

On the possibility of the excitation of oscillations mediated by magnetomechanical coupling in van der Waals antiferromagnets

M.A. Koliushenkov, A.P. Pyatakov

DOI: <https://doi.org/10.3367/UFNe.2025.12.040071>

Contents

1. Introduction	596
2. Forced resonance in oscillatory systems based on van der Waals antiferromagnets	598
2.1 Excitation of oscillations via flexomagnetic interaction; 2.2 Electrically induced Einstein–de Haas effect;	
2.3 Electric- and magnetic-field-induced spin-reorientation phase transitions in van der Waals antiferromagnets	
3. Parametric generation of oscillations by a magnetic field via magnetostriction	603
4. Conclusion	605
References	605

Abstract. In this paper the excitation mechanisms for mechanical oscillations mediated by magnetomechanical coupling in van der Waals antiferromagnets are discussed. It is shown that, in spite of the compensation of direct magnetic field effect due to the presence of two magnetic sublattices in antiferromagnets, the inverse flexomagnetic effect, the electric field-induced gyromagnetic effect, and the magnetostrictive interaction can cause detectable mechanical oscillations due to the phenomena of forced or parametric resonance. The results of analysis demonstrate the potential of such materials as a basis for new-generation of controllable micro- and nanomechanical systems, they are also of interest from a methodological point of view as an application of classical methods of oscillation theory, such as the harmonic balance method and slowly varying amplitude approximation, to analyze phenomena discovered in van der Waals magnets in the past few years.

Keywords: van der Waals magnets; flexoelectric effect, flexomagnetic effect, gyromagnetic effect, magnetoelectrics, multiferroics, straintronics

1. Introduction

The discovery of two-dimensional (2D) magnetic materials is one of the main events in magnetism of the last decade [1, 2], that has already led to a revision of the established concepts in the theory of phase transitions and magnetic ordering from

the reduced dimensions perspective, just as the discovery of graphene radically expanded the understanding of band theory and conduction mechanisms in semiconductor physics.

2D magnetics are frequently referred to as van der Waals magnetic materials, since the molecular layers within the latter are separated by significant gaps and can be readily exfoliated from the surface: the interlayer interaction via van der Waals forces is weak compared to covalent or ionic bonds. Van der Waals forces make it possible not only to detach monolayers from the surface of a bulk crystal, but also to fabricate heterostructures. A common type of magnetic ordering in such structures is A-type antiferromagnetism, with molecular monolayers magnetized in opposite directions, while the magnetic ordering of ions within each monolayer remains ferromagnetic [3]. A vivid example of such layered antiferromagnetic ordering is chromium triiodide, CrI₃ (Fig. 1): with an even number of layers, the net magnetization of the material is zero, even though the magnetic moment of a single chromium ion is about 3 Bohr magnetons. The material remains insensitive to the magnetic field and up to the critical value of 0.5 T [4], where a metamagnetic transition occurs.

Due to the specific geometry of van der Waals antiferromagnets, their properties can be modulated by applying a gate voltage [5] or by mechanical deformations [6], rendering them promising materials for practical applications in spintronics and straintronics. From a fundamental perspective, these media are of interest because of the strong coupling between their electronic, mechanical, and magnetic subsystems.

The best-known examples of this type of coupling are the piezoelectric effect and magnetostriction, which arise under homogeneous deformations. However, in van der Waals materials, effects induced by gradient deformations occurring upon bending of monolayers and multilayer heterostructures are equally important [7], for example, gradient deformations naturally appear in a ripples that form at adhesion of van der Waals materials to a substrate. When the radius of curvature falls below a critical value of 16 Ångströms [6], van der Waals magnets undergo phase transition into new magnetic states

M.A. Koliushenkov^(1, a), A.P. Pyatakov^(1, 2, b)

⁽¹⁾ MIREA — Russian Technological University,
prosp. Vernadskogo 78, 119454 Moscow, Russian Federation

⁽²⁾ Lomonosov Moscow state University, Faculty of Physics,
Leninskie gory 1, str. 2, 119991 Moscow, Russian Federation

E-mail: ^(a) koliushenkov.ma19@physics.msu.ru,

^(b) pyatakova@my.msu.ru

Received 29 October 2025, revised 8 December 2025

Uspekhi Fizicheskikh Nauk 196 (6) 638–648 (2026)

Translated by the authors

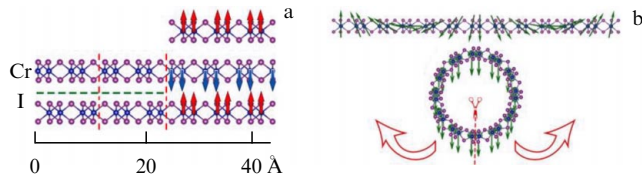


Figure 1. (a) Antiferromagnetic structure of CrI₃. The vertical dashed lines indicate the unit cell of the bilayer. Blue circles represent chromium ions, purple circles represent iodine ions. Red and blue arrows denote the spins of the magnetic sublattices (magnetic moment of a single ion $\sim 3\mu_B$); (b) bending-induced 'plumblane' state (with parallel spin alignment) in a CrI₃ monolayer, corresponding to a spin cycloid in the coordinate system associated with the nanotube surface. (The art version of figure from Ref. [6].)

(Fig. 1b) different from the original state with spins oriented perpendicular to the layer surface (Fig. 1a). Such effects can be grouped under the umbrella of 'flexo-' effects, from the Latin 'flexura' — bending.

The flexoelectric effect, unlike the piezoelectric effect, imposes no restrictions on the crystal symmetry group [8], because the bending itself has a polar asymmetry: some layers are stretched while others are compressed (Fig. 2a). Hence, the flexoelectric effect is present in all solids; however, its magnitude in bulk materials is relatively small: at characteristic strain gradients arising during epitaxial film growth (10^5 m^{-1}), polar ionic displacements within the crystal unit cell are on the order of 0.01 pm [9], i.e., the magnitude of the electric polarization is thousands of times smaller than that of ferroelectrics.

The magnetic analogue of the flexoelectric effect is bending-induced magnetization, i.e., the flexomagnetic effect [10]. In contrast to flexoelectric phenomena, flexomagnetic effects are not as universal, since the violation of time-reversal symmetry accompanying the emergence of magnetization does not follow directly from the violation of spatial inversion symmetry generated by bending deformation. The flexomagnetic effect is possible in crystals in which both spatial and time-reversal symmetries are broken, but combined space-time parity (PT) is preserved, i.e., in magneto-electric media [11]. An illustrative example of the flexomagnetic effect is the emergence of an uncompensated magnetization in the antiferromagnetically ordered bilayer structure of CrI₃ (Fig. 2b): in the layer subjected to compressive strain, the distance between magnetic ions decreases, leading to an increase in the exchange interaction; in the opposite, tensile-strained layer, the exchange interaction weakens: an uncompensation of the magnetic sublattices in the antiferromagnet occurs, resulting in a nonzero net magnetization that changes direction upon bending of the opposite sense [12]. Through the mechanism described, the violation of spatial parity leads to a violation of temporal parity, while the combined PT-parity remains preserved.

When considering both flexoelectric and flexomagnetic effects in van der Waals materials, it is necessary to account for the specific character of their deformations, namely the possibility of interlayer sliding relative to each other when the bending angle exceeds a critical value (about 30 degrees): in this case, the localization of regions with high curvature at kink points becomes energetically favorable, while the other regions remain almost unbent [13].

The flexomagnetic interaction is of interest because it allows mechanical actuation by a magnetic field in antiferro-

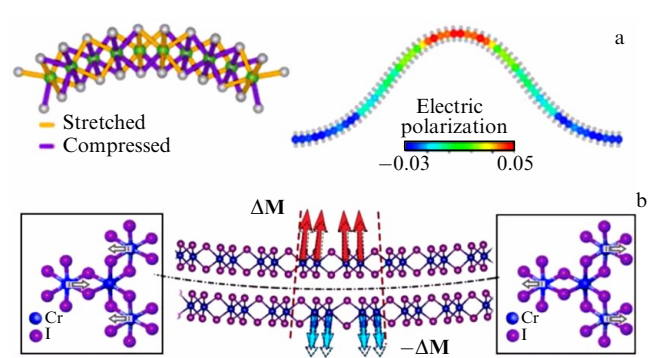


Figure 2. Flexoinduced effects in two-dimensional materials: (a) flexoelectric effect in VSe₂: emergence of an electric polarization whose direction is determined by the sign of curvature. (Figure adapted from Ref. [7].) (b) Flexomagnetic effect in CrI₃: dashed arrows indicate the initial magnetization magnitudes in the layers [12]; the left inset shows a top view of the upper compressed layer, the right inset shows the lower stretched layer. For illustrative purposes, the displacements of chromium ions are shown exaggerated as gray arrows, and the scale of magnetization change is also exaggerated: even at such a pronounced bending, the change in magnetization in each layer does not exceed 5% of its initial magnitude (the magnetic moment of a single chromium ion in this compound, according to [12], is about $3\mu_B$).

magnetic materials where the magnetizations of the sublattices are compensated: field-controlled decompensation is accompanied by bending of the material.

Decompensation of magnetization in antiferromagnetically ordered layers can also manifest itself as a gyromagnetic effect (the Einstein–de Haas effect), i.e., the acquisition of a macroscopic mechanical angular momentum by the body as a whole. An indirect manifestation of such magnetomechanical effect was observed in the van der Waals antiferromagnet FePS₃ in the form of shear oscillations arising under the action of demagnetizing femtosecond laser pulses [14]. However, the total angular momentum integrated over the entire volume of such a magnetic film was zero. In this regard, an electric field-induced magnetomechanical effect may prove more promising: an electric field reduces the symmetry of the antiferromagnet and causes an uncompensation of the magnetic sublattices, generating a nonzero magnetization and angular momentum in the van der Waals material sample as a whole [15].

The Einstein–de Haas effect observation, and the practical application in particular, in van der Waals materials is associated with a certain difficulty. Namely, a problem is the poor compatibility of the Ising-type magnetic ordering characteristic of van der Waals samples with the possibility of detecting their gyromagnetic motion (Fig. 3): rotation of a two-dimensional material about the normal to its surface will be hindered by the tangential component of the very same van der Waals forces. Therefore, to observe gyromagnetic phenomena in van der Waals materials, the presence of an in-plane magnetic moment in the film is necessary.

The aim of this work is to examine various mechanisms for exciting oscillations of van der Waals antiferromagnet plates, for which direct action of a magnetic field becomes impossible owing to the compensation of the layer magnetizations. Estimates of the oscillation parameters will be made on the basis of literature data on the mechanical properties of van der Waals antiferromagnets: there is enough data for a brief review on the relevant topic.

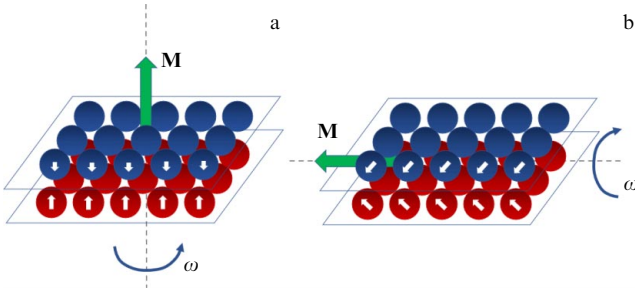


Figure 3. Influence of the magnetic ordering geometry on the detectability of the gyromagnetic effect. (a) Standard direction of the uncompensated magnetization in 2D magnetic materials (Ising-type magnetic ordering). (b) Required direction of the uncompensated magnetization should be in the plane of the sample.

2. Forced resonance in oscillatory systems based on van der Waals antiferromagnets

2.1 Excitation of oscillations via flexomagnetic interaction

The effects bearing the prefix ‘flexo’ are based on the coupling between the strain gradient and some internal order parameter. Typically, the flexoelectric and flexomagnetic effects are considered.

For the flexoelectric effect, the additional polarization of the sample in response to its inhomogeneous deformation [8] is determined by:

$$P_i = f_{ijkl}^{\text{FE}} \frac{\partial \varepsilon_{jk}}{\partial x_l}, \quad (1)$$

where f_{ijkl}^{FE} is the flexoelectric tensor and $\partial \varepsilon_{jk} / \partial x_l$ is the strain gradient.

The flexomagnetic effect [10] has a similar form but manifests itself as the appearance of additional magnetization in the presence of a strain gradient (Fig. 4a):

$$M_i = f_{ijkl}^{\text{FM}} \frac{\partial \varepsilon_{jk}}{\partial x_l}. \quad (2)$$

From general considerations a kind of converse flexo-effect relating gradients of either field or strain should arise.¹ A relatively small number of works have been devoted to flexomagnetic phenomena: after the introduction of the term linking inhomogeneous deformation and magnetization in Ref. [10], first-principles calculations of this effect in the antiperovskite Mn_3GaN [16] were performed, showing relatively small magnitudes of the effect, on the order of $2\mu_B \text{ \AA}$; i.e., the magnetic moment per formula unit in this antiferromagnet would reach a value comparable to the magnetization of a ferromagnet only at a radius of curvature of $1 \text{ \AA}$. For realistic strain gradients of $\sim 10^{-5} \text{ \AA}^{-1}$ (such as arise during epitaxial film growth), the bending-induced magnetic moment would accordingly be 100,000 times smaller. Calculations [17] predict a possible deflection of an elastic cantilever due to the inverse

flexomagnetic effect in the range of units to tens of nanometers (depending on the beam geometry), which can be readily detected by standard probe microscopy methods using a four-segment photodiode scheme [18]. Comparable values were obtained via analytical estimates in [19]. Slightly higher values of $\sim 40\mu_B \text{ \AA}$ were given by estimates of the flexomagnetic effect at phase boundaries in the antiferromagnet bismuth ferrite [20]. Unexpectedly high effect was observed in an experimental study on a ferromagnetic NiFe alloy film [21]: the flexomagnetic effect in the form of unidirectional anisotropy induced by bending a 70-nm-thick plate exceeded the values considered above by 6–7 orders of magnitude. In antiferromagnets, various mechanisms for enhancing the flexomagnetic effect have also been proposed (Fig. 4):

- via the dependence of the Néel temperature on strain [22]: in the presence of a strain gradient, some layers remain in a magnetically ordered state while the others undergo transition to a paramagnetic state. The magnitude of the effect per formula unit is $\sim 100\mu_B \text{ \AA}$ (Fig. 4a);

- via the strong dependence (up to a sign change) of the RKKY exchange on interatomic distance: when ripples arise in membranes of the Heusler alloy GdPtSb, a transition from an antiferromagnetic to a ferromagnetic state occurs [23]; the magnitude of the effect increases by a factor of 100,000, but at the cost of losing the linear dependence of magnetization on the strain gradient (Fig. 4b);

- via the creation of vortex magnetic inhomogeneities (skyrmions): in the undeformed state, the magnetizations in two antiferromagnetically ordered layers compensate each other, while upon bending, due to the mechanical stretching of one skyrmion relative to the other, the exact compensation is broken (Fig. 4c) [24];

- finally, the sample thickness plays an important role. Indeed, according to numerical calculations for a CoFe_2O_4 plate [17], both for the direct flexomagnetic effect (magnetization induced by bending) and for the inverse effect in the form of bending induced by a magnetic field, flexomagnetic phenomena become more pronounced as the plate thickness decreases. In this regard, the study of flexomagnetic effects in van der Waals antiferromagnets is of particular interest. Calculations of the flexomagnetic effect in a CrI_3 bilayer [12] show that the flexomagnetic effect therein exceeds reference values in bulk materials by two orders of magnitude [16]. The analysis of the enhancement mechanisms carried out in [12] reveals the role of two factors: the relatively large distance between layers with oppositely magnetized ions (distance d in Fig. 4d), as well as the high sensitivity of the exchange interaction within each layer to tensile and compressive strains. Therefore, when considering the possibility of exciting oscillations of a van der Waals sample via the inverse flexomagnetic effect, we will base our analysis on the results of Ref. [12].

Let’s write the expression for the energy of a bilayer of the antiferromagnet CrI_3 , taking into account the contributions to energy per formula unit from bending energy (F_{bend}) and the flexomagnetic interaction (F_{flexo}):

$$F = F_{\text{bend}} + F_{\text{flexo}} = \frac{D(\nabla \varepsilon)^2}{2} - f^{\text{FM}} \mathbf{H} \nabla \varepsilon, \quad (3)$$

where D is the bending stiffness of the sample per formula unit, ε is the strain, f^{FM} the flexomagnetic coupling coefficient, and H is the magnetic field.

¹Two types of converse effects are possible: the converse flexoelectric effect in the original sense, i.e., a homogeneous strain induced by a nonuniform electric field [8, 36], as well as bending induced by a uniform electric field (inverse flexoelectric effect) [37–39]. In the literature, confusion often arises between these two concepts.

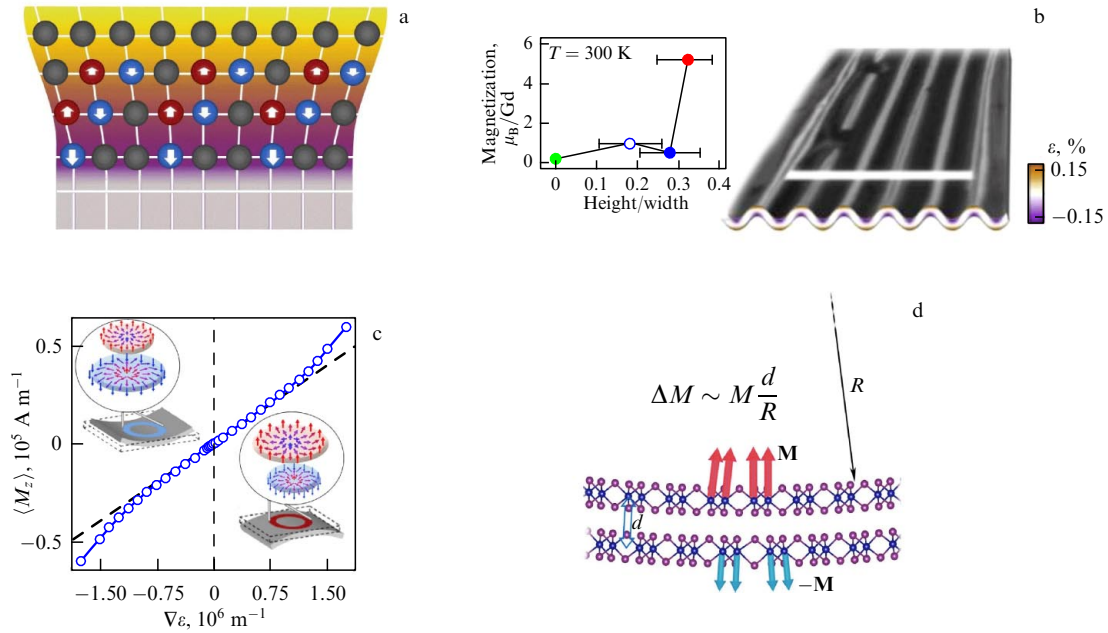


Figure 4. Various mechanisms for enhancing the flexomagnetic effect: (a) dependence of the Néel temperature on strain: red and blue colors indicate ions of the magnetic sublattices; for the upper layers, T_N is lower and they have already transitioned to a paramagnetic state, shown in gray (figure is based on Ref. [22]); (b) dependence of RKKY exchange on strain: the horizontal axis of the graph shows the ripple aspect ratio (height of the ripple divided to its period). The right panel combines an optical microscope image of the surface (the white rectangle corresponds to a scale of 100 μm) and a strain distribution map across a membrane cross section (the scale of folds and membrane thickness is exaggerated for illustrative purposes, the figure is based on Ref. [23]); (c) use of magnetic inhomogeneities [24]: decompensation of magnetization due to changes in the relative sizes of skyrmions in antiferromagnetic layers; (d) enhancement in van der Waals materials in prototypical 2D antiferromagnet, CrI_3 bilayer [12].

Minimizing the expression for the energy:

$$\frac{\partial F}{\partial \nabla \varepsilon} = D \nabla \varepsilon - f^{\text{FM}} \mathbf{H} = 0, \quad (4)$$

we obtain an expression for the magnitude of bending induced by the inverse flexomagnetic effect:

$$\nabla \varepsilon = \frac{f^{\text{FM}} \mathbf{H}}{D}. \quad (5)$$

Estimating the magnitude of the effect for a van der Waals CrI_3 sample with $f^{\text{FM}} = 200 \mu_B \text{ \AA}$ per formula unit [12] consisting of 10 bilayers (thickness 13.2 nm, elastic modulus per formula unit $D = 680 \text{ eV \AA}^2$), we obtain for a magnetic field $H = 100 \text{ Oe}$ a value of $\nabla \varepsilon = 1.8 \times 10^{-7} \text{ \AA}^{-1}$, which, for a cantilever 5 μm in length, yields a vertical deflection of $\sim 50 \text{ nm}$. These deflections can be readily detected using modern methods employed in interferometry or probe microscopy.

It is also useful to compare this effect with direct magnetic excitation for the case of a van der Waals magnet CrI_3 with an odd number of layers: in this case, the magnetic moment per unit area corresponding to a formula unit will be uncompensated and amounts to approximately three Bohr magnetons. As follows from the very dimensionality of the flexomagnetic effect [$\mu_B \text{ \AA}$], the direct effect of the magnetic field on the magnetic moments of the uncompensated odd layer becomes comparable to the flexomagnetic effect at rather short length of an oscillating segment, about 6.5 nm. Increasing the length of the cantilever made of a van der Waals material does not lead to an increase in the flexomagnetic effect, whereas the direct magnetic action will grow proportionally to the cantilever length in the case of an uncompensated van der Waals antiferromagnet with an odd number of layers.

2.2 Electrically induced Einstein–de Haas effect

Another example of magnetomechanical interaction is provided by gyromagnetic effects, which couple the spin subsystem of a crystal to its rotation as a whole. In the textbook experiment by Einstein and de Haas [25], a macroscopic ferromagnetic sample in the form of a rod was suspended by a thin thread and placed in an external alternating magnetic field. The field, acting on the magnetic moments of the atoms, ordered them, resulting in the sample acquiring an additional magnetization. Consequently, due to the conservation of total angular momentum, the rod began to rotate about its axis. In an alternating magnetic field, mechanical torsional oscillations were observed, demonstrating a coupling between the mechanical and magnetic moments of the atoms. In modern times, this effect has been reproduced on the nanometer scale using cantilevers [26]; subsequently, downscaling to the limit of a single molecule [27] and to subpicosecond timescales [28].

Typically, the gyromagnetic effect is observed in materials with ferromagnetic ordering, while the question of its detection in materials of other types remains open and relevant. In Ref. [15], the possibility of an effect analogous to the Einstein–de Haas effect in materials with antiferromagnetic (AFM) ordering was theoretically considered. Direct magnetic field actuation proves inefficient in this case owing to the low magnetic susceptibility of antiferromagnets; therefore, alternative methods of excitation are proposed to generate an additional magnetization in the sample.

Recently, indirect evidence of the Einstein–de Haas effect induced by optical excitation was obtained in a van der Waals antiferromagnet [14]. In that work, the authors studied the dynamics of a free AFM film of FePS_3 by monitoring the time evolution of the intensity of higher-order peaks in ultrafast electron diffraction. This motion corresponds to an interlayer

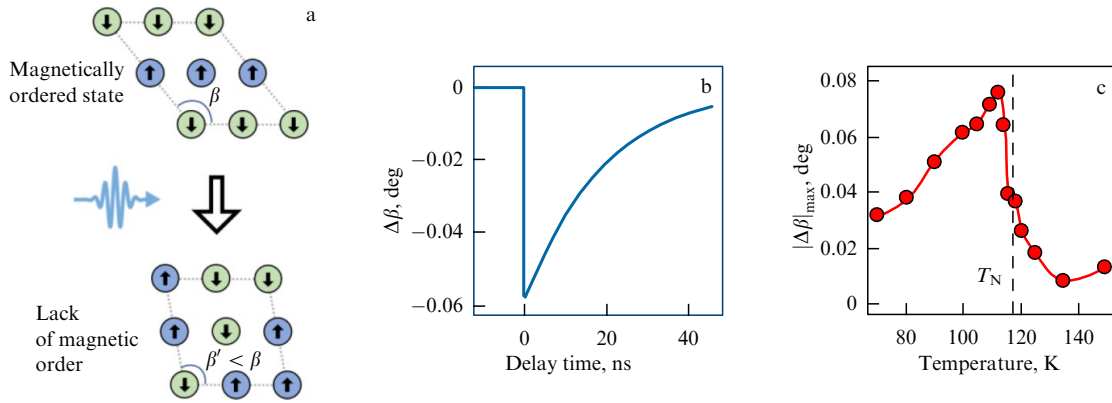


Figure 5. Magnetomechanical effects leading to shear oscillations in the van der Waals antiferromagnet FePS₃. (a) Illustration of the crystal lattice changes upon transition from the magnetically ordered state (top) to the disordered state (bottom). (b) Change in the monoclinic angle β after excitation by a 400 nm laser pulse with fluence 1.1 mJ cm⁻² at a temperature of 100 K. (c) Maximum photoinduced change in the monoclinic angle $|\Delta\beta|_{\max}$ at different temperatures. (Figures adapted from Ref. [14].)

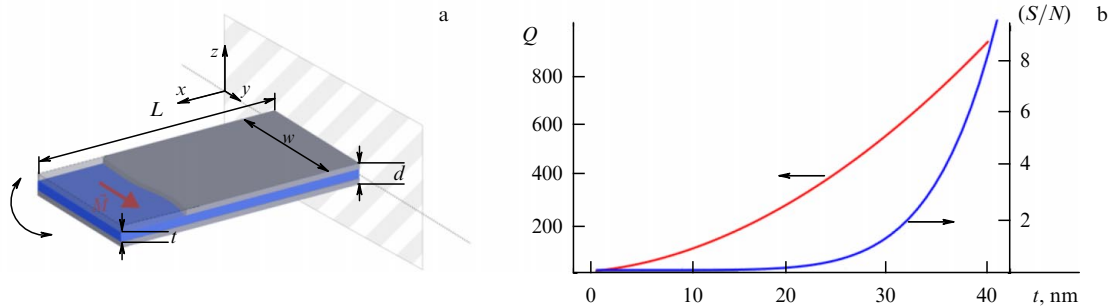


Figure 6. The system under consideration: (a) cantilever of thickness d made of a thin antiferromagnetic film (blue) of thickness t , sandwiched between two graphene electrodes (gray); (b) dependence of the quality factor and the Signal-to-Noise ratio of the system on the cantilever thickness ($L = 5 \mu\text{m}$, $w = 3 \mu\text{m}$) [15].

shear in real space (Fig. 5a), arising from angular momentum exchange between spins and the lattice. As is the case of the most van der Waals materials, the ground state of the antiferromagnet FePS₃ corresponds to spins oriented perpendicular to the layer plane. However, for shear to occur, as well as for rotation about an axis lying in the layer plane (Fig. 3b), a projection of the spins onto the sample plane is required. The authors of Ref. [14] suggest that a photoinduced spin canting into the layer plane does indeed take place. The effects are observed at temperatures below the Néel temperature and almost disappear upon heating above it (Fig. 5c).

On the other hand, a nonzero inplane magnetic moment can be induced via the magnetoelectric effect: an electric field lowers the crystal symmetry and induces a magnetic moment due to the decompensation of the antiferromagnetic sublattices. However, to create electric fields of sufficient magnitude, a small sample thickness is necessary; therefore, van der Waals materials are the logical choice as objects of study. Linear magnetoelectric effect is typical for many antiferromagnets; in particular, it has been observed in the AFM bilayer of CrI₃. The 2D geometry of the samples also favors electrical modulation of magnetic properties: a gate voltage can be applied to graphene sheets above and below the van der Waals materials, and a strong electric field of 10⁷ V cm⁻¹ can be generated without screening or breakdown effects [4].

Consider harmonic oscillations of the deflection angle ϕ of an elastic cantilever (Fig. 6a) under the action of a periodic

external torque N_0 :

$$\ddot{\phi} + 2\delta\dot{\phi} + \omega_0^2\phi = \frac{N_0}{J} \cos(pt), \quad (6)$$

$$N_0 = \frac{MV}{\gamma\tau} \left(\frac{t}{d} \right), \quad (6a)$$

where γ is the gyromagnetic ratio, ω_0 is the resonance frequency, J is the moment of inertia, M is the uncompensated magnetization arising from the magnetoelectric (ME) effect, t is the thickness of the van der Waals antiferromagnet, d is the total thickness of the cantilever including the graphene electrodes, and $\tau = 1/\omega_0$ is the characteristic time over which the magnetic moment is modulated.

We seek a solution in the form of harmonic oscillations $\phi = \phi_{\max} \cos(\omega_0 t + \phi_0)$ for the amplitude under resonant driving:

$$\phi_{\max} \approx \frac{3Q}{\gamma} \frac{M}{\rho\omega_0 L^2} \left(\frac{t}{d} \right), \quad (7)$$

where Q is the quality factor of the system and ρ is the average density of the cantilever [15].

Estimates [15] performed using formula (7) show that employing an ultrathin single bilayer sample ($d \approx 1.5$ nm) is unfavorable for detecting this effect, since the small amplitude and quality factor of such a system do not allow

the signal-to-noise ratio (S/N) to exceed unit value. However, the strong dependence of (S/N) on the sample thickness (exceeding the sixth power of the film thickness) makes it possible to use quasit-2D samples consisting of several bilayers, for which (S/N) $\gg 1$. For instance, the quality factor of an oscillatory system based on a micro-sized graphene flake with a thickness of 15 nm approaches 100 at a pressure of 10^{-6} Torr and room temperature [29]. For a van der Waals antiferromagnet sample consisting of 30 bilayers, ($d \approx 40$ nm) under similar conditions, the quality factor already approaches one thousand, the signal-to-noise ratio (S/N) ~ 10 , and the maximum oscillation amplitude upon application of an AC voltage with an amplitude of about 10 V is $\phi_{\max} \sim 10^{-5}$ radians, which is a sufficient magnitude for detecting oscillations using modern optical schemes (an equivalent cantilever oscillation amplitude in a magnetic field would be achievable only with an AC field amplitude of several Tesla (due to the low magnetic susceptibility of the antiferromagnet), which is a challenging task at the characteristic oscillation frequencies of microelectromechanical systems of hundreds of kHz and above.

As already noted in the introduction, perpendicular Ising-type anisotropy can be an obstacle to observing the gyromagnetic effect (see Fig. 3). However, it is worth noting that although Ising-type exchange is characteristic of van der Waals materials, 2D magnets with Heisenberg-type exchange and continuous symmetry, such as Ti_2NO_2 [30], have recently been discovered. Of particular interest is the magnetoelectric coupling in multiferroics such as NiBr_2 [31] and NiI_2 [32], since the in-plane spin orientation in them persists down to the monolayer limit. Furthermore, the required inplane orientation of the magnetic moment can arise during spin-reorientation magnetic phase transitions in antiferromagnets induced by magnetic and electric fields. These will be discussed in more detail in the following section.

2.3 Electric- and magnetic-field-induced spin-reorientation phase transitions in van der Waals antiferromagnets

In Ref. [33], an analysis of the possibility of deflecting magnetic moments away from the normal to the sample plane was performed, using as an example a bilayer of the antiferromagnet CrI_3 , which exhibits A-type AFM ordering (with opposite orientation of magnetizations in adjacent layers) [2] (Fig. 7). Alongside mechanically induced transitions typical for two dimensional magnetic materials, transitions induced by the application of magnetic and electric fields are also observed in it.

The ability to control the magnetic properties of a CrI_3 bilayer using an electric field was demonstrated in Ref. [4]. Figure 7b shows the dependence of the normalized magnetization on the magnetic field for a sample consisting of two antiferromagnetically ordered molecular layers. It can be seen that at small applied magnetic fields the sample is in an AFM state with zero net magnetization; however, upon application of a field of about 0.5 T, a first-order phase transition to a ferromagnetic (FM) state occurs, in which all atomic magnetic moments in both layers are aligned parallel.

Application of an electric field to the sample affects the shape of the magnetic phase diagram (Fig. 8). In the antiferromagnetic phase, the electric field E induces a net magnetization, which increases with increasing E and exhibits hysteresis upon variation of the magnetic field. The phase transition is also shifted toward higher magnetic fields in the presence of an electric field. The observed changes may be attributed to various factors, including the contribution of the magnetoelectric effect: the electric field can lower the free energy of the antiferromagnetic phase via the magnetoelectric effect and lead to an increase in the critical field H_C required for the spin-flip transition. The dependence of the critical transition field on the applied electric field, according to Ref. [4], can be described by the following formula:

$$\mu_0 H_C = \frac{2J + (\mu_0 M_0^2)/2t}{M_0 - (\alpha_{zz}E)/\mu_0}, \quad (8)$$

where J is the exchange energy density, t is the interlayer distance in CrI_3 , and α_{zz} is the component of the magnetoelectric tensor for electric and magnetic fields along the normal to the film.

In Ref. [33], the phase transition between the antiferromagnetic and ferromagnetic states of a van der Waals antiferromagnet was considered for two different models of exchange interaction: Heisenberg and Ising (Figs 9a, b). The expression for the normalized free energy of a bilayer in a magnetic field H_z , taking into account the ME interaction with the electric field E_z , is expressed in the Ising model as:

$$U(\theta, \varepsilon) = -2H_z \sin \theta \sin \varepsilon - 0.5H_e(\cos 2\theta + \cos 2\varepsilon) - H_a \cos 2\theta \cos 2\varepsilon - \alpha E_z \sin 2\theta(H_z) \sin \varepsilon(H_z). \quad (9a)$$

In the Heisenberg model:

$$U(\theta, \varepsilon) = -2H_z \sin \theta \sin \varepsilon - (H_e + H_a \cos 2\theta) \cos 2\varepsilon - \alpha E_z \sin 2\theta(H_z) \sin \varepsilon(H_z), \quad (9b)$$

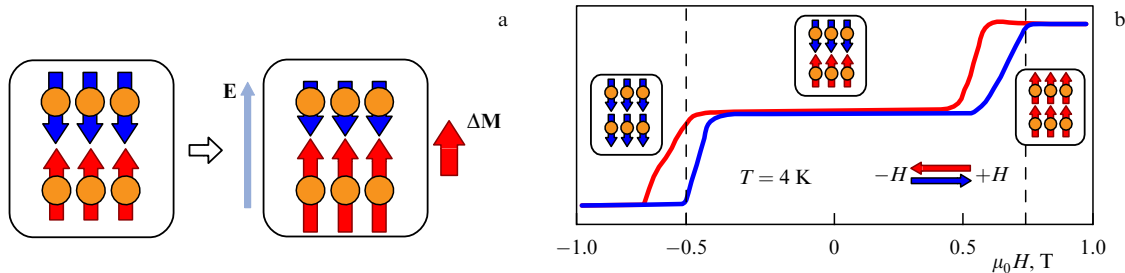


Figure 7. Crystal structure and magnetic properties of CrI_3 : (a) antiferromagnetic ordering characteristic of a CrI_3 bilayer and decompensation of magnetization due to application of an electric field; (b) magnetization plot (in arbitrary units) for a CrI_3 bilayer as a function of applied magnetic field. (Figures adapted from Ref. [4].)

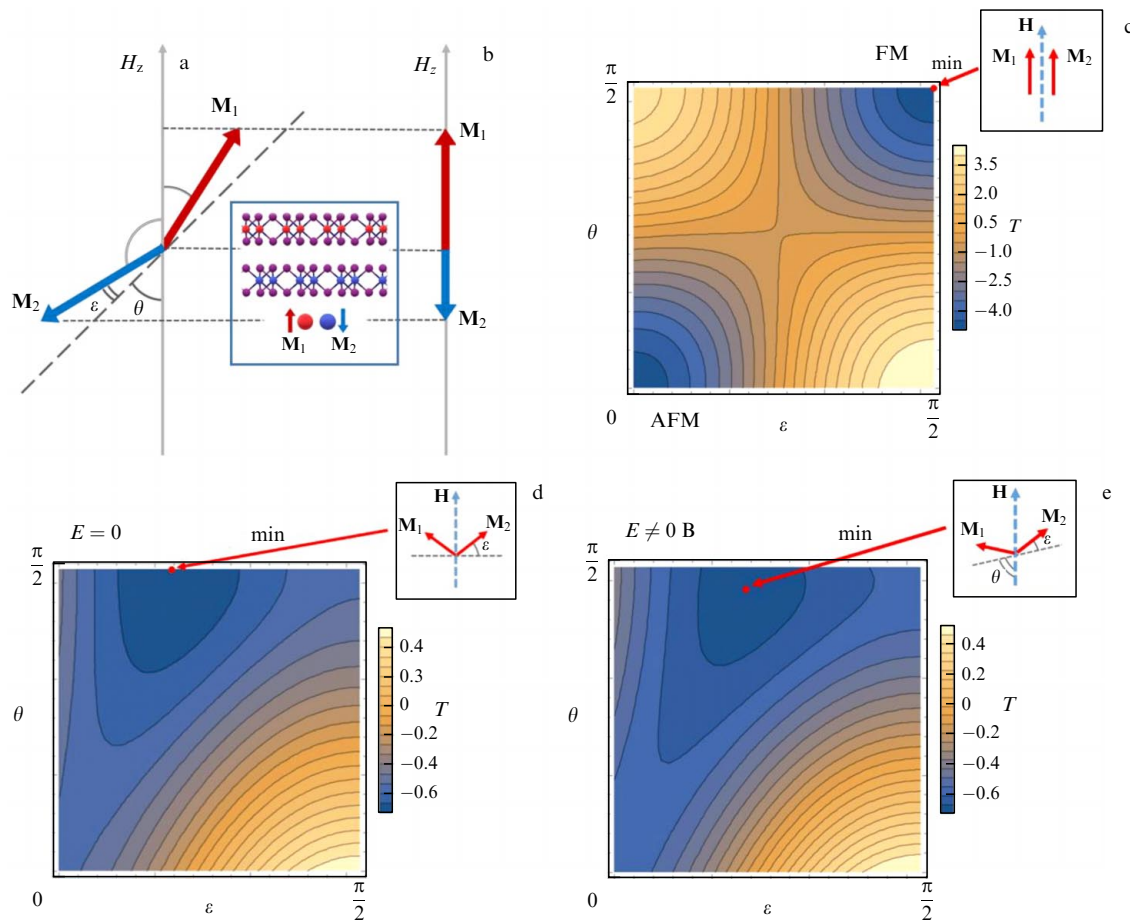


Figure 8. Spin-reorientation transitions in CrI_3 : (a) Heisenberg-type exchange interaction — under the action of an external magnetic field H_z , the magnetization vectors of both sublattices deviate from the antiferromagnetic axis by an angle ε ; (b) Ising model — the length of the magnetization vectors changes; inset: schematic representation of a CrI_3 bilayer. Contour plots of energy in coordinates of the sublattice canting angle ε and the rotation angle of the antiferromagnetism axis θ : (c) case of dominant anisotropy and weak interlayer exchange coupling; (d) transition via a spin-flop scenario — the anisotropy field is much smaller than the interlayer exchange field, the external field equals the exchange field, and the electric field is zero; (e) the same as in (d), but in the presence of an electric field contribution (0.2 V nm^{-1}) to the energy [33].

where H_e and H_a are the interlayer exchange field and the magnetic anisotropy field, respectively, α is the magneto-electric coefficient, and ε and θ are the angles of the sublattice magnetizations and the deviation of the antiferromagnetism axis, respectively.

Analysis of these expressions shows that the energy has extrema: at $(\theta, \varepsilon) = (0, 0)$ corresponding to an antiferromagnetic state with spins oriented normal to the film surface, and at $(\theta, \varepsilon) = (\pi/2, \pi/2)$ corresponding to a ferromagnetic state also with normal spin orientation. The transition between them occurs via a first-order phase transition scenario: when the external magnetic field increases to a value exceeding H_e , the ferromagnetic state becomes more favorable compared to the initial antiferromagnetic state $(\theta, \varepsilon) = (0, 0)$ (Fig. 8d). In the presence of an electric field, the position of the energy minimum shifts from the top edge of the diagram, which corresponds to a symmetry breaking in the arrangement of the sublattice magnetizations (Fig. 8e).

The magnetization dependences on the applied magnetic field for the above-considered cases exhibit a characteristic hysteresis shape and are shown in Figs 9a, b. A comparison of the approaches to the theoretical description of the influence of the electric field on the critical transition field

from Refs [4] and [33] is presented in Fig. 9d. Comparing experimental data with theoretical models, it can be concluded that none of the models quantitatively describes the experimental curves; nevertheless, formulas (9) allow one to explain the existence of two curves for the forward and reverse transitions and reproduce the general upward trend.

As can be seen from Figs 9a and b, in the case of Ising ordering, the sample transits from the antiferromagnetic to the ferromagnetic state and back without passing through intermediate stages, and the linear magnetoelectric effect leads to a shift of the critical magnetic field of the phase transition. In the case of weak magnetic anisotropy and Heisenberg-type exchange, the transition to the ferromagnetic state proceeds via a spin-flop phase, i.e., the material remains in an AFM state, but the antiferromagnetic vector lies in the film plane (Fig. 8d). This creates the possibility of inducing an in-plane magnetization in the magnetic system under the action of an interlayer electric voltage (Fig. 8e). The emergence of an uncompensated in-plane magnetization via the Einstein–de Haas effect generates a mechanical torque parallel to the film plane, which can be used to excite mechanical oscillations in microelectromechanical systems based on van der Waals antiferromagnets.

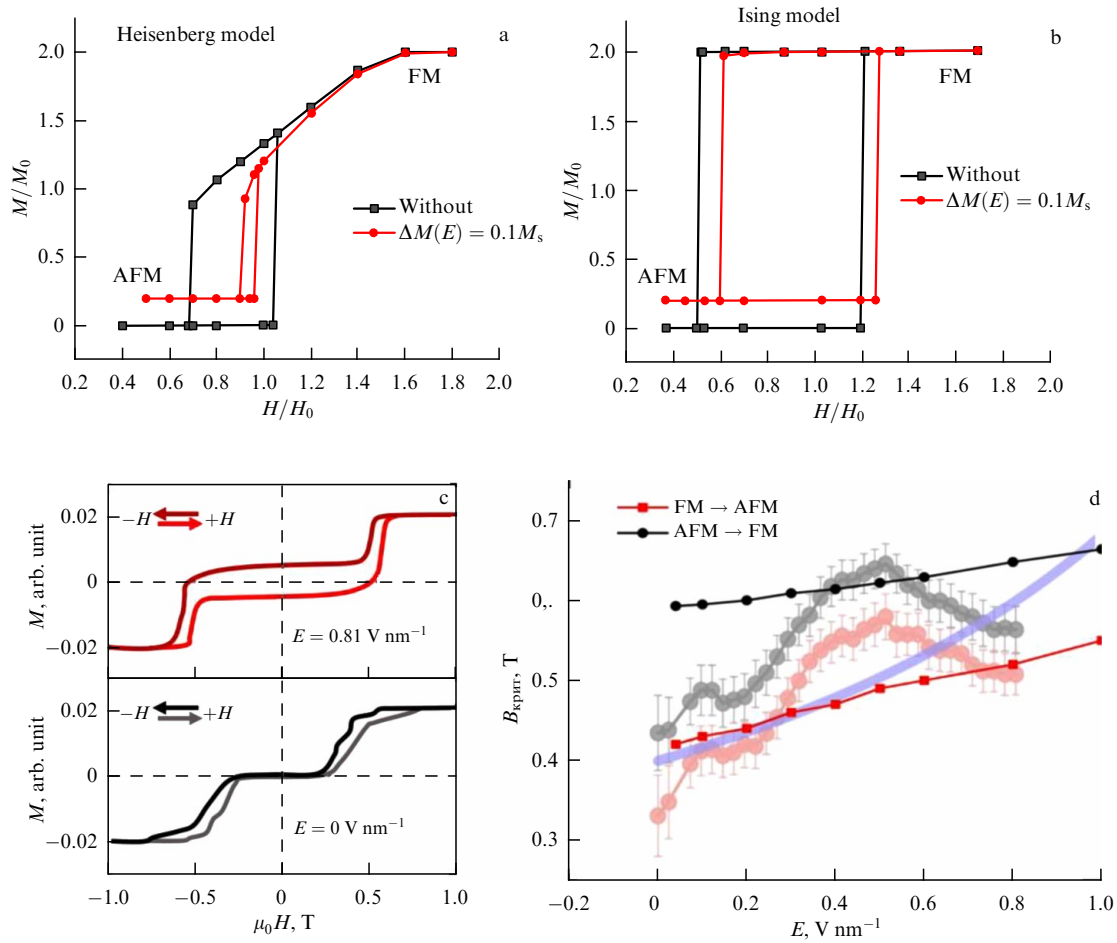


Figure 9. Influence of an electric field on the transition in a CrI₃ bilayer. Magnetization plots for a CrI₃ bilayer as a function of applied magnetic field in the absence and presence of an electric field: (a) in the Heisenberg exchange model (data from [33]); (b) in the Ising exchange model (data from [33]); (c) experimental dependences (data from [4]); (d) dependence of the critical transition field on the electric field strength. Blue line shows theoretical dependence according to formula (8); thin red and black lines show the theoretical dependences (for the forward and reverse transitions) according to formula (9a); large red and black dots correspond to experimental data [4].

3. Parametric generation of oscillations by a magnetic field via magnetostriction

Previously, we considered cases of forced resonance in antiferromagnets via flexomagnetic and gyromagnetic effects, which are proportional to the applied field, with oscillations occurring at the frequency of the external driving force. The most well-known case of coupling between the magnetic and mechanical subsystems in a crystal is magnetostriction. It differs from the effects discussed above in that it is quadratic in the magnetic order parameter; therefore, its influence on the mechanical motion of antiferromagnets can manifest as a parametric drive at the double frequency. Evidence of such an influence of magnetostriction on the mechanical stiffness of a sample can be found in [34], manifesting as a shift in the resonance frequency of a van der Waals AFM membrane made of CrI₃ upon a magnetically induced transition from the antiferromagnetic to the ferromagnetic state (Fig. 10a). When a magnetic field is applied in the plane of the sample, a smooth frequency shift is observed (Fig. 10b) due to spin canting, which confirms the strong magnetic anisotropy of the material (Fig. 1a).

In that work, oscillations were excited by an electric voltage at the mechanical resonance frequency of the membrane (approximately 25.5 MHz).

We now estimate the threshold values and oscillation amplitudes arising from parametric resonance due to periodic modulation of the membrane stiffness.

Consider the oscillations of a nonlinear oscillator in the presence of harmonic modulation of a certain parameter:

$$\ddot{\varphi} + 2\delta\dot{\varphi} + \omega_0^2(1 + m \cos 2\omega_0 t)(\varphi - \gamma\varphi^3) = 0, \quad (10)$$

where δ is the damping decrement of the system, m is the modulation depth of the elastic parameter due to the external magnetic field, ω_0 is the natural frequency of the system, and γ is the coefficient of cubic nonlinearity of membrane stiffness.

Introducing dimensionless time $\tau = \omega_0 t$, $\theta = \delta/\omega_0$, we obtain dimensionless oscillation equation with perturbations due to the dissipation, parametric drive and nonlinearity at the right side of the equation (small inhomogeneity):

$$\varphi'' + \varphi = -2\theta\varphi' - \varphi m \cos 2\tau + \gamma\varphi^3, \quad (11)$$

where the derivative with respect to dimensionless time τ is denoted as φ' .

We seek a solution in the form:

$$\varphi = u(\tau) \cos \tau + v(\tau) \sin \tau, \quad (12)$$

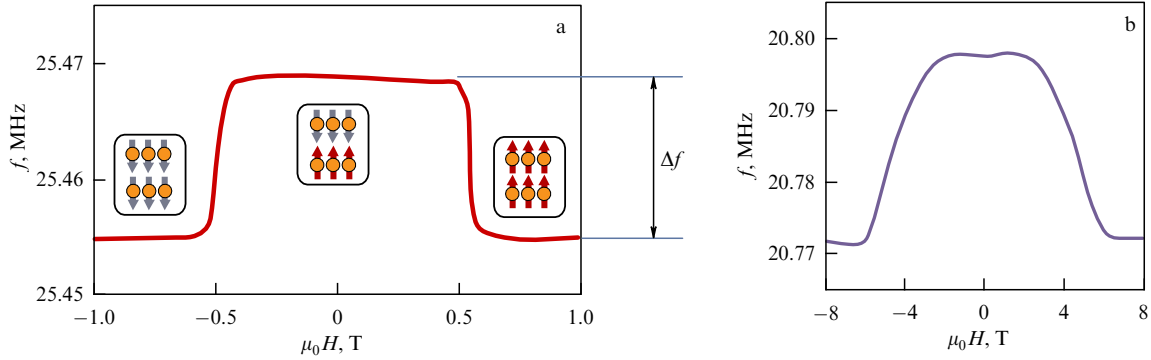


Figure 10. Dependence of the resonance frequency on applied magnetic field for a membrane made of the van der Waals antiferromagnet CrI₃: a change of $\sim 0.06\%$ for a bilayer CrI₃ sample (for a sixlayer sample, a shift greater than 0.2% was observed). (a) Field perpendicular to the sample plane; (b) field in the plane. (Graphs are reproduced from Ref. [34].)

where u and v are slowly varying amplitudes ($u' \ll u, v' \ll v$). Finding the stationary solution with the period-averaged values of u', v' are zero, we obtain the amplitude of parametric oscillations:

$$A_{\text{param}}^2 = \frac{4}{3\gamma} \sqrt{\frac{m^2}{4} - \frac{1}{Q^2}}, \quad (13)$$

where A denotes the total amplitude: $A^2 = u^2 + v^2$.

As can be seen from Fig. 10, the application of a magnetic field allows one to change the resonance frequency of the system due to the transition from the antiferromagnetic to the ferromagnetic state. According to [34], the magnitude of the observed changes is $\Delta f_0 \sim 0.01$ MHz, while the resonance frequency is $f_0 \approx 25$ MHz. From these data, the possible modulation depth in such a system can be estimated as:

$$m = \frac{\Delta(f^2)}{f_0^2} \approx \frac{2\Delta f_0}{f_0} \sim 10^{-3}. \quad (14)$$

According to the same work, the quality factor of the system under vacuum condition was $Q \sim 2000$; using this value and formula (14), it can be concluded that the modulation depth is near the threshold for parametric oscillation excitation.

The amplitude of the resulting parametric oscillations, according to (13), will be determined by the nonlinearity parameter γ . To estimate it, we use data from Ref. [35], in which mechanical oscillations in membranes based on FePS₃ heterostructures were excited by thermal action of a laser beam. From the temperature dependence of the oscillation amplitude, a connection was demonstrated between the magnetic state and the mechanical characteristics of the membranes, both linear and nonlinear.

The authors [35] explained these experimental observations using an analytical model of magnetostriction, in which, below the Néel temperature, the nonlinear stiffness (γ^*) and nonlinear damping (η_{nl}^*) become dependent on both the mechanical and magnetic parameters of the membrane:

$$\gamma^* = \gamma_0 - \frac{\lambda^2}{12B} \frac{E^2 c_3^2}{r^4} \frac{1}{1 + 4\omega^2 \tau^2}, \quad (15)$$

$$\eta_{nl}^* = \eta_{nl} + \frac{\lambda^2}{2B} \frac{E^2 c_3^2}{r^4} \frac{1}{1 + 4\omega^2 \tau^2}, \quad (16)$$

where γ_0 and η_{nl} are the nonmagnetic parts of the nonlinear stiffness and damping, respectively, λ is the magnetostriction

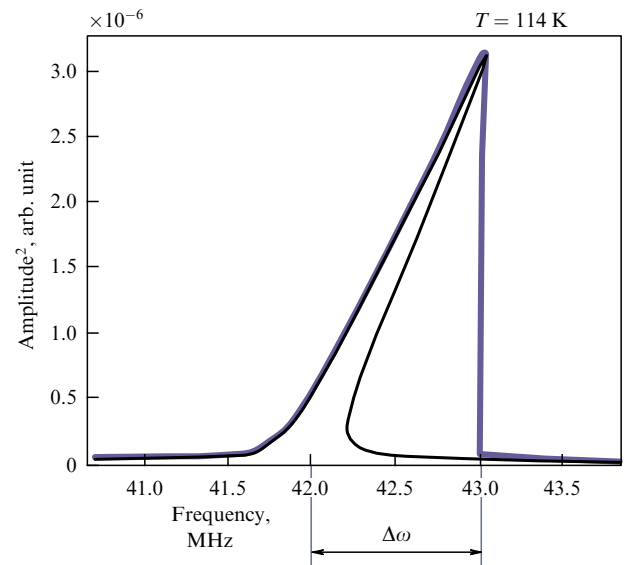


Figure 11. Nonlinear dynamics of an MLG/FePS₃ membrane. Influence of nonlinearity on the amplitude branches of the resonance peak. (Graph reproduced from Ref. [35].) MLG is the multi-layer graphene.

coefficient, E is the Young's modulus of the membrane, c_3 is a geometric factor, r is the membrane radius, B is a phenomenological positive constant, τ is the magnetic relaxation time, and ω is the frequency.

To estimate the magnitude of forced resonance in such systems, we use the well-known expression for the dependence of the resonance frequency on amplitude in a nonlinear system [35]:

$$\omega_{\text{res}}^2 = \omega_0^2 \left(1 + \frac{3}{4} \gamma A^2 \right), \quad (17)$$

and will also employ the amplitude-frequency characteristic for a quasi-two-dimensional membrane from Ref. [35], shown in Fig. 11.

Using expression (17), the oscillation amplitude under forced resonance in such a system can be approximately estimated as:

$$A_f^2 = \frac{2\Delta\omega}{\omega_0} \frac{4}{3\gamma}. \quad (18)$$

Comparing the expressions for the oscillation amplitudes under forced resonance (18) and under parametric excitation (13), (14), we find that for a high quality factor, their squares scale as the relative frequency shift of nonlinear oscillations $\Delta\omega/\omega_0 \sim 0.025$ (see Fig. 11) to one-quarter of the modulation coefficient m (14); i.e., the amplitude of the parametric resonance is an order of magnitude smaller than the forced-resonance amplitude, and is only slightly above the noise level in Ref. [35].

Thus, it is better to observe the parametric excitation in the absence of forced resonance, i.e., when the second harmonic in the spectrum of the membrane's natural oscillations is suppressed; otherwise, parametric resonance manifests as a parasitic effect against the background of forced excitation via the flexomagnetic interaction.

4. Conclusion

In the present work, various mechanisms for exciting mechanical oscillations in two-dimensional van der Waals antiferromagnets based on magnetomechanical coupling have been considered. As the main channels of interaction between the magnetic and mechanical subsystems in such materials, the flexomagnetic effect, the electrically induced Einstein–de Haas effect, and magnetostriction have been examined. An analysis has been carried out of the features of van der Waals antiferromagnets: how to use the factors favoring the generation and detection of mechanical oscillations and to circumvent unfavorable ones:

— The very geometry of van der Waals magnets promotes their extreme flexibility and the pronounced manifestation of flexoelectric and flexomagnetic interactions, which induce bending deformations and mechanical oscillations of membranes of micron dimensions and nanometer thickness. The performed estimates show that the inverse flexomagnetic effect in the van der Waals antiferromagnet CrI_3 can lead to detectable amplitudes of bending oscillations (~ 10 nm) at the moderate alternating magnetic field amplitudes (~ 100 Oe).

— Furthermore, it has been shown that the electrically induced Einstein–de Haas effect and the magnetoelectric interaction can serve as an alternative mechanism for exciting oscillations in antiferromagnetic systems. The Ising-type magnetic ordering with perpendicular anisotropy as a factor unfavorable for excitation of van der Waals flake's vibration can be eliminated in materials with Heisenberg-type exchange or in a magnetic field-induced state with in-plane spin ordering. Due to magnetoelectric effect an electric field induces a decompensation of the magnetic sublattices and gives rise to a net magnetization in antiferromagnetic samples, which can lead to mechanical oscillations via the Einstein–de Haas effect. The quality factor of the system, and consequently the amplitude of such oscillations, will depend strongly on the number of monolayers in the material, and for thicknesses exceeding 10 nm (tens of antiferromagnetic bilayers), it reaches detectable magnitudes.

— The analysis shows that magnetostriction enables parametric excitation of oscillations at an alternating magnetic field frequency twice the frequency at which forced resonance of the system occurs.

The obtained results indicate the feasibility of realizing new types of micro- and nanomechanical devices controlled by magnetic or electric fields, as well as the promise of van der Waals antiferromagnets as a platform for magnetomechanical and magnetoelectromechanical systems.

Acknowledgments. We express our gratitude to Professor A.K. Zvezdin for collaboration and valuable discussions, and to Z.A. Pyatakova for assistance with figure preparation.

Funding. This research was supported by a grant from the Russian Science Foundation no. 25-79-30019, <https://rscf.ru/project/25-79-30019/>.

References

1. Gong C et al. *Nature* **546** 265 (2017)
2. Huang B et al. *Nature* **546** 270 (2017)
3. Rahman S et al. *ACS Nano* **15** 17175 (2021)
4. Jiang S, Shan J, Mak K F *Nature Mater.* **17** 406 (2018)
5. Liang S et al. *Nature Electron.* **6** 199 (2023)
6. Edström A et al. *Phys. Rev. Lett.* **128** 177202 (2022); Qiu G et al. *npj Quantum Mater.* **8** 15 (2023)
7. Shi W et al. *J. Phys. Chem. C* **123** 24988 (2019)
8. Zubko P, Catalan G, Tagantsev A K *Annu. Rev. Mater. Res.* **43** 387 (2013)
9. Peng W et al. *Nature Phys.* **20** 450 (2024)
10. Eliseev E A et al. *Phys. Rev. B* **79** 165433 (2009)
11. Landau L D, Lifshitz E M *Elektrodinamika Sploshnykh Sred* (Moscow: Fizmatlit); Translated into English: *Electrodynamics of Continuous Media* (Oxford: Pergamon Press, 1984)
12. Qiao L et al. *Phys. Rev. B* **109** 014410 (2024)
13. He R et al. *Phys. Rev. Lett.* **134** 076101 (2025)
14. Zong A et al. *Nature* **620** 988 (2023)
15. Koliushenkov M A, Pyatakov A P *Europhys. Lett.* **147** 36002 (2024)
16. Lukashev P, Sabirianov R F *Phys. Rev. B* **82** 094417 (2010)
17. Sidhardh S, Ray M C J. *Appl. Phys.* **124** 244101 (2018)
18. Mironov V L *Osnovy Skaniruyushchei Zondovoi Mikroskopii* (Basics of Scanning Probe Microscopy) (Moscow: Tekhnosfera, 2004)
19. Eliseev E A et al. *Phys. Rev. B* **84** 174112 (2011)
20. Zhang J X et al. *Nanoscale* **4** 6196 (2012)
21. Belyaev B A et al. *Phys. Status Solidi RRL* **14** 1900467 (2020)
22. Makushko P et al. *Nature Commun.* **13** 6745 (2022)
23. Du D et al. *Nature Commun.* **12** 2494 (2021)
24. Liu L, Chen W, Zheng Y *Phys. Rev. Lett.* **128** 257201 (2022)
25. Einstein A, de Haas W J *Deutsche Phys. Gesellschaft, Verhandlungen* **17** 152 (1915)
26. Wallis T M, Moreland J, Kabos P *Appl. Phys. Lett.* **89** 122502 (2006)
27. Ganzhorn M et al. *Nature Commun.* **7** 11443 (2016)
28. Dornes C et al. *Nature* **565** 209 (2019)
29. Bunch J S et al. *Science* **315** 490 (2007)
30. Frey N C et al. *ACS Nano* **12** 6319 (2018)
31. Bikaljević D et al. *ACS Nano* **15** 14985 (2021)
32. Song Q et al. *Nature* **602** 601 (2022)
33. Zvezdin A K, Koliushenkov M A, Pyatakov A P *JETP Lett.* **121** 577 (2025); *Pis'ma Zh. Eksp. Teor. Fiz.* **121** 605 (2025)
34. Jiang S et al. *Nature Mater.* **19** 1295 (2020)
35. Šiškins M et al. *Nature Commun.* **16** 2177 (2025)
36. Shu L et al. *Appl. Phys. Lett.* **104** 232902 (2014)
37. Zalesskii V G, Rumyantseva E D *Phys. Solid State* **56** 1352 (2014); *Fiz. Tverd. Tela* **56** 1301 (2014)
38. Bhaskar U K et al. *Nature Nanotechnol.* **11** 263 (2016)
39. Wen X et al. *Phys. Rev. Applied* **15** 014032 (2021)