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On gravitational frequency shift derived from energy conservation

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Contents

Abstract. **In physics, thought experiments are impressive heuristic tools. They are valuable instruments to help scientists find new results and to teach students known ones. However, as we shall show, they should always be accepted cautiously, even when they are a shortcut to 'prove' well-established results. Here, we show that the most widely known thought experiments devised to derive the gravitational frequency shift from energy conservation are, in fact, problematic. When properly set and correctly read, these thought experiments reveal that the existence of the gravitational frequency shift is, in fact, at odds with energy conservation. We also propose two new simple thought experiments, one using energy conservation and the other the conservation of linear momentum, that corroborate that conclusion, showing that these conservation principles do not imply a gravitational frequency shift. Our results may be of some epistemological interest and could serve as a warning sign on how thought experiments should be accepted and trusted.**

Keywords: special relativity, general relativity, gravitational frequency shift, conservation of energy, linear momentum conservation, thought experiments

1. Introduction

In 1907, Einstein introduced the equivalence principle [1]. He used that principle to 'extrapolate' the effects of special relativity to systems at rest in a gravitational field via their alleged equivalence to uniformly accelerated systems. In that paper, Einstein first derived the gravitational redshift, the gravitational time dilation, and other effects of gravity on electromagnetic processes, like the variable speed of light and gravitational light deflection.

His first attempt to extend special relativity to gravitation was, according to Einstein himself, not particularly satisfying, and he returned to the topic in 1911, providing a much simpler derivation of the gravitational time dilation, redshift, and light deflection.

Let us briefly review his second derivation of the gravitational redshift [2]. Consider two material systems, S_1 and S_2, at rest in a local, uniform gravitational field $\mathbf{a}$ (Fig. 1). S_1 and S_2 are separated by a distance d. Consider further a reference frame K_0. System K_0 is a free-falling (gravitation-free) system located near S_2 with an initial instantaneous velocity relative to S_2 equal to zero.

Suppose further that a ray of light of frequency ν_2 is emitted by S_2 towards S_1 when the velocity of the free-falling frame K_0 relative to S_2 and S_1 is still equal to zero. The ray of light reaches S_1 after a time nearly equal to d/c, where c is the speed of light. According to the principle of equivalence, this situation is physically equivalent to one in which K_0 is at rest, and S_2 and S_1 accelerate with acceleration $-\mathbf{a}$ and initial velocity equal to zero. When the ray of light arrives at S_1, the velocity of S_1 relative to the stationary frame K_0 is equal to $v = ad/c$. Therefore, *in the view of any observer in frame* K_0, the ray of light received at S_1 is Doppler-shifted and has a frequency ν_1 as follows:

$$\nu_1 = \nu_2 \left(1 + \frac{v}{c}\right) = \nu_2 \left(1 + \frac{ad}{c^2}\right), \tag{1}$$

where $\nu_1 = \nu_2(1 + v/c)$ is the Doppler formula for $v \ll c$.

For ad, Einstein substituted the gravitational potential Φ of S_2, that of S_1 taken as zero, and assumed that relation (1),

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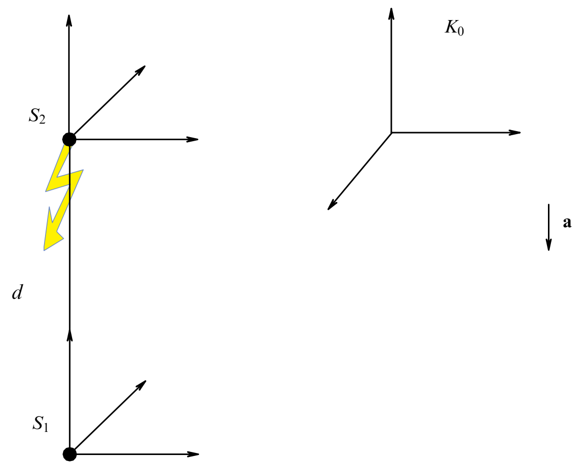

Figure 1. Material systems S_1 and S_2 are stationary in a local, uniform gravitational field **a**. Reference frame K_0 is a free-falling (gravitation-free) system located near S_2 with zero initial velocity relative to S_2. According to equivalence principle, this is equivalent to systems S_1 and S_2 accelerating upward with acceleration $-\mathbf{a}$ and frame K_0 being inertial and at rest.

deduced for a homogeneous gravitational field, would also hold for other forms of the field. Then, Einstein arrived at the well-known (approximated) formula for the gravitational redshift (in this example, it is actually a blueshift):

$$\nu_1 = \nu_2 \left(1 + \frac{\Phi}{c^2}\right). \qquad (2)$$

From this formula, Einstein also derived the gravitational time dilation formula. Suppose that, during the time interval Δt_2 (as measured by a clock at rest at S_2), S_2 emits n waves. Then, from the definition of frequency, we have $n = \nu_2 \Delta t_2$. Let S_1 receive these same n waves during the time interval Δt_1 (as measured by a clock at rest at S_1). Then, again, according to the definition of frequency, we have $n = \nu_1 \Delta t_1 = \nu_2 \Delta t_2$. Hence, equation (2) leads to the gravitational time dilation formula

$$\Delta t_2 = \Delta t_1 \left(1 + \frac{\Phi}{c^2}\right). \qquad (3)$$

In Section 2 of the 1911 paper, Einstein showed that, in general, energy is affected by a gravitational field and that, like the inertial mass, the gravitational mass of a body increases by E/c^2 when the body absorbs an amount of energy equal to E. In that derivation, the setup is the same as in Fig. 1. Einstein used the approximated relativistic energy transformation law $E_1 = E_2(1 + v/c)$ [3] and, again, the equivalence principle. Moreover, by devising a clever thought experiment, he proved the following:

"[...] hence, energy must possess a *gravitational* mass which is equal to its *inertial* mass. If a mass M_0 is suspended from a spring balance in the system K' [the system moving with acceleration $-a$], the balance will indicate the apparent weight $M_0 a$ because of the inertia of M_0. If the quantity of energy E is transferred to M_0, the spring balance will indicate $(M_0 + E/c^2)a$, in accordance with the principle of the inertia of energy. According to our basic assumption [the principle of equivalence], exactly the same thing must happen if the experiment is repeated in the system K, i.e., in the gravitational field [emphasis in the original]."

It cannot escape us that the discovery that energy is affected by a gravitational field, together with the Planck–Einstein relation for the energy of a photon of frequency ν, $E = h\nu$, and some algebra, again gives relation (2) for the gravitational frequency shift. However, although Einstein's discovery of mass and energy dependence on gravitation is a crucial assumption, and his idealized experiment inspired the subsequent thought experiments analyzed in this paper, our last interpretation of that discovery does not strictly count as a derivation of the gravitational frequency shift from energy conservation. In fact, it is still a derivation from special relativity and the principle of equivalence.

Instead, the typical (archetypal) derivations from energy conservation can be found, for instance, in the books by Born [4], Misner, Thorne, and Wheeler [5], Weinberg [6], Feynman, Leighton, and Sands [7], Rindler [8], and Schutz [9] (and many minor physics textbooks).

In the following section, we first explicitly list all the assumptions necessary for the derivation of the gravitational frequency shift from energy conservation and outline the typical thought experiment widely used in literature, which extends those assumptions to the photon. In Section 2.1, we briefly discuss the criticism of this kind of derivation advanced, among others, by Weinberg [6] and Okun, Selivanov, and Telegdi [10, 11]. We agree with their reservations in applying the mentioned assumptions to the photon.

However, in Sections 3 to 5, we recall that a derivation from energy conservation that does not require these assumptions for the photon is possible. As representatives of such derivations, we analyze in more detail the thought experiments presented in Misner, Thorne, and Wheeler [5], Weinberg [6], and Feynman, Leighton, and Sands [7]. We then show that, when revised and corrected, they, too, appear unfortunately troublesome for the very existence of the gravitational frequency shift (Sections 3.1, 4.1, and 5.1).

That early attempts at a clear derivation of the gravitational redshift were fraught with errors and ambiguities is not new, as pointed out by Scott [13], but here we can say more. In Section 6, we prove that, in fact, energy conservation does not imply a gravitational frequency shift. We do that with a simple thought experiment that does not need to assume any gravitational mass or gravitational potential energy of the photon. In that proof, we do not even need to assume that the gravitational potential energy of a body contributes to the total mass of the body. In Appendix A, we also present a more direct thought experiment that uses the conservation of linear momentum and arrives at the same conclusion.

In Section 7, we briefly recap the significance of our results.

Before we proceed, we would like to reiterate the scope of this paper and outline what it does and does not address. The primary objective is to illustrate the inherent difficulties in deriving gravitational redshift using simple energy conservation arguments without fully invoking general relativity, as is commonly done in many introductory textbooks. Despite these difficulties, gravitational redshift is a well-established phenomenon. We believe that the paradoxes encountered in deriving redshift from energy conservation can be resolved through a comprehensive treatment within the framework of general relativity. For instance, throughout the entire article, we assume the classical principle of energy conservation, and, in all of the thought experiments involving photons (as is standard in such derivations in the literature), no distinction is made between proper frequency and coordinate frequency, as is done in general relativity. While this distinction could be crucial for solving the paradoxes we will discuss, it is beyond the scope of the present work.

2. Gravitational frequency shift and 'gravitational potential energy' of a photon

The derivation of the gravitational frequency shift from energy conservation is generally seen as a confirmation of that phenomenon as an alternative to (and independent of) the classical derivation from special relativity and the principle of equivalence, and it can be found in many textbooks on general relativity.

Let us first list all the premises and commonly held beliefs explicitly or tacitly assumed in the derivation from energy conservation. They are crucial for the acceptance of its physical validity:

1. Not only can mass be converted into energy, but *every kind of energy* has mass as well (or can always be converted into mass) via the mass-energy equivalence formula $E = mc^2$, where m is the rest mass [1, 2].
2. Inertial mass is equivalent to gravitational mass.
3. The energy of a light photon with frequency ν is $E = h\nu$, where h is Planck's constant;
4. The principle of conservation of energy.

One example of a derivation that uses all the previous assumptions is the following (an 'infinitesimal' version of it can be found, for instance, in the book by Rindler [8]). A receiver $\mathcal{R}$ is placed straight above an emitter of photons $\mathcal{E}$ at a distance d. Both of them are stationary in a uniform gravitational field $\mathbf{g}$. The emitter $\mathcal{E}$ releases a photon of frequency ν, and energy $E = h\nu$, towards $\mathcal{R}$. Although photons do not have rest mass, for the sake of derivation, it is assumed that the emitted photon has an 'effective' gravitational mass m equal to its inertial mass obtained from the mass-energy equivalence, $m = E/c^2 = h\nu/c^2$ (assumptions 1, 2, and 3). Since the emitted photon needs to climb a height d in the uniform gravitational field, its energy E' at the receiver $\mathcal{R}$ is lower than E. Due to the conservation of energy (assumption 4), we necessarily have

$$E' = E - mgd\,, \tag{4}$$

where the potential energy mgd is the energy 'spent' by the photon in climbing the distance d.

Equation (4) can be rewritten as follows:

$$\nu' = \frac{E'}{h} = \frac{E - mgd}{h} = \frac{h\nu - (h\nu/c^2)gd}{h} = \nu\left(1 - \frac{gd}{c^2}\right), \tag{5}$$

which is the sought-after gravitational frequency shift formula (1) (if the positions of $\mathcal{E}$ and $\mathcal{R}$ are reversed, the minus sign becomes a plus sign in the equation).

2.1 Shared criticism of 'photon gravitational potential energy' approach

To the best of this author's knowledge, the previous type of derivation received little criticism. There are some notable exceptions, though, like Weinberg, who affirms that the concept of gravitational potential energy for a photon is without foundation [6]. Similarly, Okun, Selivanov, and Telegdi argue that any explanation of the gravitational frequency shift in terms of gravitational mass and gravitational potential energy of the photon is incorrect and misleading [10, 11].

We fully agree with these authors, since the photon has no rest mass, and the appeal to its gravitational mass and gravitational potential energy is a weak and illegitimate argument.

3. Misner's, Thorne's, and Wheeler's derivation

It is possible to come up with a gravitational frequency shift derivation from energy conservation that does not appeal to the concept of gravitational potential energy or the gravitational mass of the photon. We refer to, for instance, the derivations in Misner, Thorne, and Wheeler [5], Weinberg [6], Feynman, Leighton, and Sands [7], Schutz [9], Koks [12], and Earman and Glymour [14].

Let us start with the thought experiment by Misner, Thorne, and Wheeler. They recount Einstein's 1911 realization of the interaction between light and gravity as follows (the speed of light is set as $c = 1$):

"That a photon must be affected by a gravitational field Einstein (1911) showed from the law of conservation of energy, applied in the context of Newtonian gravitation theory. Let a particle of rest mass m start from rest in a gravitational field g at point $\mathcal{A}$ and fall freely for a distance h to point $\mathcal{B}$. It gains kinetic energy mgh. Its total energy, including rest mass, becomes

$$m + mgh\,.$$

Now, let the particle undergo an annihilation at $\mathcal{B}$, converting its total rest mass plus kinetic energy into a photon of the same energy. Let this photon travel upward in the gravitational field to $\mathcal{A}$. If it does not interact with gravity, it will have its original energy on arrival at $\mathcal{A}$. At this point it could be converted by a suitable apparatus into another particle of rest mass m (which could then repeat the whole process) plus an excess energy mgh that costs nothing to produce. To avoid this contradiction of the principal [*sic*] of conservation of energy, which can also be stated in purely classical terms, Einstein saw that the photon must suffer a red shift."

In this derivation, as in those considered in the following sections, no reference is made to the gravitational mass or gravitational potential energy of the photon. Energy has a mass only after absorption by a nonrelativistic and macroscopic material body (the apparatus that converts it into a particle in the last part of the process). That is allowed by the widely-held interpretation of the mass-energy equivalence.

3.1 Main issues with Misner's, Thorne's, and Wheeler's derivation

Unfortunately, Misner's, Thorne's, and Wheeler's argument is problematic [25]. If a particle of rest mass m starts from rest in a gravitational field $\mathbf{g}$ at point $\mathcal{A}$ and falls freely for a distance h to point $\mathcal{B}$, that particle also possesses an energy equal to mgh already at point $\mathcal{A}$. It is called gravitational potential energy. Therefore, owing to mass-energy equivalence (assumption 1), at point $\mathcal{A}$, that particle already has a total mass/energy equal to $m + mgh$. Heuristic proof of this last statement is given in Appendix A. Now, if the energy of the photon produced in the particle annihilation at point $\mathcal{B}$ and traveling upward does not have its original value on arrival at $\mathcal{A}$ (i.e., $m + mgh$), the mass of the particle created by the suitable apparatus at the end of the process will not have the same mass as the original particle (again, $m + mgh$), and the total energy/mass will not be conserved. When Misner, Thorne, and Wheeler say that the particle "gains kinetic energy *mgh*" on arrival at point $\mathcal{B}$, and "its total energy, including rest mass, becomes $m + mgh$," they seem to forget that the particle already has gravitational potential energy *mgh* and total energy $m + mgh$ just before starting to fall. That is demanded by the principle of conservation of energy. The same analysis, with a few adjustments, also applies to the derivations in Schutz [9] and Koks [12], leading to the same conclusion.

Even if Misner, Thorne, and Wheeler do not explicitly mention the Planck–Einstein relation $E = h\nu$, the fact that the energy can be converted into a single photon or a finite (and definite) number of photons is a tacit but important further assumption. For if it were possible to convert energy into light in a 'continuous' way, the conservation of energy could still be re-established: in principle, if the emitter at point $\mathcal{B}$ continuously emitted higher frequency radiation (higher intrinsic energy) for an interval Δt and the receiver at point $\mathcal{A}$ continuously received lower frequency radiation (lower intrinsic energy) for a suitably longer interval $\Delta t' > \Delta t$, the total amount of energy could still be conserved (and this would also bring gravitational time dilation back into the picture). However, the quantization of energy in light transmission has strong theoretical and experimental evidence.

4. Weinberg's derivation

Weinberg [6] presented a derivation from energy conservation slightly different from that given in Misner, Thorne, and Wheeler [5] but equally problematic. It could be of some interest to go into detail. Weinberg writes (again, the speed of light is set as $c = 1$):

"Incidentally, the gravitational red shift of light rising from a lower to a higher gravitational potential can to some extent be understood as a consequence of quantum theory, energy conservation, and the 'weak' Principle of Equivalence. When a photon is produced at point 1 by some heavy nonrelativistic apparatus, an observer in a locally inertial coordinate system moving with the apparatus will see its internal energy and hence its inertial mass change by an amount related to the photon frequency ν_1 he observes, that is, by

$$\Delta m_1 = -h\nu_1 \,,$$

where $h = 6.625 \times 10^{-27}$ erg s is Planck's constant. Suppose that the photon is then absorbed at point 2 by a second heavy apparatus; an observer in a freely falling system will see the apparatus change in inertial mass by an amount related to the photon frequency ν_2 he observes, that is, by

$$\Delta m_2 = h\nu_2 \,.$$

However, the total internal plus gravitational potential energy of the two pieces of apparatus must be the same before and after these events, so

$$0 = \Delta m_1 + \phi_1 \Delta m_1 + \Delta m_2 + \phi_2 \Delta m_2 \,,$$

and therefore

$$\frac{\nu_2}{\nu_1} = \frac{1+\phi_1}{1+\phi_2} \simeq 1 + \phi_1 - \phi_2$$

in agreement with our previous result. (Also, it makes no difference whether the photon frequencies are measured in locally inertial systems, because the gravitational field in any other frame will affect the rate of the observer's standard clock in the same way as it affects the ν's.)"

4.1 Main issues with Weinberg's derivation

Leaving aside the reference to the free-falling observer who will necessarily see a Doppler shift due to the motion relative to the stationary emitting apparatus, a thing that, in the humble opinion of this author, unnecessarily complicates the picture, Weinberg's derivation seems to violate the conservation of energy from the very beginning. First, he states that, upon photon emission, the apparatus will change its *internal* energy by an amount $h\nu_1 = |\Delta m_1|$. But then, he says that the variation in the *total energy* of the apparatus to consider upon emission is $|\Delta m_1 + \phi_1 \Delta m_1|$. Namely, the apparatus emits energy equal to $|\Delta m_1|$, but its total energy variation is $|\Delta m_1 + \phi_1 \Delta m_1| \neq |\Delta m_1|$. That already represents a violation of energy conservation. If we reestablish the conservation of energy ($|\Delta m_{1/2} + \phi_{1/2} \Delta m_{1/2}| = |h\nu_{1/2}|$), no gravitational frequency shift is implied.

5. Feynman's, Leighton's, and Sands's derivation

A further application of energy conservation is present in the derivation by Feynman, Leighton, and Sands [7]. They write:

"We know that the gravitational force on an object is proportional to its mass M, which is related to its total internal energy E by $M = E/c^2$. For instance, the masses of nuclei determined from the energies of nuclear reactions which transmute one nucleus into another agree with the masses obtained from atomic weights.

Now think of an atom which has a lowest energy state of total energy E_0 and a higher energy state E_1, and which can go from the state E_1 to the state E_0 by emitting light. The frequency ω of the light will be given by

$$\hbar\omega = E_1 - E_0 \,. \tag{42.7}$$

Now suppose we have such an atom in the state E_1 sitting on the floor, and we carry it from the floor to the height H. To do that we must do some work in carrying the mass $m_1 = E_1/c^2$ up against the gravitational force. The amount of work done is

$$\frac{E_1}{c^2}\, gH \,. \tag{42.8}$$

Then we let the atom emit a photon and go into the lower energy state E_0. Afterward we carry the atom back to the floor. On the return trip the mass is E_0/c^2; we get back the energy

$$\frac{E_0}{c^2}\, gH \,, \tag{42.9}$$

so we have done a net amount of work equal to

$$\Delta U = \frac{E_1 - E_0}{c^2}\, gH \,. \tag{42.10}$$

When the atom emitted the photon it gave up the energy $E_1 - E_0$. Now suppose that the photon happened to go down to the floor and be absorbed. How much energy would it deliver there? You might at first think that it would deliver just the energy $E_1 - E_0$. But that can't be right if energy is conserved, as you can see from the following argument. We started with the energy E_1 at the floor. When we finish, the energy at the floor level is the energy E_0 of the atom in its lower state plus the energy E_{ph} received from the photon. In the meantime we have had to supply the additional energy ΔU of Eqn (42.10). If energy is conserved, the energy we end up with at the floor must be greater than we started with by just the work we have done. Namely, we must have that

$$E_{ph} + E_0 = E_1 + \Delta U ,$$

or

$$E_{ph} = (E_1 - E_0) + \Delta U . \qquad (42.11)$$

It must be that the photon does not arrive at the floor with just the energy $E_1 - E_0$ it started with, but with a *little more energy*. Otherwise some energy would have been lost. If we substitute in Eqn (42.11) the ΔU we got in Eqn (42.10) we get that the photon arrives at the floor with the energy

$$E_{ph} = (E_1 - E_0)\left(1 + \frac{gH}{c^2}\right) . \qquad (42.12)$$

But a photon of energy E_{ph} has the frequency $\omega = E_{ph}/\hbar$. Calling the frequency of the emitted photon ω_0 — which is by Eqn (42.7) equal to $(E_1 - E_0)/\hbar$ — our result in Eqn (42.12) gives again the relation of (42.5) between the frequency of the photon when it is absorbed on the floor and the frequency with which it was emitted."

5.1 Main issues with Feynman's, Leighton's, and Sands's derivation

The weak link in Feynman's, Leighton's, and Sands's chain of inference appears to be what is written between equations (42.8) and (42.9), including the latter. The total energy of the atom sitting on the floor is E_1. After being carried to the height H, its total energy becomes $E_1 + (E_1/c^2)gH$ (its rest energy plus the work done on the atom). With the emission of a photon of energy $\hbar\omega = E_1 - E_0$, the total energy changes to

$$\left(E_1 + \frac{E_1}{c^2} gH\right) - (E_1 - E_0) = E_0 + \frac{E_1}{c^2}\, gH .$$

According to the conservation of energy, that total energy must be conserved after the atom is carried back to the floor. Now, if we subtract the new rest energy E_0 of the atom from this total energy, we get back the correct energy, $(E_1/c^2)gH$, and the net amount of work we have done is $\Delta U = 0$. Therefore, according to equation (42.11), the photon must arrive at the floor with just the energy $E_1 - E_0$ it started with at the height H.

6. Proof that energy conservation does not imply gravitational redshift

Here, we prove that energy conservation does not imply gravitational frequency shift. We do this with a simple thought experiment that does not need to assume any gravitational mass or gravitational potential energy of the photon. This thought experiment has already been applied to sound waves to show they can escape any gravity well [16]. In this proof, we do not even need to assume that the gravitational potential energy of a body contributes to the total mass of the body (assumption 1), as we have done in the revision of some previous demonstrations. However, in the Appendix, we show that this must always be the case.

Consider a body of mass m stationary at point $\mathcal{B}$ and a macroscopic apparatus stationary at point $\mathcal{A}$, which is at a height h above point $\mathcal{B}$ in a gravitational field $\mathbf{g}$ (Fig. 2). Let the apparatus perform the following cycle. The first step consists of the apparatus emitting a photon of energy E' (frequency ν') that is suitably lower than mgh. The original energy E' is such that, when the photon arrives at a receiver at the bottom, it becomes equal to $E_b = mgh$ (where $E_b > E'$) due to the standard gravitational redshift (blueshift in this case). In this way, E_b is exactly what is needed to raise the mass m to the apparatus at height h. Then, the mass is released back to the initial position, and the energy from that release (mgh) goes into the apparatus's reservoir. At the end of the cycle, the apparatus will gain positive energy ($mgh - E' > 0$) out of nowhere. To resolve this inconsistency, energy E' must be equal to mgh, implying that ν' and ν should always be equal to each other.

To emphasize the above conclusion, consider the cycle in reverse. Let the apparatus perform mechanical work on body m, raising it to point $\mathcal{A}$. The work done by the apparatus is equal to mgh, which is also equal to the gravitational potential energy of the body m relative to point $\mathcal{B}$. Now, if the mass is lowered back to point $\mathcal{B}$ and its potential energy conventionally (and entirely) converted into a single photon of energy mgh (ultimately emitted by the receiver, now transformed into a beacon), the energy of the photon must always be the same while climbing up the gravitational field back to point $\mathcal{A}$. The photon energy at point $\mathcal{A}$ must still be equal to mgh. That is demanded by the conservation of energy. Through photon absorption, the apparatus must regain the same energy expended at the beginning of the cycle on m. Therefore, owing to the Planck–Einstein relation, the photon frequency must be the same at points $\mathcal{A}$ and $\mathcal{B}$.

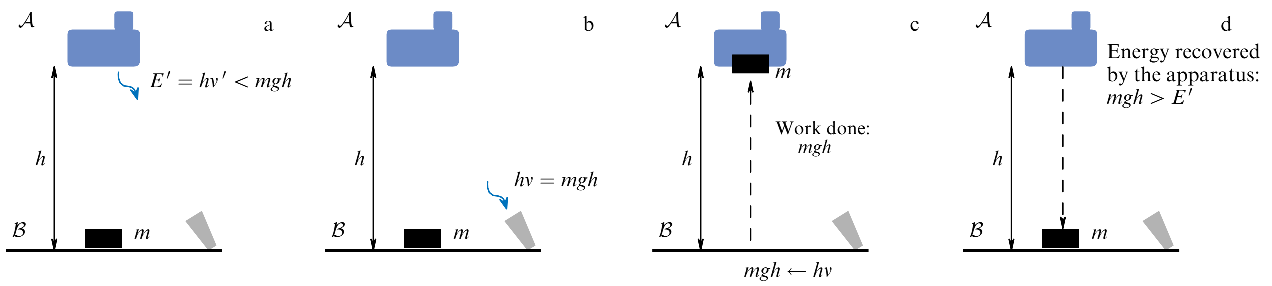

Figure 2. Pictorial representation of thought experiment described in Section 6. In this case, mass-energy equivalence is not needed.

7. Concluding remarks

Thought experiments are a fascinating aspect of physics that can serve as powerful heuristic tools. However, it is important to approach them with caution, as we have seen in the derivation of the gravitational frequency shift from energy conservation. Sometimes, these experiments can be used as a rhetorical device to uphold our pre-existing beliefs or what we perceive to be reasonable.

Our analysis and revision of, in particular, Misner's, Thorne's, and Wheeler's, Weinberg's, and Feynman's, Leighton's, and Sands's thought experiments but, above all, the proofs given in Section 6 and the Appendix ultimately pit the existence of the gravitational frequency shift or, better yet, the dependence of the photon energy on gravitation against the conservation of energy (and linear momentum).

However, in light of the well-known and widely accepted experimental proofs of gravitational redshift (e.g., the oft-quoted results by Pound and collaborators [17, 18]), it is hard to believe that what we have derived in the present paper will be seen as a refutation of the phenomenon. On the contrary, this strengthens the fact that gravitational redshift is a peculiar general relativistic effect, and we need general relativity to consistently derive it (or, at least, special relativity plus the equivalence principle, as Einstein did in 1911). Nevertheless, we believe that our findings can serve as a cautionary tale regarding the interpretation of thought experiments. As such, they may have some epistemological interest and practical utility.

Acknowledgments

Although not always in agreement with them, I am indebted to Nils Erik Bomark, Andrea Erdas, Espen Gaarder Haug, and Gianfranco Spavieri for stimulating and fruitful discussions on the topic. I would like to express my appreciation to the anonymous reviewer whose insightful and constructive suggestions helped me refine my work.

8. Appendix. Conservation of energy and linear momentum

Here, we provide a heuristic proof that the gravitational potential energy of a body contributes to the total mass of the body.

Consider the following ideal experiment. A closed wagon of mass M moves horizontally without friction in a vertical uniform gravitational field $\mathbf{g}$ at a constant velocity v (Fig. 3). Inside the wagon, attached to floor $\mathcal{B}$, there is a particle of mass $m_{\mathcal{B}}$. At a specific moment, mass $m_{\mathcal{B}}$ annihilates into a photon of energy

$$h\nu_{\mathcal{B}} = m_{\mathcal{B}} c^2 \qquad (6)$$

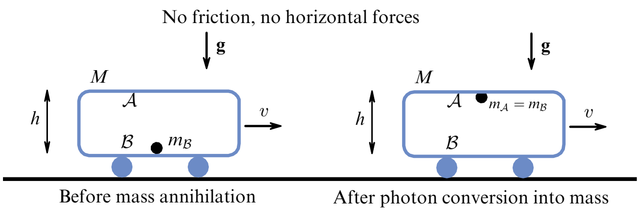

Figure 3. Pictorial representation of thought experiment described in Appendix.

(owing to the Planck–Einstein relation $E = h\nu$, where h is the Planck constant).

Then, the photon travels upward toward ceiling $\mathcal{A}$ at a height h and is absorbed and converted by a suitable apparatus into another particle of mass $m_{\mathcal{A}}$. This particle also ends up stuck to the wagon frame. The whole process happens exclusively inside the closed wagon. Owing to the conservation of energy, we must have

$$h\nu_{\mathcal{B}} = m_{\mathcal{A}} c^2 + m_{\mathcal{A}} g h \,, \qquad (7)$$

namely, the initial energy is equal to the rest energy of the new particle, $m_{\mathcal{A}} c^2$, plus the potential energy of that particle at the height h in the gravitational field $\mathbf{g}$ relative to floor $\mathcal{B}$.

According to the common understanding today (e.g., [19]), the mass of the generated particle at point $\mathcal{A}$ does not include the equivalent mass of its gravitational potential energy $m_{\mathcal{A}} g h / c^2$ and then $m_{\mathcal{A}} < m_{\mathcal{B}}$.

In reality, we shall show that the total mass of the particle generated at point $\mathcal{A}$, $m_{\mathrm{tot}\mathcal{A}}$, must be equal to $m_{\mathcal{B}}$, namely, using equations (6) and (7),

$$m_{\mathrm{tot}\mathcal{A}} = m_{\mathcal{B}} = \frac{h\nu_{\mathcal{B}}}{c^2} = m_{\mathcal{A}} + \frac{m_{\mathcal{A}} g h}{c^2} \,, \qquad (8)$$

and, therefore, the total mass of the particle generated at point $\mathcal{A}$ *must include the equivalent mass of its gravitational potential energy* $m_{\mathcal{A}} g h / c^2$.

Any different scenario seems to violate the conservation of (the horizontal) linear momentum of the closed system wagon + particle. No horizontal external forces act upon the system, and no mass is ejected. Therefore, the total velocity v must be the same before and after the entire process. However, if the mass at point $\mathcal{A}$ is less than the mass at point $\mathcal{B}$, $m_{\mathcal{A}} < m_{\mathcal{B}}$, the total horizontal linear momentum before the annihilation is

$$P_{\mathrm{i}} = (M + m_{\mathcal{B}}) v \,, \qquad (9)$$

while, after the conversion of the photon energy into mass, the total horizontal linear momentum becomes

$$P_{\mathrm{f}} = (M + m_{\mathcal{A}}) v < P_{\mathrm{i}} \,. \qquad (10)$$

This is quite bizarre. On the other hand, by imposing the conservation of the horizontal linear momentum even with $m_{\mathcal{A}} < m_{\mathcal{B}}$, we would have an equally strange consequence. Without any horizontal external force acting upon the wagon and without any mass ejection, we would see the wagon increase its velocity by itself at the end of the whole process.

It is worth noting that the above argument confirms that there is no gravitational redshift with energy conservation: if the total mass of the particle generated at point $\mathcal{A}$ is still $m_{\mathcal{B}}$, the energy of the photon from which it originates is $m_{\mathcal{B}} c^2 = h\nu_{\mathcal{B}}$, namely, the frequency of the photon at point $\mathcal{A}$ is the same as that at point $\mathcal{B}$, $\nu_{\mathcal{A}} = \nu_{\mathcal{B}}$.

At this point, it is possible to derive the exact expressions of the total mass and energy of a body in a uniform gravitational field $\mathbf{g}$ after external work is done on that body. When an agent, external to the system body-gravitational field, raises mass m by a distance $\mathrm{d}h$, the infinitesimal work performed on the body is $mg\,\mathrm{d}h$. Then, according to the result of this Appendix, that energy is stored in the body's mass, which increases by

$$\mathrm{d}m = \frac{mg\,\mathrm{d}h}{c^2} \,. \qquad (11)$$

By integrating differential equation (11) and applying appropriate boundary conditions, the mass m_h of the body at height h is $m_h = me^{gh/c^2}$, where m is the mass at height zero. The total energy $E_{\rm tot}$, proper mass plus gravitational potential energy, at height h is then given by $E_{\rm tot} = m_h c^2 = mc^2 e^{gh/c^2}$. For small distances h, we have $m_h \approx m + mgh/c^2$ and $E_{\rm tot} \approx mc^2 + mgh$. A similar result is also present in [10, 11]. Therefore, if we set $c = 1$, we recover the classical (and special-relativistic) expressions of total energy and mass of a stationary body in a gravitational field used in our review of Misner's, Thorne's, and Wheeler's argument in Section 3.

It may be helpful to notice that the formula $E_{\rm tot} = mc^2 e^{gh/c^2}$ for the total energy, mass plus gravitational potential energy, of a steady body in a gravitational field can be considered the static analogue of the formula

$$E = m_0 c^2 \sqrt{1 + \frac{p^2}{m_0^2 c^2}}$$

for the total energy, proper mass plus kinetic energy, of a free particle [20]. In that last relation, m_0 is the proper mass of the free particle, and p is the magnitude of its linear momentum. If the velocity v of the particle is much less than the speed of light, $v \ll c$, the total energy approximates to $E \approx m_0 c^2 + (1/2) m_0 v^2$, much like the previous relation $E_{\rm tot} \approx mc^2 + mgh$ is the approximation of the total energy of a static mass at a height h in a weak gravitational field ($gh/c^2 \ll 1$).

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