THE THEORY OF HYPERNUCLEI

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I. INTRODUCTION

RECENTLY there have been many studies of so-called hypernuclei (the name "hyperfragments" is sometimes used). Hypernuclei are bound nuclear systems consisting of nucleons and hyperons (hypernuclei known up to the present time contain only one hyperon).

By the study of hypernuclei it is possible to get some information about the characteristics of elementary particles (Λ, Σ, K, etc.), for example their spins, their parities, and their interactions with each other.

By now a comparatively large amount of experimental information on this subject has been accumulated. This opens up wide possibilities for the theoretical treatment of hypernuclei, both from the phenomenological point of view and on the basis of quantum field models. On its own side, the theory of hypernuclei can assist the development of the theory of ordinary nucleonic nuclei, and in particular the theory of nuclear forces.

The purpose of this survey article is a brief exposition of the main experimental facts, and also of the main theoretical investigations relating to hypernuclei.

II. EXPERIMENTAL STUDIES

1. The Discovery of Hypernuclei

In 1953 the Polish physicists Danysz and Pniewsky[1] called attention to an interesting event discovered in emulsion studies; this event is shown in Fig. 1. It shows a nuclear fragment f, produced by cosmic rays, with charge between 4 and 6, starting at the star A (21 + 18p) and decaying at the point B ($\sim 90\,\mu$ from A), where a second star appears, with 4 tracks. The first track has length $\sim 9\,\mu$, the second $\sim 123\,\mu$, and the fourth $\sim 2\,\mu$. The length of the third track could not be determined, since it passes out of the emulsion. The analysis showed that tracks 1 and 2 could be produced by the particles p, d, t, and α; track 3, by p, d, t, or π; evidently track 4 is a heavy nuclear fragment.

Excluding the very improbable case of simple coincidence of the end of the track f with the center of the star B, Danysz and Pniewsky

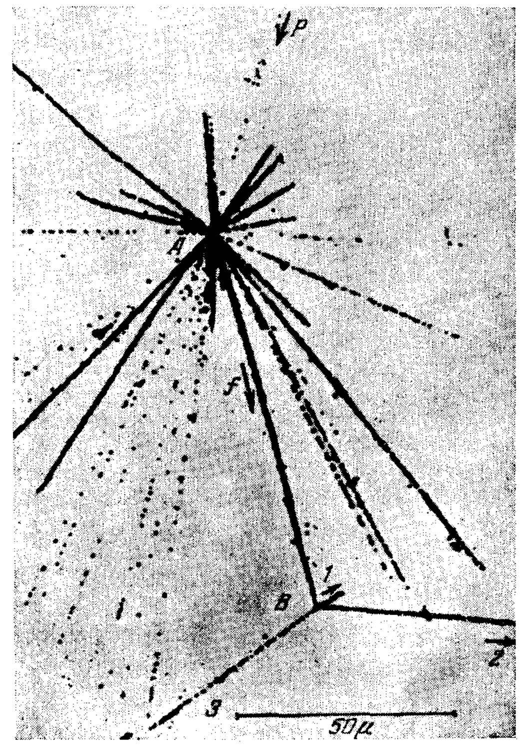

FIG. 1

pointed out that the star was produced by the decay of a nuclear fragment after it had travelled in the emulsion a distance of about $90\,\mu$ ($\sim 10^{-12}$ sec). The fact that the fragment had stopped, or nearly stopped, excludes the possibility of the production of the star B as the result of a simple nuclear collision between f and an atom of the emulsion. The large time between the production of the fragment and its decay excludes the possibility that f might be some ordinary nucleus in an excited state (if this had actually been the case, this time could not have been longer than 10^{-20} sec). One can conceive of the following explanations of this phenomenon.

a) The decay was brought about by the nuclear fragment's absorbing a π meson, which had been in a Coulomb orbit of a mesic atom.

b) The decay occurred as the result of the absorption of some other, heavier, meson from a Coulomb orbit.

c) There was a decay of some new particle (from energetic considerations this was most probably a Λ) which had been inside the nucleus.

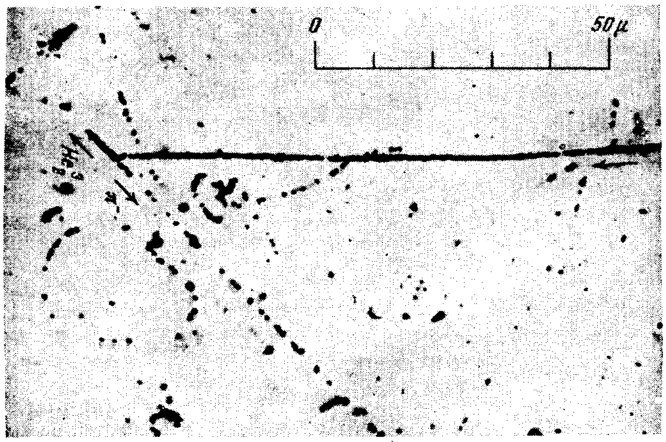

FIG. 2

Danysz and Pniewsky assumed that the last case was what had occurred.

In the case pointed out by Danysz and Pniewsky it was, however, hard to make the decision between a) and c). Soon afterward a similar case was observed by Crussard and Morellet.[2] They showed that, at least in their case, explanation a) could not be correct, by energetic considerations. A particularly important case was observed by Bonetti et al.[3,4] It is shown in Fig. 2. Here it was possible to measure directly the mass of the fragment, $M_f = (5150 \pm 1000)\, m_e$, and its charge, $Z = 1$. The secondary star consisted of two tracks (a long one, and a much shorter one). It was found that the second track was due to a meson, and the assumption was made that the following decay occurred:

$$H^3_\Lambda \rightarrow He^3 + \pi^- + Q.$$

From the conditions of energy and momentum balance the value of Q and the mass of the fragment were found. It turned out that $Q = 41.7 \pm 1$ Mev, $M_f = 5860\, m_e$, in agreement with direct measurements. The value of Q was considerably smaller than the value that would have occurred in the case of the absorption of a K meson. This excludes the possibility b), and if we assume that the process was c), this leads to a value of the binding energy of the Λ particle in the H^3_Λ nucleus of about 1 Mev.

Thus if the three events that have been described are of the same nature, this seems to be the most plausible uniform interpretation. Many further studies of this sort have reliably confirmed this assumption.

2. The Identification of Hypernuclei

Since at present the predominant mechanism of the formation of hypernuclei is unknown, the experimental studies are made with different sources of high-energy particles. Sensitive photographic plates are irradiated, for example, with cosmic rays, in proton beams with energies 3 and 6 Bev, in beams of π^- mesons (1.5 Bev), and so on. Hypernuclei are, however, a comparatively rare effect in nuclear reactions. Some data on the frequencies of their production for various sources of fast particles and K mesons are collected in Table I.

TABLE I

Nature of excitation	Number of hypernuclei observed	Number of stars studied	Frequency of hypernuclei per 1000 stars	Literature reference
3 Bev	14	14 480	1	36
Cosmic rays	6	24 000	0.25	9
Cosmic rays	5	25 000	0.2	35
3 Bev	21	20 000	1	
6 Bev	7	10 000	0.7	
K-meson capture	28	1 152	24	38
,,	2	52	26	39
,,	4	70	57	40
,,	2	52	39	39
,,	4	70	57	40
,,	46	1 001	46	66
,,	28	1 152	24	38
,,	11	319	34	70
,,	12	239	50	68
,,	28	815	34	69
Interactions of K mesons of energy 5-150 Mev in flight.				
,,	5	106	47	69
,,	17	415	40	70

As can be seen from the table, for protons of energy 3 – 6 Bev the frequency of production of hypernuclei is about 10^{-3} per star. In cosmic rays it is somewhat lower. Particularly large numbers of hypernuclei are produced by the absorption of K mesons, which, by the way, agrees with theoretical predictions. Up to now hypernuclei have been found only in nuclear emulsions. In studies of hypernuclei great difficulties are encountered in their identification. These difficulties are mainly in the separation of those cases that are decays of hypernuclei from other kinds of nuclear collisions or decays that occur with ordinary nuclei in the emulsions. For example, it sometimes happens that the total charge of the fragments from a secondary star is larger than the charge of the connecting track. This clearly indicates that in such cases ordinary nuclear collisions have occurred. The tracks of heavy fragments are sometimes very short ($< 15\,\mu$), so that in these cases it is difficult or quite impossible to determine the mass and charge of the fragment. Sometimes tracks can be so short that the secondary star practically coincides with the primary star, and it is almost impossible to observe it. On the other hand, light hypernuclei can have such long tracks that they do not decay in the emulsion, and this also makes their detection difficult. Light hypernuclei mainly decay with the emission of π mesons. Many π^0 meson decays can be erroneously interpreted as scattering. For example, it is extraordinarily difficult to establish the decay $H^3_\Lambda \rightarrow H^3 + \pi^0$, which can easily be confused with nuclear scattering, or the decay $He^4_\Lambda \rightarrow He^3_\Lambda + n + \pi^0$.

As a result of these and many other difficulties, even if one has a guaranteed case of the decay of a hypernucleus, it is sometimes almost impossible to determine unambiguously all of the particles and thus to find the exact scheme of the decay. In spite of this, however, several hundred cases of hypernuclei are known by now, although the number of cases in which the decay scheme has been exactly determined is much smaller.[1-74]

3. The Lifetimes of Hypernuclei

According to present experimental data, the majority of hypernuclei decay after stopping in the emulsion. Because of this it is not possible to determine their lifetimes with much accuracy. Only certain estimates can be made. It is found that the lifetimes are somewhat larger than 10^{-11} sec and somewhat smaller than 10^{-10} sec. This means that the lifetimes of hypernuclei are not very different from that of the free Λ particle ($t = 3 \times 10^{-10}$ sec). It is true that in some cases one can determine the lifetimes of hypernuclei by observing their decay in flight. Unfortunately at present such cases are few. Some of them are given in Table II.[135]

TABLE II

Case	Lifetime (10^{-10} sec)	Literature reference
H^3_Λ (7)	3.3	32
H^3_Λ	1.3	33
H^3_Λ	0.01	12
He^4_Λ	0.8	30
$He^{3,4}_\Lambda$	0.1	34
He^4_Λ	5.4	7
Li^7_Λ	0.1	34
B (?)	0.2	35

4. The Frequency of Hypernuclei as a Function of the Nuclear Charge

The data of the present section are from all cases known to be hypernuclei, even though the decay scheme may not be exactly established. This still allows us to gain some fairly definite ideas about the decay of hypernuclei.

Figures 3 and 4 show the Z-dependences of the numbers of cases of mesonic and nonmesonic decay.[135] Here the charges have been determined to accuracy ± 1. The first plot is more reliable than the second; the shape of the latter may suffer various changes as new experimental data are obtained, for the following reasons: a) Very heavy and highly charged ($Z \geq 5$) hypernuclei are usually emitted with small speeds and have short tracks, which naturally makes it hard to determine their charges; moreover, to produce heavy hypernuclei one needs more powerful sources of nuclear excitation. b) The efficiency of detection of non-

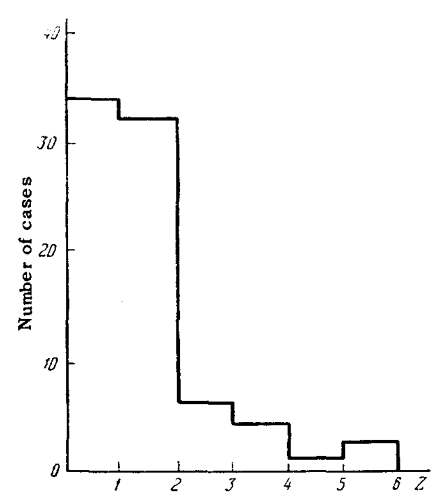

FIG. 3

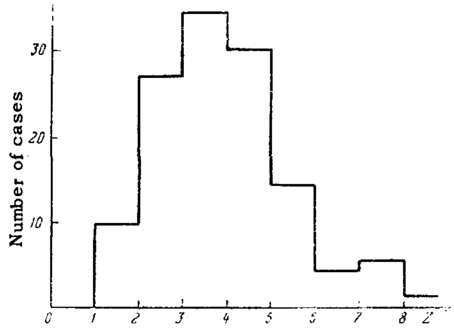

FIG. 4

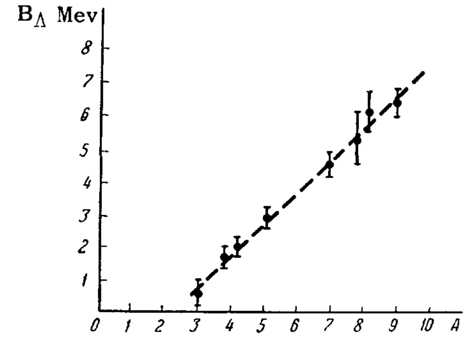

FIG. 5

mesonic and π^0-mesonic decays is comparatively low.

Nevertheless it can be seen from the plots that light hypernuclei decay mainly with meson emission, whereas the nonmesonic type of decay predominates for heavy hypernuclei. As we shall see later, the determination of the ratio of the nonmesonic and mesonic types of decay makes it possible to draw some conclusions about the spin of the Λ particle and the spins of hypernuclei.

5. The Binding Energy

The binding energy of the Λ particle in a hypernucleus is determined from the relation

$$B_\Lambda = M_\Lambda + M_A - \sum_i m_i - Q,$$

where M_Λ and M_A are the respective masses of the Λ particle and the nucleus containing A nucleons, m_i are the masses of the products of the reaction, and Q is the sum of their kinetic energies. The data relating to the binding energies of the Λ particle in hypernuclei are presented in Table III and Fig. 5.[98] The smallest binding energy is that in H^3_Λ; here the binding energy is close to zero. The absence of H^2_Λ and of the hyperhelium isotope He^3_Λ leads to the idea that the interaction of the Λ particle with nucleons is weaker than the nucleon-nucleon interaction. This is also confirmed by the fact that in the nuclei H^3_Λ, He^4_Λ, and Li^7_Λ, for example, the binding energy of the Λ particle is weaker than that of the last neutron in the corresponding ordinary nuclei. It is true that in H^4_Λ, He^5_Λ, and Be^9_Λ the Λ particle is more strongly bound than the corresponding last neutron ($B_n \leq 0$ for H^4 and He^5, since these nuclei do not exist, and for Be^9 $B_n = 1.67$ Mev), but this is explained by the fact that the Λ particle is in a 1S state, since it does not obey the Pauli principle with respect to nucleons.

TABLE III

Hypernucleus	Number of cases	B_Λ	Hypernucleus	Number of cases	B_Λ
H^3_Λ	9	0.6 ± 0.4	Li^7_Λ	3	4.5 ± 0.4
H^4_Λ	21	1.8 ± 0.3	Li^8_Λ	1	5.4 ± 0.8
He^4_Λ	9	2.0 ± 0.3	Be^8_Λ	1	6.2 ± 0.6
He^5_Λ	15	2.9 ± 0.3	Be^9_Λ	3	6.4 ± 0.4

It is interesting to note that the binding energy B_Λ of the Λ particle in hypernuclei has an obvious tendency to increase with increase of the atomic weight. This behavior can be qualitatively understood in the following way. Since the depth of the potential well for a Λ particle in nuclear matter depends only on the density of the nuclear matter (for given Λ-N forces), it is clear that in heavy nuclei the Λ particles will have a well-depth D independent of A, but the radius of the nucleus will increase with A, as $r_0A^{1/3}$. Since the binding energy of the lowest 1S state increases with increasing radius of the nucleus, it is natural to expect that B_Λ will increase monotonically with increase of A, approximately as the quantity $[D - \frac{1}{2}M_\Lambda \Gamma_0^2 A^{2/3}]$ for sufficiently large A.[98]

Later we shall examine in more detail the problem of the interaction of the Λ particle with the nucleons and the variation of the binding energies of the Λ particle in hyperfragments.

6. Some Anomalous Cases

One of the characteristic features of the decay of hypernuclei composed of a Λ particle and nucleons is the release of an energy Q (the kinetic energy of the decay fragments of the hypernucleus), which is 30 to 50 Mev for mesonic decays and 150 to 170 Mev for nonmesonic decays. A number of cases have been observed, however, which can be treated as cases of hypernuclei with anomalously large values of Q.[9,41-43] For example, Franzinetti et al.[9] have reported a case of nonmesonic decay. Analysis of the plate showed that the decay can be described as

$$^{10}B^* \rightarrow He^4 + He^3 + H' + 2n + Q.$$

The value of Q, however, turns out very large: $Q \cong 500$ Mev. In the case of reference 43 a π-mesonic decay was registered. The Q, however, was anomalously large for π-mesonic decays: $Q = 110$ Mev. In the case of reference 41 the secondary star has three tracks; one could be identified with the recoil of the nucleus and a second with a hydrogen isotope, and the third was similar to a K meson. The energy Q released in this case is ≥ 550 Mev. Since the nature of the connecting track could not be determined, these cases can be interpreted either as decays of hypernuclei or as captures of negative mesons. In the former case the hyperon would have to have a mass $\sim 2910\, m_e$, and in the second case the negative meson would have to have a mass $\sim 1075\, m_e$. Although it is still impossible to draw any serious conclusions on the basis of these isolated cases, they must be kept in mind in further studies.

A paper by Fry, Baldo-Ceolin, et al.[44] gives a not very clear indication that a bound system of a Σ^+ particle and a proton has been observed. These same writers recently stated that they have evidently finally succeeded in experimentally discovering the existence of a bound system Σ^+-p.[45]

III. THEORETICAL DISCUSSION OF HYPERNUCLEI

7. Basic Properties of Hyperons and K Mesons

Here we shall only recall some of the most important characteristics of the elementary particles and their interactions. Various aspects of this matter are treated in more detail in a number of reviews.[134-136]

As is well known, it is now generally agreed that all interactions (except the ultraweak gravitational interactions) can be classified in the following three groups:

a) Strong interactions. These interactions manifest themselves in processes of production and scattering of π and K mesons, hyperons, and nucleons. They are also responsible for the nuclear interaction of nucleons and hyperons. The strong interactions are characterized by a dimensionless coupling constant of the order of unity, $f^2/\hbar c \sim 1$.

b) Electromagnetic (intermediate-strength) interactions. These are characterized by the constant $e^2/(4\pi\hbar c) = 1/137$.

c) Weak interactions. These include the interactions that cause the β decay of neutrons and the decays of π, μ, and K mesons and of hyperons. They are characterized by the Fermi coupling constant $G^2/\hbar c \sim 10^{-13}$.

Here we shall be interested only in the strong interactions. A fundamental fact in the theory of elementary particles at the present time is the possibility of extending the principle of charge independence to the domain of phenomena that includes the strong interactions of K mesons and hyperons.

As is well known, the study of ordinary nuclei has led to the conclusion that the nuclear forces acting between neutrons are the same as those between protons (excluding the Coulomb interaction). This conclusion has come, in particular, from the equality of the binding energies of "mirror" nuclei such as, for example, H^3 and He^3, C^{13} and N^{13}, and so on.

In 1936 Breit et al.[75] advanced the hypothesis that nuclear forces obey a stronger requirement, namely, that they are charge-independent. This postulate asserts that the nuclear forces between any two nuclei are the same, if the particles are in states with the same spin and orbital angular momentum. Subsequent experiments have fully confirmed this conclusion.

Cassen and Condon[76] showed that this principle can be especially simply expressed in the language of isotopic spin, which had been first introduced by Heisenberg.[77] In particular, the charge of a nucleon is expressed by the formula $Q = T_3 + \frac{1}{2}$, where T_3 is the component of the isotopic spin along the z axis in an auxiliary isotopic space, and is $\frac{1}{2}$ for a proton and $-\frac{1}{2}$ for a neutron. Kemmer extended the concept of isotopic spin to π mesons, assigning to π^+, π^0, and π^- mesons the respective values $T_3 = +1, 0, -1$.

For π mesons the formula for the charge is $Q = T_3$.

Thus we can write

$$Q = T_3 + \frac{n}{2}, \tag{1}$$

where $n = 1$ for nucleons, $n = -1$ for antinucleons, and $n = 0$ for π mesons. All the experimental data existing at present (including those relating to hypernuclei) are not in contradiction with the hypothesis of charge independence as applied to hyperons and heavy mesons. Unquestionably the confirmation or disproof of this hypothesis will play a large part in the understanding of the nature of the elementary particles. Charge independence for the strong interactions can be expressed by the

TABLE IV

Particle	T	T_3	S
p, n	$\frac{1}{2}$	$\frac{1}{2}$, $-\frac{1}{2}$	0
π^+, π^0, π^-	1	+1, 0, −1	0
Λ	0	0	−1
Σ^+, Σ^0, Σ^-	1	+1, 0, −1	−1
K^+, K^0	$\frac{1}{2}$	$+\frac{1}{2}$, $-\frac{1}{2}$	1
K^-, $\widetilde{K}^0$	$\frac{1}{2}$	$-\frac{1}{2}$, $+\frac{1}{2}$	−1
Ξ^0, Ξ^-	$\frac{1}{2}$	$+\frac{1}{2}$, $-\frac{1}{2}$	−2

requirement that under the strong interactions there must be conservation of the magnitude of the isotopic spin of a system of particles. All particles now known to us can be distributed in a pattern of charge multiplets. A possible scheme, one of the simplest, is shown in Table IV. In this case, however, the charges of the particles are no longer given by the formula (1). Gell-Mann[78] and Nishijima[79] have generalized Eq. (1) by the introduction of an additional number S, which has been given the name of the "strangeness:"

$$Q = T_3 + \frac{1}{2}n + \frac{1}{2}S.$$

The values of S for the various particles are shown in the last column of Table IV. It is not hard to see that the strangeness must be conserved in the strong interactions along with the conservation of the isotopic spin (and consequently also its projection) and the baryon number (the number of baryons minus the number of antibaryons). The explanation of the physical meaning of "strangeness" in the framework of three-dimensional isotopic space has been the object of papers by d'Espagnat and Prentki,[80] Racah,[81] Salam,[82] and other authors. We may note that in addition to this some authors have considered a four-dimensional isotopic space, which leads to a somewhat different classification of the particles, in particular to the prediction of a fourth π meson, and so on.[131-133] Here, however, we can confine ourselves to three-dimensional isotopic space. Besides the requirements of invariance of the interaction with respect to rotations in the three-dimensional isotopic space, one also takes into account invariance with respect to reflections. Thus the natural requirement is imposed that the Lagrangian function that describes the interaction must be a scalar in the isotopic space. In addition it is assumed that the Lagrangian function is invariant: a) with respect to the proper Lorentz group and the group of reflections in ordinary space, b) with respect to charge conjugation, and c) with respect to a certain type of gauge transformation that assures the conservation of "strangeness."

On the basis of these requirements one sorts out all the possible fields and constructs the general Lagrangian function for all the fields and the interactions between them. It has the following form:

$$\begin{aligned} L = {} & g_1\overline{N}\,(i\gamma_5)\,\boldsymbol{\tau\pi}N + g_2[\overline{\Lambda}\Omega\boldsymbol{\pi\Sigma} + \overline{\boldsymbol{\Sigma}}\boldsymbol{\pi}\Omega\Lambda] + g_3[\overline{\boldsymbol{\Sigma}}\Omega \times \boldsymbol{\Sigma}]\boldsymbol{\pi} \\ & + g_4\overline{\Xi}\Omega\boldsymbol{\tau\pi}\Xi + g_5[\overline{N}\Omega K\Lambda + \overline{\Lambda}\Omega K^*N] \\ & + g_6\{[\overline{N}\Omega\boldsymbol{\tau\Sigma}K] + [K^*\overline{\boldsymbol{\Sigma}}\boldsymbol{\tau}N]\} + g_7[\overline{\Xi}\Omega\tau_2K^*\Lambda + \overline{\Lambda}\Omega\tau_2\Xi] \\ & + g_8[\overline{\Xi}\Omega\tau_2\boldsymbol{\tau\Sigma}K^* + K\boldsymbol{\tau\Sigma}\Omega\tau_2\Xi]. \end{aligned} \qquad (2)$$

Here $\Omega = 1$ or $i\gamma_5$, depending on the parity of the K mesons and the relative parities of the baryons. It is also assumed that all baryons have the spin $S = \frac{1}{2}$. On the basis of the experimental data on the large "up-down" asymmetry of the decay products of the Λ particle it can be concluded that its spin is actually equal to $\frac{1}{2}$.[83] The same is evidently true of the Σ particle. At the present time the spin of the Ξ particle has not been determined.

It can also be regarded as finally established that the spin of the K meson is 0.

On the basis of the system of Gell-Mann and Nishijima the possibility of the existence of metastable Λ nuclei is quite clear. Since among all the strange baryons the Λ particle has the smallest mass, Λ nuclei have lifetimes roughly equal in order of magnitude to that of the free Λ particle. According to the system that has been described it is impossible, for example, for Σ nuclei of certain types to exist because of the following fast processes:

$$\Sigma^- + p \rightarrow \Lambda^0 + n,\quad \Sigma^+ + n \rightarrow \Lambda^0 + p,\quad \Sigma^0 + n \rightarrow \Lambda^0 + n.$$

It is, however, possible (in principle) for the bound systems Σ^+-p or Σ^--n to exist.

Similarly the existence of a number of types of Ξ-nuclei is impossible because of the processes

$$\Xi^- + p \rightarrow \Lambda^0 + \Lambda^0,\quad \Xi^0 + n \rightarrow \Lambda^0 + \Lambda^0.$$

But in principle there is a possibility for the existence of the systems

$$\Xi^- - n \text{ or } \Xi^0 - p.$$

It is interesting to note that calculations on the basis of field theory with reasonable constants for the coupling of π and K mesons with baryons allow the existence of a bound system Σ^+-p (or Σ^--n).[119] At one time there was discussion of the question of the existence of K-nuclei, i.e., bound systems consisting of $K^+(K^0)$ mesons and nucleons.[84] It is now established that there is repulsion between K^+ mesons and nucleons, and K^+-nuclei cannot exist. The same is evidently true of K^0 mesons.

A very curious question is that of the existence of hypernuclei containing two Λ particles.[98] Such nuclei could be formed in cases in which a Ξ particle gets into an ordinary nucleus. Typical interactions leading to the production of nuclei with two Λ particles could be

$$\Xi^- + Li^7_\Lambda \longrightarrow He^6_{\Lambda\Lambda} + n$$

$$\Xi^- + Be^9 \begin{cases} \nearrow Li^9_{\Lambda\Lambda} + n \\ \searrow Li^8_{\Lambda\Lambda} + n + n \end{cases}$$

These nuclei could exist even if there is a not very strong repulsion between Λ particles. Calculations on the basis of a field model on the assumption of "global symmetry" show that the Λ-Λ forces are forces of attraction, though somewhat smaller than the Λ-N forces, so that the existence of nuclei with two Λ particles seems quite possible.

Experimental studies of nuclei of this type would be very interesting. Since the Λ particles in the nucleus would decay independently of each other, a chain of decays would be observed in an emulsion. The study of such decays will be one of the first possibilities for experimental investigation of the forces between two Λ particles.

8. Mesonic and Nonmesonic Modes of Decay of Hypernuclei. Spin of the Λ Particle

The lifetime of the Λ particle is larger than the characteristic time associated with the strong interactions by a factor of 10^{13}. Attempts have been made to explain such an anomalously long lifetime of the Λ particle by assuming that it has a very large spin, say $S \geq {}^{11}\!/_2$.[85-87] This can be understood qualitatively in the following way.

The wave function of the π meson produced in the decay of a Λ particle with angular momentum l and momentum k in a region $r \ll 1/k$ has the form

$$\frac{1}{\sqrt{v\tau_0}} \frac{(2l-1)!!}{r\,(kr)^l},$$

where $v = k/m$, and τ_0 is the lifetime of the Λ particle. Joining this solution onto the solution for $r > r_0$, where r_0 is the radius of the Λ particle, we get

$$\frac{1}{\sqrt{v\tau_0}} \frac{(2l-1)!!}{r_0\,(kr_0)^l} \sim \frac{1}{r_0^{3/2}}.$$

For π-meson energy $\epsilon = 37$ Mev, $k = 2\,(\mu_\pi \epsilon)^{1/2}$ $r_0 \sim (M\mu_\pi)^{-1/2}$, and $l = 5$ we find that $\tau_0 \sim 10^{-10}$ sec. As shown in reference 88, however, this hypothesis is in contradiction with the very fact of the existence of hypernuclei. In fact, the momentum of a nucleon absorbing the decay π meson is $p \sim (M\mu_\pi)^{1/2} \gg k$, so that the centrifugal barrier for a nucleon is considerably more transparent than for the π meson. Consequently the lifetime of a hypernucleus against nonmesonic decay is reduced by a factor $(p/k)^{2l}$. For $l = 5$ this factor is about 10^7; that is, the lifetime of a Λ particle with spin $\frac{1}{2}$ (sic — transl.) in a nucleus would be less than that observed experimentally by a factor $10^6 - 10^7$. Thus even these simple arguments show the inacceptability of the hypothesis of a large spin for the Λ particle, which has also been disproved by direct experiments (Alvarez et al.).

As was remarked in Sec. 6, for hypernuclei with $Z \geq 3$ the decay is predominantly through nonmesonic modes. Consequently, in heavy hypernuclei there must exist some mechanism that leads to the predominance of nonmesonic decay over mesonic decay. Primakoff and Cheston[89] have suggested that the virtual π mesons produced in the decay of the particle are absorbed by one of the nucleons of the nucleus, and that thus the mass difference $M_\Lambda - M_N$ is converted into kinetic energy of nucleons. The following processes occur:

$$\Lambda^0 + p \longrightarrow (p + \pi^-) + p \longrightarrow p + n,$$
$$\Lambda^0 + N\,(p) \longrightarrow (n + \pi^0) + n\,(p) \longrightarrow n + n\,(p). \qquad \text{(a)}$$

As is well known, in the decay of a free Λ particle, $\Lambda^0 \rightarrow p + \pi^-$, the relative momentum of the π meson is $k_d \cong 95$ Mev/c, whereas a virtual π meson can transfer to each nucleon a momentum $k_c \cong 420$ Mev/c. It is clear that in light hypernuclei, where the Λ particle is more weakly bound, nonmesonic decays will be less frequent. In heavy nuclei, however, where the Λ particle is mainly inside the core nucleus, being more strongly bound to it, nonmesonic processes will predominate. The matrix element for the processes (a) depends on the amplitude M_k of the interaction $\Lambda^0 \rightarrow p + \pi^-$. As Ruderman and Karplus[90] have pointed out, if we assume a sufficiently small effective radius for the Λ particle (less than 0.5×10^{-13} cm), the decay π meson has to overcome a centrifugal barrier caused by its orbital angular momentum l. Then, as is well known, the amplitude for penetration through the barrier is proportional to k^l; therefore $M_k = C_l k^l$, where C_l does not depend strongly on the values of the momentum and of l. On this basis the ratio of the nonmesonic mode of decay to the mesonic mode corresponding to the emission of a π meson with angular momentum l is

$$R_l = D\left(\frac{k_c}{k_d}\right)^{2l} \cong (17)^l D,$$

where D does not depend on l.

Usually the value of D depends on the interaction between the meson and the nucleon and on the nuclear density. Ruderman and Karplus found D in the following two cases:

a) for hypernuclei with $Z \geq 3$, in which the Λ particle is mainly inside the nuclear volume,

$$R_l(\pi^-) \cong 4 \times (17)^l.$$

We note that for $Z \geq 3$ two mesonic (π^-) decays were found experimentally, and 86 nonmesonic decays.[11]

b) For light hypernuclei, in which the Λ particle mainly occupies a volume larger than the core nucleus, the quantity $R_l(\pi^-)$ naturally depends on the binding energy, since the probability of capture is proportional to the probability for the Λ particle to be inside the core nucleus. In this case the result was

$$R_l(\pi^-) \cong 0.6\sqrt{B_{\Lambda,\,\mathrm{Mev}}} \times (17)^l.$$

For hyperhelium the experimental value of this ratio is ~ 1, and the value found for B_Λ was $B_\Lambda \sim 2$ Mev. Thus in these two cases good agreement with the experimental values is obtained for $l = 0$, although $l = 1$ also cannot be excluded. From this, on the basis of the arguments given above, Ruderman and Karplus also concluded that the spin of the Λ particle is most probably equal to $\frac{1}{2}$. A similar conclusion was reached independently by Nishijima,[91] though from a less rigorous treatment.

It has now been established from studies of angular correlations that the spin of the Λ particle is $\frac{1}{2}$. As we see, however, it has also been possible to reach conclusions about the spin of the Λ particle on the basis of arguments about hypernuclei.

9. Phenomenological Treatment of Hypernuclei with $A \leq 5$

As was pointed out earlier, the nature of the dependence of the binding energy B_Λ of the Λ particle on the mass number shows that the interaction between a Λ particle and a nucleon is weaker than the nucleon-nucleon interaction. This fact was established in the very first theoretical papers on hypernuclei.[92-95] For example, in reference 92 the simplest model was considered, according to which a self-consistent potential for the Λ particle is introduced in the form of a rectangular well of depth V_0, independent of the spins. Since the Λ particle is in the 1S state, we have for the binding energy B_Λ

$$B_\Lambda = V_0 - \frac{z_{1S}^2 \hbar^2}{2M_\Lambda R_0^2},$$

where z_{1S} is a quantity depending on the product $V_0R_0^2$; here $R_0 = r_0A^{1/3}$ is the radius of the well, and V_0 is its depth.

By varying V_0 one can find that the best agreement with experiment is obtained for $V_0 = 25$ Mev. We recall that in ordinary nuclei $V_0 \sim 40$ Mev. The simplest of the Λ-hypernuclear systems is that consisting of two nucleons and a Λ particle. It can exist in states with $T = 0$ or $T = 1$. In the singlet state this gives only the system H^3_Λ; in the triplet state, He^3_Λ, H^3_Λ, n^3_Λ. If for H^3_Λ the state with $T = 1$ has the smaller energy, the binding energy of He^3_Λ will be equal to that of H^3_Λ (it is usually somewhat smaller because of the Coulomb repulsion). If the state with $T = 0$ lies lower, then the state of H^3_Λ with $T = 1$ will go over into that with $T = 0$ by the emission of a γ-ray quantum (M1). In this case the B_Λ for He^3_Λ will be smaller than for H^3_Λ. Consequently, if He^3_Λ exists, its binding energy does not exceed the B_Λ for the system H^3_Λ, if charge independence holds.[96] According to present data H^3_Λ is evidently an isotopic singlet. As was indicated above, the nucleus He^3_Λ has not been discovered, although its decay $He^3_\Lambda \rightarrow p + p + p + \pi^-$ would have been comparatively easy to detect. These remarks could be applied also to n^3_Λ, although its decays by the schemes $n^3_\Lambda \rightarrow p + n + n + \pi^-$, or, less probably, $n^3_\Lambda \rightarrow H^3 + \pi^-$, would be harder to detect. From the fact that the nucleon-nucleon interaction in the state $T = 1$ is considerably smaller than that in the state $T = 0$, it is shown by calculation in reference 97 that for a three particle system the possibility is evidently excluded that there could exist a bound state with $T = 1$, i.e., the hypernuclei He^3_Λ, H^3_Λ, n^3_Λ. The assumption of charge independence of the Λ-N forces also requires that the binding energies of H^4_Λ and He^4_Λ should be approximately equal. This is indeed confirmed by the experiments. The difference of the energies B_Λ for these two nuclei can be explained by various electromagnetic effects, and also by the change of the energy of the core nucleus on account of the Coulomb repulsion.

The nuclei H^3_Λ, H^4_Λ, He^4_Λ, and He^5_Λ are those most studied up to the present time. The values of the binding energy of the Λ particle in these nuclei enable us, following the work of Dalitz, to draw some conclusions about the character of the interaction of Λ particles with nucleons.

Let V_S and V_T be the potentials for the interaction of a Λ particle with a nucleon in the singlet and triplet states, respectively. Then by introducing projection operators, we can write the expression for the potential of the Λ-N forces in the form (not including exchange forces)

$$V=\frac{3+(\sigma_\Lambda\sigma_N)}{4}V_T+\frac{1-(\sigma_\Lambda\sigma_N)}{4}V_S.$$

Assuming that the Λ particle moves in a certain averaged field produced by the nucleons of the rest of the nucleus, and that the remaining structure is not strongly deformed by the interaction with the Λ particle, we get for the effective potential of the Λ particle in the nucleus

$$U(\mathbf{r})=\left\langle S\left|\sum_{i=1}^{A}\int V_{\Lambda N_i}(|r-r_i|)\,\rho(r_i)\,d^3r_i\right|S\right\rangle.$$

Here the sum is taken over all the nucleons of the core nucleus, A in number. $\rho(r_i)$ is the density distribution of the core nucleus, normalized to unity. If the radius of the Λ-nucleon forces is small in comparison with the size of the hypernucleus, we can write

$$U(r)=U_A\rho(r), \tag{3}$$

where

$$U_A=\left\langle S\left|\sum_{i=1}^{A}\int V_{\Lambda N_i}(R_i)\,d^3R_i\right|S\right\rangle, \qquad R_i=r-r_i. \tag{4}$$

Thus this expression is an approximation of zero radius of action of the Λ-nucleon forces.

The hypernucleus He^5_Λ. In this case the core nucleus is an α particle. There is a high degree of probability that the spin of He^5_Λ is $\frac{1}{2}$. In this case use of the spin wave function of He^5_Λ to perform the averaging in Eq. (4) leads to the following expression:

$$U_4=3U_T+U_S, \tag{5}$$

where U_T and U_S are the volume integrals of V_T and V_S:

$$U_T=\int V_T(r)\,d^3r \text{ and } U_S=\int V_S(r)\,d^3r. \tag{6}$$

For $\rho(r)$ let us take the Gaussian distribution:

$$\rho(r)=\frac{1}{a^3\pi\sqrt{\pi}}e^{-\frac{r^2}{a^2}}. \tag{7}$$

In accordance with the experiments of Hofstadter, and taking account of the radius of the proton, we have for the parameter a[98]

$$a=1.18\cdot10^{-13}\text{ cm}.$$

We can find U_4 by solving the Schrödinger equation for a particle moving in the potential well determined by the expressions (3), (4), and (6). Such a problem has been solved, for example, in reference 123. Omitting the intermediate calculations, we give the value of U_4 obtained with $B_\Lambda = 2.9$ Mev: $U_4 = 630$ Mev f^3, where 1f (fermi) $= 10^{-13}$ cm.

In reference 98 an evaluation of the volume integral U_4 has been made with account taken of the finiteness of the radius of action of the Λ-nucleon forces and the deformability of the core nucleus. The potential taken for the Λ-nucleon forces was of the form

$$V(r)\sim\frac{e^{-\frac{r}{r_0}}}{r}$$

with

$$r_0=\frac{1}{m_K}\approx0.4f \text{ and } r_0=\frac{1}{2m_\pi}\approx0.7f.$$

With these assumptions the following values of the volume integrals were obtained:

$$U_4=925\text{ Mev}f^3 \qquad \text{for } r_0=\frac{1}{2m_\pi};$$

$$U_4=715\text{ Mev}f^3 \qquad \text{for } r_0=\frac{1}{m_K}.$$

The hypernuclei H^4_Λ and He^4_Λ. For these hypernuclei two values of the spin are possible: I = 0 and I = 1. The experimental facts available at present show that the spins of these nuclei are apparently equal to 0, and we shall assume this here. In this case

$$U_3=\frac{3}{2}U_T+\frac{3}{2}U_S. \tag{8}$$

In this case also let us choose the distribution (7) with the value $a = 1.38 \times 10^{-13}$ cm, which is obtained on the assumption that the observed difference of the binding energies of these nuclei, 0.76 Mev, is due to the Coulomb interaction of the protons in He^3. Corrections for the finite size of the proton have also been taken into account.

The following values are obtained for the volume integrals:

$$U_3=750\text{ Mev}f^3 \qquad \text{for } r_0=0;$$

$$U_3=660\text{ Mev}f^3 \qquad \text{for } r_0=\frac{1}{m_K};$$

$$U_3=850\text{ Mev}f^3 \qquad \text{for } r_0=\frac{1}{2m_\pi}.$$

The hypernucleus H^3_Λ. This nucleus is an isotopic singlet. If, according to our assumption, its spin is $\frac{1}{2}$, then

$$U_2 = \frac{3}{2} U_S + \frac{1}{2} U_T. \tag{9}$$

This nucleus has been treated in reference 97 by the methods of the three-body problem developed for the analogous Coulomb problem.

The calculations led to the following values of the volume integrals:

$$U_2 = 500\,\text{Mev}\,f^3 \qquad \text{for } r_0 = \frac{1}{m_K};$$

$$U_2 = 795\,\text{Mev}\,f^3 \qquad \text{for } r_0 = \frac{1}{2m_\pi}.$$

Starting with these estimates of the values of U_2, U_3, U_4, one can obtain some information about the values of U_T and U_S by using any two of the equations (5), (8), and (9).

The calculations show that in all cases we have $U_T < U_S$:

$$U_T \cong (0.2 \text{ to } 0.5)\, U_S.$$

These estimates are obtained on the assumption of central forces. Because of the short-range character of the Λ-nucleon forces, tensor forces will not bring about any important changes. There could, however, be decided modifications of the results if many-particle forces are important.

As has been shown by calculations,[127,128] many-particle forces indeed make a perceptible contribution to the interaction of a Λ particle with nucleons. This is due to the fact that, for example, the three-particle 2π-meson forces decrease more slowly with distance than the two-particle 2π-meson forces. The three-particle forces will evidently make a large contribution in the case of heavy hypernuclei.

10. Treatment of Light Hypernuclei on the Basis of the Field Model

In the theoretical analysis of hypernuclei, as in that of ordinary nuclei, there is also a second approach that is of interest; this involves the use of the field model for the interaction of the elementary particles. Without question the present theory of the elementary particles and their interactions is far from perfection. For example, one cannot obtain from the theory the values of the charges and masses of the elementary particles, which are mostly taken from experiment. Possessing this information, however, we can try to give a description of the effects involving the elementary particles by means of the relativistic quantum field theory, which is the only apparatus suitable for such a description. To be sure, the question remains open as to whether quantum theory in its existing form is applicable to the domain of phenomena occurring at distances $\sim 10^{-14}$ cm, or to the corresponding ultrahigh-energy processes. Furthermore, there are cogent arguments for the necessity of decided changes in the existing theory, for example by using nonlinear forms or by other changes. For phenomena, however, for which relatively large distances are characteristic, it has been possible to get many correct results. For example, quantum electrodynamics is in good agreement with experiment. The situation in mesodynamics has also considerably improved in recent years, in particular in connection with the development of the theory of dispersion relations. It has been possible to give a satisfactory treatment of the scattering of π mesons by nucleons and even to make advances in the understanding of the decays of π and K mesons. For the meson-nucleon system in the Tamm-Dancoff approximation, and for photoproduction, it has been possible to get a semiquantitative description in the simplified theory proposed by Chew.[101] The use of field-theory methods has also proved fruitful in the treatment of the problem of nuclear forces. Here we must first of all mention a paper by Levy,[102] who showed that the theory gives an interaction potential for two nucleons that contains a strong repulsion at small distances. Thus the theory approached a justification of the phenomenological potential of Jastrow, who had postulated this repulsion at small distances in order to explain the peculiar behavior of proton-proton scattering at energies from 100 to 400 Mev. Levy's paper contained some mistakes, which were afterward corrected in papers by Klein,[103] Hamada and Sugawara,[104] and Drell and Huang.[105]

In 1953 Brueckner and Watson[106] applied a number of new considerations in the treatment of the problem of nuclear forces; they took into account the scattering of virtual mesons by the two interacting nucleons with "suppression" of the effects of pairs of particles. Various considerations on the value of the pair-suppression constant affected the results only slightly. It turned out, however, that effects of multiple scattering of the virtual mesons are important in the region $r \lesssim \frac{1}{2}\hbar/\mu c$. Brueckner and Watson eliminated this effect by introducing a repulsive wall at small distances.

Working with the Brueckner-Watson method, Gartenhaus[107] used for the elimination of singularities at small distances the cutoff in the momentum of the virtual mesons, proposed by Chew, which is equivalent to the introduction of a form-factor for the nucleons.

As is customary, Gartenhaus replaced the pseudoscalar coupling of nucleons and mesons by a pseudovector coupling, using the well known Foldy-Dyson transformation and dropping all terms involving powers of the coupling constant higher than the first. The interaction Hamiltonian takes the form

$$H_{\rm int} = (4\pi)^{\frac{1}{2}} \frac{f_0}{\mu} \sum_{N=1}^{2} \sum_{\lambda} \int d\mathbf{r} \rho(\mathbf{r} - \mathbf{r}_N) \mathbf{r}_\lambda^N \sigma^N \nabla \varphi_\lambda (r).$$

Here a system of units is used in which $\hbar = c = 1$; μ is the mass of the π meson; $\rho(r)$ is a source function with the property $\int \rho(r)(d\mathbf{r}) = 1$ and f_0 is the unrenormalized coupling constant.

In the calculation of the interaction potentials one uses the Fourier transform of the source function,

$$v(k) = \int e^{ik\mathbf{r}} \rho(\mathbf{r})(d\mathbf{r}).$$

In addition to this, the constant f_0 is replaced by f, which is now the renormalized, "actual" constant, and is taken from experiment. Using this Hamiltonian, Gartenhaus obtained the nucleon-nucleon forces in the second and fourth orders of perturbation theory, associated with the exchange of one and two virtual π mesons. The numerical results obtained by Gartenhaus for the deuteron are in good agreement with the experimental data. Values used for the pseudovector interaction constant were $f^2 \sim 0.089 - 0.093$, which agree with the values obtained from the scattering and photoproduction of π mesons on nucleons. It must be pointed out that the Gartenhaus potential also gives quite satisfactory agreement with experiment for nucleon-nucleon scattering at low energies. We note that very recently Marshak and Signell,[120] by supplementing the Gartenhaus potential with a spin-orbit term, have got good agreement with experiment for nucleon-nucleon scattering for energies up to 150 Mev. In view of these facts we may suppose that the potentials of Brueckner and Watson and of Gartenhaus give a basically correct description of the interaction of nucleons, at least in the low-energy domain. This leads to the thought that they can also be used for the analysis of the interaction of hyperons with nucleons, and in particular for the discussion of the binding energies of Λ-nuclei. Whereas the forces between nucleons are mainly due to the transfer of π mesons, contributions to the Λ-nucleon forces can be expected from the exchange of both π and K mesons. These contributions can be examined in the following forms:[98]

a) Exchange of π mesons. The exchange of one π meson is forbidden ($\Lambda^0 \to \Lambda^0 + \pi^0$) because of the law of conservation of isotopic spin in the strong interactions. The absence of this process at once explains the fact that the forces between Λ particles and nucleons are weaker than nucleon-nucleon forces. Consequently, this part of the interaction can be caused only by the exchange of two π mesons, for example:

$$\Lambda + N \to \Lambda + \pi + \pi + N \to \Lambda + N$$

$$\Lambda + N \to \Sigma + \pi + N \to \Lambda + \pi + \pi + N \to \Lambda + N.$$

The effective radius of this interaction is

$$\sim \frac{1}{2m_\pi} \cong 0.7\, f.$$

b) Exchange of one K meson

$$\Lambda + N \to N + \tilde{K} + N \to N + \Lambda$$

$$\Lambda + N \to \Lambda + K + \Lambda \to N + \Lambda$$

with a radius of interaction $\sim 1/m_K \cong 0.4$ f.

Interactions of a combined type, by the exchange of π and K mesons together, are also possible; for them the radius $\sim 1/(m_\pi + m_K) \cong 0.3$ f. These two processes cause a transfer of "strangeness" from the Λ particle to the nucleon, and are thus exchange interactions.

c) More complex processes, including the exchange of more than one K meson, with or without π mesons. This will give an exchange interaction in the case of an odd number of K mesons. Its radius $\sim {}^1/_2\, m_K \cong 0.2$ f or less. An example of another, more complex, process that may play an important part is given by many-particle forces of the form:[98,109]

$$N + \Lambda + N \to N + (\pi + \Lambda + \pi) + N \to N + \Lambda + N.$$

The exchange of one π meson can occur along with an electromagnetic interaction:

$$\Lambda^0 \to \begin{Bmatrix} p + K^- \to \pi^0 + p + K^- \\ n + \tilde{K}^0 \to \pi^0 + n + \tilde{K}^0 \end{Bmatrix} \to \pi^0 + \Lambda^0.$$

This process may partly explain the difference of the binding energies of He^4_Λ and H^4_Λ. The effective value of the coupling constant for such processes is of the order of $(e^2/\hbar c)^2 G^2$, where G is the coupling constant for the π-N interactions.

Analyses of the binding energies B_Λ in light hyperfragments have been made in many other papers.[110-116] In our calculations and in the papers by other authors[110-116] forces have been considered that are associated with the exchange of one K meson, two π mesons, and a π meson and a K meson together. To eliminate the singularities at small distances a repulsive core was introduced,[111-114] with the radius of the repulsion chosen so that for the ordinary nucleon-nucleon forces $r_c \sim 0.45\,f$. Because of the short range character of the πK and 2K forces the main contribution from them will lie in the region of the repulsion. Since at the present time nothing is known about the size of the repulsive core, one can try to treat the binding energies of light hypernuclei by using the Gartenhaus method, but then the question arises as to the value of the cutoff for the K mesons. The calculation shows, however, that the K-mesonic forces make only a comparatively small contribution ($\sim 20-25$ percent), and to get qualitative agreement with experiment we can take the same cutoff value as for the ordinary forces. Of course, the statistical approximation cannot be regarded as particularly satisfactory for the K mesons (since the mass of the K meson is smaller than that of the nucleon by only a factor two); therefore the results are of a semiqualitative character.

The Λ-nucleon forces were obtained by using the Hamiltonian in the d'Espagnat-Prentki-Salam form (2), with the assumption that the coupling constants of the π mesons with the baryons are equal, in accordance with global symmetry.[117] It is assumed that the relative parities of all the baryons are positive. For simplicity we assume the scalar type of interaction of K mesons with baryons. Denoting the potentials of the forces associated with the exchange of one K meson, two K mesons, and a K and a π meson, by V^{1K}, V^{2K}, and $V^{\pi K}$, respectively, we have for these potentials the following expressions:[115-116]

$$V^{1K} = g_\Lambda^2 \frac{4\pi}{(2\pi)^3} p_x p_\sigma \int v^2(k) \frac{e^{ik(r_1-r_2)}}{\omega^{(K)2}} (dk),$$

$$V^{2K} = -g_\Lambda^2 (g_\Lambda^2 + 3g_\Sigma^2) \frac{(4\pi)^2}{(2\pi)^6}$$

$$\times \iint v^2(k_1) v^2(k_2) e^{i(k_1+k_2)(r_1-r_2)} \frac{(dk_1)(dk_2)}{\omega_1^{(K)3}\omega_2^{(K)2}},$$

$$V^{K\pi} = \left(\frac{f}{m_\pi}\right)^2 \frac{g_\Lambda^2 + g_\Sigma^2}{2} \frac{3(4\pi)^2}{(2\pi)^6} \frac{M_N}{M_\Lambda} p_x p_\sigma \iint (\sigma_1 k_2)(\sigma_2 k_2)$$

$$\times \left[\frac{1}{\omega_2^{(\pi)3}\omega_1^{(K)2}} + \frac{1}{\omega_2^{(\pi)2}\omega_1^{(K)3}}\right]$$

$$\times v^2(k_1) v^2(k_2) e^{i(k_1+k_2)(r_1-r_2)} (dk_1)(dk_2).$$

We assume that $g_{\Xi Y} = g_\Lambda$. This is not in contradiction with the experimental data available at present.

For the forces associated with the exchange of two π mesons we have:

$$V^{2\pi} = -\left(\frac{f}{m_\pi}\right)^4 \frac{3(4\pi)^2}{(2\pi)^6} \frac{M_N^2}{M_\Lambda^2}$$

$$\times \iint \left\{\frac{(\mathbf{k}_1\mathbf{k}_2)^2}{\omega_1^{(\pi)2}\omega_2^{(\pi)3}} + \frac{\sigma_1[\mathbf{k}_1\times\mathbf{k}_2]\,\sigma_2[\mathbf{k}_1\times\mathbf{k}_2]}{\omega_1^{(\pi)3}(\omega_1^\pi+\omega_2^\pi)\,\omega_2^{(\pi)2}}\right\}$$

$$\times v^2(k_1) v^2(k_2) e^{i(\mathbf{k}_1+\mathbf{k}_2)(\mathbf{r}_1-\mathbf{r}_2)} (d\mathbf{k}_1)(d\mathbf{k}_2).$$

In the derivation of the potentials $V^{2\pi}$ and $V^{K\pi}$ we have neglected the difference of the masses of the Λ and Σ particles, and in the derivation of V^{2K}, the differences of the masses of Λ, Σ, and Ξ; this introduces only a small error. Using the expressions that have been obtained for the forces, we can calculate the volume integrals.

If we take for the coupling constants the values $f^2 = 0.08$, $g_\Lambda^2 = g_\Sigma^2 = 1$, we get the following estimates of the values of U_T and U_S:

$$U_S = (U^{1K} + U^{2\pi} + U^{K\pi} + U^{2K})_S = +620\,\mathrm{Mev}\,f^3,$$
$$U_T = (U^{1K} + U^{2\pi} + U^{K\pi} + U^{2K})_T = +220\,\mathrm{Mev}\,f^3,$$
$$U_T = 0.35 U_S.$$

Thus without the introduction of additional parameters the quantum field theory of the Λ-nucleon forces gives results that agree fully with the phenomenological treatment. For the case also of the pseudoscalar interaction of K mesons with baryons the calculations lead to a stronger interaction in the singlet state.[114,115]

11. On the Spins of Some Hypernuclei

The determination of the spins of hypernuclei is a matter of great interest. It could confirm (or disprove) the conclusions drawn on the basis of the quantum-field calculations of Λ-nucleon forces, and the estimates of the interactions in the singlet and triplet states. The spins of hypernuclei can be determined by studying the angular correlations in nuclear reactions involving hypernuclei. Interesting reactions are cascade processes of the type

$$a + b \rightarrow c + d, \quad c \rightarrow e + f.$$

The products of the first reaction are in general polarized. The angular distribution of the products of the reaction depends on their spins and on the nature of the polarization. By comparing the distributions theoretically calculated for various spins with the experimentally observed distributions, one can try to determine the value of the spin. If particles c and d decay, the correlation of their decay products also depends on the value of the spin and its polarization. If particles a and b are unpolarized or have spins zero, one can examine the correlation of the following vectors: the direction of the incident beam, the direction of the reaction product c, the directions of its decay products. The angular distribution of the decay products of a particle with spin i in their center-of-mass system is given by the following formula:

$$I(\theta, \varphi) = \left| \sum_m Y_{im}(\vartheta, \varphi) \psi_m \right|^2,$$

where ψ_m is the spin function of the particle.

Let us consider the reaction $K^- + He^4 \rightarrow H^4_\Lambda + \pi^0$, $H^4_\Lambda \rightarrow He^4 + \pi^-$, which we shall examine near threshold. This cascade process can give some information about the spin of H^4_Λ and about the parities of the elementary particles. Furthermore it is easy to study it, since the spins of all of the particles involved (except the H^4_Λ) are known and are equal to zero; we assume that the spin of the K meson is also zero. From the law of conservation of parity we get

$$\Pi_\pi \Pi_{He} (-1)^l = \Pi_H \Pi_K (-1)^{l'},$$

where the Π_Y are the intrinsic parities of the corresponding particles, and l and l' are the orbital angular momenta of the particles before and after the reaction. Let us write this equation in the form

$$\Pi (-1)^{l+l'} = 1,$$

where Π is the product of all the intrinsic parities of the particles. Let us assume that the forces between the K meson and nucleon have a short radius of action, so that the He^4_Λ and K (sic — transl.) are produced in an S state, i.e., $l' = 0$. Then if $\Pi = -(-1)^i$, we have $l = i$, and the angular distribution takes the form

$$I_i(\gamma) \propto |P_i(\cos\gamma)|^2,$$

where γ is the angle between the axis (which we take in the direction of the incident π-meson (sic — transl.) beam) and the direction of the decay products of the hypernucleus in their center-of-mass system.

TABLE V

Parity \ Spin	0	1	2
$\Pi = (-1)^i$	1	$3\cos^2\gamma$	$\frac{5}{4}(1 - 6\cos^2\gamma + 9\cos^4\gamma)$
$\Pi = -(-1)^i$	Reaction forbidden	$\frac{3}{2}(1 - \cos^2\gamma)$	$\frac{15}{2}(\cos^2\gamma - \cos^4\gamma)$

If $\Pi = -(-1)^i$, the value $l' = 0$ is forbidden, since in this case $l = i$. Let us assume that in this case H^4_Λ and K (sic — transl.) are produced in the p state. In this case $l = i$, which follows from the law of conservation of parity. Table V shows the angular correlations for the values i = 0, 1, 2 of the spin of the hypernucleus.[121,122] The following reactions can be useful in the study of hypernuclei:[117]

$$K^- + He^4 \rightarrow H^3_\Lambda + n, \quad H^3_\Lambda \rightarrow He^3 + \pi^-,$$
$$K^- + He^4 \rightarrow He^4_\Lambda + \pi^0, \quad He^4_\Lambda \rightarrow He^4 + \pi^-,$$
$$\left.\begin{array}{l} K^- + He^4 \rightarrow \Sigma^0(\Lambda^0) + H^3 \\ K^- + He^4 \rightarrow \Sigma^- + He^3 \end{array}\right\}, \quad \Sigma \rightarrow N + \pi.$$

Some conclusions about the spins of hypernuclei can be obtained by studying the ratios of the numbers of decays assignable to various channels. Let us consider, for example, the decay

$$H^4_\Lambda \rightarrow \pi^- + He^4.$$

The Λ particle is in a 1S state relative to the core nucleus H^3. If I = 0 for H^4_Λ, then its two-particle π^--meson decay can be due only to the S-wave channel of the decay of the Λ particle. If I = 1, this decay can only be due to the p-wave channel. But not all p states lead to such a decay. If we choose the z axis along the direction of the emerging π meson, then only the states with m = 0, i.e., ⅓ of all the p states, lead to such a decay. Owing to parity nonconservation in the weak interactions the free Λ particle can decay by the S and p channels with certain branching coefficients. It has been shown in reference 98, by the analysis of some experimental data, that the p-state fraction is small and is about ⅓. Since the Λ particle in the nucleus H^4_Λ is relatively weakly bound, this value is also correct for the decay of the bound Λ particle. If the spin of H^4_Λ is unity, the number of two-particle π-meson decays should be a small fraction of the number of all π-meson decays, about 5 percent. Experiment, however, gives a much higher value, about 60 percent; this indicates that the spin of H^4_Λ is evidently ½ (sic — transl.). This in turn requires that the Λ-nucleon forces be stronger in the singlet state than in the triplet, which is in agreement with the direct calculations.

Let us consider the reactions

$$K^- + He^4 \begin{array}{l} \nearrow H^4_\Lambda + \pi^0 \\ \searrow He^4_\Lambda + \pi^- \end{array}$$

From the law of conservation of orbital angular momentum it follows at once that if the reactions actually occur and the H^4_Λ and He^4_Λ are produced in the ground state, the parity of the K meson is opposite to that of the Λ particle (if the latter has a positive parity relative to the nucleon). These reactions will be forbidden if the K meson is a scalar. Therefore the study of reactions in which K mesons interact with helium is of great importance for determining the properties of elementary particles.

In conclusion we can emphasize that the study of hypernuclei from all points of view will certainly be one of the important fields of nuclear physics and elementary-particle physics for a long time to come.

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