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THE PRESENT STATE OF THE THEORY OF BETA DECAY

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1. INTRODUCTION

THE discovery of parity nonconservation in 1956 is one of the most surprising occurrences in the history of physics. The experiment of Wu (F1) with polarized cobalt, which has already become a classic, could without doubt have been done long before that time, and the only reason it had not actually been done was the deep conviction of physicists of the absurdity of the very thought that parity might not be conserved. Only the riddle of the decay of the K meson, which decayed as an even particle into two pions and as an odd particle into three pions gave rise to the question: what experimental foundations are there for such a deep conviction of the existence of such a general law? And suddenly it turned out that such foundations exist only for the strongly interacting particles — nucleons, for which processes that do not conserve parity have been shown to be nonexistent to an accuracy of 10^{-6} to 10^{-7}. In the case of the weak interactions, on the other hand, there were no such data on this point, and Lee and Yang advanced the bold hypothesis that parity is not conserved at all in such processes. One of the experiments they proposed was at once carried out by Wu, and about the end of the year 1956 parity conservation was struck out of the list of the basic laws of the physical world.

Further events developed very rapidly. At the end of this same year Landau (B6) and Salam (B7), and a few weeks later also Lee and Yang (B8) developed a new theory of the neutrino. At the same time Landau (B5) stated the bold hypothesis of the conservation of combined parity — a hypothesis that allows the retention of our ideas about the symmetry of space-time.

Right after Wu's experiments there appeared notes about measurements of the longitudinal polarization of electrons (E1–6), the magnitude of which was in agreement with the predictions of the theory.

The further experimental developments in 1957 went through a stage full of contradictions. The confusion that arose after the publication of experiments by Allen and others (C11) on the correlation in the positron decay of Cl^{34} was so great that there were even doubts as to whether positron and electron decays obey the same laws. Subsequently it was found that the root of the contradictions lay in the insufficiently careful analysis of the results of the experiments on the correlation in the decay of He^6.

The end of 1957 and the year 1958 saw gradual elimination of the contradictions and the emergence of an organized picture of the phenomena.

A new direction was given to research by papers of Sudarshan and Marshak (B41) and Gell-Mann and Feynman (B40) which presented very interesting ideas about the universal character of the weak interactions.

These papers formulated the hypothesis of the V-A type of interaction, which was at once confirmed by many experiments. The study of He^6, which contradicted the proposed theory, was declared by these theorists to be unreliable, and experimenters later agreed with this view* (C16).

*The only experimental result remaining not in accordance with the scheme was then the absence of decay of the π meson into an electron and a neutrino.

Even Gell-Mann and Feynman were afraid to doubt this result. But evidently this last obstacle was also an illusory one. At the conference in Geneva in September 1958 an account was given of the discovery of such a decay [Fazzini, Fidecaro, From that time on there has been a new quantum number in physics, chirality; conservation of chirality has been found to be a fundamental feature of β-decay processes.

After this work the β-decay theory fitted very well into the general scheme of a universal interaction, which had as another case the decay of the μ meson.

At this same time Goldhaber, Grodzins, and Sunyar (D5) made a direct measurement of the chirality of the neutrino, which confirmed all the postulates of the theory.

Finally, as early as the Geneva Conference in the summer of 1958 Telegdi and his coworkers (F7) announced the results of experiments with polarized neutrons which demonstrated the correctness of the theory in terms of a more elementary example and made it possible to determine the relative sign in Fermi and Gamow-Teller transitions. At this same time the measurement of the fundamental constants was completed. Finally, the experiments of Spivak and his group led to the determination of the lifetime of the neutron with adequate accuracy. In the summer of 1958 we for the first time found it possible to write the β-decay Hamiltonian with the numerical values of all the constants contained in it.

The only question remaining completely unsettled was that of the conservation of the combined parity; the experiments on this are not yet finished.

Preliminary results of experiments of Clark and others (F8) on the e-ν correlation in the decay of polarized neutrons gave a first confirmation of the Landau hypothesis (though still with very low accuracy).

The conservation of combined parity is favored by the negative result of experiments seeking to detect a dipole moment of the μ meson (H3).

Thus a new picture of the β interaction has come into being. But the accuracy of the experiments performed recently is still not great enough. Strictly speaking, there still remains the question: do there perhaps exist new effects (they may be concealed in the 10 to 15-percent range of the experimental errors) that will show that the new picture gives only an approximate description of the phenomena? Is the neutrino a precisely longitudinal particle? Does the electron behave in a strictly two-component way? This must be answered by further experiments.

Merrison, Paul, and Tollestrup, Phys. Rev. Letters 1, 247 (1958)].

The ratio of the number of $\pi \to e + \nu$ to the number of $\pi \to \mu + \nu$ decays was $> 4 \times 10^{-5}$. A similar value ($\sim 10^{-4}$) was also obtained by an American group [Impeduglia, Plano, Prodell, Samios, Schwartz, and Steinberger, Phys. Rev. Letters 1, 249 (1958)].

With this reservation, it is now useful to gather together the existing theoretical arguments and experimental results, and discuss them from a single (if not yet definitively proved) point of view.

This task is undertaken in the present survey. In it an attempt is made to give a systematic exposition of the theory of allowed β decays on the basis of the model of the two-component neutrino. Since the main task of the survey is not to give formulas for the concrete analysis of experimental data, but only to give the physical picture, only the clearest cases are considered; attention is given to all sorts of complications (forbidden transitions, the Coulomb field) only to the extent that they affect the qualitative aspect of the phenomena.

The survey does not include problems connected with parity nonconservation in μ-meson and K-meson decays, nor the problem of the universal weak interaction. It is confined to problems relating to the β decay of nuclei.

2. THE DIRAC EQUATION

We begin with a brief summary of the properties of Dirac's equations. As is well known, in the absence of external fields the equation of a relativistic particle with spin $\frac{1}{2}$ and nonvanishing rest mass has the form*

$$(-W + \boldsymbol{\alpha}\mathbf{p} + \beta m)\psi = 0, \tag{2.1}$$

where W is the energy of the particle (including the rest mass); $\mathbf{p} = -i\nabla$ is the momentum operator; and α and β are matrices which can be expressed in terms of the four two-rowed Pauli matrices σ_x, σ_y, σ_z, and 1:

$$\boldsymbol{\alpha} = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2.2}$$

This way of writing them presupposes that each element in an array is a two-rowed matrix.

The solution of Eq. (2.1) is written in the form of a plane wave multiplied by a four-component quantity (bispinor):

$$\psi = u \exp i\mathbf{pr} \tag{2.3}$$

Moreover, it is convenient to represent u in terms of two two-component quantities φ and χ:

$$u = \begin{pmatrix} \varphi \\ \chi \end{pmatrix}. \tag{2.4}$$

Then from Eq. (2.1) we get for φ and χ the algebraic equations

*In units with $\hbar = c = 1$.

$$W\varphi = \boldsymbol{\sigma}\mathbf{p}\chi + m\varphi,$$
$$W\chi = \boldsymbol{\sigma}\mathbf{p}\varphi - m\chi, \tag{2.5}$$

where **p** is now an ordinary vector, and not an operator. Using the second of the equations (2.5) we can express χ in terms of φ:

$$\chi = \frac{\sigma \mathbf{p}}{W+m}\varphi. \tag{2.6}$$

To get a normalized function, we shall suppose that the two-component quantity φ is normalized:

$$|\varphi|^2 = \varphi_1^*\varphi_1 + \varphi_2^*\varphi_2 = 1.$$

For the normalization of u we require

$$|u|^2 = |\varphi|^2 + |\chi|^2 = 1.$$

Noting that $(\boldsymbol{\sigma}\mathbf{p})^2 = p^2$, we now get

$$u = \left(\frac{W+m}{2W}\right)^{\frac{1}{2}} \begin{pmatrix} \varphi \\ \frac{\sigma\mathbf{p}}{W+m}\varphi \end{pmatrix}. \tag{2.7}$$

The Dirac equation is also often written in a more symmetrical form. Let us change the sign of the second of the equations (2.5) and multiply both these equations by i. Then it is easy to see that they can be written in the form

$$(iW\gamma_4 + \boldsymbol{\gamma}\mathbf{p} - im)\,u = 0 \tag{2.8}$$

or in the four-dimensional form

$$\gamma_k \nabla_k \psi_k + m\psi = 0 \qquad (\nabla_k = +ip_k). \tag{2.9}$$

The four-rowed matrices γ_k are given by

$$\boldsymbol{\gamma} = -i\gamma_4\boldsymbol{\alpha} = \begin{pmatrix} 0 & -i\sigma \\ i\sigma & 0 \end{pmatrix}, \quad \gamma_4 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2.10}$$

The significance of the multiplication by i is that thus we make all four γ matrices Hermitian.

The matrices γ_k (k = 1, 2, 3, 4) have obvious properties: they anticommute with each other, and their squares are equal to unity,

$$\gamma_i\gamma_k + \gamma_k\gamma_i = 2\sigma_{ik}. \tag{2.11}$$

The product of all four of the γ_i is denoted by γ_5:*

$$\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4 = -\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \tag{2.12}$$

γ_5 anti-commutes with all the other γ_k:

$$\gamma_5\gamma_k + \gamma_k\gamma_5 = 0. \tag{2.13}$$

It is easy to see that $\gamma_5^2 = 1$ and

$$\boldsymbol{\alpha} = -\boldsymbol{\sigma}\gamma_5. \tag{2.14}$$

*In some papers this product is denoted by $i\gamma_5$.

To set up the interaction Hamiltonian we have to form bilinear expressions from the quantities ψ_α^* and ψ_β ($\alpha, \beta = 1, 2, 3, 4$). In nonrelativistic mechanics there exists only one such quantity (a three-dimensional scalar), the probability density $\psi^*\psi$. Since for a particle with spin $\frac{1}{2}$ in relativistic mechanics there are four quantities ψ_α^* and four quantities ψ_β, we can form in all 16 products $\psi_\alpha^*\psi_\beta$. It is convenient to introduce instead of these 16 quantities 16 linear combinations formed from them, which are to have explicitly expressed tensor properties.

It is obvious that in relativistic mechanics the probability density is the fourth component of the current-density four-vector. We have normalized the function ψ so that $|\psi|^2 = 1$. Then $W|\psi|^2$ can be regarded as the fourth component of a four-vector. In this case the other three (spatial) components must be equal to the momentum **p** of the particle. It is not hard to see that they will be the vector

$$W(\psi^*\boldsymbol{\alpha}\psi).$$

Let us insert instead of the function ψ its representation in terms of the two-component functions; then, using Eq. (2.14), we get:

$$W(\psi^*\boldsymbol{\alpha}\psi) = \frac{1}{2}(W+m)\,[\chi^*\boldsymbol{\sigma}\varphi + \varphi^*\boldsymbol{\sigma}\chi]$$
$$= \frac{1}{2}\varphi^*\,[(\boldsymbol{\sigma}\mathbf{p})\,\boldsymbol{\sigma} + \boldsymbol{\sigma}\,(\boldsymbol{\sigma}\mathbf{p})]\,\varphi. \tag{2.15}$$

Using the properties of the Pauli matrices, we get from Eq. (2.15)

$$W(\psi^*\boldsymbol{\alpha}\psi) = \mathbf{p}. \tag{2.16}$$

Thus the four quantities

$$\begin{matrix} \psi^*\psi \\ \psi^*\boldsymbol{\alpha}\psi \end{matrix} \tag{2.17}$$

are proportional to the components of a four-vector.

The meaning of the factor W can be understood if we use the fact that $\psi^*\psi$ is the probability referred to the momentum interval $dp_x\,dp_y\,dp_z$. The product $dp_x\,dp_y\,dp_z$ is not invariant with respect to Lorentz transformations; the expression $W^{-1}\,dp_x\,dp_y\,dp_z$ is an invariant. Therefore the components (2.17) form a four-vector only after multiplication by W. The other quantities introduced in the theory have similar meanings.

The quantities (2.17) can be written more symmetrically as

$$V \rightarrow \begin{Bmatrix} \bar{\psi}\gamma_4\,\psi \\ \bar{\psi}\boldsymbol{\gamma}\,\psi \end{Bmatrix} \rightarrow \bar{\psi}\gamma_i\psi, \tag{2.18}$$

where

$$\bar{\psi} = \psi^*\gamma_4. \tag{2.19}$$

The components (2.18) form a vector in the Minkowski representation (with imaginary fourth component).

Thus we have found four bilinear combinations out of the 16. Now by using the Dirac equation (2.19), which can be written

$$(\gamma_k p_k - im)\psi = 0 \qquad (2.20)$$

(summation over k from 1 to 4), we can obtain also the other 12 quantities. To do this we multiply Eq. (2.20) by $\bar{\psi}$ on the left. Since

$$\bar{\psi}\gamma_k p_k \psi = (\bar{\psi}\gamma_k\psi)\, p_k$$

is a scalar (the four-dimensional product of two vectors), the second term $im\bar{\psi}\psi$ will also be a scalar, and consequently the quantity

$$S = \bar{\psi}\psi \qquad (2.21)$$

(by which the scalar m is multiplied) is a scalar. We now multiply Eq. (2.20) by $\bar{\psi}\gamma_i$ on the left. Arguing in the same way, we find that $i(\bar{\psi}\gamma_i\gamma_k\psi)p_k$ is a vector, and consequently the quantity

$$T = i\bar{\psi}\gamma_i\gamma_k\psi, \qquad i \neq k, \qquad (2.22)$$

or

$$T = \frac{-i}{2}\bar{\psi}(\gamma_i\gamma_k - \gamma_k\gamma_i)\psi = \bar{\psi}\sigma_{ik}\psi$$

is an antisymmetric tensor.*

Multiplying Eq. (2.20) on the left by $\bar{\psi}\gamma_m\gamma_k$, we further find that

$$A = i\bar{\psi}\gamma_m\gamma_l\gamma_k\psi \qquad (m \neq l \neq k)$$

or

$$A = i\bar{\psi}\gamma_i\gamma_5\psi \qquad (2.23)$$

is an antisymmetric tensor of the third rank — a pseudovector. Finally, multiplying Eq. (2.20) on the left by $\bar{\psi}\gamma_m\gamma_l\gamma_k$, we find (for the indices all different) that

$$P = \bar{\psi}\gamma_5\psi \qquad (2.24)$$

is a pseudoscalar.

Multiplication of Eq. (2.20) by $\bar{\psi}\gamma_5$ now gives nothing new, since $\bar{\psi}\gamma_5\gamma_4\psi$ reduces to the components of the pseudovector. (There cannot be more than four γ matrices in a product). It is not hard to see that we have exhausted all the possibilities. In fact,

S has 1 component,
V has 4 components,
T has 6 components,
A has 4 components,
P has 1 component,

which is 16 components in all.

If we express the components of these quantities in terms of the matrices $\sigma\gamma_4$ and γ_5, we get the following table:

scalar S: $\psi^*\gamma_4\psi$;
vector V: $\psi^*\psi$; $i\psi^*\sigma\gamma_5\psi$;
tensor T: $\psi^*\gamma_4\sigma\psi$; $\psi^*\gamma_4\sigma\gamma_5\psi$;
pseudovector A: $i\psi^*\gamma_5\psi$; $\psi^*\sigma\psi$
and pseudoscalar P: $\psi^*\gamma_4\gamma_5\psi$. (2.25)

The tensor consists of two vectors, one composed of the components i, k (i, k = 1, 2, 3), and the other of the components i4.

From the formulas (2.25) we see that the scalar differs from the pseudoscalar by the replacement of ψ by $\gamma_5\psi$. The components of the vector and the pseudovector differ by a similar replacement. Finally, the two vectors that form the tensor go over into each other by this replacement.

This property of the matrix γ_5 means that it has a pseudoscalar character, i.e., that multiplication of a wave function by γ_5 changes the parity of the wave function.

We call attention to the fact that $(-\gamma_5)$ acting on a wave function interchanges its two pairs of components:

$$-\gamma_5\begin{pmatrix}\varphi\\ \chi\end{pmatrix} = \begin{pmatrix}\chi\\ \varphi\end{pmatrix}. \qquad (2.26)$$

This means that φ and χ have opposite parity, as also follows from the pseudoscalar character of the scalar product $\boldsymbol{\sigma}\mathbf{p}$ in Eq. (2.7).

For a particle with mass zero (a neutrino) the action of the matrix $(-\gamma_5)$ is equivalent to multiplication of the wave function by $\boldsymbol{\sigma\nu}$, where $\boldsymbol{\nu}$ is the unit vector in the direction of the neutrino's momentum,

$$\nu = \frac{\mathbf{p}_\nu}{|\mathbf{p}_\nu|}. \qquad (2.27)$$

This follows from the fact that $(\boldsymbol{\sigma\nu})^2 = 1$ and

$$-\gamma_5\begin{pmatrix}1\\ \sigma\nu\end{pmatrix} = \begin{pmatrix}\sigma\nu\\ 1\end{pmatrix}. \qquad (2.28)$$

There remains for us to say a few words about the inclusion of Coulomb-field effects. In the general case this leads to cumbersome calculations (cf. reference B25).

If we can suppose that $Ze^2/p \ll 1$ (light nuclei and fast electrons), the effect of the Coulomb field reduces just to a change of the phase relation between the two components φ and χ of the wave function. Namely, the wave function for an elec-

*The coefficient i is introduced in order to make the matrix $i\gamma_i\gamma_k$ Hermitian: $(i\gamma_i\gamma_k)^+ = -i\gamma_k^+\gamma_i^+ = i\gamma_i\gamma_k$.

tron can be written in the form

$$\left(\frac{W+m}{2W}\right)^{\frac{1}{2}}\left(\begin{array}{c}\varphi \\ \left(1+i\,\frac{Ze^2}{p}\right)\frac{\sigma \mathbf{p}}{W+m}\,\varphi\end{array}\right). \tag{2.29}$$

(For a positron the sign of the term in e^2 is reversed.) This function is normalized to unity up to the order $(Ze^2/p)^2$. Hereafter we shall not, as a rule, introduce corrections for the Coulomb field. The effect of the Coulomb field is treated in detail in references B25, B27, B37, B38, to which we refer the reader.

3. THE TWO-COMPONENT NEUTRINO

For many years the question remained open as to whether the mass of the neutrino is exactly zero or simply very small.* It was not clear whether there might be some sort of difference in principle between the two possibilities. It was only at the end of 1956 that Salam (B7), Landau (B5), and Yang and Lee (B8) showed that the exact vanishing of the neutrino mass together with refusal to assign a definite sign of intrinsic parity to the neutrino leads to a new model of the neutrino — to the two-component or longitudinal neutrino.†

We go back to the Dirac equation, in which we set the mass $m = 0$:

$$W\varphi = (\sigma \mathbf{p})\chi, \tag{3.1}$$
$$W\chi = (\sigma \mathbf{p})\varphi. \tag{3.2}$$

Instead of the functions φ and χ we introduce two new functions

$$\begin{aligned}\psi_+ &= 2^{-1/2}(\varphi+\chi),\\ \psi_- &= 2^{-1/2}(\varphi-\chi).\end{aligned} \tag{3.3}$$

Noting that (in our chosen representation of the matrices) according to Eq. (2.26) interchange of φ and χ is equivalent to multiplication by the matrix $(-\gamma_5)$, we can also write

$$\begin{aligned}\psi_+ &= 2^{-1/2}(1-\gamma_5)\psi,\\ \psi_- &= 2^{-1/2}(1+\gamma_5)\psi.\end{aligned} \tag{3.4}$$

The functions (3.4) are eigenfunctions of the matrix $(-\gamma_5)$:

$$\begin{aligned}(-\gamma_5)\psi_+ &= \psi_+,\\ (-\gamma_5)\psi_- &= -\psi_-.\end{aligned} \tag{3.5}$$

*We recall that in electrodynamics the vanishing of the rest mass of the photon is a consequence of the gauge invariance of the theory.

†We remark that it is not entirely consistent to call the two-component neutrino a longitudinal particle. The photon, for which the angular-momentum component in the direction of the momentum is ± 1 (left and right circularly polarized photons) is usually called transverse (from the direction of the vector potential).

The eigenvalues of this matrix are called the "helicity" or "chirality." The replacement $\psi \rightarrow \pm\gamma_5\psi$ in the Dirac equation changes the sign of the mass; this can be seen from Eq. (2.20) and the fact that γ_5 anticommutes with the other γ's. Therefore the interaction is invariant with respect to this replacement only if the mass of the particle is zero. Conversely, the requirement of invariance with respect to the transformation $\psi \rightarrow \pm\gamma_5\psi$ (definite chirality of the particle) leads to the vanishing of the mass of the particle (cf. references B41, B42). Each of the functions ψ_+ and ψ_- has only two independent components; these functions can be written in the form of the columns

$$\left.\begin{aligned}\psi_+ &= \frac{1}{2}\begin{pmatrix}(1+\sigma\nu)\varphi\\(1+\sigma\nu)\varphi\end{pmatrix};\\ \psi_- &= \frac{1}{2}\begin{pmatrix}(1-\sigma\nu)\varphi\\-(1-\sigma\nu)\varphi\end{pmatrix},\end{aligned}\right\} \tag{3.6}$$

where ν is the unit vector in the direction of the momentum of the particle and φ is the two-component Pauli spin function. The functions ψ_+, ψ_- in the form (3.6) are normalized to unity. In fact,

$$|\psi_+|^2 = \frac{1}{2}|(1+\sigma\nu)\varphi|^2 = 1. \tag{3.7}$$

Here we have used the fact that $(\sigma\nu)^2 = 1$ and have set

$$\varphi^*\sigma\nu\varphi = 0.$$

This means that we take the average value of the spin component in the state φ (but not ψ_+) to be zero.

Thus multiplication of ψ by the matrix $(1\pm\gamma_5)$ makes ψ into the wave function of the two-component neutrino.

In the original paper of Lee and Yang (B8) a somewhat different representation of the neutrino wave function is chosen. Instead of Eq. (3.6) they set

$$\left.\begin{aligned}\psi_+ &= \frac{1}{\sqrt{2}}\begin{pmatrix}1+\sigma\nu\\0\end{pmatrix}\varphi;\\ \psi_- &= \frac{1}{\sqrt{2}}\begin{pmatrix}1-\sigma\nu\\0\end{pmatrix}\varphi.\end{aligned}\right\} \tag{3.8}$$

This of course does not change any of the results. We shall use the representation (3.6).

The wave functions ψ_+ and ψ_- satisfy the equations

$$\begin{aligned}\psi_+ &= \sigma\nu\psi_+,\\ \psi_- &= -\sigma\nu\psi_-,\end{aligned} \tag{3.9}$$

which follow directly from Eq. (3.1).

The equations (3.9) show that ψ_+ corresponds to a state with the spin component in the direction of the momentum equal to $+\frac{1}{2}$, and ψ_- corresponds to a state with the spin component in that direction equal to $-\frac{1}{2}$. In the language of the vector model such a state corresponds to strong coupling between the spin and the momentum. In this state the spin precesses around the momentum of the particle.

We note an obvious fact: when the signs of all the spatial coordinates are changed the functions ψ_+ and ψ_- interchange their roles. In fact, under such a transformation the components of the vector $\boldsymbol{\nu}$ change sign, but the components of $\boldsymbol{\sigma}$ (a pseudovector) remain unchanged. Consequently the product $\boldsymbol{\sigma\nu}$ changes its sign also. A change of the sign of the time, on the other hand, does not change the sign of the product $\boldsymbol{\sigma\nu}$, since under this transformation both the momentum components and the spin components change sign (the spin transforms like an angular velocity)

The functions ψ_+ and ψ_- describe particles completely polarized respectively with and against the direction of motion. This property does not depend on the coordinate system, and is therefore relativistically invariant. It is not hard to see that such states can be realized in an obvious way only for extreme relativistic particles ($m = 0$).

If the rest mass of a particle is not equal to zero one can always go over to a coordinate system in which the particle is at rest; in this system the momentum is zero, and the spin has an arbitrary direction. Now going back again to a moving system, one can get arbitrary relative directions of the spin and momentum. Therefore the existence of longitudinal polarization of the particles is a consequence of the nonexistence of a rest system for such a particle.

As is well known, an analogous situation exists also for photons. The absence of a rest mass of the photon is directly connected with the fact that there are only two polarizations of a photon: the angular momentum of a photon can be directed either along its wave vector (left-circularly polarized photon) or else in the opposite direction (right-circularly polarized photon).

The representation of the neutrino wave function by means of the operators $1 \pm \gamma_5$ is not the only way of choosing a two-component representation, if we renounce the postulate that the neutrino mass is identically zero. We shall not consider the other representations in detail, and refer the reader to the original papers (see the papers of Case (B22) and Pauli (B19)).

The wave equation of a free neutrino (with zero mass and charge) is obviously invariant under the two transformations

$$\psi \longrightarrow \psi^c$$

and

$$\psi \longrightarrow \gamma_5 \psi, \tag{3.10}$$

where ψ^c is the wave function of the charge-conjugate particle (the antineutrino).

Therefore the state of a free neutrino is a degenerate state. This means that an arbitrary linear combination

$$a\psi + b\psi^c + c\gamma_5\psi + d\gamma_5\psi^c \tag{3.11}$$

can describe the state of a free neutrino.* The requirement that the wave functions of neutrino and antineutrino must be eigenfunctions of the operator $(-\gamma_5)$ is a step which destroys this ambiguity.

A different approach was suggested by Majorana, who proposed a theory based on the idea that the neutrino and antineutrino are identical.

Such a theory corresponds to the choice of the neutrino wave function in the form

$$\psi \longrightarrow \frac{1}{2}(\psi + \psi^c). \tag{3.12}$$

In this scheme ψ is an eigenfunction of the charge-conjugation operator, and the neutrino on the average has no longitudinal polarization.

The Majorana theory is obviously not compatible with the principle of conservation of leptonic charge (the impossibility of conversion of neutrino into antineutrino) and describes a purely neutral particle. The theory of the longitudinal neutrino represents the other limiting case of completely polarized neutrinos and antineutrinos that cannot be converted into each other. Furthermore in the Majorana theory a mass of the neutrino can appear as a result of virtual neutrino-antineutrino transitions, whereas the mass of the longitudinal neutrino is identically equal to zero because of the rigorous exclusion of such transitions.

It is obvious that one may construct any intermediate scheme, in which the neutrino would be partially polarized (B19, 20).

Although strictly speaking at the present time the longitudinal character of the neutrino has not been rigorously proved (the experimental errors amount to 15 to 20 percent), still this scheme is so attractive from the theoretical point of view that we shall not consider the other possibilities here.

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*We agree to say that the equation of the neutrino is invariant with respect to the Pauli transformations:

$$\psi \longrightarrow \alpha\psi + \beta\gamma_5\psi, \tag{I}$$
$$\psi \longrightarrow a\psi^c + b\gamma_5\psi^c, \tag{II}$$

where $|\alpha|^2 + |\beta|^2 = |a|^2 + |b|^2 = 1$ (cf. Lüders[B13]).

Thus the hypothesis that now appears most probable is that the neutrino and antineutrino are completely longitudinally polarized. The experiment of Goldhaber, Grodzins, and Sunyar (D5), who measured the circular polarization of the γ-ray quanta that follow the process of electron capture in Eu (cf. Sec. 10) has shown that the neutrino is polarized opposite to its direction of motion. The antineutrino must then be polarized in the direction of motion. Thus the neutrino has the symmetry of a left-handed screw (left chirality), and the antineutrino the symmetry of a right-handed screw (right chirality).* Hereafter we shall simple speak of left-handed and right-handed particles.

Let us consider some more formulas.

First of all we turn our attention to the properties of the matrices $(1-\gamma_5)$ and $(1+\gamma_5)$. It is obvious that

$$(1\pm\gamma_5)^2 = 2(1\pm\gamma_5),$$
$$(1+\gamma_5)(1-\gamma_5) = (1-\gamma_5)(1+\gamma_5) = 0. \qquad (3.13)$$

By means of these matrices we can resolve any wave function into two components,

$$\psi = \frac{1}{2}(1-\gamma_5)\psi + \frac{1}{2}(1+\gamma_5)\psi. \qquad (3.14)$$

In the case of a particle with mass zero this corresponds to separating the states into the two longitudinal states ψ_+ and ψ_-. Moreover, since γ_5 anticommutes with each of the four matrices γ_i, if we denote by γ_B (B = S, V, T, A, P) the sixteen matrices that can be made from the γ_i, we can write the commutation rules for $(1\pm\gamma_5)$ with the γ_B:

$$(1\mp\gamma_5)\gamma_B = \begin{cases}\gamma_B(1\mp\gamma_5) & B = S, T, P\\ \gamma_B(1\pm\gamma_5) & B = A, V\end{cases}$$

Let us introduce the adjoint functions for the longitudinal particles. Since $\bar{\psi} = \psi^*\gamma_4$, we have

$$\bar{\psi}_\pm = \psi_\pm^*(1\mp\gamma_5)\gamma_4.$$

From this we get

$$\bar{\psi}_+ = \bar{\psi}(1+\gamma_5), \qquad (3.15)$$
$$\bar{\psi}_- = \bar{\psi}(1-\gamma_5). \qquad (3.16)$$

It can be seen from Eqs. (3.15) and (3.16) that for the longitudinal particles we can construct only the vector and the pseudovector, and

$$\bar{\psi}_+\gamma_B\psi_+ = \bar{\psi}_-\gamma_B\psi_- = 0 \quad (B = S, T, P). \qquad (3.17)$$

*We note that in optics left-circularly polarized light is the name given to light having the symmetry of a right-handed screw; this is due to the use of a different system of coordinates (the observer looks in the direction opposite to the wave vector).

From these equations it follows in particular that the two-component neutrino cannot have a magnetic moment.*

The impossibility of forming a scalar for the longitudinal neutrino corresponds to the fact that its mass is zero. The impossibility of forming a pseudoscalar for it is due to the same fact, since for a particle with given chirality $\bar{\psi}\gamma_5\psi = \pm\bar{\psi}\psi$.

It is curious to note that nothing prevents such a neutrino's having a charge, because the current vector $\bar{\psi}_+\gamma_i\psi_+$ is different from zero.

As we have already said, for a particle with nonvanishing mass one cannot introduce invariant states with longitudinal polarization. Nevertheless it is useful to examine the results of the action of the matrices $(1-\gamma_5)$ and $(1+\gamma_5)$ on the wave functions of such particles.

Using the wave function (2.8), we can write in analogy with Eq. (3.5)

$$(1-\gamma_5)\psi = \begin{pmatrix}\left(1+\frac{\sigma\mathbf{p}}{W+m}\right)\varphi\\ \left(1+\frac{\sigma\mathbf{p}}{W+m}\right)\varphi\end{pmatrix}. \qquad (3.18)$$

Let us introduce the vector $\mathbf{e} = \mathbf{p}/p$ (the unit vector along the momentum of the electron). It is obvious that an electron with complete longitudinal polarization must be described by one of the functions

$$(1+\boldsymbol{\sigma}\mathbf{e})\varphi \qquad (3.19)$$

(polarization along the momentum), or

$$(1-\boldsymbol{\sigma}\mathbf{e})\varphi \qquad (3.20)$$

(polarization opposite to the momentum).

Let us resolve the operator that appears in Eq. (3.18) into two operators

$$1+\frac{\sigma p}{W+m} = a(1+\boldsymbol{\sigma}\mathbf{e}) + b(1-\boldsymbol{\sigma}\mathbf{e}). \qquad (3.21)$$

It is obvious that the squares of the quantities

$$a = \frac{1}{2}\left(1+\frac{p}{W+m}\right); \quad b = \frac{1}{2}\left(1-\frac{p}{W+m}\right) \qquad (3.22)$$

determine the respective fractions of electrons with polarizations along and opposite to the momentum, and

$$P = \frac{a^2-b^2}{a^2+b^2} \qquad (3.23)$$

is, by definition, the average polarization of the electrons in the state (3.18).

From Eq. (3.22) we have

*It has been established experimentally that in any case the magnetic moment of the neutrino is smaller than 10^{-9} Bohr magnetons (cf. Sec. 10).

$$a^2+b^2=\frac{W}{W+m},$$
$$a^2-b^2=\frac{p}{W+m}, \qquad (3.24)$$

from which we have

$$P=\frac{p}{W}=\beta \qquad (3.25)$$

(the speed of the electron in units of the speed of light). Thus an electron in the state (3.18) has the average polarization β in the direction of its momentum. Analogously an electron in the state

$$\psi_-=(1+\gamma_5)\psi=\begin{pmatrix}\left(1-\frac{\sigma\mathbf{p}}{W+m}\right)\varphi, \\ -\left(1-\frac{\sigma\mathbf{p}}{W+m}\right)\varphi\end{pmatrix} \qquad (3.26)$$

will have the polarization $-\beta$.

For $\beta \rightarrow 1$ the electron becomes a longitudinally polarized particle.*

We remark for completeness that the Dirac equation for the electron can be written in two-component form if we go over to the second-order equation (Gell-Mann and Feynman (B40)). Namely, if we write the Dirac equations including an electromagnetic field for the functions ψ^+ and ψ^-, from Eq. (2.9)

$$\gamma_k(\nabla_k-ieA_k)\psi^+ + m\psi^-=0,$$
$$\gamma_k(\nabla_k-ieA_k)\psi^- + m\psi^+=0, \qquad (3.27)$$

and eliminate one of these functions, we get for the other one the equation

$$(\nabla_k-ieA_k)^2\psi^{\pm}+\frac{1}{2}\sigma_{kl}F_{kl}\psi^{\pm}=m^2\psi^{\pm}, \qquad (3.28)$$

where $\sigma_{kl}=(-i/2)(\gamma_k\gamma_l-\gamma_l\gamma_k)$, and F_{kl} is the electromagnetic field tensor. For further discussion see the paper of Feynman and Gell-Mann (B40).

4. PARITY

In order to describe the production and absorption of particles, one carries out a second quantization. Its meaning is that the state of the system is described by the occupation numbers (the numbers of particles in given states), and the wave function ψ_a is regarded as an operator that reduces the number of particles in the state a by unity (absorption operator). The production of a particle in the state a is described by the operator $\bar{\psi}_a$. The operator ψ_a also describes the production of an antiparticle in a certain state a′, and the operator $\bar{\psi}_a$ the absorption of an antiparticle from that same state a′. Here the state a′ differs from the state a by the reversal of the direction of the particle's spin (see below).

The operators ψ_a anticommute with each other, i.e., the interchange of two operators changes the sign of the expression in question.

Productions and destructions of particles obey the quantum-mechanical conservation laws. As is well known, the conservation laws are associated with definite symmetry properties of space; they impose definite limitations on the form of the interaction Hamiltonian.

Let us examine the properties of the Hamiltonian with respect to reflections. Usually three types of reflections are considered:

1) mirror reflection (P) — change of the signs of all spatial coordinates and momenta;

2) time reflection (T) — change of sign of the time and interchange of absorption and emission;

3) charge reflection (C) — change of the signs of all charges, or interchange of particles and antiparticles.

The invariance of the Hamiltonian with respect to these operations leads to the well known laws of conservation of spatial parity, time parity, and charge parity, respectively.

Let us examine how the wave functions of particles with spin $\frac{1}{2}$ and the corresponding operators transform under the reflections.

These transformations can be defined in the following forms:

$$P:\psi\rightarrow\gamma_4\psi,$$
$$T:\psi\rightarrow T\bar{\psi}, \qquad (4.1)$$
$$C:\psi\rightarrow C\bar{\psi}.$$

Of these three operations only the mirror reflection is represented by the matrix γ_4. The other two reflections involve a change from ψ to $\bar{\psi}$; this is a nonlinear operation, which cannot be expressed in terms of the matrices γ_i in a way independent of the choice of the representation of the γ's. The matrices C and T are defined by their commutation relations with the γ's:

$$T\gamma_iT^{-1}=-\gamma_i^T \quad (i=1,2,3), \qquad (4.2)$$
$$T\gamma_4T^{-1}=\gamma_4^T,$$
$$C\gamma_iC^{-1}=-\gamma_i^T \quad (i=1,2,3,4), \qquad (4.3)$$

where γ^T is the matrix obtained by transposing the matrix γ.

To get an understanding of the meaning of the transformations (4.1) we write down the Dirac equations for a particle in an electromagnetic field,

*It is interesting to note that this sort of representation of a relativistic electron was used by Yennie, Ravenhall, and Wilson for calculations of the scattering of electrons in a Coulomb field [Yennie, Ravenhall, and Wilson, Phys. Rev. 95, 500 (1954)].

$$[\gamma_k(\nabla_k - ieA_k) + m]\,\psi = 0, \tag{4.4}$$

$$[\gamma_k^T(\nabla_k + ieA_k) - m]\,\bar{\psi} = 0, \tag{4.5}$$

where Eq. (4.5) is obtained from Eq. (4.4) if we note that $\gamma_i^* = \gamma^T$ because of the Hermitian character of the matrices γ_i.

Using the commutation relations of γ_4 with the other γ's we easily see that $\gamma_4\psi$ satisfies the equation

$$[\gamma_4(\nabla_4 - ieA_4) - \gamma(\nabla - ie\mathbf{A}) + m]\,(\gamma_4\psi) = 0. \tag{4.6}$$

But this equation is on the other hand obtained from Eq. (4.4) if we make the replacements $x_i \to -x_i$, $A_i \to -A_i$ $(i = 1, 2, 3)$, that is, if we change the signs of all the spatial coordinates.

Using the properties of the matrix T [Eq. (4.2)], we also find that $T\bar{\psi}$ satisfies the equation (obtained from Eq. (4.5) by multiplication by T):

$$[-\gamma_4(\nabla_4 - ieA_4) + \gamma(\nabla - ie\mathbf{A}) + m]\,T\bar{\psi} = 0. \tag{4.7}$$

This equation differs from Eq. (4.4) by the replacements $x_4 \to -x_4$, $A_4 \to -A_4$, i.e., by reversal of the time.

Finally, by means of Eq. (4.3) we find that $C\bar{\psi}$ satisfies the equation

$$[\gamma_k(\nabla_k + ieA_k) + m]\,(C\bar{\psi}) = 0. \tag{4.8}$$

This equation differs from Eq. (4.4) by change of the sign of the charge of all particles.

With regard to the operation of time reversal we must make some remarks. In the very definition of such an operation there is an arbitrary feature, which lies in whether or not we include in this operation passage from emission to absorption (passage to $\bar{\psi}$). In the unquantized theory one can define the time reversal by the formula $\psi \to \gamma_1\gamma_2\gamma_3\psi$.* With this definition of the reversal we note in particular that the sign of the momentum of a particle is not changed. This definition is less suitable, and we shall not use it.

An important conclusion follows from the relations (4.1). If we now consider the simultaneous reflection of all four coordinates, the operation PT (strong reflection), we see that is can obviously be written in the form

$$PT : \psi \to \gamma_4 T\bar{\psi}. \tag{4.9}$$

By means of Eq. (4.2) we easily get the commutation relations of (PT) with the γ's. We have

$$(PT)\gamma_i(PT)^{-1} = PT\gamma_i T^{-1}P = \gamma_i^T \quad (i = 1, 2, 3, 4). \tag{4.10}$$

*The operation of time reversal was defined in this form by Racah [G. Racah, Nuovo Cimento 14, 322 (1937)]. The definition presented in the text was introduced by Wigner [E. P. Wigner, Gött. Nachr, Phys. Math. Klasse, p. 549 (1932)].

The right member of Eq. (4.10) differs from that of Eq. (4.3) by the sign. Therefore the commutation rules of PT will be the same as those for the operator $\gamma_5 C$. This means that apart from an immaterial phase factor we can write:

$$PT = \gamma_5 C \tag{4.11}$$

or

$$PTC = \gamma_5. \tag{4.12}$$

This formula was found by Pauli (B15) and is a special case of a theorem which we shall discuss later.

Let us now determine the concrete forms of the matrices T and C for the case of γ matrices chosen in the form (2.10).

It can be seen from Eq. (2.10) that out of the four γ matrices two, γ_1 and γ_3, are composed of purely imaginary elements, and two, γ_2 and γ_4, of real elements. Therefore in this representation

$$\gamma_{1,3}^T = -\gamma_{1,3}, \qquad \gamma_{2,4}^T = +\gamma_{2,4}. \tag{4.13}$$

Then it follows from Eq. (4.2) that

$$T\gamma_i T^{-1} = \begin{cases} \gamma_i \,(i = 1, 3, 4), \\ -\gamma_i \,(i = 2). \end{cases} \tag{4.14}$$

From this we have

$$T = \gamma_1\gamma_3\gamma_4. \tag{4.15}$$

In just the same way we find from Eq. (4.3)

$$C = \gamma_2\gamma_4. \tag{4.16}$$

We emphasize once again that with a different choice of the γ's the forms of T and C would be different; only the commutation relations (4.2) and (4.3) are invariant.

For completeness we also give the commutation relations that are obtained from the formulas given above:

$$PT = TP; \; PC = -CP; \; TC = -CT. \tag{4.17}$$

It follows from this, in particular, that for particles with spin $\frac{1}{2}$ the spatial and time parities of particle and antiparticle are opposite.

Let us now go back to Eq. (4.12). It is not hard to see that since replacement of ψ by $\gamma_5\psi$ in the Dirac equation is a passage to particles of the opposite parity, if one makes this replacement for all the particles in nature this will not affect any phenomena, since the relative parities of the particles are not changed. This conclusion, which is an obvious one for free particles and for particles in the electromagnetic field, turns out to be a very general property of the physical world.

Pauli (B15), Lüders (B14, B21), and Zumino (B16) (also Schwinger (B17, B18) and Jost (B57) have proved a theorem according to which the product of all three operations PCT commutes with any Hamiltonian, and therefore invariance under PCT does not impose any new limitations on an interaction. We can illustrate the physical meaning of this assertion by the following argument. Suppose ψ_a^+ describes the absorption of a particle from the state a with positive longitudinal polarization. Then P takes this operator over into the operator ψ_a^- for absorption of a particle with negative longitudinal polarization:

$$P\psi_a^+ \to \psi_a^-.$$

The operation T converts ψ_a^+ into the operator for production of a particle, $T\psi_a^+ \to \overline{\psi}_a^+$, and accordingly $PT\psi_a^+ \to \psi_a^-$. Thus from the one operator ψ_a^+ we have obtained four operators:

$$\psi_a^+,\ \psi_a^-,\ \overline{\psi}_a^+,\ \overline{\psi}_a^-.$$

Finally, the charge reflection C leads to the appearance of four more operators describing the production and absorption of antiparticles:

$$\psi_a^{+c},\ \psi_a^{-c},\ \overline{\psi}_a^{+c},\ \overline{\psi}_a^{-c}.$$

All told there are obtained 8 operators, and 8 corresponding operations, by means of which these operators can be obtained from a single one, for example ψ_a:

$$\begin{array}{l} 1 \\ P,\ PC, \\ C,\ PT, \\ T,\ TC,\ PCT. \end{array} \tag{4.18}$$

The invariance of the Hamiltonian under reflections is invariance under the operations (4.18). But not all of these operations are independent. Quantum mechanics leads to the result that absorption of a particle in a state of negative energy and production of an antiparticle in a state of positive energy (and the opposite spin direction) are one and the same process. Therefore there must be a relation between the operators for production of the particle and absorption of the antiparticle. This requirement has the result that only 4 of the 8 operators (4.18) are independent, and if, for example, we require invariance of the Hamiltonian with respect to P and C, then it is automatically invariant with respect to T.

Thus there remains to be considered the question of the invariance of the interaction with respect to the two reflections P and C. Lee and Yang* were the first to note that whereas in strong interactions parity is conserved there is no experimental basis for such an assertion about the weak interactions. As the result of a great series of experiments (begun by the classic experiment of Wu (F1) it has been proved that neither spatial nor charge parity is conserved in the weak interactions.

Parity nonconservation leads to fundamental conclusions about the properties of space.

At first glance it would seem that in the new situation it is impossible to regard the world as symmetrical and that, on the contrary, the distinction between right and left has an absolute character. But Landau (B5) and Yang and Lee (B8)† pointed out that this conclusion is not a necessary consequence of the nonconservation of the spatial and charge parities.

If we assume that the laws of nature are invariant with respect to the combined reflection PC (or the time reversal T), the symmetry of the world is preserved.

In this case there is no way to distinguish between the right-hand direction in the world and the left-hand direction in an antiworld, and the perceived asymmetry of the directions is due to the asymmetry of the charges in our world, which consists of positively charged nuclei and negatively charged electrons. In this case a truly neutral system would also be symmetrical with respect to the two directions of rotation.

In such a world with invariance under PC it is obvious that the operation P and the operation C lead to the same results. This means that passage from particles to antiparticles is change of the sign of the chirality.

The law of conservation of the combined parity has not as yet been experimentally confirmed with adequate accuracy, although the existing experiments indicate that it is apparently valid (cf. Secs. 9, 10). It is obvious that an experimental check of the conservation of combined parity will give us information about one of the most fundamental properties of nature.

An interesting example of the difference between worlds with different parity properties is the question of the existence of dipole moments of elementary particles (B5) and of asymmetry in the decay of polarized particles. The average dipole moment of a particle in a stationary state must be parallel to its spin

*See the discussion at the Sixth Rochester Conference (Reports of Wigner, Feynman, Yang).

†Cf. also Wigner, Revs. Modern Phys. 29, 255 (1957). Russian transl. Usp. Fiz. Nauk 65, No. 2 (1958).

$$[\mathbf{d}]_{av} = a[\boldsymbol{\sigma}]_{av}. \tag{4.19}$$

An asymmetry in the decay of a polarized particle can be characterized by the fact that the average value of the momentum of an emerging particle is not zero, but is given by an equation of the type

$$[\mathbf{p}]_{av} = b[\boldsymbol{\sigma}]_{av}. \tag{4.20}$$

where $\mathbf{p}$ is the momentum of the emerging particle and a and b are scalars. These two equations cannot be correct it spatial parity is conserved, since they connect quantities of different symmetries. Furthermore, Eq. (4.19) is noninvariant with respect to the time reversal T, since this changes the sign of σ but not that of $\mathbf{d}$. On the other hand Eq. (4.20) is not invariant under PT = C: the inversion P changes the sign of $\mathbf{p}$ but not that of σ. Therefore the dipole moment can exist in a world that is noninvariant with respect to P and T, and the asymmetry can exist only in a world that is noninvariant with respect to P and C. In particular, if the hypothesis about the combined parity is correct, elementary particles cannot have dipole moments. In a world invariant under C the dipole moment could exist, but there would be no asymmetry. Finally, both effects could exist only in a completely asymmetrical world.

The properties of the interaction with respect to time reversal enable us to reach qualitative conclusions not only about the properties of the stationary states of a system, but also about the properties of reactions.

On time reversal the roles of the initial and final states of the system are interchanged. Therefore such invariance imposes limitations on properties of the system in two cases: for the stationary states and for the elastic scattering of particles.

If we have some sort of nuclear reaction or decay process in which the particles present in the final state are not the same as in the initial state, then in the strict sense time-reversal invariance does not lead to any limitations on the properties of either of these states. This invariance imposes only certain restrictions on the relations between the direct and inverse reactions, for example on the relations between the polarizations in these reactions.*

We get much more information if the effect in question is described in first-order perturbation theory, so that the initial and final wave functions of the particles are plane waves (Born approximation). In this case the transition amplitude is proportional to the matrix element of the Hermitian interaction Hamiltonian, and this leads to useful new relations.

From general theory it is known that time reversal means that in the scattering-matrix element the initial and final states are interchanged and the signs of the spins and momenta are reversed for all the particles of the system:

$$T: (\mathbf{k},\ \boldsymbol{\sigma}\,|\,S\,|\,\mathbf{k}',\ \boldsymbol{\sigma}') \rightarrow (-\mathbf{k}',\ -\boldsymbol{\sigma}'\,|\,S\,|-\mathbf{k},\ -\boldsymbol{\sigma}) \tag{4.21}$$

Symbolically this transformation can be written in the form

$$T: \mathbf{k} \rightarrow -\mathbf{k}'\quad \boldsymbol{\sigma} \rightarrow -\boldsymbol{\sigma}'. \tag{4.22}$$

In the first-order perturbation expression S is proportional to the interaction Hamiltonian and the interchange of the initial and final states in the labels of the matrix element (transposition) is equivalent to complex conjugation and does not change the transition probability.* Therefore in this case the properties of the system are not changed by this interchange. This in turn, together with Eq. (4.21), means that in the first approximation of perturbation theory time reversal is equivalent to the changes:

$$T_{approx}: \mathbf{k} \rightarrow -\mathbf{k},\ \boldsymbol{\sigma} \rightarrow -\boldsymbol{\sigma}. \tag{4.23}$$

The transformation (4.23) affects only the separate states, and according to our argument is valid when the interaction can be neglected in the final state. In the case of β decay this means that the transformation (4.23) is equivalent to time reversal if we can neglect the Coulomb interaction for the light particles, i.e., if their energy is high enough or if the nuclear charge is small.

We shall make use of the transformation (4.23) later on; here we shall consider as an example the problem of the polarization of the products of a reaction.

Let us consider a reaction with two particles in the initial state and two in the final state:

$$a + b \rightarrow c + d.$$

The particles on the left and right can be the same (scattering) or different (reaction). The plane of the scattering is defined by the two vectors $\mathbf{k}$ and $\mathbf{k}'$ (in the center-of-mass system).

Let us see whether there can be polarization normal to the plane of the reaction, if the initial

*We note that the reversibility of all reactions is assured by the Hermitian nature of the Hamiltonian – the reality of the eigenvalues of the energy of the system.

*If in addition the matrix element does not depend on the velocities, the matrix element is in general real.

particles were not polarized and if the interaction is invariant under time reversal. This is connected with whether there can exist a relation of the type

$$[\sigma]_{av} = \alpha \mathbf{k} \times \mathbf{k}', \qquad (4.24)$$

where α is a scalar.

Under the exact time reversal (4.22) both sides of the equation transform in the same way.

Equation (4.24) expresses, for example, the well known properties of the polarization in nucleon-nucleon scattering, for which the interaction conserves the time parity.

It is easy to see, however, that Eq. (4.24) is not invariant with respect to the transformation (4.23). This means that there is no normal polarization in the Born approximation, provided time parity is conserved.

We see that the normal polarization of an electron arises either because of nonconservation of the time parity or on account of the Coulomb interaction. For light nuclei and relativistic electrons it is small.

5. THE TYPES OF β INTERACTION

In order to write the expression for the interaction we require that: (1) the interaction energy density (Hamiltonian) must be a scalar for processes with parity conservation or a pseudoscalar for processes involving change of parity, and (2) derivatives of the wave functions of the particles shall not occur in the interaction expression. The introduction of derivatives is equivalent to the assumption that the interaction depends on the energies and momenta. Since the light particles are relativistic, there is no basis for a restriction to just a linear dependence (first derivatives). The introduction of a completely arbitrary dependence on the momenta, on the other hand, leaves the theory without content. As is well known, experiment confirms the lack of energy-dependence of the theory in ordinary β decays.

For definiteness we begin with the process of β decay of the neutron.

For greater symmetry the β decay of the neutron (or of a nucleus) is usually written not in the form

$$N \rightarrow P + \tilde{\nu} + e^-, \qquad (5.1)$$

but in the formally equivalent form

$$N + \nu \rightarrow P + e^-. \qquad (5.2)$$

This means that instead of speaking of the emission of an antineutrino we shall speak of the absorption of a neutrino.* Furthermore the basic assumption is made that β decay cannot occur with the emission of a neutrino, i.e., the neutrino and antineutrino are different particles and can never go over into each other. This means, for example, that the process $N + \tilde{\nu} \rightarrow P + e^-$ is impossible. This fact is called the law of conservation of light particles, or the conservation of the neutrino charge (A4).

We introduce the notations:

P is the operator for absorption of a proton;
N is the operator for absorption of a neutron;
e is the operator for absorption of an electron (negative);
ν is the operator for absorption of a neutrino.

In the usual way of writing formulas the particles in the final state are represented in the Hamiltonian by the adjoint of the wave function (it corresponds to the production of a particle). The terms in the Hamiltonian that describe this decay must have the form

$$H = C(\bar{P}N)(\bar{e}\nu). \qquad (5.3)$$

Here C is a certain matrix with four indices such that H is a scalar or a pseudoscalar. The usual procedure for finding an explicit expression for H is as follows: one forms all possible tensor expressions from the two pairs of wave functions, and then multiplies scalar by scalar, vector by vector, and so on. This gives the scalar part of the Hamiltonian. By multiplying scalar by pseudoscalar, vector by pseudovector, and so on, we find the pseudoscalar part of the Hamiltonian.

There still remains an arbitrariness in the choice of the pairs of functions. Obviously the choice can be made in three ways, which can be schematically represented in the forms

$$(NP)(e\nu); \ (Ne)(P\nu); \ (N\nu)(Pe).$$

Following the usually accepted course, we group the heavy particles in one pair, the light in the other, and multiply together the like expressions (cf. end of Sec. 10).

Since the Hamiltonian must be Hermitian we must also add to the expressions formed so far their Hermitian adjoints. Then by using the formulas (2.25) we get (introducing convenient numerical factors) five scalar expressions

*Equations (5.1) and (5.2) are to be regarded as the definition of the neutrino and antineutrino.

$$H_S = (P^*\gamma_4 N)(e^*\gamma_4\nu) + \text{эрм. сопр., Herm. adj.} \tag{5.4}$$

$$H_V = (P^*\gamma_4\gamma_i N)(e^*\gamma_4\gamma_i\nu) + \text{Herm. adj.}$$

$$= (P^*N)(e^*\nu) - (P^*\sigma\gamma_5 N)(e^*\sigma\gamma_5\nu) + \text{Herm. adj.} \tag{5.5}$$

$$H_T = \frac{1}{2}(P^*\gamma_4\sigma_{ik}N)(e^*\gamma_4\sigma_{ik}\nu) + \text{Herm. adj.}$$

$$= (P^*\gamma_4\sigma N)(e^*\gamma_4\sigma\nu) + (P^*\gamma_4\sigma\gamma_5 N)(e^*\gamma_4\sigma\gamma_5\nu)$$

$$+ \text{Herm. adj.} \tag{5.6}$$

$$H_A = (P^* i\gamma_4\gamma_i\gamma_5 N)(e^* i\gamma_4\gamma_i\gamma_5\nu) + \text{Herm. adj.}$$

$$= (P^*\sigma N)(e^*\sigma\nu) - (P^*\gamma_5 N)(e^*\gamma_5\nu) + \text{Herm. adj.} \tag{5.7}$$

$$H_P = (P^*\gamma_4\gamma_5 N)(e^*\gamma_4\gamma_5\nu) + \text{Herm. adj.} \tag{5.8}$$

According to general rules, the operator for production of a particle also describes the absorption of the antiparticle. Therefore each of the expressions written describes not only β^- decay but quite a number of different decays (in which either a neutron is converted into a proton or an antiproton is converted into an antineutron*) (here a bar over a letter denotes the antiparticle):

$$N \to P + e^- + \tilde{\nu},$$
$$\overline{P} \to \overline{N} + e^- + \tilde{\nu},$$
$$N + e^+ \to P + \tilde{\nu},$$
$$N + \nu \to P + e^- \text{ and so on.}$$

The Hermitian adjoint terms not written out in Eq. (5.4) describe inverse processes in which a proton is converted into a neutron or an antineutron into an antiproton, or, finally, $\overline{N} + P$ annihilation:

$$P \to N + e^+ + \nu,$$
$$\overline{N} \to \overline{P} + e^+ + \nu,$$
$$P + e^- \to N + \nu,$$
$$P + \tilde{\nu} \to N + e^+ \text{ and so on}$$

The Hamiltonians (5.4) – (5.8) are spoken of as the types (scalar, vector, and so on) of interaction. If in all these expressions we replace the neutrino wave function φ by $-\gamma_5\varphi$, we get five new expressions H'_S, H'_V, H'_T, H'_A, and H'_P which are pseudoscalars.

The general form of the interaction leading to β decay will be a linear combination of all 10 types. The general form of the β interaction is written:

$$H = \sum_{k=1}^{5} (C_k H_k + C'_k H'_k). \tag{5.9}$$

Thus in the general case the β interaction is characterized by 10 complex (or 20 real) constants C and C'.*

For the longitudinal neutrino the constants C and C' are obviously not independent. If the neutrino is polarized along its momentum its wave function has the form $(1-\gamma_5)\nu$. Then $C_i = -C'_i$, $(i = 1, \ldots 5)$, but if the neutrino is polarized opposite to its momentum we have $C_k = C'_k$. It can be regarded as established by the direct experiments of Goldhaber, Grodzins, and Sunyar (D5) and also by the totality of other data that the neutrino has spin component $-\frac{1}{2}$ in the direction of its momentum. Therefore we set

$$C_k = C'_k.$$

in the Hamiltonian.

In what follows we shall maintain this assertion and assume that the neutrino is a left-handed particle.

It is convenient to introduce interaction constants g_i such that for the two-component neutrino

$$C_i = C'_i = g_i/\sqrt{2}. \tag{5.10}$$

With this choice the old numerical value of g is not changed. We still have to determine what values of the constants g_i will correspond to the process of positron decay. This is most simply done by constructing the Hamiltonian from the wave functions of the positron and the neutrino. Then we would get the same five quantities g_i, the only difference being that the sign of V and T relative to S, A, and P would be opposite to that for the case of β^- decay.†

This can also be proved in the following way from the Hamiltonian written out above.

The positron decay is described by the Hermitian adjoint terms not written out explicitly. The general form of such a term is

$$(\overline{N}\gamma_B P)(\bar{\nu}\gamma_B e). \tag{5.11}$$

In order for it to describe the emission of a positron and the absorption of a neutrino [cf. Eq. (5.2)] we must replace $\bar{\nu} \to \nu' C^{-1}$ and $e \to C\bar{e}'$, where ν' and e' are the operators for the antiparticles and C is the charge-conjugation operator. The nucleon factor does not need to be changed, for it describes what we want, production of a neutron and disappearance of a proton. Since we can show from Eq. (4.3) that

*In addition the same matrix element describes the annihilation process $N + \overline{P} \to e^- + \tilde{\nu}$.

*We note that if the neutrino charge were not conserved the number of constants would be twice as large.

†Physically this means that with the same mass and spin the signs of the current and magnetic moment are reserved.

$$C^{-1}\gamma_B C = \begin{cases} +\gamma_B^T & \text{for } S,\ A,\ P) \\ -\gamma_B^T & \text{for } V,\ T), \end{cases} \tag{5.12}$$

we get a change of sign for the vector and tensor terms (the index T, for transposition, disappears if we shift $\bar{e}'$ to the first place and ν' to the last).

Therefore all calculations for positron decay can be made with the same Hamiltonian, but with the signs of g_V and g_T changed and the wave function of the neutrino replaced by that of the antineutrino according to the scheme:

Electron decay	Positron decay
$C_S,\ C_V,\ C_T,\ C_A,\ C_P$	$C_S,\ -C_V,\ -C_T,\ C_A,\ C_P$
$C'_k = C_k$	$C'_k = -C_k.$

It was assumed above that the constants g_i are complex. If the combined (or the time) parity is conserved, these constants must be real. In the case of conservation of charge parity all the constants C must be real and the constants C′ imaginary. In this case obviously there can be no two-component neutrino (since we cannot at the same time have $C_k = \pm C'_k$).

Let us verify this assertion. We write out the general term of the scalar and pseudoscalar parts of the Hamiltonian, together with their Hermitian adjoints:

$$C_B(\bar{P}\gamma_B N)\ (\bar{e}\gamma_B\nu) + C_B^*(\bar{N}\gamma_B P)\ (\bar{\nu}\gamma_B e), \tag{5.13}$$

$$C'_B(\bar{P}\gamma_B N)\ (\bar{e}\gamma_B\gamma_5\nu) + C_B^*(\bar{N}\gamma_B P)(\bar{\nu}\gamma_5\gamma_B\nu). \tag{5.14}$$

Under the inversions $\psi \to T\bar{\psi}$ and $\psi \to C\bar{\psi}$ the two terms are interchanged. Since in Eq. (5.13) each term consists of two identically formed factors, a change of the order of the operators alters nothing and the condition of invariance for the two inversions reduces to $C_B = C_B^*$, or the reality of these constants (we ordinarily set a common phase factor equal to unity).*

In the case of the terms (5.14) there is an extra factor γ_5 in the brackets describing the light particles. Therefore the conditions for invariance are determined by the commutation relations of this factor with T and C.

From Eqs. (4.2) and (4.3) one can get

$$T\gamma_5 T^{-1} = \gamma_5^T, \tag{5.15}$$

$$C\gamma_5 C^{-1} = -\gamma_5^T. \tag{5.16}$$

(We note that in our representation $\gamma_5^T = \gamma_5$.)

It follows from Eq. (5.15) that there is no change of sign on time reversal and the condition for invariance is $C'_B = +C'^*_B$, i.e., that the constants be real. For charge conjugation there is a change of sign and the condition for invariance is $C'_B = -C'^*_B$, or that the constants be imaginary.

We summarize all these types in a table.

Table of possible types (as to parities) of β-interaction theories (with conservation of neutrino charge)

Hamiltonian invariant with respect to	Condition on the constants	Number of independent real constants	Possibility of longitudinal neutrino
P, C, T	$C'_k = 0$, C_k real	5	No
C, PT	C_k real, C'_k imag.	10	No
T, PC	C_k, C'_k real	10	Yes
The same with longitudinal neutrino	$C_k = C'_k$	5	–
P, TC	$C'_k = 0$	10	No
PCT	None	20	Yes
The same with longitudinal neutrino	$C_k = C'_k$	10	–

Let us now return to the Hamiltonian. Inserting the neutrino function in the form $(1+\gamma_5)\nu$ into Eqs. (5.4) – (5.8), we can rewrite the five interaction types for the β decay of the neutron:

$$H_S = 2^{-1/2}g_S(P^*\gamma_4 N)(e^*\gamma_4(1+\gamma_5)\nu) + \text{Herm. adj.} \tag{5.13'}$$

$$H_V = 2^{-1/2}g_V[(P^*N)(e^*(1+\gamma_5)\nu) + (P^*\sigma\gamma_5 N)(e^*\sigma(1+\gamma_5)\nu)] + \text{Herm. adj.} \tag{5.14'}$$

$$H_T = 2^{-1/2}g_T[(P^*\gamma_4\sigma(1+\gamma_5)N)(e^*\gamma_4\sigma(1+\gamma_5)\nu)] + \text{Herm. adj.} \tag{5.15'}$$

$$H_A = 2^{-1/2}g_A[-(P^*\gamma_5 N)(e^*(1+\gamma_5)\nu) + (P^*\sigma N)(e^*\sigma(1+\gamma_5)\nu] + \text{Herm. adj.} \tag{5.16'}$$

$$H_P = 2^{-1/2}g_P[(P^*\gamma_4\gamma_5 N)(e^*\gamma_4(1+\gamma_5)\nu)] + \text{Herm. adj.} \tag{5.17'}$$

Summing over all the neutrons in a nucleus, one can get from this the expression for the Hamiltonian H for decay of the nucleus.

We call attention to the fact that with the longitudinal neutrino all the terms are naturally grouped by twos. The types S and P differ only in the matrix elements of the heavy particles and go over into each other by the replacement $N \to \gamma_5 N$. The same can be said of the types V and A. This replacement does not affect the tensor type.

Let us consider the factor that relates to the nucleons. Since in β decay the nucleons can to

*It is not hard to see that in processes of first order in the weak interaction it is impossible to determine a common phase factor.

first approximation be regarded as nonrelativistic particles, we can neglect the last two components of their wave functions and regard the wave functions as Pauli two-component functions. Then for the decay of the neutron we set

$$(P^*\gamma_4 N) = (P^* N) = \langle 1 \rangle, \tag{5.18}$$

$$(P^*\gamma_4 \sigma N) = (P^* \sigma N) = \langle \sigma \rangle. \tag{5.19}$$

In this approximation the other matrix elements are equal to zero.

We have introduced the notations $\langle 1 \rangle$ and $\langle \sigma \rangle$, which are easily generalized to the case of β decay of nuclei.* Introducing the operator τ_+^i, which turns the i-th nucleon in the nucleus into a proton, if it has formerly been a neutron, we can write

$$\langle 1 \rangle = \Psi^* \sum \tau_+^{(i)} \Phi, \tag{5.20}$$

$$\langle \sigma \rangle = \Psi^* \sum \tau_+^i \sigma^{(i)} \Phi, \tag{5.21}$$

where the summation is extended over all the nucleons in the nucleus, and Φ and Ψ are the wave functions of the nucleus before and after the decay. The matrix elements $\langle 1 \rangle$ and $\langle \sigma \rangle$ are real. This follows from the fact that the states of the nucleus have definite parity and are invariant under time reversal. In fact, these matrix elements are calculated between two stationary states of the nucleus. The wave functions of such states can be made real by a simple choice of their phase factors. This last follows from the fact that ψ and ψ^* in this case describe the same state (there is no degeneracy). These functions are not affected by the replacement $\mathbf{k} \rightarrow -\mathbf{k}$, $\sigma \rightarrow -\sigma$ (there is no favored direction). According to a general theorem about time reversal it follows from this that the matrix elements taken between two stationary states are not altered by interchange of the initial and final states; since such interchange quite generally changes the matrix elements of Hermitian operators into their complex conjugates, it follows that they must be real.

The calculation of these matrix elements and the comparison with experiment have been carried out, for example, in reference A3, a survey of the β decays of light nuclei.

If we now replace the factor $\exp i\mathbf{pr}$ in the wave functions of the light particles by 1 (its value at $r = 0$), we can write down the expressions for all the types of β-decay interaction, in the form they take for allowed transitions:

*Other notations are often used in the literature:

$$\langle 1 \rangle \equiv M_F \equiv \int 1; \ |\langle \sigma \rangle| \equiv M_{GT} \equiv \left| \int \sigma \right|.$$

$$H_S = 2^{-1/2} g_S \langle 1 \rangle (e^* \gamma_4 (1 + \gamma_5) \nu) + \text{Herm. adj.} \tag{5.22}$$

$$H_P = 0, \tag{5.23}$$

$$H_V = 2^{-1/2} g_V \langle 1 \rangle (e^* (1 + \gamma_5) \nu) + \text{Herm. adj.} \tag{5.24}$$

$$H_A = 2^{-1/2} g_A \langle \sigma \rangle e^* \sigma (1 + \gamma_5) \gamma + \text{Herm. adj.} \tag{5.25}$$

$$H_T = 2^{-1/2} g_T \langle \sigma \rangle e^* \gamma_4 \sigma (1 + \gamma_5) \nu + \text{Herm. adj.} \tag{5.26}$$

where e^* and ν now do not depend on the coordinates.

Thus the general form for the β-decay Hamiltonian can be written

$$H = 2^{-1/2} \langle 1 \rangle [e^* (g_S \gamma_4 + g_V)(1 + \gamma_5) \nu] + \langle \sigma \rangle [e^* (g_T \gamma_4 + g_A) \sigma (1 + \gamma_5) \nu]. \tag{5.27}$$

The physical difference between the various types of β-decay interaction can be clearly seen from this expression. If we resolve the wave function of the electron into two parts, $e^* = \frac{1}{2}(1 + \gamma_5) e^* + \frac{1}{2}(1 - \gamma_5) e^*$, then by using the commutation rule $\gamma_4 \gamma_5 = -\gamma_5 \gamma_4$ we find that in the types S, T only the part $(1 - \gamma_5) e^*$ remains, and in the types V and A only the part $(1 + \gamma_5) e^*$. This shows that the terms of types S and T describe processes in which the polarizations of the emitted electron and antineutrino are the same,* and V and A, processes in which the polarizations of the emitted electron and antineutrino are different.

Let us now introduce instead of the electron wave function its expression in terms of the two-component function:

$$e = \left(\frac{W + m}{2W} \right)^{1/2} \begin{pmatrix} \varphi \\ \dfrac{\sigma \mathbf{p} \varphi}{W + m} \end{pmatrix};$$

$$e^* \gamma_4 = \left(\frac{W + m}{2W} \right)^{1/2} \begin{pmatrix} \varphi^* \\ -\dfrac{\varphi^* \sigma \mathbf{p}}{W + m} \end{pmatrix}, \tag{5.28}$$

where $\varphi^* \sigma \mathbf{p} \equiv \sigma^T \mathbf{p} \varphi^*$. And an analogous expression is used for the neutrino wave function.

Let us substitute these in Eq. (5.22):

$$\begin{aligned} H = \left(\frac{W + m}{8W} \right)^{1/2} \Big\{ & \langle 1 \rangle g_S e^* \left(1 + \frac{\sigma \mathbf{p}}{W + m} \right) (1 - \sigma \mathbf{v}) \nu \\ & + \langle 1 \rangle g_V e^* \left(1 - \frac{\sigma \mathbf{p}}{W + m} \right) (1 - \sigma \mathbf{v}) \nu \\ & + \langle \sigma \rangle g_T e^* \left(1 + \frac{\sigma \mathbf{p}}{W + m} \right) (1 - \sigma \mathbf{v}) \nu \\ & + \langle \sigma \rangle g_A e^* \left(1 - \frac{\sigma \mathbf{p}}{W + m} \right) (1 - \sigma \mathbf{v}) \nu \\ & + \text{Herm. adj.} \Big\}. \end{aligned} \tag{5.29}$$

We may remark for completeness that if the neutrino were not a longitudinal particle we would have to add to this expression a similar one con-

*More precisely it follows that the polarizations of the emitted electron and the <u>absorbed</u> neutrino are different, which is the same thing.

taining different constants g and with the sign of $\sigma\nu$ changed.

Equation (5.24) can be written in a more compact and symmetrical form

$$\left(\frac{8W}{W+m}\right)^{1/2} H = e^*\left(1+\frac{\sigma\mathbf{p}}{W+m}\right) \times (g_S\langle 1\rangle + g_T\langle\sigma\rangle\mathbf{c})(1-\sigma\mathbf{v})\,\mathbf{v} + e^*\left(1-\frac{\sigma\mathbf{p}}{W+m}\right)(g_V\langle 1\rangle + g_A\langle\sigma\rangle\sigma)(1-\mathbf{c}\mathbf{v})\,\mathbf{v}. \quad (5.30)$$

We recall once more that $\langle\sigma\rangle$ is a matrix element relating to the nucleus, and σ is a matrix acting on the light-particle functions.

We shall study the properties of this Hamiltonian in the following section. According to the foregoing, we get for positron decay instead of the expression (5.27)

$$H = 2^{-1/2}[\langle 1\rangle e^*(g_S\gamma_4 + g_V)(1-\gamma_5)\nu + \langle\sigma\rangle e^*(-g_T\gamma_4 + g_A)\sigma(1-\gamma_5)\nu + \text{Herm. adj.}] \quad (5.31)$$

and an analogous formula instead of Eq. (5.29).

Although we shall not deal with forbidden transitions in what follows, we shall point out here their main properties.

From Eqs. (5.27) and (5.28) it can be seen that in allowed transitions the terms S and V do not involve the change of spin or parity of a nucleon; the matrix element is $\langle 1\rangle$.* This means that the two light particles do not carry away any angular momentum. The terms T and A can give reversal of the spin of a nucleon – the matrix element involved for them is $\langle\sigma\rangle$. For the decay of a nucleus this means that the spin can change by ± 1 or 0 (but the transition $0 \rightarrow 0$ is forbidden). The parity of the nuclear state does not change.

In the discussion of allowed transitions three approximations have been made:

(1) The nuclear dimensions have been taken to be zero and the light-particle wave functions have been replaced by their values at the origin.

(2) The matrix elements for the nucleons have been calculated in nonrelativistic approximation; i.e., we have set $\gamma_4 N = N$ and $\gamma_5 N = 0$.

(3) The Coulomb field has been neglected.

The first of these approximations amounts to an expansion in powers of the ratio of the nucleon wavelength (sic) to the dimensions of the nucleus. For the decay of a nucleon (of dimensions in any case $< 10^{-13}$ cm) this approximation holds with high accuracy.

For nuclei, however, and transitions with spin change larger than 1, the decay probability is determined by the subsequent terms of the expansion. It is obvious that including them leads to the appearance of powers of the coordinates in the nuclear matrix elements and changes the selection rules.

Neglect of the relativistic terms for the nucleons can be incorrect for heavy nuclei. As we know, inclusion of the other two components leads to the appearance of the operator $\sigma\mathbf{p}$ in the matrix elements (σ and $\mathbf{p}$ are the spin and momentum of the nucleon). This operator describes the coupling of the spin of a nucleon in the nucleus with its orbit. Since it changes its sign on space inversion, it changes the parity selection rules. Inclusion of effects of the Coulomb field also leads to the appearance of new matrix elements, which play an important part in the decay of heavy nuclei. A detailed account of forbidden transitions can be found in references B25, B26, B27, and B38.

*Since the parity of a nuclear level is determined by nuclear forces, which come from strong interaction, it has a definite meaning.

6. THE SPECTRUM OF ALLOWED TRANSITIONS AND THE ELECTRON-NEUTRINO CORRELATION

According to the general rules of perturbation theory the probability of β decay is given by

$$w = 2\pi |H|^2 \rho_E. \quad (6.1)$$

We find the density of states of the electron-antineutrino system by the usual rules* (we shall sum explicitly over the spins, and therefore shall not include a factor for their statistical weight in ρ_E):

$$\rho_E = \frac{p^2\,dp}{(2\pi)^3\,dW}\,dW\,\frac{W_\nu^2}{(2\pi)^3}\,d\Omega_\nu\,d\Omega_e; \quad (6.2)$$

W, p are the energy and momentum of the electron, W_ν is the energy of the neutrino, and $d\Omega_\nu$, $d\Omega_e$ are the respective solid angles.

Introducing the magnitude W_0 of the decay energy (the limit of the β-ray spectrum plus the mass of the electron) and setting $W_\nu = (W_0 - W)$, we get

$$d^3w = \frac{1}{(2\pi)^5}|H|^2(W^2-m^2)^{1/2} \times (W_0 - W)^2 W\,dW\,d\Omega_\nu\,d\Omega_e. \quad (6.3)$$

We shall retain only two variables: the energy of the electron and the angle ϑ between the momenta of the electron and neutrino:

*The momentum of the recoil nucleus is fixed by the selection rules. We neglect the recoil energy.

$$\frac{d^2w(W,\vartheta)}{d\cos\vartheta\, dW} = \frac{2}{(2\pi)^3}|H|^2 f(W), \tag{6.4}$$

where f(W) is the so-called Fermi function,

$$f(W) = (W^2 - m^2)^{1/2}(W_0 - W)^2 W. \tag{6.5}$$

We shall now calculate $|H|^2$.

In this calculation we encounter expressions of the type $|e^*O\nu|^2$, where O is some matrix.

This expression is put in the following form:

$$|e^*O\nu|^2 = (e^*O\nu)(\nu^*O^+e). \tag{6.6}$$

Summing over the two components of ν and using the normalization of ν, we get

$$|e^*O\nu|^2 = e^*OO^+e. \tag{6.7}$$

The right member is the average value of the operator OO^+ calculated for the state of the emitted electron described by the Pauli two-component function e.

Thus, denoting the average by square brackets, we get the basic formula

$$\sum_{\text{spin of }\nu} |e^*O\nu|^2 = [OO^+]_{\text{av}}. \tag{6.8}$$

Let us apply this rule to Eq. (5.29). For an unpolarized nucleus the product $\langle 1\rangle\langle\sigma\rangle$ averaged over the spin of the nucleus vanishes. Therefore we can consider the two terms separately:

$$H = \langle 1\rangle H_1 + \langle\sigma\rangle H_\sigma \tag{6.9}$$

and for an unpolarized nucleus

$$|H|^2 = \langle 1\rangle^2 |H_1|^2 + \langle\sigma\rangle^2 |H_\sigma|^2 \tag{6.10}$$

(the nuclear matrix elements are real).

If we do not as yet concern ourselves with the polarization, we must sum the value of $|H|^2$ calculated by the formula (6.8) over the two spin states of the electron. Since on this summation expressions in which the matrix σ occurs linearly vanish, and the averages of expressions not containing σ do not depend on the state of the electron, the summation reduces to omission of terms linear in the spin and multiplication of the result by 2.

The further steps are obvious. For the square of the absolute value of the matrix element H_1, summed over the spin states of the electron, we find:

$$\begin{aligned}\sum |H_1|^2 = {} & \frac{W+m}{2W}|g_S|^2 \left[\left(1+\frac{\boldsymbol{\sigma}\mathbf{p}}{W+m}\right)(1-\boldsymbol{\sigma}\boldsymbol{\nu})\left(1+\frac{\boldsymbol{\sigma}\mathbf{p}}{W+m}\right)\right]_{\text{av}} \\ & + \frac{W+m}{2W}|g_V|^2\left[\left(1-\frac{\boldsymbol{\sigma}\mathbf{p}}{W+m}\right)(1-\boldsymbol{\sigma}\boldsymbol{\nu})\left(1-\frac{\boldsymbol{\sigma}\mathbf{p}}{W+m}\right)\right]_{\text{av}} \\ & + \frac{W+m}{2W}\, 2\,\mathrm{Re}\, g_S g_V^* \left[\left(1+\frac{\boldsymbol{\sigma}\mathbf{p}}{W+m}\right)(1-\boldsymbol{\sigma}\boldsymbol{\nu})\left(1-\frac{\boldsymbol{\sigma}\mathbf{p}}{W+m}\right)\right]_{\text{av}}.\end{aligned} \tag{6.11}$$

For the further calculation we need two formulas:

$$\left(1-\frac{\sigma\mathbf{p}}{W+m}\right)\left(1+\frac{\sigma\mathbf{p}}{W+m}\right) = \frac{2m}{W+m}, \tag{6.12}$$

$$\left(1 \pm \frac{\sigma\mathbf{p}}{W+m}\right)^2 = \frac{2W}{W+m}(1 \pm \beta\sigma\mathbf{e}), \tag{6.13}$$

where **e** is the unit vector in the direction of the electron momentum, and β is the speed of the electron.*

Then we get from Eq. (6.11), after summation over the spin of the electron

$$\sum |H_1|^2 = |g_S|^2 + |g_V|^2 - \beta(|g_S|^2 - |g_V|^2)\cos\vartheta + \frac{2m}{W}\,\mathrm{Re}\, g_S g_V^*. \tag{6.14}$$

Here we have used the formula

$$[(\boldsymbol{\sigma}\mathbf{e})(\boldsymbol{\sigma}\boldsymbol{\nu})]_{\text{av}} = \mathbf{e}\boldsymbol{\nu} = \cos\vartheta. \tag{6.15}$$

We must now fix the meaning of the direction ν.

The Hamiltonian contains the wave operator for the absorption of a neutrino, and we have to use the momentum of the emitted antineutrino. We have already said, however, that the operator for absorption of ν also describes the emission of $\tilde{\nu}$ with the same momentum and the opposite direction of the spin. Therefore we can regard ν as the unit vector along the direction of emission of the $\tilde{\nu}$.

This decision can be checked by examining, for example, the term corresponding to the scalar interaction type in Eq. (5.29). Setting the speed of the electron equal to 1 for simplicity, we rewrite the factor describing the light particles in the form

$$e^*(1+\sigma e)(1-\boldsymbol{\sigma}\boldsymbol{\nu})\nu. \tag{6.16}$$

The electron-antineutrino pair does not carry away any angular momentum; therefore the two particles cannot emerge in one direction (their chiralities are the same).

On the other hand, the expression (6.16) goes to zero for $\mathbf{e} = \boldsymbol{\nu}$; thus it is verified that ν is

*Equation (6.12) means that the two two-component electron functions are not orthogonal. They become orthogonal only for $m/W \to 0$. At the same time the four-component functions $(1+\gamma_5)\psi$ are of course orthogonal.

the direction of emission of the antineutrino.*

The calculation of $|H_\sigma|^2$ is only a little bit longer. We denote by $\boldsymbol{\sigma}\mathbf{l}$ the component of the electron spin in the direction of polarization of the nucleus. Then $|H_\sigma|^2$, summed over the polarization of the electron, is given by

$$\frac{2W}{W+m}\sum|H_\sigma|^2=|g_T|^2\left[\left(1+\frac{\sigma p}{W+m}\right)^2\sigma l(1-\sigma\nu)\sigma l\right]_{av}$$
$$+|g_A|^2\left[\left(1-\frac{\sigma p}{W+m}\right)\sigma l(1-\sigma\nu)\sigma l\right]_{av}$$
$$+\frac{2m}{W}\operatorname{Re}g_Tg_A^*\left[\sigma l(1-\sigma\nu)\sigma l\right]_{av}. \qquad (6.17)$$

Working out the products, we get

$$\sum|H_\sigma|^2=|g_T|^2+|g_A|^2-\beta(|g_T|^2-|g_A|^2)$$
$$\times[(\sigma e)(\sigma l)(\sigma\nu)(\sigma l)]_{av}. \qquad (6.18)$$

To calculate the fourfold product we resolve the unit vector $\mathbf{l}$ into two components: $\mathbf{l}_{||}$ along $\boldsymbol{\nu}$ and $\mathbf{l}_{\perp}$ perpendicular to $\boldsymbol{\nu}$. Then $\boldsymbol{\sigma}\mathbf{l}_{||}$ commutes with $\boldsymbol{\sigma\nu}$, and $\boldsymbol{\sigma}\mathbf{l}_{\perp}$ anticommutes with it; so

$$(\sigma e)(\sigma l)(\sigma\nu)(\sigma l)=(\sigma e)(\sigma\nu)[(\sigma l_{||})^2$$
$$-(\sigma l_{\perp})^2]=(\sigma e)(\sigma\nu)(l_{||}^2-l_{\perp}^2). \qquad (6.19)$$

Averaging over the directions of $\mathbf{l}$ and noting that

$$[l_{||}^2]_{av}=\frac{1}{3},\quad [l_{\perp}^2]_{av}=\frac{2}{3},$$

we find that the average value of the brackets in Eq. (6.19) is $-\frac{1}{3}$.

Substituting in Eq. (6.18), we have

$$\sum|H_\sigma|^2=|g_T|^2+|g_A|^2+\frac{1}{3}\beta(|g_T|^2-|g_A|^2)+\frac{2m}{W}\operatorname{Re}g_Tg_A^*. \qquad (6.20)$$

The formula for β^+ decay is obtained from Eq. (6.20) by changing the sign of the last term (change of sign of g_T).

Thus we find for the β-decay probability

$$\frac{d^2w(W,\vartheta)}{d\cos\vartheta\, dW}=\frac{f(W)}{4\pi^3}\Big\{\langle 1\rangle^2(|g_S|^2+|g_V|^2)+\langle\sigma\rangle^2(|g_T|^2+|g_A|^2)$$
$$-\beta\cos\vartheta\left[\langle 1\rangle^2(|g_S|^2-|g_V|^2)-\frac{1}{3}\langle\sigma\rangle^2(|g_T|^2-|g_A|^2)\right]$$
$$+\frac{2m}{W}[\langle 1\rangle\operatorname{Re}g_Sg_V^*+\langle\sigma\rangle\operatorname{Re}g_Tg_A^*]\Big\}, \qquad (6.21)$$

where $f(W)$ is defined by Eq. (6.5), and $<\sigma>^2 \equiv <\boldsymbol{\sigma}>^2$.

It can be seen from Eq. (6.26) that in the decay of unpolarized nuclei there is interference only between the decay types S and V, and between T and A. Therefore there are three cases in which there are no interference terms in the spectrum: (1) the products $g_Sg_V^*$ and $g_Tg_A^*$ are purely imaginary — in this case the combined parity is not conserved; (2) the interaction is a sum of the scalar and tensor types — then the electron is polarized like an antineutrino; (3) the interaction is a sum of vector and pseudovector types — then the electron is polarized like a neutrino.

In the last two cases Eq. (6.21) can be rewritten in the form

$$\frac{d^2W}{d\cos\vartheta\, dW}=\frac{f(W)}{4\pi^3}\{\langle 1\rangle^2|g_{S,V}|^2+\langle\sigma\rangle^2|g_{T,A}|^2\}$$
$$\times(1+\lambda\beta\cos\vartheta), \qquad (6.22)$$

where

$$\lambda_{,T}=-\frac{1-\frac{1}{3}R}{1+R};\quad R=\frac{\langle\sigma\rangle^2}{\langle 1\rangle^2}\left|\frac{g_T}{g_S}\right|^2 \qquad (6.23)$$

and

$$\lambda_{V,A}=\frac{1-\frac{1}{3}R}{1+R};\quad R=\frac{\langle\sigma\rangle^2}{\langle 1\rangle^2}\left|\frac{g_A}{g_V}\right|^2. \qquad (6.24)$$

The total probability of β decay is obtained from Eq. (6.22) and the formula

$$f=\int_m^{W_0}f(W)\,dW=(W_0^2-m^2)^{1/2}\left[\frac{W_0^4}{30}-\frac{3W_0^2m^2}{20}-\frac{2m^4}{15}\right]$$
$$+\frac{W_0m}{4}\ln\frac{W_0+(W_0^2-m^2)^{1/2}}{m}. \qquad (6.25)$$

Tables commonly give the half-life

$$t_{1/2}=\frac{\ln 2}{w}.$$

We get for the interaction types (S, T) and (V, A)

$$ft_{1/2}=\frac{2\pi^3}{|g_{S,V}|^2}\cdot\frac{1}{\langle 1\rangle^2+R\langle\sigma\rangle^2}\equiv\frac{A}{\langle 1\rangle^2+R\langle\sigma\rangle^2}. \qquad (6.26)$$

Good tables of values of $ft_{1/2}$ have been compiled by Feingold (A7); other information about β-radioactive nuclei is given in the tables of King (A8).

7. THE POLARIZATION OF THE ELECTRONS

As we have seen, the correlation between the momenta of the electron and neutrino, considered in the preceding section, enables us to distinguish between the types of decay interaction, but gives no information about parity nonconservation. Such information is given only by the polarization phenomena of β decay. Since the polarization vector transforms under reflections differently from the

*Arguing quite formally we can say: the momentum changes sign on space inversion and on time reversal; consequently, in virtue of the PCT theorem, it does not change sign on charge conjugation.

momentum vector, the correlation between the directions of the spins and momenta of the particles depends on the invariance properties of the system with respect to reflections.

If we consider the decay of a stationary nucleus, then in virtue of the law of conservation of momentum the decay pattern is determined by two momenta (for definiteness, the momentum p_e of the electron and the momentum p_ν of the antineutrino) and four spins. The spin of the neutrino is always directed along its momentum, so that the independent polarizations are those of the nuclei (initial and final) and that of the electron.

Thus we have at our disposal five vectors. Accordingly the expression for the transition amplitude contains scalars and pseudoscalars formed from these vectors, and the coefficients of these quantities (functions of the electron energy) determine the various distribution functions that are experimentally measurable. Without the use of the spins one can construct only three scalars, p_e^2, p_ν^2, and $\mathbf{e}\boldsymbol{\nu}$. The corresponding parts of the transition amplitude determine the spectra of the electrons and neutrinos (recoil nuclei)* and the electron-neutrino correlation. This clearly exhausts all the independent quantities that can be formed without considering the spins.

We now include in the treatment the spin of the electron, without as yet fixing the spins of the initial and final nuclei.

In this section we shall deal with the correlation between the direction of the election spin and the momenta of the electron and neutrino — with the pseudoscalars $\boldsymbol{\sigma}\mathbf{e}$, $\boldsymbol{\sigma}\boldsymbol{\nu}$, and $\boldsymbol{\sigma}(\mathbf{e}\times\boldsymbol{\nu})$. For unpolarized nuclei this clearly exhausts the list of quantities characterizing the process.† In the following sections we shall discuss the effects associated with polarization of the nucleus.

Let us introduce a right-handed rectangular coordinate system convenient for our purposes in the following way. We take one of the axes along the momentum of the electron (unit vector $\mathbf{e}$); we take the second axis normal to the plane of the decay, in the direction of the vector product $\mathbf{e}\times\boldsymbol{\nu}$ (unit vector $\mathbf{n}$). We locate the third axis in the plane of the decay (the plane determined by the vectors $\mathbf{e}$ and $\boldsymbol{\nu}$, since the vector of the recoil nucleus lies in this same plane) perpendicular to $\mathbf{e}$ and parallel to the vector product $\mathbf{n}\times\mathbf{e}$ (unit vector $\mathbf{m}$). It is obvious that $\mathbf{e}\times\mathbf{m}=\mathbf{n}$ and $\mathbf{m}\times\mathbf{n}=\mathbf{e}$.

*It is obvious that the spectra of the electrons and neutrinos are connected by the law of conservation of energy.

†Since $\sigma^2 = 3$, $e^2 = 1$, and $\nu^2 = 1$, these vectors can occur in the amplitude only linearly.

In general the average polarization of the electron can have components along all three axes:

$$\langle\sigma\rangle = P_e\mathbf{e}+P_m\mathbf{m}+P_n\mathbf{n}. \qquad (7.1)$$

It is not hard to see that the three unit vectors on the right transform differently under reflections. Let us examine how the terms in Eq. (7.1) transform under the reflections P and T. Since we are neglecting the Coulomb interaction, the operation of time reversal will be understood in the sense of the first-order perturbation theory (change of the signs of momenta and spins, cf. Sec. 4); We shall denote this operation by T_1, to distinguish it from the exact transformation T. Later we shall also consider the effect of the Coulomb field. Under either the spatial or the time reflection (P or T_1) the momenta change sign. Therefore the signs of the unit vectors $\mathbf{e}$ and $\mathbf{m}$ change under these reflections.

On the other hand, the polarization $\langle\sigma\rangle$ does not change on the reflection P and has its sign changed by either T or T_1 (it transforms like an angular velocity). It can be seen from the table that if parity is conserved, i.e., if the properties of the system remain unchanged under all

Transformations under reflections

Vector \ Reflection	P	T_1
$\mathbf{e}$	—	—
$\mathbf{n}$	+	+
$\mathbf{m}$	—	—
$\langle\sigma\rangle$	+	—

reflections, to the first order of the perturbation theory the polarization cannot have a component along any of the axes, since $\boldsymbol{\sigma}$ transforms differently from any of the unit vectors. Parity nonconservation is necessary if there is to be a nonvanishing polarization.

If spatial parity is not conserved but time parity is, components of the electron's polarization appear along the unit vector $\mathbf{e}$ (longitudinal polarization) and along the unit vector $\mathbf{m}$. There can still be no polarization along the unit vector $\mathbf{n}$ (compare signs in the T_1 column). In this case the polarization of the electron lies in the plane of the decay. If the time parity is also not conserved, the polarization of the electron can have a component along the unit vector **n**.

We note that if we average over all directions of the vector (we shall not register the direction of the recoil nucleus), the only preferred direction will be the vector **e**, and the polarization vector will be along it, independently of the conservation of P or T. Since the nonconservation of the time parity is to be detected from the complex character of the constants of the interaction (or, more precisely, from the phase differences of the various interaction types), it can manifest itself only in phenomena associated with the interference of the various types. Therefore for pure interaction types the polarization always lies in the plane of the decay.

We shall now derive formulas for the polarization of the electrons.

We begin with the scalar interaction. The probability for production of an electron is given by Eq. (6.10), not averaged over the spin. This probability is proportional to

$$e^*\left(1+\frac{\sigma\mathbf{p}}{W+m}\right)(1-\sigma\mathbf{v})\left(1+\frac{\sigma\mathbf{p}}{W+m}\right)e. \qquad (7.1')$$

In our further calculations we shall usually omit common numerical factors, since we shall not be interested in the absolute values of the decay probabilities, which have been computed earlier. The operator between e* and e determines the probabilities of production of an electron with various values of the spin component. If we bring this operator into a form proportional to

$$1+\sigma\mathbf{P}, \qquad (7.2)$$

then the probability of production of an electron with spin component $+\frac{1}{2}$ in the direction of the vector **P** will be

$$w_+ = 1+|\mathbf{P}|$$

and the corresponding value for the spin component $-\frac{1}{2}$ will be

$$w_- = 1-|\mathbf{P}|,$$

so that **P** is, by definition, the polarization of the electron:

$$|\mathbf{P}| = \frac{w_+ - w_-}{w_+ + w_-}. \qquad (7.3)$$

Let us transform the operator in Eq. (7.1′). For the calculation we resolve $\boldsymbol{\nu}$ into two vectors in the directions of **e** and **m**. We get:

$$(\sigma\mathbf{v}) = (\mathbf{e}\mathbf{v})(\sigma\mathbf{e}) + (\mathbf{m}\mathbf{v})(\sigma\mathbf{m}). \qquad (7.4)$$

Noting that the different components of σ anticommute, we get for the operator in Eq. (7.1)

$$\left(1+\frac{\sigma\mathbf{p}}{W+m}\right)^2[1-(\mathbf{e}\mathbf{v})(\mathbf{e}\sigma)]$$
$$-\left(1+\frac{\sigma\mathbf{p}}{W+m}\right)\left(1-\frac{\sigma\mathbf{p}}{W+m}\right)(\mathbf{m}\mathbf{v})(\sigma\mathbf{m}). \qquad (7.5)$$

Expanding the products and discarding a common factor, we get

$$1+\beta(\sigma\mathbf{e})-(\beta+\sigma\mathbf{e})\,\mathbf{e}\mathbf{v}-\frac{m}{W}(\mathbf{m}\mathbf{v})(\sigma\mathbf{m}) \qquad (7.6)$$

and, dividing by $1-\beta\mathbf{e}\boldsymbol{\nu}$, we find:

$$1+\frac{\beta-\mathbf{e}\mathbf{v}}{1-\beta\mathbf{e}\mathbf{v}}(\sigma\mathbf{e})-\frac{m}{W}\frac{\mathbf{m}\mathbf{v}}{1-\beta\mathbf{e}\mathbf{v}}\sigma\mathbf{m}. \qquad (7.7)$$

Comparing this with Eq. (7.2), we see that for the scalar interaction the polarization of the electron has two components:*

longitudinal

$$P_e(S)=\frac{\beta-\mathbf{e}\mathbf{v}}{1-\beta\mathbf{e}\mathbf{v}} \qquad (7.8)$$

and transverse

$$P_m(S)=-\frac{m}{W}\frac{\mathbf{m}\mathbf{v}}{1-\beta\mathbf{e}\mathbf{v}}. \qquad (7.9)$$

The vector interaction type differs from the scalar in that, in Eq. (7.1) and correspondingly also in Eq. (7.5), we must change the sign of $\sigma\mathbf{p}$; this reduces effectively to a change of the sign of β, which leads to the following formulas for the polarization:

$$P_e(V)=-\frac{\beta+\mathbf{e}\mathbf{v}}{1+\beta\mathbf{e}\mathbf{v}}, \qquad (7.10)$$

$$P_m(V)=-\frac{m}{W}\frac{\mathbf{m}\mathbf{v}}{1+\beta\mathbf{e}\mathbf{v}}. \qquad (7.11)$$

In the cases of the tensor and pseudovector types we must consider instead of the operator (7.1) the operator

$$\left(1\pm\frac{\sigma\mathbf{p}}{W+m}\right)\sigma\mathbf{l}\,(1-\sigma\mathbf{v})\,\sigma\mathbf{l}\left(1\pm\frac{\sigma\mathbf{p}}{W+m}\right), \qquad (7.12)$$

where the upper sign refers to the tensor and the lower to the pseudovector type, and where **l** is the vector in the direction of the spin of the nucleus, over which we have to average.

The average value (in the sense of averaging over the directions of **l**) of the triple product $(\sigma\mathbf{l})(\sigma\boldsymbol{\nu})(\sigma\mathbf{e})$ has been calculated in Section 6; it is equal to $-(\sigma\boldsymbol{\nu})/3$. After the averaging the expression (7.12) becomes

$$\left(1\pm\frac{\sigma\mathbf{p}}{W+m}\right)\left(1+\frac{1}{3}\sigma\mathbf{v}\right)\left(1+\frac{\sigma\mathbf{p}}{W+m}\right). \qquad (7.13)$$

Comparing this with the expression (7.2) we see that the polarization for the tensor interaction

*This is in agreement with the fact that for the scalar interaction the polarization lies in the plane of the decay.

type is obtained from that for the scalar type by the replacement $\nu \rightarrow -\nu/3$. This same change in the expression for the polarization for the vector type gives that for the pseudovector type.

Thus for the tensor interaction we have

$$P_e(T) = \frac{\beta + \frac{1}{3}\mathbf{e}\mathbf{\nu}}{1 + \frac{1}{3}\beta\mathbf{e}\mathbf{\nu}}, \tag{7.14}$$

$$P_m(T) = \frac{m}{3W}\frac{\mathbf{m}\mathbf{\nu}}{1 + \frac{1}{3}\beta\mathbf{e}\mathbf{\nu}}, \tag{7.15}$$

and for the pseudovector type

$$P_e(A) = -\frac{\beta - \frac{1}{3}\mathbf{e}\mathbf{\nu}}{1 - \frac{1}{3}\beta\mathbf{e}\mathbf{\nu}}, \tag{7.16}$$

$$P_m(A) = \frac{m}{3W}\frac{\mathbf{m}\mathbf{\nu}}{1 - \frac{1}{3}\beta\mathbf{e}\mathbf{\nu}}. \tag{7.17}$$

We shall also write out the formulas for the combined types (S, T) and (V, A). Since it is the transition amplitudes, not the polarizations, that are added, we find that if we write the probability for emission of an electron in the form

$$w \sim F(\vartheta) + G(\vartheta)\,\boldsymbol{\sigma}\mathbf{e} + \frac{m}{W}H(\vartheta)(\boldsymbol{\sigma}\mathbf{m}), \tag{7.18}$$

the polarization of the electron is given by the formulas

$$P_e = \frac{G(\vartheta)}{F(\vartheta)}, \tag{7.19}$$

$$P_m = \frac{m}{W}\frac{H(\vartheta)}{F(\vartheta)}. \tag{7.20}$$

For type (S, T)

$$F(\vartheta) = 1 - \beta\mathbf{e}\mathbf{\nu} + A\left(1 + \frac{1}{3}\beta\mathbf{e}\mathbf{\nu}\right),$$

$$G(\vartheta) = \beta - \mathbf{e}\mathbf{\nu} + A\left(\beta + \frac{1}{3}\mathbf{e}\mathbf{\nu}\right),$$

$$H(\vartheta) = \left(\frac{A}{3} - 1\right)\mathbf{m}\mathbf{\nu},$$

$$A = \left|\frac{g_T}{g_S}\right|^2 \frac{\langle\sigma\rangle^2}{\langle 1\rangle^2} \tag{7.21}$$

and for type (A, V)

$$F(\vartheta) = 1 + \beta\mathbf{e}\mathbf{\nu} + A\left(1 - \frac{1}{3}\beta\mathbf{e}\mathbf{\nu}\right),$$

$$G(\vartheta) = -\beta + \mathbf{e}\mathbf{\nu} - A\left(\beta + \frac{1}{3}\mathbf{e}\mathbf{\nu}\right),$$

$$H(\vartheta) = \left(\frac{A}{3} - 1\right)\mathbf{m}\mathbf{\nu},$$

$$A = \left|\frac{g_A}{g_V}\right|^2 \frac{\langle\sigma\rangle^2}{\langle 1\rangle^2}. \tag{7.22}$$

To obtain a formula including all four types, we must substitute in Eq. (7.21) the sum of the respective terms of (7.21) and (7.22), multiplied by $|g_{S,V}|^2 \langle 1\rangle^2$, respectively. We have also to add on the terms that arise from interference of (S, V) and (T, A) (the other combinations do not interfere in the case of an unpolarized nucleus). We shall not write these terms out in detail, but shall just briefly indicate their properties. In order to find the interference of S and V, we obviously have to replace the operator in Eq. (7.3) by the operator obtained by cross-multiplying H_S and H_V

$$g_S g_V^*\left(1 + \frac{\boldsymbol{\sigma}\mathbf{p}}{W+m}\right)(1 - \boldsymbol{\sigma}\mathbf{\nu})\left(1 - \frac{\boldsymbol{\sigma}\mathbf{p}}{W+m}\right) + \text{comp. conj.} \tag{7.23}$$

Transforming the operators and dropping (as before) a factor $2W/(W+m)$, we get

$$2\mathrm{Re}\, g_S g_V^* \frac{m}{W}[1 + (\mathbf{\nu}\mathbf{e})(\boldsymbol{\sigma}\mathbf{e})] + 2\beta\mathrm{Re}\, g_S g_V^*(\mathbf{m}\mathbf{\nu})(\boldsymbol{\sigma}\mathbf{m}) - 2\mathrm{Im}\, g_S g_V^*(\mathbf{n}\mathbf{e})(\boldsymbol{\sigma}\mathbf{n}). \tag{7.24}$$

The interference between T and A gives analogous terms, in which we must make the replacements

$$\beta \rightarrow -\frac{1}{3}\beta, \quad g_S g_V^* \rightarrow g_T g_A^*.$$

In accordance with the statement made earlier, the interference terms lead to the appearance of a component of the polarization along the third unit vector **n**. In the perturbation theory a polarization in this direction arises when the interaction constants are complex (more exactly, when the phases of g_S and g_T or of g_S and g_V are different). This can occur only if there is nonconservation of time parity.

This sort of effect is also produced by the Coulomb interaction when there is nonconservation of time parity. In fact, in this case the term in $\boldsymbol{\sigma}\mathbf{p}$ is multiplied by the complex factor (for small Z) $a = 1 + iZe^2/hv$. Then

$$\left(1 + a\frac{\boldsymbol{\sigma}\mathbf{p}}{W+m}\right)\left(1 - a^*\frac{\boldsymbol{\sigma}\mathbf{p}}{W+m}\right) \rightarrow \frac{2m}{W}(1 + 2i\beta\,\mathrm{Im}\, a\boldsymbol{\sigma}\mathbf{e}) \tag{7.25}$$

instead of $2m/W$ (for $Z = 0$). This leads to the appearance of a polarization in the direction **n** for all the types (even without interference of the different types). In the interference terms this polarization, associated with the Coulomb interaction, will be proportional to $\mathrm{Re}\, g_S g_V^*$ and $\mathrm{Re}\, g_T g_A^*$, whereas the polarization associated with nonconservation of time parity is proportional to the imaginary parts of these products. This feature is obviously due to the fact that on time reflection $g \rightarrow g^*$, i.e., $\mathrm{Re}\, g_S g_V^*$ and $\mathrm{Re}\, g_T g_A^*$ do not change sign, but $\mathrm{Im}\, g_S g_V^*$ and $\mathrm{Im}\, g_T g_A^*$ do. Therefore only the latter terms violate time-reflection invariance.

We can draw a general conclusion from Eq. (7.25) about the properties of the Coulomb interaction. Namely, it can be seen from this formula that the polarization σe that arises from the Coulomb interaction is shifted in phase by $\pi/2$ (a factor i) relative to the other terms of the same kind. Therefore the Coulomb interference will always be displaced in phase from the non-Coulomb interference. From what has been said about time parity it is clear, furthermore, that if the effect in question is not invariant with respect to the approximate (Born-approximation) version of time reflection, then without the Coulomb interaction terms appear involving the imaginary parts of products of the decay constants, and the Coulomb interaction gives terms proportional to their real parts. On the other hand, in effects that are invariant under the approximate time reflection, the non-Coulomb terms will be proportional to the real parts of products of the constants, and the Coulomb terms to the imaginary parts. These considerations are useful for understanding the structure of the various formulas.

8. THE DECAY OF POLARIZED NUCLEI

In the preceding sections we have dealt with all the effects that are not associated with polarization of the nucleus in its initial or final state. Let us now consider the decay of a polarized nucleus. In this case the distribution of the electrons among the various directions is no longer isotropic; there is a correlation between the momentum of the electron and the direction of the nuclear spin. The experimental demonstration of this effect by the work of Wu was the first confirmation of the ideas of Yang and Lee. If we at first do not take into account the polarization of the nucleus after the decay, then there are three new effects not connected with the spin of the electron: (1) a correlation of the momentum of the electron with the polarization of the nucleus — the pseudoscalar $\mathbf{el}$ ($\mathbf{l}$ is the unit vector in the direction of the polarization); (2) a correlation of the momentum of the neutrino with the polarization of the nucleus — the pseudoscalar $\boldsymbol{\nu}\mathbf{l}$; and (3) an effect of the polarization of the nucleus on the electron-neutrino correlation — the scalars $\mathbf{l}(\mathbf{e}\times\boldsymbol{\nu})$ and $Q_{ik}(e_i\nu_k + e_k\nu_i)$, where Q_{ik} is the (quadrupole) polarization tensor of the nucleus.

It is not hard to see that the correlations $\mathbf{el}$ and $\boldsymbol{\nu}\mathbf{l}$ involve nonconservation of spatial parity (they are scalars with respect to T_1), and $\mathbf{l}(\mathbf{e}\times\boldsymbol{\nu})$ involves nonconservation of time parity (a scalar with respect to P). The tensor effect does not change the parities. In allowed transitions these effects are absent for types S and V, since for these types the matrix elements do not depend on the spin of the nucleus. Thus an anisotropy of the electron distribution will be due to contributions from types T and A. Only in transitions without change of spin and parity ($j \to j$ transitions) will there also be a contribution from terms coming from types S and V, owing to their interference with the main terms. Since no averaging is carried out over the spin of the nucleus, the arguments used earlier do not apply, and in the absence of the Coulomb field S interferes with T and V with A (there is interference between types with the same chirality). We note, however, that if $I \to I$ transitions occur that are not between mirror nuclei, they are accompanied by a change of the isotopic spin of the nucleus. Then the Fermi matrix element $\langle 1 \rangle$ is commonly very small, and the interference terms are practically nonexistent (in the approximation of charge invariance they are zero).

We begin with the transitions in types T and A; we shall examine the interference terms later.

Let us go back to Eq. (7.12). This formula describes the distribution of the light particles for prescribed spin states of the nuclei, defined by the vector $\mathbf{l}$. The vector $\mathbf{l}$ came from the matrix element calculated by using the wave functions of the initial and final nuclear states. It can be written in the form

$$M = \Phi^*_{\rm fin} \sum_i \sigma^{(i)} \Phi_{\rm init}, \qquad (8.1)$$

where the summation is extended over all the neutrons in the nucleus, if for definiteness we again take the case of electron β decay. This matrix element depends on the nuclear spin components in the initial and final states. The square of the vector (8.1) is proportional to the quantity $\langle\sigma\rangle^2$, which was introduced earlier. We write Eq. (8.1) in the form

$$M = \langle\sigma\rangle\, \mathbf{a}(I, M;\ I', M'), \qquad (8.2)$$

separating off the angular (vector) factor of the matrix element M as the unit vector $\mathbf{a}(a_x, a_y, a_z)$. The quantities a_x, a_y, a_z depend on the spin of the nucleus and its component in the initial (I, M) and final (I′, M′) states (they are proportional to Clebsch-Gordan coefficients); by definition

$$|a_x|^2 + |a_y|^2 + |a_z|^2 = 1. \qquad (8.3)$$

It follows from the normalization that $M^2 = \langle\sigma\rangle^2$, and the probability of a transition with a given change of the spin and the spin component is proportional to the square of the absolute value of the corresponding coefficient a_k.

In allowed transitions the nuclear spin component can change by 0 or ± 1. Obviously this means that the light particles (electron and neutrino) carry away a (total) spin component of 0, ±1, respectively, so that the spin component of the whole system does not change.

In the old picture, in which the light particles did not have the spin coupled to the momentum, this led to a dependence of their polarizations on the direction of emission, but did not affect the angular distribution. In the two-component-neutrino picture the momentum of the antineutrino follows its spin, and therefore the anisotropy of the spin orientation leads to an anisotropy in the direction of emission as well.

Let us consider a nucleus with prescribed spin component M_0 and a transition with a prescribed change of the nuclear spin, $I \rightarrow I'$.

The transition amplitude will be proportional to

$$\mathbf{a}e^*\left(1 \pm \frac{\sigma\mathbf{p}}{W+m}\right)\sigma(1-\sigma\mathbf{v})\,e, \tag{8.4}$$

where the signs + and − refer to the types T and A respectively. Squaring this and averaging over the spin of the electron and the direction of emission of the neutrino (leaving out of account for the time being the electron-neutrino correlation), we get for the decay probability

$$w \sim \sum_{i,k} a_i a_k^* [(1 \pm \beta\sigma\mathbf{e})\,\sigma_i\sigma_k]_{\mathrm{av}}. \tag{8.5}$$

We note that

$$[\sigma_i\sigma_k]_{\mathrm{av}} = \delta_{ik},$$
$$[\sigma\mathbf{e}\sigma_i\sigma_k]_{\mathrm{av}} = i\mathbf{e}_{i\times k}, \tag{8.6}$$

where the third unit vector is denoted by $\mathbf{i}\times\mathbf{k}$. Then

$$w \sim 1 \pm i\beta(\mathbf{a}\times\mathbf{a}^*)\,\mathbf{e}. \tag{8.7}$$

Introducing instead of a_x and a_y the components a_+ and a_-,

$$a_\pm = 2^{-1/2}(a_x \pm ia_y), \tag{8.8}$$

corresponding to the change $|\Delta M| = 1$, and taking as the axis of quantization the polarization axis of the nucleus, $\mathbf{l}$, we get

$$w \sim 1 \mp \beta(\mathbf{el})(a_+^2 - a_-^2). \tag{8.9}$$

It can be seen from Eq. (8.1) that the coefficients a_+ and a_- arise from the compounding of the spin I of the initial nucleus with the spin I′ of the final nucleus ($I' = I \pm 1$ or I), which gives the vector $\mathbf{1}$. According to the general rules they are proportional to the Clebsch-Gordan coefficients corresponding to the composition scheme $I + 1 = I'$.

We give here a table of the Clebsch-Gordan coefficients we shall require.

Spin of final nucleus	$\Delta M = 1$ $a_+ \sim$	$\Delta M = 0$ $a_0 \sim$	$\Delta M = -1$ $a_- \sim$	Sum of squares of coefficients: for given M	Sum of squares of coefficients: for given M_0
$I+1$	$\left(\frac{(I+M)(I+M+1)}{(2I+1)(2I+2)}\right)^{1/2}$	$\left(\frac{(I-M+1)(I+M+1)}{(2I+1)(I+1)}\right)^{1/2}$	$\left(\frac{(I-M)(I-M+1)}{(2I+1)(2I+2)}\right)^{1/2}$	1	$\frac{2I+3}{2I+1}$
I	$\left(\frac{(I+M)(I-M+1)}{2I(I+1)}\right)^{1/2}$	$-\frac{M}{I^{1/2}(I+1)^{1/2}}$	$-\left(\frac{(I-M)(I+M+1)}{2I(I+1)}\right)^{1/2}$	1	1
$I-1$	$\left(\frac{(I-M)(I-M+1)}{2I(2I+1)}\right)^{1/2}$	$-\left(\frac{(I-M)(I+M)}{I(2I+1)}\right)^{1/2}$	$\left(\frac{(I+M+1)(I+M)}{2I(2I+1)}\right)^{1/2}$	1	$\frac{2I-1}{2I+1}$

The Clebsch-Gordan coefficients are usually normalized so that the sums of squares across the rows are equal to 1. This normalization corresponds to summation with a constant value of the nuclear spin component M' in the final state. We need coefficients a_k normalized for a prescribed initial-state spin component M_0. We therefore set $M = M_0 + 1$ in the first column, $M = M_0$ in the second, and $M = M_0 - 1$ in the third, and find that the sums of squares of the coefficients for fixed M_0 along the rows have the values shown in the last column. One must divide the squares of the Clebsch-Gordan coefficients by these numbers in order to get a^2 and a_-^2. Simple calculations from the table then lead to the following values:

$$a_+^2 - a_-^2 = \begin{cases} \frac{M_0}{I+1} & \text{for the transition } I \to I+1, \\ -\frac{M_0}{I(I+1)} & \text{for the transition } I \to I, \\ -\frac{M_0}{I} & \text{for the transition } I \to I-1. \end{cases} \tag{8.10}$$

Let us average these values over all the nuclei in the target. Since

$$\frac{\langle M_0 \rangle}{I} = P \tag{8.11}$$

is the polarization of the nucleus, we finally get the angular distributions for types T and A:

$$w(T) \sim 1 - \beta \mathbf{el} P \omega, \tag{8.12}$$

$$w(A) \sim 1 + \beta \mathbf{el} P \omega, \tag{8.13}$$

where

$$\omega = \begin{cases} \frac{I}{I+1} & \text{for the transition } I \to I+1, \\ -\frac{1}{I+1} & \text{for the transition } I \to I, \\ -1 & \text{for the transition } I \to I-1. \end{cases} \tag{8.14}$$

The formulas (8.12) and (8.13) describe the effect for transitions with change of the spin. For the transition $I \to I$ we still have to calculate the interference term. The interference of the vector and pseudovector types is described by the expression

$$+ a_0 g_V g_A^* \left(1 \frac{\sigma \mathbf{p}}{W+m}\right)(1 - \boldsymbol{\sigma}\boldsymbol{\nu})(\boldsymbol{\sigma}\mathbf{l})\left(1 - \frac{\sigma \mathbf{p}}{W+m}\right)$$
$$+ a_0 g_V^* g_A \left(1 - \frac{\sigma \mathbf{p}}{W+m}\right)(\boldsymbol{\sigma}\mathbf{l})(1 - \boldsymbol{\sigma}\boldsymbol{\nu})\left(1 - \frac{\sigma \mathbf{p}}{W+m}\right)$$
$$= + a_0 2 \operatorname{Re} g_V g_A^* \left(1 - \frac{\sigma \mathbf{p}}{W+m}\right)$$
$$\times (1 - \boldsymbol{\sigma}\boldsymbol{\nu})(\boldsymbol{\sigma}\mathbf{l})\left(1 - \frac{\sigma \mathbf{p}}{W+m}\right). \tag{8.15}$$

Averaging over $\boldsymbol{\nu}$ and removing the factor $2W/(W + m)$, we arrive at the expression

$$+ 2a_0 \operatorname{Re} g_V g_A^* (1 - \beta \boldsymbol{\sigma} \mathbf{e}) \boldsymbol{\sigma} \mathbf{l}. \tag{8.16}$$

Equation (8.16), unlike (8.7), contains only the first power of $\mathbf{l}$. Averaging over the spin of the electron, we bring Eq. (8.16) to the form

$$- 2\beta a_0 \operatorname{Re} g_V g_A^* (\mathbf{el}). \tag{8.17}$$

Substituting the value of a_0 from the table,

$$a_0 = \frac{M}{I^{1/2}(I+1)^{1/2}}, \tag{8.18}$$

averaging over the spins in the target, and introducing the polarization P of the nuclei, we now bring Eq. (8.17) to the form

$$- 2\beta P \operatorname{Re} g_V g_A^* \left(\frac{I}{I+1}\right)^{1/2}. \tag{8.19}$$

We get the expression for the interference of types S and T from Eq. (8.16) by the replacement $\mathbf{p} \to -\mathbf{p}$. Since this change does not affect the result (because of the averaging over the electron spin), we get for the interference term:

$$+ 2\beta P \operatorname{Re} g_S g_T^* \left(\frac{I}{I+1}\right)^{1/2}. \tag{8.20}$$

Now we can write the expression for the distribution of the electrons from the transition $I \to I$. Introducing the nuclear matrix elements $\langle 1 \rangle$ and $\langle \sigma \rangle$ and combining Eqs. (8.19) and (8.12), we get for the A, V type

$$w(VA) = 1 - \beta \frac{B}{1 + |A|^2} (\mathbf{el}) P, \tag{8.21}$$

where

$$A = \frac{g_V \langle 1 \rangle}{g_A \langle \sigma \rangle}, \tag{8.22}$$

$$B = \frac{1}{I+1} + \left(\frac{I}{I+1}\right)^{1/2} \cdot \operatorname{Re} A. \tag{8.23}$$

For the S, T type we get by a similar procedure

$$w(ST) = 1 + \beta \frac{B}{1 + |A|^2} (\mathbf{el}) P, \tag{8.24}$$

$$A = \frac{g_S \langle 1 \rangle}{g_T \langle \sigma \rangle}, \tag{8.25}$$

$$B = \frac{1}{I+1} + \left(\frac{I}{I+1}\right)^{1/2} \operatorname{Re} A, \tag{8.26}$$

For positron decays there are two changes in the formulas. First, the sign of β is changed (opposite sign of the polarization of the positron), and second, because of the change of sign of the V and T interaction constants, the interference term changes sign. The result is that in the formulas we must change the sign of β, and for the $I \to I$ transition we must also change the sign of the interference term. A combined treatment of all four types does not introduce anything essentially new, since V and A do not interfere with T, and S does not interfere with V and A, if we neglect the Coulomb interaction. An interference does arise, however, if we include this interaction. Furthermore, this effect will be proportional to the imaginary parts of the products of constants that come in ($\operatorname{Im} g_S g_V^*$, etc.) and will therefore exist only if there is nonconservation of time parity and if the types (V, A) and (S, T) are present together. Since we have agreed to confine ourselves in the main to the possibilities V, A and S, T, we shall not concern ourselves with this point in any more detail.

Let us now examine the correlation between the direction of emission of the antineutrino and the spin of the nucleus. Since in the V, A type the chirality of the antineutrino is opposite to that of the electron, and its speed is the speed of light, in the formulas obtained for the electron in type V, A we can simply make the formal changes

$$\mathbf{e} \to \boldsymbol{\nu}, \quad \beta = -1. \tag{8.27}$$

It can be shown, moreover, that the sign of the interference term remains unchanged. Thus we get for the antineutrino with right-handed chirality:

$$w(\tilde{\nu}) \sim 1 + \boldsymbol{\nu}\mathbf{l}P\omega \qquad (I \to I \pm 1), \tag{8.28}$$

$$w(\tilde{\nu}) \sim 1 + \frac{\frac{1}{I+1} - \left(\frac{I}{I+1}\right)^{1/2} \operatorname{Re} A}{1 + |A|^2} \cdot \boldsymbol{\nu}\mathbf{l}P \qquad (I \to I). \tag{8.29}$$

The formulas for the neutrino are obtained by changing the signs of the second term and of Re A.

A case of the greatest interest is that of the decay of polarized neutrons (Telegdi et al. (F7)). In this case the nuclear matrix elements are known: $\langle 1 \rangle = 1$, $\langle \sigma \rangle = 3^{1/2}$.

Denoting the absolute value of the ratio of the Fermi and Gamow-Teller constants by

$$\lambda = \frac{g_V}{g_A} \text{ or } \frac{g_S}{g_T}, \tag{8.30}$$

we get from Eqs. (8.21) – (8.23) for the correlation between the electron and the neutron spin direction (type V, A):

$$w(e) \sim 1 - \frac{\frac{2}{3} + \frac{2}{3} \operatorname{Re} \lambda}{1 + \frac{1}{3} |\lambda|^2} \mathbf{e}\mathbf{l} \tag{8.31}$$

and for the antineutrino

$$w(\tilde{\nu}) \sim 1 + \frac{\frac{2}{3} - \frac{2}{3} \operatorname{Re} \lambda}{1 + \frac{1}{3} |\lambda|^2} \boldsymbol{\nu}\mathbf{l}. \tag{8.32}$$

Let us now calculate the effect of polarization of the nucleus on the correlation. We return to Eq. (8.4). If in going from this to Eq. (8.5) one does not average over the direction of emission of the neutrino, but averages only over the electron spin, there are added to Eq. (8.5) the terms

$$-\sum_{i,k} a_i a_k^* [(1 \pm \beta\boldsymbol{\sigma}\mathbf{e}) \sigma_i (\boldsymbol{\sigma}\boldsymbol{\nu}) \sigma_k]_{\text{av}}. \tag{8.33}$$

The e-ν correlation is described by just the second term

$$\mp \beta \sum_{i,k} a_i a_k^* [(\boldsymbol{\sigma}\mathbf{e}) \sigma_i (\boldsymbol{\sigma}\boldsymbol{\nu}) \sigma_k]_{\text{av}}. \tag{8.34}$$

The first term obviously describes the correlation between the momentum of the neutrino and the polarization of the nucleus.

Using the formula

$$(\boldsymbol{\sigma}\mathbf{A})(\boldsymbol{\sigma}\mathbf{B}) = \mathbf{AB} + i\boldsymbol{\sigma}(\mathbf{A} \times \mathbf{B}), \tag{8.35}$$

where **A** and **B** are any vectors, we reduce Eq. (8.34) to the form

$$\mp \beta \sum_{i,k} a_i a_k^* (e_i \nu_k + e_k \nu_i - \delta_{ik} \mathbf{e}\boldsymbol{\nu}). \tag{8.36}$$

Separating out the term that describes the correlation independent of the polarization of the nucleus,

$$\mp \beta \sum_{i,k} a_i a_k^* \left(e_i \nu_k + e_k \nu_i - \frac{2}{3} \mathbf{e}\boldsymbol{\nu}\right) \pm \frac{\beta}{3} \mathbf{e}\boldsymbol{\nu} \tag{8.37}$$

and averaging over the various nuclei, we see that the effect of the polarization of the nucleus is described by the sum

$$\mp \beta \sum_{i,k} [a_i a_k^*]_{\text{av}} \left(e_i \nu_k + e_k \nu_i - \frac{2}{3} \mathbf{e}\boldsymbol{\nu}\right)_{\text{av}} \tag{8.38}$$

Equation (8.38) can be rewritten in the form

$$\mp \beta \sum_{i,k} R_{ik} e_i \nu_k, \tag{8.39}$$

where the tensor

$$R_{ik} = \frac{1}{2} \left[a_i a_k^* + a_k a_i^* - \frac{2}{3} \delta_{ik}\right]_{\text{av}} \tag{8.40}$$

is proportional to the quadrupolarization tensor of the target. From this it follows that a tensor correlation can exist only for nuclei with spin greater than $\frac{1}{2}$.

If now we take as our coordinate system the principal axes of the symmetric tensor R_{ik}, this tensor will have only three components, with their sum equal to zero. Introducing the components $a_\pm$, we find

$$\left.\begin{aligned} R_{xx} = R_{yy} &= \frac{1}{2}(a_+^2 + a_-^2) - \frac{1}{3} = -\frac{1}{2}\left(a_0^2 - \frac{1}{3}\right), \\ R_{zz} &= a_0^2 - \frac{1}{3}. \end{aligned}\right\} \tag{8.41}$$

Substituting in Eq. (8.36) and using the value of a_0^2, we get

$$\mp \beta [2e_z \nu_z - e_x \nu_x - e_y \nu_y] \cdot \left[\frac{[M^2]_{\text{av}}}{I(I+1)} - \frac{1}{3}\right] \eta$$

$$= \mp \beta \left[\frac{1}{3} e\nu - (el)(\nu l) \right] \left[\frac{I(I+1) - 3[M^2]_{av}}{I(I+1)} \right] \eta, \qquad (8.42)$$

where $[M^2]_{av}$ is the mean square value of the spin component. The factor in square brackets vanishes when all values of M^2 are equally probable (absence of quadrupolarization).

$$\eta = \begin{cases} -\frac{I}{2I+3} & \text{for the transition } I \to I+1, \\ 1 & \text{for the transition } I \to I, \\ -\frac{(I+1)}{(2I-1)} & \text{for the transition } I \to I-1. \end{cases} \qquad (8.43)$$

The expression (8.39) must be added to $\pm(\beta/3)\mathbf{e}\boldsymbol{\nu} = \pm(\beta/3)\cos\vartheta$, the corresponding expressions for the correlation in types T and A (Eq. (6.22)).

The formula remains the same for the correlation in positron decays.

There remain to be considered the interference terms that appear in $\mathrm{I} \to \mathrm{I}$ transitions.

These terms lead to a correlation of the vector type. From Eq. (8.15) we find in a similar way that the correlation is described by the additional terms

$$\left.\begin{array}{l} -\operatorname{Re} g_S g_T^* \\ +\operatorname{Re} g_V g_A^* \end{array}\right\} \times \beta[(\boldsymbol{\sigma}\mathbf{e})(\boldsymbol{\sigma}\boldsymbol{\nu})(\boldsymbol{\sigma}\mathbf{l})]_{av} a_0. \qquad (8.44)$$

The average value of the product is $\mathbf{i}(\mathbf{e}\times\boldsymbol{\nu})\mathbf{l}$, and therefore the whole expression becomes

$$\left.\begin{array}{l} +2\operatorname{Im} g_S g_T^* \\ -2\operatorname{Im} g_V g_A^* \end{array}\right\} \times \beta P(\mathbf{e}\times\boldsymbol{\nu})\mathbf{l}\left(\frac{I}{I+1}\right)^{1/2}. \qquad (8.45)$$

In this case the effect exists only for nonconservation of the combined parity, in agreement with the statement made earlier.

The expressions (8.45) obviously change sign when we go over to the positron decay.

The decay of polarized nuclei gives rise to a complicated pattern of electron polarization.

If in Eq. (8.4) we do not average over the spin, several terms proportional to $\boldsymbol{\sigma}$ occur. These terms will be of two types, which differ in that those of the first type give a polarization proportional to the polarization of the nuclei, and the others give a polarization proportional to the quadrupolarization. The former terms determine components of the electron polarization

$$\langle\sigma_i\rangle = \sum_k \alpha_{ik} P_k \qquad (8.46)$$

as linear functions of the components of the polarization of the nuclei, while the latter give a connection of the type

$$\langle\sigma_i\rangle = \sum_{kl} \beta_{ikl} R_{kl}. \qquad (8.47)$$

The coefficients α_{ik} and β_{ik} are formed from the vectors $\mathbf{e}$ and $\boldsymbol{\nu}$. The properties of these coefficients with respect to reflections can be studied by just the same methods as used for previous cases.

In Eq. (8.46) the components of the polarizations of the nucleus and electron transform in the same way. Therefore it is obvious that if any parity is conserved the coefficients α_{ik} must remain unchanged under the corresponding reflection. If we choose the coordinate system $\mathbf{e}$, $\mathbf{n}$, $\mathbf{m}$ that we used in Sec. 7, we can use the table on page 19 to write down the coefficients for the transformation of binary products of the unit vectors, which will obviously also be the transformation coefficients for the quantities (α_{ik})

	P	T_1
ee, nn, mm	+	+
en	—	—
em	+	+
nm	—	—

We see that invariance under P brings with it vanishing of the components **em** and **nm** (effect of the polarization of the nucleus along **n** on the longitudinal polarization of the electron, and so on).

In an analogous way one can also study the properties of the coefficients β_{ik}. We shall not present the detailed and rather cumbersome formulas here.

9. THE POLARIZATION OF THE NUCLEUS AFTER THE DECAY. THE β-γ CORRELATION

Let us now bring into our discussion the last of the parameters — the polarization of the nucleus after the decay.

If an unpolarized nucleus decays, then owing to the fact that the electron and neutrino carry away angular momentum the nucleus is polarized after the decay.

If we do not record the direction of emission of the neutrino and average over the directions of the electron spin, then the axis of polarization in allowed decays can only be the momentum of the electron. In allowed transitions the light particles carry away an angular momentum not larger than unity, and therefore in the decay the degree of polarization of the nucleus can change only by unity — an unpolarized nucleus becomes linearly polarized nucleus becomes quadrupolarized, etc. This too, however, can occur only if parity is not conserved; with parity conservation a polarization obviously cannot be proportional to a momentum.

The final polarization of the nucleus is usually measured by the angular distribution of γ-ray quanta or their circular polarization.*

It is not hard to see that since the electromagnetic interaction conserves parity, the angular distribution of the γ-rays cannot depend on a linear polarization (nor, in general, on any odd-degree polarization) of the target. In fact, in virtue of parity conservation a target with the polarization P radiates just like one with the polarization $-$ P. But the sum of the two targets is an unpolarized target, which radiates isotropically. Therefore the angular distribution of the γ rays is determined only by the quadrupolarization of the target (by the even polarizations). A formal expression of this fact is the requirement that the wave vector of the γ-ray quantum must always occur in the amplitude raised to an even power.

The effect of interest to us, the polarization of the nucleus, can be determined from the circular polarization of the γ rays.

A left-circularly polarized† quantum carries away an angular-momentum component unity; such quanta are emitted by a completely polarized target in the direction parallel to its polarization. Therefore the degree of (left-circular) polarization of a quantum emitted by a target with degree of polarization P is given by

$$r = P \cos \vartheta. \tag{9.1}$$

General formulas for the angular distribution and polarization of γ-ray quanta emitted by polarized nuclei can be found in, for example, the book edited by Siegbahn (A2).

Since the direction of the polarization arising from the β decay of an unpolarized nucleus is parallel to the momentum of the electron, the angle ϑ in Eq. (9.1) is at the same time the angle between the directions of emission of the electron and the γ-ray quantum. Therefore the correlation between the polarization of the final nucleus and the polarization of the γ-ray quantum is called β-γ-polarization correlation.

The calculation of the polarization of the nucleus after β decay is worked out from the same formulas as for the decay of polarized nuclei.

According to Eqs. (8.9), (8.18), and (8.19) the probability of decay of a nucleus, not averaged over the polarizations of the nucleus in the initial and final states, is given for $I \to I \pm 1$ transitions by the relation

$$w \sim 1 \mp \beta (a_+^2 - a_-^2). \tag{9.2}$$

We have set the scalar product **el** equal to unity, choosing the axis of quantization in the direction of emission of the electron. For $I \to I$ transitions we have also to include the interference effect, which according to Eqs. (8.18) and (8.20) is proportional to

$$\left.\begin{array}{l} -\operatorname{Re} g_S g_T^* \\ -\operatorname{Re} g_V g_A^* \end{array}\right\} \times \beta a_0. \tag{9.3}$$

The coefficients of $a_+^2 - a_-^2$ and a_0 determine the relative probabilities of transitions into a final state with a prescribed value of M from various initial states. In order for the total transition probability to be normalized to unity, as in Eq. (9.2), it is necessary to have $a_+^2 + a_-^2 + a_0^2 = 1$ for a specified final M. This corresponds to the usual normalization of the Clebsch-Gordan coefficients. By means of the table on page 23 we can write Eq. (9.2) in the form

$$w \sim 1 \mp \beta B(M), \tag{9.4}$$

where

$$B(M) = \begin{cases} \dfrac{M}{I+1} & \text{for the transition } I \to I+1, \\ \dfrac{M}{I(I+1)} & \text{for the transition } I \to I, \\ -\dfrac{M}{I} & \text{for the transition } I \to I-1. \end{cases} \tag{9.5}$$

The polarization of the nucleus in the final state is equal to the average value of M divided by the spin of the nucleus in the final state:

$$P' = \frac{1}{I'(2I'+1)} \sum_{M=-I'}^{I} B(M)\, M. \tag{9.6}$$

The first term in Eq. (9.4) makes no contribution to (9.6). As the result we get for the polarization in $I \to I \pm 1$ transitions, for types T and A:

$$P'(T) = \left\{\begin{array}{ll} -\dfrac{\beta}{3}\dfrac{I+2}{I+1} & \text{for the transition } I \to I+1, \\ +\dfrac{\beta}{3} & \text{for the transition } I \to I-1, \end{array}\right\} \tag{9.7}$$

$$P'(A) = \left\{\begin{array}{ll} \dfrac{\beta}{3}\dfrac{I+2}{I+1} & \text{for the transition } I \to I+1, \\ -\dfrac{\beta}{3} & \text{for the transition } I \to I-1. \end{array}\right\} \tag{9.8}$$

For the $I \to I$ transition we also need the value of a_0:

*An interesting case is the decay of C^{17}. The N^{17} nucleus resulting from the decay emits a polarized neutron, whose polarization can, in principle, be measured.[B65]

†According to the optical terminology this is a quantum with angular-momentum component along the wave vector m = 1, corresponding to a right-handed screw.

$$a_0 = \frac{M}{\sqrt{I(I+1)}} \text{ for the transition } I \to I. \quad (9.9)$$

Introducing the nuclear matrix elements, we get for the types (T, S) and (V, A):

$$P'(T, S) = -\frac{\beta}{3}\frac{\frac{1}{I}+2\sqrt{\frac{I}{I+1}}\operatorname{Re}A}{1+|A|^2}, \quad A = \frac{g_S\langle 1\rangle}{g_T\langle\sigma\rangle}, \quad (9.10)$$

$$P'(V, A) = +\frac{\beta}{3}\frac{\frac{1}{I}-2\sqrt{\frac{I}{I+1}}\operatorname{Re}A}{1+|A|^2}, \quad A = \frac{g_V\langle 1\rangle}{g_A\langle\sigma\rangle}. \quad (9.11)$$

It may be pointed out that the signs of the terms in Re A in the formulas for P′ are opposite to the signs of these terms in the formulas for the Wu effect, Eqs. (8.21) — (8.26). Therefore the two effects are not proportional to each other, and measurements of both on the same nucleus can be used to get additional information about the nucleus (about the spin or the value of A). Just as before, the results for the positron decay are obtained by the changes

$$\beta \to -\beta \quad A \to -A. \quad (9.12)$$

It is not hard to get also the general formula for an arbitrary combination of the four types. We shall not do this here, however. Remarks on this point are given after Eq. (8.26).

One can study in the same way more complicated effects, which involve all five of the vectors belonging to the system. The formulas one gets are, however, rather cumbersome, and no physically new results apppear (at least for allowed transitions). Therefore we shall only deal briefly with the qualitative pattern of one of these effects — the e-γ correlation in polarized nuclei. This effect is interesting because it provides a possibility of checking the conservation of the combined parity (B30, B32). The angular distribution for the electron momentum contains a term depending on the direction **q** of emission of the γ-ray quantum and the vector **l** that defines the direction of polarization of the nucleus.

We are interested in a relation of the form

$$\mathbf{e} \sim \mathbf{l} \times \mathbf{q}, \quad (9.13)$$

which is noninvariant with respect to the approximate time reflection (the left side changes sign, the right side does not). Since, however, the γ-ray quantum conserves parity, **q** must appear to an even power.* Therefore the lowest power of **l** that can appear in the term we require is the second. Accordingly Eq. (9.13) must contain a scalar coefficient proportional to an odd power of **q**, i.e., an odd power of the scalar product **lq**.

Thus the connection between **e**, **l**, and **q** will be given (along with terms that conserve the time parity) by terms of the type

$$\mathbf{e} \sim \mathbf{l} \times \mathbf{q}\,(\mathbf{lq})^{2n+1}. \quad (9.14)$$

The power of **q** is associated with the multipole radiation. Obviously the effect consists of an asymmetry of the electron distribution relative to the plane defined by the axis of polarization of the nucleus and the direction of emission of the quantum.*

As with the other effects, such an asymmetry occurs even without time-parity nonconservation, on account of the Coulomb interaction. Up to now only one experiment has been done [cf. Sec. 11 (i)], with polarized Co^{58}; the results are not in contradiction with the conservation of the combined parity.

10. THE V-A INTERACTION

The experimental data are now coming into better and better agreement with the pattern given by an interaction consisting of a linear combination of V and A. Marshak and Sudarshan (B41) and Gell-Mann and Feynman (B40) have called attention to the interesting properties of this interaction and have pointed out its attractiveness from the theoretical point of view. The starting point of these arguments was the fact that if one writes the Hamiltonian for the μ-meson decay, $\mu^- \to e^- + \nu + \bar{\nu}$ in the same form as for the β decay of the neutron with types V and A and sets $g_V = -g_A = g_\mu$:

$$H_\mu = 2^{-1/2} g_\mu [\bar{\nu}\gamma_\alpha(1+\gamma_5)\mu][\bar{e}\gamma_\alpha(1+\gamma_5)\nu^-], \quad (10.1)$$

then this Hamiltonian correctly describes the spectrum of the electrons, and the constant g_μ determined from the lifetime of the μ meson agrees to within experimental errors (1 to 2 percent) with the vector β-decay constant.

This way of writing the Hamiltonian presupposes that all fermions enter into the weak interactions as two-component particles with definite chirality; only the strong interactions can change the chirality of particles — convert them into four-component particles.

It is obvious, furthermore, that such a Hamiltonian conserves the combined parity (the phase difference between g_V and g_A is $\pi/2$).

These arguments have served as the basis for

*This corresponds to the fact already mentioned, that an asymmetry of the angular distribution of the quanta is caused only by even polarizations of the final nucleus.

***l** and **q** must not be perpendicular; if they are, according to Eq. (9.14) the effect vanishes.

the hypothesis that for the β decay also one must write this same type of interaction, with equal constants g_A and g_V. Actually, however, β decay is described not by the Hamiltonian (10.1), but by

$$H_\beta = 2^{-1/2} g\,[\bar{P}\gamma_\alpha (1+\Lambda\gamma_5) N]\,[\bar{e}\gamma_\alpha (1+\gamma_5)\nu] \qquad (10.2)$$

with Λ equal not to unity, but to about 1.2 (cf. Sec. 11). It is natural to explain the difference between the pseudovector constants in the μ decay and the β decay by saying that owing to the presence of the strong interaction of the nucleons with the vacuum (π mesons, K mesons) there is a renormalization of the decay constants and the nucleons cease to have a definite chirality. The natural question then arises: why does the strong interaction lead to renormalization of only the pseudovector constant, while the vector constant remains unchanged? A possible answer to this lies in an analogy with the electromagnetic interaction. [Cf. S. S. Gershteĭn and Ya. B. Zel'dovich, J. Exptl. Theoret. Phys. (U.S.S.R.) **29**, 698 (1955), Soviet Phys. JETP **2**, 576 (1956)].

The electric charges of all particles — strongly interacting and weakly interacting — are the same, despite the polarization of the vacuum. In this case the constancy of the charge is guaranteed, as is well known, by the conservation of electric charge. Evidently also in the case of the vector interaction there must exist a conserved quantity.

In order to see what form such a conservation law can have, let us compare the electromagnetic interaction of a system of particles with the vector β-decay interaction.

If we introduce the matrices τ_+, τ_-, and τ_0 for the isotopic spin of the nucleon,* the electromagnetic interaction of the nucleon can be written in the form

$$H_{el} = e j_\alpha A_\alpha. \qquad (10.3)$$

The four-vector current j_α can be written in the form (n is the wave function)

$$j_\alpha = \frac{1}{2}\bar{n}\gamma_\alpha\,(1+\tau_0)\,n + j_\alpha\ \text{(meson)} \qquad (10.4)$$

where the second term is due to the mesons surrounding the nucleon (the meson cloud). The factor $\frac{1}{2}(1+\tau_0)$ vanishes for the "bare" neutron. The current j_α can be written in the form of two terms: an isotopic scalar

$$j_\alpha^{(0)} = \frac{1}{2}\bar{n}\gamma_\alpha n \qquad (10.5)$$

and the third component of an isotopic vector

$$j_\alpha^{(3)} = \frac{1}{2}\bar{n}\gamma_\alpha\tau_0 n + j_\alpha\ \text{(meson)} \qquad (10.6)$$

*τ_0 multiplies the proton by + 1 and the neutron by − 1, τ_+ turns neutron into proton, and τ_- turns proton into neutron.

For each of these currents one can write a conservation law:

$$\frac{\partial j_\alpha^{(0)}}{\partial x_\alpha} = \frac{\partial j_\alpha^{(3)}}{\partial x_\alpha} = 0. \qquad (10.7)$$

The vector interaction for β decay can be written in a form like Eq. (10.3):

$$H_V = \sqrt{2}\,g k_\alpha B_\alpha, \qquad (10.8)$$

where*

$$k_\alpha = \frac{1}{2}\bar{n}\gamma_\alpha\tau_+ n \qquad (10.9)$$

and

$$B_\alpha = \bar{e}\gamma_\alpha(1+\gamma_5)\,\nu. \qquad (10.10)$$

We see that B_α plays a role analogous to that of the electromagnetic potential, and k_α a role analogous to that of the isotopic vector current (10.6).

If we assume that for a real nucleon the interaction must contain not the expression (10.9) but the first component of the isotopic vector whose third component is the expression (10.6), then in virtue of the isotopic invariance of the strong interaction it follows from the conservation of the current [Eq. (10.7)] that the current (10.9) is also conserved (apart from radiative corrections):

$$\frac{\partial k_\alpha}{\partial x_\alpha} = 0. \qquad (10.11)$$

Thus a current arises that is at the same time a vector in the isotopic space

$$\mathbf{J} = \frac{1}{2}\bar{n}\gamma\boldsymbol{\tau}n + \mathbf{j}\ \text{(meson)} \qquad (10.12)$$

For the nucleon surrounded by the π-meson cloud

$$\mathbf{J} = \frac{1}{2}\bar{n}_0\gamma\boldsymbol{\tau}n_0 + \boldsymbol{\pi}\times\frac{\partial\boldsymbol{\pi}}{\partial t}, \qquad (10.13)$$

where $\boldsymbol{\pi}$ is the wave function of the π meson (a vector in the isotopic space). Possibly we should include in the current (10.13) a contribution from the K mesons and perhaps some other interactions (cf. discussion in reference B63).

A paper by Gell-Mann (B43) discusses possible methods for testing experimentally the nature of the vector interaction (cf. references B54, B55, B62).

The basic idea of such a test is that certain nuclear matrix elements that affect the properties of β decay have a form analogus to the matrix elements of the electromagnetic interaction. Then it follows from the analogy between the vector interaction in β decay and the electromagnetic interaction that these elements must be identical. This fact should be confirmed by experiment.

*The coefficient $2^{1/2}$ comes from the normalization $\tau_\pm = 2^{-1/2}(\tau_x \pm i\tau_y)$.

Gell-Mann considers the matrix elements that give the first-order corrections that appear in the formulas if one does not replace the neutrino wave function by unity, but keeps the term of first order in the coordinate (ikr).

In this case it can be shown that the corrections to the shape of the spectrum and the β-ν correlation will be determined by a nuclear matrix element analogous to the nuclear matrix element that determines a magnetic dipole transition. (Gell-Mann calls this phenomenon weak magnetism.)

In the case of the β^- transition $B^{12} \rightarrow C^{12}$ and the β^+ transition $N^{12} \rightarrow C^{12}$ this matrix element can be evaluated from the γ-ray transition from the isotopically similar level $C^{12*} \rightarrow C^{12}$.

In this way a value of the order of 20 percent is obtained for the correction to the spectrum, which can be measured.

Another effect of a similar kind can be obtained by considering the correlation between the β ray and the γ-ray polarization in allowed transitions, in which it can occur only owing to the effect of the change of the wave function over the volume of the nucleus. The value of this effect also can be estimated from isotopically similar electromagnetic transitions.

Preliminary experimental results are not in contradiction with such estimates (cf. references G16, G18).

The theory of the V-A interaction also makes it possible to make a number of predictions about weak-interaction decays of other particles, in particular hyperons; in these cases also there are evidently no contradictions with existing experimental information. These problems, however, are beyond the scope of this survey.

The idea of the universal interaction leads to the possibility in principle of two new effects (in first order in g^2) — the scattering of neutrinos by electrons and parity nonconservation in the scattering of neutrons by protons.

Unfortunately, the accuracy of experiments is at present insufficient for the detection of these effects (cf. the report by Roberts (B59) and the work on measuring ionization losses of neutrinos (D7)).

11. THE EXPERIMENTAL DATA ON THE FORM OF THE β-DECAY INTERACTION

This section contains a brief survey of the main data bearing on the choice of the form of the β-decay interaction Hamiltonian.

(a) The Absence of Interference Terms

The data on β decays with $I = 0 \rightarrow I = 0$ (without change of parity) make possible an estimate of the possible size of the Fermi interference terms (S, V). Such an analysis was carried out by Gerhart (C4) (cf. reference C19). From the spectra of O^{14}, A^{26*}, and Cl^{34} he found for the relative magnitude of the interference term

$$b_F = \frac{\mathrm{Re}\,(C_S C_V^* + C_S' C_V'^*)}{|C_S|^2 + |C_{S'}|^2 + |C_V|^2 + |C_V'|^2} = 0.00 \pm 0.12. \quad (11.1)$$

The best estimate of the interference between T and A has been made by Sherr and Miller (C5) from the ratio of the probabilities of K capture and positron decay of Na^{22} (the interference terms occur in different ways in the two processes). Their result is

$$b_{GT} = -0.01 \pm 0.02. \quad (11.2)$$

Several other papers (C6 — 9) give various values of b_{GT} in the range $0.015 \leq b_{GT} \leq 0.093$. If the imaginary part corresponding to Eq. (11.1) is also zero (conservation of combined parity), it then follows that the choice is between the combinations (ST) and (VA).

(b) The e-ν Correlation

The choice between the combinations (ST) and (VA) can be made on the basis of measurements of the electron-neutrino correlation.

The old experiments on the decay of He^6 (C10), on the basis of which the conclusion favoring the tensor interaction was drawn, have now been refuted. The experiments of Allen and others (C11) on the decay $Cl^{34} \rightarrow A^{34}$ agree only with the combination (V, A). This choice is also not contradicted by the data on the decay of Ne^{19} (C12 — 13), which agree with both possibilities, and on Na^{24} and Ne^{23} (C14a).

The exact values of the correlation coefficients depend on the nuclear matrix elements, and we shall not discuss them here. The results of experiments with He^6 (C16) and Li^8 (C17, 20) agree with the combination (V, A).

(c) The Magnitudes of the Decay Constants

If there are no interference terms, then independently of the choice between the combinations (ST) or (VA) the probability of β decay is determined by two constants g_F and g_{GT} or the two constants A and R that appear in the formula for $ft_{1/2}$ (cf. Sec. 6)

$$ft_{1/2} = \frac{A}{\langle 1\rangle^2 + R\langle\sigma\rangle^2}. \tag{11.3}$$

The constant A is determined from the decay of O^{14} ($0 \rightarrow 0$ transition) (C16)

$$A = 6550 \pm 150 \text{ sec}; \tag{11.4}$$

in the latest paper (C4) a somewhat smaller value*

$$A = 6200 \pm 120 \text{ sec} \tag{11.4'}$$

is obtained from an analysis of the data on the decay of three mirror nuclei O^{14}, Al^{26*}, and Cl^{34}. The decay of the neutron has been most carefully studied by Sosnovskiĭ, Spivak, Prokof'ev, Kutikov, and Dobrynin (C3). Their result is

$$t_{1/2} = 11.7 \pm 0.3 \text{ sec}. \tag{11.5}$$

This gives the result

$$ft_{1/2} = 1180 \pm 40 \text{ sec}. \tag{11.6}$$

From this and Eq. (11.4′) we get

$$R = 1.52 \pm 0.08. \tag{11.7}$$

If we adopt the value (11.4), then

$$R = 1.42 \pm 0.08.$$

The interaction constants are found to have the values (C18)

$$\left.\begin{aligned} g_F &= 1.400 \pm 0.009 \text{ erg cm}^3 \cong 2.9\cdot 10^{-12}(mc^2)\left(\frac{\hbar}{mc}\right)^2, \\ g_{GT} &= 1.7 \pm 0.05 \text{ erg cm}^3 \cong 3.5\cdot 10^{-12}(mc^2)\left(\frac{\hbar}{mc}\right)^2. \end{aligned}\right\} \tag{11.8}$$

(d) Conservation of the Neutrino Charge

It has been shown experimentally that different neutral particles are emitted in β^- and β^+ decays. This follows from the absence of the reaction† $\tilde{\nu} + Cl^{37} \rightarrow A^{37} + e^-$ (D1) (an experiment suggested by Pontecorvo) under bombardment by the antineutrinos coming from a pile. Furthermore the reaction inverse to neutron decay, $\tilde{\nu} + p \rightarrow n + e^+$, has definitely been established (D2, D11).

The same conclusion follows from the absence of the neutrinoless double β decay $Ca^{48} \rightarrow Ti^{48} + e^- + e^-$ (see reference A4).

(e) The Chirality of the Neutrino

The chirality of the neutrino has been determined by the direct experiments of Goldhaber, Grodzins, and Sunyar (D5), on the polarization* of the nucleus produced by K capture from the nucleus Eu^{152m}. Since in the K-capture process the polarization of the recoil nucleus is the same as that of the emitted neutrino, this is the most direct experiment for measuring the chirality of the neutrino. Their experiments showed that the chirality of the neutrino is -1.

The two-component property of the neutrino can be verified from the ratio between the decay of the neutron and the inverse reaction $\tilde{\nu} + p \rightarrow n + e^+$. If the antineutrino is completely polarized (a two-component particle) the ratio of the probabilities

$$\frac{w(\tilde{\nu} + p \rightarrow n + e^+)}{w(n \rightarrow p + e^- + \tilde{\nu})}$$

will be twice as large as for a four-component neutrino (for the same energies of the particles). Formally this is due to the fact that the statistical weight for the decay of the neutron has been reduced by a factor two (there is no summation over the spins of the antineutrino). Then from the ratio between the direct and inverse processes

$$\frac{w(1 \rightarrow 2)}{\rho_2} = \frac{w(2 \rightarrow 1)}{\rho_1} \tag{11.9}$$

it follows that the probability of the inverse process is increased by a factor two.

Such experiments have been performed by Reines and Cowan (A6, D11). The experimental value of the cross-section, referred to a single neutrino in the flux from the reactor was found to be

$$\sigma = 11 \pm 4\cdot 10^{-44}\ \text{cm}^2/\nu,$$

or referred to a single act of fission, on the assumption that each fission in the reactor produces 6.1 neutrinos,

$$\sigma = 67 \pm 24\cdot 10^{-44}\ \text{cm}^2/\text{fission}.$$

The main difficulty in the theoretical treatment of the data lies in the determination of the energy spectrum of the neutrinos. A very careful determination was made by Carter and others (D12). (Cf. also reference D6). In the same paper the theoretical value of the cross-section for the longitudinal neutrino was found on the basis of these measurements to be

$$\sigma = 60 \pm 10\cdot 10^{-44}\ \text{cm}^2/\text{fission},$$

which is in good agreement with experiment. It may be noted that the number of neutrinos with

*The decrease of the value of A is associated with a new value of the limit of the β-ray spectrum of O^{14} (D. A. Bromley, unpublished work cited in reference C4).

†Data on the existence of this reaction that appeared in 1957 have been found to be incorrect.

*The polarization of the nucleus was measured through the polarization of the subsequent γ ray.

enough energy for the reaction (threshold 1.804 Mev) was found to be $2.0 \pm 0.2\ \nu$/fission. This gives for the cross-section per neutrino with energy greater than 1.8 Mev the value $(31 \pm 4) \times 10^{-44}\ \text{cm}^2/\nu$.

Besides the data on the properties of the neutrino that have been mentioned we may add references to estimates of the upper limit on the mass of the neutrino ($m_\nu < 1/500$ electron mass (D9, D10); cf. reference A7) and of the upper limit on its magnetic moment ($\mu_\nu < 10^{-9}$ electron Bohr magneton (D7)).

(f) The Chirality of the Electrons

If the electron has a definite chirality, then in all allowed transitions and in many forbidden transitions it must be longitudinally polarized and the value of the polarization must be $-\beta(-v/c)$. The corresponding polarization for positrons must be $+\beta$. Although there have been papers in which other values of the polarization were found, they were later shown to be incorrect, and at present all existing experiments confirm these values of the polarization for electrons and positrons. The errors in all the experiments are, however, comparatively large (say 15 to 20 percent), and it is of great importance that more accurate results be obtained. Since no new information besides the chirality can be obtained from the existing data, we shall not present the numerical data here, and refer the reader to the original papers (Section E of the list of literature).

(g) The Decay of Polarized Nuclei

Information about the chirality of the electron is also given by measurements of the angular distribution of the electrons from the decay of polarized nuclei — the Wu effect. These experiments have been performed with three cobalt isotopes: Co^{60}, Co^{58}, and Co^{56}. The transitions in Co^{58} and Co^{56} are of the type $I \rightarrow I$, so that the amount of polarization of the electrons from these transitions must depend on the interference of the V and A terms — Eqs. (8.21) — (8.23). For Co^{58} $(I = 2)$ the value of the coefficient B in Eq. (8.23) has been found to be about $1/3$, which can agree with theory only if it is assumed that there is no interference term. (In this case $B = 1/(I+1)$.) A similar result is found for Co^{56} $(I = 4)$. The experimental value is $B = 0.222 \pm 0.021$, which is again in good agreement with the theoretical value, $B = 1/5$.

For the transition $I \rightarrow I-1$ — the case of Co^{60} $(I = 5)$ — experiment also confirms the theoretical value, $B = 1$.

The absence of the interference terms gave rise to the idea of nonconservation of time parity. In this case, assuming that $g_V = ig_A$, we could explain the absence of interference.

The actual situation, however, turned out to be simpler. Measurement of the ratio of matrix elements $\langle 1 \rangle / \langle \sigma \rangle$ from the angular distribution of the γ rays from polarized cobalt has shown that this ratio is extremely small, and it is this that is the cause of the observed effect. For the square of the ratio of these elements in the case of the Co^{58} decay experiment gave the value -0.003 ± 0.005 (F5) instead of the previously assumed value 0.12.

The small value of $\langle 1 \rangle$ is explained by the great difference between the structures of the initial and final nuclei in the decays of Co^{56} and Co^{58}, since in these nuclei the neutrons and protons are in different shells. Thus on the shell model and the assumption of isotopic invariance the matrix element $\langle 1 \rangle$ should be equal to zero.

The study of this effect for polarized neutrons is of the greatest interest. Although the first experiments (F6) were in evident contradiction with other data on the chirality of the electron, later improvements eliminated the disagreement.

It follows from Eqs. (8.31) and (8.32) that for real λ the correlation coefficients for the electron and neutrino should be equal; for $|\lambda| = 1.2$ we have: for the electron -1.00 $(\lambda > 0)$ or -0.09 $(\lambda < 0)$, and for the antineutrino $+0.09$ $(\lambda > 0)$ or $+1.00$ $(\lambda < 0)$. Experiment (F7) gave for these two quantities the values -0.11 ± 0.02 (electron) and 0.88 ± 0.15 (antineutrino)* which agrees with the (V, A') interaction and real negative λ.

If the real character of λ (the conservation of the combined parity) is confirmed by experiment, then all the constants for β decay are known.

If we go back to the four-dimensional way of writing the Hamiltonian, we get for the (V, A) interaction

$$H = 2^{1/2} g_V (\bar{P}\gamma_i N)[\bar{e}\gamma_i(1+\gamma_5)\nu] - g_A[\bar{P}(\gamma_i\gamma_5)N][\bar{e}\gamma_i(1+\gamma_5)\nu] + \text{Herm. adj.} \quad (11.10)$$

With the notation $g_A/g_V = \Lambda$ $(\Lambda > 0)$, we have the Hamiltonian

$$H = 2^{1/2} g_V [\bar{P}\gamma_i(1+\Lambda\gamma_5)N][\bar{e}\gamma_i(1+\gamma_5)\nu] + \text{Herm. adj.} \quad (11.11)$$

introduced in the theory of Feynman and Gell-Mann (Eq. (10.2)).

*Values given at the Geneva conference in July 1958. These values are somewhat different from those given in reference F7.

(h) The Correlation between the Electron and the Polarization of the γ Ray

In principle the e-γ-polarization correlation gives the same information as the Wu effect. For allowed transitions (the only ones we shall consider) the same picture is obtained. For Co^{60} ($5 \rightarrow 4$ transition) (G6) experiment gives -0.41 ± 0.08 as the value of the asymmetry coefficient. The theoretical value, from Eq. (9.2), is 0.33. The most complete paper (G5) deals with a number of ($I \rightarrow I$) transitions. For the nuclei Na^{24} ($4^+ \rightarrow 4^+\beta^-$) and Co^{58} ($2^+ \rightarrow 2^+\beta^+$) there is agreement between experiment and theory if we assume that the interference term is small. In this case the theoretical asymmetry coefficient, by Eq. (9.1), is $\pm 1/3$. Experiment gives for Na^{24} $+0.07 \pm 0.04$ and for Co^{58} -0.14 ± 0.07. For both Na^{24} and Co^{58} the small value of the interference effect is due to the smallness of the matrix element $\langle 1 \rangle$. It is interesting that for the nuclei Sc^{44}, Sc^{46}, and V^{48} there is considerable interference effect. It is estimated (G5) that in these decays the ratio of matrix elements $\langle 1 \rangle / \langle \sigma \rangle$ is ~ 0.45 for Sc^{46}, $\sim 1/5$ for Sc^{44}, and $\sim 1/5$ for V^{48}. This fact is in agreement with other data which indicate that after the nucleus Ca^{40} the $f_{5/2}$ shell is filled up in a regular way, so that the decays of all three of these nuclei occur without change of the number of nucleons in the shell, which means that the matrix element $\langle 1 \rangle$ is rather large. Considerable interference effects are also observed in the decays of Mn^{52} (G6, G7) and Zr^{95} (G8).

(i) The Conservation of the Combined Parity

In principle the conservation of the combined parity has perhaps been verified in two ways. One way consists of measurements of energy-dependences of the effects. When interference terms are present this makes it possible to determine the imaginary or real parts of products, expressions such as $\mathrm{Re}\, g_V g_A^* \langle 1 \rangle \cdot \langle \sigma \rangle$. Since the nuclear matrix elements are real, one can thus find the relative phase of g_V and g_A. Up to now the accuracy of the measurements is still insufficient for such an analysis. The second method is associated with the asymmetry of emission that appears in the Born approximation when there is violation of symmetry with respect to the time. These effects are always masked by the Coulomb interaction, and to study them one must use light nuclei and fast electrons, unlike the conditions of the former method, which is obviously more sensitive for low-energy electrons and large nuclear charges. The first experiment made was that with polarized Co^{58} (F4). In this experiment measurements were made of the asymmetry of the emission of the electron relative to the plane defined by the direction $\mathbf{l}$ of polarization of the nucleus and the direction $\mathbf{q}$ of emission of the γ-ray quantum (the angle between $\mathbf{l}$ and $\mathbf{q}$ was 37°, cf. Sec. 9). The observed asymmetry did not exceed the possible effect of the Coulomb interaction.

In another experiment, with polarized Mn^{52} (G7), the result was also not in contradiction with conservation of the combined parity.

As we have already stated (Sec. 4) the conservation of the combined parity is incompatible with the existence of electric dipole moments of elementary particles.

There have been attempts to detect dipole moments of neutrons and of μ mesons (H3); the results were negative: the values of the electric dipole moment (in units $e\hbar/2mc$) were found to be $< 2.5 \times 10^{-9}$ for the neutron and $< 10^{-2}$ for the μ meson.

The experiment cleanest in principle was that of Clark and others (F8), who measured the e-ν correlation for polarized neutrons (the pseudovector effect, cf. Sec. 8). In this experiment the authors did not detect any appreciable pseudovector effect, and thus also did not find nonconservation of the combined parity. But the accuracy of these experiments is also still very low. This effect has been studied with higher accuracy by Burgy, Krohn, Novey, and others (F9), who found that the ratio of the constants has a phase differing from π by not more than 8°.

An interesting possibility for checking the conservation of the combined parity is provided by a study of the spectrum and polarization of RaE (Alikhanov and others (E2)). A theoretical analysis (B64) shows that the parity nonconservation does not exceed 10 percent. A study of other effects in RaE would be of much interest.

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E5. Goldhaber, Grodzins, and Sunyar, Phys. Rev. 106, 826 (1957) (Y^{90}).
E6. S. S. Hanna and R. S. Preston, Phys. Rev. 108, 160 (1957) (N^{13}, Cu^{64}).
E7. Benczer-Koller, Schwarzschild, Vise, and Wu, Phys. Rev. 109, 85 (1958) (Au^{198}, Y^{90}).
E8. Lipkin, Cuperman, Rothem, and de-Shalit, Phys. Rev. 109, 223 (1958) (Au^{198}).
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E20. Mann, Miskel, and Bloom, Phys. Rev. Lett. **1**, 34 (1958) (same as E19).

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Translated by W. H. Furry