BRITTLE FRACTURE OF METALS

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1. THE THEORETICAL STRENGTH OF METALS

IT is the fundamental problem of the theory of metal fracture to explain the low actual strength of metals compared with their theoretical strength as well as to account for the two types of fracture — brittle fracture, which occurs without very appreciable plastic deformation, and ductile fracture, which occurs after considerable plastic deformation.

Since fracture results in the appearance of new surfaces the calculations are naturally concerned with the change of surface energy. This change provides the simplest means of calculating the theoretical strength of materials:

$$\sigma_{theor} = \sqrt{\frac{E\gamma}{a}}, \tag{1}$$

where E is Young's modulus, 2γ is the surface energy associated with the formation of two new surfaces and a is the separation of atomic planes in the crystal lattice. If for metals we assume $E = 10^{11}$ dynes/cm^2, $\gamma = 10^3$ ergs/cm^2 and $a = 3 \times 10^{-8}$ cm the theoretical strength becomes $\sigma_{theor} = 10^{11}$ dynes/cm^2, or about 10^3 kg/mm^2, which is considerably greater than the actual strength of metals.

2. GRIFFITH'S THEORY

Griffith[1] advanced one of the first explanations of the contradiction between experiment and theory for amorphous materials, particularly glass; he assumed the existence of cracks at the tips of which tension rises above the theoretical strength, making it possible for the cracks to propagate. A calculation of the stress distribution in a crack[2] shows that the presence of the crack causes reduction of the elastic energy by $\pi(1-\nu)\sigma^2 c^2/8G$, where G is the shear modulus, ν is Poisson's ratio, c is the crack length, and σ is the stress normal to the plane of a two-dimensional crack. It was Griffith's basic assumption that a crack grows only if its propagation reduces the free energy of the system, i.e., if the loss of potential energy of deformation exceeds the increase of surface energy within the crack cavity. On this basis we obtain the following relation between the critical size of an elliptical crack (the so-called Griffith crack) and the stress required for its growth:

$$\sigma = \sqrt{\frac{2E\gamma}{\pi c}}. \tag{2}$$

A calculation for a more probable crack shape whose tip width is of the order of the interatomic distance yielded a result which is considerably different from (2).[3]

Griffith's theory was entirely satisfactory for amorphous materials but could not be extended to metals in principle because of the different nature of the plastic deformation in metals.

Basic factors in the problem of fracture are the temperature dependence of the resistance to plastic deformation and the degree of homogeneity of the deformation. In amorphous materials the relation between the deformation rate and the applied stress is expressed by the viscosity equation according to which the deformation rate is proportional to the applied stress. The viscosity varies exponentially and increases rapidly with falling temperature; when the temperature of glass is reduced from 80° to 60°C its viscosity increases by a factor of 10,000. Thus for all amorphous materials there exists a temperature at which plastic deformation is practically impossible. The absence of plastic deformation is decisive for fracture; in this case a local stress concentration cannot be eliminated. This accounts for the fact that even friction can produce cracks on the surface of glass.

In metals the resistance to plastic deformation depends considerably less on temperature. With cooling to 4.2°K the yield point is multiplied only a few times, i.e., even close to absolute zero the resistance to plastic deformation is considerably below the theoretical strength and local stress concentrations can be eliminated through plastic deformation.

Finally, in an amorphous material the deformation rate in each volume element is completely determined by the state of stress in that element. In metals, as in ionic crystals, deformation is not homogeneous. In crystalline substances deformation is heterogeneous both within single grains (along slip planes) and in a specimen as a whole. This heterogeneous character of plastic deforma-

tion in metals evidently accounts for the formation of microcracks.

Orowan[4] has proposed a modification of Griffith's equation to make it applicable to metals. He finds it necessary to take into account the plastic deformation energy of the layer adjacent to the fracture surface; this energy is about 10^3 times greater than the surface energy in Griffith's equation. According to Orowan

$$\sigma = \sqrt{\frac{2Ep}{\pi c}}, \tag{3}$$

where p is the plastic deformation energy of the surface layer and depends mainly on the test conditions.

A. V. Stepanov has suggested that the discrepancy between the theoretical and practical strength of metals can be accounted for by defects arising during extension as a result of plastic flow.[5] According to the similar theory of Frenkel strength is not determined by cracks already existing in a body but by those which result from the application of a stress.[6]

Numerous investigations, especially during recent years, have shown that even brittle fracture is preceded by plastic deformation.[7-15] An x-ray study by Tsobkallo[14] found 5% deformation of the layer adjacent to the fracture surface in notched mild steel rupturing within liquid nitrogen. Chang[15] also used x-rays in detecting 2% deformation of a smooth sample undergoing brittle fracture at 38° K. These results confirmed the theory that fracture results from the plastic deformation which is required to produce microcracks.

A similar conclusion is reached from an analysis of experiments on the brittle strength of metals,[16] from which it follows that all processes which increase the stress required for plastic flow also increase the brittle strength. Finally, it is shown by some data that fracture can only occur after the stress exceeds the yield point.[17]

3. DISLOCATION THEORY OF FRACTURE

The idea that fracture results from plastic deformation is the basis for the dislocation theory of metal fracture. In 1948 Zener[16,18] pointed out that the relaxation of shear stresses within a slip band formed during plastic deformation results in a stress ahead of the edge of the band, which is calculated by Inglis'[2] formula to be proportional to $\sqrt{L/r}$, where L is the length of the slip band and r is the radius of curvature of its edge. After the slip band stops at the edge of a grain (since in the general case the slip planes of adjacent grains are oriented unfavorably) or a plate of

some solid (precipitate) phase further relaxation of stresses occurs within the slip band. As a result the stress ahead of the band edge continues to increase and may reach the limit of theoretical strength, after which cracks are formed. Further relaxation of stresses continues the growth of cracks already formed.

Zener also shows that from the point of view of the plastic deformation mechanism this stopping of a slip band at an obstruction is equivalent to the formation of a dislocation cluster (or array) whose density increases as it approaches the obstruction; such clusters of dislocations were subsequently discovered experimentally.[19,20] It thus follows from Zener's hypothesis that crack formation and fracture result from plastic deformation.

Zener's ideas were subsequently developed mainly by Petch[21,22] and Stroh.[23,24] Eshelby et al.[25] computed the positions of dislocations in a cluster around an obstacle and obtained the values of the shearing stresses produced by the dislocations. Koehler[26] used the positions of the dislocations given in reference 25 to obtain the normal stresses, and Petch[22] used dislocations to account for the relation between grain size and brittle strength.
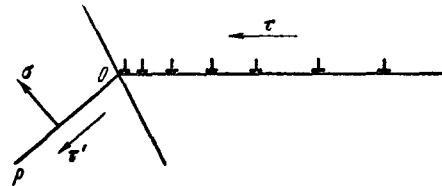

FIG. 1. Schematic diagram of a dislocation array.[23]

Stroh[23,24] considered the following conditions for crack formation. At distance r from the apex of the dislocation array the normal stress to the plane OP (Fig. 1) is given by

$$\sigma = \tau \left(\frac{L}{r}\right)^{\frac{1}{2}} f(\theta), \tag{4}$$

where L is the length of the slip band, τ is the shearing stress along the slip band and $f(\theta)$ is a function depending on the orientation of the OP plane. When a crack forms at the point O and propagates the distance c along OP the mean stress is

$$\sigma = \alpha \left(\frac{L}{c}\right)^{\frac{1}{2}} \tau, \tag{5}$$

where α is a constant.

The total energy of crack formation as determined by the $\pi(1-\nu)\sigma^2 c^2/8G^2$ reduction of elastic energy and by the surface energy 2γ of the crack cavity is

$$W = \frac{-\pi(1-v)\sigma^2 c^2}{8G} + 2c\gamma. \qquad (6)$$

Inserting σ from (5) into (6), we find that the formation of a crack leads to reduced energy of the system when

$$\tau^2 = \frac{16\gamma G}{\pi(1-v)\alpha^2 L}. \qquad (7)$$

After the calculation of α (7) becomes

$$\tau^2 = \frac{3\pi\gamma G}{8(1-v)L}. \qquad (8)$$

(7) and (8) give the conditions for the formation and growth of a crack to a length comparable with the length of a slip band. After a crack reaches the Griffith crack length [Eq. (2)] its further growth becomes possible through the normal stress alone.

According to Eshelby[25] a cluster of n dislocations establishes a slip line of length

$$L = \frac{Gbn}{\pi(1-v)\tau}, \qquad (9)$$

where b is the interatomic distance. Substituting this value into (8), we obtain

$$n\tau b = \frac{3}{8}\pi^2\gamma. \qquad (10)$$

Let fracture begin at a tensile stress of 32 kg/mm^2, i.e., at shear stress 16 kg/mm^2 (1.6 × 10^9 dynes/cm^2). With b = 2 × 10^{-8} cm and γ = 10^3 ergs/cm^2 we obtain n $\simeq$ 100 from (9). This number of dislocations creates a step of the order 200 A on the free surface. The step formed by a slip band in iron is estimated to be of height ~ 800 A, so that the required dislocation clusters can evidently be formed in iron.

The degree of plastic deformation which, according to the theory under consideration, must necessarily precede brittle fracture is very small. Let a grain of diameter d have a single slip line; the plastic shear will then be nb/d. Since the length L of a slip band is approximately equal to d, we obtain from (9) a plastic deformation of the order nb/L $\simeq$ τ/G, which is of the same order as for elastic deformation. In actuality the plastic deformation preceding brittle fracture is considerably greater[7,27] than the theoretically required magnitude; this discrepancy is still unaccounted for.

If the length of a slip line is taken to equal the grain diameter and we consider that in uniaxial tension the shear stress τ is one half the tensile stress σ, (8) enables us to obtain the relation between grain size and stress in brittle fracture. We must also insert into the left-hand side of (8) the difference $(\tau - \tau_0)$, in order to take into account the resistance to motion of dislocations (τ_0)

because of crystal lattice distortions resulting from the presence of dissolved atoms and dispersed precipitates. Following the various substitutions we have

$$\sigma = k \cdot d^{-1/2} + \sigma_0, \qquad (11)$$

where

$$k = \left[\frac{6\pi\gamma G}{(1-\gamma)}\right]^{1/2}. \qquad (12)$$

(11) is in good agreement with experimental results obtained for a number of steels, zinc, magnesium and molybdenum, and the values of k determined from experimental data are in good agreement with those calculated from (12). Also, the experimentally determined dependence of σ_0 on nitrogen and carbon concentration in iron (Fig.2) confirms the foregoing explanation of this quantity.[28]
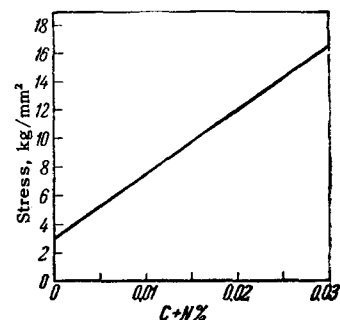

FIG. 2. σ_0 as a function of carbon and nitrogen concentration in ferrite.[28]

The critical shearing stress obtained from (8) may not cause crack formation if the stress ahead of the dislocation cluster is reduced through plastic deformation. Similarly, a crack already formed cannot propagate when plastic deformation eliminates the stress concentrations at the tips of the crack. The formation and propagation of a brittle crack is therefore possible only when plastic deformation is inhibited.

According to the dislocation theory, plastic deformation is associated with the movement of dislocations formed at Frank-Read sources and is impossible when these sources are blocked and cannot generate new dislocations.[29] The blocking of sources may result from the segregation of impurities at dislocations (occurring, according to Cottrell, in body-centered cubic metals) as well as from a shortage of slip planes (in metals with hexagonal structure).

In face-centered cubic metals where impurities are unable to block sources firmly, and in metals with body-centered cubic and hexagonal lattices in a ductile state, the growth of cracks is stopped by plastic deformation, but in the course of the deformation there is an increase in the number of

cracks, which eventually coalesce to form a single fracturing crack.[30]

The dislocation theory is in satisfactory agreement with experiment regarding the relation between fracture stress or the lower yield point and grain size, as well as the relation between the ductile-to-brittle transition temperature and deformation rate, grain size etc.

The lower yield point. With rising temperature the blocking of Frank-Read sources by impurity atoms becomes impossible and the metal becomes ductile. The stress generated by a dislocation cluster on a grain boundary does not induce a crack in an adjacent grain; instead it produces slip at a stress equal to the lower yield point. A number of experiments with ductile low carbon steel[31] showed that the maximum stress in ductile fracture of iron as well as brittle fracture is linearly dependent upon $d^{-1/2}$ (where d is the grain diameter), with the same slope for both lines (Fig. 3). This clearly indicates that the same mechanism is effective in both instances — the formation of dislocation clusters on grain boundaries.[28,22]
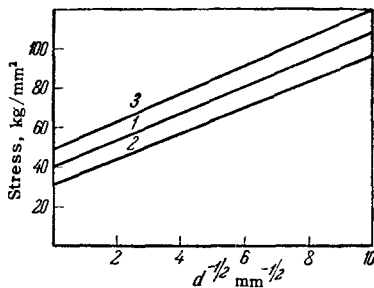
FIG. 3. Fracture strength of mild steel as a function of grain size:[31] 1 — test at — 196° (brittle fracture), 2 — test at 18° (ductile fracture), 3 — test at — 115° (brittle fracture).

While brittle fracture is associated with normal stresses in a grain resulting from the action of a dislocation cluster in an adjacent grain, slip results from tangential stresses in the grain produced by the same dislocation cluster. At a distance r from the apex of a dislocation cluster the shearing stress τ' (Fig. 1) is given by

$$\tau' = \beta\tau\left(\frac{L}{r}\right)^{1/2}. \tag{13}$$

From (13) we obtain the lower yield point:

$$\sigma_{l.\,y.\,p.} = K^* d^{-1/2} + \sigma_0, \tag{14}$$

where

$$K^* = 4\,(\alpha\sigma_u G_u)^{1/2}\beta^{-1}, \tag{15}$$

where σ_u is the upper yield point, and α and β are coefficients. Thus the same kind of relation is obtained for plastic flow as for brittle fracture.

FIG. 4. Lower yield point of mild steel as a function of grain size:[28] 1 — annealed steel, 2 — nitrited steel, 3 — quenched from 650°, 4 — quenched from 650° and and aged at 150° for 1 hour, 5 — quenched from 650° and aged at 200° for 100 hours, 6 — annealed Swedish iron.
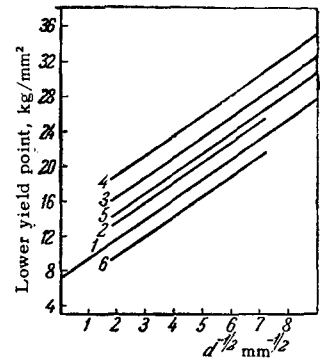
Equation (14) provides a good description of experimental results[28] (Fig. 4). For two temperatures T_1 and T_2 we have from (15):

$$\frac{K_1^*}{K_2^*} = \frac{(\sigma_u G)_1^{1/2}}{(\sigma_u G)_2^{1/2}}. \tag{16}$$

According to references 22 and 28 for mild steel at 20° C we have $K_{20}^* = 0.72 \times 10^8$ dynes/cm$^{-3/2}$ and at — 196° C, $K_{-196}^* = 1.9 \times 10$ dynes/cm$^{-3/2}$; thus $K_{20}^*/K_{-196}^* = 2.6$. It is known from experiment that in this temperature range the upper yield point varies by a factor of 4.1, while the elastic modulus changes by 10%. With these values in (16) we obtain 2.1, which agrees well with 2.6 from the slope of the experimental curves.

The ductile-to-brittle transition temperature. Recent data[7] have shown that values of impact ductility are distributed in a scattering zone around the ductile and brittle fracture levels. This is the so-called bimodal distribution (Fig. 5), which provides evidence that both brittle and ductile fracture exist in the scattering region at the same temperature. The probability of brittle fracture in the dislocation model is equivalent to the probability that blocking of dislocations is maintained.
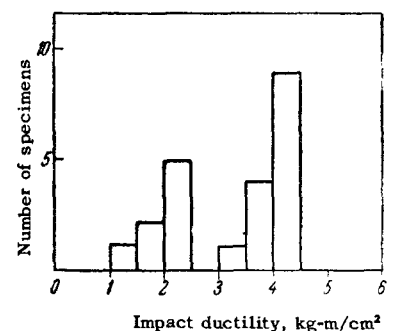
FIG. 5. Bimodal distribution of impact ductility in scattering zone.[7]

Impact ductility, kg-m/cm^2

The freeing of a blocked dislocation requires the activation energy U (σ), which depends on the stress σ acting on the dislocation. The probability that a dislocation is freed in unit time is given by ν exp $[-$U$(\sigma)/$kT$]$, where ν is a constant with the dimensions of frequency. Hence the mean

time that elapses before the freeing of a disloca-
tion is

$$\delta = \left(\frac{1}{v} \right) \exp \left[\frac{U(\sigma)}{kT} \right] . \qquad (17)$$

This equation expresses the relation between the
stress and the time required before freeing of a
dislocation. The existence of this relation is con-
firmed experimentally by the fact that the yield
point varies with duration of the load.[33]

The probability that a dislocation is not freed
in time τ is given by $p = \exp(-\tau/\delta)$ or, by in-
serting δ from (17):

$$p = \exp \left[-vt \exp \left\{ -\frac{U(\sigma)}{kT} \right\} \right] . \qquad (18)$$

In (18) σ is the maximum stress acting on a
blocked dislocation because of a dislocation cluster
and t is the duration of the maximum stress value.
Thus (18) is the probability of brittle fracture at a
given temperature, and the "critical" transition
temperature is

$$T_{tr} = \frac{U(\sigma)}{k} \ln vt. \qquad (19)$$

For comparison with experimental results we
may regard p as the fraction of the samples which
undergo brittle fracture at a given temperature and
(18) can be rewritten as

$$\ln \ln \left(\frac{1}{p} \right) = \ln vt - \frac{U}{kT} . \qquad (20)$$

This relation is confirmed by experiment.[32]

From (19) we derive the relation between the
transition temperature T_{tr} and the deformation
rate v:

$$\frac{1}{T_{tr}} = -\left(\frac{k}{U} \right) \ln v + C. \qquad (21)$$

The linear relation between $1/T_{tr}$ and $\ln v$ has
been observed by Wittman and Stepanov in a num-
ber of steels.[34]

The dependence of T_{tr} on grain size is deter-
mined by the probability of plastic flow in a grain
adjacent to a dislocation cluster; this probability
is equal to the flow probability $v \exp(-U kT)$ of
such a grain multiplied by the number of grains.
The number of grains in a given volume is in-
versely proportional to the grain volume and is
thus proportional to d^{-3}. We thus obtain

$$\frac{1}{T_{tr}} = -\frac{7}{2} \left(\frac{k}{U} \right) \ln d + C. \qquad (22)$$

Experimental results obtained with steels[35] con-
firm Eq. (22).

It is thus assumed by the dislocation theory that
Griffith cracks in metals result from plastic de-

formation. Griffith cracks are associated with
tangential stresses and their development is asso-
ciated with normal stresses applied to an entire
sample. The dependence of the transition tem-
perature on the stressed state can be considered
from this point of view.

We know that a triaxial stressed state (result-
ing from a notch) increases the ratio of the nor-
mal stress to the tangential stress. This usually
accounts for the higher transition temperature
(fracture strength is reached before the yield
point).

In the dislocation theory fracture can occur
only after the tengential stresses reach the re-
quired level. Therefore Petch[21] and Stroh[23] as-
sume that the triaxial stressed state cannot pro-
duce an approach of the fracture strength and
yield point, which in the dislocation theory are
determined by tangential stresses; thus variation
of the stressed state cannot vary the transition
temperature. Petch and Stroh account for the
higher transition temperature in notched samples
by an increased deformation rate in the notch.
This is unlikely since the change of the deforma-
tion rate in the notch cannot account for the ob-
served rise of the transition temperature.

It must evidently be taken into account that
tangential stresses control the first stage of
brittle fracture, which consists in the formation
of Griffith cracks that grow as a result of normal
stresses. The length of a Griffith crack is in-
versely proportional to the normal stress and is
determined by (2). It is clear that in a triaxial
stressed state the length of a Griffith crack re-
sulting from higher normal stress is consider-
ably small than in a uniaxial stressed state. This
means that whereas in a uniaxial stressed state a
crack ceases to grow because of plastic deforma-
tion and the crack cannot reach the Griffith size
required for brittle fracture, in a triaxial stressed
state plastic deformation does not prevent attain-
ment of the shorter Griffith crack length required
for brittle fracture. In the latter case the transi-
tion temperature must evidently be higher than
in tests on smooth samples.

4. TEMPERATURE DEPENDENCE OF YIELD POINT

The variation of the yield point down to $-196°$ C
in iron, tantalum tungsten, and molybdenum has re-
cently been investigated by Bechtold.[36] Wessel[37]
has made a similar study down to $4.2°$ K for nickel,
zirconium, β brass and steel alloys (with sphe-
roidized structure). In nickel (with a face-cen-
tered cubic lattice) the yield point varied by a

factor of 2, and in high-purity zirconium (with a close-packed hexagonal lattice) by a factor of only 3. The yield point of β brass (with a body-centered cubic lattice) increased by a factor of 7. According to Wessel the relatively small increase of the yield point of steels (by a factor of only 2) can be accounted for by their high yield point — of the order 100 kg/mm^2 — at room temperature. A similar result was obtained somewhat earlier by Uzhik,[38] who showed that the degree of increase of the yield point of steels with decreasing temperature is inversely proportional to their yield point at room temperature: whereas a temperature reduction to $-196°$C raises the yield point by only $10 - 20\%$ for high-strength steels ($\sigma_S = 160$ kg/mm^2), the yield point of Armco iron increases by a factor of $3 - 4$ for the same temperature reduction.

The investigations of Wessel[37] and of Bechtold[36] have shown that the yield point does not increase monotonically as the temperature is reduced. The yield points of steels, tantalum, molybdenum, and tungsten reach definite levels which do not change with further temperature reduction (Fig. 6).
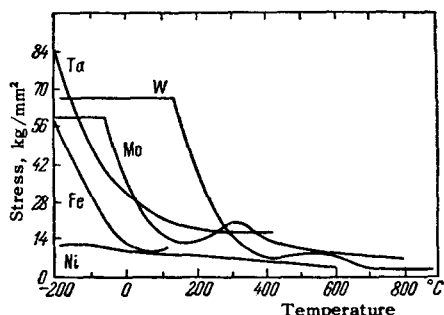

FIG. 6. Temperature variation of yield point and fracture strength of a few metals.[36]

This behavior of the yield point is not accounted for by existing dislocation theories, which require a rise of the yield point as the temperature drops down to absolute zero. According to Barrett[39] a possible cause of this effect is the fact that beginning with a certain temperature thermal oscillations do not play a part in the tearing away of dislocations from their "atmospheres."

The behavior of stress-strain curves at $4.2°$ K is noteworthy. Thus the stress-strain curves of all metals tested by Wessel exhibited sharp "drops" (Fig. 7). Such drops have been found in the curves of a large number of metals and alloys — iron, cadmium, austenitic steels, copper, neutron-irradiated copper, aluminum, nickel, Monel metal, high-purity aluminum, tantalum etc.[39] These effects can obviously not always have the same cause. The drops may result from twinning (in

FIG. 7. Stress-strain curves of steel alloy No. 4340 at low temperatures.[37]
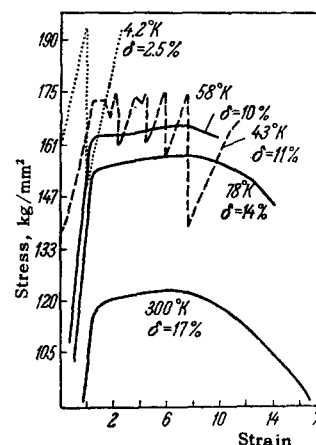
iron, for example) or martensite transformation, which evidently occurs in β brass at low temperatures and as a result of deformation;[39] but it clearly results most frequently from the thermal instability of metals at low temperatures. With low specific heat and thermal conductivity at low temperatures a localized deformation can result in a sharp local temperature rise which facilitates further slip. In Wessel's experiments on steel alloys a sharp drop of the stress-strain curve corresponded to a temperature rise of about $10°$C in a liquid helium bath, but no twins were observed in the samples. However, this mechanism can account for such drops only for metals with a body-centered cubic structure. Calculations show no considerable temperature rise due to localized slip in face-centered cubic metals.[40]

Numerous experiments have established a sharp rise of the yield point at low temperatures in body-centered cubic metals. A considerably smaller increase is observed in close-packed hexagonal metals and a very small rise in face-centered cubic metals (Fig. 6).

Thus far very little experimental work has been done on the mechanism of fracture, especially of its initial stages, most work having been devoted to an investigation of the conditions for the transition to the brittle state. It has already been mentioned that even brittle fracture is preceded by a certain amount of plastic deformation. It therefore seems necessary to investigate the initial stages of plastic deformation, especially those associated with the yield points of body-centered cubic metals. This has been done by Wessel[27] for mild steel, tantalum, niobium, and molybdenum. The results confirm earlier findings[41-44] that just before the upper yield point is reached very appreciable plastic deformation (of the order $0.1 - 1.0\%$) is observed. This means that a large number of dislocations move just before the upper yield point is reached. The liberation of the first disloca-

tions from the Cottrell "atmospheres" evidently occurs at stresses below the elastic limit. The motion of dislocations results in the formation of clusters at different barriers. This explanation provides a correct estimate of the magnitude of plastic deformation preceding the upper yield point.

The clustering of groups of dislocations around barriers and the corresponding rise of applied stress in the regions of clusters produces high stress concentrations. "Abrupt yielding" (AB in Fig. 8) results from stress relaxation around dislocation clusters because new sources of dislocations begin to be effective in adjacent regions, i.e., plastic flow begins in these regions. The relaxation of stresses in a single cluster results in greater microstresses in neighboring clusters; this produces general flow of the material. Figure 8 I gives a schematic idea of the initial plastic-flow stage above the critical temperature. The rounded segment of the stress-strain curve around the upper yield point is associated with an incubation period during which the first plastic slip occurs as a result of the formation of dislocation clusters. After the incubation period sudden flow of the metal begins.
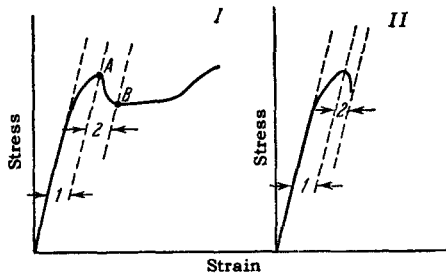

FIG. 8. Stress-strain curves of metals with abrupt flow at a temperature above (I) and below (I I) the temperature:[27] 1— Formation of dislocation cluster around obstructions, 2 — Stage of "abrupt yielding."

Below the transition temperature fracture occurs even during the period of flow. At low temperatures the motion of dislocations requires higher stresses, which are also required to initiate plastic flow through the action of dislocation clusters in adjacent grains. Consequently, very high local stresses may appear ahead of dislocation clusters and may exceed the theoretical strength and thus produce a crack (Fig. 8 II). This interpretation is confirmed by Wessel's experimental results for the initial stages of plastic flow.

A lower temperature increases the stress required to more dislocations; at the same time there is a sharp rise in the stress (the upper yield point) required to begin plastic flow or

crack formation. The higher stresses needed to begin plastic flow (or a crack) require the formation of denser dislocation clusters, i.e., an increased degree of plastic deformation preceding flow. For a number of metals investigated by Wessel (such as molybdenum — Fig. 9) the higher degree of plastic deformation preceding flow has been confirmed through cooling to the transition temperature. There is a corresponding increase of the plastic deformation which accompanies abrupt yielding.
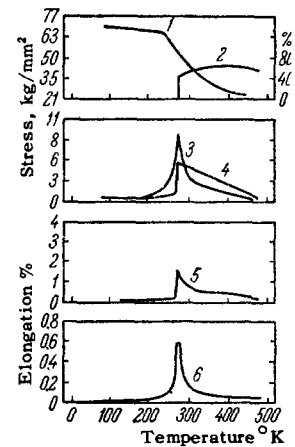

FIG. 9. Temperature dependence of mechanical properties in an early stage of plastic deformation of molybdenum:[27] 1—Upper yield point, 2—Elongation (in 25 mm), 3—Upper yield point minus proportional limit, 4—Upper yield point minus lower yield point, 5—Plastic deformation during abrupt yielding, 6—Plastic deformation preceding upper yield point.

With further cooling below the transition temperature the resistance to plastic deformation rises sharply. The formation of dense dislocation clusters is inhibited but the concentration of stresses ahead of such clusters is strongly enhanced; this accounts for the increased ability of dislocation clusters to form microcracks at low temperatures. It is evident that the degree of plastic deformation preceding flow must be reduced below the critical temperature; this was observed by Wessel (Fig. 9).

Different materials are characterized by different degrees of plastic deformation preceding brittle fracture; mild steel undergoes considerable plastic deformation, whereas molybdenum and niobium exhibit very insignificant deformation. This is evidently associated with the different abilities of materials to deform plastically under stresses produced by dislocation clusters.

Since plastic deformation is associated with the displacement of dislocations it is entirely possible that after sufficiently great plastic deformation (sometimes accompanied by cold working) dislocation clusters may be formed which are sufficient, through the resulting stresses, to produce a brittle crack. This is evidently the brittle fracture mechanism in the case of metals without distinct yield points.

It is clear that the physical state of a metal,

especially the nature of the distribution, shape and size of its structural elements which form barriers for dislocation clusters, exercises considerable influence on the ability of dislocations to form clusters and to initiate brittle cracks.

Thus brittle fracture requires a certain amount of plastic deformation; abrupt yielding and the ductile-to-brittle transition are related and determined by the same cause. The fracture mechanism can be described briefly as follows:

(1) Prior to abrupt yielding or brittle fracture dislocations move and finally pile up at barriers. This process is associated with considerable plastic deformation, the degree of which increases with decreasing temperature, reaches its maximum at the transition temperature, and is reduced at still lower temperatures.

(2) Above the transition temperature increase of the stress ahead of a dislocation cluster to the level required to overcome resistance to deformation produces flow of the material.

(3) Below the critical temperature the resistance to plastic deformation is so great that a stress ahead of dislocation clusters can induce cracks. The presence of high local stresses and the formation of microcracks, accompanied by some plastic deformation, lead to the development of brittle fracture.

5. THE ROLE OF IMPURITIES

Investigations of the temperature dependence of the yield point have shown that a sharp increase of the yield point at lower temperatures is observed in body-centered cubic metals and alloys, to a lesser extent in close-packed hexagonal metals and to an insignificant degree in face-centered cubic metals. It is known that the latter metals are not cold brittle, but conclusions regarding the first two groups are inconsistent.

It is usually assumed that body-centered cubic metals are inclined to cold brittleness; however, β brass does not undergo brittle fracture and high-purity tantalum undergoes ductile fracture in impact tests at $-196°$ C[45] and under static straining at $4.2°$ K.[39] With a sufficient oxygen content tantalum becomes brittle even at room temperature.[46] On the other hand, after zone melting iron was produced which at $4.2°$ K ruptured with as much as 80% necking.[47] It should be noted that at lower temperatures the yield point of such iron rises much less than in ordinary technical iron and that it approaches the yield point curves for face-centered cubic metals (Fig. 10).

We also know how the plasticity of other body-centered cubic metals — chromium, vanadium,
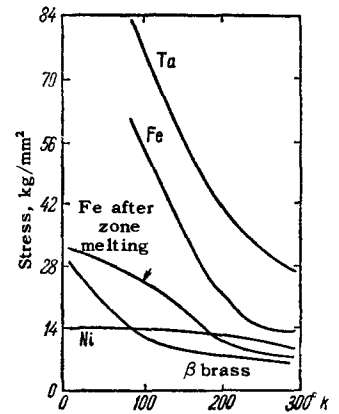

FIG. 10. Temperature dependence of the yield point of iron after zone melting.[47]

niobium, molybdenum, and tungsten — depends on purity. Plasticity in these metals depends on nitrogen, oxygen and carbon impurities, which form interstitial solid solutions. It is evident that the influence of these elements depends on how they distort the lattice, i.e., on the geometry of the lattice. A considerable role can also be played by the distribution of these elements inside grains and especially their segregation at grain boundaries.

A similar picture is observed in titanium and zirconium (with close-packed hexagonal structure). Thus impact tests of notched high-purity titanium at low temperatures did not produce brittle fracture,[48] whereas higher impurity content leads to brittle fracture even at room temperature.

The influence of impurities which form interstitial solid solutions is accounted for in the dislocation theory of metal fracture by their interaction with dislocations and dislocation sources. Dislocations that have been blocked by impurity atoms cannot participate in plastic deformation; thus instead of producing plastic flow a dislocation cluster generates microcracks and initiates brittle fracture.

The especially strong influence of impurities in body-centered cubic metals is evidently accounted for by the fact that such lattices with their relatively small interstices are more strongly distorted by oxygen, nitrogen, and carbon atoms. Moreover, foreign atoms in a body-centered cubic lattice produce asymmetrical tetragonal distortions. Carbon atoms in α-iron occupy octahedral interstices and can be located either at the centers of faces or at the midpoints of cell edges, thus producing (001) or (010) tetragonal distortion. According to Cottrell[29] a linear dislocation is blocked because the hydrostatic pressure around it is reduced as the unit cell volume changes in an interstitial solid solution. Screw dislocations are blocked because

shear stresses are removed with changing positions of interstitial atoms and tetragonal distortion along their axes.

Bechtold[36] used the fact that the deformation rate derived from internal friction measurements using free torsional oscillations agrees with the rate from static tensile tests, to infer that the temperature of the internal friction peak associated with the diffusion of interstitial elements must agree with the temperature at which the abrupt rise of the yield point begins. This hypothesis has been confirmed by experiment.

In face-centered cubic metals interstitial atoms distort the lattice uniformly in all directions, affect only the hydrostatic component of the stress field and can block only linear dislocations without affecting screw dislocations. Thus complete blocking of dislocations is impossible in face-centered cubic metals.

Hexagonal metals — cadmium ($c/a = 1.886$), zinc ($c/a = 1.856$), and magnesium ($c/a = 1.624$) — are evidently inclined to cold brittleness for another reason: at low temperatures the slip systems in these metals are insufficient in number for plastic deformation. Reduction of the ratio c/a in magnesium to 1.61 (close to titanium and zirconium) by adding lithium raises the number of slip planes and increases plasticity; a magnesium alloy containing 14.5% lithium ruptures at 4.2° K by necking.[49]

The causes of other anomalies have been investigated less thoroughly. Thus according the Barrett[39] body-centered cubic alkali metals are ductile at 5° K;[39] Kostenets has found that the plasticity of sodium does not decrease down to − 253° C.[50] This result may be associated with the fact that alkali metals do not form interstitial solid solutions, and this is apparently the reason why brittle fracture of β brass is not observed.

Some investigators relate the tendency to brittle fracturing with the change to a different deformation mechanism at low temperatures (twinning). Twinning alone can clearly not be a cause of brittleness, but like slip lines it leads to the formation of stress concentrations which can induce brittle fracture in metals which have a tendency to brittleness. Thus it is known that iron and zinc, which exhibit twinning at low temperatures, are cold brittle. On the other hand copper exhibits ductile fracture at 4.2° K although it is deformed by twinning.[51]

6. THE STRUCTURE OF STEEL AND ITS TENDENCY TOWARD BRITTLE FRACTURE

The modern theory of fracture in metals has still not found a theoretical relationship between metal structure and the degree of plastic deformation followed by fracture. Specifically, it is not possible at the present time to relate the concept of barriers for slip bands in dislocation theory with the structural components of steels and alloys (except for grain boundaries).

However, sufficient experimental work has already been done to enable us to determine the resistance of the various structures to plastic deformation and their tendency to brittle fracture. Thus it has been established that the resistance to plastic deformation of an alloy consisting of a soft phase that contains a dispersed hard phase is inversely proportional to the logarithm of the mean distance between particles of the hard phase and is independent of the amount of the latter.[52]

Impact tests have shown the especially low impact ductility of pearlite steel.[53] Steel becomes brittle after the formation of relatively small amounts of pearlite ($\sim 20\%$) on grain boundaries; subsequent high annealing does not appreciably enhance the impact ductility. The critical brittle temperature of pearlite steels is ordinarily 100 − 150° higher than for steels with spheroidized structure. However, the ductility of steels of pearlite structure can depend essentially on grain size, the separation of cementite plates and their size, etc.; when this structure is broken up the tendency to brittle fracture is reduced. Low ductility and brittle crystalline fracture are also characteristic of bainite steel; here also brittleness is not corrected by high annealing.

An investigation of the effect of residual austenite has shown that the ductility of steel is determined by the temperature during austenite decay in the course of an anneal preceding a test.[53,54] The kinetics of residual austenite decay resembles the kinetics of transformation in the original austenite, i.e., residual austenite can be transformed in pearlite or intermediate regions. The transformation of residual austenite in a pearlite or upper portion of an intermediate region sharply reduces the impact ductility. The formation of bainite from the decay of residual austenite does not worsen and sometimes even improves the impact properties of steel.

The negative effect of residual austenite on plastic properties appears when the austenite is transformed into martensite during tests of low-annealed steel, in which case unannealed hard martensite resulting from the decay of residual austenite, combined with softer annealed martensite, reduces plasticity in tensile tests as well as during static bending of notched samples.[55]

Total transformation in the lower region of an intermediate transformation results in the forma-

tion of a structure which is almost indistinguishable from martensite in its impact properties. Small quantities of upper bainite do not effect the transition temperature of steel with the structure of lower bainite, and reduce the impact ductility only slightly above the transition temperature. Increase of upper bainite above 30 — 40% raises the transition temperature and reduces impact ductility at room temperature. This negative effect of upper relatively soft bainite is evidently associated with the different hardnesses of upper and lower bainite. The same worsening of plastic properties has been observed during the simultaneous presence of soft and hard martensite in very strong samples.[56]

The brittleness of steel is also well known in cases where the martensite is surrounded by a fine ferrite network. When two phases with highly different yield points are combined a stressed state similar to that in a notch appears in the phase with the lower yield point (in this case the ferrite). Therefore brittle fracture occurs in the normally plastic component.[57]

According to Pashkov[58] an increase of the original austenite grain size raises the transition temperature of high-annealed steel. A similar result was obtained for martensite; the transition temperature of fine-grained martensite (obtained from fine-grained austenite) was 40° lower than for coarser martensite (obtained from coarser austenite grains).

Austenitic steel in its usual state is highly ductile, but after its partial martensite transformation, which produces hard martensite plates in a plastic matrix, this steel becomes brittle even at room temperature.

Crussard et al.[7] give the following classification of the different structures in the order of their increasing importance for the reduction of impact ductility: (1) a cementite network on grain boundaries, (2) martensite with troosite borders, (3) pure martensite, (4) pearlite columns.

These examples show that brittle fracture is furthered by the presence of two phases differing strongly in hardness. In some cases the shapes and distribution of the phases play a part; plates of precipitates are especially dangerous.

The importance of precipitate dimensions is well known from the brittleness of titanium in the presence of hydrogen.[48] When the temperature is lowered from 300° to 20° the solubility of hydrogen in α-titanium is reduced from 8 to 0.1 atomic percent; here lamellar titanium hydride is precipitated. When titanium containing hydrogen is slowly cooled from 400° coarse titanium hydride platelets appear accompanied by a sharp reduction of impact ductility; quenching from 400° in water produces thin titanium hydride platelets which have little effect on impact ductility. Aging at room or higher temperature, which is accompanied by the growth of hydride lamellae, is followed by lower impact ductility.

Small structural changes in steel containing 0.01 or 0.02% carbon can strongly affect brittle fracture.[59,60] The transition temperature in such steels depends strongly on the cooling rate at 500 — 600° after heating to A_{c_1}. A very low transition temperature after quenching from 700° is raised by 100° following slow cooling in the 500 — 600° interval. The cooling rate in this temperature range has no effect when the carbon content is small (of the order 0.001%), when the carbon is associated with a titanium admixture and when the carbon content is above 0.05%; the transition temperature of steel with 0.015% carbon, quenched from 700°, is considerably lower than in the same steel containing titanium.

This effect is evidently associated with a change of the carbide distribution. Thus the presence of carbides on ferrite grain boundaries and block boundaries, which is observed after slow cooling in the 500 — 600° range, raises the transition temperature considerably compared with uniform carbide distribution over entire grains (after solution at 700°, rapid cooling and aging). A higher transition temperature also accompanies an extension of the block boundaries.[60]

Similar results have been obtained for mild steel,[7] the impact ductility of which reached its maximum after rapid cooling around $A_{c_1} = 550°$; the cooling rate at lower temperatures had no effect on impact ductility.

An increase of the nitrogen content of iron from 0.005 to 0.02% raises the transition temperature,[61] a linear dependence being observed.

The influence of alloy components on the transition temperature has been well investigated phenomenologically,[62] but almost nothing is known about the mechanism involved. Alloy components may not only affect grain size but can also interact with interstitial atoms of nitrogen and carbon. It is known, for example, that manganese strongly affects the precipitation of nitrogen from a solid solution in alpha iron.[63]

The appearance of microcracks during deformation is often associated with a carbide phase. Bruckner[64] has found that during impact tension, cracks can form in pearlite columns and in carbide precipitates along ferrite grain boundaries. The small cracks appearing after ferrite fracture combine to form a brittle crack.

Fracture often begins at a grain boundary and then moves along an interphase boundary beginning at the grain boundary. Lorig[65] gives photomicrographs which illustrate the origin of a crack at a grain boundary where a sulfide is present. The crack either passes through the sulfide splitting it, or along the boundary between it and the grain, with the simultaneous formation of a few cracks that subsequently combine. Lorig also demonstrates crack propagation along lamellar precipitates of aluminum nitride in steel (along the boundary between the nitride and ferrite) as well as the generation of cracks in carbides and fracture along carbide boundaries (located along grain boundaries) in molybdenum. These examples show the favorable conditions for crack generation at inclusions along grain boundaries, but such cracks do not lead to brittle fracture when the matrix is able to deform plastically and prevent the development of the cracks.

Electron microscope studies of fracture surfaces by Crussard[7] and by Orlov and Utevskiĭ[66] have revealed carbide particles on ductile fracture surfaces, oriented regularly in ductile fracture "cups," in the case of high-annealed notched steel samples under impact. Carbide particles are also present on fracture surfaces in intergranular fracture. Carbide particles are ordinarily not observed in brittle fractures.

It is well known that the transition temperature of mild steel rises as a result of aging, especially strain aging. This is especially interesting because in strain aging there is apparently no formation of particles in a new phase but carbon and nitrogen atoms segregate at dislocations.

Zener[16] notes that the "abrupt" stopping of a slip band promotes the development of a brittle fracture. Thus in hexagonal metals as a result of an insufficient number of possible slip systems grain boundaries become "abrupt" obstructions for slip bands. Therefore spherical inclusions in these metals may enhance plasticity (for example, aluminum and zinc admixtures).

7. INTERGRANULAR FRACTURE

In some instances fracture along grain boundaries may result from the presence of continuous or discontinuous films consisting of particles of a second phase, such as have been detected in molybdenum, for example. However, the precipitation of continuous networks on grain boundaries is a very infrequent phenomenon, and the presence of individual particles does not as a rule induce brittleness. Therefore many instances of embrittlement are associated with impurity segregation

on grain boundaries, which weakens the bonds between grains. Since grain boundaries comprise a very small fraction of the total metal volume a very small number of harmful impurities can fill them. In practice atoms differing greatly in size from those of the solvent metal are most likely to segregate on grain boundaries.[67]

The development of brittleness along boundaries has been observed in iron-phosphorus alloys (up to 0.3% phosphorus) and in iron-nitrogen (containing 0.01% nitrogen).[68,69] Precipitates at grain boundaries were not observed in these alloys but the strong enrichment of grain boundaries with phosphorus was established radiographically in the case of iron-phosphorus.[70] Brittleness develops with continuous cooling from the austenitic region but especially strongly with isothermal maintenance at 600 — 700°. With increasing phosphorus content the temperature range of isothermal development of brittleness is broadened. Brittleness of both alloys appears both in impact tests and in tests of brittle strength at − 196°C (brittle fracture occurring at grain boundaries). These alloys are characterized by reduced strength when the test temperature is lowered from + 100° to − 196°C. The addition of 0.05% carbon to an iron-phosphorus alloy sharply modifies the embrittlement picture. Brittleness no longer develops abruptly but during a considerably longer period than the few minutes previously required. The effect of carbon is accounted for by the assumption that carbon also segregates on grain boundaries, hindering the subsequent segregation of phosphorus.

The mechanism of embrittlement resulting from the presense of phosphorus or nitrogen at grain boundaries has been described as follows. The expansion coefficient of grain boundaries enriched with phosphorus or nitrogen differs from that of metal in grain interiors; temperature reduction induces a stress system which leads to brittleness at grain boundaries. This accounts for the reduced strength of brittle specimens at lower temperatures, but the explanation can hardly be regarded as adequate. Specifically, it is not clear why when two specimens with identical phosphorus content on grain boundaries are maintained at 400° and 700°, respectively, the latter specimen is especially brittle.

Bismuth and antimony, each of which possesses an atomic radius much larger than that of copper, induce brittleness in copper.[71,72] The embrittlement of copper requires as little as 0.002% bismuth segregating at grain boundaries. Metallographic examination of a brittle specimen containing bismuth shows protuberances at grain bounda-

ries which were attributed to the presence of bismuth. Increased grain size furthered the embrittlement. Lowered impact ductility was observed in specimens that had been cooled rapidly from 350 — 550°. Slow cooling (but not rapid cooling) from above 600°, at which temperature bismuth becomes much more soluble in copper, resulted in embrittlement. Brittleness is evidently associated with the diffusion of bismuth atoms to grain boundaries.

It is assumed in reference 71 that a layer of bismuth atoms on a grain boundary varies in thickness at different places; therefore heating above 600° would dissolve the thinnest portions, making the film discontinuous, with the specimen thus becoming ductile. The bismuth film is also disrupted by 0.04% phosphorus, in the presence of which individual bismuth agglomerates are found on grain butt junctions.

Kê,[73] who investigated copper-bismuth alloys by measuring internal friction, also concluded that the bismuth is distributed nonuniformly on grain boundaries. In high-purity copper at 500° he found an internal friction peak associated with stress relaxation at grain boundaries; the introduction of bismuth led to a peak dependent on bismuth content at 290°, which grew as the peak at 500° diminished. Kê postulates that the part of the grain boundary not containing bismuth atoms produces the peak at 500°; the remainder is responsible for the internal friction peak at 290° resulting from the loosening action of bismuth atoms on intergranular cohesion.

An impact ductility test of copper specimens containing antimony also revealed embrittlement after slow cooling from 600°.[72] Antimony is highly soluble in copper and comprises 2.1% at 200°; no precipitates were found by microscopic examination, but grain boundaries revealed the same protuberances as in copper-bismuth alloys.

Copper-antimony alloys yield the characteristic impact ductility transition curves of cold brittle metals (Fig. 11).

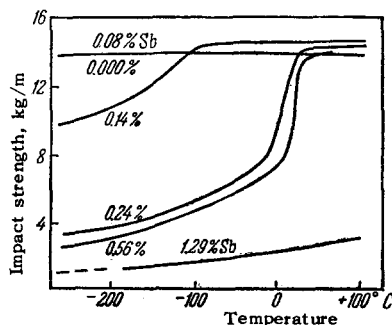It is evident that the general factor which re-

lates the brittleness of copper (a face-centered cubic metal) with that of body-centered cubic metals is the presence of impurities which distort the lattice. The different between copper and body-centered cubic metals evidently lies in the fact that in copper the impurity atoms can produce distortions only at grain boundaries, whereas in body-centered cubic metals impurities can distort the lattice even within grains, which leads to fracture along cleavage planes.

It was established in reference 74 that decarbonized iron becomes more brittle with a relatively small oxygen content. A metallographic analysis shows that with more than 0.003% oxygen brittle fracture occurs predominantly at grain boundaries, but the grain boundaries showed no oxide films or excess of oxide precipitation compared with grain interiors. The segregation of oxygen on grain boundaries may also possibly occur here. Low[17] has found that intercrystalline fracture of iron containing oxygen shows the same dependence of the fracture strength on grain size as in transcrystalline fracture, but, unlike the latter, does not involve appreciable plastic deformation even in the case of the smallest grains. The transcrystalline fracture strength of this steel is approximately equal to the yield point. The intercrystalline fracture strength coincides approximately with the yield point and fracture occurs immediately after the first shears, as a result of the low strength of grain boundaries in iron containing oxygen.

A microscopic examination of the fracture zone reveals very insignificant deformation at the fracture. The work p calculated in the Griffith-Orowan equation is nearly equal to the surface energy of a crack, which is shown to vary by the hydrogen embrittlement of steel. We know that 0.0001% hydrogen is sufficient to develop brittle fracture of steel along grain boundaries at room temperature, but that this brittleness will develop only in a polycrstalline specimen and not in a single crystal. Petch and Stables[75] attribute this effect to the adsorption of hydrogen on fracture surfaces, thus reducing their surface energy. Since the diffusion of hydrogen to a crack requires time, the crack grows slowly at first, but sudden fracture can occur after the crack reaches a certain size. Intercrystalline fracture is evidently accounted for by the lower diffusion of hydrogen along grain boundaries as a result of stresses.

The slow embrittlement predicted by this theory has been confirmed experimentally and accounts for the more pronounced effect of hydrogen in ordinary tensile tests than in impact tests. This

FIG. 11. Impact ductility of copper-antimony alloys as a function of test temperature.[72]

explanation is supported by the fact that the depend-
ence of brittle strength on grain size in hydroge-
nated steel can be expressed by an equation similar
to (12) but with a smaller value of K.[28]

A different theory of hydrogenated steel brittle-
ness has recently been proposed.[76] Following an
investigation of varying tendency to brittle frac-
ture during the aging of hydrogenated steel after
deformation, it was concluded that brittleness is
determined by hydrogen concentrations in stressed
zones created by microcracks, since the latter can
be regarded as crack concentrators. Thus hydro-
gen is responsible for the brittleness while still in
solid solution rather than after being precipitated
and adsorbed on crack surfaces.

8. THE PROPAGATION OF CRACKS

The investigation of cleavage planes has shown
that they are always studded with steps produced
as follows. Several of separate cracks are usu-
ally formed ahead of the continuously propagating
principal crack; they then unite as the bridges be-
tween them are broken down, accompanied by con-
siderable plastic deformation.[77] Steps can also be
formed when a crack passes through a screw dis-
location with its Burgers vector not lying in the
plane of crack propagation. Finally, large steps
can be formed when a crack intersects the boun-
dary between two grains of highly different orien-
tation.

As has been shown by dislocation theory, slip
planes can produce cracks, which in turn serve
to concentrate stresses. This is a quite likely
mechanism in fracture in view of the high speed
of brittle crack propagation.

In addition to the plastic deformation associ-
ated with the breakdown of bridges between indi-
vidual cracks, plastic deformation is also pos-
sible in the region that has not yet been reached
by the principal crack. Petch[21] believes that plas-
tic deformation can both precede crack propagation
and follow it. In the latter case plastic deforma-
tion results from the relaxation of the elastic
stresses associated with a brittle crack.

Robertson[78] has measured the temperatures at
which brittle crack propagation was stopped under
different stresses. He kept one end of a mild steel
plate 25 mm thick in liquid nitrogen while the other
end was heated to produce a temperature differen-
tial. A weak detonation in the liquid nitrogen bath
induced a brittle crack, which ceased to propagate
at a definite temperature. For sufficiently high
stresses the crack-stopping temperature was in-
dependent of the stress, but beginning at a certain
stress this temperature fell off sharply (Fig. 12).

FIG. 12. Stress required
for the propagation of a
brittle crack in mild steel
as a function of temperature.

Crack-stopping temperature
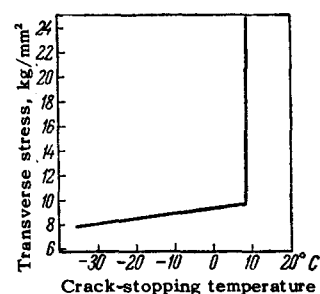
Robertson determined that a brittle crack can-
not propagate below a certain minimum stress,
which is usually 8 kg/mm^2 and lies considerably
below the yield point. This effect has been ac-
counted for as follows.[23] Dislocations are scat-
tered throughout the cleavage plane and whenever
the crack intersects a screw dislocation a step is
formed on the plane. Steps increase the surface
area of the fracture and the corresponding surface
energy. When ρ dislocations are present in unit
area the surface energy is increased by $\frac{1}{2}\rho c^2 b\gamma$.
When this quantity is added to the right-hand side
of (6) we find that the stress required for propa-
gation of a very large crack does not become in-
finitesimally small; instead it approaches a limit
which for $\rho \cong 10^{10}$ cm^{-2} is about 6.3 kg/mm^2.
This is approximately equal to the critical stress
that is observed experimentally.

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