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On characteristic times for kinetics of scattering

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Contents

Abstract. Although kinetic equations for various scattering processes have proven their practical significance, the question about their area of applicability and derivation from the initial dynamic equations is satisfactorily solved only for the case of gases. This question remains relevant for kinetic equations describing both nonlinear wave processes and transport phenomena in semiconductors. Using the simplest example of elastic scattering, we show that the validity of the kinetic equation for the distribution function $n_{\mathbf{k}}$ implies the existence of an internal characteristic time scale τ_s much smaller than standard scattering times such as the transport time τ_{tr}. The time τ_s corresponds to the rapid stochastization of the phases of the waves (quasiparticles). As show our estimates, this time satisfies the inequalities $1 \ll \omega_k\tau_s \ll \omega_k\tau_{tr}$, where $\omega_k = \varepsilon_k/\hbar$ is the characteristic frequency of the quasiparticles. This means that over time τ_s the density matrix $\rho_{\mathbf{k},\mathbf{k}'}$ relaxes to the diagonal form $n_{\mathbf{k}}\delta_{\mathbf{k}-\mathbf{k}'}$, so that its contribution to transport phenomena is small in the parameter τ_s/τ_{tr} compared to the contribution of $n_{\mathbf{k}}$.

Keywords: kinetic equations, scattering processes, density matrix, stochastization time, transport time

1. Introduction

Kinetic equations for the distribution function $n_{\mathbf{k}}$ (occupancy numbers), where $\mathbf{k}$ is the quasi-momentum, have long proven their significance and are widely used to describe a big number of classical and quantum physical phenomena. These include a variety of transport phenomena in gases and solids [1–7], including photovoltaic phenomena [8–11], as well as nonlinear wave phenomena in hydrodynamics and plasma [12–14]. A general property of kinetic equations (KE) is that they are time irreversible (dissipative) and contain, in addition to $n_{\mathbf{k}}$, only the first time derivative $\partial n_{\mathbf{k}}/\partial t$. The relative simplicity of KE allows one to calculate such properties of nonequilibrium statistical systems as relaxation times, $n_{\mathbf{k}}$ spectra, photocurrents, etc. A more general description within the framework of the single-particle approximation is the description of kinetic phenomena using the density matrix $\rho_{\mathbf{k},\mathbf{k}'}$, the diagonal elements of which give $n_{\mathbf{k}}$. The efficiency of the KE method is due to the fact that, as a rule, the off-diagonal elements of the density matrix (with $\mathbf{k} \neq \mathbf{k}'$) can be considered relatively small. The present scientific development places steadily growing demands to the methods of kinetic description.

Unfortunately, a consistent derivation of KE from the initial conservative dynamic equations is known only for the case of gases [1]. In the overwhelming majority of cases, the KE are postulated using intuitive considerations about the probabilities of elementary processes [1–7]. This can lead to misunderstandings and elementary errors [15]. The question about the nature and status of the relation $\rho_{\mathbf{k},\mathbf{k}'} \simeq n_{\mathbf{k}}\delta_{\mathbf{k}-\mathbf{k}'}$ does not arise in this case. In the literature, one can find only indirect evidence that the smallness of the off-diagonal elements $\rho_{\mathbf{k},\mathbf{k}'}$ is associated with a rapid stochastization of the quasiparticle phases (loss of coherence) [14, 16]. In nonlinear optics, a phenomenological method for taking kinetic effects into account in equations for the density matrix is widely used [17]. In the framework of this method, the relaxation times of diagonal and off-diagonal elements of the density matrix are distinguished (but not calculated). A similar phenomenological approach was recently used in a study of the bulk photovoltaic effect [18], while the relaxation times for $n_{\mathbf{k}}$ and $\rho_{\mathbf{k},\mathbf{k}'}$ did not differ. Thus, the question about the need to proceed beyond the KE remains open. Accordingly, the question about the magnitude of the observed effects also remains open.

The aim of this work is to show, with regard to elastic and quasi-elastic scattering, that the phase stochastization time τ_s (i.e., the relaxation time of the off-diagonal elements $\rho_{\mathbf{k},\mathbf{k}'}$) is much shorter than the times associated with the relaxation of $n_{\mathbf{k}}$, including the transport time τ_{tr}. This not only substantiates the validity of the corresponding quantum equations, but

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also proves the validity of the kinetic description of photovoltaic phenomena [8–11] and the smallness of the corrections related to the off-diagonal elements of the density matrix. Our approach is based on the classical Hamiltonian formalism for normal wave (quasiparticle) amplitudes [12, 14], which is easily transferred to the quantum case.

2. Derivation of the kinetic equation

The simplest and most important for the subsequent is the KE for elastic scattering

$$\frac{\partial n_{\mathbf{k}}}{\partial t} = \sum_{\mathbf{k}'} \left(W_{\mathbf{k},\mathbf{k}'} n_{\mathbf{k}'} - W_{\mathbf{k}',\mathbf{k}} n_{\mathbf{k}} \right), \tag{1}$$

where $W_{\mathbf{k},\mathbf{k}'} \equiv W_{-\mathbf{k}',-\mathbf{k}}$ is the probability of transition per unit time from state $\mathbf{k}'$ to state $\mathbf{k}$, proportional to the concentration of scattering centers N_0 and the energy delta function $\delta(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'})$. In this form, Eqn (1) is applicable to any particles of the same type — electrons, phonons, etc., — even in the absence of central symmetry, when $W_{\mathbf{k},\mathbf{k}'} \neq W_{\mathbf{k}',\mathbf{k}}$, i.e., there is no detailed equilibrium [15]. In the Born approximation $W_{\mathbf{k},\mathbf{k}'} = W_{\mathbf{k}',\mathbf{k}}$. The absence in KE of the quantum factors $(1 \mp n_{\mathbf{k}})$ and $(1 \mp n_{\mathbf{k}'})$ for fermions and bosons, which appear erroneously in most books, corresponds to a correct accounting of the occupation numbers in intermediate states beyond the Born approximation. The presence of such factors would lead to an obvious absurd: in the absence of detailed equilibrium, the kinetic equation for phonons would be nonlinear (quadratic) in $n_{\mathbf{k}}$ in the classical limit $n_{\mathbf{k}} \gg 1$. The simple form of KE simplifies the subsequent consideration.

To achieve the goal of this paper — the justification of a rapid phase stochastization — we will first consider the derivation of KE from conservative dynamic equations in their simplest Hamiltonian formulation. Our derivation is an adaptation (as applied to elastic scattering) of the method presented in [12–14], but does not repeat it. We assume that the normal complex amplitudes of the waves $a_{\mathbf{k}}, a_{\mathbf{k}}^*$, which are independent, obey the classical Hamiltonian dynamic equations [12, 14]

$$\mathrm{i}\,\frac{\partial a_{\mathbf{k}}}{\partial t} = \frac{\partial H}{\partial a_{\mathbf{k}}^*}\,. \tag{2}$$

In the case of elastic scattering, the Hamiltonian H can be chosen in the form

$$H = \sum_{\mathbf{k}_1} \omega_{\mathbf{k}_1} |a_{\mathbf{k}_1}|^2 + \sum_{\mathbf{q},\mathbf{k}_{1,2}} \left(V_{\mathbf{q}}\, a_{\mathbf{k}_1} a_{\mathbf{k}_2}^* b_{\mathbf{q}}\, \delta_{\mathbf{k}_1 - \mathbf{k}_2 + \mathbf{q}} + \text{c.c.} \right), \tag{3}$$

where $\omega_{\mathbf{k}} = \varepsilon_{\mathbf{k}}/\hbar$ is the frequency, $V_{\mathbf{q}} = V_{-\mathbf{q}}^*$ are coupling coefficients with the frequency dimension, $b_{\mathbf{q}}$ are static complex amplitudes with random phases (relevant to random positions of the scattering centers) and a unit modulus. This corresponds to a nonzero statistic average $\langle b_{\mathbf{q}}^* b_{\mathbf{q}'} \rangle = \delta_{\mathbf{q}-\mathbf{q}'}$. In our approach, the distribution function $n_{\mathbf{k}} = \langle |a_{\mathbf{k}}|^2 \rangle$, and the density matrix is the correlation function, $\rho_{\mathbf{k},\mathbf{k}'} = \langle a_{\mathbf{k}}^* a_{\mathbf{k}'} \rangle$.

In the standard transition from summation to integration over $\mathbf{k}$ in Eqn (2), the partial derivative with respect to $a_{\mathbf{k}}^*$ has to be understood as the variational derivative $\delta/\delta a_{\mathbf{k}}^*$. The normal amplitudes $a_{\mathbf{k}}, a_{\mathbf{k}}^*$ can be viewed as the classical analogue of quantum creation and annihilation operators. A slight generalization of Eqn (3) allows one to describe electron-phonon scattering with phonon amplitudes $b_{\mathbf{q}}(t)$ and frequencies $\Omega_{\mathbf{q}}$. It is sometimes useful to add an infinitely small wave damping 0 to the left-hand side of the conservative dynamic equations (2), ensuring the correct choice of the time direction.

Next, it is important to follow the assumptions by which from the dynamic equation

$$\mathrm{i}\,\frac{\partial a_{\mathbf{k}}}{\partial t} = \omega_{\mathbf{k}} a_{\mathbf{k}} + 2 \sum_{\mathbf{k}_1} V_{\mathbf{k}-\mathbf{k}_1} a_{\mathbf{k}_1} b_{\mathbf{k}-\mathbf{k}_1}\,, \tag{4}$$

relevant to Hamiltonian (3), we obtain the kinetic equation (1). For the distribution function, we have from (4):

$$\left\langle \frac{\partial |a_{\mathbf{k}}|^2}{\partial t} \right\rangle \simeq \frac{\partial n_{\mathbf{k}}}{\partial t} = 4\,\mathrm{Im} \sum_{\mathbf{k}_1} V_{\mathbf{k}-\mathbf{k}_1} \langle b_{\mathbf{k}-\mathbf{k}_1} a_{\mathbf{k}}^* a_{\mathbf{k}_1} \rangle\,. \tag{5}$$

Replacement the average of the derivative with the derivative of the average on the left-hand side is a common practice in deriving various KE in [12–14]. This replacement requires a justification, which, however, is outlined only in [14]. This justification is the possibility of averaging on a time scale which is much shorter than the characteristic time of the change of $n_{\mathbf{k}}$. This small time is associated with the time of phase stochastization τ_{s}, i.e., the loss of coherence. In the absence of such a small time scale, the transition to the KE would be impossible. In the presence of the small scale τ_{s}, replacing the average of the derivative by the derivative of the average is analogous to that in deriving, for example, Maxwell's macroscopic equations from microscopic ones. In contrast to [14], we do not assume that $\tau_{\mathrm{s}} \sim \omega_{\mathbf{k}}^{-1}$. Such an assumption appears unnatural in the case of elastic scattering. Moreover, it would imply that the irreversibility of the kinetic process is due to the conservative dispersion effect. Below, we relate τ_{s} directly to the properties of the random scattering process.

The triple average $\langle b_{\mathbf{k}-\mathbf{k}_1} a_{\mathbf{k}}^* a_{\mathbf{k}_1} \rangle$ appearing in Eqn (5) depends on the wave phases. Owing to the rapid phase relaxation, this average adiabatically follows relatively slow changes in $n_{\mathbf{k}}(t)$. This means that when taking the average from the equation for $b_{\mathbf{k}-\mathbf{k}_1} a_{\mathbf{k}}^* a_{\mathbf{k}_1}$, which follows from Eqn (4), we must neglect the time derivative. In addition, one should take into account the obvious relationship $\langle b_{\mathbf{q}}^* b_{\mathbf{q}'} a_{\mathbf{k}}^* a_{\mathbf{k}'} \rangle = n_{\mathbf{k}} \delta_{\mathbf{q}-\mathbf{q}'} \delta_{\mathbf{k}-\mathbf{k}'}$. As a result we obtain

$$\langle b_{\mathbf{k}-\mathbf{k}_1} a_{\mathbf{k}}^* a_{\mathbf{k}_1} \rangle = \frac{V_{\mathbf{k}-\mathbf{k}_1}^* (n_{\mathbf{k}_1} - n_{\mathbf{k}})}{\omega_{\mathbf{k}_1} - \omega_{\mathbf{k}} - 2\mathrm{i}0}\,. \tag{6}$$

Keeping the seed decay 0 in the denominator is crucial even though its magnitude is negligible compared to $\omega_{\mathbf{k}}$. When substituting Eqn (6) into (5), this denominator yields a delta function according to the relation $\mathrm{Im}\,(\omega_{\mathbf{k}_1} - \omega_{\mathbf{k}} - \mathrm{i}0)^{-1} = \pi\delta(\omega_{\mathbf{k}} - \omega_{\mathbf{k}_1})$. As a result, we obtain the desired kinetic equation (1), in which $W_{\mathbf{k},\mathbf{k}'} = 4\pi |V_{\mathbf{k}-\mathbf{k}'}|^2 \times \delta(\omega_{\mathbf{k}} - \omega_{\mathbf{k}'})$. The symmetry property $W_{\mathbf{k},\mathbf{k}'} = W_{\mathbf{k}',\mathbf{k}}$ is associated with the use of the Born approximation; this approximation is sufficient for the purpose of our work.

3. Stochastization time

Now we consider the question about the rate of phase stochastization, i.e. the inverse stochastization time τ_{s}^{-1}. To do this, we select the random part of the phase $\varphi_{\mathbf{k}} = \arg a_{\mathbf{k}} + \omega_{\mathbf{k}} t$ and obtain, using the statistical properties of the amplitudes $b_{\mathbf{q}}$, from the dynamic equation (4) for the

quadratic average of the time derivative:

$$\langle \dot{\varphi}_{\mathbf{k}}^2 \rangle = 2 \sum_{\mathbf{q}} \left| \frac{a_{\mathbf{k}-\mathbf{q}}}{a_{\mathbf{k}}} \right|^2 |V_{\mathbf{q}}|^2 . \tag{7}$$

The initial distribution $a_{\mathbf{k}}$ can be considered coherent or partially coherent. It is noteworthy that the obtained expression does not contain the frequency delta function. Note also that the absolute value of the ratio in Eqn (7) tends to 1 as $\mathbf{q} \to 0$. The stochastization rate can be defined as $\gamma_s = \tau_s^{-1} = [\langle \dot{\varphi}_{\mathbf{k}}^2 \rangle]^{1/2}$. It should be compared with the standard relaxation rates (inverse times) defined using the kinetic equation (1). This is the inverse escape time $\gamma_0 = \sum_{\mathbf{k}'} W_{\mathbf{k}',\mathbf{k}}$ and the inverse transport time $\gamma_{\text{tr}} = \tau_{\text{tr}}^{-1} = \sum_{\mathbf{k}'} W_{\mathbf{k}',\mathbf{k}}(1 - \cos\theta)$, where θ is the scattering angle [3]; the time τ_{tr} determines such an important characteristic as the mobility of charge carriers at constant current. It should be kept in mind that the used approximation of nearly free particles makes sense only under the condition $\gamma_{0,\text{tr},s} \ll \omega_{\mathbf{k}}$. Violation of this condition most likely leads to localization effects.

The fundamental difference between the expressions for $\gamma_{0,\text{tr}}$ and γ_s is noteworthy. While the rates $\gamma_{0,\text{tr}}$ can be roughly estimated as $|V_{\mathbf{q}_*}|^2/\omega_{\mathbf{k}}$, the rate γ_s is roughly estimated as $|V_{\mathbf{q}_*}|$, where $\mathbf{q}_*$ is some characteristic value of $\mathbf{q}$. In this case, the ratio $\gamma_{0,\text{tr}}/\gamma_s$ is roughly estimated as $|V_{\mathbf{q}_*}|/\omega_{\mathbf{k}}$, which is a small parameter within the concept of almost free particles. Moreover, it is important to keep in mind that the probability of elastic electron scattering $W_{\mathbf{k}',\mathbf{k}}$ usually increases sharply as $\theta \to 0$ (i.e., for forward scattering). This is especially important for scattering by charged defects; here $W_{\mathbf{k},\mathbf{k}'} \propto \theta^{-4}$ as $\theta \to 0$, necessitating a 'cutoff' at small angles [2, 3]. The presence of the factor $(1 - \cos\theta) \simeq \theta^2/2$ for $\theta \to 0$ should lead to an additional decrease in the ratio $\gamma_{\text{tr}}/\gamma_s$.

4. Quantitative estimates

To make more accurate estimates, we consider the case of electrons, switch in the standard way from summation to integration over $\mathbf{k}$ or $\mathbf{q}$, and adopt the assumptions of isotropic scattering, $V_{\mathbf{q}} = V(q)$, and a quadratic dispersion law, $\omega_{\mathbf{k}} = \hbar k^2/2m$, where m is the effective mass. In addition, we restrict ourselves to the case of scattering by charged defects. Under the assumptions made, the summation is replaced by the integration over $\mathrm{d}\mathbf{k}$ with $|V_{\mathbf{q}}|^2$ replaced by $(N_0 \hbar^2/4\pi m^2)\,\sigma(\theta)$, the differential scattering cross section $\sigma(\theta) = 1/4 \sin^4(\theta/2) k^4 a_B^2$, Bohr radius $a_B = \epsilon_0 \hbar^2/me^2$, and the static permittivity ϵ_0 [3]. Note that $\sin(\theta/2) = q/2k$, and the exact expression for $\sigma(\theta)$ used above coincides with that calculated in the Born approximation. When calculating γ_{tr}, the integral over θ diverges logarithmically at the lower limit. This divergence can be resolved by 'cut off' in θ. Taking into account the decrease of the scattering angle with increasing impact parameter, we obtain that $\theta_{\min} = 4N_0^{1/3}/a_B k^2$; this corresponds to an impact parameter equal to half the average distance between scattering centers [3]. As a result, $\gamma_{\text{tr}} \approx 8\pi\omega_k \times \Lambda N_0/a_B^2 k^5$, where $\Lambda = \ln(2/\theta_{\min})$. The dimensionless parameter $\eta = N_0/a_B^2 k^5$ has to be very small to ensure the inequality $\gamma_{\text{tr}} \ll \omega_k$. Note also the useful relation $\theta_{\min}^3 = 64\eta/ka_B$; it emphasizes the importance of small values of η and sufficiently large values of ka_B.

When estimating γ_s from Eqn (7), we assume that the function $|a_{\mathbf{k}-\mathbf{q}}/a_{\mathbf{k}}|^2$ decreases in q rapid enough for $q \gtrsim k$, such that the singularity $|V(q)|^2 \propto 1/q^4$ for $q \to 0$ is of prime importance. In this case, it is easy to obtain that

$$\gamma_s \approx 2\sqrt{2}\,\omega_k \times \theta_{\min}^{-1/2} \eta^{1/2} . \tag{8}$$

The factor $\theta_{\min}^{-1/2} > 1$ comes from our 'cut off' procedure. A necessary condition for the applicability of Eqn (8) is obviously the inequality $\sqrt{\eta} \ll 1$, it is stronger than the condition for the validity of the above estimate for γ_{tr}. For the ratio $\gamma_{\text{tr}}/\gamma_s$, we obtain up to a numerical coefficient:

$$\frac{\gamma_{\text{tr}}}{\gamma_s} \approx \eta^{1/2} \times \theta_{\min}^{1/2} \ln\left(\frac{2}{\theta_{\min}}\right) . \tag{9}$$

Both factors on the right-hand side are small, and the smallness of $\sqrt{\eta}$ is of a more fundamental nature. The smallness of the product $\theta_{\min}^{1/2} \ln(2/\theta_{\min})$ is due to the Coulomb divergence of the scattering cross section. If we ignore the influence of this factor, then the estimate (9) can be rewritten as $\gamma_{\text{tr}}/\gamma_s \sim \gamma_s/\omega_k \ll 1$, emphasizing the importance of the condition for the applicability of the concept of nearly free electrons. The analysis performed confirms the estimate made above from general considerations and indicates that the phase stochastization time is always much shorter than the transport time.

At a sufficiently low concentration N_0, scattering by phonons can become the dominant scattering mechanism. The derived kinetic equation can be easily generalized to this case as well. To do this, it is necessary to consider $b_{\mathbf{q}}$ as the phonon amplitude, add the phonon contribution $\sum_{\mathbf{q}'} \Omega_{\mathbf{q}'} |b_{\mathbf{q}'}|^2$ to the Hamiltonian (3), and include the dynamic equation $\mathrm{i}\partial_t b_{\mathbf{q}} = \partial H/\partial b_{\mathbf{q}}^*$, see also [12, 14]. The phonon distribution function $N_{\mathbf{q}} = \langle b_{\mathbf{q}}^* b_{\mathbf{q}} \rangle$ can then be considered thermodynamic equilibrium (Rayleigh) function. The main conclusions about the status of the stochastization time are naturally transferred to this case.

It is also important to touch upon the quantum mechanic derivation of the kinetic equations. In this case, it is necessary to use the commutation dynamic equation for the density matrix $\rho_{\mathbf{k},\mathbf{k}'} = \langle \hat{a}_{\mathbf{k}}^+ \hat{a}_{\mathbf{k}'} \rangle$, in which $\hat{a}_{\mathbf{k}}^+$ and $\hat{a}_{\mathbf{k}'}$ are the creation and annihilation operators. The quantum approach leads to some complication of the derivation procedure because of the use of quantum commutation relations, but leads to the same conclusions, see [15]. The relation (7) in this case corresponds to the probability amplitudes of scattered electrons.

5. Discussion

In essence, the above discussion suggests the existence of a simple alternative: If the kinetic equation for scattering processes is valid, then there is a rapid stochastization of the phases, $\tau_s \ll \tau_{\text{tr}}$ and $\rho_{\mathbf{k},\mathbf{k}'} \simeq n_{\mathbf{k}} \delta_{\mathbf{k}-\mathbf{k}'}$. If such a rapid stochastization is absent, then the KE for $n_{\mathbf{k}}$ is not applicable. Thus, the validity of the KE implies that the response of the system at times $t \lesssim \tau_s$ (and, accordingly, at frequencies $\omega \gtrsim 2\pi/\tau_s$) is controlled not by the transport time τ_{tr}, but by τ_s.

The same is applicable to the recent work [18]. In this work, the photocurrent for transitions from the valence to conduction band in a noncentrosymmetric cubic crystal of class T_d was calculated not as previously [9, 19–21] using the KE for the electron and hole distribution functions $n_{\mathbf{k}}^{\mathrm{e}}$ and $n_{\mathbf{k}}^{\mathrm{h}}$, but using the electron-hole density matrix $\rho_{\mathbf{k},\mathbf{k}'}^{\mathrm{e-h}}$. This was motivated by the presence of significant off-diagonal elements of the velocity operator $\mathbf{v}_{\mathbf{k},\mathbf{k}'}^{\mathrm{e-h}}$ in the basis of Coulomb wave functions. In this case, the momentum relaxation was

phenomenologically described by a single relaxation time τ_{tr}. In the case of a discrepancy with the previous kinetic approach, the work [18] would have claimed a significant revision of the theory. However, the results of current calculations within the framework of a single-band model with and without taking into account off-diagonal effects coincided. Leaving aside the reasons for this coincidence, it can be argued that there is no reason to revise the theory by taking into account the off-diagonal elements of the density matrix. This result is important in the context of the ongoing discussion about the ratio of contributions to the photocurrent caused by the asymmetry of the $n_{\mathbf{k}}^{e,h}$ distributions and those not associated with this asymmetry — the so-called ballistic and shift currents, see [11, 22–24].

What are possible manifestations of the short stochastic time τ_s? Most probably, the simplest of them is the spectral dependence of radiation absorption by free charge carriers, accessible to experiment. We are talking about the linear response of the system. For a parabolic zone and radiation frequency $\omega \ll k_B T/\hbar$, when the classical kinetic description is valid, the imaginary part of the permittivity responsible for absorption is traditionally expressed in terms of τ_{tr}: $\epsilon''_\omega = -\tau_{tr}\omega_p^2/\omega(\omega^2\tau_{tr}^2+1)$, where ω_p is the plasma frequency [4, 5]. Accordingly, the absorption coefficient proportional to $\omega\epsilon''_\omega$ should have a Lorentzian spectrum and be proportional to the concentration of free carriers. In the context of our consideration, the given expression for ϵ''_ω is valid only for $\omega \ll 2\pi/\tau_s$. At higher frequencies, the modification of the Lorentzian dependence associated with the stochasticization of the carrier phases can become important. Experimental data conducted at different carrier concentrations show significant deviations from the predicted Lorentzian dependence [5]. This effect deserves attention, but consideration of its theoretical and experimental details is beyond the scope of this work.

In addition to the KE for scattering by impurities and phonons, there is an important class of classical kinetic equations describing the interaction (decay and merge) of waves owing to various conservative three- and four-wave nonlinear processes [12–14]. These kinetic equations cover a wide range of observable phenomena in plasma and hydrodynamics. Unlike the KE considered above, they do not explicitly contain elements responsible for the violation of time reversal invariance, such as scattering by randomly located impurities or thermal phonons. Instead, they incorporate various kinds of instabilities leading to spatiotemporal amplification of waves from noise and, thus, to phase stochastization. The question of the value of time τ_s for such equations is not fully resolved, although the role of dispersion seems to be important [14].

It should be noted that elementary mechanisms illustrating the transition to chaos in dynamic systems have been actively considered in the literature, see, for example, [16, 25]. However, they do not reveal a direct connection with the properties of kinetic equations.

6. Conclusions

We have shown that the condition for the applicability of the kinetic equations describing the scattering of quasiparticles on impurities and phonons is the smallness of the phase stochastization time τ_s compared to the transport time τ_{tr} dictated by the KE itself. The ratio τ_s/τ_{tr} in this case turns out to be small with respect to the main parameter of the theory $(\tau_s\omega)^{-1} \ll 1$. Because of this, the density matrix $\rho_{\mathbf{k},\mathbf{k}'}$ quickly relaxes to the diagonal form $n_{\mathbf{k}}\delta_{\mathbf{k}-\mathbf{k}'}$, such that the contribution of the off-diagonal elements to the observable quantities turns out to be small compared to the contribution from the distribution function. This circumstance not only clarifies the status of the KE, but is also important for the analysis of the actual photovoltaic effects.

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