Personalia

VLADIMIR ALEKSANDROVICH FOCK ON HIS SIXTIETH BIRTHDAY

M. G. VESELOV

Uspekhi Fiz. Nauk **66**, 695-699 (December, 1958)

DECEMBER 22, 1958, will be the sixtieth birthday of one of our greatest theoretical physicists, academician Vladimir Aleksandrovich Fock. V. A. Fock was born in St. Petersburg, into the family of a surveying engineer, A. A. Fock, well-known among specialists for his work in the science of forestry. After he finished high-school in Petrograd in 1916, Fock entered the mathematical physics faculty of Petrograd University, but then volunteered for the artillery school and after a few short courses there was sent to the front. In 1918 he was demobilized, returned to Petrograd, and resumed his studies at the university.

At that time the state institute of optics had just been organized in Petrograd, and Fock with a group of the best students in the physics faculty was invited by D. S. Rozhdestvenskiĭ to serve as "laborant" at the institute. Among the "laborants," who later formed the nucleus of the scientific division of the optics institute, were many names which are now famous: A. N. Terenin, E. F. Gross, S. É. Frish and others. The exceptional abilities of Fock in theoretical physics and mathematics became obvious during his student years, and in 1922 he had already completed two research papers, one in quantum theory and the other in applied mathematics.[2] These two fields, quantum theory and applied mathematics in the wide sense of the term, together with the theory of relativity, remained Fock's chief scientific interests.

The twenties were years of vigorous post-revolutionary development of research in physics in the newly founded physics institutes. At the beginning of the twenties the theoretical physicists in the Soviet Union could be counted with the fingers, and Fock took an active part in the work of several institutes. In 1924-36 he was working in the theoretical division of the physico-technical institute. In 1928-41 he was director of theoretical research at the state institute of optics. Fock's scientific and teaching activities continued uninterruptedly at Lenigrad university, where he was successively student, aspirant, lecturer and professor. In recent years he has held the chair of theoretical physics. By his own research and by his personal guidance he has played a great part in the development of theoretical physics in the Soviet Union and especially in Leningrad.

Fock's research was strongly influenced by his close relations with the applied mathematicians of the famous St. Petersburg school. In the first years after he finished his studies (1924-26) he was mainly occupied in solving mathematically difficult problems of applied physics. The varied and practical character of these investigations can be seen from a list of some of the titles: calculation of radiation from a surface of arbitrary form, theory of the mixing of optical glass during heating, solution of a diffusion problem by finite differences with applications to the diffusion of light, calculation of the thermal resistance of a cable with multiple core, etc.

In these and later papers in mathematical physics Fock showed his characteristic physical intuition, which allowed him to discard the inessential features of a physical phenomenon and construct mathematical machinery appropriate to its understanding. He showed also his mastery of mathematical manipulations, which allowed him to carry through the solution of the problem and reduce the results to explicit formulae and tables.

Fock's enthusiasm and skill in the creation of general methods and theories found expression also in his solutions of concrete problems of practical physics. For example, in his calculations of the illumination problem,[3] Fock introduced the new concept of an illumination vector, and built upon it a theory of the vector illumination field, which was developed further by other scientists and has been largely instrumental in establishing the high level of lighting technology in the Soviet Union. A second example is Fock's paper in the theory of elasticity,[3] in which he reduced a plane elasticity problem to a Fredholm integral equation. The solution of this problem, which was obtained seven years later by N. I. Muskhelishvili, has been widely exploited by the Georgian school.

Fock's mastery of mathematical technique may be seen particularly clearly in his study of the conformal representation of a zero-angle quadrilateral upon a half-plane.[6] The solution of this problem is needed in calculating the distribution of heat and of electric field in a cable, and was considered by many mathematicians to be practically impossible. Fock obtained it in an analytic form suitable for computation.

These first investigations of Fock in the field of applied mathematics made him well-known among Soviet physicists and mathematicians.

We cannot here mention all the mathematical papers of Fock. His work on integral equations,[2,28] together with G. A. Grinberg, has led to interesting and important results in the theory of the scattering of light and in the theory of shore effect.[32]

After 1926, Fock's main interest was in quantum theory, and until 1941 he turned to problems of applied mathematics only sporadically, stimulated by practical questions which arose from time to time. In 1927 he worked out a thermal theory of electrical break-down in dielectrics.[7] In 1930 he gave an elegant solution to the problem of the skin-effect in a curved conductor.[11] Fock published several papers concerned with electrical methods of ore-prospecting, and in 1935 published a monograph on the theory of electrical core-sampling.[17] In 1935 he solved a difficult problem of internal ballistics, the motion of the gases in the barrel of a gun as it is discharged. In 1940 Fock solved an acoustical problem in the theory of resonant absorption.[27]

Fock's first papers in quantum mechanics appeared in 1926, immediately after the discovery of the wave equation by Schrödinger. His first paper[4] generalized Schrödinger's equation to the case of a magnetic field and deduced a formula for the normal Zeeman effect. In the same paper he applied perturbation theory to obtain the splitting of the levels of the hydrogen atom in an electric field. His second paper[5] generalized the wave equation to the case of a relativistic charged particle in an electromagnetic field, and verified the gauge-invariance of the equation. These papers appeared at the beginning of the creation of wave mechanics, and played an important part in its development. In 1927-1928 Fock lived abroad in Göttingen and Paris. There he completed several major researches in quantum mechanics,[8,9] and became one of the leading theoretical physicists in the world.

In 1929 he published a series of papers, partly in collaboration with D. D. Ivanenko, interpreting geometrically the Dirac equation. He established the peculiar geometrical properties of the Dirac wave-field, and used the notion of parallel displacement of a spinor to generalize the Dirac equation to Riemannian geometry.

Fock's book[13] "Principles of Quantum Mechanics," the first Soviet book on quantum mechanics, was published in 1932. It contains discussions of many problems using Fock's original methods.

Fock devoted a long series of investigations to the theory of many-electron systems, and to the creation of approximate methods in this field. This work began in 1930 with the invention of the method of the self-consistent field with exchange.[12] The basic idea of this method is to apply a variation principle to obtain equations for the one-electron wave functions. In this way he justified the Hartree equations, which had been derived earlier from intuitive considerations, but he found that a correct treatment of the Pauli principle led to a system of equations which differed from the Hartree equations by the presence of additional exchange terms. Fock with his collaborators[19] applied the Hartree-Fock method to calculations of atomic structure. The method was immediately and widely used in the theory of many-electron systems. In a series of papers, Fock studied the effects of the exchange terms, and explained various spectroscopic phenomena, the inversion of doublets, violation of sum-rules for oscillator strengths, and some anomalies in the intensities of series of lines. In the same group of papers he published a semi-classical method for calculating wave-functions[20] and a generalization of the Fock method to situations in which the one-electron approximation can be partially avoided.[25]

In 1940 Fock studied the problem of constructing many-electron wave-functions not containing spin-variables, and he was the first to formulate the cyclic symmetry properties of these wave-functions.[26] In 1935 he published an exquisite paper on the hydrogen atom,[22] in which he showed that the degeneracy of the levels of an electron in a Coulomb field is connected with a symmetry of the field with respect to a group rotations in four dimensions. We may mention also his 1954 paper,[36] in which he obtained an exact solution of the Schrödinger equation for helium-like atoms in the form of a series expansion. This problem has become important in recent years in connection with the verification of the radiative corrections predicted by modern quantum electrodynamics.

In the thirties Fock published his fundamental investigations in quantum field theory. The ideas and methods which he then developed have found an extraordinarily wide application in the post-war development of quantum field theory. His great work of 1932 on second quantization[16] established a method for describing in configuration-space systems with a variable number of particles. This paper was the first to develop clearly and consistently the method of second quantization, and to show its connection with a configuration-space picture. A system with a variable number of particles is described by an infinite set of equations for functions which describe states with a fixed number of particles. This method is now widely used in meson theory, and was generalized by several physicists in the early fifties.

Fock published in 1932, in collaboration with B. Podol'skiĭ, several papers working out a new version of quantum electrodynamics which had been proposed by Dirac in a one-dimensional model. In this version the Coulomb interaction appears as a result of eliminating the longitudinal electromagnetic field. The invariant version of electrodynamics was perfected in a joint paper by Dirac, Fock and Podol'skiĭ. They developed the so-called many-time formalism, in which each particle, as well as the field, has its own time. The many-time theory was further generalized by Tomonaga and Schwinger in 1946-1947, when it became the super-many-time formalism.

Fock's last work in quantum field theory was the creation of a rigorous theory for systems of a variable number of Bose particles, using the so-called method of functionals. Already in 1928,[9] Fock had considered the idea of describing systems with a variable number of particles by means of generating functions, instead of by an infinite set of functions referring to systems with a fixed number of particles. The final form of this idea,[21] with applications to electrodynamics, was worked out in 1934. Fock's method of functionals exhibits the wave-function of the field, or the the state-functional, as the generating function of the probability amplitudes of the field in states with a fixed number of particles.

Fock's original paper, and other papers by his pupils, applied the method successfully to various problems of quantum electrodynamics. The idea of a generating functional has been widely used in quantum field theory in recent years. One may describe the field by a sequence of functions of a different type, for example Feynman amplitudes, instead of probability amplitudes in configuration-space. Each such set of functions has its own generating functional.

Fock's 1937 paper on proper time[23] is a singular and highly original piece of work. He introduced into the Dirac equation a parameter representing the proper-time of the electron, and in this way solved the Cauchy problem for the Dirac equation. The solution of the equation is obtained as a contour integral over the proper-time. Since the proper time is invariant, the proper-time method has been particularly fruitful in problems of quantum field theory where considerations of relativistic invariance play an essential role. It is remarkable that the importance of this method was understood only in the early fifties.

A large part of Fock's scientific work dealt with the theory of relativity. In the twenties he paid his tribute to the prevailing fashion by attempting to construct a unified five-dimensional field theory. But he quickly abandoned this attempt, considering it formalistic and fruitless.

The earliest of his well-known work in relativity theory was a long paper on the motion of extended masses.[24] This paper appeared in 1939, and developed an approximate method for solving the Einstein equations with extended masses of spherical shape, assuming the space to be Euclidean at infinity. He showed that the equations of motion of Newton are obtained as integrability conditions of the Einstein field equations. In this paper he introduced for the first time a harmonic coordinate system, defined uniquely up to a Lorentz transformation, and generalizing the Newtonian inertial system to general relativity.

Afterwards Fock studied various other important problems of gravitation theory: the derivation of equations of motion of bodies taking account of their structure and rotation, the deduction of the ten integrals of the equations of motion, the theory of astronomical aberration, etc. As a result of these investigations, Fock arrived at some general views concerning the physical interpretation of Einstein's theory. He observed that the covariant form of the equations is not related to the idea of relativity. The departure from uniformity of space-time in Riemannian geometry is not a generalization but a specialization of the idea of relativity, since relativity is a consequence of the uniformity of the space-time continuum. Thus the name "general relativity" throws a false light on the theory of gravitation. Fock also emphasized the local character of the equivalence between acceleration and gravitational fields. He recognized the great importance of boundary conditions, and of the introduction of harmonic coordinate systems, in ensuring the uniqueness of solutions.

Fock summarized all these researches in his book "Theory of Space, Time and Gravitation,"[37] which appeared at the end of 1955. His book contains a thorough and original exposition of the special theory of relativity and of gravitation theory. Besides discussing general questions, he also worked out many concrete problems, some of which were published there for the first time.

Fock's work in the theory of diffraction, which occupied him during the war and post-war years, is of importance both theoretically and practically. Here again he worked out powerful mathematical methods, and obtained practical results, which are basic to the whole modern theory of diffraction and its applications. He found an approximate method for calculating the infinite series and integrals which represent the exact solutions of diffraction problems, and hence constructed a rigorous theory of the propagation of radio waves over the surface of the earth, neglecting the effects of the atmosphere.[29] His paper "Propagation of a Plane Wave around the Earth, Including Effects of Diffraction and of Refraction"[33] solved the same problem in an atmosphere with a non-uniform distribution in height. Fock also established the important principle of the locality of the electromagentic field in the penumbral region. He was able to obtain approximate but sufficiently good solutions to the diffraction problem for convex conductors of arbitrary shape,[35] using the well-known conditions of M. A. Leontovich to define the problem. Fock's work in diffraction is receiving greater recognition among scientists and engineers who work in radio-technology, both here and abroad.

This cursory review naturally cannot give an adequate picture of the richness of Fock's scientific work. A large number of highly original and interesting papers cannot even be mentioned. A complete list of Fock's books and papers, including his contributions to general and philosophical discussions of physical theory, contains more than two hundred titles.

Fock's scientific services received manifold recognization. His work in atomic theory was awarded a Mendeleev prize in 1936. His work on the propagation of radio-waves earned him a Stalin prize, first class, in 1946. He received two orders of Lenin and the order of the Workers' Red Banner. In 1932 he was elected a corresponding member, and in 1939 a full member, of the U.S.S.R. Academy of Sciences.

A great quantity of difficult and important research witnesses to Fock's exceptional talent and industry. And this is not all that he did. For many years he directed the theoretical physics faculty of Leningrad university, devoting much time to guiding the work of his staff. For many years he gave courses on quantum mechanics and relativity. Finally, he took an active part in discussions of general problems concerning the physical and philosophical interpretation of the new theories. He many times spoke out against incorrect and unscientific papers on relativity and quantum theory, defending the modern progressive scientific view. His articles and papers on general problems of theoretical physics are always distinguished by the depth and rigor of his arguments, and always contribute to the clarification of the questions and to the correct public understanding of the nature of modern theories. His writings always show his characteristic directness and honesty, those native qualities which, together with his scientific genius, have earned Vladimir Aleksandrovich universal admiration.

Bibliography of Fock's Most Important Scientific Publications

1. Illumination from a Surface of Arbitrary Shape, Труды ГОИ (Trans. State Optics Institute) **3**, No. 28 (1924).
2. A Class of Integral Equations, Math. Z. **21**, 161 (1924).
3. Reduction of a Plane Problem of Elasticity Theory to a Fredholm Integral Equation, Журн. Русск. физ.-хим. об-ва (J. Russ. Phys. Chem. Soc.) **58**, 11 (1926).
4. Schrödinger's Wave Mechanics, Z. Physik **28**, 242 (1926).
5. An Invariant Form of the Wave Equation and the Equations of Motion of a Charged Particle, Z. Physik **29**, 226 (1926).
6. Conformal Mapping of a Zero-Angle Quadrilateral Onto a Half-Plane, Ж. Лен. мат. об-ва (J. Len. Math. Soc.) **1**, 147 (1927).
7. Thermal Theory of Electrical Breakdown,

Тр. Лен. физ.-техн. лаб. (Trans. Leningrad Physico-technical Lab.) No. 5, 52 (1928).
8. The Relations Between the Integrals of Quantum-Mechanical Equations of Motion and the Schrödinger Equation, Z. Physik **49**, 323 (1928).
9. Generalization and Solution of the Statistical Equation of Dirac, Z. Physik **51**, 165 (1928).
10. The Dirac Wave Equation and Riemannian Geometry, Журн. Русск. физ.-хим. об-ва (J. Russ. Phys. Chem. Soc.) **62**, 133 (1930).
11. Skin Effect in a Ring with Circular Cross Section, ibid. **62**, 281 (1930).
12. Approximate Method for Solving the Quantum-Mechanical Many-Body Problem, Z. Physik **61**, 126 (1930).
13. Principles of Quantum Mechanics, Leningrad, Publ. by Commission for Improving the Living Conditions of Students, 251 pp., 1932.
14. Quantization of Electromagnetic Waves and Interaction of Charges in Dirac's Theory, J. Phys. (U.S.S.R.) **1**, 801 (1932), (with B. Podol'skiĭ).
15. Quantum Electrodynamics, J. Phys. (U.S.S.R.) **2**, 468 (1932), (with P. A. Dirac and B. Podol'skiĭ).
16. Configuration Space and Second Quantization, Z. f. Physik **75**, 622 (1932).
17. Theory of Measurement of the Resistance of Strata by Electrical Core Sampling, Moscow-Leningrad, GTTI, (1933).
18. Quantum Exchange Energy, Z. Physik **81**, 195 (1933).
19. Numerical Solution of Generalized Self-Consistent Field Equations, J. Exptl. Theoret. Phys. (U.S.S.R.) **4**, 295 (1934) (with M. I. Petrashen').
20. Approximate Representation of Wave Functions of Penetrating Orbits, Dokl. Akad. Nauk, SSSR, **1**, 241 (1934).
21. Quantum Electrodynamics, J. Phys. (U.S.S.R.) **6**, 425 (1934).
22. The Hydrogen Atom and Non-Euclidean Geometry (preliminary report), Izv. Akad. Nauk SSSR, Div. Techn. Sci., No. 2, 169 (1935).
23. Proper Time in Classical and Quantum Mechanics, Izv. Akad. Nauk, SSSR, Ser. Fiz. No. 4-5, 551 (1937).
24. Motion of Finite Masses in General Relativity, J. Exptl. Theoret. Phys. (U.S.S.R.) **9**, 375 (1939).
25. Incomplete Separation of Variables for Divalent Atoms, J. Exptl. Theoret. Phys. (U.S.S.R.) **10**, 723 (1940), (with M. G. Veselov and M. I. Petrashen').
26. Wave Functions of Many-Electron Systems, J. Exptl. Theoret. Phys. (U.S.S.R.) **10**, 961 (1940).
27. Theoretical Investigation of the Admittance of a Circular Hole in a Barrier Across a Pipe, Dokl. Akad. Nauk SSSR **31**, 875 (1941).
28. Some Integral Equations of Mathematical Physics, Матем. сб. (Math. Collection) **14**, 3 (1944).
29. Дифракция радиоволн вокруг земной поверхности (Diffraction of Radio Waves Around the Surface of the Earth) Moscow-Leningrad, Acad. Sci. Press, 1946.
30. Solution of a Problem in the Propagation of Electromagnetic Waves along the Earth's Surface by the Method of Parabolic Equations, J. Exptl. Theoret. Phys. (U.S.S.R.) **16**, 557 (1946) (with M. A. Leontovich).
31. The Two Interpretations of the Uncertainty Principle for Energy and Time, J. Exptl. Theoret. Phys. (U.S.S.R.) **17**, 93 (1947) (with N. S. Krylov).
32. Theory of Shore Effect in Electromagnetic Waves (with G. A. Grinberg). (Studies in the Propagation of Radio-waves) Vol. 2, Moscow-Leningrad, Acad. Sci. Press, 1948.
33. Propagation of a Plane Wave around the Earth with Diffraction and Refraction, Izv. Akad. Nauk SSSR, Ser. Fiz. **12**, 81 (1948).
34. Theory of Propagation of Radio Waves from an Elevated Source in an Inhomogeneous Atmosphere. Izv. Akad. Nauk SSSR, Ser. Fiz. **14**, 70 (1950).
35. Generalization of Reflection Formulae to the Case of Reflection of an Arbitrary Wave from a Surface of Arbitrary Shape, J. Exptl. Theoret. Phys. (U.S.S.R.) **20**, 961 (1950).
36. The Schrödinger Equation for the Helium Atom, Izv. Akad. Nauk SSSR, Ser. Fiz. **18**, 161 (1954).
37. Теория пространства, времени и тяготения (Theory of Space, Time, and Gravitation) Moscow, Gostekhizdat, 1955.

Translated by F. J. Dyson