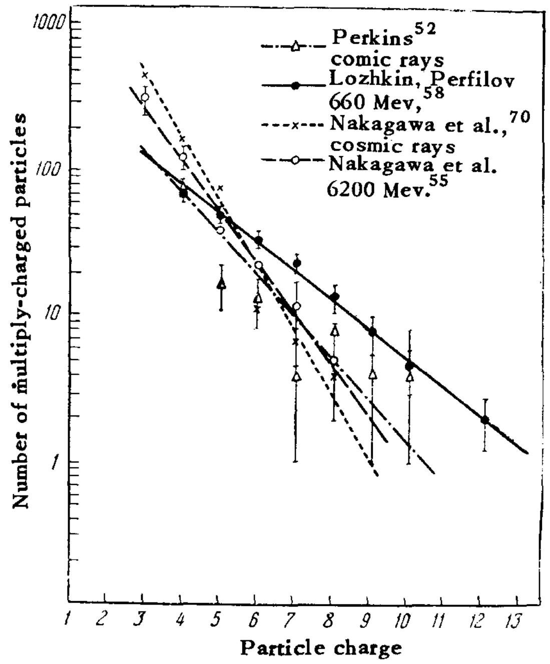

FIG. 10. Dependence of the total cross section of production of fragments on their charge at different incident-particle energies in the disintegration of silver and bromine nuclei.

TABLE II

Comparative data on the cross sections of production of fragments with given Z and isotopes having the same Z

Method of nuclear emulsions, nuclei Ag and Br (reference 53)			Radiochemical method, Ag nuclei		
Z of fragment	E of proton (Mev)	Cross section ($10^{-29}cm^2$)	Isotope	E of protons (Mev)	Cross section ($10^{-29}cm^2$)
4	350	100	Be^7_4	335	10 [36]
	660	350		1000	250 [50]
6	350	50	C^{11}_6	340	1.0 [46]
	460	100		480	~3.0 [41]
9	350	12	Fe^{18}_9	340	1.0 [46]
	460	20		420	1.6 [37]
	660	40		1000	20 [44]
11	350	8	Na^{24}_{11}	340	1.0 [46]
	460	12		480	~3.0 [41]
	660	24		1000	30 [41]

observations in nuclear emulsions.[53,58] The table also lists the cross sections for given isotopes at a given Z in bombardment of silver as obtained from radiochemical investigations.[36,37,41,44,46,50]

Although the comparison of relative yields of fragments with given Z and a given isotope having this value of Z must at present be confined to only a few isotopes, one significant feature does nevertheless manifest itself, namely that for a given Z the total cross section for multiply-charged particles is always considerably greater than the cross sections of the individual radioactive isotopes having the same Z. This difference is caused by the fact that the total yield of fragments with a given Z is made up of a sum of yields of several isotopes, and the principal part of the cross section is that due to isotopes which are not registered by the radiochemical method. This explains in part the difference in the distributions of the isotope yields and in the yields of the entire sum of fragments as functions of their Z and A, which was mentioned earlier. In addition, the great difference (from 10 to 50 times) between the total cross section for multiply-charged particles for a given Z and cross sections for the isotopes listed in Table II also leads to the conclusion that certain definite isotopes with a definite Z/A ratio are favored. In fact, owing to the small number of possible isotopes of light nuclei, such a large difference in the cross section cannot be explained by assuming that all the possible isotopes have approximately the same yield. Since this difference is explained by the presence of isotopes which are not caught by radiochemical means, these isotopes may be either very short lived (such as B^{12}, C^{10}, F^{17}, etc.), or too long lived (such as Be^{10}, C^{14}), or else stable isotopes. In radiochemical measurements it is essentially the stable and short-lived isotopes of light nuclei that are lost, since there are very few long-lived isotopes with half-lives that are not suitable for measurement. Thus, for the light nuclei considered in Table II, there are only three such isotopes: Be^{10}, C^{14}, and Na^{22}. There are approximately as many short-lived neutron-rich isotopes as there are neutron-deficient ones, with respect to the stable isotopes. Thus, generally speaking, the loss may be due either to the former or to the latter nuclei. In this case, however, when we speak of short-lived β-active isotopes, we can hope to obtain information on the relative fraction among all other fragments from the data obtained by the nuclear-emulsion methods. Since usually a time on the order of several hours elapses between the exposure of the nuclear emulsion and its development, the β decay of the multiply-charged particles stopped in the emulsion is expected to be registered with a probability near unity. It follows, however, from the data obtained by Grigor'ev and Sidorov,[63] that the majority of the fragments emitted in the disintegrations are stable with respect to β decay (only 15 out of 65 fragments with $Z > 3$ experience β decay, of which 14 were isotopes of Li^8 and B^8). In addition, the data available on the relative probability of the observation of three isobars Li^8, and Be^8, and B^8 in the disintegration of the silver and bromine nuclei in nuclear emulsion[53] are on hand. The probability of observing the isobar Be^8 is found to be higher than that of observing Li^8 and B^8, although the total yield of fragments with charge 3 is greater than the yield of fragments with charge 4. Thus, judging from the multiply-charged fragments registered in the experiment, it can be concluded that the greatest yield is that of stable isotopes with a ratio $Z/A = 0.5$ in the region of the lightest nuclei.

Let us consider furthermore the ratio of the yields of different isotopes of a single element, observed in the disintegration of certain nuclei. At present data of this kind are still very scanty (see Table I) and pertain essentially to the relatively heavy fragments. The paper by Wolfgang et al.[43] contained data on the cross sections of production of the isotopes Na and P at proton energies greater than 1 Bev. Whereas the isotopes Na^{22} and Na^{24} have approximately equal yields, the isotope P^{32} is produced in all cases with a much greater probability than P^{33}. If the hypothesis that mostly stable isotopes are produced, i.e., Na^{23} and P^{31}, then in the former case the Na^{22} and Na^{24} are on opposite sides of the stability line, and in the latter case P^{32} and P^{33} are on the same side of the stability line. The great difference between the cross sections for P^{32} and P^{33} may indicate a sharp decrease in the cross section in the region of neutron-rich nuclei. The yields of Na^{22} and Na^{24} may be quite close to each other if the dependence of the cross section of production of the fragment on its mass number is symmetrical.

Caretto et al.[44] give the yield ratios Na^{25}/F^{18} for various targets and different particle energies. In most cases the ratio $\sigma_{Na^{24}}/\sigma_{F^{18}}$ is found to be greater than unity (ranging from 2 to 8). This coincides also with data by others.[42,43] In reference 44 the authors considered this fact as an indication of the preferred production of neutron-rich isotopes of light nuclei in the disintegrations and consequently of a greater probability of production of fragments with charge $Z = 11$ than of fragments with charge $Z = 9$. We have already mentioned, however, that the experimental data obtained by the nuclear-emulsion method leads to the conclusion that the fragments are stable and of lower yield as the fragment charge is increased. This discrepancy may be due to errors in both methods. But if such a difference between the cross sections of production of Na^{24} and F^{18} and the cross sections for the productions of fragments with $Z = 11$ and $Z = 9$ does exist, it can be explained by the fact that Na^{24} and F^{18} make up different fractions of all fragments with $Z = 11$ and $Z = 9$ respectively, and furthermore the relative yield of F^{18} among fragments with $Z = 9$ is considerably lower than the relative yield of Na^{24} among the fragments with $Z = 11$.

The neutron to proton ratio (n/p) in the silver and bromine nuclei amounts to approximately 1.3. The n/p ratio for the stable isotopes of light nuclei ranges from 1 to 1.25. Nuclear disintegrations in which fragments are produced with the same n/p ratio as in the initial nucleus could lead to the appearance, in the most cases, of unstable isotopes with a certain excess of neutrons (from 1 to 2). One can hardly expect a greater neutron excess in the fragments than in the initial nucleus.

Were the fragments to have sufficient excitation energy for nucleon evaporation, this evaporation could lead to the experimentally observed stable isotopes. And since the lifetime with respect to such a decay is definitely less than 10^{-14} seconds, in this case multiply-charged particles will not leave a visible track in the emulsion prior to decaying, which agrees with the fact that the observed multiply-charged fragments, which leave tracks in the emulsion, are stable with respect to the emission of heavy particles (p or α).

One can assume, however, that in most cases the fragments emitted from the nuclei do not have any considerable excitation energy. In fact, the presence of such neutron-rich isotopes as Li^8 among the fragments is direct evidence that the fragment has no excitation energy, for in the opposite case it would be impossible to observe it (the Li^8 2.28-Mev level is unstable with respect to neutron emission).

For the same reason, the presence of the isotopes B^8 and N^{13}_{7} among the disintegration products is also evidence of the low excitation of the emitted fragments. We thus have, on the one hand, a great predominance of stable isotopes among the fragments observed in experiment, and on the other hand evidence in favor of the low excitation energy of the fragments as they are emitted from the nucleus. This leads to only one conclusion, namely that the fragments emitted in the disintegrations are stable and that furthermore this stability is apparently one of the characteristics of the fragmentation process. It must be noted, however, that sometimes the fragment is emitted from the nucleus with sufficient excitation energy to make it unstable relative to the decay into its component particles. The cases of emission from nuclei of excited fragments B^9, C^{12}, and O^{16}, which experience decay into α particles, is given in the paper by Perkins.[52] Similar cases of production in nuclear emulsions of excited fragments are relatively rare, as follows from work performed with the aid of the method of nuclear emulsions. It can therefore be assumed that the known properties of the multiply-charged particles observed in nuclear disintegrations (their distribution by mass and charge, their excitation energy) can be used to characterize the very process of production of multiply-charged particles.

A few words should be said concerning the nature of multiply-charged particles in disintegrations with two and more fragments. It was shown by Lozhkin[53,67] that in these disintegrations there are observed various charge combinations of two fragments, and the majority of fragments have $Z < 8$. A certain predominance of fragments with smaller charges (in particular fragments of Li^8) has been observed compared with disintegrations with a single multiply-charged particle.

The characteristics of disintegrations with two fragments have also led to the conclusion that in this case the two fragments are not the decay products of one larger fragment emitted from a nucleus, but are produced directly in the disintegration process.

4. Energy Distribution of Fragments

The greatest amount of experimental information on the energy distribution of the multiply-charged particles produced in nuclear disintegrations has been obtained up to now with the aid of the method of nuclear emulsions, i.e., principally for the disintegrations of silver and bromine nuclei. It is known from references 53, 55, 58, 71 and 73 that independent of the charge of the resultant multiply-charged particles, their energy distribution is characterized by the presence of fragments with energies that are both considerably greater and lower than the nominal Coulomb barrier. The greatest number of fragments is produced here with energies near the nominal Coulomb barrier of the nucleus. These features are clearly seen from Fig. 11, which shows the energy distributions of particles with charges 3, 4, and 5, taken from references 53 and 55. It is seen that the energy distributions of the fragments shift towards the region of higher energies with increasing Z of the fragment, in accordance with the value of the Coulomb barrier. Figures 11 and 12 (which shows the energy distribution of Li^8 at different proton energies) display clearly the weak dependence of the energy distribution of the fragments on the energy of the bombarding particles.

The existence of multiply-charged particles with energies much greater than the energy of the Coulomb repulsion has been noted earlier in investigation of disintegrations produced by cosmic rays.[22 – 26,52,69] The conclusion that there exist such super-barrier multiply-charged particles was also reached in an investigation of secondary nuclear reactions in the bombardment of targets with high energy protons.[36,60,76] Bombardment of many elements (Cu, Sn, Pb) has resulted in the production of isotopes that have three or four more charge units than the target nucleus. The appearance of such isotopes can be attributed only to secondary reactions of the initial nuclei of the target with the fragments of charge Z = 3 and 4 that are produced in the disintegrations, these fragments having energies considerably greater than the Coulomb barrier of the initial nucleus.

The presence of multiply-charged particles with $E \gg E_{Coul}$ is one of the most interesting features of their production. As the energy of the incoming particles that induce the disintegration is increased, the

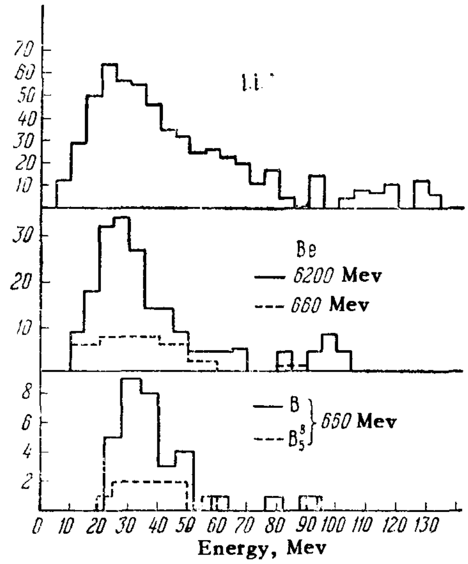

FIG. 11. Energy distribution of multiply-charged particles (Be, B) in the disintegration of silver and bromine nuclei at proton energies 660 Mev[53] and 6.2 Bev.[70] For Li the proton energy is only 6.2 Bev.

energy distribution of the resultant multiply-charged particles changes essentially owing to the increase in the relative fraction of particles with $E \gg E_{Coul}$. On the other hand, the most probable fragment energy remains the same. This is clearly seen in Figs. 11

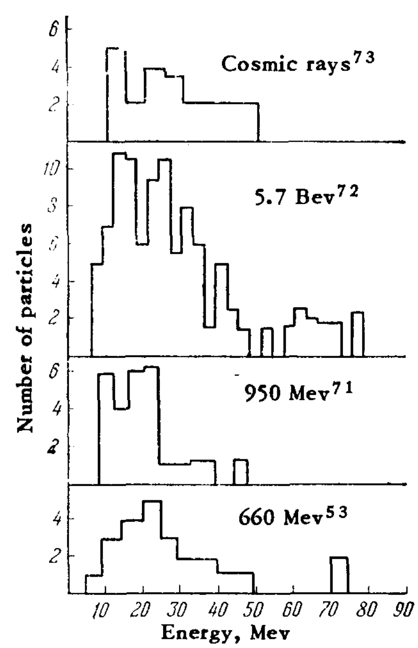

FIG. 12. Energy distribution of Li^8 in the disintegration of nuclei of silver and bromine at various bombarding-particle energies.

and 12. In spite of the wide range of incident-particle energies (from 660 to 6200 Mev), no substantial change in the most probable energy of fragments with charge 3 – 5 is to be seen. The latter is confirmed also by measurement of the ranges of fragments of all charges at proton energies of 300, 460, and 660 Mev.[53] As the proton energy changes, the most probable range of fragments remains the same; all that changes is the relative fraction of the long-range fragments. Figure 13 shows the energy distributions obtained by Katkoff for Li^8 fragments produced in the interaction between 2.2 Bev protons with nuclei of aluminum, copper, silver, gold and uranium. As seen from the diagram, the position of the maximum of the energy distribution of the Li^8 fragments from gold, silver, and uranium is in good agreement with the values of the effective Coulomb barrier, determined from the formula

$$V_{\text{eff}} = \frac{V}{1+\frac{E}{1000}},$$

where

$$V = p\cdot\frac{Z_1\cdot Z_2\cdot e^2}{r_0\,(A_1^{1/3}+A_2^{1/3})},$$

$r_0 = 1.4 \times 10^{-13}$ cm and p is the penetrability of the barrier.

At the same time the energy spectrum of the Li^8 fragments, emitted in the disintegration of the copper nuclei, is strongly shifted towards the larger energies. Actually the spectrum of the Li^8 fragments from copper ($Z = 29$) lies in the region of larger energies than the spectrum of Li^8 from silver ($Z = 47$), and this represents so great an anomaly, that further research is required. It is interesting to note that the position of the maximum of the energy distribution of the Li^8 fragments emitted from uranium indicates that there can be hardly any noticeable contribution from the emission of Li^8 from the excited fission fragments, but that the Li^8 fragments are emitted directly from the uranium nuclei.

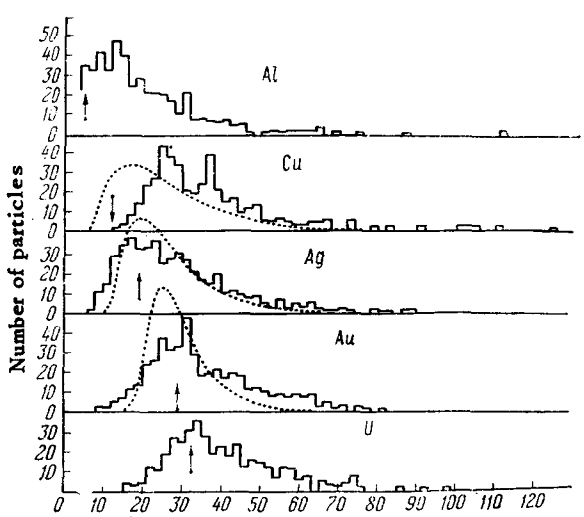

FIG. 13. Energy distribution of Li^8, formed of various nuclei at proton energy 2.2 Bev.[75] The arrows indicate the magnitude of the effective Coulomb barrier. The dotted lines show the calculated evaporation spectrum.

When the fragment energies are close to the value of the effective Coulomb barrier, their energy distribution, in the case of fragments with small charges, is described satisfactorily by the thermodynamic formula of the evaporation process

$$N\,(E)\cdot dE = \frac{E-V_{\text{eff}}}{T^2}\cdot\exp\left(-\frac{E-V_{\text{eff}}}{T}\right)dE$$

with a suitable choice of temperature.

However, this coincidence may be fortuitous in view of the fact that the fragmentation process has certain peculiarities that do not fit the framework of evaporation theory. The discrepancy between a dependence of this type and the experimental one is greatest in the region of small fragment energies and in the region $E \gg E_{\text{Coul}}$. According to Mekhedov[77] the energy spectrum for Li fragments, in the region $E \gg E_{\text{Coul}}$, obeys a relation of the form

$$N\,(E)\cdot dE = \frac{\text{const}}{E^n}\,dE,$$

with $1 < n < 2$. The energy distribution of multiply-charged particles in the energy region $E \gg E_{\text{Coul}}$ does not have a clearly pronounced maximum boundary, although the presence of such a boundary is to be expected at a specified energy of incident particles. In nuclear disintegrations one observes, for a given incoming-particle energy, the emission of multiply-charged particles (with energy greater than E_{Coul}) even with momenta considerably greater than the momenta of the incident particles.[78] The position of the upper boundary of the energy spectrum consequently remains unclear for the time being. It is interesting to note, in connection with the peculiarities of the energy distribution of multiply-charged particles, that the energy spectrum of the protons and α particles in the same disintegrations differs little from the analogous distributions in ordinary disintegrations, with the exception of the relatively large fraction of slow particles in disintegrations with multiply-charged particles.[58]

The energy distribution and the ratio of the fragment energies in disintegrations with two fragments was investigated in references 53 and 67. It was shown that in most cases the ratio of the energy of the heavier fragment to that of the lighter one is greater than unity. Sometimes one fragment or both have energies greater than the Coulomb-repulsion energy.

5. Angular Distribution of Fragments

The angular distribution of multiply-charged particles in nuclear disintegrations is a characteristic which is of great importance to a theoretical interpretation of the process. As shown already by Perkins[52] in an investigation of disintegrations produced by cosmic rays, the angular distribution of multiply-charged particles is anisotropic with respect to the direction of the incident particles, and the degree of anisotropy depends substantially on the speed of the fragments. It was shown by Perkins that the fragments with velocity $\beta > 0.2$ are observed only in the forward hemisphere relative to the direction of the incident particle, while the fragments with velocities $\beta < 0.14$ have a tendency to be emitted at an angle of $60 - 90^{\circ}$ to the direction of the incident particle.

The angular distribution of multiply-charged particles with $Z > 4$ and $Z > 3$, produced in the disintegration of silver and bromine nuclei, were investigated by many workers.[30,53,55,58,65] Figure 14 shows the angular distribution of multiply-charged particles in the laboratory system, at proton energies of 660 and 6200 Mev. It is seen from the diagram that as the energy of the incident particle increases, the angular distribution of the fragments becomes more isotropic. This fact is confirmed by an examination of Table III, which gives the ratios of the numbers of multiply-charged particles of different nature, emitted in the forward and backward hemisphere with respect to the direction of the incident proton at different energies of incident particles. The same table lists data that characterize the angular distribution of multiply-charged particles with energies greater than the energy of the Coulomb repulsion. In spite of the small statistical accuracy of the results, the preferred emission of fast multiply-charged particles into the forward hemisphere is clearly seen.

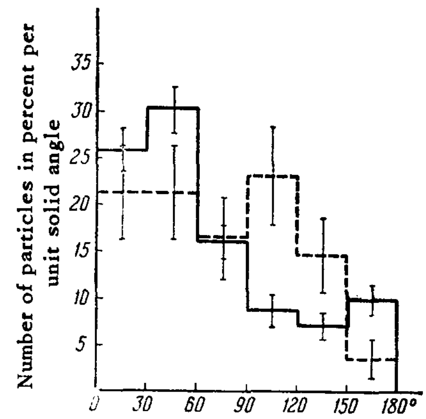

FIG. 14. Angular distribution of multiply-charged particles with $Z > 4$ at a proton energy 660 Mev[58] (solid line) and multiply-charged particles with $Z > 3$ at proton energy 6.2 Bev[70] (dotted line).

The anisotropy of the angular distribution of multiply-charged particles is a characteristic inherent in the mechanism of their production. Corrections for the motion of the center of mass at incident particle energies of $300 - 660$ Mev amount to approximately 0.2, considerably less than the magnitude of the observed anisotropy, but at incident-particle energy in the Bev region these corrections may lead to the conclusion that the light fragments (such as Li and Be) have an isotropic distribution in the center-of-mass system.

TABLE III
Anisotropy of angular distribution of fragments in the laboratory system in the disintegration of Ag and Br nuclei

Fragment charge	Fragment energy	Energy of bombarding particles	Forward/backward anisotropy	References
$Z \geqslant 4$	$\geqslant 2$ Mev/nucleon	P; 350 Mev	3.1 ± 0.6	53, 58
»	»	P; 460 »	3.0 ± 0.5	
»	»	P; 660 »	2.8 ± 0.3	
»	»	π^+ 280 »	3.2 ± 1	65
$Z \geqslant 3$	»	P; 6.2 Bev	1.44 ± 0.5	70
$Z \geqslant 4$	»	Cosmic Rays	1.25 ± 0.5	70
$Z \geqslant 3$	»	»	1.96 ± 0.6	52
Li	»	»	1.43 ± 0.5	70
Li	$\geqslant 5$ Mev	P; 6.2 Bev	1.52 ± 0.3	55
Li^8	$\geqslant 6$ Mev	P; 5.7 »	1.53 ± 0.5	72
Li^8	$\geqslant 8$ Mev	P; 950 Mev	2.5 ± 1.6	71
Li	$\geqslant 60$ Mev	P; 6.2 Bev	~ 16	55
Li^8	$\geqslant 36$ Mev	P; 5.7 »	~ 3	72
$Z \geqslant 4$	$\geqslant E_{Coul}$	P; 660 Mev	5.6 ± 2	53
$Z \geqslant 4$	$\geqslant 60$ Mev	P; 6.2 Bev	~ 14	55

It is impossible to establish from the published data[53] a clear-cut dependence of the angular distribution of multiply-charged particles in nuclear disintegrations on the total number of particles in the disintegration, i.e., on the total energy transferred to the nucleus in collision. It is noted in reference 72 that the angular distribution of Li^8 exhibits a reduction in angular anisotropy upon going to multiple-pronged stars, but the observed effect does not exceed the statistical error.

No definite conclusions can be drawn as yet regarding the dependence of the angular distribution of fragments emitted during disintegrations on the fragment charge.

In connection with the angular distribution of multiply-charged particles, it is interesting to note that there exists a definite angular correlation between the multiply-charged particles and the residual nuclei, and also between these particles and protons and α particles in the disintegration. As was shown in references 53 and 80, the fragment and residual nuclei are emitted mostly in opposite directions, and the α particles and protons are emitted, in disintegrations with fragments, mostly at angles close to 90° relative to the direction of motion of the fragment. The existence of an angular correlation between the fragments and some of the remaining particles in the disintegration may be evidence that they are all produced at the same time in the disintegration.

6. Properties of Residual Nuclei in Disintegrations with Fragments

A study of the residual nuclei in a certain definite region of mass numbers in interactions between high energy particles and nuclei gives additional information on the character of the process, which leads to the appearance of multiply-charged particles. The production in nuclear disintegrations of fragments with masses in the region $A < 40$ corresponds to a fully defined region of masses of the residual nuclei; when the incident particles have energies less than 1 Bev, this region lies between the masses of the products of fission and those of spallation. Starting from the known properties of the emitted fragments, one can expect these residual nuclei to have clearly outlined properties. Since the observed fragments are for the most part stable isotopes of the light nuclei, the residual nuclei in these disintegrations will first have a considerable excess of neutrons. If the residual nucleus does not have sufficient excitation energy, the evaporation process cannot change the situation and the observed residual nucleus should be expected to have a neutron excess. This assumption explains the dependence observed by Kruger and Sugarman[46] of the most probable charge of residual nuclei on their mass numbers when holmium is bombarded with 450-Mev protons. In this case the residual nuclei in the region of mass numbers 99 – 115 have a considerable neutron excess, which cannot be explained by the spallation mechanism (i.e., nucleon cascade plus evaporation of nucleons).

At greater incident-particle energies, evaporation in the case of increasing excitation energy of the residual nuclei can lead to the formation of neutron-deficient nuclei, owing to the preferred evaporation of the neutrons. Among these products one can no longer distinguish the residual nuclei from the fragmentation products, but the dependence of the cross section for their production on the energy of the incident particles is confirmation of the fact that some of these products, in a definite region of masses, is connected with the production of fragments with $A < 40$. Figure 15 shows the dependence, taken from the paper by Wolfgang et al.,[43] of the cross section of production of neutron-deficient isotopes of barium (Ba^{128}, Ba^{129}, Ba^{131}) and cadmium (Cd^{107}), and of the neutron rich isotopes Ba^{140} and Cd^{115}, on the proton energy in the bombardment of lead. One can see from a comparison of Figs. 15 and 2 the extent to which the variations of the cross sections for the production of neutron-deficient isotopes of barium agree with the cross sections for the formation fragments, whereas an entirely different variation is observed for the isotopes of cadmium. The rapid increase in cross sections for the production of neutron-deficient isotopes can be considered as evidence of the increase in the probability of large energy transfers from the proton to a nucleus with increasing proton energy. This again leads to the conclusion that the process considered here for the production of multiply-charged particles is connected with the magnitude

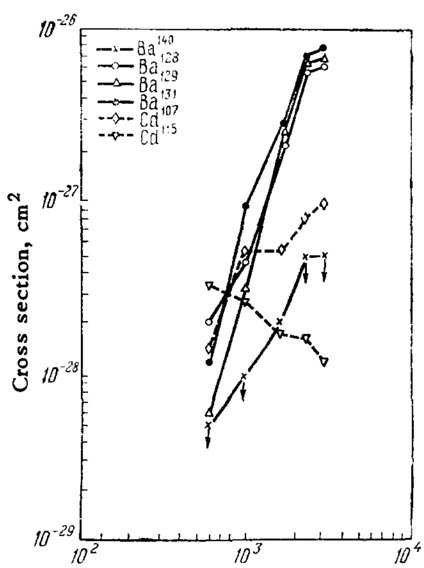

FIG. 15. Dependence of the yield of barium and cadmium isotopes on the incident-proton energy in the irradiation of lead.[43]

of the energy transferred to the nucleus. The same conclusion concerning the large energy transferred to the nucleus in the case of production of neutron-deficient barium isotopes was arrived at by Sugarman et al.[102,112] from a study of the range characteristics of the recoil nuclei in disintegrations of bismuth by protons of various energies. This is confirmed by the investigation of recoil nuclei produced by irradiation of bismuth and tantalum with 450-Mev protons.[81,82]

7. On the Mechanism of Fragmentation

As already noted, the disintegrations of the silver and bromine nuclei, in which multiply-charged particles are observed, are similar in many of their properties to the ordinary disintegrations. They have approximately the same α/p ratio, the anisotropy in the angular distribution of the α particles and protons may be evidence of a nuclear cascade mechanism of nuclear excitation, while the angular distribution of the residual nuclei is direct evidence of the excitation of the residual nucleus. Thus the production of α particles and protons in these disintegrations can be understood from the point of view of ordinary concepts concerning the course of the nuclear reaction at high energies.

Let us now consider the principal distinguishing features of the emission of multiply-charged particles in nuclear disintegrations, from the point of view of the processes which at the present time appear to be already the customary ones for the interpretation of the interaction between high energy particles and nuclei, and which at the same time may be responsible in principle for the production of multiply-charged particles.

A. Nuclear-cascade process. The strong anisotropy of the angular distribution of multiply-charged particles with respect to the direction of the incident particles, the increase in yield of multiply-charged particles with increasing number of cascade particles in the disintegration,[53,55] the features of the energy distribution of the multiply-charged particles, all give grounds for assuming that they appear in a nuclear-cascade process. Similar assumptions were made by the authors of many papers.[69,71,72,83] There are, however, other distinguishing features of disintegrations with multiply-charged particles, such as multiplicity of the process of production of multiply-charged particles, which are difficult to understand from the point of view of the nuclear-cascade process. It is therefore necessary to approach such conclusions with caution. We can attempt to estimate the probability of production of multiply-charged particles in nuclear cascade processes, by assuming the presence of stable (at least sufficiently long-lived ones) clusters of nucleons inside the nucleus[84] and the existence of a quasi-elastic scattering of cascade nucleons on these clusters. In such a quasi-elastic scattering the energy E_M obtained by a cluster of nucleons, is determined wholly by the recoil angle φ or by the scattering angle ϑ of the fast nucleon

$$E_M = 2Mc^2 \cdot \frac{\beta^2 \cdot \cos^2\varphi}{\left(1 + \frac{M}{m}\sqrt{1-\beta^2}\right)^2 - \beta^2\cos^2\varphi}.$$

The probability that a cluster (M) will obtain an energy E_M by collision from a nucleon (m) is determined wholly by the differential scattering cross section

$$w(E_M)\cdot dE_M = \sigma(\vartheta)\cdot d\vartheta.$$

In elastic scattering of high-energy nucleons by light nuclei (of the multiply-charged-particle type considered here), the differential scattering cross section is well accounted for by the theory of diffraction scattering.[85] It is known that the cross section for elastic scattering decreases very rapidly with increasing scattering angle. The total cross section for elastic scattering is determined almost in its entirety by angles $\vartheta < \lambda\!\!\!^{-}/R$ ($\lambda\!\!\!^{-}$ is the de Broglie wavelength of the particle and R the nuclear radius). Since there exists for any incident-nucleon energy a certain minimum value of the scattering angle ϑ_{min}, at which the considered nucleon cluster still acquires enough energy to leave the nucleus, $E_M = E_{Coul} + E_{binding}$, it is clear that the probability of production of multiply-charged particles capable of leaving the nucleus will be maximum precisely at the scattering angle ϑ_{min} and will diminish rapidly with increasing angle, ϑ i.e., with increasing energy imparted to the recoil nucleus in the collision. The existence of a lower limit in the energy of multiply-charged particles, capable of leaving the nucleus, leads to the conclusion that there exists a certain minimum energy of cascade nucleons, below which they can no longer knock fragments out of the nucleus. In elastic scattering the nucleon energy E_m and the energy of the recoil nucleus E_M are related by

$$E_m = E_M \cdot \frac{(m+M)^2}{4\cdot m\cdot M}\cdot\frac{1}{\cos^2\varphi}.$$

For values $\varphi = 0$, $M = 10m$, $E_M = 40$ Mev we obtain $E_m^{min} = 121$ Mev.

Since the maximum probability of production of a recoil nucleus with $E = 40$ Mev corresponds to a certain minimum scattering angle ϑ_{min} or to a certain maximum recoil angle φ_{max}, the energy E_m^{min} will be even greater for φ.

Thus, only the fastest cascade nucleons will be responsible for the knock-out of multiply-charged fragments. The angular and energy distributions of such nucleons is known from Monte-Carlo calculations of the nuclear cascade process.[9] It can be shown that for the cascade-nucleon energies considered we have angles $\varphi_{max} \lesssim 40°$ and $\vartheta_{min} \gtrsim 90°$. Consequently to estimate the probability of production of multiply-charged particles in the nuclear-cascade process it is necessary to know the differential cross section for elastic scattering for angles $\theta > 90°$. Unfortunately, there are still no experimental data on the scattering of high energy particles by light nuclei (of the type of the fragments considered here) at such angles. It becomes necessary to confine ourselves to only a qualitative consideration. Assuming that the fast nucleons in the nucleus experience scattering only by nucleon clusters and are not scattered by individual nucleons, which leads to a maximum estimate, the cross section for the production of multiply-charged particles can be found from the formula

$$\sigma_{fr} = N_M \cdot N_m \cdot w \cdot \sigma_{el} \; (\vartheta > 90°),$$

where N_M is the number of possible clusterings of the nucleons in the nucleus, w is the probability of their existence, N_m the number of fast cascade nucleons capable of knocking out fragments, and σ_{el} the cross section for elastic scattering of fast nucleons at angles $\vartheta > 90°$.

It was shown in reference 53 that even at the extreme assumptions σ_{fr} will be found to be less than that observed experimentally. In addition, in the production of multiply-charged particles during the process of the development of the nuclear cascade the angular distribution should be much more anisotropic than is observed in experiment.

In addition to these expected contradictions with the assumptions regarding the appearance of the observed multiply-charged particles in nuclear cascades, there are certain experimental facts which are difficult to understand from this point of view. These include: 1) the presence of particles of energy less than the Coulomb barrier among the multiply-charged particles emitted during the nuclear disintegration, 2) the multiplicity of production of multiply-charged particles in the disintegrations, 3) the favored production of multiply-charged particles in disintegrations with a large number of "evaporation" α particles and protons. In fact, in the nuclear-cascade process, developing at T = 0, there is no reduction of the Coulomb barrier at all, and the energy of fragments upon emission will be determined by the Coulomb barrier of the original nucleus, all the more in view of the fact that the fragments can be knocked out only at the very start of development of the nuclear cascade, while the cascade-particle energy is still sufficiently large. In connection with the latter, it can be concluded that the emission of multiply-charged particles, if they are produced in a nuclear-cascade process, should not be related to the excitation energy of the residual nucleus, i.e., to the number of evaporation α particles and protons.

The production of fragments as a result of quasi-elastic collision of fast nucleons with clusters of nucleons inside the nucleus is also difficult to reconcile with the observed correlation between the energy of the fragments and their angles of emission,[66,53,86] and also with direct measurements of the scattering cross section of fast protons on carbon nuclei at angles close to 180°.[87]

However, in spite of the foregoing, we cannot reject completely this mechanism of fragment production. At the present time we can consider it reliably established that in addition to nucleon-nucleon collisions a high particle, upon entering the nucleus, experiences sometimes collisions with small clusters of the type H^2, H^3, He^3 and He^4. This follows from experiments on the investigation of the features of scattering of fast protons by nuclei,[7,88,89,90,95] from experiments on the study of production of pions in the interaction between fast protons and nuclei,[91,92] from direct observations of fast deuterons in bombardment of nuclei by high energy protons,[93,94] and from experiments on the study of the spectra of α particles in the disintegration of nuclei.[69,96,98,99] Similar conclusions concerning the interaction between the fast bombarding particle and nucleon clusters inside the nucleus, the momenta of which are correlated, are obtained also from a study of the recoil nuclei Na^{24} in the disintegration of aluminum, silicon, and phosphorous, induced by high energy protons.[97]

There exists in principle still another possibility of production of fast multiply-charged particles during the process of development of a nuclear cascade. This is a phenomenon analogous to the pick-up process, but more complicated than in the case of the pick-up of a proton by a neutron. However, as indicated in reference 53, the experimental data on the production of multiply-charged particles do not confirm this assumption.

B. Evaporation of Particles from an Excited Nucleus. Many experimental investigations[27,73,72,100] indicate that the emission of multiply-charged particles may be explained by the theory of evaporation of particles from strongly excited nuclei. As arguments in favor of the evaporation nature of multiply-charged particles, data are given here on the probability of emission of multiply-charged particles compared with the probability of emission of protons[19,27,73,74] and on the dependence of the probability on the nuclear excitation energy,[73,100] data on the energy spectra of multiply-charged particles[55,73] and on their angular distributions.[72] However, such a coincidence, as shown in reference 53, exists only for fragments with small charges ($Z = 3$ and 4) and furthermore basically only for the relative probability of emission of the fragments Li and Be relative to protons. For other characteristics of fragment production, the agreement here is poor. Thus, for example, the anisotropy of the angular distribution is considerably greater than that expected from evaporation theory, the energy spectrum has a long tail on the side of high-energy fragments, etc.

The most thorough comparisons of the probability of emission of fragments with the predictions of evaporation theory have been made up to now for the fragments Li^8 and Be^7. Figure 16 shows the experimental and theoretical yields of Li^8 per single disintegration vs. the average excitation energy of the nucleus (for the nuclear Ag and Br), as given by Goldsack et al.[72] The experimental data of Wright[24] and Munir[71] are also used here.

As can be seen from the diagram, the agreement between experiment and the Le Couteur calculations is in general quite poor, but the existing difference may be due to some extent both to inaccuracy in the theory and to inaccuracy in the estimate of the excitation energy in the experiment.

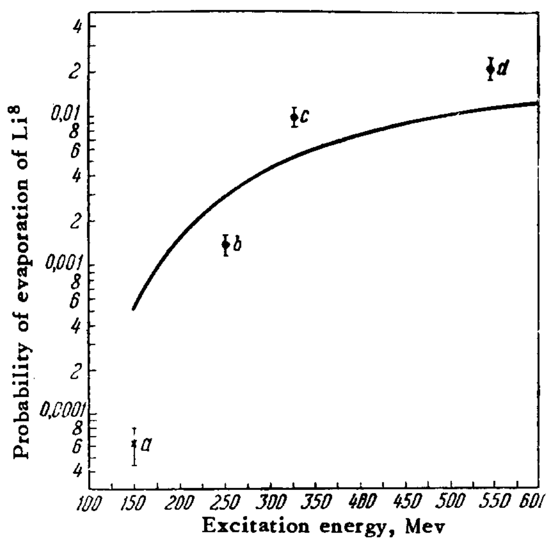

FIG. 16. Dependence of the cross section of production of Li^8 on the excitation energy of the nucleus: a) data of Wright,[27] b) data of Munir,[71] c) and d) data of Goldsack et al.[72] Solid line – calculated by evaporation theory.

A much better agreement between the calculated and experimental data was found for the production of Be^7 fragments. In the paper by Hudis and Miller[100] the evaporation of Be^7 from copper, silver and gold nuclei was calculated with an electronic computer, with allowance for the distribution of the residual nuclei after the nuclear cascade in A, Z and E_{exc}. For all three nuclei good agreement was found between the calculated dependence of the production cross section on the incident-proton energy with the experimental data of Baker, Friedlander, and Hudis.[50]

As regards the energy spectrum of the fragments produced, a comparison between the observed spectra with the calculated ones does for the most part not give good agreement. As shown by Katkoff[75] (see Fig. 13) a more or less satisfactory agreement between theory and experiment for the production of Li^8 fragments can be obtained only for disintegrations of silver nuclei (and furthermore only for fragments with $E < 40$ Mev). This is confirmed also by data obtained in an investigation of the disintegrations of silver and bromine nuclei in nuclear emulsion.[55,72] The spectrum of Li^8 from gold, calculated by the evaporation formula, is considerably narrower than the experimental one and gives too small a number of high energy fragments Li^8. The calculated form of the spectrum of Li^8 from copper is in good agreement with observations, but its position on the energy scale is approximately 10 Mev lower.

Thus, judging from the energy spectra of Li^8, the evaporation mechanism cannot describe the emission of fragments from different nuclei with equally good results.

For multiply-charged particles with $Z > 5$, there is no agreement with evaporation theory even as regards the relative probability of their emission from the nucleus. The relative probability of their observation as compared to protons is much greater than in accordance with the evaporation theory.[53,58]

There are, in addition, direct experimental data which are difficult to explain from the point of view of the appearance of fragments in the evaporation process. These include, first, the rather considerably multiplicity of the production of multiply-charged particles and secondly the growth in the cross section for production of multiply-charged particles with increasing atomic number of the target nucleus. The large angular anisotropy of multiply-charged particles is also impossible to understand from the point of view of evaporation.

C. Process of Asymmetrical Fission of the Nucleus. It is well known from radiochemical research on products of disintegration of nuclei by fast particles, and also from direct measurements,[102,103] that the ratio of the cross section of the fission reaction to the cross section of the inelastic interaction increases monotonically with increasing bombarding-particle energy. At the same time, the experimental data indicate that as the nuclear excitation energy is increased, more and more asymmetrical forms appear.[104,105] These facts have been used in several papers as grounds for assuming that the process that leads to the appearance of multiply-charged particles in disintegrations is a prolonged one.[31,38,47,107] Furthermore, such an assumption was made also simply because the emission of fragments with $A \sim 20$ from a nucleus with $A \sim 100$ was interpreted most sensibly as fission of the nucleus.[32-35,108]

There are also theoretical premises for such a point of view. As shown by Fujimoto and Yagamuchi,[108] when the nucleus is highly excited the reduced "viscosity" of nuclear matter causes the energy to be concentrated preferably in the surface and volume oscillations, rather than going into heating the nucleus as a whole. In this case the fission may become a very probable process. When the temperature of the nucleus becomes of the same order as the binding energy of the nucleon, the fission width becomes comparable with the neutron width, owing to the reduction in the surface tension; this conclusion holds for both symmetrical and asymmetrical fission.

The absence of a more or less rigorous theory for fission processes does not allow us to analyze the process of production of multiply-charged particles from the point of view of the strongly asymmetrical nuclear fission. One can only indicate the following principal features of the process of nuclear fission, which contradict the known facts in production of disintegrations with multiply-charged particles. The experimental facts known at present indicate the following:

1) In nuclear fission the kinetic energy of the fragments is determined by their Coulomb interaction. In the emission of multiply-charged particles the kinetic energy of the fragments is in a considerably percentage of the cases greater than the energy of Coulomb repulsion.

2) The fission process is characterized by a decrease in yield with increasing degree of asymmetry of nuclear fission. In the emission of multiply-charged particles, to the contrary, their yield increases with decreasing fragment mass.

3) Nuclear fission is a process that requires a time interval much greater than the nuclear time.[109,110] The process of emission of multiply-charged particles, one might think, is a process that takes place within the nuclear time.[53]

4) The formation of more than two fragments in nuclear fission is a very rare event. The multiplicity of formation of multiply-charged particles, which is a substantial aspect of the process under consideration, is a rather frequent event.

5) Fragments produced during fission have in most cases the same n/p ratio as the nucleus undergoing fission. Multiply-charged particles, emitted during disintegration of the nuclei, are for the most part stable isotopes, and the residual nuclei produced thereby have initially a neutron excess.

6) In nuclear fission the angular distribution of fragments, while it does have an anisotropy with respect to the direction of the incident particle, this anisotropy is much less pronounced than the distribution of multiply-charged particles, which, as is known, are emitted preferably in the forward hemisphere.

7) The fission cross section of nuclei increases relatively slowly with increasing energy of the incident particles and increases sharply with increasing A of the target (approximately 10^3 times from Ho to U). The cross section for the process of production of multiply-charged particles increases greatly with increasing energy of the incident particles (see Figs. 2 and 3) and increases relatively slowly with increasing A of the target (only by a factor of a few times in the same range of mass numbers).

8) The fragments produced in fission are for the most part excited. The multiply-charged particles produced in nuclear disintegrations do not have as a rule an excitation energy.

D. Hypotheses on the Mechanism of Fragmentation. The foregoing discussion of the known processes of production of nuclear disintegration products leads to the conclusion that not one of the analyzed processes can explain the entire aggregate of experimental data. The same can be said as regards any combination of knock-on, evaporation, and fission processes, although there exist many adherents of such a point of view.[38,41,71,72,79] If we insist on this point of view we cannot explain such aspects of fragmentation as, for example, the multiplicity and the relatively high cross section of production of multiply-charged particles.

These known difficulties in the interpretation of the production of multiply-charged particles in nuclear disintegrations have lead many experimenters and theoreticians to advance various hypothesis at different times. Thus, as early as in 1949 Telegdi[52] proposed, to explain the high probability of emission of multiply-charged particles, the presence of a large angular momentum in the excited nucleus. The emission of heavy particles will help the nucleus get rid of the large angular momentum acquired by collision with the fast particle, since the angular momentum carried away by the fragment (orbital momentum plus intrinsic spin) will be usually considerably greater than the angular momentum carried away by the nucleons. Telegdi notes that such a process explains the large angular anisotropy of the emitted fragments. Unfortunately the Telegdi hypothesis received no further development in subsequent years. In particular, it is not clear how to explain with the aid of this model the existence of anisotropy in the angular distribution of the fragments relative to the forward and backward directions. When particles are evaporated from a nucleus with large angular momentum, the anisotropy should be produced only with respect to the directions of 0 to 90° to the motion of the incident particle. There is no doubt, however, that the angular momentum acquired by the nucleus may influence the course of the process of fragment production. It is known that an increase in the energy imparted to the nucleus by the collision is accompanied by an increase in the angular momentum of excitation of the nucleus.[111,112] Also, as shown above, the fragment production cross section increases with imparted energy. A connection can thus exist between the probability of fragment emission and the magnitude of the angular momentum of the excited nucleus.

It is assumed in the Telegdi hypothesis that the emission of fragments occurs during the process of ordinary evaporation, but under special initial conditions (large angular momentum of the excited nucleus). A hypothesis of this type, in which it is assumed that the processes responsible for the production of multiply-charged particles are known ones, but operating under specified conditions, is also the Heisenberg "turbulent effect" hypothesis,[114] the multiple meson exchange hypothesis,[83] the fluctuation of nuclear matter hypothesis,[84] and the hypothesis of asymmetric fission of a nucleus with large angular momentum.[115,116] The first two hypotheses have not been confirmed experimentally. The "turbulent effect" arising in multiple meson production induced by a particle entering the nucleus would be expected to yield a correlation between the fragment production and meson showers, but this is not the actual case.[52,69] The hypothesis of asymmetric fission of a nucleus with large angular momentum, like the Telegdi hypothesis, received no further development.

A few other hypothesis, which are to some extent departures from the ordinary concepts, have also been advanced. The foremost of these is the Perkins hypothesis[52] on the long-range nuclear forces, advanced to explain the interaction between a fast nucleon and a large group of nucleons. In light of the present notions, this hypothesis is allied with the point of view of collective interaction of the group nucleons with the incident fast nucleon.[94] Hypotheses of this second group include several in which the fragment production process is considered as a peculiar disintegration of the nucleus along with knock-on, evaporation, and fission.[43,46,53]

There exist several points of view concerning the distinguishing features of the fragmentation process, as a peculiar nuclear disintegration process. Kruger and Sugarman[46] consider the essential characteristic of the fragmentation process to be the stability of the fragments produced. Wolfgang, Baker et al.[43] consider the distinguishing aspect of this new process to be the high speed of the process. The fragments produced in a fast process have the same n/p ratio as the initial nucleus and have sufficient excitation for particle evaporation, Lozhkin[53] considers fragmentation as a fast process that leads to the formation of essentially stable light-nuclei isotopes. In light of the aforementioned peculiarities of the process of production of multiply-charged particles in nuclear disintegrations, the latter hypothesis appear to be fruitful in that sense, that they attempt to explain as a whole all the phenomena, without separating at first the parts that are possibly due to other processes.

Let us consider in greater detail the proposed mechanisms for the fragmentation process. In American papers the cause of the appearance of a new type of nuclear transformation, wherein a fragment is emitted from the nucleus, is the meson mechanism of transferring energy to the nucleus, by which the pion produced during nucleon-nucleon collisions in a nuclear cascade is absorbed in the nucleus. If this does not take place, the fragment cannot appear in the disintegration. A simple model of nucleon-nucleon collisions leads to a weak increase in energy transferred with increasing energy of the incident particle. At particle energies above 300 Mev, however, the pion production becomes noticeable and it has been assumed[42,117] that meson absorption in the parent nucleus becomes an important means of transferring large energies to the nucleus. Contributing to this is the circumstance that the most probable energy of the pions produced in such collisions[118] is close to energy of resonant capture of a pion by a nucleon pair[119] and corresponds to the maximum of pion-nucleon elastic scattering.[120] Consequently a pion produced inside the nucleus has a small probability of being emitted to the outside. An estimate made in reference 121 shows that when 1000-Mev protons interact with a nucleus of $A = 100$ approximately 1/3 of all the pions produced are emitted to the outside. As is well known,[122] the cross section for meson production in nucleon-nucleon collisions increases very rapidly from threshold to 1000 Mev, and then remains approximately constant. The increase in the energy transferred in the region of energy of incident particles greater than 1000 Mev may be due to multiple production of mesons in a single nucleus, and also to multiple production of mesons in one collision.

The meson mechanism of transferring energy to the nucleus is assumed to be responsible not only for the large magnitude of the energy transferred, but also for the strong localization of nuclear heating. In view of the large cross section of pion-nucleon scattering and the high probability of pion capture, the pion mean free path amounts to approximately 0.1 of the nuclear radius,[43] causing all of its energy to be transferred to the nucleons located in a small region of the nucleus. Under these conditions, as proposed by Wolfgang et al.[43] considerable local disturbances are produced in the nucleus, many nucleon-nucleon bonds are broken, and the joint influence of the tension forces, Coulomb repulsion, and the momenta acquired during the development of the cascade lead to a rapid disintegration of the nucleus. This creates a high probability that the fragment will break off from the nucleus. To explain the increase in fragment yield with increase in atomic number of the target, later on the same authors[44] advanced the hypothesis that the fissioning ability of the nucleus (i.e., the parameter Z^2/A) influences the fragment-production process. However, this is in definite contradiction with the assumed high speed and non-equilibrium nature of the process, proposed by the same authors. The parameter

Z^2/A could become significant only if it were assumed that a constant equilibrium exists between the volume and surface energy of the nucleus, i.e., if the process is a slow one.

Lozhkin[53] gives a somewhat different treatment of the fragmentation process. Although the mechanism of meson absorption in the parent nucleus explains fairly well the transfer of large energy to the nucleus, no direct connection can be established between the fragment production during nuclear disintegration and the absorption of the meson in the given nucleus. A direct connection exists between the probability of fragment production and the magnitude of the energy transferred to the nucleus, but the method by which this energy is transferred to the nucleus is apparently not important. Actually, whether a pion is absorbed in the parent nucleus or the incident nucleon is completely "tangled up" in the nucleus, a whole cascade of fast nucleons is produced in the nucleus, and their total energy will be same in either case. As to their spatial distribution in the nucleus, any random combination is possible in either case. Disintegrations with production of multiply-charged particles may also occur when the meson mechanism of energy transfer does not take place, all the more since in the meson mechanism of energy transfer one can expect also small transfers of energy, when the pion is absorbed by the nucleons near the surface of the nucleon. This is confirmed by the existence of disintegration fragments with small numbers of α particles and protons, by the slow variation of the cross section for the production of multiply-charged particles in the region of low energy of incident protons (there is no sharp decrease of the cross section at the meson-production threshold), by the absence of a large difference in the cross section for fragment production in disintegrations induced by fast pions and protons, and by the fact that both a multiply-charged particle and a pion are simultaneously emitted in disintegration, indicating that the produced pion was not absorbed in the nucleus.

In the same reference, the process of multiple-charged particle production is represented as a fast process of nuclear disturbance with simultaneous production of a multiply-charged particle and several lighter ones (n, p, α). It is suggested that such a character of nuclear disturbance is caused by strong distortion in its shape, the disturbance to the bonds of the component nucleons and their groupings, which are due to special conditions of the development of the nuclear cascade. If under special conditions of primary interaction between a fast nucleon and the nucleus many fast nucleons appear in a relatively small portion of the nucleus within the nuclear time (10^{-22} sec), and possibly clusters of nucleons, it is difficult to imagine how the nucleon bonds that exist in the normal state can be preserved. In addition, owing to the strong interaction between nucleons, the development of the intranuclear cascade will be simultaneously accompanied by local volume and surface distortions of the nucleus, which become aggravated by the action of surface tension and Coulomb repulsion forces. In this case there is high probability of individual nucleons or nucleon clusters breaking away from the nucleus if the directions of their momenta are suitable. Such a process should be fast (on the order of nuclear time) since in subsequent instants of time the necessary nuclear deformation will no longer prevail. The greater the energy liberated in the process of development of nuclear cascade, the greater the disturbance to the integrity of the nucleus and, quite naturally, the greater the expected fragment yields. From the point of view of such a fast disintegration it becomes possible to explain qualitatively many peculiarities of the observed disintegrations accompanied by fragments.

However, even these mechanisms of the fragmentation process called for special assumptions, if certain peculiarities of the emission of fragments from the nuclei are to be understood. The principal difficulty lies in explaining the existence of fragments with energies considerably greater than the Coulomb repulsion energy. To explain the emission of such multiply-charged particles it becomes necessary to assume that they possess even inside the nucleus energies and a momentum of the same order of magnitude, as can result from adding up the energies and the momentum acquired by the nucleon cluster during the time of development of the nuclear cascade, and the energy and momentum of the intrinsic motion of the nucleon clusters in the nucleus. The latter must be assumed to account for fragments with $E > E_{Coul}$ in the back hemisphere relative to the incident particle. Another difficulty lies in explaining the nature of multiply-charged particles. In a fast disturbance to the nucleus it is difficult to assume in any regrouping whatever in the nucleus. To explain the stability of the observed fragments it is therefore necessary either to suggest the existence of nucleon clusters with definite properties, in the nucleus or else to propose that the fragments break away from the nuclear surface, which has an equal number of neutrons and protons.

The difficulty in interpreting the experimental data on the emission of multiply-charged particles in nuclear disintegration is due to the incompleteness of the experimental data themselves and the clear inadequacy of the existing notions of the nuclear structure and the character of interaction between fast particles and nuclei. But even now we can consider as worthy of attention the conclusions that lead to the existence of a certain fast process of nuclear disturbance, differing in its nature from the known fission, evaporation, and knock-on processes, and that the process of production of multiply-charged particles is connected with the nuclear structure.

II. FISSION OF HEAVY NUCLEI AT LARGE EXCITATION ENERGIES

1. Fission Cross Section

One of the most important parameters that characterize the fission process is the fission cross section σ_f. This fission parameter has been investigated by now to a sufficient degree by many workers for different nuclei and different fissioning particle energies.[1–51]

The fission cross section of the heaviest nuclei, such as uranium and thorium (nuclei in which the critical value of the fission barrier U_f is approximately equal to the neutron binding energy ϵ_n even in the initial state), increases rapidly immediately after the threshold and at excitation energies that exceed the fission threshold by only several Mev, reaches values on the order of several tenths of a barn (10^{-24} cm^2). Upon further increase in the excitation energy the fission cross section varies rather slowly. A typical curve for the variation of the cross section of uranium fission by fast deuterons,[24] is shown in Fig. 17. The curve shows that at deuteron energy above 40 Mev the uranium fission cross section remains practically constant and amounts to approximately 1.3 – 1.4 barn. Analogous curves were obtained by several workers who investigated the excitation function for uranium fission by fast particles of other types.[24,32,33,34]

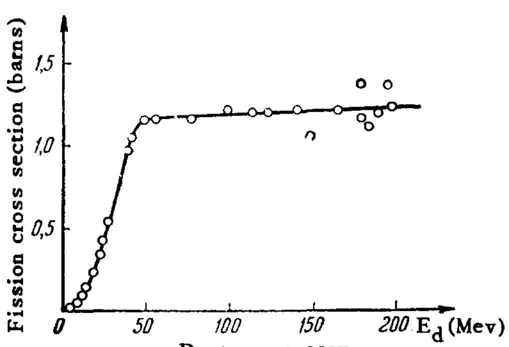

FIG. 17. Variation of the uranium fission cross section as a function of the deuteron energy.

In the analysis of fission events of lighter nuclei (in which $U_f > \epsilon_n$) the fission excitation function has a somewhat different form. Firstly, the rapid increase in the fission cross section begins not immediately after the nominal fission threshold, but at considerably larger excitation energies. Secondly, we fail to see a tendency on the part of the cross section to saturate with increasing incident-particle energy, for a wide range of incident-particle energies. The excitation function for the fission of bismuth,[23] which is characteristic for this group of nuclei, is shown in Fig. 18. Curves of analogous shape are obtained in the fission of bismuth by protons, deuterons, and α particles.[25,27]

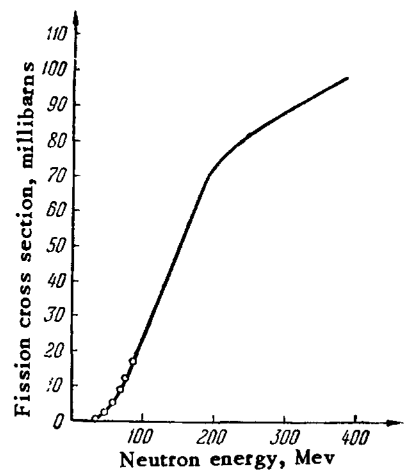

FIG. 18. Variation of the bismuth fission cross section as a function of the neutron energy.

If we consider the behavior of the fission cross section of different nuclei at equal excitation energies, then, as demonstrated by numerous experiments, the fission cross section decreases rapidly with decreasing charge of the fissioning nucleus. Table IV lists the fission cross section for different nuclei bombarded with 450-Mev protons.[20]

Analogous relations, which indicate that the fission cross section decreases with decreasing charge of the fissioning nucleus, are observed also in the fission of various nuclei by other types of particles.[23]

Table V lists the cross sections for fission of heavy nuclei by neutrons of different energies.

Many authors have attempted to derive an analytical expression for the fission cross sections of various nuclei. Thus, for example, Sugarman[20] gives the following relation for the fission cross section of various nucleis by 450-Mev protons.

$$\sigma_f = 0.07 e^{0.41(\epsilon_n - U_f)},$$

where ϵ_n is the neutron binding energy and U_f is the critical fission energy.

This relation describes quite satisfactorily the variation of the cross section of fission of various nucleons by 450-Mev protons. However, if it is attempted to use this equation for other incident-particle energies, it no longer yields correct results.

From the point of view of the emission hypothesis of fission, such a variation of the fission cross section with the charge of the fission nucleus is qualitatively quite understandable. The lighter the nucleus that experiences fission by the emission mechanism, the larger the number of neutrons N_n that it must emit to reduce its barrier to the value of the neutron binding

TABLE IV

Fission cross section of heavy nuclei by 450-Mev protons[20], 10^{-24} cm^2

Nucleus	U^{238}	Th	Bi	Au	Re	W	Ta	Ho	Ag^{35}
Cross Section	1.35	0.67	0.21	0.061	0.019	0.004	0.005	0.002	0.0003

energy in the residual nucleus. However, a large excitation energy is necessary to emit a large number of neutrons, and as the excitation energy is increased the relative probability of emission of a charged particle increases in relation with that for emission of a neutron, and consequently the probability of emission of neutrons only begins to decrease, and with it the fission cross section, with diminishing charge of the fissioning nucleus.

In addition, the rapid increase in the dependence of the fission cross section of nuclei with charges $Z = 83 - 74$ on the bombarding-particle energy also brings to mind the emission character of the fission of these nuclei.[23] Actually, the upper limits for the fission cross section of nuclei ranging from Bi to W, fissioned by 14-Mev neutrons, are exceedingly small, on the order of 10^{-29} cm^2. On the other hand, it follows from the statistical model that with increasing excitation energy (for energies considerably in excess of the critical value of the fission energy), the neutron width and the fission width increase in approximately the same manner. However, even at neutron energies $E_{n0} = 84$ Mev the fission cross section of bismuth increases by more than three orders of magnitude. In the case of high-temperature fission, the increase in fission cross sections should not be so fast. It is therefore natural to assume that at large excitation energies the fission cross section is determined by the fission width not for the levels of the initial nucleus, but for the levels of the nucleus formed after the emission of a considerable number of neutrons.

TABLE V

Fission cross section of heavy nuclei by neutrons of various energies, 10^{-24} cm^2

E_n Mev	U	Th	Bi	Pb	Tl	Au	Pt	Re	W
84	1,4	1	0,019	0,0055	0,0032	0,002	0,00095	—	—
120	1,14	1	0,036	0,02	0,01	0,01	—	0,0017	0,0011
380	1,03	0,9	0,074	0,033	0,019	0,02	0,012	—	0,0038

2. Angular Distributions of Fission Fragments

The data available at present on the angular distribution of fragments in the fission of heavy nuclei[52-69] show that a certain anisotropy exists in the distribution of the fragments relative to the direction of the beam of particles that induce the fission. The character of this anisotropy depends substantially on the nature of the particles interacting with the nucleus. As early as in 1953, V.I. Ostroumov[52] has shown that in the fission of uranium by protons of energy $E_p = 460$ Mev a noticeable anisotropy is observed in the angular distribution of the fragments. The ratio of the number of fragments in the direction of the beam to those perpendicular to it is considerably less than unity.

Somewhat later there appeared in literature communications on investigations of the angular distribution of fragments in the fission of heavy nuclei by neutrons up to 20 Mev,[54,56,67,68] protons of 22 Mev,[53,69] deuterons up to 22 Mev, and α particles up to 45 Mev.[55] It is indicated in reference 53 that when thorium nuclei are fissioned by 22-Mev protons the angular distribution of the fission fragments is satisfactorily described by a formula of the type $J(\varphi) = a + b\cos^2\varphi$ where in the ratio b/a increases with increasing ratio of the fragment masses in fission. Brolley et al.[56] investigated the dependence of the angular distribution of fission fragments on the energy of the incident particle.

For fission by neutrons of energy 10^{-7}, 2.5, 4.6, 7.5, 14.3, and 20.4 Mev the ratio of fragments at 0° to that at 90° is respectively 0.99, 1.02, 1.13, 1.36, 137, and 1.11. Thus, as the neutron energy increases, in this energy region, the anisotropy first increases and then diminishes, but the distribution is described for all energies in this interval by the formula

$$J(\varphi) = (1 + a\cos^2\varphi + b\cos^4\varphi).$$

In investigating angular distribution of uranium fission fragments at high excitation energy (excitation energies in excess of 60 Mev), Lozhkin et al.[61] reached the conclusion that as the excitation energy increases, the angular distribution changes very little, and if an attempt is made to describe this distribution in terms of the sine of the angle φ, this function should contain at least the fourth power of sin φ. In the general form, the angular distribution fits quite satisfactorily the relation of the type

$$J(\varphi) = 1 + 0.29\sin^4\varphi.$$

If the anisotropy is characterized by the ratio of the number of fissions in the interval 60 – 90° to the number of fissions in the interval 0 – 30°, the anisotropy of the angular distribution for fissioning-nuclei excitation energies of 60, 150 and 320 Mev is respectively 1.13 ±0.1, 1.31 ±0.15 and 1.35 ±0.16.

The theory of angular distribution of fission fragments is still very little developed. There are only a few theoretical papers devoted to the angular distribution of fragments in the fission by γ and relatively slow nucleons. Thus, for example, Strutinskiĭ[57] shows, on the basis of the conservation of angular momentum that in the particular case when the fragment spins are equal to zero, the angular distribution in photofission has a maximum at 90°, whereas in fission by neutrons the maximum is at 0°.

A more complete picture of the angular distribution of fission fragments was given by Bohr[59] on the basis of the generalized model.

Let us consider, for example the fission of even-even nuclei by fast neutrons. The compound nucleus that results from the neutron capture has different values of spin J, defined as the sum of the nuclear spin, the neutron spin, and the neutron angular momentum

$$\bar{J} = \bar{I}_{\text{nuc}} + \bar{I}_{n0} + \bar{l}.$$

If the quantization axis is chosen along the direction of the primary beam, then the projection of the angular momentum of the compound nucleus on the given axis J_{oz} can assume only two values, $J_z = \pm 1/2$ (since $\bar{J} = \bar{I}_{\text{nuc}} + \bar{I}_{n0} + \bar{l}$ and $I_{\text{nuc}} = 0$, $I_{n0} = 1/2$, $\bar{l} \perp z$). If the nuclear spin J is much greater than 1/2 (the case of capture of neutrons with large l), it follows from the inequality $J >> J_z$ that the spin of the compound nucleus will be oriented preferably perpendicular to the neutron beam axis.

On the other hand, at neutron energies exceeding the fission threshold by several Mev, the nuclei may pass through a great variety of different states at the saddle point, but with a considerable probability of small values of K (K is the projection of J on the axial-symmetry axis of the nucleus). Thus, the axial-symmetry axis is also in essence perpendicular to the angular momentum. Since the preferred direction for the scattering of fragments should coincide with the axis of nuclear symmetry,[59] and since the location of the latter, as shown above, coincides with the direction of the primary neutron beam, the scattering of the fragments should be in this case preferably along the beam.

Upon further increase in the energy of the incident nucleon, the degree of orientation of J during the instant of fission is considerably reduced, both because of the considerable contribution of the direct interactions, and because of the removal of part of the spin by the neutrons evaporated prior to fission. Consequently, the value of the coefficient that characterizes the angular anisotropy should go through a maximum and then approach zero asymptotically.

However, many experiments on the investigation of the angular anisotropy of the fission of uranium by ultra-high-speed particles yield a noticeable anisotropy in the backward direction[61,62] $N_{90}/N_0 > 1$ – the so-called "negative" anisotropy. If one adheres to the foregoing concept, then this means that in the given energy range of incident nucleons ($E_p > 100$ Mev) the resultant nuclear spin is oriented at the instant of fission predominantly along the direction of the primary beam, although it follows from general considerations that at such high incident-particle energies the spin of the compound nucleus should be determined essentially by the angular momentum of the incident particle relative to the center of the nucleus, i.e., the resultant spin should be oriented perpendicular to the direction of the incident nucleon.

Quite recently Halpern[66] proposed a qualitative model to explain the "negative" anisotropy in the fission of uranium by super-fast particles. This model is essentially as follows. In that range of incident-nucleon energies, where the nuclei began to exhibit transparency properties (E > 100 Mev), the various nucleon-nucleon interactions include also such that result in one of the colliding nucleons having an energy close to the energy of the primary nucleon, and this nucleon moving in a direction close to that of the primary beam leaves the nucleus without further collision with the nucleons of the nucleus. The second nucleon, having a low velocity, moves in the perpendicular plane and has a considerable probability of sticking in the nucleus. In Halpern's opinion, it is precisely such interactions that are responsible for the "negative" anisotropy effect. Actually, if one considers the given group of interactions, the expected picture should be analogous to that expected when uranium is bombarded by nucleons with considerably less energy, moving in a plane perpendicular to the direction of the primary fast-particle beam.

This model explains quite successfully the form of the anisotropy when uranium is fissioned by super fast particles, and makes it possible to justify logically the reversal of the sign of the angular anisotropy with increasing energy of the incident particles (the change in the sign of the anisotropy should appear in that energy region, in which the transparency of the nuclei begins to manifest itself) and even makes it possible to draw certain quantitative estimates of the anisotropy coefficients.

The weakest part of this model, in our opinion, is the fact that the entire effect of "negative" anisotropy is due to weakly excited nuclei (the fast nucleon leaves the nucleus, and the slow one which has a perpendicular momentum loses its energy to the excitation of the fissioning nucleus), whereas the experimental data presently available indicate that the coefficient of "negative" anisotropy increases with increasing excitation energy of the fissioning nucleus.[61]

Summarizing all that has been said above on this question, we can state that it is difficult at present to propose a reliable explanation for either the angular distribution itself at high excitation energies, or for its dependence on the excitation energy.

3. Mass Spectra in Fission

Numerous experimental data on the fission of uranium by thermal neutrons indicate a unique distribution of individual fragments by masses and velocities. In many cases, the spectrum of individual fragments is described by a double-hump distribution curve. The very shape of the distribution curve shows that the fission of U^{235} by thermal neutrons has an asymmetric character. As the fission-inducing particle energies increase, the trough between the humps is gradually filled in.[70,72] An example of a transition spectrum from the case of fissioning by slow neutrons to the case of fast-particle fissioning is the mass spectrum for the fission of U^{235} by 14-Mev neutrons (Fig. 19).

An analogous picture of the "degeneracy" of the double-hump spectrum is observed in an investigation of the energy spectrum of individual fission fragments. Table VI lists certain summary results from many papers, which characterize the gradual filling of the trough between the humps with increasing incident-particle energy. From the values listed in Table VI it can be seen that a neutron energy E_{n0} = 90 Mev the energy distributions of the fragments are single-maximum curves.[71] Upon further increase in the energy of the incident particle, the shape of the curve changes insignificantly, remaining a curve of one maximum, the placement of which shifts, depending on the energy of the fission-inducing particle.

Figure 20 shows the mass spectra of the fission products[24] of uranium bombarded by deuterons of 20, 50, 100 and 190 Mev. From an examination of the shapes of the curves it is clearly seen that as the incident-particle energy increases the trough between the humps is gradually filled, and at a deuteron energy on the order of 100 Mev the curve has a single maximum. Analogous pictures of the degeneracy of a double-humped curve were obtained also in bombardment with other charged particles,[24] and also with neutron[33] and γ quanta.[34] With further increase in the energy of the incident particles, the shape of the mass curve remains analogous, the only difference being that the curve begins to broaden.[24,11]

While the filling in of the trough between the humps indicates that with increasing energy of the incident particles the contribution of the fissions that are symmetrical in mass increases, the broadening of the mass curve in the region of very high excitation energies

Mass spectrum of U^{235} fission fragments
No. 1 – thermal neutrons
No. 2 – neutrons with 14 Mev energy

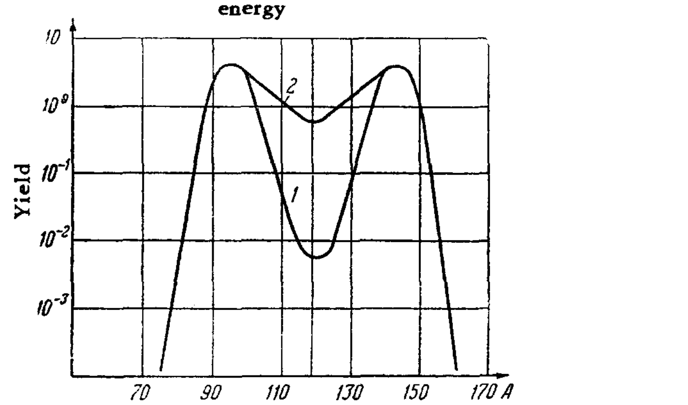

FIG. 19. Mass spectrum of fragments of uranium fission by thermal neutrons and by 14-Mev neutrons (curve 2).

TABLE VI

Characteristics of energy spectrum of fission fragments

Type of fission nucleus and energy of fission-inducing particle	Position of maximum, Mev: In the region of large energies	Position of maximum, Mev: In the region of small energies	Ratio of minimum to maximum in the region of large energies, percent
U^{235} + thermal n'_0 (Fowler)[70]	93	61,8	21
(Friedland)[72]	93	60,1	23
U^{235} + 2.5 Mev neutrons (Friedland)[72]	91	59	36
U^{235} + 14-Mev neutrons (Friedland)[72]	91	59	57
Th + 45-Mev neutrons (Jungerman)[71]	104	73	83
U^{238} + 45 Mev neutrons (Jungerman)[71]	89	68	84
$U^{235}+E_n$ (90 Mev)	80		Energy spectrum of individual fragments has one maximum
$U^{238}+E_n$ (90 Mev)	83		
$Th^{232}+E_n$ (90 Mev)	83		
$Bi^{209}+E_n$ (90 Mev) (Jungerman)[71]	75		

brings to mind that in this excitation-energy region the contribution of the asymmetric fission form begins to increase. Corroborating this point of view are the investigations on the distribution of the fission asymmetry with the excitation energy of the fissioning nuclei.[73] Figure 21 shows the distribution of individual fragments of uranium fission by ranges in emulsion as a function of the excitation energy of the fissioning nucleus – the uranium-filled nuclear emulsions were bombarded by 660-Mev protons. All the fission events were broken up into groups based on the number of charged particles connected with the fission point. For each such group the average initial excitiation energy was determined from the angle between fragments.[16] The curves shown in the diagram correspond to an initial excitation energy of 60, 240 and 540 Mev. For comparison we show the distribution of individual fragments by ranges for a given type of emulsion in the fission of uranium by slow neutrons.

The curves of Fig. 21 lead to the following conclusions:

1. The most probable range of fragments diminishes in fission with increasing excitation energy, this being

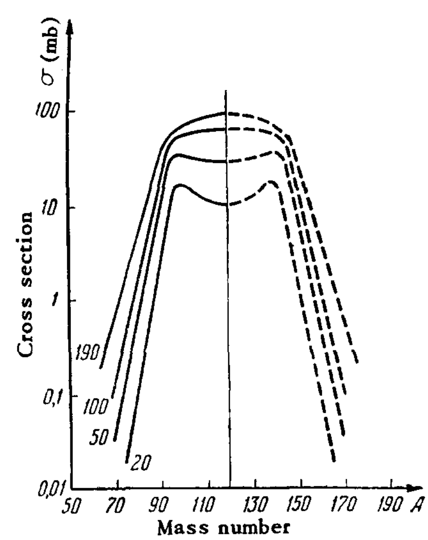

FIG. 20. Mass spectra of uranium fission fragments in bombardment by deuterons of energy 20, 50, 100, and 190 Mev.

connected apparently with the reduction in the kinetic energy of the fragments due to the reduction in the charge of the fissioning nucleus (the number of charge particles emitted prior to fission increases with increasing excitation energy[16]).

2. The distribution of fragments by ranges exhibits one clearly expressed maximum, unlike the fission by slow neutrons, but the half-width of the distribution curve increases substantially upon going to larger excitation energies.[28,100,73]

Inasmuch as the increase in the half-width of the fragment range distribution can serve as an argument in favor of the increased fraction of asymmetric fissions with increasing excitation energy, Shamov and Lozhkin[73] investigated the distributions of the range ratios of two complimentary fragments at different excitation energies of uranium, bismuth, and tungsten. Table VII lists the ratios l_1/l_h (l_1 and l_h are respectively the ranges of the light and heavy fission fragments) for different excitation energies of uranium, bismuth, and tungsten nuclei. The table lists also the distribution of the range ratios of the fission fragments of U^{235} fissioned by thermal neutrons.

From an analysis of the values given in Table VII we can draw the following conclusions:

1. At relatively small excitation energies (on the order of 100 Mev) the fraction of symmetrical fissions is greater for bismuth than for uranium.

2. With increasing excitation energy, the character of fission changes more radically for bismuth than for uranium.

3. At excitation energies on the order of 400 Mev the character of fission, from the point of the asym-

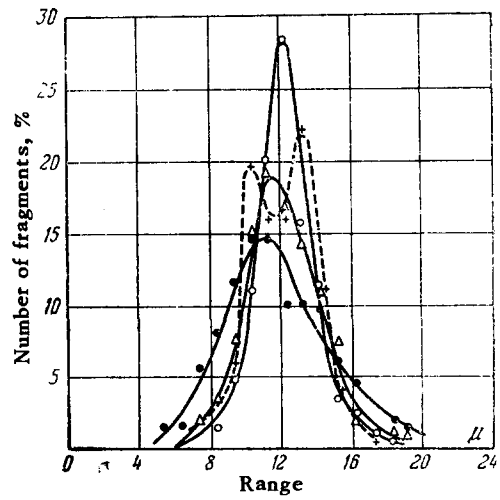

FIG. 21. Range distribution of individual fission fragments of uranium at various initial excitation energies (energy of incident proton E_p = 660 Mev = const): +) fission of U^{235} by thermal neutrous, — o) fission of uranium at an excitation energy E_{exc} = 60 Mev, — △) fission of uranium at E_{exc} = 240 Mev, — •) fission of uranium at E_{exc} = 540 Mev.

TABLE VII

Distribution of the ratios of ranges of light and heavy fission fragments at various excitation energies of uranium, bismuth and tungsten nuclei

Ratio of ranges, $\frac{l_1}{l_h}$	Fraction of fission with given $\frac{l_1}{l_h}$, percent							
	Uranium			Bismuth			Tungsten	U^{235}
	Excitation energy, Mev							Thermal neutrons
	60	240	540	150	240	380	400	
1—1.15	46	32	28	54.5	38.3	29.8	32	28
1.15—1.3	32	27	26	23.6	28	17.3	21	45.5
1.3—1.45	8	19	19	8.2	14.7	22	15	17.3
1.45—1.6	10	11	9	7.25	7.35	11.5	13	4
1.6—1.75	2	6	6	2.7	7.35	9.6	4	4
1.75—1.9	3	1	3	3.6	2.94	4.92	4	1.34
1.9—2.05		1	4.6		1.47	0.96	2	
2.05—2.2		2	1.46			2.88	2	
2.2—2.35		—	0.3			1.92	3	
2.35—2.5		1	1.27			1.96	2	
2.5—2.65			0.99			—	1	
2.65—2.8			0.67			0.96		
2.8—2.95			0.16					

metry of the process, is approximately the same for such differing nuclei as uranium, bismuth, and tungsten.

The conclusions obtained by the authors of reference. 73 pertain to asymmetry of fragment ranges, but are apparently equally reliable as regards the character of mass distribution. A basis for this conclusion is the fact that in the analysis of the fission of uranium by thermal neutrons the fragment range distribution recalls in its character the fragment mass distribution, and the range asymmetry in fission may denote merely the mass asymmetry in fission.

4. The Mechanism of Fission

Even in the early papers on fission of bismuth by deuterons with energies E_d = 190 Mev, the authors of reference 3 have advanced very interesting hypotheses to explain the character of fission of heavy nuclei by fast nucleons. In the simplified form, the mechanism proposed for the fission is as follows. The compound nucleus (or the excited nucleus, if the incident particle is not captured) is gradually cooled by emission of nucleons, and by virtue of the presence of the Coulomb potential barrier the most probable is emission of neutrons. As the neutrons are emitted, the fission barrier is gradually decreased, and the binding energy of the neutron in the residual nucleus, to the contrary, increases. In the case when approximately ten neutrons are emitted by the nucleus, the fission barrier U_f drops to a value on the order of the neutron binding energy ϵ_n in the residual nucleus, and consequently the residual nucleus becomes sufficiently unstable against fission.[74]

Breaking up the fission process into three stages (production of the compound nucleus, emission of neutrons by the compound nucleus, fission of the residual nucleus), the authors arrive to the following specific scheme for the fission of bismuth by 190-Mev deuterons:

$$Bi_{83}^{209}\ (d,\ 12n)\ Po_{84}^{199} \longrightarrow \text{fission fragments}$$

By virtue of the preferred symmetrical fission of bismuth by fast nucleons, the maximum of the mass spectrum of individual fragments should in this case be in the region of 100 mass units, which is confirmed by experiment.

The treatment proposed by Murin et al.[13] of the mechanism of fission of bismuth by fast deuterons was later on called the emission mechanism of fission, thereby emphasizing the fact that the fission is preceded by a multiple emission of neutrons. However the emission mechanism for fission proposed by the authors must be considered more readily as one of the possible fission mechanisms, rather than the only one possible.

Actually, the location of the peak in the mass spectrum of individual fission fragments near 100 mass units is far from proving the fact that the fissioning nucleus has a mass of nearly 200 mass units (i.e., after the emission of 11 – 12 neutrons). Were the Po^{211} nucleus to be fissioned from the upper excitation level ("high-temperature" fission), one would obtain in this case two strongly excited fragments, which upon subsequent cooling would emit approximately five or six neutrons each (if there is enough initial excitation energy for the emission of 10 – 12 neutrons), as a result of which, in the case of symmetrical fission, the peak of the mass curve for the yield of individual fragments would be near 100 mass units. Thus, the form of the mass curve by itself does not allow us to draw a specific conclusion on the mechanism of fission of heavy nuclei at large excitation energies.

The mass spectrum of the fragments obtained by fission of uranium by α particles of energy E_α = 380 Mev was obtained by O'Connor and Seaborg.[4] The cross section $\sigma_f = 2 \times 10^{-24}$ cm^2 obtained by them for the fission of uranium is quite close to the geometric cross section, indicating that the fission processes predominate over the spallation reaction. The maximum of the mass spectrum of the fragments lies in the range $A < \frac{1}{2}(A_U - A_\alpha) = 121$, which allows the authors to advance the hypotheses that the excited nucleus emits several neutrons prior to fission. It is impossible to establish the number of emitted neutrons from this investigation, since the mass spectrum is highly smeared and the exact position of the maximum is difficult to establish.

While the emission theory finds additional corroboration, in the case of bismuth fission, in the small value of the fission cross section (σ_f (Bi) $\sim 0.2 \times 10^{-24}$ cm^2), in the case of uranium fission this argument loses its validity, since the fission cross section obtained by the authors, σ_f (U) $= 2 \times 10^{-24}$ cm^2 is close to the geometric one and does not contradict the Bohr and Wheeler theory on fission from the strongly-excited level.[75] It is impossible to decide, by means of time relationships, which of the two possibilities is actually realized (emission fission or high-temperature fission), since the difference between them is substantial only during the initial instant of time $\tau \sim 10^{-16}$ sec. It is quite obvious, however, that the fission cross section for these processes should differ substantially. In the former case (emission fission) the fission cross section σ_f will be of order of the geometric cross section, multiplied by a factor $(\rho_n)^{N_{n0}}$ where ρ_n is a relative probability of emission of the neutron relative to that of the proton (owing to the Coulomb barrier, $\rho_n > 0.5$), and N_{n0} is the number of neutrons which must be emitted prior to fission.[23] In the second case the fission will proceed in accordance with the Bohr and Wheeler theory and consequently the fission cross section will be close to the geometric cross section.[75] It must be noted at the same time that the identification of the character of fission from the value of the cross section σ_f is valid apparently only for that group of heavy nuclei, in which the barrier exceeds noticeably the neutron binding energy. Let us illustrate the above with an example. In order for the bismuth to fission by emission, the excited nucleus should emit prior to fission approximately ten neutrons, as a result of which a barrier for fission is reduced to the value of the binding energy of the neutron in the residual nucleus, which becomes in this state sufficiently unstable against fission. The cross section of such a process can be estimated from the expression

$$\sigma_f \simeq \sigma_{\mathrm{geom}}(\varrho_n)^{N_{n_0}}.$$

In this case the quantity N_{n0} has a fully defined physical meaning, for it represents the number of neutrons that the nucleus must emit in order to reduce its barrier to the value of the neutron binding energy in the residual nucleus.

In the fission of a strongly excited uranium nucleus, the quantity N_{n0} does not have a well defined physical meaning, for even at a nominal value of the barrier the uranium nucleus is sufficiently unstable against fission, and the latter competes successfully with the disintegration process. On the other hand, if the strongly excited uranium nucleus first reduces its excitation energy, for various reasons, by neutron emission, then it becomes impossible to identify this process by determining the fission cross section σ_f or by determining the position of the maximum in the fission-fragment mass spectrum, and other characteristics must be sought, connected with the given process, which would permit a more reliable determination of the character of the fission of uranium at high excitation energies.

We shall dwell below on an analysis of the results on the mechanism of fission of heavy nuclei, obtained by the following methods:

a) Investigation of the energy spectrum and the number of charged particles emitted by the fissioning nucleus (photomethod).

b) Analysis of the ranges of definite fission fragments, produced under different initial excitation energies.

c) Method of angular correlations of the emitted particles with the fission fragments.

d) Method of random tests (Monte Carlo method).

a) To investigate fission reactions with the aid of nuclear photoemulsions, the latter are usually filled with the investigated element either by impregnation in a suitable solution, or by introducing the investigated element. (U, Bi, or W) in the form of suspensions in the photographic emulsion. The identification of the fission events in this method is practically 100% correct. In each fission event the following are determined: the angle between the fission fragments, the lengths of the fission fragments, the number of charged particles, the angular distribution of charged particles, and also their charge and energy. Next, the angle between fragments is used to determine the initial excitation energy of the fissioning nucleus,[52,76] E_{f0}, which is determined by the loss of kinetic energy of the incident nucleon (ΔE_{p0}) inside the nucleus

$$E_{f_0} = E_{p_0} - E_k - Q - E_{\mathrm{nuc}}$$

where E_k is the kinetic energy of the knock-on nucleons, Q the binding energy of the knock-on nucleons, and E_{nuc} the kinetic energy of the fissioning nucleus ($E_{\mathrm{nuc}} \ll Q$, E_k and therefore E_{nuc} is usually neglected).

To determine the value of E_k it is usually assumed that the entire momentum is carried away by a single nucleon in the direction of the incident particle. Thus, if the velocity of the fissioning nucleus is denoted by U and the mass by M, the momentum of the knock-on particles will be $P_k = P_0 - MU$, and the kinetic energy of the cascade particles is

$$E_k = \sqrt{P_k^2c^2 + m^2c^4} - mc^2.$$

The value of the translational velocity of the fissioning nucleus U is determined from the measured angle between fragments

$$U = \frac{v_1 \cdot v_2 + \sin(\varphi_1 + \varphi_2)}{v_1 \sin\varphi_1 + v_2 \sin\varphi_2}.$$

The relations between the quantities v and π are defined in Fig. 22.

A comparison of the initial excitation energy of the fissioning nucleus E_{f0} with the number of charged particles $n_{\alpha p}$ emitted during fission shows that as the number of charged particles increases, the initial excitation energy also increases, and the relationship $E_{f0} = f(n_{\alpha p})$ can be expressed analytically for various fissioning nuclei and initial energies of the incident nucleons.[16,77] By virtue of the foregoing, the analysis is facilitated by breaking up the fission events into groups based on the number of charged particles, connected with the point of fission, each such group having a fully defined average value of initial excitation energy. On the other hand, an analysis of the energy and angular distributions of the charged particles connected with the fission point (U, Bi) allows us to conclude that in practice all the

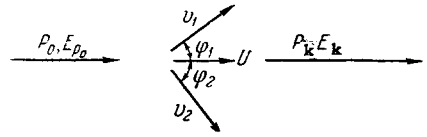

FIG. 22. Diagram of momenta and angles.

charged particles are emitted by the fissioning nucleus, and not by the excited fragments.[77,78] Thus, an analysis of the fission of U and Bi by fast nucleons gives grounds for assuming that the fission of these nuclei at large excitation energy proceeds via the emission mechanism, i.e., it occurs after preliminary cooling of the nucleus by multiple emission of nucleons. This conclusion can be verified in the following manner. Knowing the initial excitation energy of the fissioning nucleons and the number of charged evaporation particles (from the angular distribution of the charged particles it is possible to segregate from the total number of charged particles connected with the fission point those particles which result from evaporation) it is possible to calculate the number of charged particles that must be evaporated from a given nucleus under a given assumption concerning the excitation level from which fission takes place.

Figures 23 and 24 show the dependence of the number of evaporation charged particles emitted in the fission of uranium and bismuth on the initial excitation energy of the fissioning nucleus.[77] The crosses in the same figures indicate values obtained by calculation from evaporation theory[89] under the assumption that the entire initial excitation energy is removed prior to fission by emission of neutral and charged particles. From a comparison of the experimental data with the computed values $(n_{\alpha p})_{ev} = f(E_{f0})$ it can be concluded that the fissioning uranium nuclei emit as many charged particles (and consequently as many neutrons) as would be emitted by a nucleus with a given initial excitation energy, but under the assumption that the entire excitation energy is removed by emission of neutral and charged particles. Thus, the fission is the final result, which occurs after preliminary cooling of the nucleus.

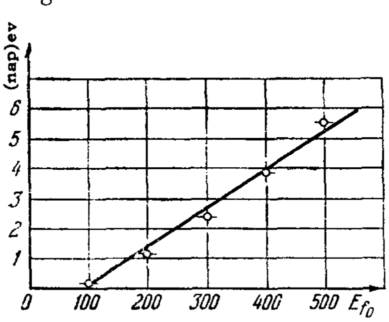

FIG. 23. Dependence of the number of charged particles (evaporation particles) emitted in the fission of uranium on the initial excitation energy of the fissioning nucleus (solid curve).

The circles denote the calculated values for the number of evaporation particles under the assumption that the entire energy of excitation is removed by emission of particles.

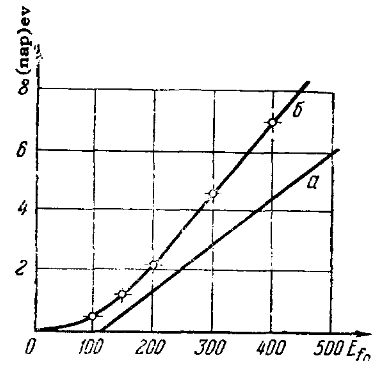

FIG. 24. Dependence of the number of charged particles (evaporation particles) emitted in fission on the initial excitation energy of a fissioning bismuth nucleus (curve a).

The diagram shows also the values obtained by calculation under the assumption that the entire excitation energy is removed by emission of particles (curve b).

An analysis of bismuth fission events shows that the fissioning nucleus emits fewer charged particles than would follow from the calculated curve. This is evidence that the only fissioning nuclei are those from which fewer charged particles and more neutrons are emitted during the process of preliminary cooling, i.e., which increase the parameter Z^2/A. In the emission of a larger number of charged particles (say the number that follows from the calculated curve) there was not enough initial excitation energy to bring the residual nucleus to such a state, at which the fission can compete successfully with the disintegration process $(U_f = \epsilon_n)$.

Thus, in the fissioning of Bi nuclei by fast nucleons, the latter evaporate during the first stage of the cooling process of nucleons (essentially neutrons), as a result of which the fissioning ability of the residual nuclei increases. The fissioning act itself occurs predominantly after the removal of practically all the exexcitation energy, i.e., it is, as in the case of uranium fission, the final act in the cooling of the nucleus.

An analysis of the fission of lighter nuclei, such as tungsten, shows that the fission occurs from an excitation level that exceeds the binding energy of the neutron in the fissioning nucleus.[78] For nuclei in the tungsten region and lighter nuclei, the cross section of pure emission fission, $\sigma_f = \sigma_{geom} (\rho_n)^N n_0$ becomes so small, that fission from excited levels, i.e., temperature or barrier fission, begins to compete with it. [In this case to attain the condition for emission fission $U_f = \epsilon_n$ it becomes necessary to emit more than 20 neutrons, provided that one charged particle is emitted during the cooling process. And since the emission of a charged particle becomes comparable in the case of large excitation energies for nuclei with charge $Z = 73$ with the probability of emission of a neutron, the probability of evaporation of a large number of neutrons only becomes very low.)

b) An interesting attempt to solve the problem of the mechanism of uranium fission by fast particles was undertaken by Templeton.[79] The idea of the experiment was as follows: if we measure the ranges of a definite fission product, (for example, Ag^{111}) in the fission of uranium by slow and fast particles, then the larger the number of neutrons emitted prior to fission, the shorter the ranges of the Ag^{111}. Indeed, since the summary kinetic energy of the fragments is determined by the Coulomb repulsion, and the distribution of energies among the fission fragments is inversely proportional to the masses, the highest energy will correspond to the fragment with the smallest mass. If we segregate a fully defined mass (Ag^{111}) in the fission of uranium by slow and fast particles (i.e., at small and large excitation energies), then in the case of uranium fission by fast particles (under the condition of prior emission of neutrons) the range of Ag^{111} will be found to be shorter, since the second complementary fragment has in this case the smaller mass and will take the larger share of the energy.

Templeton has investigated the ranges of uranium fission fragments in aluminum foils in fission induced by 335-Mev protons and 18-Mev deuterons. It was found that the ranges in the case of proton-induced fission were shorter than those of the same fragments in the deuteron experiments, and that the lighter the investigated fragment, the greater the observed difference in range. The results of the experiment on the determination of the ranges of definite fission fragments in proton and deuteron experiments are as follows:

$$Sr^{89}:\ \Delta R = 0{,}249 \text{ mg/cm}^2, \quad Sr^{91}:\ \Delta R = 0{,}192 \text{ mg/cm}^2$$
$$Ag^{111}:\ \Delta R = 0{,}153 \text{ mg/cm}^2, \quad Ba^{140}:\ \Delta R = 0{,}099 \text{ mg/cm}^2.$$

If E is the total kinetic energy of fission, A the mass of the nucleus prior to the emission of the neutrons, M the mass of the fission product (subject to the analysis) and N the number of neutrons emitted prior to fission, then the difference in energy for a definite product M in the case of deuteron and proton fission is given by the relation.

$$\Delta E = \frac{EMN}{A(A-N)}.$$

Knowing ΔE_M, M, and A, the authors calculate the value of N.

The values obtained by the authors for the number of neutrons emitted prior to fission in the case of fission of uranium by protons with energy $E_p = 335$ Mev, are listed in Table VIII.

TABLE VIII

Number of neutrons emitted by uranium fission prior to fission

Separated fission products	Reduction in energy of the given product ΔE, Mev	Number of neutrons N emitted prior to fission
Sr^{89}	7	25
Sr^{91}	5.4	19
Ag^{111}	4.3	13
Ba^{140}	2.8	8

The results of Table VIII indicate that in the case of fission of uranium by high energy protons the excited nucleus emits prior to fission a considerable number of neutrons, i.e., the fission is of emission character. While fully agreeing with the authors conclusion on the mechanism of the fission of uranium by fast nucleons, we still would like to call attention to certain details of the work, which may influence if not the conclusion itself, at least the quantitative determinations of the number of neutrons prior to emission.

Let us turn to the experimental setup (Fig. 25). Here A is a source of fissionable material U^{238}, B and C are aluminum foils, the thickness of which is chosen to satisfy the condition $t < R < 2t$, where R is the range of the fission fragments, P is a proton or deuteron beam. If the fission products have an isotropic distribution, it can be readily shown that the range of a definite fission product is given by

$$R_x = \left(1 + \frac{A_2}{A_1}\right)t,$$

where A_1 and A_2 are the activities in the first and second foils, due to the sticking of the given fission product.

Measurements of the ranges with the aid of the formula $R_x = (1 + A_2/A_1)\,t$, as correctly pointed out by the authors themselves, are valid only when the fission-fragment distribution is isotropic with respect to the incident particle beam.

However, experiment on the fission of uranium by deuterons with energies $E_d = 22$ Mev indicate that fission has a noticeable anisotropy.[55] The number of

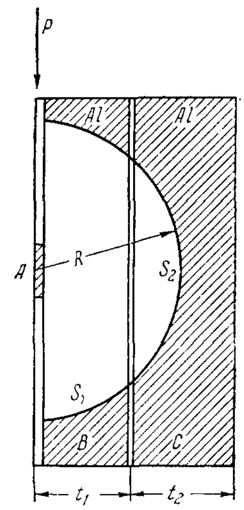

FIG. 25. Arrangement of Templeton's experiment.

fragments traveling in the direction of the beam exceeds considerably the number of fragments in the perpendicular direction. This anisotropy should lead to a certain overestimate in the ranges in deuteron experiments, for in this case the range should be determined from the relation

$$R_x = \left(1 + \frac{A_2 - \Delta A}{A_1 + \Delta A}\right) t$$

and the actual ranges will be shorter.

On the other hand, many experiments on the fission of uranium by fast protons also indicate a certain anisotropy in the angular distribution of the fission fragments, but of opposite character,[61,69] i.e., in this case the fragments are scattered mostly in a perpendicular direction. If these experiments represent the true picture, then the ranges determined by Templeton in the case of proton experiments will be underestimated, and since he uses for his final calculations the difference $\Delta R = R_d - R_p$, then actually this difference will be less (if it exists at all), and consequently the number of neutrons emitted prior to fission will be less. In addition, Templeton did not take into account the variation of the charge of the fissioning nucleus in the case of proton experiments, a variation that may be due both to the knock-out of charged particles by the cascade, or as a result of the subsequent evaporation process. In this case a change in the charge of the fissioning nucleus by two or three units is quite sufficient to explain the observed reduction in the ranges observed in proton experiments.

c) One of the most direct methods of determining the level from which the fission of heavy nuclei takes place at large initial excitation energies is the study of the angular correlations between the direction of motion of the fission fragments and the evaporated neutrons.[90]

Indeed, if the neutrons are evaporated prior to fission, they will be isotropically distributed relative to the fission fragments. But if the neutrons are evaporated by the excited fission fragments, then in a coordinate system connected with the moving fragments, the distribution will be isotropic, whereas in the laboratory system a noticeable anisotropy should be observed. If we define the anisotropy coefficient as the ratio of number of neutrons at 0 and 90° ($K = N_0/N_{90}$) to the direction of the moving fragment, then in the case of neutron emission from the excited fragments, this coefficient is greater than unity. The magnitude of this coefficient will depend on the velocity of motion of the fission fragment and on the velocity of the evaporated neutrons. Furthermore, the greater the fragment velocity and the less the emitted-neutron velocity, the greater the value of this coefficient.

Using the method of measuring the coincidence between neutrons and fission fragments, Harding[90] determined the value of the coefficient K for fission of uranium by protons of energy $E_p = 147$ Mev. The value obtained by Harding is $K = 1.27 \pm 0.11$. Using this value and making certain supplementary assumptions, one can determine the number of neutrons emitted prior to fission, and the number of neutrons emitted after fission, or, in the end result, the level of excitation from which the fission of the residual nucleus takes place. The author bases further calculations on the following assumptions:

1) the fission produces two fragments of equal masses, $A = 119$;

2) the average fragment excitation energy is $E_f = 20$ Mev;

3) in all fission cases the velocity of the fragments is $v_f = 1.2 \times 10^9$ cm/sec;

4) the energy spectrum of the neutrons emitted from the fission fragments is described by the evaporation formula.[89]

Using the foregoing assumptions, the author obtained the following values for the anisotropy coefficients:

1) all neutrons evaporated after fission $K > 2.25$;

2) all neutrons evaporated prior to fission $K = 1.01$;

3) 1.5 neutrons evaporated after fission and remaining prior to fission, $K = 1.29 \pm 0.05$;

4) 3.5 neutrons evaporated after fission, and the remaining prior to fission, $K = 1.42 \pm 0.07$.

The average number of neutrons per event of uranium fission by 147-Mev protons was determined by Harding[80] and found to be 13.1 ±1.6.

Comparing the value obtained for the anisotropy coefficient with the experimentally determined value $K = 1.27 \pm 0.11$, Harding reached the conclusion that in fission of uranium by 147-Mev protons an average of 13 neutrons was evaporated, 11.5 ±1 prior to fission and 2.5 ±1 after fission.

Considering the fact that upon fission of uranium by thermal neutrons an average of 2.5 neutrons are emitted from the fission fragments, the result obtained by Harding shows that in uranium fission at high excitation energies ($E_{f0} \sim 100$ Mev) almost all the excitation energy is released prior to fission by multiple emission of neutrons.

However, there is still an opposing point of view regarding the mechanism of fission of heavy nuclei at large excitation energies.[81 – 84] Thus, for example, Marquez,[84] using the numerical material given by Harding, concludes that when uranium is fissioned by 147-Mev protons all 13 neutrons are evaporated after fission from the excited fragments.

Since we have dwelled in detail on Harding's work, it would be proper to pay no less attention to the work of Marquez. Unlike Harding, who determines the energy spectrum of the neutrons (in the final analysis – the mean kinetic energy of the neutrons, $E_n = 2T$) by computation, using the relation given by Le Couteur, Marquez bases his calculations on the law of conservation of energy

$$E_{p0} + m(U^{238}) + m(H_1^1) = E_{kf} + \overline{E}_{kp} + \overline{E}_{\gamma} + m(Pd^{112}) + m(Ag^{114}) + 13m(n_0) + \overline{E}_{n0},$$

where E_{p0} is the kinetic energy of the incident protons ($E_{p0} = 147$ Mev), $m(U^{238})$ is the mass of uranium in Mev units, $m(H_1^1)$ is the proton mass in Mev units, E_{kf} is the kinetic energy of fission fragments in symmetrical fission of uranium, $\overline{E}_{kp}$ – the average kinetic energy, carried away by the protons per fission event ($\overline{E}_{kp} = 0.15$ Mev), $\overline{E}\gamma$ is the average energy carried away by the γ quanta ($\overline{E}\gamma = 5$ Mev), $m(Pd^{112})$ and $m(Ag^{114})$ are the masses of the fission fragments in Mev units, $13\,m(n_0^1)$ is the mass of the 13 neutrons in Mev units, and $\overline{E}_{n0}$ is the kinetic energy of all the 13 neutrons due to the fission of the uranium by 147-Mev protons.

Solving this equation for $\overline{E}_{n0}$, Marquez obtained a value of 80 Mev, and knowing the number of neutrons due to fission ($N = 13$), he obtained an average neutron kinetic energy of 6.15 ±1.5 Mev in the laboratory system, and respectively 5.4 ±1.5 Mev in the system of coordinates connected with the moving fragments.

After calculating the anisotropy coefficient for an average neutron energy of 5.5 Mev, and assuming that all the neutrons are evaporated from the fragments, the Marquez obtains $K = 1.23$, which is in good agreement with the experimental values of Harding ($K = 1.27 \pm 0.11$), and consequently allows him to conclude that all neutrons are evaporated after fission by moving fragments. Since two authors, analyzing the same experimental material, obtained different results, we allow ourselves several critical remarks, which in our opinion, will eliminate the existing misunderstanding.

The value obtained by Marquez for the average kinetic energy of the neutrons due to fission ($\overline{E}_{n0}$) undoubtedly pertains to two groups of neutrons, which differ substantially in their production mechanism, i.e., to the cascade and evaporation neutrons.

Although the number of cascade neutrons is considerably less than the number of evaporation neutrons, nevertheless the averaging of the energy of these two groups of neutrons undoubtedly leads to a considerable overestimate of the average kinetic energy of the evaporation neutrons, which must be known as accurately as possible in order to determine the anisotropy. Actually, as is known, the average excitation energy of the fissioning nuclei of uranium fissioning by protons of energy ~ 140 – 150 Mev is approximately 100 Mev,[85] the average number of cascade particles (n, p) is 1.2 (0.4 protons and 0.8 neutrons).[85] Thus an incident proton of energy $\overline{E}_{p0} = 147$ Mev loses only part of its energy (~ 100 Mev) in exciting the nucleus, and the balance of the energy (147 - 100 = 47 Mev) is in the form of kinetic energy of the cascade particles ($n_{cp} = 1.2$). If the average number of neutrons due to fission is 13.1 ±1.6, this means that 12 particles must be classified as evaporative, and one (but of energy ~ 50 Mev) as a cascade particles.

Inasmuch as the kinetic energy of all the neutrons (cascade and evaporation) is 80 Mev in accordance with the energy equation written out by Marquez, then only approximately 30 Mev fall to the 12 evaporation neutrons; consequently, the average energy of the neutrons produced in the evaporation process is not 6.15 Mev (as determined by Marquez), but merely 2.5 Mev. But at this value of the average evaporation neutron energy, the anisotropy coefficient in the case of emission of neutrons from excited fragments should be $K > 2.25$. On the other hand, the presence of a single cascade nucleon in Harding's experiment cannot influence greatly his conclusions, because in his experiment the correlation between the neutrons and fission fragments was determined in a plane perpendicular to the direction of motion of the incident proton, and the cascade nucleons have a preferred direction along the direction of incident proton and are distributed isotropically with respect to this direction. However, if a very rough approximation is made by assuming that the detector registered all cascade nucleons (i.e., that they are in a plane perpendicular to the motion of the incident proton), then in this case too the anisotropy coefficient should increase by merely 1.5%. Thus in analyzing the results obtained by the foregoing authors,[84,99] preference should be given, in our opinion, to the Harding's conclusions.

An investigation with ideas analogous to those of Harding was performed by Ostroumov and Filov[86] by the method of nuclear photoemulsions. They investigated the angular correlation between fission fragments and charged particles connected with the fission point. Only "black" tracks of charged particles were chosen for the analysis, i.e., of those particles which from energy consideration can be classified as evaporation particles. The result of the analysis shows that the coefficient of anisotropy is equal to one for protons and to 0.8 for α particles. The author of this paper concludes that the protons are evaporated by the fissioning nucleus prior to fission. He draws no specific conclusion as regards the mechanism of production of the α particle, but the value obtained for the anisotropy coefficient ($K < 1$) apparently gives grounds for assuming that these particles, like the protons, are not emitted by the fission fragments.

d) An interesting attempt at solving the problem of the level from which fission takes place at high excitation energies was undertaken by Dostrovsky, Frankel, and Rabinovitz who used the Monte Carlo method.[87] To solve this problem by this method it is necessary to estimate the probability of fission in relation to the probability of neutron emission at different excitation energies.

In the Bohr and Wheeler liquid-drop model[75] (as applied to the fission process) it is assumed that the point at the saddle is in static equilibrium. If this assumption is true for large excitation energies, then the Bohr and Wheeler classical formula can be used for fission at high excitation energies. However, since the number and nature of the fission channels differs here from those at small excitation, the fission parameters, such as E_s and $(Z^2/A)_{cr}$ should be determined experimentally at large excitation energies.

The correct energy dependence of the fission width over a wide range of excitation energies can be a sensitive test of this assumption and a test of the liquid-drop model at large excitation energies.

According to the Bohr and Wheeler model[75]

$$\Gamma_f = \frac{1}{2\pi\omega(E)} \int^{E-E_f} \omega^*(E - E_f - \xi)\, d\xi, \tag{A}$$

where $w(E)$ is the level density in the excited nucleus prior to fission, $\omega^*(E)$ the level density in the saddle, E_f the fission barrier. The width for neutron emission, according to Weisskopf[88] is given by

$$\Gamma_n = h \int_0^{E_A - E_n} \sigma(E_A, \xi) \frac{gm}{\pi^2 h^3} \xi \exp\left[S_B(E_A - E_n - \xi) - S_A(E_A)\right] d\xi, \tag{B}$$

in which E_A is the initial excitation energy of the nucleus A, E_n the binding energy of the neutron, $\sigma(E_A, \xi)$ the cross section of the inverse process, g the statistical weight of the spin state ($g = 2$ for neutrons), m the neutron mass, S_A the entropy of the initial nucleus, S_B the entropy of the residual nucleus, ξ the kinetic energy of the neutron.

If it is assumed that the nucleus is a degenerate Fermi gas, then

$$\omega_i(E) = \text{const} \cdot \exp \int_0^E \frac{dE}{T_i(E)},$$

where T_i is the nuclear temperature, $E_i = a_i T_i^2$ and consequently

$$\omega_i(E) = \text{const} \cdot \exp\left[2(a_i E)^{1/2}\right].$$

Assuming that $\sigma(E_A, \zeta)$ is equal to the geometric cross section at large excitations, we obtain by integrating (A) and (B)

$$\frac{\Gamma_f}{\Gamma_n} = \frac{K_0}{4A^{2/3}(E - E_n)} \cdot \left(2\sqrt{a(E - E_f)} - 1\right) \exp \times \times \left(2\sqrt{a(E - E_f)} - 2\sqrt{a(E - E_n)}\right), \tag{C}$$

where $K_0 = n^2/2mr_0^2$, r_0 the parameter of the nuclear radius, $E_f = E_s A^{2/3} f(x)$, $f(x)$ a function of $(Z^2/A) / (Z^2/A)_{cr}$, and E_f is determined after Frankel-Metropolis.

In the Bohr and Wheeler theory the surface-energy parameter E_s = 14 Mev, $(Z^2/A)_{cr}$ = 47.8; these quantities pertain to a radius r_o = 1.45 x 10^{-13}. Furthermore, for the liquid-drop model as applied to fission, the ratio $(Z^2/A)_{cr}/E_s$ is a constant quantity dependent only on the radius:

$$\frac{\left(\frac{Z^2}{A}\right)_{cr}}{E_s} = \frac{10 r_0}{3e^2} = 2{,}31 \cdot 10^{13} r_0 \ Mev^{-1}$$

A comparison of the various combinations of $(Z^2/A)_{cr}$ and E_s indicates that the best agreement with the experimental results is obtained at $(Z^2/A)_{cr}$ = 47.8 and E_s = 17.5 Mev. This combination yields r_o = 1.18 x 10^{-13} cm. Using formula (C), the authors obtain the ratio σ_f/σ_{inel} by the Monte Carlo method, and found it to be in good agreement with the experimental data, as can be seen from Table IX.

The good agreement between the calculated and experimental data is evidence that the parameters have been rather successfully chosen in formula (C). Using this dependence, the authors of that paper obtained the values of the fission probability from different excitation levels for several elements and at different initial excitation energies.

The results of the analysis show that for heavy nuclei Th^{229} and Fr^{219} at initial excitation energies up to 200 Mev the fission takes place from all excitation levels, but is more probable towards the end of the evaporation process. At considerably greater excitation energies, fission from lower levels is also more probable, but the process is less clearly pronounced. At excitations on the order of 100 Mev, fission at the final stage of the evaporation process is overwhelming. Thus, for example, at an excitation energy of Th^{229} of 100 Mev, 60% of the fission takes place after emission of more than seven neutrons, whereas for Fr^{219} at E_{exc} = 100 Mev, approximately 60% of all the fissions are for Fr^{211}, i.e., after the emission of eight neutrons.

Summarizing the various investigations devoted to a study of the mechanism of fission at large excitation energies, it must be noted unfortunately that there is still no fully established point of view on this question. For that group of heavy nuclei, for which the fission barrier is approximately equal to the neutron binding energy even in the initial state (U, Th), the interpretation of the results is the most complicated and the opinions of various authors who have investigated this question are still quite contradictory.

TABLE IX

Probability of fission of nuclei Bi and U, calculated by the Monte Carlo method and determined experimentally

Nucleus	E_n Mev	σ_f/σ_c (Monte Carlo)	Jungerman[26]	N. Ivanova[46]
Bi^{209}	286	0.11	0.125±0.02	
U^{238}	82	0.77	0.76	
U^{238}	156	0.78	0.76±0.1	0.77
U^{238}	236	0.76	0.76±0.1	
U^{238}	450	0.8		0.74

On the one hand, the relatively large fissioning ability of this group of nuclei naturally brings to mind that the fission should compete successfully with neutron emission over the entire stage of cooling of a strongly excited nucleus. On the other hand (as shown above), an analysis of many investigations of uranium fission at high excitation energies shows quite convincingly that the fission process is the final act in the cooling of the nucleus. Finally, there still exists an opposite point of view regarding the mechanism of the fission of uranium at high excitation energies. According to this point of view the fission is predominatnly from the upper excitation level (high temperature fission). However, the arguments in favor of this point of view are much weaker, and this gives grounds for preferring the emission mechanism of uranium fission at high excitation energies.

As regards the mechanism of fission of nuclei near the bismuth region, most experimenters agree in the opinion that the fission proceeds here via the emission mechanism.

The fission of nuclei in the tungsten region has not yet been sufficiently investigated. However, those few experimental facts that have accummulated by now indicate that in this case fission proceeds from excitation levels that exceed considerably the binding energy of the neutron in the residual nucleus (barrier fission).

Finally, the information concerning the mechanism of fission of nuclear located in the middle of the periodic system are worst than scanty and any specific conclusions regarding them would still be premature.

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