

TEMPERATURE DEPENDENCE OF THE INTERNAL FRICTION OF METALS AND ALLOYS

V. S. POSTNIKOV

 Usp. Fiz. Nauk **66**, 43-77 (September, 1958)

I. INTRODUCTION

As is well known, oscillations induced in a solid are damped out rapidly even in the absence of external resistance. It is customary to explain such damping by the presence of "internal friction forces" or simply "internal friction" within the solid itself. This term must be taken to mean the ability of the solid to convert, in an irreversible way, the energy of mechanical oscillations into heat. These "forces of internal friction" amount up to 70% (or more) of all the resistive forces that act on a vibrating solid in an actual situation.

The magnitude of internal friction may be defined in various ways. In the simplest and most direct way it may be expressed by means of a quantity δ defined, if the internal friction is not great, as the ratio

$$\delta = \frac{\Delta U}{U}, \quad (1)$$

where ΔU is the energy dissipated per cycle, while U is the energy of the oscillations.

At room temperature for materials with low internal friction $\delta \sim 10^{-7}$, while for materials with high internal friction the values of δ may reach 0.1 or more. The value of δ is determined not only by the nature of the materials, but depends also, as we shall see later, on many other factors and above all on the amplitude of the oscillations. In the case of well tempered metals and of alloys, an amplitude dependence of the magnitude of internal friction determined, for example, by the method of low frequency rotational oscillations, can be observed for angles of twist exceeding 10^{-5} radian. The appreciable dependence of the magnitude of δ on the amplitude of oscillation should be taken into account in comparing the results of measurements obtained by different investigators. A direct measurement of δ , possible only in the case when $\delta \geq 0.1$, has been carried out by Hopkinson¹ and Föppl.²

Since in the deformation of a real solid one always observes a phase lag of the deformation behind the stress, i.e., mechanical hysteresis is present, it is also possible to adopt as a measure of internal friction the phase shift angle φ .^{3,4,5,6}

A direct determination of φ is possible only when $\tan \varphi$ is greater than 0.1.¹⁶

Most frequently⁷⁻³² the magnitude of internal friction is determined from the damping of free vibrations of a sample, since for metals and alloys $\tan \varphi < 0.1$. To do this experimentally one obtains (if the frequency of vibrations exceeds 2 cps) a time display of the free oscillations of the sample (Fig. 1a), and from it one computes the logarithmic decrement of the damped vibrations θ (taken to be a measure of internal friction), which by definition is equal to

$$\theta = \ln \frac{a_n}{a_{n+1}}. \quad (2)$$

Here a_n and a_{n+1} are the amplitudes of oscillations in the n -th and $(n+1)$ -th cycles.

Usually, to increase the accuracy of the determination of θ (particularly in the case of visual observations, which are possible only when the frequency of the free vibrations does not exceed 2 cps), one employs the formula

$$\theta = \frac{1}{N} \ln \frac{a_n}{a_m}, \quad (3)$$

which gives the mean value of the logarithmic decrement for the portion of the time display bounded by the n -th and m -th cycles ($N = m - n$). If within this interval the value of the decrement varies slowly as the amplitude decreases, formula (3) introduces negligible errors. But if this formula is applied to the whole time display ($m = N, n = 0$), the calculated value of θ may differ appreciably from the mean values of the logarithmic decrement for a small portion of the time display, in view of its possible dependence on the amplitude of the oscillations.

Logarithmic decrements lower than 10^{-4} cannot be measured, since the losses in the apparatus, even in vacuo, are of the same order of magnitude. It is also impossible to measure the internal friction by this method when $\theta \geq 0.3$, for at these values of θ the motion of the sample becomes aperiodic.

The next method³⁴⁻³⁸ of measuring internal friction consists of observing the amplitude of forced oscillations while the frequency of the external ex-

cing force is slowly varied and its amplitude is kept constant. The amplitude of the oscillation of the sample (Fig. 1b) reaches a maximum when the frequency of the external force is equal to the resonance frequency ν_0 of the sample, and falls off sharply if the frequency of the external force is either greater or less than this frequency. As a measure of internal friction in this method we adopt the quantity

$$B = \frac{\Delta\nu}{\nu_0}, \quad (4)$$

where $\Delta\nu$ is the half-width of the resonance peak.

This method gives good results when the internal friction determined by the quantity B is not smaller than 10^{-5} , since it is difficult to obtain resonance curves when the internal friction is very small, owing to the great sharpness of resonance.

The internal friction can also be measured by other methods (cf. a review of these methods in references 3, 4, 49-52, 87, 250, and 251), but they are not often used.

The quantities indicated above as measures of internal friction are interrelated by simple equations, which are valid when the internal friction does not depend on the amplitude and when the magnitude of the internal friction, defined for example as $\tan \varphi$, does not exceed 0.1. Since in the preponderant majority of known cases the internal friction in metals and alloys does not exceed 0.1, these relations between δ , $\tan \varphi$, θ , and B do have a practical meaning: they enable us to compare results of investigations obtained by different methods. In references 3, 4, 5, and 51 the following relations between the measures of internal friction have been obtained:

$$\tan \varphi = \frac{\theta}{\pi} = \frac{\delta}{2\pi} = \frac{B}{\sqrt{3}}. \quad (5)$$

When the internal friction is large, the relations between these different measures of internal friction depend on the mechanism of internal friction and must be derived separately for each case.⁵

We note that in the theory of oscillatory circuits the quantity $\sqrt{3}/B$ is usually referred to as the figure of merit and is denoted by Q . Therefore many authors use the symbol Q^{-1} for the magnitude of internal friction.

In conclusion we should say a few words about the earlier work (approximately prior to 1930) on internal friction. In these investigations⁵⁴⁻⁷⁶ quite rough static and dynamic methods were used to find the coefficient of viscosity, which was used to characterize internal friction in materials. However, as Pines⁷⁷ has correctly noted, this should

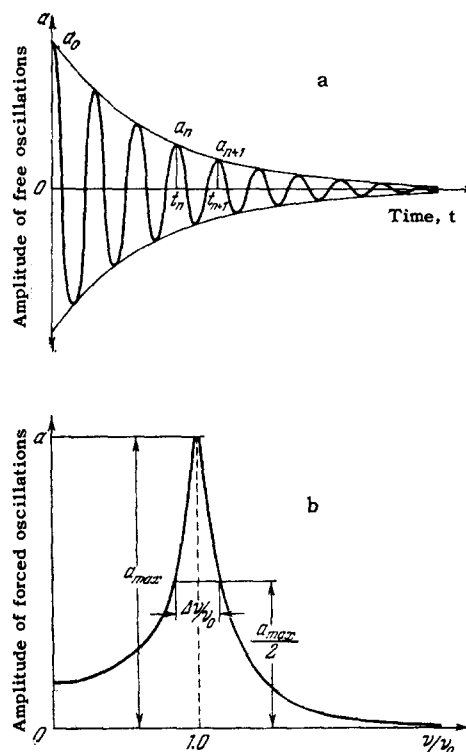


FIG. 1. Methods of determining internal friction: (a) by time display of free oscillations, (b) by width of the amplitude resonance peak.

not be done, since the coefficient of viscosity and the internal friction are by no means the same. Therefore, unfortunately, it does not appear to be possible to make use of the results of most of the earlier research on internal friction.

Recently there has been a sharp increase in interest in the study of the dependence of internal friction on various factors, particularly with a view toward studying various problems of physical and chemical kinetics. Special attention has been devoted to the temperature dependence of internal friction, and the present review is devoted to this topic.

II. EXPERIMENTAL RESULTS

1. Pure Metals

Figures 2 and 3 show the temperature dependence, obtained by high frequency methods, of internal friction in tin,^{79,252} lead,^{80,81,82} aluminum,^{81,83,249} silver,^{80,81} and copper^{80,81,84,85} at low temperatures. The maximum friction in tin is associated with the transition of tin into the superconducting state²⁵¹⁻²⁵³ at 3.73°K. A similar maximum occurs in the case of lead.²⁵¹ The peak of internal friction in the case of lead, aluminum (at approximately 105°K), and silver is explained largely by the motion of dislocations in the stress field. The peak for aluminum at approximately

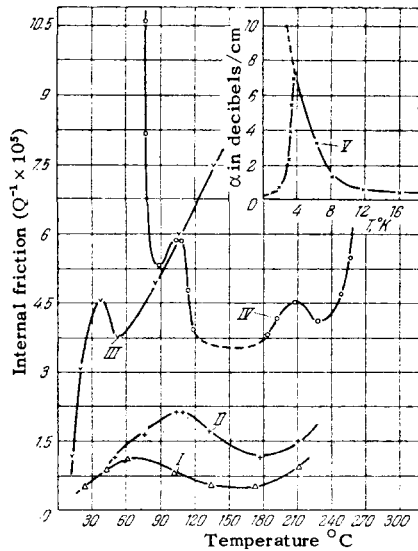


FIG. 2. Internal friction in polycrystalline lead (I—99.9% Pb), aluminum (II, III—99.99% Al), silver (IV—99.5% Ag), and in a single crystal of tin (V—>99.99% Sn) at low temperatures. Frequency of oscillations I, II, IV— 10^4 cps, III— 10^3 cps, V— 10^7 cps.

235°K is explained by the motion of single vacancies.²⁴⁹ Near room temperature, the internal friction of all metals investigated so far increases monotonically. It is of interest to note the good agreement between the results of investigations by different authors^{80,84} in the case of single crystals of copper, shown in Fig. 3 by the curves III' and III'', which enables us to join the two branches into a single curve (dotted line in Fig. 3).

Figures 4 to 9 give the temperature dependence, obtained by various methods, of internal friction in tin,⁸⁶⁻⁸⁷ lead,⁸⁸ zinc,⁸⁹ magnesium,^{15,47} aluminum,^{15,47,90,94} silver,^{97,98} gold,^{99,100} copper,^{93-97,121} nickel,^{103,106} cobalt,^{103,104,107} iron,^{103,104,108,109} titanium,¹⁰²⁻¹⁰⁴ platinum,^{98,101} zirconium,¹¹² molybdenum,^{103,104} tantalum,^{100,112} and tungsten^{103,104} at

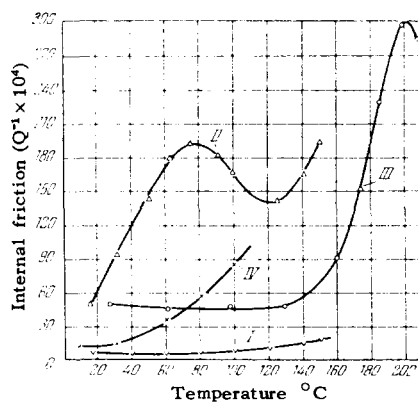


FIG. 4. Temperature dependence of internal friction in single-crystal and polycrystalline tin (I, II—99.99% Sn, $\nu \sim 300$ cps), polycrystalline lead (III—99.9% Pb, $\nu \sim 950$ cps), and zinc (IV—99.6% Zn, $\nu \sim 180$ cps).

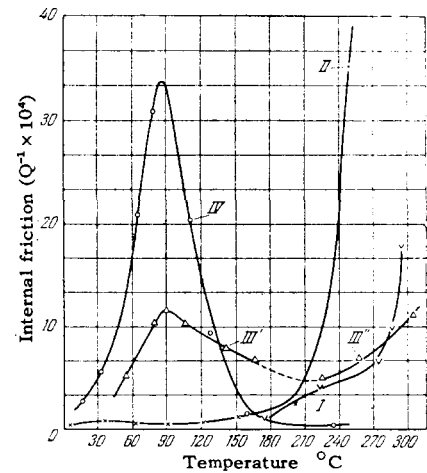


FIG. 3. Internal friction in single crystals of tin (I—99.93% Sn, $\nu \sim 1.5 \times 10^3$ cps), copper (III'—99.9% Cu, $\nu \sim 4 \times 10^4$ cps, III''—99.86% Cu, $\nu \sim 3.9 \times 10^4$ cps) and in polycrystalline lead (II—99.9% Pb, $\nu \sim 10^4$ cps), copper (IV—99.9% Cu, $\nu \sim 3 \times 10^4$ cps) at low temperatures.

temperatures above room temperature. The temperature dependence of internal friction of tin, lead, zinc, silver, and copper was measured at frequencies from 180 to 800 cps; in the case of aluminum, magnesium, and cobalt the frequency range was 1355 to 8×10^4 cps; in the case of magnesium, aluminum, gold, copper, nickel, cobalt, iron, titanium, platinum, zirconium, molybdenum, tantalum, and tungsten the frequency was on the order of 1 cps. All the metals were in polycrystalline form. Tin, aluminum, gold, copper, and iron were also used in single-crystal form. All the metals were annealed at high temperatures for a time long enough to remove internal stresses and to obtain a more or less equilibrium state.

It is seen from the figures that the internal fric-

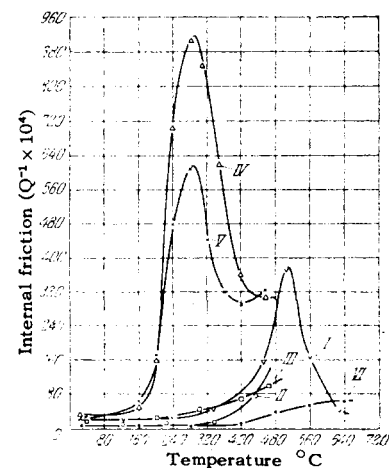


FIG. 5. Temperature dependence of internal friction in aluminum—single crystal (II—99.99% Al, $\nu \sim 4 \times 10^4$ cps; III—99.5% Al, $\nu \sim 1$ cps) and polycrystalline (I—99.99% Al, $\nu \sim 10^4$ cps; IV—99.991% Al, 1 cps; V—99.98% Al, $\nu \sim 1$ cps; VI—99.99% Al, $\nu \sim 5 \times 10^3$ cps).

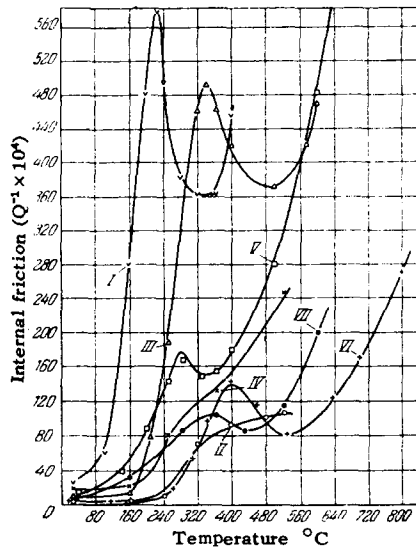


FIG. 6. Temperature dependence of internal friction in polycrystalline magnesium (I – 99.97% Mg, $\nu \sim 1$ cps; II $\approx 99.99\%$ Mg, $\nu \sim 5 \times 10^3$ cps), copper (III – 99.999% Cu, $\nu \sim 1$ cps; IV – 99.999% Cu, very large crystals, $\nu \sim 1$ cps), and silver (V – 99.99% Ag, $\nu \sim 1$ cps).

tion of single crystals of metals increases monotonically with increasing temperature, without giving any maxima. On the other hand, the internal friction of polycrystalline metals (tin, lead, magnesium, aluminum, silver, gold, copper, nickel, iron, and zirconium) shows a maximum at temperatures (with the exception of tin and lead) equal to approximately $(0.4 \text{ to } 0.6) T_{\text{melt}}$. Maxima in the internal friction of tin and lead are observed at temperatures $\sim (0.7 \text{ to } 0.8) T_{\text{melt}}$. A shift of the maximum towards comparatively higher tempera-

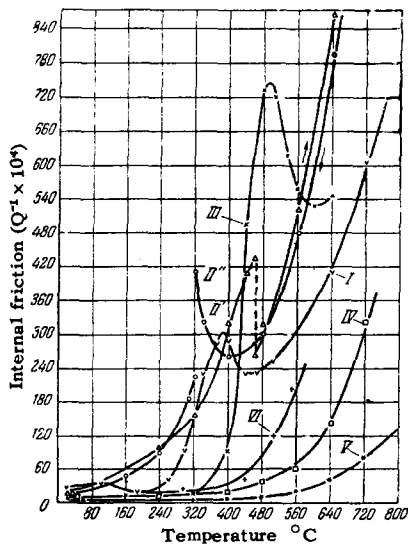


FIG. 8. Temperature dependence of internal friction in polycrystalline nickel (I – 99.99% Ni), cobalt (II' – heating, II'' – cooling, 99.98% Co), iron (III, IV – 99.95% Fe, VI – very large grains), molybdenum (IV 99.92% Mo), and tungsten (V – 99.61% W). Frequency of oscillation $\nu \sim 1$ cps.

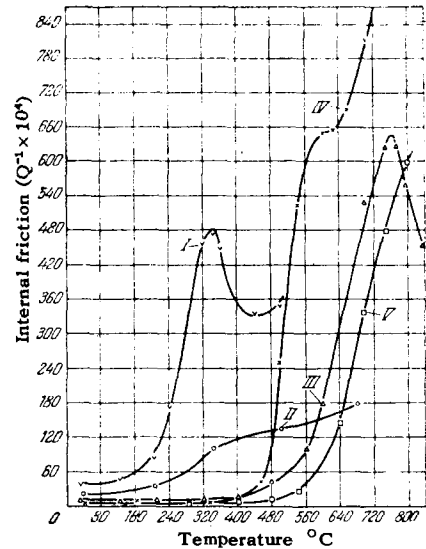


FIG. 7. Temperature dependence of internal friction in polycrystalline and single-crystal gold (I, II – 99.9998% Au, $\nu \sim 1$ cps), polycrystalline platinum (III – 99.97% Pt, $\nu \sim 1$ cps) and titanium (IV – 99.81% Ti, $\nu \sim 1$ cps; V – 99.81% Ti, very large grains, $\nu \sim 1$ cps).

tures is associated with an increase in the frequency of oscillations of the sample. The internal friction of titanium, molybdenum, tantalum, and tungsten does not have a maximum in the temperature region above $0.4 T_{\text{melt}}$. For these not very pure metals the only characteristic feature is the existence of an inflection point in the curve $Q^{-1}(T)$. In Figs. 8 and 9 the inflection points for molybdenum, tantalum, and tungsten are not shown, since they lie above 800°C . The curve of $Q^{-1}(T)$ for tantalum in the low temperature region (~ 0.14 to

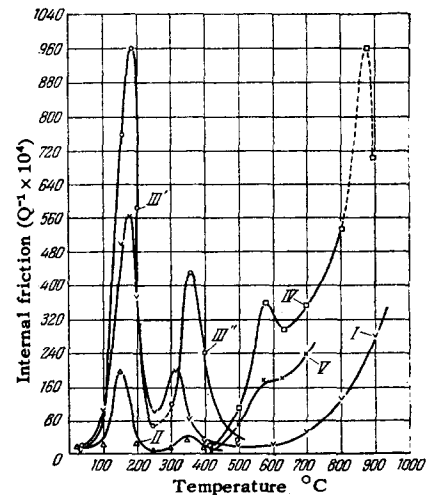


FIG. 9. Temperature dependence of internal friction in polycrystalline tantalum (I – 99.76% Ta + 0.17% Al, enriched by carbon, oxygen and nitrogen; II, III, III' – 99.9% Ta, enriched by carbon, oxygen and nitrogen) and zirconium (IV – 97.5% Zr + 2.4% Hf; V – the same sample, saturated with oxygen). Frequency of oscillation $\nu \sim 1$ cps.

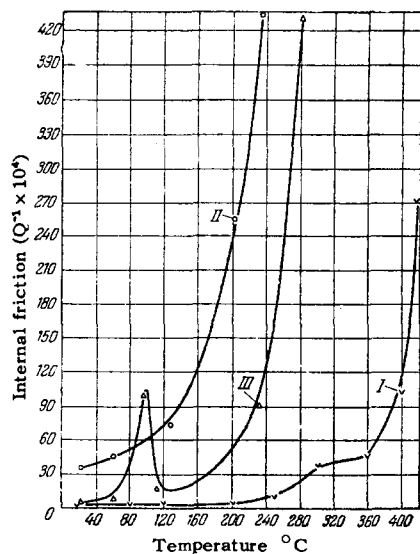


FIG. 10. Temperature dependence of solid solutions Cu-Bi (I—technical copper + 0.01% Bi, $\nu \sim 10^3$ cps), Pb-Sn (II—99.9998% Pb + 0.058% Sn, $\nu \sim 3 \times 10^4$ cps), Pb-Bi (III—99.9998% Pb + 0.053% Bi, $\nu \sim 3 \times 10^4$ cps).

0.19 T_{melt}) shows two sharply pronounced maxima, whose nature, as we shall see, differs from the nature of the maxima encountered at higher temperatures. Metals like cobalt and zirconium exhibit a very interesting dependence of internal friction on the temperature. For example, in the case of cobalt the internal friction rises rapidly with increasing temperature until it reaches a maximum of 450°C. In the neighborhood of this temperature (if the sample is maintained at constant temperature) the internal friction decreases rapidly and attains a certain stable value within approximately 30 minutes. As the temperature is

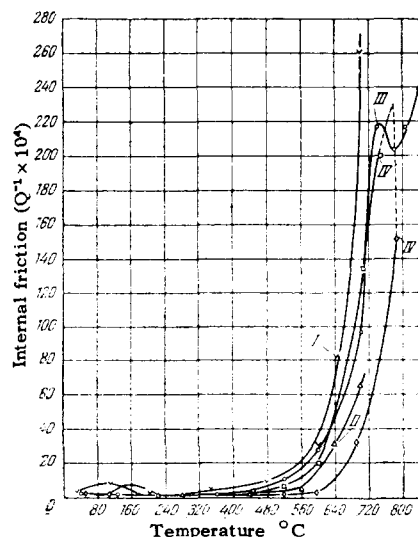


FIG. 12. Temperature dependence of Armco-iron (I) and of solid solutions Fe-Si (II—Armco-Fe + 4% Si), Fe-C (III—Armco-Fe + 0.84% C) and Fe-Co-Cr (IV', IV''—Armco-Fe + 40% Co, 10% Cr; IV' heating, IV'' cooling). Frequency of oscillation $\nu \sim 5 \times 10^3$ cps.

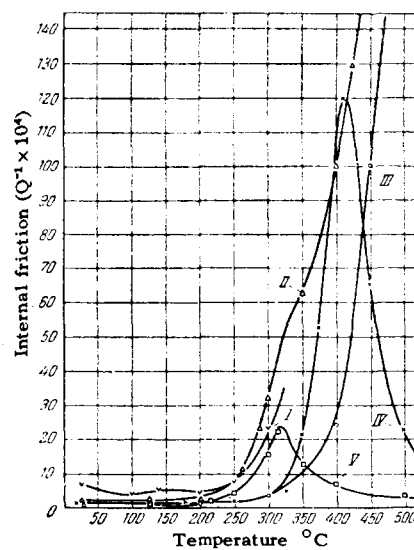


FIG. 11. Temperature dependence of solid solutions Al-Mg (I—99.99% Al + 21.18% Ag, $\nu \sim 13 \times 10^3$ cps) and Cu-Zn (II—99.9% Cu + 28% Zn, $\nu \sim 5 \times 10^3$ cps; III—Cu + 30% Zn, $\nu \sim 800$ cps; IV—Cu + 30% Zn, $\nu \sim 600$ cps, single crystal; V—Cu + 45% Zn, $\nu \sim 21 \times 10^3$ cps, single crystal).

increased further the internal friction continues to increase without showing any further peculiarities. On cooling, the internal friction decreases to 400°C, after which it begins to grow rapidly, reaching a maximum at 320°C. Isothermal conditioning (during 30 minutes) at this temperature leads to a rapid decrease in internal friction to a stable value. A further decrease in temperature leads to a smooth decrease in internal friction to $\sim 20 \times 10^{-4}$ at 20°C. In the temperature range from 300 to 450° a kind of "hysteresis" occurs in the temperature variation of internal friction.

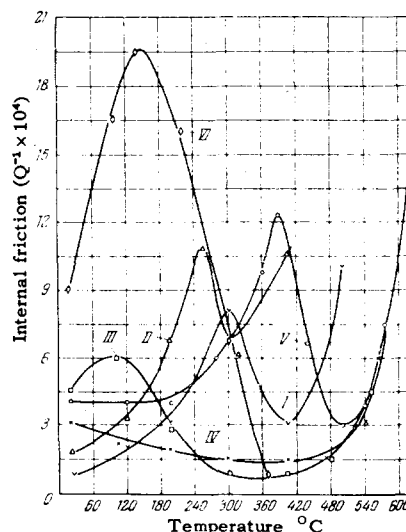


FIG. 13. Temperature dependence of internal friction in solid solutions Fe-Ni (I—Armco-Fe + 9.88% Ni + 0.67% Mn; II—Fe + 28.84% Ni + 0.38% Mn; III—Fe + 40% Ni; IV—Fe + 60% Ni; V—Fe + 80% Ni; VI—technical nickel). Frequency of oscillation $\nu \sim 10^3$ cps.

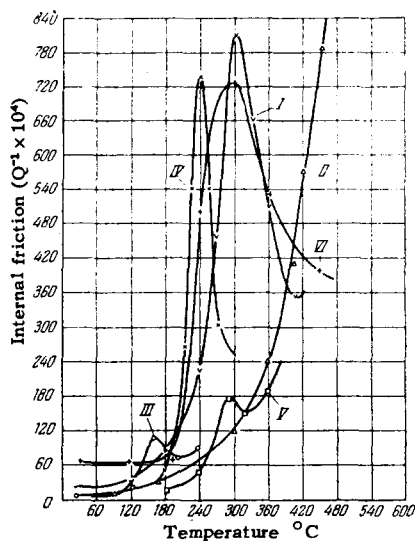


FIG. 14. Temperature dependence of internal friction in solid solutions Al - Cu (I - 99.991% Al + 0.5% Cu; II - Al + 4% Cu), Al - Ag (III - 99.99% Al + 20.1% Ag) and Ag - Zn (IV - 99.99% Al + 30.2% Zn; V - Al + 15.8% Zn). Frequency of oscillation $\nu \sim 1$ cps.

Consequently, within a certain temperature range the internal friction in cobalt depends not only on the temperature, but also on the duration of the experiment. Zirconium behaves in a similar manner in the temperature range from 800 to 900°C. As is well known, these metals undergo transitions from one allotropic modification into another within these temperature ranges. The internal friction of titanium above 840°C should behave in a manner similar to that of cobalt and zirconium, but there are no experimental data.

The following conclusions can be drawn from the results of measurements shown in Figs. 4 to 9:

1. Well annealed metal samples exhibit temperature dependence of internal friction that is constant in time (reproducible in repeated measurements, if the oxidation of the samples is held down to a minimum).

2. The temperature dependence of internal friction of annealed single-crystal metal samples does not exhibit any maxima. The magnitude of the internal friction of single crystal samples is always less than the magnitude of the internal friction of corresponding polycrystalline samples.

3. Each polycrystalline metal (at a constant frequency of free vibrations) shows at least one maximum or inflection point on the curve of the temperature dependence of internal friction, provided no allotropic transitions take place on heating.

4. The internal friction of metals which may undergo allotropic changes on heating (for example, cobalt or zirconium), is not constant in time in the

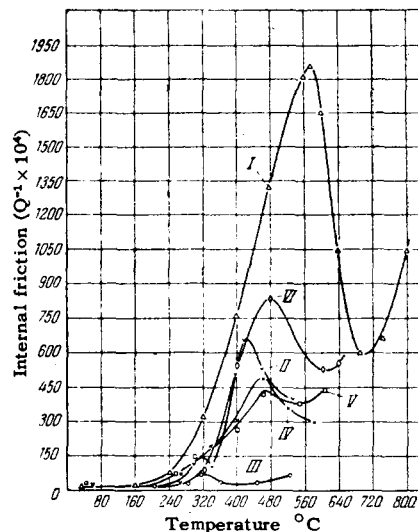


FIG. 15. Temperature dependence of internal friction in solid solutions Cu - Be (I - 99.992% Cu + 1.9% Be), Cu - Zn (II, III - cathode copper + 29.86% Zn; III - single crystal), Cu - Si (IV - 99.999% Cu + 0.1% Si), Cu - Al (V - 99.999% Cu + 0.1% Al) and Cu - Sn (VI - 99.9993% Cu + 6% Sn). Frequency of oscillation $\nu \sim 1$ cps.

temperature range in which an allotropic change occurs. An "hysteresis" of internal friction is observed within this temperature range.

5. The maximum (or inflection point) of internal friction lies for the most part in the temperature range of recrystallization of the metal. Its position for a given metal, as shown by experiment, depends on the magnitude of the preceding plastic deformation: an increase in the degree of plastic deformation shifts the maximum towards lower temperatures. For different metals subjected to the same degree of plastic deformation, the position of the maximum (or inflection point) of internal friction is determined to some extent by the melting point of the metal: for metals of higher melting points the maximum of internal friction occurs at higher temperatures. The position of the maximum is also influenced by the frequency of oscillations: as the frequency of oscillations of the sample is increased the maximum is observed at increasingly higher temperatures.

6. At a given temperature metals of higher melting point exhibit a lower value of internal friction compared to that for metals of lower melting point. This difference increases as the temperature increases.

7. The magnitude of internal friction depends also on the preceding deformation. As a rule, the internal friction is considerably greater in the plastically deformed metals than well annealed ones.

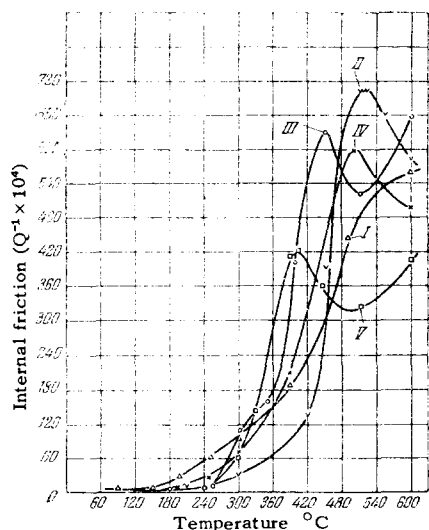


FIG. 16. Temperature dependence of internal friction in solid solutions Cu-Zn (I—spectral purity Cu + 1.17% Zn; II—8.01% Zn; III—32.1% Zn) and Cu-Ga (IV—0.97% Ga; V—16.2% Ga). Frequency of oscillation $\nu \sim 1$ cps.

2. Alloys

Figures 10 to 13 present results of investigations of the temperature dependence of internal friction of binary alloys Pb-Sn,⁸² Pb-Bi,⁸² Al-Ag,⁴⁷ Cu-Bi,⁴³ Cu-Zn,¹¹⁴⁻¹¹⁷ Fe-Si,¹¹⁸ Fe-C,^{11,118,119} and Fe-Ni,¹²⁰⁻¹²¹ and of the ternary alloy Fe-Co-Cr¹²¹ at temperatures above room temperature, obtained by different methods at frequencies from 600 to 3×10^4 cps.

Figures 14 to 23 present the temperature dependence of the internal friction of the following binary alloys: Al-Zn,¹²³ Al-Mg,^{124,125} Al-Ag,^{126,127} Al-Cu,^{128,129} Cu-Ga,¹²² Cu-Sn,⁹⁵ Cu-Zn,^{122,130,131}

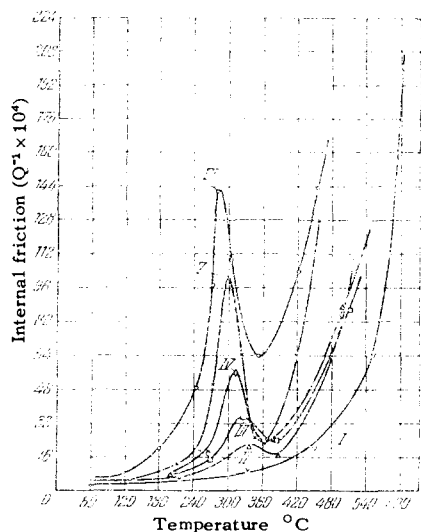


FIG. 18. Temperature dependence of internal friction in solid solutions Cu-Zn (I—spectrally pure Cu + 5% Zn; II—10%, III—15%, IV—20%, V—25%, VI—30% Zn; I and II very large grains). Frequency of oscillation $\nu \sim 1$ cps.

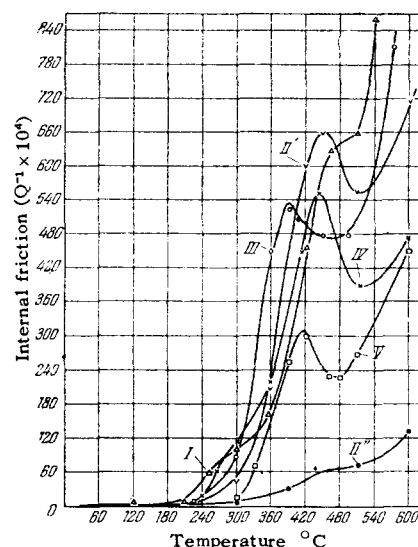


FIG. 17. Temperature dependence of internal friction in solid solutions Cu-Ge (I—spectral purity Cu + 1.02% Ge; II', II''—4.94% Ge; II—very large grains), Cu-As (III—3.93% As) and Cu-Si (IV—0.9% Si; V—9.0% Si). Frequency of oscillation $\nu \sim 1$ cps.

Cu-Al,¹³² Cu-As,¹²² Cu-Ge,¹²² Cu-Si,^{122,132} Cu-Be,¹³³⁻¹³⁵ Au-Ni,¹³⁶ Ni-C,^{103,105,106} Ni-B,¹³⁷ Ni-Be,¹³⁷ Ni-Mn,¹³⁷ Ni-Fe,¹³⁷ Ni-Ti,¹³⁷ Ni-Cr,^{137,168} Ni-Zr,¹³⁷ Ni-Nb,¹³⁷ Ni-Mo,¹³⁷ Ni-W,¹³⁷ Fe-B,¹³⁸ and Fe-W,¹⁴⁶ and of the complex alloys: kovar,^{137,139} nichrome,¹³⁷ alloy No. 2,¹³⁷ elinvar,¹³⁷ elgiloy,¹³⁷ Nimo alloy,^{137,139} nimonik-80,¹³⁷ alloy 38KhMYuA,¹⁴⁰ Fe-Gr-Ni alloy,¹⁴¹⁻¹⁴⁴ and austenite steel 25-20.¹⁴⁵ The measurements have been carried out over a wide range of temperatures by various low-fre-

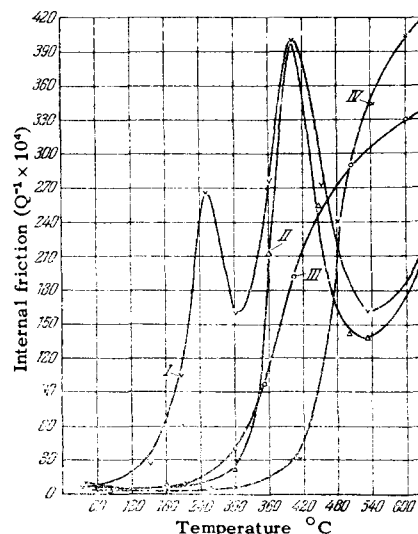


FIG. 19. Temperature dependence of internal friction in solid solutions Au-Ni (I—99.98% Au + 30% Ni, sample is quenched in oil at 800°C; II—the sample is heated at 400° and is rapidly cooled in vacuo; III—the sample is kept at 400° for 240 hours, and is then slowly cooled), and Ni-C (IV—99.99% Ni + 0.21% C). The frequency of oscillation is $\nu \sim 1$ cps.

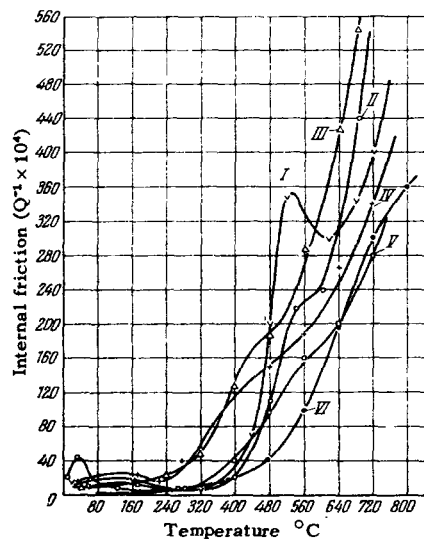


FIG. 20. Temperature dependence of internal friction in solid solutions Ni-B (I - 99.99% Ni + 0.25% B), Fe-B (II - Armco-Fe + 0.006% B), Ni-Fe (III - 99.99% Ni + 2.6% Fe), Ni-Zr (IV - 99.99% Ni + 5.5% Zr) and Ni-Nb (V - 99.99% Ni + 1.76% Nb). Frequency of oscillation $\nu \sim 1$ cps.

quency methods. All the alloys, with a few exceptions (cf. Figs. 14, 15, and 17), were investigated in their polycrystalline state.

We also note that within a narrow temperature range (from 20 to 300 or 400°) investigations have been carried out for the pure metals zinc,^{29,49} silver,¹⁴⁹ copper,^{147,149} and nickel,³⁵ and also for the following alloys: Sb-Bi,¹⁵⁰ Al-Cu,¹⁵¹ Al-Cu-Si,¹⁵¹ Al-Cu-Mg,¹⁵¹ Al-Mg-Si,¹⁵¹ Al-Cu-Mg-Si,¹⁵¹ Ag-Cd,¹⁸⁶ Ag-Zn,¹⁸⁶ Au-Cd,¹⁸⁷ Au-Zn,¹⁸⁶ Au-Cu,¹⁸⁵ Cu-Zn,^{185,186} Mn-Cu,⁴⁵ Fe-B,¹⁵² Fe-C,^{153-164,217,247,248} Fe-N,^{153,155-159} Fe-Ni,¹⁶⁵ 105-Kh-12 steel,¹⁶⁶

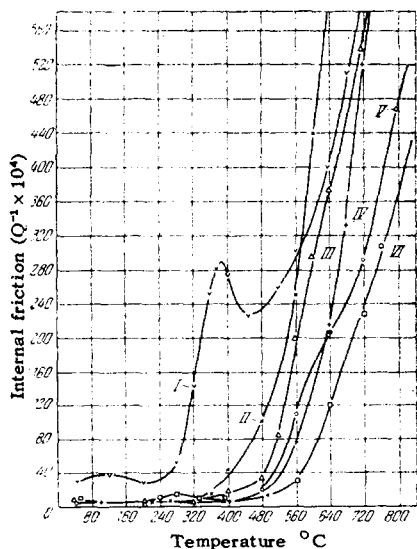


FIG. 22. Temperature dependence of internal friction in pure nickel (I - 99.99% Ni), kovar (II), technical nichrome (III), alloy No. 2 (IV), elinvar (V), elgiloy (VI - alloy K = 40). Frequency of oscillation $\nu \sim 1$ cps.

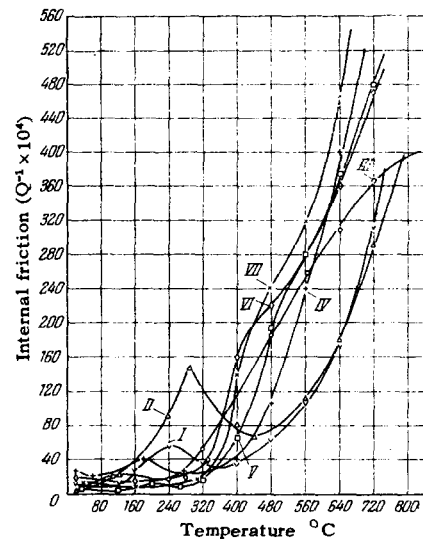


FIG. 21. Temperature dependence of internal friction in solid solutions Ni-Be (I, II - 2.9% Be; I - slowly cooled, II - quenched in water from 950°C), Ni-Ti (III - 1.04% Ti), Ni-Mn (IV - 22.8% Mn), Ni-Cr (V - 1.19% Cr), Ni-Mo (VI - 0.86% Mo) and Ni-W (VII - 1.54% W) based on 99.99% nickel. Frequency of oscillation $\nu \sim 1$ cps.

Pd-H,¹⁶⁷ Pd-Cu,¹⁸⁵ and Pt-Cu.¹⁸⁵ The investigations were carried out by different methods, primarily in order to find out the behavior of the element dissolved in the alloy.

The measurements of the temperature dependence of internal friction of the pure metals and alloys enumerated above lead to the following conclusions:

1. The internal friction of alloys increases with increasing temperature (over a wide range

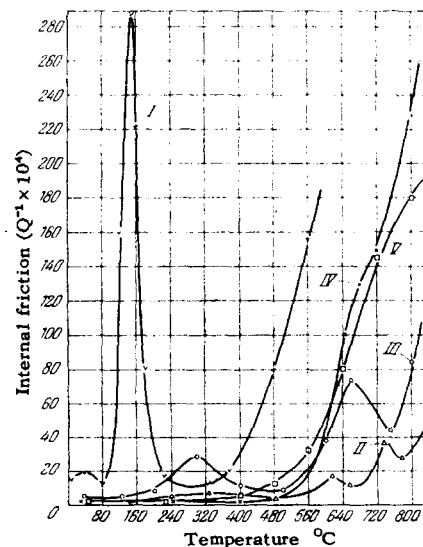


FIG. 23. Temperature dependence of internal friction in alloy ZVKhMYuA (I), austenite steel 18-18 (II), austenite steel 25-20 (III - containing 0.3% C, sample quenched in water from 1230°C), Nimo alloy (IV), and alloy nimonik-80 (V). Frequency of oscillation $\nu \sim 1$ cps.

of temperatures) much more slowly than the internal friction of those pure metals of which the alloys are formed.

2. The magnitude of internal friction, particularly at high temperatures, is less for the alloys than for the pure metals.

3. At high temperatures the magnitude of internal friction turns out to be related to the stability of the alloy at the same temperatures. In all the investigations, the magnitude of internal friction was found to be lower for the less refractory alloys.

4. All the binary alloys investigated exhibit at a certain temperature one maximum or inflection point in the curve of $Q^{-1}(T)$. More complex alloys may have several such points.

5. Just as in the case of pure metals, the maxima of internal friction in the case of an alloy are displaced towards higher temperatures (or else disappear entirely) as the frequency of oscillations of the sample is increased.

6. As a rule, at higher temperatures deformed alloys have a greater internal friction the same samples when previously well annealed.

III. THE THEORY OF INTERNAL FRICTION

Any atomic system taken out of the state of equilibrium and left to itself must either return to the initial state or go over into a new equilibrium state, if several stable states correspond to the given conditions. The transition of a system from one equilibrium state to another is characterized, as is well known, by a relaxation time (one or several). If a periodic force is applied to the system, there will be observed a dissipation of elastic energy which depends on the frequency of the applied force. This dissipation of elastic energy of the oscillations is determined by the totality of all the relaxation phenomena called into play when the atomic system is subjected to a periodic deformation, and therefore cannot as yet be determined in general form. In the first approximation (when the relaxation phenomena occur independently of one another) we can assume that the total internal friction, which characterizes the dissipation of the energy of the oscillations, is determined by the sum of the contributions made by the different relaxation phenomena.^{78,93,137}

Let us consider the basic relaxation phenomena which occur in a solid when it is deformed.

1. The General Thermodynamic Theory of Internal Friction

We consider a homogeneous isotropic solid. For small deformations, for small temperature changes,

and for small deviations from the equilibrium state the stress tensor σ_{ik} is determined by the following equation:¹⁶⁹⁻¹⁷¹

$$\begin{aligned} \sigma_{ik} = & K\varepsilon_{il}\delta_{ik} + 2\mu \left(\varepsilon_{ik} - \frac{1}{3}\varepsilon_{ll}\delta_{ik} \right) - \alpha K(T - T_0)\delta_{ik} \\ & + \frac{2\eta_1}{\tau_1} \int_{-\infty}^{t-t} e^{-\frac{t-t'}{\tau_1}} \left[\dot{\varepsilon}_{ik}(t') - \frac{1}{3}\dot{\varepsilon}_{ll}\delta_{ik} \right] dt' \\ & + \frac{\eta_2}{\tau_2} \int_{-\infty}^t e^{-\frac{t-t'}{\tau_2}} \varepsilon_{ll}(t') \delta_{ik} dt'. \end{aligned} \quad (6)$$

In the case of isothermal pure shear (for example, small torsional oscillations of a rod undergoing relaxation) $\varepsilon_{ll} = 0$, $\Delta T = 0$, and therefore

$$\sigma_{ik} = 2\mu\varepsilon_{ik} + \frac{2\eta_1}{\tau_1} \int_{-\infty}^t e^{-\frac{t-t'}{\tau_1}} \varepsilon_{ik}(t') dt'. \quad (7)$$

If the deformation is a periodic function of the time,

$$\varepsilon_{ik} = \varepsilon_{ik}^{(0)} e^{i\omega t} \quad (8)$$

and the generalized Hooke's law can be written

$$\sigma_{ik} = \mu^* \varepsilon_{ik}, \quad (9)$$

where the complex shear modulus is given by

$$\mu^* = \mu + \frac{i\omega\eta_1}{1 + i\omega\tau_1}. \quad (10)$$

On multiplying the numerator and denominator of the second term in the right hand side of (10) by $(1 - i\omega\tau_1)$ we shall obtain

$$\mu^* = \mu + \frac{(\omega\tau_1)^2}{1 + \omega^2\tau_1^2} \frac{\eta_1}{\tau_1} + \frac{i\omega\tau_1}{1 + \omega^2\tau_1^2} \frac{\eta_1}{\tau_1}. \quad (11)$$

For the dynamic modulus one usually takes the real part of the complex modulus, i.e.,

$$\mu_\omega = \mu + \frac{(\omega\tau_1)^2}{1 + \omega^2\tau_1^2} \frac{\eta_1}{\tau_1}. \quad (12)$$

From the last expression we obtain

$$\left. \begin{aligned} \mu_\infty &= \mu + \frac{\eta_1}{\tau_1}, \\ \mu_0 &= \mu. \end{aligned} \right\} \quad (13)$$

Taking (12) and (13) into account, we obtain the shear-modulus decrement that defines the degree of relaxation:

$$\Delta_\omega = \frac{\mu_\infty - \mu_\omega}{\mu_0} = \frac{\Delta_0}{1 + \omega^2\tau_1^2}, \quad (14)$$

where

$$\Delta_0 = \frac{\mu_\infty - \mu_0}{\mu_0}$$

is the maximum value in the decrement of the shear modulus. For the measure of internal friction one usually takes^{3,4,51,52,78} $\tan \varphi$, which is equal to the

ratio of the imaginary part of the complex modulus to its real part. By taking this ratio from (11) we obtain

$$Q^{-1}(\omega) = \frac{\Delta_0 \omega \tau_1}{1 + \frac{\mu_\infty}{\mu_0} \omega^2 \tau_1^2} \quad (15)$$

Expressions (14) and (15) agree [up to a factor μ_∞/μ_0 in the denominator of (15)] with the expressions obtained by Zener,³ where μ_∞ denotes the "unrelaxed" while μ_0 denotes the "relaxed" modulus. A similar result was recently obtained by V. T. Shmatov.²³⁴

Figure 24 shows the variation in the internal friction $Q^{-1}(\omega\tau)$ and of the ratio of the dynamic modulus μ_ω to the unrelaxed modulus μ_∞ as functions of the parameter $\omega\tau$ in the case when $\mu_0/\mu_\infty = 0.6$. It can be seen that the magnitude of the internal friction attains a maximum (whose value depends on Δ_0 and on the ratio μ_∞/μ_0 for the given material) at $\omega\tau = \sqrt{\mu_0/\mu_\infty} \approx 1$, i.e., when the frequency ω of the oscillations of the sample approximately coincides with the reciprocal value of the relaxation time τ for the process. The ratio μ_ω/μ_∞ at the point where $Q^{-1}(\omega\tau)$ is a maximum has the greatest rate of variation with respect to the parameter $\omega\tau$.

For $\omega\tau > 10^{-1}$ the internal friction is insignificant, and the variation of the modulus is practically equal to zero, i.e., the dynamic modulus is practically equal to the "unrelaxed" modulus μ_∞ . Consequently, in this frequency range [$\omega > (10\tau)^{-1}$] there is practically no relaxation, and therefore there is no appreciable dissipation of the elastic energy of oscillations. For $\omega\tau < 10^{-1}$, the dynamic modulus is practically equal to the "relaxed" modulus μ_0 , while the internal friction is again very small. In the intermediate frequency range we have a partial relaxation of the dynamic modulus and a considerable internal friction, which reaches a maximum at $\omega\tau \approx 1$.

Thus, according to the thermodynamic theory, in the case of a homogeneous isotropic solid, an appreciable dissipation of elastic energy accompanying the periodic deformation occurs whenever the reciprocal value of the relaxation time of the relaxation process under consideration coincides in order of magnitude with the frequency of oscillation of the samples. Phenomena, with one or several relaxation times may be atomic diffusion, relaxation of magnetic flux, heat conductivity, interaction of phonons with the conduction electrons, etc. Thus, by varying the frequency of oscillation of the sample from very low values ($\sim 10^{-10}$ cps in the case of atomic diffusion) up to very high ones ($\sim 10^{10}$ cps

in the case of interaction of phonons with conduction electrons) one can obtain a discrete relaxation spectrum even for a homogeneous isotropic solid.

Real solids, as a rule, are neither homogeneous nor isotropic. Therefore in the case of each of these phenomena we may find for these solids not one relaxation time, but a whole set, and in the general case of a continuous spectrum. If several relaxation processes are taking place in the solid simultaneously, each of which can be characterized by its own particular relaxation time τ_i , then the internal friction in such a solid is defined by the expression^{3,4,78,93,103,104,137}

$$Q^{-1}(\omega) = \sum_{i=1}^n \frac{\Delta_i^{(0)} \omega \tau_i}{1 + \left(\frac{\mu_\infty}{\mu_0}\right)_i (\omega \tau_i)^2} \quad (16)$$

The frequency dependence of the internal friction of such a solid may have not a single maximum, but several, or, if the relaxation times τ_i are grouped around some one mean value τ_{av} , then it may have a single maximum which is considerably "smeared."^{93,103,104,137} Such "smeared" maxima can be observed in the case of a curve of the temperature dependence of internal friction of polycrystalline metals. To explain such maxima, the idea of a "two component system" was introduced.^{3,180,181,193} This idea consists of assuming that the solid comprises two phases, one amorphous and the other perfectly elastic. In such a solid the stress relaxes with a relaxation time

$$\tau = \frac{\eta}{\mu}, \quad (17)$$

where η is the coefficient of viscosity of the amorphous phase, and μ is the shear modulus of

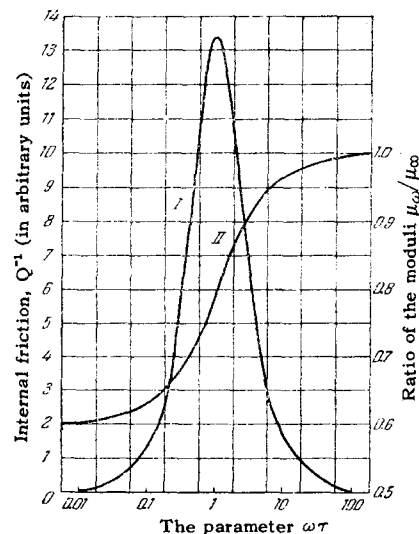


FIG. 24. Dependence of internal friction (I) and of ratio μ_ω/μ_∞ (II) on the parameter $\omega\tau$.

the elastic phase. This relaxation time will depend on the physical nature of the amorphous region, and also on its dimensions and shape. It may be easily shown that if the amorphous region has a width c and a length l , the coefficient of viscosity of this region is related to the relaxation time by the following equation¹⁵

$$\eta = \frac{\mu_{\infty} \tau}{l} c, \quad (18)$$

from which it can be seen that the relaxation time depends on the dimensions of the amorphous region (and on its shape).

If amorphous regions are embedded into an elastic matrix which is the base of the solid, then each such region will relax the stress according to its own relaxation time τ_i . The behavior of such a solid will thus be determined by the totality of all the relaxation times τ_i . This is what causes the "smearing" of the maxima.

Experiments show that the role of amorphous regions can be played by slip bands or by grain boundaries.

Finally, in the case of a continuous distribution of relaxation constants τ , the internal friction is given by the expression^{3,37}

$$Q^{-1}(\omega) = \int_{-\infty}^{+\infty} \frac{\psi(\tau) \omega \tau}{1 + \left(\frac{\mu_{\infty}}{\mu_0}\right)_{\text{av}} (\omega \tau)^2} d(\ln \tau), \quad (19)$$

where $\Psi(\tau)$ is an unknown distribution function, and difficult to determine experimentally. It can be found from the condition

$$(\Delta_0)_{\text{av}} = \int_{-\infty}^{+\infty} \psi(\tau) d(\ln \tau), \quad (20)$$

where $(\Delta_0)_{\text{av}}$ is the experimentally-known mean decrement in the modulus; this requires the inversion of the integral, which introduces a considerable inaccuracy into the final result.

In the case of relaxation phenomena associated with the diffusion displacement of atoms in the stress field, the relaxation time depends on the temperature, as is shown by numerous experiments, in accordance with the following law

$$\tau = \tau_0 e^{H/RT}, \quad (21)$$

where H is the heat of activation of the process under investigation. Since the internal friction Q^{-1} depends on the frequency and on the relaxation time through the parameter $\omega\tau$, we may leave the frequency ω unchanged, and vary the relaxation time τ by varying the temperature: at a certain temperature T we obtain the equality $\omega\tau \cong 1$ at which we observe the maximum internal

friction due to the existence of the relaxation phenomenon under consideration. Thus it becomes possible to understand why at a certain temperature (for a given frequency ω) a maximum appears on the curve of the temperature dependence of internal friction. It also becomes possible to understand the shift of the maximum as the frequency of oscillation of the sample is increased.

The heat of activation H is determined in the following manner. Suppose that we have two curves of the dependence of internal friction on the temperature, one obtained at a constant frequency ν_1 and the other at a constant frequency ν_2 . If we select on these two curves two points corresponding to the maxima of internal friction, this will mean that

$$\nu_1 \tau_1 = \nu_2 \tau_2$$

or, according to relation (21),

$$\nu_1 e^{H/RT_1} = \nu_2 e^{H/RT_2}.$$

By taking logarithms of both sides of the above equation we obtain, after a simple rearrangement,

$$H = R \frac{T_1 T_2}{T_2 - T_1} \ln \frac{\nu_2}{\nu_1}. \quad (22)$$

As we shall see below, everything said with respect to the determination of H applies also to the case when the relaxation process is characterized not by a single relaxation time, but by a whole set of relaxation times grouped around some mean value τ_{av} . In this case we obtain H_{av} for the complex process under consideration. The quantity Δ_0 is determined experimentally^{235,236} or is calculated from expression (15) in terms of known values of τ , μ_{∞}/μ_0 , ω , and Q_{max}^{-1} . In certain cases, as we shall see below, τ and Δ_0 can be estimated theoretically.

We now consider individual processes which give rise to internal friction, and start with processes whose relaxation times at room temperature are very large (of the order of 10^{10} sec and greater).

2. Internal Friction Due to the Ordering of Atoms under Stress

(a) **Substitution Alloys.** In annealed substitution solutions not undergoing ordering, the dissolved atoms are distributed isotropically in the lattice of the solvent. If a stress is applied to such an alloy, a certain amount of ordering in the distribution of the dissolved atoms is introduced.⁴ The relaxation time τ associated with ordering under stress must be simply related to the mean frequency of atomic

transitions, since the mechanism of ordering is the same as in the case of volume diffusion. Therefore the relaxation time of the ordering process may be estimated by means of the following formula²³⁷

$$\tau = \tau_0 e^{H/RT}, \quad (23)$$

where H is the activation energy for diffusion. A noticeable maximum of internal friction in such solutions, due to a change of order in the distribution of the dissolved atoms under the influence of an alternating stress, can be observed only at comparatively high concentrations (of the order of 10 atomic per cent and higher) of the dissolved component and at comparatively high temperatures, when the reciprocal of the relaxation time τ^{-1} is comparable with the frequency of variation of the applied stress. Ordinarily low-frequency methods ($\nu \sim 1$ cycle/sec) are used in such cases. The internal friction in substitution alloys due to ordering under stress was first observed by Zener¹¹⁶ in the case of single crystals of α -brass at ~ 600 cps. Later Kê and Nowick^{131,182,238} confirmed the existence of such a peak in α -brass and in the Ag-Zn alloy at lower frequencies. In a detailed paper²³⁸ Nowick studied the dependence of the internal friction of the Ag-Zn alloys on composition and on temperature.

Zener²³⁷ gave an explanation of the above effect, based on the violation of cubic symmetry by a pair of different atoms forming the solid solution. Under the action of the applied stress, these pairs of atoms take up a preferred orientation. The tendency of a lattice containing such pairs to go over to a new equilibrium state is what gives rise to the appearance of internal friction when it is periodically deformed. The degree of relaxation in this case can be determined by means of the formula²³⁷

$$\Delta_0 = nE_T \left(\frac{\partial \epsilon}{\partial n_p} \right)^2 \frac{1}{4kT}, \quad (24)$$

where n is the number of atom pairs per unit volume, E_T is the "isothermal" modulus of elasticity, ϵ is the tension strain, and

$$n_p = n \frac{e^{-u/kT}}{1 + e^{u/kT}}$$

is the number of pairs oriented along the new direction of p under the action of the applied stress.

If in the absence of stresses a certain degree of long range order exists in the solid solution, then the applied stress will give rise to a change in the degree of ordering. Relaxation processes of such type were first considered by V. S. Gorskii^{183,184} in the case of the Cu-Au alloy.

The degree of relaxation for alloys undergoing ordering may be approximately defined³ by

$$\Delta_0 = E_S \left(\frac{\partial S_c}{\partial T_c} \right) \left(\frac{\partial \epsilon}{\partial S_c} \right)^2, \quad (25)$$

where E_S is the "adiabatic" modulus of elasticity, S_c is the entropy of displacement, T_c is the temperature equal to the derivative $(\partial H / \partial S_c)_\sigma$ at constant stress σ , H is the heat content per unit volume, and ϵ is the tension strain. The relaxation time is estimated by means of formula (23). In contrast to the case discussed above in which the solid solution is not ordered in the absence of stresses, we can obtain here a characteristic peak of internal friction considered only as a function of frequency (at $T = \text{const}$), but not as a function of temperature (for $\omega = \text{const}$), because the degree of ordering in the absence of stresses is itself a function of the temperature. It is therefore difficult to explain the results of experiments in which a dependence of internal friction on the temperature has been obtained.^{94,185,187} In these experiments, carried out using alloys undergoing ordering, it was shown that the internal friction increases appreciably near the Curie point for long-range order and continues to grow as we go to the temperature of disordering, reaching very large values. The explanation of relaxation phenomena in substitution alloys in terms of the reorientation of pairs of dissolved atoms has met objections on the part of Nowick⁴ who, however, did not advance any serious arguments against the admissibility of such an explanation. In essence all his remarks reduced to certain improvements of the relaxation mechanism proposed by Zener. According to Nowick⁴ the relaxation time is equal to

$$\tau = \alpha \Gamma_A^{-1}, \quad (26)$$

where Γ_A is the mean frequency of atomic transitions of the less mobile atoms of type A in the solution A-B, and α is a coefficient of proportionality. The value of Γ_A depends on the temperature according to the formula

$$\Gamma_A = \Gamma_0 e^{-H/RT}, \quad (27)$$

where H is the activation energy of volume diffusion. This energy depends strongly on the concentration of atoms of type A in the solution.^{4,123,128} The degree of relaxation Δ_0 was not determined by Nowick, but it can be estimated approximately by using formula (30).

(b) **Interstitial Solutions.** In an interstitial solid solution free of stress the distribution of dissolved atoms is isotropic with respect to each atom of the

basic lattice. This means that the nearest dissolved atom has the same probability of being situated along any one of the possible crystallographic directions. For example, in a body-centered cubic lattice the interstitial atoms are mostly found not at points with the greatest free volume, but in positions of the type $(\frac{1}{2}, 0, 0)$ (i.e., in the middle of the edges of the cell) or in the centers of the faces $(\frac{1}{2}, \frac{1}{2}, 0)$ which are equivalent to the former positions. The interstitial atoms situated at such points deform the lattice, giving rise to deformations having tetragonal symmetry. The greatest deformations occur in the direction between two nearest atoms of the solvent, i.e., along one of the principal axes $(1, 0, 0)$. In the absence of stresses all three types of interstitial positions are occupied in a disordered way, and therefore each type of these interstitial positions is associated with one-third the total number of interstitial atoms. If tension stress is applied, for example, along the z axis, the isotropic distribution of atoms will be violated; the equilibrium distribution now will be such that the greater number of the dissolved atoms is found in positions with the tetragonal axis z .

The transition to the new equilibrium situation is characterized by a certain relaxation time τ comparable with the lifetime of the interstitial atom in the interstitial position.^{4,181}

$$\tau \cong \frac{1}{\nu_0} e^{H/RT}, \quad (28)$$

where ν_0 is the frequency of oscillation of the interstitial atom in the interstitial position, H is the heat of activation of a mol of interstitial atoms, which depends on the type of interstitial position and on the nature of both the interstitial atoms and the atoms of the solvent.

The following expression has been proposed⁴ for the degree of relaxation of this process

$$\Delta_0 = \frac{2}{9} E_\infty \frac{\lambda^2 n}{kT}. \quad (29)$$

Here $\lambda = \partial \epsilon / \partial p$; p is an internal parameter, equal in our case to $n_z - n/3$, n is the total number of interstitial atoms per unit volume, n_z is the number of atoms in position z , k is the Boltzmann constant, E_∞ is the unrelaxed modulus of elasticity, and T is the absolute temperature. The internal friction due to the rearrangement of interstitial atoms under stress was first observed²³⁹ and explained¹⁵⁹ in the case of α -iron. Later the same phenomenon was observed also in the case of other solid interstitial solutions having a cubic body centered lattice similar to solutions in α -iron. Thus, Kê investigated relaxation effects

associated with the presence of carbon, oxygen,¹¹¹ and nitrogen¹¹² in tantalum. The behavior of carbon in niobium has been investigated by Wert.²⁴⁰ Comparatively recently, papers have appeared on the study of relaxation phenomena associated with the behavior of interstitial atoms in face centered cubic lattices.^{105,106,141-144,241}

In considering relaxation phenomena in solid solutions we have assumed that the dissolved atoms form a homogeneous solid solution at the temperature at which the investigations were made. However, certain metals that are completely soluble in each other at high temperatures no longer mix below a certain critical temperature. In such a case the degree of relaxation, which is defined in the general case by means of^{3,83,184}

$$\Delta_0 = E_\infty \left(\frac{\partial \epsilon}{\partial c} \right)_\sigma^2 : \left(\frac{\partial^2 F}{\partial c^2} \right)_\sigma, \quad (30)$$

(E_∞ is the unrelaxed modulus of elasticity, c is the atomic concentration of the dissolved component, and F is the free energy per unit volume) may be either anomalously small or very large, depending on the extent of the region of solubility, and on the temperature at which the measurement is made.

The curvature $(\partial^2 F / \partial c^2)_\sigma$ is always anomalously large in those phases which have a narrow range of solubility. Consequently, in such phases the relaxation effects associated with atomic diffusion under stress are anomalously small. Conversely, in the case of a wide range of solubility the curvature at a certain value of concentration, particularly near the critical temperature, is close to zero, and the degree of relaxation is anomalously large.

3. Internal Friction Due to Ferromagnetism

Because of the well-known connection between magnetic and mechanical phenomena, all types of magnetic relaxation lead to additional appreciable dissipation of elastic energy of oscillation. It is customary¹⁷⁵⁻¹⁷⁸ to divide "magnetic" losses into:

- (1) losses due to macroscopic eddy currents;
- (2) losses due to microscopic eddy currents;
- (3) losses associated with magnetomechanical hysteresis.

Losses of the first type are due to the relaxation of magnetic flux caused by the appearance of eddy currents produced by the change in the induction in the sample as a whole as a result of its periodic deformation. The relaxation time is determined in this case by the following formula³

$$\tau \cong \frac{d^2}{D}, \quad (31)$$

where d is the transverse dimension of the sample, D is the coefficient of magnetic diffusion given by the following expression³

$$D = \frac{10^8}{0.4\pi k\mu}, \quad (32)$$

k is the specific electric conductivity, and μ is the magnetic permeability.

The degree of relaxation due to the diffusion of magnetic flux is determined by the following expression³

$$\Delta_0 = 4\pi\mu^{-1}\lambda^2 E_{\infty}, \quad (33)$$

where $\lambda = \partial\epsilon/\partial H$ is the tension strain produced by a change in the intensity of the field. As Becker and Döring¹⁷⁷ have shown, the degree of relaxation Δ_0 has its maximum value at a magnetization that is close in magnitude to the remanent magnetization; in this case

$$(\Delta_0)_{\max} \cong 0.3\lambda_s \frac{E_{\infty}}{\sigma_1}, \quad (34)$$

where λ_s is the tension strain, which corresponds to a change in magnetization from zero to saturation, while σ_1 is a measure of the remaining microstresses which may be due to previous plastic deformations or to the presence of impurity atoms.

Losses of the second kind are due to the fact that each domain reacts to the deformation produced by the oscillation of the sample, and consequently local variations of magnetic flux and local eddy currents.

The third type of losses arises when the deformations produce an irreversible displacement of domain boundaries. The relaxation times have not been determined for the second and third types of losses. For the second type of losses it has been found that^{177,178}

$$Q_1^{-1} = \frac{42.6 l^2 J_s^2 \nu^2 \sigma^2}{\pi \rho \sigma_1^2}, \quad (35)$$

where l is the periodicity of the distribution of internal stresses σ_1 , J_s is the limiting magnetization, ν is the oscillation frequency, σ is the applied stress, and ρ is the density. In the case of magnetomechanical hysteresis¹⁷⁸

$$Q_2^{-1} = \frac{4b\sigma E_{\infty}}{3\pi}, \quad (36)$$

where

$$b = \frac{d(E^{-1})}{d\sigma}.$$

The losses of elastic energy accompanying oscillations of a ferromagnetic rod may be divided into their component parts fairly simply. Indeed,

the first type of losses is excluded if the material is in a demagnetized state. In order to separate losses of the second and the third type one can make use of their different dependence on the oscillation frequency.¹⁷⁹

4. Internal Friction Due to Heat Conductivity

Thermal conductivity gives rise the best known relaxation mechanism in metals and alloys, first investigated in detail by Zener.³ According to Zener, the degree of relaxation for a homogeneous isotropic solid is given by

$$\Delta_0 = E_s \frac{T\alpha^2}{\rho c_p}, \quad (37)$$

where E_s is the "adiabatic" modulus of elasticity, T is the absolute temperature, α is the coefficient of linear thermal expansion, c_p is the specific heat at constant pressure, and ρ is the density.

The relaxation time for attaining temperature equilibrium is given by the formula

$$\tau \cong \frac{d^2}{\chi}, \quad (38)$$

where d is the distance (usually the size of the sample), comparable in magnitude with the distance that must be traversed by the heat flux in order to establish temperature equilibrium, and χ is the coefficient of temperature conductivity given by the formula

$$\chi = \frac{\kappa}{\rho c_p}, \quad (39)$$

where κ is the coefficient of heat conductivity, c_p and ρ have the same meaning as before.

Zener has also investigated³ the case when the sample is not isotropic microscopically, i.e., when it consists of separate randomly-oriented individual crystallites whose average linear dimension is d . In this case a macroscopically homogeneous stress applied to the sample will result in a microscopic inhomogeneity of the stress from one crystal to the next. This will lead to a change of temperature in the crystals and will consequently (owing to the temperature gradient in the grains) produce thermal fluxes. The relaxation time will also be given in this case by expression (38), if we take d to mean the average linear dimension of the grain. The degree of relaxation may be estimated¹⁷⁴ by means of formula (37), multiplied by the correction factor

$$R = \frac{(E^{-2}) - (E^{-1})^2}{(E^{-2})}, \quad (40)$$

which determines the relative value of the mean

square deviation of the modulus of elasticity of neighboring crystallites. The factor R shows that the degree of relaxation (and the maximum of internal friction) will increase as the elastic anisotropy of individual crystallites increases.

In going over to anisotropic solids Zener, as was first correctly noted by Isakovich,²⁴² makes a large simplification by taking into account only the equalization of temperatures within the boundaries of each individual crystallite and not taking into account heat exchange between them. Such a simplification leads to a change in the nature of the dependence of internal friction on frequency.

From problems of damping of oscillations of finite bodies investigated by Zener we can go over to an investigation of the absorption of sound in an unbounded medium. The physical problems associated with this, and the asymptotic estimates of the damping coefficient for sound in anisotropic unbounded media have been investigated by Landau and Lifshitz,¹⁷³ and later by Lessen²⁴⁶ in a form which is more compact and tractable than Zener's treatment.

In conclusion we note that internal friction due to thermal conductivity is as a rule of small magnitude, since the degree of relaxation Δ_0 , for example, in the case of α -brass¹⁷⁴ amounts to only 0.0036, while relaxation due to atomic diffusion in the case of the same alloy is ~ 0.5 .

5. Internal Friction Due to the Interaction of the Acoustic Field with Phonons and Conduction Electrons

In the absence of an acoustic field the numbers of phonons and of electrons in any arbitrary state are determined respectively by the Planck and the Fermi distribution functions. When an acoustic field is imposed, for example, by exciting free vibrations of the sample, the distribution functions are altered because of the interaction of the sound waves with the phonons and the electrons. As a result of such interactions, the energy of acoustic vibrations is gradually distributed among all the phonons and all the electrons. The amount of the energy of sound vibrations that is dissipated depends above all on the ratio of the wavelength of sound to the mean free path of the phonons or for electrons.¹⁸⁸

The scattering of sound by phonons when the wavelength of sound is considerably greater than the phonon mean free path was first investigated in references 188 to 191. For experiments carried out at temperatures above the characteristic temperature, the internal friction turned out to be proportional to the frequency of sound vibrations

ν and to the temperature T . The second part of reference 190 treats the scattering of sound due to the interaction not only with phonons but also with conduction electrons. It was under the same conditions found that $Q^{-1} \sim \nu T$. If the temperature at which the experiment is carried out is $T < \theta$, then $Q^{-1} \sim \nu/T^5$.

Comparatively recently the question of the dissipation of energy of sound vibrations as a result of the interaction with conduction electrons, which are assumed to be free, has been again considered in reference 253. For the case of longitudinal waves it was found that

$$Q_1^{-1} = \frac{2Nm}{\rho\omega\tau} \left(\frac{1}{3} \frac{k^2 l^2 \tan^{-1} kl}{kl - \tan^{-1} kl} - 1 \right). \quad (41)$$

Here N is the number of electrons per unit volume, m is the electron mass, ρ is the density of the metal, ω is the circular frequency of the sound, k is the wave number, while τ and l are the mean collision time and the mean free path for the electron, respectively. In the case of transverse waves the magnitude of internal friction is given by

$$Q_2^{-1} = \frac{2Nm}{\rho\omega\tau} \frac{1-g}{g}, \quad (42)$$

where

$$g = 3 \left(2k^2 l^2 \frac{k^2 l^2 + 1}{kl \tan^{-1} kl - 1} \right)^{-1}. \quad (43)$$

In the case of low frequencies, when $\lambda \gg l$ and $kl \ll 1$, we obtain from (41) and (42):

$$Q_1^{-1} \cong \frac{8Nm v_0^2 \tau}{15\rho v_l^2}, \quad (44)$$

$$Q_2^{-1} \cong \frac{2Nm v_0^2 \omega \tau}{5\rho v_s^2}, \quad (45)$$

where ν_0 is the electron velocity, while ν_l and ν_s are the velocities of the longitudinal and the transverse waves respectively.

In the case of high frequencies, when $\lambda \approx l$ and $kl \approx 1$,

$$Q_1^{-1} \cong \frac{2\pi Nm v_0}{6\rho v_l} \quad (46)$$

and

$$Q_2^{-1} \cong \frac{16Nm v_0}{3\rho v_s \omega}. \quad (47)$$

The last condition is fulfilled in the region of ultrahigh frequencies ($\nu \sim 10^6$ cps) at very low temperatures, when the electron mean free path becomes comparatively large. Experimental work^{250,251} on the dissipation of ultrasound in pure

metals at very low temperatures (Figs. 2 and 3) is in satisfactory agreement with the theory. However, Pippard's theory²⁵³ does not explain the sharp falling off in the absorption sound when the metal goes over into the superconducting state (Fig. 2, curve V). An attempt to treat this interesting problem was made by Mason.²⁵²

At very high frequencies ($\nu \sim 10^9$ cps) the interaction of phonons with conduction electrons plays an essential role. This interaction has been taken into account by Paranjape,¹⁹² who found for the case of longitudinal waves:

$$Q^{-1} = 4 \left(\frac{\pi}{6} \right)^{\frac{1}{3}} \frac{v_l n^2 n_0^{\frac{2}{3}} e^2}{k} \frac{\partial \sigma}{\partial T}; \quad (48)$$

here v_l is the velocity of sound, n is the number of electrons in an atom, n_0 is the number of atoms per unit volume, e is the electronic charge, k is the wave number, and σ is the specific electric resistance. In this case, too, the dissipation of energy turns out not to depend on the frequency. It is not possible to check this theory since experiments at such frequencies have not yet been performed.

6. Internal Friction Due to the Migration of Atoms in a Stress Field

An essential role may also be played by irreversible processes other than relaxation. Some of these have been considered in the preceding section. In Secs. 6 and 7 we shall extend our investigation of the dissipation of energy due to various irreversible processes.

As is well known, the crystal lattice of a real crystal may possess various types of imperfections (defects). These imperfections appear either during crystallization from a melt, or as the result of a preceding plastic deformation, and also of thermal motion of atoms.¹⁹⁴ The presence of defects produces continuous displacements of atoms in the crystalline lattice. Each transition of an atom is associated with overcoming a certain potential barrier u_i , whose magnitude depends on the type of the transition and on the state of the sample as a whole. The probabilities of individual transitions are proportional¹⁹⁵ to $e^{-u_i/kT}$

On considering various transitions of atoms within the lattice of the sample we find¹³⁷ that the total number of different transitions per unit time per unit volume is

$$\omega' = \omega_0 e^{-u_i/kT}. \quad (49)$$

In an unstressed state such transitions occur at random and in such a way that the mean thermal

energy of the crystal at constant temperature remains constant. This means that there is no spontaneous dissipation of energy by a crystal which is in thermal equilibrium with the surroundings.

In a deformed sample each volume element of the sample will experience a shearing stress $\theta(t)$ (for example, in torsional oscillations). In this case the number of irreversible transitions leading to the dissipation of elastic energy will be determined by the difference between the total number of transitions in the presence of the stress field and the number of transitions which are caused by thermal fluctuations in the absence of the field.

Therefore, the number of irreversible transitions per unit time due to the presence of the stress field is equal to

$$\Delta\omega = \omega'' \left(\frac{u}{kT} - \frac{\beta\bar{\theta}}{kT} \right) - \omega' \left(\frac{u}{kT} \right), \quad (50)$$

where $\beta\bar{\theta}$ is the average change in the potential barrier u caused by the shearing stress $\bar{\theta}$. Since these stresses are small we have $\beta\bar{\theta} \ll kT$, and the parameter $\beta\bar{\theta}/kT$ will be small compared with unity. On expanding ω'' into a Taylor's series in powers of the small parameter $\beta\bar{\theta}/kT$, and on restricting ourselves to first-order terms, we obtain

$$\Delta\omega \cong \frac{\beta\bar{\theta}}{kT} \omega_0 e^{-u_i/kT}. \quad (51)$$

Introducing the mean "activation" energy H per mole, we obtain

$$\Delta\omega \cong \frac{\beta\bar{\theta}}{kT} \omega_0 e^{-H/RT}. \quad (52)$$

The above expression for $\Delta\omega$ may be easily related to the magnitude of the internal friction Q^{-1} for this process. Indeed, the relative energy $\Delta U/U$ dissipated per cycle of oscillations is equal to $(a_n^2 - a_{n+1}^2)/a_n^2$ where a_n and a_{n+1} are two successive amplitudes separated by a time interval equal to one period. The energy dissipated per unit time will be larger by a factor ν . Assuming that this energy is proportional to the number of irreversible transitions $\Delta\omega$, we obtain

$$\frac{a_n^2 - a_{n+1}^2}{a_n^2} = \alpha \Delta\omega.$$

On taking account of the fact that $Q^{-1} = \frac{\ln \frac{a_n}{a_{n+1}}}{\pi}$, we obtain

$$\Delta\omega = \frac{\nu}{\alpha} (1 - e^{-2\pi Q^{-1}}).$$

Since $Q^{-1} < 0.1$, we can expand the exponential

within the brackets of the preceding expression into a series in powers of $2\pi Q^{-1}$ and restrict ourselves to the first two terms of the expansion. Taking (52) into account, we then obtain^{93,137}

$$Q^{-1} = \frac{\alpha\beta\bar{\theta}\omega_0}{\omega kT} e^{-H/RT}. \quad (53)$$

From this expression for Q^{-1} it can be seen that internal friction is inversely proportional to the frequency ω and directly proportional to the average value of the external shearing stress $\bar{\theta}$. Moreover, internal friction will decrease as the "activation energy" H , characterizing the possibility of various types of atomic transitions within the lattice, increases. The internal friction determined by (53) increases with increasing temperature, reaching a maximum at a temperature

$$T_m = \frac{H}{R}, \quad (54)$$

whose magnitude depends on the value of H .

7. Internal Friction Due to Static Hysteresis

In contrast to dynamic hysteresis⁵³ which occurs in an oscillating sample when the deformation is not a single valued function of the stress, hysteresis can also occur in several solids when they are statically stressed. Such a possibility arises only in that case when there remains after the removal of the stress a constant remanent deformation that can be reduced to zero only by applying a stress in the opposite direction. When the stress is varied between the limits $\pm\sigma$ the result is a static hysteresis loop whose area characterizes the energy losses (similar to the magnetomechanical hysteresis discussed earlier).

Static hysteresis is possible principally¹⁹⁶ in the case of stresses that lead to plastic deformation. Therefore the latter are of importance in the study of fatigue and are not very important in the study of internal friction at small amplitudes. However, it has been shown^{84,197} comparatively recently that the damping of oscillations in single crystals at deformations on the order of 10^{-7} , a damping that can be ascribed to the motion of dislocations, is also accompanied by static hysteresis. Thus, static hysteresis is possible also in the case of very small amplitudes as a result of a particular kind of atomic rearrangement (or of magnetic rearrangement in the case of magnetoelastic hysteresis). The speed of such rearrangements is of the order of the speed of sound, and they can therefore be considered to be practically "instantaneous" in the case of low-frequency measurements of internal friction. In con-

trast to the relaxation type of internal friction, static hysteresis shows a noticeable amplitude dependence even at small amplitudes.

In the case of static hysteresis the internal friction is given by the following formula^{4,53}

$$Q^{-1} = \frac{E_\infty}{\pi} \left[(A_0 - a_0) + \frac{2}{3}(A_1 - a_1)\sigma_1 + \frac{1}{2}(A_2 - a_2)\sigma_2^2 + \dots \right], \quad (55)$$

where E_∞ is the unrelaxed modulus of elasticity, A_0, A_1, A_2 are the coefficients in the expansion of $d\epsilon''/ds = F(\sigma)$ (as the load is applied) into a power series, a_0, a_1, a_2 are the coefficients in the expansion of $d\epsilon''/d\sigma = f(\sigma)$ (as the load is removed) into a power series; ϵ'' is the inelastic part of the total deformation ϵ ; $\sigma = \sigma_1 \cos \omega t$ is the periodically varying stress which appears in the sample when the load is applied, and σ_1 is the amplitude of this stress.

In the case of a loop which has the form of a parallelogram in which F and f are constant, the magnitude of the internal friction Q^{-1} will not depend on the stress amplitude σ_1 . In general, if there are no grounds for assuming such a shape for the loop, the existence of static hysteresis is closely related to the amplitude-dependent dissipation of energy even at very small amplitudes. This characteristic amplitude dependence of Q^{-1} , together with its independence of the frequency in the case of static hysteresis, makes it possible to distinguish relaxation phenomena from phenomena belonging to this class.

8. Internal Friction and Dislocations

The internal friction of a well annealed material is usually of the order 10^{-5} , although its magnitude depends on the purity of the metal and on the amplitude of the oscillations. However, this minimum of internal friction also depends strongly on the experimental conditions, particularly in the case of single crystals. In references 42, 199, and 200 it was reported that internal friction increases strongly as a result of weak shocks, and even as a result of different manipulations of the crystal. Moreover, single crystals prepared from the same material and by the same method differ sometimes by more than a factor ten with respect to their internal friction.^{201,202} Even different parts of the same crystal have unequal internal friction.¹⁹⁷ An even greater scattering of the values of internal friction is observed in the case of materials that had been previously deformed.^{24,32,38,42,85,90,94,96,98,100,101,108,114,118,128,199,200,202-225}

The amplitude dependence of internal friction observed in the case of annealed materials and of

"cold worked" materials is difficult to explain within the framework of the relaxation theory of internal friction. In connection with this, attempts have been made recently to utilize the theory of dislocations to explain the phenomena mentioned above.^{24,80,84,147,198,214,216,222,227-232} The dislocation theory of internal friction is based on Koehler's model.²²⁹ According to this model the dissipation of energy is due to the oscillations under the influence of a periodic external force of the dislocation loop associated with impurities.

It is assumed that the pure single crystal contains a network of dislocations prior to the deformation. When an external stress is applied a deformation due to dislocations takes place in addition to the elastic deformation, and is called a deformation dislocation. The qualitative behavior of a dislocation under the influence of an increasing external force is shown²³² in Fig. 25. In the case of zero stress the length L_N of the dislocation is linked by impurity particles (Fig. 25, A). For very small stresses (B) the loops L_c sag and continue to sag until the disruptive stress has been reached (C-D). As the stress is increased further the total length of the dislocation increases (D-F), until finally a closed dislocation loop (G) is formed. Because of this process, the dislocation deformation must be irreversible and can therefore be classed as a plastic deformation.

A consistent examination of this mechanism leads to the conclusion that losses may be of two types: those that depend on the frequency of oscillation of the sample, and those that do not. In case of losses of the first type the internal friction can be represented by the formula²³²

$$Q_1^{-1} = \frac{\Delta_0 \Lambda L^2}{2\pi D} \left[\frac{\Omega}{(1-\Omega^2)^2 + \frac{\Omega^2}{D^2}} \right], \quad (56)$$

where Λ is the total length of the moving dislocation lines, L is the length of the dislocation loop,

$$\Delta_0 = \frac{8\mu a^2}{\pi^2 c},$$

μ is the shear modulus, a is Burgers' vector,

$$D = \frac{\omega_0 B}{A}, \quad \Omega = \frac{\omega}{\omega_0},$$

ω_0 is the resonance frequency, and ω is the frequency of oscillation of the sample. The quantities A , B , and C are the coefficients in the equation of motion of bound dislocations:

$$A \frac{\partial^2 \xi}{\partial t^2} + B \frac{\partial \xi}{\partial t} - C \frac{\partial^2 \xi}{\partial y^2} = a\sigma,$$

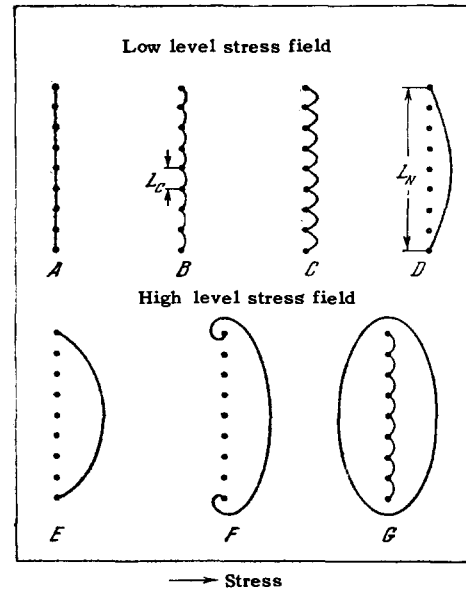


FIG. 25. Schematic diagram of the formation of dislocation loops in a stress field.

where $\xi(x, y, t)$ is the displacement of an element of the dislocation loop from its equilibrium position, y is the coordinate of this element; σ is the stress.

In the case of losses of the second type the internal friction is given by

$$Q_2^{-1} = \frac{\Delta_0 \Lambda L_N^2}{2\pi^2} \left(\frac{L_N}{L_c} \right) \left[\frac{\Gamma}{\sigma_0} - 1 \right] e^{-\frac{\Gamma}{\sigma_0}}, \quad (57)$$

where $\Gamma = L_b \sigma / L_c$, L_b is the length of the dislocation, $\sigma = \sigma_0 \cos(\omega t - kx)$ is the stress, and the remaining quantities have their previous meaning.

The theory presented briefly above does not apply to high-temperature regions. The first attempt to solve the problem of high-temperature internal friction from the point of view of dislocation theory was made by Weertman.²⁴³ His calculation is based on the results of the theory of microcreep^{244,245} and of the theory of quenching.²⁴⁴ Assuming that a low amplitude stress causes the dislocations to move in the glide plane Weertman obtains for the internal friction an approximate equation of the following form:

$$Q_1^{-1} = Q_0^{-1} e^{-H/kT}, \quad (58)$$

where the factor in front of the exponential is given by

$$Q_0^{-1} = \frac{3a\mu b^2 N v_0}{2\pi^4{}^{4/3} \nu k T c^{13/9} \beta^{2/3} \left(\ln \frac{1}{c} \right)^{2/3}}, \quad (59)$$

and the heat of activation is given by

$$H = \frac{3}{4^{2/3} \pi} b^3 c^{9/3} \mu^{2/3} \left(\ln \frac{1}{c} \right)^{2/3}. \quad (60)$$

The last two expressions contain the following quantities: α is a constant equal to $1/25$, μ is the shear modulus, b is the distance between nearest neighboring atoms, N is the density of dislocations, ν_0 is the frequency of oscillation of a dislocation line in its potential well, ν is the frequency of the oscillation of the sample, c is the impurity concentration, and $\beta = (1/a) (da/dc)$ where a is the lattice constant of the alloy. A displacement of a dislocation in the stress field created by the external periodic force may take place not only in the glide plane.

By examining other possibilities for the displacement of dislocations in a stress field, Weertman obtains another expression for the magnitude of internal friction at high temperatures:

$$Q_2^{-1} = \frac{\alpha \mu b^5 \nu_0^* N}{2\pi \nu k T} e^{\frac{H^* - ST}{kT}}, \quad (61)$$

where ν_0^* is the frequency of oscillation of a vacancy, H^* is the activation energy for self diffusion, and S is the activation entropy for self diffusion. The other symbols remain the same.

The second mechanism proposed by Weertman can also be applied to low-frequency oscillations, but again only for small amplitudes, $\sim 10^{-7}$. It is well known that all the low frequency methods ($\nu \sim 1$ cps) utilize oscillation amplitudes $\sim 10^{-5}$ and greater, which excludes the possibility of applying Weertman's theory to the numerous experimental results obtained by these methods.

A comparison of the theory with the experimental results obtained by high-frequency methods leads, for example in the case of aluminum, to the following results.

The theory yields $H = 11,000$ cal/g at., $\Delta_0 \cong 270$; while calculations based on two points of the experimental curve give $H = 12,900$ cal/g at., $\Delta_0 = 100$. The agreement between theory and experiment in this case is thus quite satisfactory. The degree of success in explaining internal friction by means of the theory of dislocations has so far not been very great, because internal friction of deformed samples is a very complicated phenomenon that depends on many factors: on the degree and nature of preceding deformation, on the time of aging, on the presence of impurities in the sample, on the frequency of oscillations, on the temperature at which measurements are carried out, etc. To attain a greater degree of success it is necessary to carry out systematic investigations, taking into account many parameters that affect internal friction, particularly at low frequencies, when static hysteresis is observed.

IV. APPLICATION OF THE THEORY TO THE EXPLANATION OF THE TEMPERATURE DEPENDENCE OF INTERNAL FRICTION OF METALS AND ALLOYS

From a survey of experimental data it is easily seen that the problem of temperature dependence of internal friction has been studied mostly at temperatures above 0°C and largely by low-frequency methods.

We shall therefore utilize the theoretical picture of internal friction developed above to explain the temperature dependence $Q^{-1}(T)$ of pure metals and alloys in the case of low-frequency oscillations ($\nu \sim 1$ cps). Of the pure metals we shall select for this purpose aluminum, the metal most investigated in all respects, and of the alloys we shall take, for example, Ni-Fe. Earlier we have said that if the total internal friction does not depend on the time and is small ($Q^{-1} < 0.1$), then it can be expressed as the sum of contributions made by the different dissipative mechanisms acting independently of one another. In computing the total value of internal friction it is necessary to take into account the constant contribution due to the various losses in the apparatus. This part of the losses depends on the construction of the apparatus and on the method of measurement. However, it does not exceed the value $\sim 1 \times 10^{-4}$. Thus

$$Q^{-1} = Q_a^{-1} + \sum_{i=1}^n Q_i^{-1}, \quad (62)$$

where Q_i^{-1} are the contributions made by the various dissipation mechanisms. Let us make an estimate of the magnitude of the various contributions for the case of low frequency torsional oscillations of an aluminum sample. In references 93 and 137 such an estimate has been made for various pure metals. In particular, in the case of a single crystal of pure aluminum (99.98% Al), an estimate of the contributions of the various dissipative mechanisms discussed above yields temperature dependence of internal friction in the form

$$Q_{Al}^{-1} = 1.1 \cdot 10^{-5} + 4.5 \cdot 10^{-6} t, \quad (63)$$

where t is the temperature in $^\circ\text{C}$ at which the measurements were made.

In Fig. 26 this quantity is represented by the straight line III which coincides with the experimental curve I for the internal friction of a single crystal aluminum sample only up to 240°C . The reason for the discrepancy, as we shall see later, is the imperfection of real single crystals. From the same diagram it can also be seen that the internal friction of polycrystalline aluminum (II) differs from the internal friction of single-crystal

aluminum (I). The appearance of a maximum on curve II may be explained on the basis of the concept of a "two-component system" discussed earlier. In the present case one of the components will be the grain (assumed to be elastic), while the other component will be the grain boundary (assumed to be viscous). When such a system undergoes periodic deformation, displacements along the grain boundaries take place and lead to dissipation of energy. This mechanism in the case of aluminum was discussed in detail for the first time in reference 15 and again in references 93, 137, and 224. Using the known values of Δ_0 and τ for polycrystalline aluminum, an evaluation was made, with the aid of formula (15), of the contribution to the internal friction due to the shift in the grain boundaries, on the assumption that this mechanism can be described by one relaxation time τ and one heat of activation H . The calculations gave the curve IV shown in Fig. 26, which differs strongly from the experimental curve II. This discrepancy can be partially eliminated by assuming that a displacement along the grain boundaries corresponds to a spectrum of relaxation times and to a set of heats of activation with the maximum density in the neighborhood of a certain mean value. By making a suitable choice of τ_i in the neighborhood of τ_{av} , of $(\mu_\infty/\mu_0)_{av}$, and of H_{av} it is possible to obtain the curve V. This curve taken together with the straight line III still does not give a complete description of the experimental curve II, particularly at high temperatures. But at high temperatures the dissipation mechanism associated with migration of atoms in the stress field becomes important. Its contribution is described by formula (57). In references 93 and 137 the following value for this contribution was obtained

$$Q_1^{-1} = \frac{1.13 \cdot 10^4}{T} e^{-\frac{8,900}{RT}}, \quad (64)$$

which is shown in Fig. 26 by the curve VI. Now the sum of all the contributions given by the curve VII agrees well with the experimental curve II.

Returning to the internal friction of single-crystal aluminum, we can explain the discrepancy between the experimental curve of internal friction (I) with the straight line for the internal friction (III) by the fact that the dissipation of energy due to various transitions of atoms in the lattice in the stress field takes place also in the single-crystal state of the material. The internal friction in the case of this mechanism is given by^{93,137}

$$Q_2^{-1} = \frac{7.12 \cdot 10^4}{T} e^{-\frac{13,800}{RT}}. \quad (65)$$

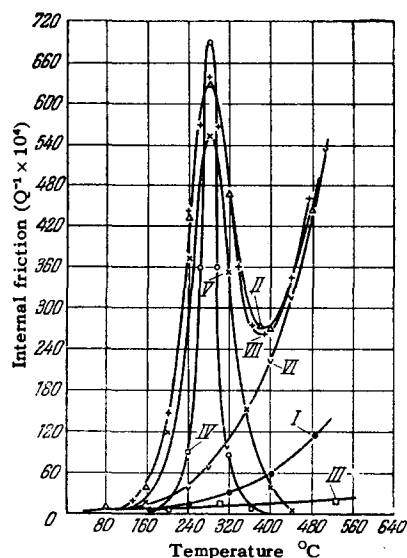


FIG. 26. Temperature dependence of internal friction in aluminum: I and II are experimental curves; III – VII are theoretical curves.

By comparing the quantity Q_1^{-1} for polycrystalline aluminum [formula (64)] with the value of internal friction Q_2^{-1} for single-crystal aluminum [formula (65)], we see that the parameter H is considerably larger for the single-crystal state of aluminum than for the polycrystalline state. Such a result does not contradict the modern concept of the single-crystal state of the metal as being a more perfect state (in the sense of the degree of order in the position of the particles), than the polycrystalline state. A similar satisfactory result was obtained^{93,137} for the internal friction $Q^{-1}(T)$ of copper, nickel, iron, and cobalt (at high temperatures, since at intermediate temperatures Q_0^{-1} is not constant because of the allotropic transitions $C_{0\beta} \rightleftharpoons C_{0\alpha}$).

In our earlier considerations of the temperature dependence of internal friction of alloys we arrived at the conclusion that, firstly, the internal friction of alloys is considerably smaller than the internal friction of pure metals of which the alloy is composed almost over the whole range of temperatures; secondly, the maximum in the internal friction due to viscous flow along grain boundaries often degenerates into an inflection point as the concentration of dissolved atoms is increased; thirdly, in the low temperature region there appear maxima which are absent in the case of pure metals. Since the nature of the internal friction of alloys changes gradually as the concentration of the dissolved atoms increases,^{103,104,137} we can say that the internal friction of alloys can also be represented (with a few exceptions²³³) as a sum of contributions Q_i^{-1} .

By analyzing the different contributions Q_i^{-1} ,

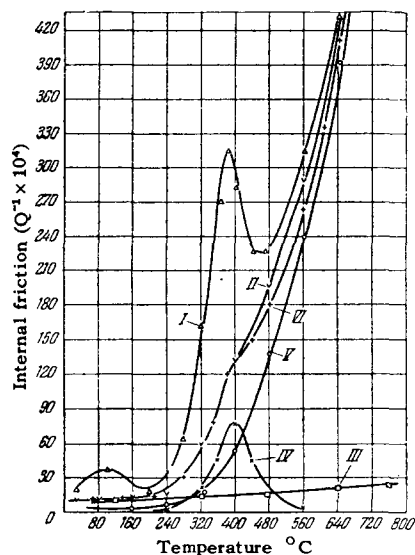


FIG. 27. Temperature dependence of internal friction in nickel (I) and in a solid solution of iron (2.6% Fe) in nickel. I and II are experimental curves; III–VI are theoretical curves.

for example, for Ni-Fe¹³⁷ in a manner similar to that employed in the case of pure metals, and by representing the results in graphical form, we obtain Fig. 27. In this diagram curve I represents the temperature dependence of internal friction of pure nickel (99.99% Ni), and curve II represents the temperature dependence of internal friction of a solid solution of iron in nickel. The straight line III represents the internal friction associated with all the processes that give a linear dependence of Q^{-1} on the temperature:

$$Q_{II}^{-1} = 8.5 \cdot 10^{-4} + 2.27 \cdot 10^{-6}t. \quad (66)$$

Curve IV represents that part of the internal friction which is due to the viscous slipping along the grain boundaries, and which is determined by formula (16); the mean heat of activation for such a process is $\sim 60,000$ cal/mole. Curve V represents the contribution due to the migration of atoms in the stress field; its value is

$$Q_V^{-1} = \frac{28.2 \cdot 10^3}{T} e^{-\frac{12,000}{RT}}. \quad (67)$$

Finally, curve VI gives the sum of all these contributions and, as can be seen, it quite satisfactorily reproduces the shape of the experimental curve II. For pure nickel the mean heat of activation of the process of relaxation of stress along the grain boundaries is 63,000 cal/mole, while the value of the maximum for this contribution is $\sim 165 \times 10^{-4}$. The contribution due to the migration of atoms is given by

$$Q_{Ni}^{-1} = \frac{2.76 \cdot 10^3}{T} e^{-\frac{8200}{RT}}. \quad (68)$$

On intercomparing the results obtained above for the internal friction of pure nickel and of the solid solution of iron in nickel (similar results have also been obtained for many other metals and alloys¹³⁷) we see that a reduction in the internal friction of alloys in comparison with pure metals is due to the decrease, in the first instance, of the magnitude of the contributions associated with viscous slipping along grain boundaries, and with the migration of atoms in the stress field (through an increase in the average "activation" energy H for the various types of transitions of the atoms in the alloy lattice).

The low temperature maxima occurring in the case of alloys and of certain not quite pure metals (iron, tantalum, and others), are associated with the presence of foreign atoms (such as hydrogen, carbon, nitrogen, boron, oxygen, etc.), and are due primarily to the preferential distribution of the atoms in the stress field.¹⁵²⁻¹⁶⁶

CONCLUSION

The material presented above enables us to say that the physical picture of the dissipation of elastic energy in oscillating samples is sufficiently clear. The existing theories describe (qualitatively, and in certain cases, also quantitatively) in a satisfactory manner the temperature dependence of internal friction of well-annealed metals and alloys which do not undergo phase transitions on heating. First attempts have been made to describe the behavior of internal friction of plastically deformed metals and alloys. However, the theory of internal friction, in view of the complexity of the phenomenon of energy dissipation itself, and of the small number of reliable systematic investigations (particularly of plastically deformed metals and alloys), which take into account the fact that internal friction always depends on many factors, is still far from complete. There are very few investigations (either theoretical or experimental) of internal friction at high temperatures. And yet investigations of internal friction at high temperatures provide us with still another possibility of gaining an understanding of the problem of the behavior of alloys at high temperatures which should not be neglected.

¹B. Hopkinson, G. T. Williams, Proc. Roy. Soc., Ser. A, 87, 502 (1912).

²O. Föppl, J. Iron a. Steel Inst. 134, 393 (1936).

³C. Zener, Elasticity and Anelasticity of Metals, 1948.

⁴A. S. Nowick, Progress in Metal Physics 4, 1 (1953).

- ⁵C. Zener, *J. Appl. Phys.* **18**, 1022 (1947).
- ⁶T. S. Kê, M. Ross, *Rev. Scient. Instr.* **20**, 795 (1949).
- ⁷W. Weber, *Poggendorf's Ann.* **34**, 247 (1937).
- ⁸K. Jokibe, S. Sukai, *Phil. Mag.* **42**, 397 (1921).
- ⁹E. A. Kopilovich, *Sov. Phys.* **3**, 561 (1933).
- ¹⁰A. Gemant, W. Jackson, *Phil. Mag.* **23**, 960 (1937).
- ¹¹F. Förster, H. Breitfeld, *Z. Metallkunde* **30**, 343 (1938).
- ¹²Е. В. Lunts, *Прикладная математика и механика (Appl. Math. and Mech.)* **1**, 331 (1938).
- ¹³J. Norton, *Rev. Sci. Instr.* **10**, 77 (1939).
- ¹⁴P. Chevenard, *Rev. Met.* **39**, 65 (1942).
- ¹⁵T. S. Kê, *Phys. Rev.* **71**, 533 (1947).
- ¹⁶Ch. Boulanger, *Rev. Met.* **46**, 321 (1949).
- ¹⁷R. Kamel, *Phys. Rev.* **75**, 1606 (1949).
- ¹⁸C. A. Wert, *J. Appl. Phys.* **20**, 29 (1949).
- ¹⁹M. E. Fine, *J. Metals* **2**, 1322 (1950).
- ²⁰F. K. Gorskii, *J. Tech. Phys. (U.S.S.R.)* **20**, 1111 (1950).
- ²¹L. Glikman and K. Shishokina, *Завод. лаб. (Plant Laboratory)* **11**, 1365 (1951).
- ²²V. I. Prosvirin and N. N. Morgunova, *Труды ЦНИИТМАШ (Trans. of Central Scientific Research Institute for Technology and Machine Building) No. 40* (1951).
- ²³J. W. Jensen, *Rev. Scient. Instr.* **23**, 397 (1952).
- ²⁴R. E. Maringer, *J. Appl. Phys.* **24**, 1525 (1953).
- ²⁵C. C. Lo and M. Gensamer, *Acta Metallurgica* **1**, 483 (1953).
- ²⁶V. S. Postnikov, *Труды Кемеровского горного института (Trans. Kemerovo Mining Inst.)* **1**, 211 (1954).
- ²⁷K. Misek, *Чех. физ. журнал (Czech. Phys. J.)* **4**, 67 (1954).
- ²⁸S. O. Tsobkallo and V. A. Chelnokov, *J. Tech. Phys. (U.S.S.R.)* **24**, 499 (1954).
- ²⁹C. Ang and C. Wert, *J. Appl. Phys.* **25**, 1061 (1954).
- ³⁰B. S. Berry, *Rev. Scient. Instr.* **26**, 884 (1955).
- ³¹V. S. Postnikov and Yu. V. Piguzov, *ИТЭИН АН СССР (Inst. Tech. Inform., Acad. Sci. U.S.S.R.) "Tools and Stands" PS - 55 - 448*.
- ³²G. B. Brook and A. H. Sully, *Acta Metall.* **3**, 460 (1955).
- ³³S. Weinig, *Rev. Scient. Instr.* **26**, 91 (1955).
- ³⁴S. L. Quimby, *Phys. Rev.* **25**, 558 (1925).
- ³⁵J. Zacharias, *Phys. Rev.* **44**, 116 (1933).
- ³⁶L. Poltavskii, *Sov. Phys.* **3**, 555 (1953).
- ³⁷W. T. Cooke, *Phys. Rev.* **50**, 1158 (1936).
- ³⁸R. L. Wegel and H. Walther, *Physics* **6**, 141 (1936).
- ³⁹D. Bencroft and R. B. Jacobs, *Rev. Scient. Instr.* **9**, 279 (1938).
- ⁴⁰R. H. Randall, F. C. Rose, and C. Zener, *Phys. Rev.* **56**, 343 (1939).
- ⁴¹A. Gemant, *J. Appl. Phys.* **11**, 647 (1940).
- ⁴²A. W. Lawson, *Phys. Rev.* **60**, 330 (1941).
- ⁴³T. S. Kê, *J. Appl. Phys.* **20**, 1226 (1949).
- ⁴⁴P. G. Bordoni, *Nuovo cimento* **7**, 162 (1950).
- ⁴⁵A. V. Siefert and F. T. Worrell, *J. Appl. Phys.* **22**, 1257 (1951).
- ⁴⁶J. K. Pättison, *Rev. Scient. Instr.* **25**, 67 (1954).
- ⁴⁷K. Tanaka, H. Abe, and K. Hirano, *J. Phys. Soc. Japan* **10**, 454 (1955).
- ⁴⁸R. Cabarat, *Akust. Beihefte* **1**, 200 (1956).
- ⁴⁹C. Peterson, *Arch. techn. Messen* **164**, 74 (1949).
- ⁵⁰E. Diepschlag and H. Müller, *Arch. Metallkunde* **3**, 400 (1949).
- ⁵¹E. S. Sorokin, *Central Scientific Research Institute for Commercial Structures No. 15* (1954).
- ⁵²G. Kol'skii, *Волны напряжения в твердых телах (Stress Waves in Solids)* GILL, 1955, Ch. V, VI.
- ⁵³G. Löschner, *Technik* **11**, 289 (1956).
- ⁵⁴V. D. Kuznetsov, *Физика твердого тела (The Physics of Solids) vol. 4, Ch. 30*, 1947.
- ⁵⁵C. Barus, *Phil. Mag.* **29**, 337 (1890).
- ⁵⁶R. Voigt, *Ann. d. Phys.* **47**, 671 (1892).
- ⁵⁷W. Heydweiber, *Wied. Ann.* **63**, 56 (1897).
- ⁵⁸Reiger, *Phys. Z.* **2**, 213 (1901).
- ⁵⁹B. P. Veinberg, *J. Russ. Phys. Soc.* **36**, 47 (1904).
- ⁶⁰Trouton and Rankine, *Phil. Mag.* **8**, 538 (1904).
- ⁶¹B. P. Veinberg, *Записки Новосибирского университета (Notes, Novosibirsk University)* **105**, 157 (1905).
- ⁶²B. P. Veinberg, *J. Russ. Phys. Soc.* **44**, 201 (1912).
- ⁶³B. P. Veinberg, *J. Russ. Phys. Soc.* **45**, 701 (1913).
- ⁶⁴K. Honda, and S. Konno, *Phil. Mag.* **42**, 115 (1921).
- ⁶⁵B. P. Veinberg, *Изв. Томского техн. ин-та (Bull. Tomsk Tech. Inst.)* **43**, 39 (1923).
- ⁶⁶V. D. Kuznetsov, *Изв. Томского техн. ин-та (Bull. Tomsk Tech. Inst.)* **43**, 47 (1923).
- ⁶⁷B. P. Veinberg, *Изв. Томского техн. ин-та (Bull. Tomsk Tech. Inst.)* **45**, 27 (1924).
- ⁶⁸V. D. Kuznetsov and L. A. Shvirk, *Сообщен. о научно-технических работах в республике (Communications on the Scientific and Technical Work in the Republic)* **14**, 1 (1924).
- ⁶⁹V. D. Kuznetsov, *Изв. Томского техн. ин-та (Bull. Tomsk Tech. Inst.)* **46**, 42 (1925).
- ⁷⁰V. D. Kuznetsov and L. A. Shvirk, *Изв. Томского государственного университета (Bull. Tomsk State Univ.)* **75**, 99 (1925).
- ⁷¹V. D. Kuznetsov, *J. Russ. Phys. Soc.* **56**, 295 (1928).

- ⁷² F. Hettwer, Wien. Ber., IIa, **134**, 51 (1925).
- ⁷³ G. Subrahmaniam and D. Gunnaija, Phil. Mag. **49**, 711 (1925).
- ⁷⁴ G. Subrahmaniam, Phil. Mag. **50**, 716 (1925).
- ⁷⁵ V. D. Kuznetsov, Fifth Congress of Russian Physicists, Moscow, 1926, p. 24.
- ⁷⁶ V. D. Kuznetsov, J. Russ. Phys. Soc. **60**, 339 (1928).
- ⁷⁷ B. Ya. Pines, J. Russ. Phys. Soc. **59**, 157 (1927).
- ⁷⁸ V. S. Postnikov, Usp. Fiz. Nauk **53**, 87 (1954).
- ⁷⁹ T. Kamoto, Nature **175**, 948 (1955).
- ⁸⁰ P. G. Bordoni, Ric. Scient. **19**, 851 (1949).
- ⁸¹ P. G. Bordoni, J. Acoust. Soc. **26**, 495 (1954).
- ⁸² J. Weertman, E. J. Salkovitz, Acta Metallurgica **3**, 1 (1955).
- ⁸³ W. B. Nowack, Phys. Rev. **85**, 710 (1952).
- ⁸⁴ A. S. Nowick, Phys. Rev. **80**, 249 (1950).
- ⁸⁵ D. H. Niblett and J. Wilus, Phil. Mag. **1**, 415 (1956).
- ⁸⁶ L. Rotherham, A. Smith, and B. and G. Grunough, J. Inst. Met. **79**, 439 (1951).
- ⁸⁷ P. G. Bordoni, J. phys. et radium **16**, 285 (1955).
- ⁸⁸ J. Barducci, Ric. Scient. **24**, 2025 (1954).
- ⁸⁹ H. Barnes, C. Zener, Phys. Rev. **58**, 87 (1940).
- ⁹⁰ E. G. Stanford, Research. **6**, 21 (1953).
- ⁹¹ E. G. Stanford, Rev. Met. **51**, 674 (1954).
- ⁹² H. Birnbaum and M. Levy, Acta Metall. **4**, 84 (1956).
- ⁹³ V. S. Postnikov, J. Tech. Phys. (U.S.S.R.) **24**, 1599 (1954).
- ⁹⁴ F. Förster and W. Köster, Z. Metallk. **29**, 116 (1937).
- ⁹⁵ K. J. March, Acta Metall. **2**, 530 (1954).
- ⁹⁶ W. Köster, L. Baugert, and W. Lang, Z. Metallkunde **46**, 84 (1955).
- ⁹⁷ W. Köster, Z. Metallkunde **39**, 9 (1948).
- ⁹⁸ A. Ya. Samoïlova and V. S. Postnikov, Труды Кемеровского пединститута (Trans. Kemerovo Pedagogical Inst.) **3** (1957) (in press).
- ⁹⁹ R. March and D. Hall, Trans. AIME **197**, 937 (1953).
- ¹⁰⁰ W. Köster, L. Baugert, and J. Hafner, Z. Metallkunde **47**, 224 (1956).
- ¹⁰¹ A. Ya. Samoïlova and V. S. Postnikov, Dokl. Akad. Nauk S.S.S.R. **114**, 1228 (1957).
- ¹⁰² J. N. Pratt, W. J. Bratina, and B. Chalmers, Acta Metallurgica **2**, 203 (1954).
- ¹⁰³ V. S. Postnikov, Труды Кемеровского пединститута (Trans. Kemerovo Pedagogical Inst.) **I**, 191 (1956).
- ¹⁰⁴ V. S. Postnikov, Физика металлов и металловедение (Phys. of Metals and Metal Res.) **4**, 344 (1957).
- ¹⁰⁵ T. S. Kê, C. T. Tsien, and K. Misek, Scientia Sinica **4**, 519 (1955).
- ¹⁰⁶ T. S. Kê, C. T. Tsien, and K. Misek, Cs. Cas. Fys. **6**, 37 (1956).
- ¹⁰⁷ W. Köster, Z. Metallkunde **39**, 1 (1948).
- ¹⁰⁸ T. S. Kê, Met. Techn. **15**, 2370 (1948).
- ¹⁰⁹ T. S. Kê, Scientia Sinica **4**, 55 (1955).
- ¹¹⁰ A. Ia. Samoïlova and V. S. Postnikov, Труды Кемеровского пединститута (Trans. Kemerovo Pedagogical Inst.) **3** (1957) (in press).
- ¹¹¹ T. S. Kê, Phys. Rev. **74**, 9 (1948).
- ¹¹² T. S. Kê, Phys. Rev. **74**, 914 (1948).
- ¹¹³ W. J. Bratina and W. C. Winegard, J. Metals **8**, 186 (1956).
- ¹¹⁴ W. Körter and K. Posenthal, Z. Metallkunde **30**, 345 (1938).
- ¹¹⁵ C. Zener, D. Winule, and H. Nielsen, Trans. AIME **147**, 98 (1942).
- ¹¹⁶ C. Zener, Trans. AIME **152**, 122 (1943).
- ¹¹⁷ R. Artman, J. Appl. Phys. **23**, 475 (1952).
- ¹¹⁸ W. Köster, Arch. Eisenhüttenw. **14**, 271 (1940).
- ¹¹⁹ R. Hasiguti and G. Kamoshita, J. Phys. Soc. Japan **9**, 646 (1954).
- ¹²⁰ E. Schell and G. Reinacher, Z. Metallkunde **36**, 63 (1944).
- ¹²¹ W. Köster, Z. Metallkunde **35**, 246 (1943).
- ¹²² L. Rotherham and S. Pearson, J. Metals **8**, 881 (1956).
- ¹²³ A. S. Nowicu, Phys. Rev. **88**, 925 (1956).
- ¹²⁴ A. V. Grin' and V. A. Pavlov, Физика металлов и металловедение (Phys. of Metals and Metal Res.) **4**, 103 (1957).
- ¹²⁵ A. V. Grin', Физика металлов и металловедение (Phys. of Metals and Metal Res.) **4**, 561 (1957).
- ¹²⁶ A. I. Iamshchikova, Thesis, Moscow Steel Inst., 1954.
- ¹²⁷ A. C. Damasus and A. S. Nowicu, J. Appl. Phys. **26**, 1165 (1955).
- ¹²⁸ T. S. Kê, Phys. Rev. **78**, 420 (1950).
- ¹²⁹ T. S. Kê, J. Appl. Phys. **21**, 414 (1950).
- ¹³⁰ B. G. Childs and A. D. LeClaire, Acta Metall. **2**, 718 (1954).
- ¹³¹ T. S. Kê, J. Appl. Phys. **19**, 285 (1948).
- ¹³² J. Wenig and E. S. Machlin, Acta Metallurgica **4**, 262 (1956).
- ¹³³ V. S. Postnikov, Dokl. Akad. Nauk SSSR **71**, 79 (1953).
- ¹³⁴ V. S. Postnikov, Труды Кемеровского горного института (Trans. Kemerovo Mining Inst.) **2**, 277 (1956).
- ¹³⁵ V. S. Postnikov and G. K. Mal'tseva, Труды Кемеровского педагогического института (Trans. Kemerovo Pedagogical Inst.) **3** (1957) (in press).
- ¹³⁶ C. L. Ang, J. Sivertman, and C. Wert, Acta Metallurgica **8**, 558 (1955).
- ¹³⁷ V. S. Postnikov, Thesis, Moscow Steel Inst., 1952.
- ¹³⁸ Pridantsev, Meshcherinov, and Piguzov, Dokl. Akad. Nauk S.S.S.R. **111**, 98 (1956).
- ¹³⁹ V. S. Postnikov, Труды Кемеровского

- педагогического института (Trans. Kemerovo Pedagogical Inst.) **3** (1957) (in press).
- ¹⁴⁰ T. S. Kê and P. T. Jung, *Scientia Sinica* **3**, 261 (1955).
- ¹⁴¹ T. S. Kê and C. M. Wang, *Scientia Sinica* **4**, 501 (1955).
- ¹⁴² T. S. Kê and C. T. Tsien, *Scientia Sinica* **5**, 625 (1956).
- ¹⁴³ T. S. Kê and J. L. Ma, *Scientia Sinica* **6**, 81 (1957).
- ¹⁴⁴ C. T. Kung and T. S. Kê, *Scientia Sinica* **6**, 223 (1957).
- ¹⁴⁵ K. M. Rozin and B. N. Finkel'shtein, *Dokl. Akad. Nauk SSSR* **71**, 811 (1953).
- ¹⁴⁶ V. S. Postnikov and R. S. Lebedev, *Труды Кемеровского педагогического института (Trans. Kemerovo Pedagogical Inst.)* **3** (1957); *Физика металлов и металловедение (Phys. of Metals and Metal Res.)* **6** (1958) (in press).
- ¹⁴⁷ T. A. Read, *Trans. AIME* **143**, 30 (1941).
- ¹⁴⁸ S. G. Holder, E. E. Stansbury, and J. H. Frye, *J. Metals* **8**, 993 (1956).
- ¹⁴⁹ R. R. Hasiguti, *Collection: Proc. Internat. Conf. Theor. Phys., Tokyo*, 577, 1954.
- ¹⁵⁰ K. Matta and J. Barducci, *Ric. Scient.* **26**, 1160 (1956).
- ¹⁵¹ K. M. Entwistle, *J. Inst. Metals* **21**, 249 (1954).
- ¹⁵² W. R. Thomas and G. M. Leak, *Nature* **176**, 29 (1955).
- ¹⁵³ J. L. Snocu, *Physica* **7**, 711 (1941).
- ¹⁵⁴ C. Wert and C. Zener, *Phys. Rev.* **76**, 1169 (1949).
- ¹⁵⁵ J. D. Fast and L. J. Dijustra, *Phil. Tech. Rev.* **13**, 172 (1951).
- ¹⁵⁶ J. D. Fast and J. L. Meijering, *Philips Research Rep.* **8**, 1 (1953).
- ¹⁵⁷ L. J. Dijustra and R. J. Sladek, *J. Metals* **5**, 69 (1953).
- ¹⁵⁸ W. Köster and L. Bangert, *Arch. Eisenhüttenw.* **25**, 231 (1954).
- ¹⁵⁹ T. S. Kê, P. T. Jung, and J. N. Wang, *Scientia Sinica* **4**, 263 (1955).
- ¹⁶⁰ P. Stark, B. L. Averbach, and M. Cohen, *Acta Metallurgica* **4**, 91 (1956).
- ¹⁶¹ T. S. Kê and J. L. Ma, *Scientia Sinica* **5**, 19 (1956).
- ¹⁶² P. T. Jung and T. S. Kê, *Scientia Sinica* **5**, 645 (1956).
- ¹⁶³ B. N. Finkel'shtein and L. F. Usova, *Труды МИС (Trans. Moscow Steel Institute)* **36**, 176 (1957).
- ¹⁶⁴ J. Barducci and P. Gence, *Ric. Scient.* **26**, 2080 (1956).
- ¹⁶⁵ E. Scheil and W. Thiele, *Arch. Eisenhüttenw.*, **12**, 103 (1938).
- ¹⁶⁶ Yu. V. Piguzov, *Dokl. Akad. Nauk S.S.S.R.* **112**, 636 (1957).
- ¹⁶⁷ F. A. Lewis, C. E. Roberts, and A. R. Ubbelohde, *Proc. Roy. Soc., ser. A*, **220**, 279 (1953).
- ¹⁶⁸ K. Bungardt and H. Preisendanz, *Arch. Eisenhüttenw.* **27**, 715 (1956).
- ¹⁶⁹ B. N. Finkel'shtein and N. S. Fastov, *Dokl. Akad. Nauk S.S.S.R.* **71**, 875 (1950).
- ¹⁷⁰ B. N. Finkel'shtein and N. S. Fastov, *Проблема металловедения и физика металлов (Problems of Metal Research and the Physics of Metals)* **2**, 245 (1951).
- ¹⁷¹ N. S. Fastov, *J. Exptl. Theoret. Phys. (U.S.S.R.)* **22**, 487 (1952).
- ¹⁷² V. S. Postnikov, *Физика металлов и металловедение (Phys. of Metals and Metal Res.)* **6**, 522 (1958).
- ¹⁷³ L. D. Landau and E. M. Lifshitz, *Механика сплошных сред (Mechanics of Continuous Media)* Gostekhizdat, 1953.
- ¹⁷⁴ C. Zener, *Met. Techn.* **13**, 1 (1946).
- ¹⁷⁵ R. Becker and M. Kornetsui, *Z. Phys.* **88**, 634 (1934).
- ¹⁷⁶ M. Kersten, *Z. techn. Phys.* **15**, 463 (1934).
- ¹⁷⁷ R. Becker and W. Döring, *Ferromagnetismus*, 1939.
- ¹⁷⁸ R. Bozorth, *Ferromagnetism*, N. Y. Van Nostrand, 1951.
- ¹⁷⁹ H. J. Williams and R. M. Bozorth, *Phys. Rev.* **59**, 939 (1941).
- ¹⁸⁰ J. C. Maxwell, *Phil. Trans. Roy. Soc.* **157**, 49 (1867); *Phil. Mag.* **35**, 129 (1868).
- ¹⁸¹ Ya. I. Frenkel', *Введение в теорию металлов (Introduction to the Theory of Metals)* Gostekhizdat, 1950.
- ¹⁸² A. S. Nowick, *Phys. Rev.* **82**, 340 (1951).
- ¹⁸³ V. S. Gorskii, *Phys. Z. Sow.* **8**, 443 (1935); *J. Exptl. Theoret. Phys. (U.S.S.R.)* **6**, 272 (1936).
- ¹⁸⁴ V. S. Gorskii, *Phys. Z. Sow.* **6**, 77 (1934).
- ¹⁸⁵ W. Köster, *Z. Metallkunde* **32**, 145 (1940).
- ¹⁸⁶ W. Köster, *Z. Metallkunde* **32**, 151 (1940).
- ¹⁸⁷ W. Köster and A. Schneider, *Z. Metallkunde* **32**, 156 (1940).
- ¹⁸⁸ L. Landau and G. Rumer, *Phys. Z. Sow.* **11**, 18 (1937).
- ¹⁸⁹ A. S. Kompaneets, *Dokl. Akad. Nauk S.S.S.R.* **14**, 267 (1937).
- ¹⁹⁰ A. Akhiezer, *J. Exptl. Theoret. Phys. (U.S.S.R.)* **8**, 1318, 1330 (1938).
- ¹⁹¹ S. N. Tkachenko, *J. Exptl. Theoret. Phys. (U.S.S.R.)* **9**, 314 (1939).
- ¹⁹² B. V. Paranjape, *Proc. Phys. Soc. ser. A*, **66**, 572 (1953).
- ¹⁹³ H. O. Kneser and K. Voelz, *Ann. d. Phys.* **11**, 377 (1953).
- ¹⁹⁴ N. S. Fastov, *Проблемы металловедения и*

- физика металлов (Problems of Metal Research and the Physics of Metals) **4**, 388 (1955).
- ¹⁹⁵ L. Gurevich, Основы физической кинетики (Fundamentals of Physical Kinetics) Gostekhizdat, 1940.
- ¹⁹⁶ J. M. Burgers, First Report on Viscosity and Plasticity, Amsterdam, 1935, p. 19.
- ¹⁹⁷ A. S. Nowick, Symposium on the Plastic Deformation of Crystalline Solids. Pittsburgh, 1950, p. 155.
- ¹⁹⁸ A. S. Nowick, J. Appl. Phys. **25**, 1129 (1954).
- ¹⁹⁹ T. A. Read, Phys. Rev. **58**, 371 (1940).
- ²⁰⁰ G. H. Faund, Trans. AIME **161**, 120 (1945).
- ²⁰¹ J. H. Swift and J. E. Richardson, J. Appl. Phys. **18**, 417 (1947).
- ²⁰² R. R. Hasiguti and T. Hirai, J. Appl. Phys. **22**, 1084 (1951).
- ²⁰³ E. Körber and M. Hempel, Inst. Eisenforschung **22**, 1 (1935).
- ²⁰⁴ L. Kukanov, Заводск. лабор. (Plant Laboratory) **6**, 722 (1937).
- ²⁰⁵ W. Köster, Z. Metallkunde **32**, 282 (1940).
- ²⁰⁶ T. Norton, Trans. AIME **137**, 49 (1940).
- ²⁰⁷ F. Seitz and T. A. Read, J. Appl. Phys. **12**, 100 (1941).
- ²⁰⁸ C. Zener, H. Clarke, and C. Smith, Trans. AIME **147**, 90 (1942).
- ²⁰⁹ T. A. Read and E. P. T. Tyndall, J. Appl. Phys. **17**, 713 (1946).
- ²¹⁰ Ch. Boulanger, C. R. Acad. Sc. **226**, 1170 (1948).
- ²¹¹ T. S. Kê, J. Metals **2**, 575 (1950).
- ²¹² T. S. Kê, J. Metals **2**, 581 (1950).
- ²¹³ T. S. Kê and C. Zener, Chinese J. Phys. **8**, 133 (1951).
- ²¹⁴ J. Weertman and J. S. Kochler, J. Appl. Phys. **24**, 624 (1953).
- ²¹⁵ H. Fusfeld, J. Appl. Phys. **24**, 1062 (1953).
- ²¹⁶ W. Köster and E. Stolte, Z. Metallkunde **45**, 356 (1954).
- ²¹⁷ W. Köster, L. Bangert, and R. Hahn, Arch. Eisenhüttenw. **25**, 596 (1954).
- ²¹⁸ L. A. Glikman and E. A. Khein, J. Tech. Phys. (U.S.S.R.) **24**, 400 (1954).
- ²¹⁹ L. A. Glikman and E. A. Khein, J. Tech. Phys. (U.S.S.R.) **24**, 560 (1954).
- ²²⁰ J. Friedel, Ch. Boulanger, C. Crussard, Acta Metallurgica **3**, 380 (1955).
- ²²¹ A. S. Nowick, Acta Metallurgica **3**, 312 (1955).
- ²²² J. Weertman, J. Appl. Phys. **26**, 202 (1955).
- ²²³ V. S. Postnikov and M. M. Belyaev, Физика металлов и металловедение (Phys. of Metals and Metal Res.) **2**, 504 (1956).
- ²²⁴ V. S. Postnikov, Труды Кемеровского горного института (Trans. Kemerovo Mining Inst.) **1**, 202 (1954).
- ²²⁵ A. V. Grin', Физика металлов и металловедение (Phys. of Metals and Metal Res.) **4**, 383 (1957).
- ²²⁶ T. A. Kontorova and Ia. I. Frenkel', J. Exptl. Theoret. Phys. (U.S.S.R.) **8**, 1340 (1938).
- ²²⁷ J. D. Esheby, Proc. Roy. Soc., ser. A. **197**, 395 (1949).
- ²²⁸ G. Leibfried, Z. Phys. **127**, 344 (1950).
- ²²⁹ J. S. Kochler, Imperfections in Nearly Perfect Crystals, 1952, p. 197.
- ²³⁰ P. G. Bordoni and M. Nuovo, Nuovo cimento **11**, 127 (1954).
- ²³¹ W. Köster and L. Bangert, Acta Metallurgica **3**, 274 (1955).
- ²³² A. Cranato and K. Lücke, J. Appl. Phys. **27**, 583 (1956).
- ²³³ A. L. Nowick, J. Appl. Phys. **22**, 925 (1951).
- ²³⁴ V. T. Shmatov, Report at the all-Union Inter-university Conference on Relaxation Phenomena in Pure Metals and Alloys, April 1958, Moscow.
- ²³⁵ T. S. Kê, Phys. Rev. **76**, 549 (1949).
- ²³⁶ V. S. Postnikov, Труды Кемеровского горного института (Publ. Kemerovo Mining Inst.) **1**, 202 (1954).
- ²³⁷ C. Zener, Phys. Rev. **71**, 34 (1947).
- ²³⁸ A. S. Nowick, Phys. Rev. **88**, 925 (1952).
- ²³⁹ E. Woordruff, Phys. Rev. **16**, 321 (1903).
- ²⁴⁰ C. Wert, J. Appl. Phys. **21**, 1196 (1950).
- ²⁴¹ T. S. Kê, P. W. Jang, Scientia Sinica **6**, 623 (1957).
- ²⁴² M. A. Isakovich, J. Exptl. Theoret. Phys. (U.S.S.R.) **18**, 386 (1948).
- ²⁴³ J. Weertman, J. Appl. Phys. **28**, 193 (1957).
- ²⁴⁴ A. H. Cottrell, Dislocations and Plastic Flow in Crystals, Oxford, 1953.
- ²⁴⁵ G. Schoeck, Phys. Rev. **102**, 1458 (1956).
- ²⁴⁶ M. Lessen, J. Appl. Phys. **28**, 364 (1957).
- ²⁴⁷ I. I. Chernikova, Физика металлов и металловедение (Physics of Metals and Metal Research) **5**, 102, 176 (1957).
- ²⁴⁸ B. N. Finkel'shtein and L. F. Usova, Сборник трудов Московского института стали (Collection of papers of the Moscow Steel Inst.) **36**, 176 (1957).
- ²⁴⁹ Pavlov, Kriuchkov and Fedotov, Физика металлов и металловедение (Physics of Metals and Metal Research) **5**, 371 (1957).
- ²⁵⁰ H. J. McSkimin, J. Appl. Phys. **24**, 988 (1953).
- ²⁵¹ H. E. Bömmel, Phys. Rev. **96**, 220 (1954).
- ²⁵² W. P. Mason and H. E. Bömmel, J. Acoust. Soc. **28**, 930 (1956).
- ²⁵³ A. B. Pippard, Phil. Mag. **46**, 1104 (1955).