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From Landau two-fluid model to de Sitter Universe

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Contents

Abstract. Condensed matter analogs are useful when considering phenomena related to the quantum vacuum. This is because, in condensed matter, we know the physics both in the infrared and in the ultraviolet limits, while, in particle physics and gravity, the physics on the trans-Planckian scale are unknown. One of the cornerstones of the connections between nonrelativistic condensed matter and modern relativistic theories is the two-fluid hydrodynamics of superfluid helium, which was developed by Landau and Khalatnikov (Isaak Markovich Khalatnikov later founded and headed the Landau Institute). The dynamics and thermodynamics of the de Sitter state of the expansion of the Universe bear some features of the multi-fluid system. There are actually three components: the quantum vacuum, the gravitational component, and relativistic matter. The expanding de Sitter vacuum serves as a thermal bath with local temperature, which is twice the Gibbons–Hawking temperature related to the cosmological horizon. This local temperature leads to the heating of the matter component and the gravitational component, which behaves like Zel'dovich stiff matter and represents dark matter. In equilibrium and in the absence of conventional matter, the positive partial pressure of the dark matter compensates the negative

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partial pressure of the quantum vacuum. That is why, in full equilibrium, the total pressure is zero. This is rather similar to the correspondingly superfluid and normal components of superfluid liquid, which together produce a zero pressure of the liquid in the absence of environment. We propose the phenomenological theory, which describes the dynamics of dark energy and dark matter. If one assumes that, in the dynamics, gravitational dark matter behaves like real Zel'dovich stiff matter, it is shown that both components undergo power law decay due to the energy exchange between these components. It then follows that their values at the present time have the correct order of magnitude. We also consider other problems through the prism of condensed matter physics, including black holes and the Planck constant.

Keywords: quantum vacuum, thermodynamics of de Sitter Universe, quantum tunneling, black holes, Unruh effect, Planck constant

1. Introduction

The de Sitter universe has no ordinary matter content and is entirely supported by a quantum vacuum (called dark energy). This state is thought to have been in the early stages of inflation, and is now being approached again. Being isotropic and homogeneous in space and time, the de Sitter state is unique in cosmology. Isotropy and homogeneity allow us to construct thermodynamics for this state that is in many ways similar to that of isotropic and homogeneous condensed matter.

The vacuum of the de Sitter spacetime is characterized by the local temperature $T = \hbar H/\pi$, where H is the Hubble parameter (see Refs [1, 2] and references therein). This temperature describes the thermal processes of decay of the composite particles and the other activation processes, which are energetically forbidden in the Minkowski spacetime but are allowed in the de Sitter background (see also Refs [3–5]). Notably, this temperature determines the probability of the ionization of an atom in the de Sitter environment, $\exp(-E/T)$, where E is the ionization potential. This activation temperature is twice the Gibbons–Hawking [6] temperature $T_{\rm GH} = \hbar H/2\pi$ of the cosmological horizon, $T = 2T_{\rm GH}$. As distinct from the $T_{\rm GH}$, the activation temperature has no relation to the cosmological horizon. It describes the local processes which take place well inside the cosmological horizon.

The factor of 2 difference between the local temperature and the temperature of the Hawking radiation from the cosmological horizon measured by the same observer has a simple explanation. In the case of Hawking radiation, the observer detects particles radiated from the horizon but cannot detect their partners, which are simultaneously (coherently) created on the other side of the horizon. This is similar to the doubling of the Hawking temperature of black holes suggested by 't Hooft [7–9]. Regarding the black hole horizon, the suggested partner is in the mirror image of the black hole spacetime — 'quantum clone inside black hole.' The factor of 2 difference between the two temperatures is also supported by topological arguments [10, 11].

The local temperature $T = \hbar H/\pi$ determines the local temperature in de Sitter spacetime, giving rise to local entropy and the modified Gibbs–Duhem relation of the de Sitter state. The latter is modified due to the gravitational degrees of freedom: the gravitational coupling $K = 1/16\pi G$ and the scalar Riemann curvature $\mathcal{R}$. These are the thermodynamically conjugate variables, which are similar to chemical potential and particle density in condensed matter. As a result, the de Sitter state can be considered a mixture of two components: the vacuum energy (dark energy) and the gravitational component, which behaves like Zel'dovich stiff matter and has some analogy with dark matter. It is this dark matter component which is responsible for the thermodynamics of the de Sitter state with equilibrium temperature $T = \hbar H/\pi$. Together with ordinary matter (which includes the real dark matter), the expanding Friedmann–Robertson–Walker Universe is represented by three components: dark energy, gravitational dark matter, and ordinary matter.

This is the analogue of the Landau–Khalatnikov two-fluid hydrodynamics, which we consider in Section 2.This hydrodynamic theory became the source of different connections between the physics of condensed matter and relativistic physics, which includes quantum field theories, quantum gravity, and the physics of the quantum vacuum. The natural consequence of the two-fluid hydrodynamics is the acoustic metric in Section 2.1, which is produced by the 'flow of the vacuum' — the superfluid motion of the liquid. The excitations of the 'superfluid vacuum' — quasiparticles — form an analogue of matter on the background of the moving 'quantum vacuum.' There are different directions in the extension of analogue gravity. Gravity emerges in systems which have point nodes in the fermionic spectrum, such as the chiral superfluid ^{3}He-A and Weyl semimetals (see Section 2.2). Another scenario of the formation of gravity comes from the elasticity tetrads, which describe deformations of crystals in Section 2.3. In this case, the quantum vacuum is considered a superplastic crystal. Superfluid ^{3}He-B suggests a scenario in which the gravitational tetrads emerge as the bilinear combination of fermion fields (see Section 2.4).

In Section 3, we consider the thermodynamics of the de Sitter vacuum. We show that matter perceives the de Sitter state of the quantum vacuum as a thermal bath with temperature $T = \hbar H/\pi$. This is demonstrated in Section 3.2 using the example of ionization of the hydrogen atom in the de Sitter environment and in Section 3.3 using the example of proton decay. In both cases, the temperature $T = \hbar H/\pi$ determines the activation process. This local temperature determines the local thermodynamics of the de Sitter state: the energy density, free energy, and entropy density in Section 3.4. In Section 3.5, a holographic bulk–surface connection is obtained between the total entropy of the Hubble volume and the Gibbons–Hawking surface entropy of the cosmological horizon.

In Section 4, the two-component thermodynamics of the de Sitter state is discussed. This state can be represented in terms dark energy (vacuum) and dark matter (gravitational degrees of freedom). The latter is represented by a pair of thermodynamically conjugate variables: the gravitational coupling $K = 1/16\pi G$ and the Riemann curvature $\mathcal{R}$. The equation of state of this dark matter is the same as that of Zel'dovich stiff matter in Section 4.2 and of Landau Fermi liquid in Section 4.3.

In Section 5, we consider the dynamics of the de Sitter state, which may follow from its thermodynamics. In the suggested phenomenological approach, it is assumed that the equations are modified in such a way that, away from equilibrium, the gravitational dark matter behaves like real stiff matter. It undergoes a cool-down due to expansion, which is partially compensated by the energy exchange with the dark energy component. The energy exchange leads to the loss of dark energy, and as a result both dark components have a power-law decay. Both components at the present time are reaching the correct order of magnitude. How this phenomenology is supported by microscopic theory is an open question.

In Section 6, it is shown how the quantum tunneling processes determine the thermodynamics of a black hole. The quantum tunneling processes of Hawking radiation of particles determine the temperature of the black hole. Macroscopic quantum tunneling, which describes processes of splitting of the black hole into smaller parts, determines the black hole entropy. This entropy is nonextensive and is described by the Thallis–Cirto entropy in Section 6.4.

In Section 7, we discuss the temperature of the Unruh effect and its relation to the Schwinger pair production. The combined process, which includes the creation of charged particles by an electric field and then their acceleration by this electric field, demonstrates that the Unruh temperature has the extra factor 2, which is in agreement with the analogue of the Unruh effect in superfluids.

In Section 8, the cosmological constant problems are discussed using the experience with superfluid liquids. The main lesson from the ground states of these liquids is that the fully equilibrium quantum vacuum has exactly zero vacuum energy. This follows from the thermodynamics, which is valid both for many-body condensed matter systems and for a relativistic quantum vacuum. This solves the main cosmological constant without any fine-tuning. The other cosmological problem is that it is not easy to reach the final equilibrium state. The de Sitter state has high symmetry, due to which it serves as the attractor in the dynamics of the Universe, and the final Minkowski vacuum can be obtained only by fine-tuning. However, it is shown that the de Sitter vacuum is unstable. According to Section 3, the de Sitter vacuum represents a heat bath for matter. This results in the thermal radiation of matter, which violates the de Sitter symmetry and thus promotes relaxation to the Minkowski state with a zero cosmological constant.

In Section 9, we discuss the dynamic origin of Planck constants $\hbar$ and $\not{h} = \hbar c$ as elements of the tetrads in Minkowski spacetime. We also discuss the space–time dimensions of the physical quantities. In particular, the Planck constants $\hbar$ and $\not{h}$ have dimensions of time and space, respectively, and thus they cannot be considered to be fundamental constants. The effective Planck constant $\not{h}_{\rm ac}$ emerging in the acoustic gravity in superfluid ^{4}He has the dimension of length and the size of the interatomic distance. Some dimensionless quantities can be quantized, which is considered using the example of the entropies of the cosmological and black hole horizons.

Among the authors of the cited papers are members of the Landau Institute: Abrikosov, Beneslavskii, Berezinskii, Dzyaloshinsky, Feigel'man, Finkel'shtein, Gehshkenbein, Iordansky, Kamenshchik, Khalatnikov, Kopnin, Larkin, Polyakov, Rashba, and Starobinsky.

2. Landau–Khalatnikov two-fluid hydrodynamics and analogue gravity

2.1 River model of quantum vacuum

The two-fluid hydrodynamics of superfluid ^{4}He developed by Landau and Khalatnikov (see the Khalatnikov book [12]) gives us important hints for understanding general relativity and the dynamics of a quantum vacuum.The relativistic character of the two-fluid hydrodynamics is manifested at low temperatures, where the normal component of a liquid moving with velocity $\mathbf{v}_{\rm n}$ is formed by the relativistic-like excitations with the linear spectrum — phonons. The coherent part of the liquid, which moves with velocity $\mathbf{v}_{\rm s}$, represents the 'quantum vacuum,' which provides the curved spacetime for these quasiparticles. This analogy can be seen already in Eqn (3.13) of the Khalatnikov book, which corresponds to the Tolman law in general relativity:

$$T(\mathbf{r}) = \frac{T}{\sqrt{-g_{00}(\mathbf{r})}} , \tag{1}$$

where the effective metric is

$$-g_{00}(\mathbf{r}) = 1 - \frac{\mathbf{v}_{\rm s}^2(\mathbf{r})}{c^2} , \tag{2}$$

and c is the speed of sound. Later, such an effective metric experienced by sound waves in liquids (or correspondingly phonons in superfluids) became known as the acoustic metric [13], and the flow of the liquid with the acoustic horizon as the river model of black holes [14].

In general relativity (GR), the flow metric with an acoustic horizon corresponds to the Painlevé–Gullstrand (PG) metric [15, 16], suggesting that the gravitational field of a black hole with mass M can be considered the result of the flow of the quantum vacuum with superfluid velocity $v_{\rm s}^2 = 2GM/r$ with the event horizon at $v_{\rm s}^2 = c^2$. It is important that the PG metric is continuous across the horizon, which allows studying the thermodynamics of vacua with horizons. Both for a real black hole and for its condensed matter analog, the Hawking radiation can be considered semiclassical quantum tunneling across the horizon (see Ref. [17] for the analog black hole and Refs [18, 19] for the real black hole).

The fact that the speed of the 'vacuum' exceeds the speed of light beyond the horizon does not mean that the laws of physics are violated. The speed of light remains the speed limit for particles moving relative to the vacuum. Similarly, a superfluid liquid can move at a speed significantly greater than the speed of sound, but the relative speed of phonons moving relative to the superfluid 'vacuum' is the speed of sound.

2.2 Tetrad gravity from topological Weyl point

Acoustic gravity is the part of so-called analogue gravity which also includes other types of condensed matter systems, such as elastic media [20–26] and topological matter with Weyl fermions. The relativistic Weyl fermions in semimetals were considered by Abrikosov and Beneslavskii [27–29] (both from the Landau Institute). The Weyl materials allow us to simulate the horizon of black and white holes by tilting the Weyl cone [30–32]. In fermionic Weyl and Dirac materials, emergent gravity is formulated in terms of tetrad fields, instead of the metric gravity emerging in bosonic condensed matter systems. The tetrad gravity emerges together with all the ingredients of the relativistic quantum field theories (relativistic spin, chiral Weyl fermions, gauge fields, Γ-matrices, etc.) [33]. The spin of Weyl fermions in particle physics and pseudo-spin in Weyl materials forms a hedgehog in momentum space — the Weyl point in the fermionic spectrum. It represents the Berry phase monopole, which acts as a source or a sink of the Berry curvature [34].

The stability of this hedgehog is supported by the topology in momentum space. It is the topological stability of the Weyl point which provides the emergence of relativistic physics at low a energy.

2.3 Superplastic vacuum and translational gauge fields

The theory of elasticity in crystals can be considered in terms of elasticity tetrads [22]. These tetrads represent so-called translational gauge fields, which are expressed in terms of a system of deformed crystallographic coordinate planes, surfaces of constant phase [35–38], $X^a(x) = 2\pi n^a$:

$$E^a_\mu(x) = \partial_\mu X^a(x)\,. \tag{3}$$

It is important that such gravitational tetrads have dimensions of inverse length and inverse time, $[E^a_i] = [L]^{-1}$ and $[E^a_0] = [t]^{-1}$. When these elasticity tetrads are applied to general relativity (the so-called superplastic vacuum [39]), it is found that the Ricci curvature scalar $\mathcal{R}$ is dimensionless. The same dimensions of tetrads, $[E^a_i] = [L]^{-1}$ and $[E^a_0] = [t]^{-1}$, are encountered in the Diakonov theory discussed in Section 2.4.

2.4 Tetrads as bilinear combination of fermions

Another condensed matter example of effective gravity is provided by the B-phase of superfluid ^{3}He, where vielbeins emerge as bilinear combinations of the fermionic fields [40]. A similar mechanism of the formation of composite tetrads in low energy physics has been suggested in relativistic quantum field theories [41–47] (see Section 9.1). The emergent tetrads give rise to the effective metric (the four fermions object), to the interval, and finally to the effective action for the gravitational field. As in the case of elasticity tetrads in Eqn (3), the composite gravitational tetrads also have unusual dimensions. The consequences of such dimensions for physics are discussed in Section 9.

3. de Sitter state as heat bath

3.1 de Sitter state vs moving vacuum

We consider the de Sitter thermodynamics using the Painlevé–Gullstrand (PG) coordinates [15, 16], where the metric is

$$\begin{aligned} ds^2 &= -c^2\,dt^2 + \big(d\mathbf{r} - \mathbf{v}(\mathbf{r})\,dt\big)^2 \\ &= -c^2\left(1 - \frac{\mathbf{v}^2(\mathbf{r})}{c^2}\right) - 2\mathbf{v}(\mathbf{r})\,d\mathbf{r}\,dt + d\mathbf{r}^2\,. \end{aligned} \tag{4}$$

Here, $\mathbf{v}(\mathbf{r})$ is the shift velocity, which in condensed matter plays the role of the superfluid velocity $\mathbf{v}_s$ in Eqn (2) — the velocity of the 'superfluid quantum vacuum' [33]. In the de Sitter expansion, the velocity of the 'vacuum' is $\mathbf{v}(\mathbf{r}) = H\mathbf{r}$, where H is the Hubble parameter, and the metric is (we use $c = 1$)

$$ds^2 = -\,dt^2 + (dr - Hr\,dt)^2 + r^2\,d\Omega^2\,. \tag{5}$$

The PG metric is stationary, i.e., does not depend on time, and it does not have unphysical singularity at the cosmological horizon. That is why it is appropriate for consideration of the local thermodynamics both inside and outside the horizon. It also allows us to consider two different phases of a vacuum with broken time reversal symmetry. They are the expanding de Sitter Universe with $H > 0$ and the contracting de Sitter Universe with $H < 0$. These two degenerate states transform into each other under the time reversal $t \to -t$. In this sense, the Hubble parameter H can be considered the order parameter of the symmetry breaking phase transition from the symmetric state — the Minkowski vacuum with $H = 0$.

3.2 Hydrogen atom in de Sitter environment and de Sitter temperature

Let us now show that matter perceives the de Sitter state of the quantum vacuum as a heat bath. To do so, let us consider an atom at the origin, $r = 0$. The atom plays the role of detector (or the role of static observer) in this spacetime. An electron bounded to an atom may absorb energy from the gravitational field of the de Sitter background and escape from the electric potential barrier. If the ionization potential is much smaller than the electron rest energy but is much larger than the Hubble parameter, $\hbar H \ll \epsilon_0 \ll mc^2$, one can use the nonrelativistic quantum mechanics to estimate the tunneling rate through the barrier.

Let us consider an electron at the nth level in a hydrogen atom. In a de Sitter gravitational field, this electron can escape from the atom with the conservation of energy, which in the classical limit is given by the classical equation

$$\frac{p_r^2}{2m} + p_r v(r) = -E_n\,, \quad E_n = \frac{me^4}{2\hbar^2}\,\frac{1}{n^2}\,. \tag{6}$$

Here, $v(r) = Hr$ and $p_r(r)\,v(r)$ is the Doppler shift, which allows the electron to reach a negative energy when it escapes from the atom. The corresponding radial trajectory $p_r(r)$ for the escape of an electron from the atom is

$$p_r(r) = -mv(r) + \sqrt{m^2 v^2(r) - 2mE_n}\,. \tag{7}$$

The integral of $p_r(r)$ over the classically forbidden region, $0 < r < r_n = \sqrt{2E_n/mH^2}$, gives the ionization rate

$$\begin{aligned} w &\sim \exp\left(-\frac{2}{\hbar}\,\mathrm{Im}\,S\right) \\ &= \exp\left(-\frac{2}{\hbar}\int_0^{r_n} dr\,\sqrt{2mE_n - m^2H^2r^2}\right) \end{aligned} \tag{8}$$

$$= \exp\left(-\frac{\pi E_n}{\hbar H}\right) \equiv \exp\left(-\frac{E_n}{T}\right), \quad T = \frac{\hbar H}{\pi}\,. \tag{9}$$

The ionization rate is equivalent to the rate of ionization in the flat Minkowski spacetime in the presence of a heat bath with temperature $T = \hbar H/\pi$. This suggests that the de Sitter state of the quantum vacuum serves as the heat bath for matter.

This heat bath temperature is twice the Gibbons–Hawking temperature $T_{\rm GH} = \hbar H/2\pi$, which is generally considered to be the temperature of the cosmological horizon. Since the electron's trajectory is deep inside the horizon, $r_n \ll r_{\rm H} = c/H$, the ionization process is fundamentally different from the process of Hawking radiation from the cosmological horizon. However, there is a relation between the ionization temperature T and the Gibbons–Hawking temperature of Hawking radiation, $T = 2T_{\rm GH}$. This follows from the symmetry of the de Sitter state [2].

The ionization rate can be obtained using a simpler method [5]. Since this process takes place well inside the horizon, one can use the classical (i.e., nonrelativistic) gravitational potential $U(r) = -mv^2(r)/2 = -mH^2r^2/2$. Then, the bound state decays by quantum tunneling of the electron from the point $r = 0$ to the point $r = r_n$, at which the electron level $-E_n$ matches the de Sitter gravitational potential, $U(r_n) = -mH^2r_n^2/2 = -E_n$. The radial trajectory $p_r(r)$ follows now from the classical equation

$$\frac{\mathbf{p}^2}{2m} - \frac{1}{2}\,mH^2r^2 = -E_n\,. \tag{10}$$

One obtains

$$p_r(r) = \sqrt{m^2H^2r^2 - 2mE_n}\,, \tag{11}$$

which again gives Eqn (9) for the WKB tunneling rate of ionization of the atom.

3.3 Proton decay in de Sitter environment

The same temperature $T = H/\pi$ determines the other processes which are forbidden in a Minkowski vacuum but are allowed in the de Sitter state. This includes, for example, the decay of a proton in the de Sitter environment, which represents inverse β-decay, $p^+ \to n + e^+ + \nu$. The decay rate of a proton is [48]

$$\Gamma(p^+ \to n + e^+ + \nu) \sim \exp\left(-\frac{\pi\Delta M}{H}\right) = \exp\left(-\frac{\Delta M}{T}\right), \tag{12}$$

$$\Delta M = M_n + M_e + M_\nu - M_p\,. \tag{13}$$

Here, M_n, M_e, and M_p are the masses (rest energies) of the neutron, electron, and proton, respectively, and M_ν corresponds to the mass eigenstates for the neutrino.

This, in turn, leads to the multiple creation of matter in the de Sitter heat bath. The created neutron experiences β-decay with the creation of a proton, electron, and antineutrino. The created proton again experiences inverse β-decay, and so on. This leads to the multiple creation of electron-positron pairs. The creation of matter leads to a decrease in the vacuum energy and thus to the relaxation of the de Sitter state towards the final Minkowski vacuum state. That is why even a single proton in the de Sitter environment triggers the instability of the de Sitter vacuum. This provides a route to solve the cosmological constant problem discussed in Section 8.

3.4 Local temperature and local entropy

The behaviour of matter in the de Sitter environment suggests that the de Sitter state can be considered a heat bath, where the local temperature $T = \hbar H/\pi$ determines the thermodynamics of the de Sitter state.

From the Friedmann equations of general relativity, one can express the energy density of the de Sitter vacuum in terms of this local temperature, and then find its free energy and the entropy density. The energy density, which is the cosmological constant Λ, is ($\hbar = c = 1$) [2]

$$\epsilon_{\text{vac}} = \Lambda = \frac{3}{8\pi G}\,H^2 = \frac{3\pi}{8G}\,T^2\,. \tag{14}$$

This determines the free energy density F of the de Sitter state. From equation $F - T\,\mathrm{d}F/\mathrm{d}T = \epsilon_{\text{vac}}$, one obtains $F(T) = -\epsilon_{\text{vac}}(T)$, and thus the entropy density s_{dS} is

$$s_{\text{dS}} = -\frac{\partial F}{\partial T} = \frac{3\pi}{4G}\,T = \frac{3}{4G}\,H\,. \tag{15}$$

This is an example of the entropy density of spacetime suggested by Padmanabhan [49] (see also review papers on gravitational entropy [50, 51]).

3.5 de Sitter thermodynamics and holography

Let us now consider the possible connection between this local thermodynamics of the de Sitter state and the global thermodynamics of black holes, where the total entropy of the black hole is determined by the area of its horizon. For this, let us find the entropy S_H of the part of the de Sitter space which is surrounded by the cosmological horizon — the total entropy of the Hubble volume $V_H = (4\pi/3)\,r_H^3$. Multiplying the entropy density in Eqn (15) by the Hubble volume, one obtains

$$S_H = V_H s_{\text{dS}} = \frac{A}{4G}\,, \tag{16}$$

where $A = 4\pi r_H^2$ is the horizon area. This bulk entropy exactly coincides with the entropy of the cosmological horizon suggested by Gibbons and Hawking. However, this global entropy comes from the local entropy of the de Sitter state, rather than from the horizon degrees of freedom. This demonstrates the specific features of the de Sitter horizon in relation to the holography discussed, e.g., in Refs [49, 52–54]. Equation (16) confirms the holographic bulk-horizon correspondence in the de Sitter state, which in turn confirms the local thermodynamics of the de Sitter state with the double Hawking temperature, $T = H/\pi = 2T_{\text{GH}}$.

3.6 Double Hawking temperature measured by free-falling detector

The doubling of the Hawking temperature was suggested for black holes by 't Hooft [7–9]. In the 't Hooft scenario, a doubling of temperature is accompanied by a reduction in the total entropy of the black hole by a factor 2. In the de Sitter case, the temperature is also twice the usually accepted value, but the entropy in Eqn (16) has the traditional value expected from the holographic principle.

Another example of the doubling of the Hawking temperature of a black hole, which was discussed in Ref. [55], is rather close to the de Sitter scenario. It is now the effective temperature measured by a free-falling observer crossing the black hole horizon, which is twice the Hawking temperature. The temperature of Hawking radiation from the black hole and the effective temperature measured by a freely-falling detector at the black hole horizon are obtained in the same quantum tunneling approach, where the radiation rate is

$$w \sim \exp\left(-\frac{2}{\hbar}\,\mathrm{Im}\,S\right),$$

$$\mathrm{Im}\,S = \mathrm{Im}\int \mathrm{d}r\,\frac{E}{1+v(r)} = \frac{\pi E}{|\mathrm{d}v/\mathrm{d}r|_{r=r_H}}\int_{r_1}^{r_2}\mathrm{d}r\,\delta(r - r_H)\,. \tag{17}$$

Here, r_H is the position of the event horizon. In the case of Hawking radiation, we have $r_1 < r_H < r_2$ and the integral $\int \mathrm{d}r\,\delta(r - r_H) = 1$. As a result, we obtain the conventional

Hawking temperature, $T_{\rm H} = |{\rm d}v/{\rm d}r|_{r=r_{\rm H}}/2\pi$. In the case of a detector crossing the horizon, the radiation is caused by the detector itself. The detector plays the role of an external object that produces radiation in the same way that an external atom provides the radiation of an electron in a de Sitter environment. In this case, r_1 is determined by the position of the detector, $r_1 = r_{\rm H} < r_2$, and the integral is $\int {\rm d}r\,\delta(r-r_{\rm H}) = 1/2$. As a result, the local temperature measured by the detector is twice as high, $T = |{\rm d}v/{\rm d}r|_{r=r_{\rm H}}/\pi = 2T_{\rm H}$.

4. Two-component thermodynamics of de Sitter state

4.1 Modified Gibbs–Duhem relation, dark energy, and dark matter

The quadratic dependence of vacuum energy on temperature in Eqn (14) is important for consideration of the thermodynamic Gibbs–Duhem relation for a quantum vacuum. It leads to the reformulation of the vacuum pressure. In the conventional approach, the vacuum pressure $P_{\rm vac}$ obeys the equation of state $P = w\epsilon$ with $w = -1$, i.e., $P_{\rm vac} = -\epsilon_{\rm vac}$ (let us recall that, for nonrelativistic matter, $w = 0$ and for radiation, $w = 1/3$). In the de Sitter state, the vacuum pressure is negative, $P_{\rm vac} = -\epsilon_{\rm vac} < 0$. This pressure $P_{\rm vac}$ does not satisfy the thermodynamic Gibbs–Duhem relation, $Ts_{\rm dS} = \epsilon_{\rm vac} + P_{\rm vac}$, because the right-hand side of this equation is zero. The reason for this is that, in this equation, we did not take into account the gravitational degrees of freedom.

It was shown that gravity contributes to the thermodynamics with a pair of the thermodynamically conjugate variables: the gravitational coupling $K = 1/16\pi G$ and the Riemann curvature $\mathcal{R}$ (see Refs [56–58]). The gravitational thermodynamic variables allow us to write the modified Gibbs–Duhem relation:

$$Ts_{\rm dS} = \epsilon_{\rm vac} + P_{\rm vac} - K\mathcal{R}\,. \tag{18}$$

This equation allows us to introduce a pressure that characterizes the gravitational degrees of freedom:

$$P_{\rm DM} = P_{\rm vac} - K\mathcal{R}\,. \tag{19}$$

Then, the Gibbs–Duhem relation becomes

$$Ts_{\rm dS} = \epsilon_{\rm vac} + P_{\rm DM}\,. \tag{20}$$

We call $P_{\rm DM}$ the pressure of the dark matter component, although it has nothing to do with real physical dark matter.

Assuming that the de Sitter entropy comes from the gravitational degrees of freedom, i.e., $s_{\rm dS} = s_{\rm DM}$, one obtains from Eqn (20) the fact that the energy density of this 'dark matter' equals the energy density of the vacuum, $\epsilon_{\rm DM} = \epsilon_{\rm DE}$. That is why our dark matter satisfies the equation of state $P = w\epsilon$ with $w = 1$. This corresponds to stiff matter introduced by Zel'dovich [59], where the speed of sound is equal to the speed of light, $c_s^2 = c^2\,{\rm d}P/{\rm d}\epsilon_{\rm vac} = c^2$.

4.2 Two dark components of de Sitter and Zel'dovich stiff matter

So, the thermodynamics of the de Sitter expansion can be described in terms of two components of the de Sitter state: the vacuum component (dark energy, DE) and stiff matter (dark matter, DM). These components have the following forms:

$$w = -1\,, \quad P_{\rm DE}(H) = -\epsilon_{\rm DE}(H) = -\frac{3}{8\pi G}\,H^2\,, \tag{21}$$

and

$$w = 1\,, \quad P_{\rm DM}(T) = \epsilon_{\rm DM}(T) = \frac{3\pi}{8G}\,T^2\,. \tag{22}$$

Then, from Eqns (18)–(20), it follows that, in equilibrium, i.e., at $T = H/\pi$, the total (thermodynamic) pressure is zero, as it should be in the absence of the external environment when the partial pressure of dark matter compensates the partial pressure of the quantum vacuum:

$$P = P_{\rm DE}(H = \pi T) + P_{\rm DM}(H = \pi T) = 0\,. \tag{23}$$

In this equilibrium state, the energy density of dark matter is equal to the dark energy density:

$$\epsilon_{\rm DM}(H = \pi T) = \epsilon_{\rm DE}(H = \pi T)\,. \tag{24}$$

The modified Gibbs–Duhem relation in Eqn (20) represents the Gibbs–Duhem relation for dark matter:

$$Ts_{\rm DM} = \epsilon_{\rm DM} + P_{\rm DM}\,, \quad s_{\rm DM} \equiv s_{\rm dS}\,. \tag{25}$$

In other words, the thermodynamics of the de Sitter state is fully represented by the thermodynamics of the gravitational dark matter, which play the role of the normal component in these two-fluid thermodynamics.

So, we have the following set of equations for the equilibrium de Sitter state, which is expressed in terms of the two dark components:

$$P_{\rm DE} = -\epsilon_{\rm DE}\,, \tag{26}$$

$$P_{\rm DM} = \epsilon_{\rm DM}\,, \tag{27}$$

$$P = P_{\rm DE} + P_{\rm DM} = 0\,, \tag{28}$$

$$Ts_{\rm DM} = \epsilon_{\rm DM} + P_{\rm DM}\,. \tag{29}$$

Since the dark energy component represents the vacuum, it acquires the nonzero value only for compensating the dark matter pressure in Eqn (28) in equilibrium. In the absence of an environment, the external pressure is zero, $P = 0$. Because of that, the energies of the two components in equilibrium have equal values in Eqn (24).

A similar situation takes place for the closed static universe with positive curvature introduced by Einstein. In this case, there are three contributions to the pressure: from ordinary matter, from the vacuum energy, and from the gravitational degrees of freedom (i.e., from the spatial curvature $\mathcal{R}$) (see Section 29.4.5 in book [33] and Ref. [60]). Since the static universe has no environment, in full equilibrium, the total pressure is zero, $P = P_{\rm vac} + P_{\rm mater} + P_{\rm gravity} = 0$. The equilibrium conditions provide the connections among matter, dark energy, and curvature of the Einstein universe.

4.3 Zel'dovich stiff matter and Landau Fermi liquid

Equation (15) demonstrates that the thermal properties of the de Sitter state are similar to those of a nonrelativistic Fermi liquid, such as liquid ^{3}He. The entropy density in the Fermi liquid is also linear in temperature:

$$s_{\rm FL} = \frac{p_{\rm F}^2}{3 v_{\rm F}}\, T\,. \tag{30}$$

The Fermi velocity $v_{\rm F}$ and Fermi momentum $p_{\rm F}$ of this cosmological analog of the Fermi liquid are on the order of the speed of light and the inverse Planck length, respectively, $v_{\rm F} \sim c$ and $p_{\rm F} \sim 1/l_{\rm P} \equiv M_{\rm P}/c$, where $M_{\rm P}$ is the reduced Planck mass, $M_{\rm P}^2 = 1/(8\pi G)$.

The Sommerfeld law in a Fermi liquid states that the entropy per atom of the Fermi liquid is

$$S = \frac{s_{\rm FL}}{n_{\rm FL}} \sim \frac{T}{E_{\rm F}}\,, \tag{31}$$

where $n_{\rm FL} \sim p_{\rm F}^3$ is the density of atoms in the Fermi liquid and $E_{\rm F}$ is the Fermi energy.

Can it be related to a quantum vacuum? We do not know what the 'atoms of the vacuum' are, but from Eqn (15) it follows that the entropy density of the vacuum $s_{\rm vac} \sim T/l_{\rm P}^2 \sim (T/M_{\rm P})/l_{\rm P}^3$. This suggests that the density of the 'atoms of the vacuum' is $n_{\rm P} \sim 1/l_{\rm P}^3$ and entropy per 'atom of the vacuum' is

$$S = \frac{s_{\rm vac}}{n_{\rm P}} \sim s_{\rm vac}\, l_{\rm P}^3 \sim \frac{T}{M_{\rm P}}\,. \tag{32}$$

Equation (32) is a full analog of the Sommerfeld law for Fermi liquids. In Fermi liquids, this corresponds to the density of states at the Fermi level $N_{\rm F} \sim m p_{\rm F}$. This analogy suggests that the corresponding density of states in a quantum vacuum is $N_{\rm P} \sim M_{\rm P}^2$. The huge density of states in the quantum vacuum leads to a very large entropy of the de Sitter state, even at a very low temperature of the vacuum.

5. Two-component dynamics of de Sitter state

5.1 Phenomenological theory of two-component dynamics

The two-fluid thermodynamics of the de Sitter state discussed in Section 4 is valid in thermal equilibrium, when $T = H/\pi$. However, the equilibrium state is impossible, since the gravitational dark matter represents a heat bath for ordinary matter, which leads to the thermal nucleation of matter and thus to decay of the vacuum energy. Here, we will not go into the details of this decay process, which depends on many different factors. We shall use the phenomenological approach suggested in Ref. [2] and apply it to the Universe with cosmological stiff matter. We assume that the Einstein equations are modified in such a way that the gravitational dark matter behaves like the real Zel'dovich stiff matter in its dynamics as well. Then, the dark matter decays with time due to expansion, but it also gets energy from the energy exchange with the dark energy. On the other hand, this energy exchange leads to a decrease in the vacuum energy density, and the Universe finally approaches the equilibrium Minkowski vacuum state.

So, let us consider how the de Sitter state relaxes if one takes into account that the temperature of dark matter changes both due to expansion and due to the energy exchange with the dark energy. The only assumption in this phenomenological approach is that, due to the energy exchange, both dark components try to reach a common temperature in the process of equilibration.

5.2 Dynamics without energy exchange

If there is no energy exchange between the two components, the real stiff matter with equation of state $w = 1$ relaxes according to the Einstein equations:

$$\partial_t\, \epsilon_{\rm DM} = -3H(w+1)\, \epsilon_{\rm DM} = -6H\epsilon_{\rm DM}\,, \tag{33}$$

while the equation for the vacuum energy density with $w = -1$ is time independent:

$$\partial_t\, \epsilon_{\rm DE} = -3H(w+1)\, \epsilon_{\rm DE} = 0\,. \tag{34}$$

Of course, for the original gravitational stiff matter, one has $\partial_t \epsilon_{\rm DM} = 0$ instead of Eqn (33). But here, we consider the toy phenomenological model, in which the real Zel'dovich matter has exactly the same energy density as the gravitational Zel'dovich matter in Eqn (22).

5.3 Dynamics with energy exchange

If there is energy exchange between the two components, it is natural to assume that these dark components tend to approach the common equilibrium state in Eqn (24). This suggests that, due to the energy exchange in the process of thermalization, the temperature of the dark matter tends to approach the temperature $T = H/\pi$ of the de Sitter thermal bath, and thus the energy density of dark matter tends to approach the dark energy density, $\epsilon_{\rm DM}(T) \to \epsilon_{\rm DE}(H)$.

For this, Eqn (33) must be modified in the following way:

$$\partial_t \epsilon_{\rm DM}(T) = -6H\left(\epsilon_{\rm DM}(T) - \epsilon_{\rm DE}(H)\right). \tag{35}$$

This phenomenological equation describes the process of thermalization by the energy exchange between the two dark components.

Now, the gain of $6H\epsilon_{\rm DE}(H)$ in the energy of dark matter in Eqn (35) must be compensated by the loss of dark energy. This modifies Eqn (34), leading to a relaxation of the vacuum energy:

$$\partial_t \epsilon_{\rm DE}(H) = -6H\epsilon_{\rm DE}(H)\,. \tag{36}$$

Equation (36) gives the following asymptotic dependence of dark energy at large times:

$$\epsilon_{\rm DE}(t) = \frac{1}{6\pi G t^2}\,. \tag{37}$$

The same behavior of the vacuum energy density was obtained using the Hawking 4-form field [61] (see Refs [56, 62]). In this case, the role of dark matter is played by the oscillations of the vacuum energy during its decay [63].

From Eqn (35), one obtains the time dependence of dark matter:

$$\epsilon_{\rm DM}(t) = \frac{\ln(t/t_0)}{3\pi G t^2}\,. \tag{38}$$

Both dark components are of the same order of magnitude and have reasonable values at present:

$$\epsilon_{\rm DE}(t_{\rm present}) \sim \epsilon_{\rm DM}(t_{\rm present}) \sim H^2 M_{\rm P}^2 \sim 10^{-120} M_{\rm P}^4\,. \tag{39}$$

Both components finally approach the fully equilibrium state with zero temperature — the Minkowski vacuum. That is why this toy phenomenological model shows the route to the solution to the cosmological constant problems, including the coincidence problem (see also Section 8.3). The key point here is the instability of the de Sitter state in the presence of an external object (for example, a proton).

6. Black hole thermodynamics from macroscopic quantum tunneling

6.1 Vortex nucleation as macroscopic quantum tunneling

The process of quantum tunneling of macroscopic objects is well known in condensed matter physics (see the original papers from the seventies [64–68]). Most of the authors of these papers were from the Landau Institute. Macroscopic tunneling can be studied using collective variables that describe the collective dynamics of a macroscopic object. This approach allows one to estimate the semiclassical tunneling exponent without considering the details of the object's structure at the microscopic (atomic) level. That is why it is important in black hole physics, where the structure of the quantum vacuum is still unknown.

The processes of macroscopic quantum tunneling include, inter alia, the process of quantum nucleation of vortices in superfluids [66]. In this process, the role of the canonically conjugate collective variables is played by the z and r coordinates of the vortex ring. This provides the volume law for the vortex instanton: the action contains the topological term $S_{\rm top} = 2\pi\hbar n V = 2\pi\hbar N$, where n is the particle density, V is the volume inside the surface swept by the vortex line between its nucleation and annihilation, and N is the number of atoms inside this volume (see Section 26.4.3 in book [33] and also the paper by Rasetti and Regge [69]).

For the other linear topological defects in condensed matter, the area law is applied instead of the volume law, i.e., the action is proportional to the area of the surface swept by the defect line, as in the case of the Polyakov string action [70]. The application of macroscopic quantum tunneling to Abrikosov vortices in superconductors can be found in review paper [71], where most authors were from the Landau Institute.

The rate of emission of the vortex loop in superfluid helium moving with respect to the walls is

$$w \propto \exp\left(-2\pi N\right), \tag{40}$$

where N the number of 'atoms of the vacuum' (atoms of ^{4}He) participating in the vortex instanton. It depends on the 'velocity of the vacuum,' $N \propto 1/v_s^3$. This is similar to the quantization of the black hole entropy in terms of microstates of Planck size, where the number of 'black hole atoms' is determined by the ratio between mass M of the black hole and the Planck mass $M_{\rm P}$. In the black hole case, the tunneling rate is related to the change in entropy after tunneling, which gives the rate of emission in terms of the number of microstates, similar to that in Eqn (40). This will be discussed in Section 9.7.

6.2 Hawking radiation as quantum tunneling

Let us start with the quantum tunneling of particles from the black hole, which describes the Hawking radiation [17–19]. The rate of emission of a particle with energy ω from a black hole with mass M has the following exponential law:

$$w(\omega, M) \propto \exp\left(-8\pi G M\omega\right) \equiv \exp\left(-\frac{\omega}{T_{\rm H}}\right). \tag{41}$$

Here, $T_{\rm H}$ is the temperature of the Hawking radiation:

$$T_{\rm H} = \frac{1}{8\pi G M}. \tag{42}$$

We use $\hbar = c = 1$ and the condition $T_{\rm H} \ll \omega \ll M$. Thus, the quantum tunneling process demonstrates that the rate of emission is determined by the Hawking temperature $T_{\rm H}$. This is important for the construction of the black hole thermodynamics.

A further important step was taken by Parikh and Wilczek [18], who obtained a correction to the Hawking radiation. This correction is due to the back reaction — the reduction in the black hole mass after emission:

$$w(\omega, M-\omega) \propto \exp\left(-8\pi G\omega\left(M - \frac{\omega}{2}\right)\right). \tag{43}$$

This back reaction can be related to a decrease in the Bekenstein–Hawking entropy $S_{\rm BH}(M) = 4\pi G M^2$ of the black hole after emission [72]. The reason is that the Hawking radiation can be considered a rare effect caused by thermodynamic fluctuations. According to Landau and Lifshitz [73], rare fluctuations lead to a decrease in entropy, and thus, the rate of Hawking radiation in Eqn (43) can be described in terms of the decrease in black hole entropy after emission of a particle:

$$w(\omega, M-\omega) \propto \exp\left[S_{\rm BH}(M-\omega) - S_{\rm BH}(M)\right]. \tag{44}$$

This shows that the quantum tunneling process may serve as a source of both the Hawking temperature and the Bekenstein–Hawking entropy. This is supported by consideration of macroscopic quantum tunneling, discussed in Section 6.3. It describes the process in which a large black hole emits smaller black holes [57].

Moreover, macroscopic quantum tunneling from a black hole to a white hole with the same mass demonstrates the curious thermodynamic properties of the white hole [74]. In particular, the entropy of the white hole is negative, $S_{\rm WH}(M) = -S_{\rm BH}(M)$. This is a consequence of a special type of white hole event horizon that also occurs in a collapsing de Sitter universe with $H < 0$. It is unclear whether negative entropy can arise in systems without an event horizon and in condensed matter analogs.

6.3 Emission of black holes as macroscopic quantum tunneling

Emitted black hole can be thought of as a type of particle, and the emission process contains a similar element of back reaction [57]. However, such 'particle' emitted by a black hole has a nonzero entropy. As a result, unlike the emission of a point particle, the emission rate of a small black hole increases by the entropy of the emitted black hole [75].

All this suggests that the general process of the splitting of black holes into several parts can be expressed in terms of the entropies of the black holes participating in this process. In particular, the rate at which a black hole splits into two smaller black holes in the process of macroscopic quantum tunneling obeys the following rule:

$$w\,(M \to M_1 + M_2) \propto \exp\big[S_{\rm BH}(M_1) + S_{\rm BH}(M_2) - S_{\rm BH}(M_1 + M_2)\big]\,. \qquad (45)$$

This Eqn (45) allows us to obtain the entropy of the black hole as a function of its mass, $S_{\rm BH}(M)$. To do so, let us consider the emission of a small black hole with mass m by a large black hole with mass $M \gg m$. The rate of emission obeys Eqn (41) with $\omega = m$. On the other hand, it obeys Eqn (45) with $M_1 = M - m$ and $M_2 = m \ll M$. Expanding this equation in small m and comparing it with Eqn (41), one obtains the function $S_{\rm BH}(M)$:

$$\frac{{\rm d}S_{\rm BH}}{{\rm d}M} = 8\pi GM \Rightarrow S_{\rm BH}(M) = 4\pi GM^2\,. \qquad (46)$$

The macroscopic quantum tunneling approach is actually another way to derive the Bekenstein–Hawking entropy of a black hole. Application of macroscopic quantum tunneling to the cosmological decay of a false quantum vacuum can be found in Refs [76, 77].

6.4 Nonextensive entropy of black holes

The black hole entropy is nonextensive, since $S_{\rm BH}(M_1 + M_2) > S_{\rm BH}(M_1) + S_{\rm BH}(M_2)$. The source of the nonextensive entropy of black holes is a special type of configuration space of the black hole ensemble, which follows from the black hole transformations discussed in Section 6.3.

The entropy of conventional thermodynamic systems is extensive. The entropy there is proportional to the volume of the system, and thus the splitting of the system with volume V into two parts with volumes $V_1 + V_2 = V$ does not change the total entropy of the system, $S(V_1 + V_2) = S(V_1) + S(V_2)$, or $S(A, B) = S(A) + S(B)$. The black hole entropy does not satisfy the additivity condition. Instead, one has the following nonadditive composition rule for black hole entropies:

$$S_{\rm BH}(M_1 + M_2) = \left(\sqrt{S_{\rm BH}(M_1)} + \sqrt{S_{\rm BH}(M_2)}\right)^2. \qquad (47)$$

It is precisely because of quantum processes that an ensemble of black holes has a special type of configuration space, where the entropy is not extensive. This nonextensivity is caused by quantum fluctuations, which determine the rare processes of macroscopic quantum tunneling between the black hole states. This is the analog of intermittency in chaotic systems [78]. Such processes require a generalization of the statistics with the corresponding nonextensive entropy. Equation (47) fully determines the type of statistic: it is described by the Tsallis–Cirto entropy with $\delta = 2$ [79] (see Section 9.7).

If a black hole is a mixed state and its entropy is the von Neumann entropy, then the quantum tunneling process of breaking the black hole into smaller black holes transforms the mixed state into a less mixed state — one with lower entropy. This is consistent with Weinberg's view [80] that it is the density matrix introduced by Landau and von Neumann that describes physical reality, not the wave function introduced by Schrödinger. The wave function approach is as convenient as the complex order parameter (the condensate wave function) introduced by Ginzburg and Landau to describe superfluidity and superconductivity. But the real physical quantities are their bilinear combinations — the density matrix and the correlation function, which are invariant with respect to phase shift (for additional symmetries of the density matrix compared to the wave function, see Ref. [80]).

An important example is a two-dimensional superfluid liquid, where the condensate wave function is absent. The phase transition to the superfluid state is determined by the behavior of the correlator [81, 82].

6.5 Extensivity of de Sitter entropy

The nonextensivity of the black hole entropy and hence the application of generalized statistics is not surprising given the presence of long-range forces [83]. The generalized statistics is applicable to the black hole entropy as the entropy of a finite closed system. It cannot be applied to such open systems as the de Sitter state, where the entropy is extensive [2, 48]. The entropy of any volume V in the de Sitter state, regardless of whether it is smaller or larger than the Hubble volume $V_{\rm H}$, is proportional to the volume:

$$S_V = s_{\rm dS} V = \frac{3\pi}{4G}\,TV\,. \qquad (48)$$

The extensive entropy in Eqn (48) determines thermal fluctuations of the thermodynamic variables. For example, according to Landau and Lifshitz [73], temperature fluctuations in a volume V are inversely proportional to the volume. In the de Sitter state, one has

$$\frac{\left\langle (\Delta T)^2 \right\rangle}{T^2} \sim \frac{1}{S_V} = \frac{1}{S_{\rm H}}\,\frac{V_{\rm H}}{V}\,. \qquad (49)$$

Here as before, $V_{\rm H}$ and $S_{\rm H}$ are correspondingly the Hubble volume and the entropy of the Hubble volume. The extensivity of the de Sitter entropy does not contradict the holographic bulk-surface correspondence in Eqn (16), which is applied only to the entropy of the Hubble volume $V_{\rm H}$.

6.6 Entropy of black hole singularity

The de Sitter thermodynamics obey a modified Gibbs–Duhem relation in Eqn (18), which includes the gravitational degrees of freedom: the gravitational coupling K (the gravitational analog of the chemical potential) and the scalar curvature $\mathcal{R}$. Let us assume that a similar relation applies to the local thermodynamics of black holes. In the case of a black hole, this relation has the following form:

$$Ts(\mathbf{r}) = \epsilon\,(\mathbf{r}) - K\mathcal{R}(\mathbf{r})\,. \qquad (50)$$

Here, the temperature T and the 'gravitational chemical potential' K are constants, while the scalar curvature $\mathcal{R}(\mathbf{r})$, energy density $\epsilon(\mathbf{r})$, and entropy density $s(\mathbf{r})$ are space dependent. We have:

$$\epsilon\,(\mathbf{r}) = M\delta(\mathbf{r})\,,\ \mathcal{R}(\mathbf{r}) = 8\pi GM\delta(\mathbf{r})\,. \qquad (51)$$

Then, with $T = 1/(8\pi GM)$ and $K = 1/(16\pi G)$, the modified Gibbs–Duhem relation (50) gives the entropy density, which agrees with the holographic bulk–horizon correspondence:

$$s(\mathbf{r}) = \frac{M}{2T}\,\delta(\mathbf{r}) = \frac{A}{4G}\,\delta(\mathbf{r})\,, \qquad (52)$$

$$S_{\rm BH} = \int {\rm d}^3 r\, s(\mathbf{r}) = \frac{A}{4G}\,. \qquad (53)$$

The same singular entropy density in Eqn (52) is obtained for a charged Reissner–Nordström (RN) black hole. According to Ref. [57], the entropy and the Bekenstein–Hawking temperature of an RN black hole are the same as for an electrically neutral black hole with the same mass. The extra term is to be added to the right-hand side of Eqn (50), $-\Phi q(\mathbf{r})$, which comes from charge density $q(\mathbf{r}) = Q\delta(\mathbf{r})$. However, this term is zero, because the corresponding electric analog of the chemical potential (the electrostatic potential) is zero, $\Phi = 0$.

So, if our assumption is correct, it follows from Eqns (52) and (53) that all the entropy of the black hole, $S_{\mathrm{BH}} = A/4G$, is concentrated in the black hole singularity. This is quite natural from the condensed matter point of view: a singularity is a compact physical object with an enormous energy density. The condensed matter analog of such an object is represented by the core of a vortex in a superfluid liquid. The vortex core contains the singularity in the superfluid velocity, $\nabla \times \mathbf{v}_s = \kappa \hat{\mathbf{z}}\, \delta(\mathbf{r}_\perp)$, where κ is the quantum of circulation of the superfluid velocity. In chiral Weyl superfluids, the vortex core has a large density of states, which is caused by the flat band in the spectrum of electrons confined in the singularity [84].

It would be interesting to apply this approach to a Kerr black hole and to a super-critical Reissner–Nordström black hole with naked singularity [74, 85].

7. Unruh effect and Schwinger pair production

7.1 Schwinger vs Unruh

In Section 3, we saw that the de Sitter state is characterized by the local temperature, which is twice the Gibbons–Hawking temperature [2, 86]. This adds more questions to the many discussions about the factor 2 problem in black hole thermodynamics and in cosmology [87–92]. The Schwinger pair creation also bears some features of thermal radiation, so one can expect the occurrence of a factor 2 problem in this phenomenon as well.

The rate of Schwinger pair creation [93, 94] of particles with mass M and charges $\pm q$ in electric field $\mathcal{E}$ per unit volume per unit time is given by

$$\Gamma^{\mathrm{Schw}}(M) = \frac{\mathrm{d}W^{\mathrm{Schw}}}{\mathrm{d}t} = \frac{q^2\mathcal{E}^2}{(2\pi)^3}\exp\left(-\frac{\pi M^2}{q\mathcal{E}}\right). \tag{54}$$

Since $a = q\mathcal{E}/M$ corresponds to the acceleration of a charged particle, one obtains the analog of the Unruh effect [95]:

$$\Gamma^{\mathrm{Schw}}(M) \propto \exp\left(-\frac{\pi M}{a}\right) \equiv \exp\left(-\frac{M}{T}\right) = \exp\left(-\frac{M}{2T_{\mathrm{U}}}\right). \tag{55}$$

Schwinger pair creation does look like the Unruh effect, although the corresponding effective temperature is twice the Unruh temperature, $T = a/\pi \equiv \bar{T}_{\mathrm{U}} = 2T_{\mathrm{U}}$. There have been many attempts to connect these two phenomena (see Refs [96–99] and references therein).

Of course, the similarity between the equations for probabilities suggests that there is some analogy between the Schwinger and Unruh effects. However, the extra factor 2 presents a serious problem. At the Landau Institute, Starobinsky became interested in solving this problem. Here, we consider this problem using the experience with the doubling of the Gibbons–Hawking temperature in the case of the cosmological horizon in Section 3.

If one tries to make a direct analogy between the Schwinger and Unruh processes, this is already problematic. The original state is a vacuum in a constant electric field. Being a vacuum, it does not provide any physical acceleration. Acceleration in an electric field appears only in the presence of a charged particle. That is why one can try to find a situation where the two effects are physically connected. The connection may arise if we split the pair creation into several steps. In the first step, a pair of particles is created by the Schwinger mechanism, and then in further steps, the Unruh process enters, which is caused by the acceleration of the created charged particles by an electric field. These quantum processes should take place in unison, i.e., coherently. This means that, in the quantum tunneling picture [17, 18], the corresponding exponents are added.

7.2 Double Unruh temperature from Schwinger and Unruh

To connect the Schwinger pair production with the Unruh effect, let us consider the following combined process in which particles with charges $\pm q$ and masses $M + m$ are created. In the first step, particles with charges $\pm q$ and masses M are created; in the second step, these charged particles are accelerated by the electric field. These particles play the role of detectors whose energy levels are excited from levels with energy M to levels with energy $M + m$ due to the Unruh effect. Since these processes occur in unison, the probability of the combined process $\Gamma^{\mathrm{Schw}}(M + m)$ is equal to the product of partial probabilities:

$$\Gamma^{\mathrm{Schw}}(M + m) = \Gamma^{\mathrm{Schw}}(M)\, \Gamma_+^{\mathrm{Unruh}}(m) \Gamma_-^{\mathrm{Unruh}}(m). \tag{56}$$

Here, $\Gamma_+^{\mathrm{Unruh}}(m)$ and $\Gamma_-^{\mathrm{Unruh}}(m)$ are the excitations rates of the corresponding detectors.

On the other hand, according to Eqn (54), the probability of creating particles at the excited level with mass $M + m$ and charge q can be expressed in terms of the probability of creating particles in the ground state with mass M and charge q with an extra term:

$$\Gamma^{\mathrm{Schw}}(M+m) = \Gamma^{\mathrm{Schw}}(M)\exp\left(-\frac{2\pi M m}{q\mathcal{E}} - \frac{\pi m^2}{q\mathcal{E}}\right). \tag{57}$$

Comparing (57) with Eqn (56), one finds that the effect of acceleration (the Unruh effect) is described by the following equation:

$$\Gamma_{\pm}^{\mathrm{Unruh}}(m) = \exp\left(-\frac{\pi m}{a}\left(1 + \frac{m}{2M}\right)\right). \tag{58}$$

In the limit $m \ll M$, this corresponds to Eqn (55) with the double Unruh temperature. So, we returned to the factor 2 problem. But now, we have a new argument in favor of the factor 2 in the Unruh effect. The combined Schwinger–Unruh process suggests that the factor 2 in Eqn (55) is natural, and thus we must reconsider the temperature of the pure Unruh effect. Condensed matter analogs, where we know both the infrared and ultraviolet limits [102], support doubling. This shows that the double Unruh temperature $\bar{T}_{\mathrm{U}} = 2T_{\mathrm{U}}$ is not an artefact but a real temperature that governs the Unruh process.

7.3 Back reaction of detector in Unruh effect

Equation (58) suggests that the Unruh effect should be written in the form

$$\Gamma^{\text{Unruh}}(m) = \exp\left(-\frac{m}{\bar{T}_{\text{U}}}\left(1+\frac{m}{2M}\right)\right), \tag{59}$$

where the correction $m/2M$ describes the recoil (or back reaction) of the detector due to its finite mass M [96, 97, 100, 101]. This is similar to the back reaction of the black hole in Hawking radiation discussed by Parikh and Wilczek [18].

To get this correction, let us consider the step by step transitions to the higher mass of the detector, i.e., to the higher level n of excitation of the detector [100, 101]. We consider N steps, each with $\Delta m = m/N$. At each step, the mass of the detector increases, $M_n = M + (n-1)\Delta m$, and thus the acceleration and the Unruh temperature decrease correspondingly:

$$M_n = M + (n-1)\Delta m\,, \ \Delta m = \frac{m}{N}\,, \tag{60}$$

$$a_n = \frac{q\mathcal{E}}{M_n}\,, \ \bar{T}_{\text{U}n} = \frac{a_n}{2\pi}\,. \tag{61}$$

Then, one obtains Eqn (59):

$$\Gamma^{\text{Unruh}}(m) = \prod_n \exp\left(-\frac{\Delta m}{\bar{T}_{\text{U}n}}\right) = \exp\left(-\sum_{n=1}^{N}\frac{\Delta m}{\bar{T}_{\text{U}n}}\right), \tag{62}$$

$$\Gamma^{\text{Unruh}}(m) = \exp\left(-\frac{m}{\bar{T}_{\text{U}}}\left(1+\frac{m}{2M}\right)\right). \tag{63}$$

7.4 Back reaction in Hawking radiation

The same step-wise procedure can be applied to the Hawking radiation, where the black hole mass M decreases after each $\Delta\omega = \omega/N$ step of Hawking radiation, which gradually raises the Hawking temperature.

$$M_n = M - (n-1)\Delta\omega\,, \ \Delta\omega = \frac{\omega}{N}\,, \tag{64}$$

$$T_{\text{H}n} = \frac{1}{8\pi G M_n}\,. \tag{65}$$

This gives the Parikh–Wilczek result [18]:

$$\Gamma(\omega) = \exp\left(-\sum_{n=1}^{N}\frac{\Delta\omega}{T_{\text{H}n}}\right), \tag{66}$$

$$\Gamma(\omega) = \exp\left(-\frac{\omega}{T_{\text{H}}}\left(1-\frac{\omega}{2M}\right)\right), \ T_{\text{H}} = \frac{1}{8\pi G M}\,. \tag{67}$$

The result is similar to that in Eqn (63) except for the opposite sign, since the mass of the black hole decreases with radiation, while the mass of the detector increases due to excitation by acceleration.

8. Landau–Khalatnikov hydrodynamics and cosmological constant problems

8.1 Vacuum energy in condensed matter

The cosmological constant problem or 'vacuum catastrophe' is the substantial disagreement between the observed values of vacuum energy density (the small value of the cosmological constant) and the much larger theoretical value of zero-point energy suggested by quantum field theory. The zero-point energy is huge, because it is determined by the ultraviolet cut-off. At first glance, the condensed matter provides a very simple solution to this problem.

In condensed matter systems, such as superfluid ^{4}He and superfluid phases of ^{3}He, we know the physics both on the ultraviolet (atomic) scale and in the infrared (hydrodynamic) limit. In this low-energy limit, the physics is described by collective variables and quasiparticles forming an analog of the Universe. The effective quantum fields in these effective universes have zero-point energies, formally diverging (in the same way as in our Universe) and requiring an ultraviolet cut-off. However, looking from the microscopic (atomic ultraviolet) side, we can see that all these divergences are formal and have no physical consequences. They do not determine the energy of the ground state of the quantum liquid — the energy of the 'quantum vacuum,' which is zero in equilibrium.

The reason is that the 'vacuum' is the many-body system. In the limit of a large number of atoms, it obeys the general laws of thermodynamics, which do not depend on details of the atomic physics. In particular, the energy density ϵ of the superfluid liquid in the zero temperature limit obeys the Gibbs–Duhem relation:

$$\epsilon - \mu n = -P\,. \tag{68}$$

Here, n is the density of atoms, $\mu = \mathrm{d}\epsilon/\mathrm{d}n$ is the chemical potential, and P is pressure.

One can see that the thermodynamic potential $\epsilon - \mu n$ plays the role of the energy density of the vacuum, ϵ_{vac}, which obeys the equation of state $w = -1$:

$$\epsilon_{\text{vac}} = -P\,, \ \epsilon_{\text{vac}} = \epsilon - \mu n\,. \tag{69}$$

8.2 Zero cosmological constant in equilibrium

Equation (69) is the general property of any vacuum state in any quantum system, including the quantum vacuum of our Universe. It is important that it is ϵ_{vac}, and not ϵ, that enters into Einstein's equation as the cosmological constant Λ. This is shown using the so-called q-theory [56, 103, 104], where the physical vacuum is described by the 4-form field — the dynamical variable introduced by Hawking [61, 105]. From the equations for the q-field and the Einstein equation, one obtains

$$\Lambda = \epsilon_{\text{vac}} = \epsilon(q) - \mu q\,, \ \mu = \frac{\mathrm{d}\epsilon}{\mathrm{d}q}\,. \tag{70}$$

Let us first consider the quantum vacuum in the absence of gravity, i.e., in special relativity [106]. In this case, if the external pressure acting on the quantum vacuum is absent, then, according to Eqn (69), the cosmological constant is naturally zero.

So, the thermodynamics naturally solves the main cosmological constant problem without any fine-tuning if gravity is absent. Note that the quantity $\epsilon(q)$ is determined by the UV physics, and is huge. But, in equilibrium, it is naturally cancelled by the term $-\mu q$, which is the analog of the counterterm in quantum field theories. The mechanism of cancellation is purely thermodynamic and does not depend on whether the vacuum is relativistic or not.

Hawking considered the quadratic dependence of ϵ on q. In this case, there is no compensation, since one obtains $\Lambda = \epsilon_{\text{vac}} = -\epsilon(q)$.

By analogy with condensed matter, this vacuum corresponds to a gas instead of a liquid. A gas cannot exist in equilibrium without external pressure, whereas a liquid is self-sustained, i.e., can exist without external pressure. This suggests that the vacuum in our Universe is a self-sustained substance, and thus $\Lambda = \epsilon_{\rm vac} = 0$ in equilibrium.

8.3 Vacuum decay without fine-tuning

Now, we consider the influence of gravitational degrees of freedom. As shown in Section 4, gravity with its thermodynamically conjugate variables (the gravitational coupling K and the scalar Riemann curvature $\mathcal{R}$) adds its contribution to the total pressure. Then, at equilibrium, the total pressure is zero according to Eqn (28). But now the equilibrium state with zero pressure corresponds to a de Sitter vacuum, which by virtue of its symmetry serves as an attractor along with the Minkowski vacuum.

The de Sitter attractor can be considered using the dynamics of the Hawking q-field. Let us first assume that the Big Bang started in an originally equilibrium Minkowski vacuum. This corresponds to the special value μ_0 of the chemical potential of the q field in Eqn (69) for which $\Lambda(\mu_0) = 0$. Then, from our equations without dissipation [104], it follows that the cosmological constant, which is very large immediately after the Big Bang, relaxes with oscillations to a zero value. Its magnitude averaged over fast oscillations reaches the present value in the present time $\langle \Lambda(t_{\rm present}) \rangle \sim M_{\rm P}^2/t_{\rm present}^2 \sim 10^{-120} M_{\rm P}^4$. A similar behaviour was discussed in Section 5.3. The oscillating decay is also present in the Starobinsky inflation [107, 108], where the magnitude of the oscillation frequency is determined by the Higgs inflaton mass, instead of the Planck scale in our case.

However, if the initial conditions are different, i.e., $\mu \neq \mu_0$, then, from the equations of dynamics (again, without dissipation), it follows that the Universe relaxes with oscillations to a de Sitter attractor instead of the Minkowski vacuum state. This demonstrates that the special choice of initial conditions, $\mu = \mu_0$, is a kind of fine-tuning. The fine-tuning is also present in the Starobinsky inflation [107, 108], due to the special choice of the parameters of the system, which leads to the Minkowski attractor. The Pauli–Zel'dovich mechanism discussed by Kamenshchik and Starobinsky [109] also relies on fine-tuning — the exact cancellation of contributions of relativistic bosons and relativistic fermions to the vacuum energy. In the general case, i.e., without fine-tuning, the de Sitter attractor is inevitable. This reflects a special symmetry of the de Sitter state that puts this state on the same level as the Minkowski vacuum.

Nevertheless, the condensed matter conclusion that the vacuum energy must be zero at full equilibrium still holds. The de Sitter vacuum is unstable and decays into the Minkowski vacuum. This happens because the de Sitter environment serves as a heat bath for matter. The thermal bath provides the thermal nucleation of matter discussed in Section 3. The created matter violates the peculiar symmetry of the de Sitter state. The energy exchange between vacuum and matter finally leads to the decay of this state and thus to the nullification of the cosmological constant in the final thermodynamic equilibrium without any fine-tuning. This was considered in Sections 4 and 5. The relaxation of the dark energy in Eqn (37) was obtained via the example of energy exchange with Zel'dovich stiff matter.

This mechanism of instability of the de Sitter vacuum with regard to the Minkowski vacuum differs from the instability considered by Polyakov [110 – 113]. In our case, the radiation of matter in the de Sitter environment takes place only if the de Sitter state contains an 'impurity,' such as a proton or hydrogen atom in Section 3. This impurity breaks de Sitter symmetry and becomes the trigger for the creation of matter.

Also note that the decay of the 'cosmological constant' cannot be obtained in the framework of the classical Einstein equations due to the Weinberg no-go theorem [114]. This process requires a quantum mechanical approach, which includes quantum tunneling.In condensed matter, this can be compared with the classical time-dependent Ginzburg–Landau equation for superfluids and superconductors. It has a very narrow range of applicability, and nonequilibrium processes such as relaxation to an equilibrium state require the Green's function approach (see Kopnin's book [115]).

9. Planck constants and dimensionless physics

9.1 Composite tetrads from relative symmetry breaking

There are various scenarios for the emergence of gravity from more fundamental fields, such as quantum fermionic fields. In particular, gravity can be constructed using bilinear combinations of the fermionic fields [41 – 47]:

$$\hat{E}^a_\mu = \frac{1}{2} \left(\Psi^\dagger \gamma^a \partial_\mu \Psi - \Psi^\dagger \overleftarrow{\partial_\mu} \gamma^a \Psi \right). \tag{71}$$

The original action in this scenario does not depend on the gravitational fields (tetrads and metric) and is described solely in terms of differential forms:

$$S = \frac{1}{24} \, e^{\alpha\beta\mu\nu} e_{abcd} \int \mathrm{d}^4 x \, \hat{E}^a_\alpha \, \hat{E}^b_\beta \, \hat{E}^c_\mu \, \hat{E}^d_\nu \,. \tag{72}$$

This action, which is the operator analog of the cosmological term, has high symmetry. It is symmetric under coordinate transformations $x^\mu \to \tilde{x}^\mu(x)$, and thus is also scale invariant. In addition, the action is symmetric under spin rotations, or under the corresponding gauge transformations when the spin connection is added to the gradients.

The gravitational tetrads e^a_μ appear as the vacuum expectation values of the bilinear fermionic 1-form $\hat{E}^a_\mu$ as a result of spontaneous symmetry breaking:

$$e^a_\mu = \langle \hat{E}^a_\mu \rangle \,. \tag{73}$$

This order parameter breaks the separate symmetries under orbital and spin transformations, but remains invariant under the combined rotations. On the level of the Lorentz symmetries, the symmetry breaking scheme is $L_{\rm L} \times L_{\rm S} \to L_{\rm J}$. Here, $L_{\rm L}$ is the group of Lorentz transformations in the coordinates space, $L_{\rm S}$ is the group of Lorentz transformations in the spin space, and $L_{\rm J}$ is the symmetry group of the order parameter e^a_μ, which is invariant under the combined Lorentz transformations $L_{\rm J}$.

A similar symmetry breaking mechanism of emergent gravity is known in condensed matter physics, where the effective gravitational vielbein also emerges as the bilinear fermionic 1-form [40]. This scenario takes place in the p-wave spin-triplet superfluid ^{3}He-B, where corresponding relative symmetry breaking [116] occurs between the spin and orbital rotations, $\mathrm{SO(3)_L} \times \mathrm{SO(3)_S} \to \mathrm{SO(3)_J}$. This means that the symmetry under the relative rotations in spin and orbital spaces is broken, while the properties of ^{3}He-B are invariant under combined rotations $\mathrm{SO(3)_J}$ and thus remain isotropic.

9.2 Dimensionful metric and dimensionless interval

In the tetrad gravity, the metric field is the bilinear combination of the tetrad fields,

$$g_{\mu\nu} = \eta_{ab} e^a_\mu e^b_\nu \,, \tag{74}$$

and thus, in this scenario of quantum gravity, the metric are the fermionic quartet. In principle, the signature of η_{ab} can also come from the dynamical variable O_{ab} [117, 118] if η_{ab} emerges as the vacuum expectation value of the corresponding symmetry breaking phase transition, $\eta_{ab} = \langle O_{ab} \rangle$.

It is important that, in this quantum gravity, the fermionic fields Ψ are dimensionless [44] (we denote this as $[\Psi] = [1]$, i.e., Ψ has the same zero dimension as numbers). Thus, the tetrads in Eqn (73) have the dimensions of inverse time and inverse length, $[e^a_0] = 1/[t]$ and $[e^a_i] = 1/[L]$, while the metric elements in Eqn (74) have dimensions $1/[t]^2$, $1/[L]^2$, and $1/[t][L]$.

Due to such dimensions of tetrads and the metric, the interval is dimensionless:

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu \,, \quad [s^2] = [1] \,. \tag{75}$$

This is not surprising: the interval is a diffeomorphism invariant, whereas, in this approach to quantum gravity, all diffeomorphism-invariant quantities (such as action (72)) are dimensionless [44].

9.3 Dimensions of gauge field and mass

Let us consider the simplest examples of the dimensionless action. The action describing the interaction of a charged point particle with the $U(1)$ gauge field is

$$S = q \int dx^\mu A_\mu \,. \tag{76}$$

As the original action (72), this action does not depend on the metric field and is described solely in terms of differential forms, now in terms of the 1-form $U(1)$ gauge field A_μ.

The $U(1)$ field is the geometric quantity, which comes from the gauging of the global $U(1)$ field. The field A_μ comes from the gauging of the gradient of the phase field, and thus has the dimension of the gradient of phase, with $[A_0] = 1/[t]$ and $[A_i] = 1/[L]$. The charge q here is dimensionless — it is the integer (or fractional) geometric charge of the fermionic or bosonic field. In the case of the electromagnetic $U(1)$-field, q is expressed in terms of the electric charge of the electron, i.e., $q = -1$ for an electron and $q = +1$ for a proton. As a result, the action (76) is naturally dimensionless, $[S] = [1]$.

Such action can be extended to objects of higher dimensions which interact with the corresponding gauge fields: $1+1$ strings interacting with the 2-form gauge field, $2+1$ branes interacting with the 3-form field, and also $3+1$ objects interacting with the 4-form field.

The action describing the classical dynamics of a point particle requires the metric field, since it is expressed in terms of the interval:

$$S = M \int ds \,. \tag{77}$$

Since both the interval ds and the action S are dimensionless, from equation (77), it follows that the particle mass M is also dimensionless, $[M] = [S] = [s] = [L]^0 = [t]^0 = [1]$.

Note that we discuss here the space and time dimensions of the physical quantities. In quantum field theories, the mass dimension is also used, which will be discussed in Section 9.5.

9.4 Two Planck constants

Let us consider the quadratic terms in the action for the classical scalar field Φ:

$$S = \int d^4x \sqrt{-g} \left(g^{\mu\nu} \nabla_\mu \Phi^* \nabla_\nu \Phi + M^2 |\Phi|^2 \right) . \tag{78}$$

Comparing the gradient term and the mass term and using the dimension of the metric, one again finds that the mass M is dimensionless, $[M] = [1]$. Then, since the action S and volume element $d^4x\sqrt{-g}$ are dimensionless, it follows that the scalar field is also dimensionless, $[\Phi]^2 = [M] = [S] = [1]$.

Using the Fourier components, one obtains the spectrum of waves:

$$g^{\mu\nu} k_\mu k_\nu + M^2 = 0 \,, \tag{79}$$

where $k_0 = \omega$ is the frequency of the mode and $\mathbf{k}$ is the wave vector. In Minkowski spacetime, one obtains

$$E^2 = M^2 + \mathbf{p}^2 c^2 \,, \tag{80}$$

where the energy and momentum of particles are

$$E = \sqrt{-g^{00}}\, \omega \,, \tag{81}$$

$$c p_x = \sqrt{g^{xx}}\, k_x \,, \quad c p_y = \sqrt{g^{yy}}\, k_y \,, \quad c p_z = \sqrt{g^{zz}}\, k_z \,. \tag{82}$$

This shows that the Minkowski elements of the metric play the role of the Planck constants $\hbar$ and $\hbar\!\!\!/ \equiv \hbar c$, which connect the energy and momentum of particles with the frequency and the wave vector. This suggests that we have the following relations between the Planck constants and the Minkowski metric:

$$-g^{00}_{\rm Mink} \equiv \hbar^2 \,, \quad g^{ik}_{\rm Mink} \equiv \hbar\!\!\!/^2 \delta^{ik} \,. \tag{83}$$

Both Planck constants, bar $\hbar$ and slash $\hbar\!\!\!/$, are elements of the metric and tetrads in the Minkowski vacuum. Constant $\hbar$ is the time component of the inverse tetrad and has the dimension of time, $[\hbar] = [t]$, while $\hbar\!\!\!/$ enters the space components of tetrads and has the dimension of length, $[\hbar\!\!\!/] = [L]$. The presence of the Planck constants in the metric elements demonstrates that the metric field describes the dynamics of the quantum vacuum rather than the space–time geometry. This is natural, since space–time itself with its metric arises from the dynamics of quantum fields. The speed of light, as an element of geometry, arises as the ratio of two Planck constants:

$$c^2 = \frac{\hbar\!\!\!/^2}{\hbar^2} \,. \tag{84}$$

In principle, there can be more than two Planck constants. For example, two Planck constants $\hbar$ and $\hbar_g$ were introduced in Ref. [119] for matter and gravity, respectively (see also [120]). This suggests that gravity will be more classical than matter if $\hbar_g \ll \hbar$. With component $\hbar\!\!\!/_g$ added, one obtains four different Planck constants: $\hbar$, $\hbar\!\!\!/$, $\hbar_g$, and $\hbar\!\!\!/_g$. Then, in principle, one may have two speeds of light: $c = \hbar\!\!\!/ / \hbar$ and $c_g = \hbar\!\!\!/_g / \hbar_g$, although $c_g = c$ is more natural.

9.5 Planck constants not entering diffeomorphism invariant expressions

The diffeomorphism invariant quantities are dimensionless. This includes the action S (example is in Eqn (72)); the scalar curvature $\mathcal{R}$; the scalar field Φ; the wave function ψ; masses M; the cosmological constant Λ; the gravitational coupling K;

etc. [121, 122]. Here, the term 'dimensionless' means the absence of space or time dimensions. However, these dimensionless quantities may have nonzero mass dimension $[M]$. For example, the action has a zero mass dimension; the mass dimension of curvature is -2, i.e., $[\mathcal{R}] = [M]^{-2}$; the mass dimension of gravitational coupling is 2, i.e., $[K] = [M]^2$; the cosmological constant Λ has mass dimension 4.

Since M is spacetime dimensionless, from equation $M = \hbar\omega$, it follows that the Planck constant $\hbar$ has the dimension of time, $[\hbar] = [t]$, and the Planck constant $\hbar\!\!/$ has the dimension of length, $[\hbar\!\!/] = [L]$. Since these 'constants' are not diffeomorphism invariant, they cannot enter the diffeomorphism invariant expressions. An example is the original action (72), which does not contain the Planck constants $\hbar$ and $\hbar\!\!/$. This is a property of any action if it is written in the diffeomorphism-invariant form. Let us consider several examples, when the Planck constants are removed from the expressions rewritten in the diffeomorphism-invariant form.

The gravitational potential must be expressed in terms of the rest energies $M = mc^2$ instead of the masses m. Thus, the Newton constant must be modified:

$$U(r) = -\bar{G}\,\frac{M_1 M_2}{r}\,, \quad \bar{G} = \frac{G}{c^4}\,. \tag{85}$$

Since the potential U has the dimension of mass M, and thus is space–time dimensionless, it is found that $\bar{G}$ has the same dimension of length as $\hbar\!\!/$, i.e., $[\bar{G}] = [\hbar\!\!/] = [L]$. This demonstrates that neither $\bar{G}$ nor $\hbar\!\!/$ is diffeomorphism invariant, and thus they cannot be the fundamental constants. In the gravitational action, they compensate each other:

$$S = K \int d^4x \sqrt{-g}\,\mathcal{R}\,, \quad K = \frac{1}{16\pi}\,\frac{\hbar\!\!/}{\bar{G}} \equiv \frac{M_{\mathrm{P}}^2}{2}\,. \tag{86}$$

As we already mentioned, the gravitational coupling K and curvature $\mathcal{R}$ are space–time dimensionless, but have mass dimensions 2 and -2, respectively. That is why, although $\hbar\!\!/$ and $\bar{G}$ have the same space–time dimension, their ratio has mass dimension 2. The length of the reduced Planck constant $\hbar\!\!/$ and the reduced Planck length $l_{\mathrm{P}} = \hbar\!\!/ / M_{\mathrm{P}}$ coincide if we choose the reduced Planck mass M_{P} as the unit of mass.

Another example is Eqn (6) for electron levels in the hydrogen atom, which can be written in the conventional form or in terms of dimensionless quantities. In the traditional form, it is

$$E_n = \frac{me^4}{2\hbar^2}\,\frac{1}{n^2}\,. \tag{87}$$

In the dimensionless form, these levels can be considered in terms of the negative corrections to the electron rest energy:

$$\frac{\Delta M_{\mathrm{e}}}{M_{\mathrm{e}}} = -\frac{\alpha^2}{2n^2}\,. \tag{88}$$

In Eqn (88), the fully dimensionless parameter α (the fine structure constant $\alpha = e^2/\hbar\!\!/$) connects the space–time dimensionless quantities: the electron rest energy M_{e} and its correction ΔM_{e}.

The traditional description in Eqn (87) reflects the historical process of development of physical ideas. Here, the dimensionless quantities are split into dimensional quantities, such as electric charge e, speed of light c, Newton constant G, Planck constant $\hbar$, mass m, and Hubble parameter H.

9.6 de Sitter as ensemble of Planck 'atoms'

In principle, some dimensionless parameters can be quantized. The numbers related to symmetry and topological charges are certainly quantized. Other possible examples are the entropy of a black hole or the entropy of a Hubble volume, which are related to event horizons.

The energy of a Hubble volume is obtained by multiplying the energy density in Eqn (14) by the volume V_{H}. It can be written in terms of the space–time dimensionless variables, Planck mass M_{P}, and de Sitter temperature $T = \hbar H/\pi$:

$$E_{\mathrm{H}} = \epsilon_{\mathrm{vac}} V_{\mathrm{H}} = \frac{1}{2\hbar H}\,\frac{\hbar\!\!/}{\bar{G}} = \frac{4M_{\mathrm{P}}}{T}\,M_{\mathrm{P}} \equiv N M_{\mathrm{P}}\,. \tag{89}$$

In this equation, the energy is presented in terms of N 'atoms of the de Sitter state' with Planck masses M_{P}. In this representation, the dimensionless entropy of the Hubble volume in Eqn (16) is quantized:

$$S_{\mathrm{H}} = \frac{A}{4\hbar\!\!/\bar{G}} = \frac{\pi}{(\hbar H)^2}\,\frac{\hbar\!\!/}{\bar{G}} = 8\,\frac{M_{\mathrm{P}}^2}{T^2} = \frac{1}{2}\,N^2\,. \tag{90}$$

This leads to quantization of the cosmological constant [74].

The entropy of the Hubble volume is equivalent to the entropy of the cosmological horizon. Quantization of the horizon entropy is considered in many theories. In the Bekenstein approach [123–129], the entropy is the adiabatic invariant, the spectrum of which is equally spaced, giving rise to the linear law $S_{\mathrm{BH}} = aN$. Here, a is a dimensionless parameter whose value depends on the microscopic theory. The quantization inspired by string theory [130, 131] gives the square-root rule, $S = 2\pi(\sqrt{N_1} \pm \sqrt{N_2})$. Note that Eqn (90) applies to the entropy of the Hubble volume, whereas in general the entropy of the de Sitter state is extensive (see Section 6.5).

9.7 Black hole as ensemble of Planck 'atoms'

The ensemble of N Planck 'atoms' in Eqn (89) is similar to the ensemble of N Planck-sized black holes inside the black hole horizon [74, 79], where the entropy is also proportional to N^2 at large N:

$$S(N) = \frac{N(N-1)}{2}\,. \tag{91}$$

The probability of the quantum tunneling process of the emission of the black hole quantum ($N = 1$ and $S(N = 1) = 0$) is, on the one hand, similar to Eqn (40) for quantum tunneling process of the vortex instanton, and, on the other hand, it satisfies Eqn (45) for quantum tunneling as random fluctuation:

$$\begin{aligned} w &\sim \exp\big(S(N-1) - S(N)\big) = \exp\big(-(N-1)\big) \\ &= \exp\left(-\frac{m_{\mathrm{P}}}{T_{\mathrm{H}}}\left(1 - \frac{m_{\mathrm{P}}}{M}\right)\right). \end{aligned} \tag{92}$$

The correction to the Hawking radiation rate corresponds to the effect of back reaction in the process of radiation considered by Parikh and Wilczek [18].

The black hole entropy is nonextensive and thus requires generalized statistics.

For large N, the entropy in Eqn (91) satisfies the composition rule of the Tsallis–Cirto $\delta = 2$ entropy for ensembles with N_1 and N_2 quanta,

$$\sqrt{S_{\delta=2}(N_1+N_2)} = \sqrt{S_{\delta=2}(N_1)} + \sqrt{S_{\delta=2}(N_2)}\,, \tag{93}$$

which reproduces Eqn (47). In rare quantum fluctuation processes leading to black hole fragmentation, $N = N_1 + N_2$, the entropy decreases sharply. Then, the entropy continuously increases in the further process of black hole recombination, $N_1 + N_2 \to N$, reaching a maximum value for a given N. In this description, a black hole is not a quantum state, but a dissipative process in which continuous evolution alternates with random quantum jumps (see, e.g., Refs [132, 133]).

It would be interesting to consider analogs of such processes and possible nonextensive statistics in superfluids. One option is to consider entropy associated with the vortex instanton in Eqn (40), which describes the creation of vortices in a moving ^{4}He superfluid. In the toroidal geometry, this instanton changes the topological charge of the flow — the number of quanta of circulation. Let us assume that Eqn (40) is related to the entropy of the flowing 'vacuum' and compare this entropy with the black hole entropy. For this, let us introduce the corresponding mass M, which is the energy of the created vortex loop, and the corresponding Planck mass $M_{\rm P}$, which is the inverse interatomic distance. Then, we find that the entropy corresponding to Eqn (40) is $S \propto M^3/M_{\rm P}^3$. This entropy has higher nonextensivity than the black hole entropy $S \propto M^2/M_{\rm P}^2$, and it corresponds to the Tsallis–Cirto entropy with $\delta = 3$. The corresponding effective temperature, which characterizes the 'flow of the vacuum' with velocity $v_{\rm s}$, is $T = {\rm d}M/{\rm d}S \propto m_4 v_{\rm s}^2$, where m_4 is the mass of the helium atom.

Another possible approach is to consider the quantum tunneling process of splitting a droplet of superfluid liquid into smaller droplets with a decrease in the total entropy. Notably, due to the surface energy of the droplets, it is possible to split a droplet with a nonzero temperature and, therefore, with nonzero entropy into smaller parts with zero temperature and zero entropy.

9.8 Acoustic Planck constants

Returning to the two-fluid hydrodynamics, let us consider quasiparticles in a ^{4}He Bose superfluid. The flow of the liquid provides the acoustic gravity for 'relativistic' quasiparticles — phonons. The effective acoustic metric and thus the acoustic Planck constants, $\hbar_{\rm ac}$ and $\not{\hbar}_{\rm ac}$, can be expressed in terms of the parameters of this liquid.

If the chosen variable in the superfluid hydrodynamics is the phase Φ of the Bose condensate, which is dimensionless, then the action for phonons propagating in moving liquid is [33, 134]

$$S_{\rm ph} = \frac{m_4}{2\hbar}\int {\rm d}^3x\,{\rm d}t\,n\left((\nabla\Phi)^2 - \frac{1}{s^2}(\dot{\Phi} - \mathbf{v}\nabla\Phi)^2\right) = \frac{1}{2}\int {\rm d}^4x\,\sqrt{-g}\,g^{\mu\nu}\nabla_\mu\Phi\nabla_\nu\Phi\,. \tag{94}$$

Here, $n = 1/a^3$ is the density of bosonic particles (^{4}He atoms, which play the role of 'atoms of the vacuum'); a is the interatomic distance, which plays the role of the Planck length; m_4 is the mass of the helium atom; s is the speed of sound, which plays the role of the speed of light; and $\mathbf{v}$ is the superfluid velocity (the velocity of the 'superfluid vacuum'). It is the shift function in the Arnowitt–Deser–Misner (ADM) approach.

Finally, $\hbar$ here is the conventional Planck constant describing the microscopic physics. All the parameters are considered in conventional units, i.e., without application of dimensionless physics. The dimensionless physics will emerge for phonons, and this is the reason why we introduced the phonon action $S_{\rm ph}$ as the action divided by $\hbar$. The corresponding acoustic interval is

$$ {\rm d}s^2 = g_{\mu\nu}\,{\rm d}x^\mu\,{\rm d}x^\nu = \frac{\hbar n}{m_4 s}\left[-s^2\,{\rm d}t^2 + ({\rm d}\mathbf{r} - \mathbf{v}\,{\rm d}t)^2\right]. \tag{95}$$

The analog of the Minkowski metric corresponds to the zero value of the shift function, $\mathbf{v} = 0$, and thus $g^{0i} = 0$. Then, the effective acoustic Minkowski metric experienced by the propagating phonons is

$$g_{00} = \frac{\hbar n s}{m_4}\,,\quad g_{ik} = \frac{\hbar n}{m_4 s}\,\delta_{ik}\,,\quad \sqrt{-g} = \frac{\hbar^2 n^2}{m^2 s}\,, \tag{96}$$

with dimensions

$$[g_{00}] = \frac{1}{[t]^2}\,,\quad [g_{ik}] = \frac{1}{[L]^2}\,,\quad [\sqrt{-g}] = \frac{1}{[t][L]^3}\,. \tag{97}$$

The acoustic interval (95) is dimensionless, $[{\rm d}s] = 1$. This demonstrates that the interval ${\rm d}s$ describes the dynamics of phonons in the superfluid 'vacuum,' rather than the distances and time intervals. The same is valid for the interval in general relativity, where it describes the dynamics of a point particle in a relativistic quantum vacuum.

The quantum mechanics of phonons demonstrate that it can be described by the effective acoustic Planck constants, $\hbar_{\rm ac}$ and $\not{\hbar}_{\rm ac}$:

$$\hbar_{\rm ac}^2 = g^{00} = \frac{m_4}{\hbar n s}\,,\quad \not{\hbar}_{\rm ac}^2 = \hbar_{\rm ac}^2 s^2 = \frac{m_4 s}{\hbar n}\,. \tag{98}$$

The ground state of superfluid ^{4}He represents a strongly interacting and strongly correlated 'quantum vacuum.' Two Planck mass scales $m_4 s^2$ and $\hbar s/a$ are of the same order. That is why, in this analog of a quantum vacuum, the acoustic Planck constant $\not{\hbar}_{\rm ac} \sim a$, i.e., the length of the effective Planck constant is on the order of the effective Planck length (the interatomic distance).

9.9 Phase boundary between universes with different $\hbar$

The so-called multiple point principle [135] suggests that the quantum vacua are degenerate, and thus there is a boundary of the first order phase transition between our Universe and one with very different properties [136]. If so, it cannot be ruled out that the two neighboring universes may have different Minkowski metrics and thus different values of Planck parameters $\hbar$ and $\not{\hbar}$. If there is thermal contact between these universes, then, in thermal equilibrium, we would have the following condition for temperatures and Planck constants:

$$\frac{T_1}{\hbar_1} = \frac{T_2}{\hbar_2}\,. \tag{99}$$

According to Eqn (83), which connects the Minkowski metric and Planck constants, Eqn (99) is a particular case of the Tolman–Ehrenfest relation, $T(\mathbf{r})\sqrt{g_{00}(\mathbf{r})} = {\rm const}$.

If there is particle exchange between these two vacua, we have the similar Tolman law for the chemical potentials:

$$\frac{\mu_1}{\hbar_1} = \frac{\mu_2}{\hbar_2} \,. \qquad (100)$$

Note that the chemical potential is space–time dimensionless, while $\hbar$ has the dimension of time. Then, the quantity $\mu/\hbar$ has the dimension of frequency. This means that, at equilibrium, when crossing a phase boundary, it is the frequency that does not change, not the temperature or chemical potential. If the two vacua are superfluid, then the ac Josephson effect between these vacua is determined by the frequency difference:

$$\omega = \left| \frac{\mu_1}{\hbar_1} - \frac{\mu_2}{\hbar_2} \right| . \qquad (101)$$

For $\hbar_1 = \hbar_2$, this equation describes the conventional ac Josephson effect in superfluids.

Another interesting question is what happens when $e^a_\mu \to 0$, i.e., when we approach the critical point at which the interval ds becomes identically zero and thus the space-time disappears. According to Eqn (83), in this limit, both Planck constants approach infinity, $\hbar \to \infty$ and $\not{\hbar} \to \infty$. This demonstrates the quantum correlations between the points separated by arbitrary large distances. However, there may be different limiting cases corresponding to different relations among $\hbar$, $\not{\hbar}$, and G in the Bronstein cGh cube [137, 138] (see Ref. [139] and references therein).

10. Discussion

It is shown that the de Sitter universe can be represented in terms of two dark components in Eqns (26)–(29), dark energy (the energy of the vacuum) and 'gravitational dark matter,' which comes from the gravitational degrees of freedom. The partial pressures of two components compensate each other in the equilibrium de Sitter state. In this representation, the gravitational dark matter is responsible for the thermodynamics of the de Sitter state, which is characterized by the local temperature $T = \hbar H/\pi$ and is similar to the thermodynamics of Zel'dovich stiff matter. Let us recall that the local temperature of the de Sitter state is twice the Gibbons–Hawking temperature related to the cosmological horizon, and the stiff matter behavior of the dark matter component is the consequence of this value of the local temperature.

We tried to extend this two-fluid approach to the dynamics of the Universe, assuming that in the dynamics the dark matter component is also equivalent to Zel'dovich stiff matter. We then obtained the power-law decay of both dark components, the dark matter component in Eqn (38) and the dark energy component in Eqn (37). While the state immediately after the Big Bang may have had an energy density of the order of the Planck scale, $\epsilon \sim M_{\rm P}^4$, the energies of both components acquire the correct order of magnitude at the present time, $\epsilon \sim 10^{-120} M_{\rm P}^4$, thus offering a possible route to solving the coincidence problem. Both components eventually decay to zero at $t \to \infty$ on the way to a full equilibrium attractor, the Minkowski quantum vacuum.

The reasonable decay law (37) for dark energy is obtained in the model with stiff matter. The question then arises: what might the physical origin of this stiff matter be and how is it related to the gravitational hard matter discussed in Section 4? In principle, it is possible, that the stiff matter comes just from the gravitational dark matter, and the modified equations reflect the microscopic physics, which supports both the thermodynamics and the de Sitter dynamics. The question then remains open as to how to describe the corresponding degrees of freedom that lead to the considered thermodynamics and dynamics.

One possibility is to introduce the proper vacuum variable, which describes the quantum vacuum and its vacuum energy density. A working example of such a variable is the Hawking 4-form field [61]. This variable was used in particular in the so-called q-theory [56, 140]. However, the suggested form of the q-theory gives rise to cold dark matter instead of stiff matter [63]. An analog of cold matter is produced by the fast oscillations of the q-field. This q-theory mechanism also gives the required power-law decay of dark energy, similar to that in Eqn (37). However, this decay to the Minkowski vacuum takes place only in the case of the proper choice of the value of the corresponding analog of the chemical potential. This represents some form of fine-tuning, while the two fluid dynamics does not require fine-tuning.

To avoid fine-tuning, the interaction with ordinary matter must be included. Since the de Sitter vacuum represents a heat bath for matter, the de Sitter expansion is unstable to the thermal radiation of matter. Matter violates the de Sitter symmetry and promotes the relaxation of the de Sitter state to the Minkowski vacuum. This is the analog of the three-fluid hydrodynamics, which describes dark energy, dark matter, and ordinary matter.

We also considered the thermodynamics of a black hole, which can be obtained using macroscopic quantum tunneling processes of splitting a black hole into smaller ones. In fact, this is another way to derive the Bekenstein–Hawking entropy of a black hole. It is shown that the statistical ensemble describing a black hole is described by the nonextensive Tsallis–Csirto entropy with $\delta = 2$.

On the other hand, the entropy of the de Sitter state is extensive, i.e., for arbitrary volume V in the de Sitter universe, the entropy is proportional to the volume, $S(V) = sV$, where s is the entropy density. But the integration of the entropy density over the Hubble volume demonstrates the holographic bulk-surface connection between the entropy of the Hubble volume and the Gibbons–Hawking surface entropy of the cosmological horizon, $S(V_{\rm H}) = sV_{\rm H} = A/4G$.

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