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ON THE 90TH ANNIVERSARY OF THE P.N. LEBEDEV PHYSICAL INSTITUTE (LPI)

Physics of strong interactions at high energies

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Contents

Abstract. **In 2024, we celebrate the 90th anniversary of the Lebedev Physical Institute of the USSR/Russian Academy of Sciences (LPI). One of the most important domains of scientific research at the LPI has been theoretical physics, undertaken in different areas, in the Theoretical Physics Department. In the present review, ideas on the physics of strong interactions developed at the Sector/Laboratory of High Energy Physics in the Theoretical Physics Department are discussed.**

Keywords: high energy physics, strong interactions, quantum chromodynamics

1. Introduction

One of the most important research avenues of the P N Lebedev Physical Institute of the Russian Academy of Sciences, celebrating in 2024 its 90th anniversary, is theoretical physics. A significant part of this research is conducted, in various areas, in the LPI Theoretical Physics Department. The present review is devoted to discussing the main results obtained in the domain of the physics of strong interactions in the Sector/Laboratory of High Energy Physics of the Theoretical Physics Department. Chronologically, this review covers the period from the 1960s to the 2020s and, therefore, all the phases of development of strong interaction physics at high energies, from hadronic degrees of freedom to gauge theory of strong interactions.

A great deal of attention is paid in the review to the physics of multiparticle production. Its study began in the period when 'multiparticle production' referred to processes in which several particles were born and continues nowadays with multiplicities of several thousand particles in nuclear collisions at the Large Hadron Collider (LHC). From the fundamental point of view, the development of a consistent theory of multiparticle production presents an extraordinary theoretical challenge. Indeed, a description of the transformation of an initial state with zero entropy into a final one with a tremendously large entropy requires development of fundamental quantum field theory methods enabling a consistent description of the processes of the birth, evolution, and formation of final hadron flow in the strongly interacting configuration of quantum fields generated in high energy collisions. This challenge is clearly understood at all the stages of research continuing today.

An important feature of research on high energy physics conducted at the laboratory was its explicit linkage to experimental results that to a large extent determined its direction.

For decades, the results of research in the high energy physics of strong interactions conducted at the laboratory were reflected in the reviews published by its staff in *Soviet Physics–Uspekhi*. Reviews [1 – 20][1] reflect all the phases of theory development from the 1960s to the 2020s.

2. Physics of strong interactions in high energy collisions

2.1 Physics of high energy hadron collisions in pre-quark epoch

Development of a theory of high energy scattering of hadrons and nuclei and, in particular, of multiparticle production processes in these collisions was the focus of research at the laboratory starting in the 1960s. Let us begin with a prophetic formulation of the main principles of describing multiparticle

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[1] Even this extensive list is incomplete.

production processes in the earliest available accessible source, the rapporteur talk of E L Feinberg [21]:

• in a description of multiparticle production processes, the principles of local quantum field theory hold true at arbitrarily small spatial scales;

• high energy collisions lead to excitation of a large number of degrees of freedom ('boiling operator liquid'), allowing a quasiclassical description;

• particle interaction is so strong that local thermodynamic equilibrium is reached.

Nowadays, after more than half a century, despite a paradigm shift in the physics of strong interactions from hadron-centric to quark-gluon, the first two statements are still believed to be true, while the third one remains a working hypothesis used in a large number of studies modeling multiparticle production processes. Realization of these principles has played a decisive role in forming a trajectory of research in hadronic and nuclear high energy physics at the laboratory since its inception.

The quantum field theory description of processes of scattering of hadrons and nuclei is, as a rule, built with the help of a diagram technique using, in particular, effective, not reducible to elementary, degrees of freedom responsible for the t-channel exchange.

In the pre-quark epoch, the role of elementary degrees of freedom was played by hadrons, and that of effective t-channel degrees of freedom, by pions (later generalized to reggeons) and clusters/fireballs.

For a field-theoretic description of peripheral hadron interactions, a model of one-pion exchange was first proposed in [22]. This model played a crucial role in the development of the multiperipheral model of multiparticle production.

The multiperipheral model of multiparticle production underwent significant development based on using clusters as fundamental degrees of freedom in the t-channel exchange responsible for particle production in which a fundamental role was also played by the Bethe–Solpeter equation [2, 24, 25]. In particular, in [2], a detailed discussion of the equivalence of cluster and reggeon formalisms in describing inelastic cross sections was presented. Important results were also obtained in [23], in which a detailed analysis of the properties of reggeon trajectories within a description of cross sections using the Bethe–Solpeter equation was performed. The results of these studies were summarized in reviews [4, 26, 27]. Later, diffraction processes were analyzed in the framework of Quantum Chromodynamics (QCD) in [28, 29].

The construction of a comprehensive global self-consistent description of high energy scattering of hadrons and nuclei is impossible without constructing a description of elastic scattering processes. A corresponding theory for scattering at large angles was developed in [30]. The results of early studies and modern progress are described in reviews [18, 19].

In recent years, significant attention has been paid to the physics of ultra-peripheral nuclear collisions. A review of theoretical and experimental results in this field is given in review [20]. The phenomenon of the growth of the dilepton production rate in such collisions was discussed in [31]. In [32], ultra-peripheral collisions were discussed as a possible mechanism of positron creation in the Universe.

A quantitative description of multiparticle production processes requires well-developed methods for describing multiplicity distributions, correlations, and fluctuations. The development of the corresponding formalism based, in particular, on the experience of describing stochastic processes, statistical physics, quantum optics, etc., is described in reviews [9, 11]. Particular attention was paid to events with anomalous distribution in the strong isospin. A possible theoretical description of such anomalous events based on directly taking into account the law of conservation of strong isospin was given in [33]. An exotic mechanism of generation of events anomalous in strong isospin based on the formation of mini black holes was considered in [34].

Let us also mention paper [35], in which significant results on the properties of long-lived hadron cascades were obtained.

2.2 Statistical physics and hydrodynamics in high energy collisions

In the absence of a consistent theory of a strongly interacting quantum field, a natural approximation to use is the strong coupling one. A convenient formalism for the realization of such an approach is consideration of the evolution of the energy-momentum tensor in the framework of the hydrodynamics of an ideal liquid, in which a regime of infinitely strong coupling is realized, augmented by an analysis of the effects of the finite coupling constant, i.e., of viscosity effects.

Equations of relativistic hydrodynamics as applied to a description of heavy ion collisions do, in fact, just reflect the conservation of energy-momentum and baryon current,

$$\partial_\mu (T^{\mu\nu}_{\text{id}} + S^{\mu\nu}) = 0\,, \qquad \partial_\mu J^\mu_{\text{bar}} = 0\,, \tag{1}$$

where $T^{\mu\nu}_{\text{id}}$ and $S^{\mu\nu}$ are ideal (describing an ideal liquid) and viscous components of the energy-momentum tensor. In particular,

$$T^{\mu\nu}_{\text{id}} = (\varepsilon + P) U^\mu U^\nu + P g^{\mu\nu}\,, \tag{2}$$

$$J^\mu_{\text{bar}} = \rho_{\text{bar}} U^\mu\,, \tag{3}$$

where ε and P are energy density and pressure, U^μ is the 4-velocity of a liquid, and ρ_{bar} is the baryon current density. In addition, there is an equation of state

$$P = P(\varepsilon, \rho_{\text{bar}})\,. \tag{4}$$

An important role in selecting a version of hydrodynamical theory for describing nucleus-nucleus high energy collisions is played by specification of the initial state of the system. This question is discussed in [38].

Let us note that a consistent description of viscosity effects in relativistic hydrodynamics faces substantial difficulties, particularly in describing propagation of sound and shock waves.

A key global particular feature of the multiparticle production process in heavy ion collisions is a pronounced imbalance of energy-momentum flows in the transverse and longitudinal directions reflected in a pronounced asymmetry of longitudinal and transverse pressure. In this situation, an application of the hydrodynamical description is naturally realized in the framework of anisotropic relativistic hydrodynamics, in which, instead of (2), one has

$$T^{\mu\nu} = (\varepsilon + P_\perp) U^\mu U^\nu - P_\perp g^{\mu\nu} - (P_\perp - P_\parallel) Z^\mu Z^\nu\,, \tag{5}$$

where $P_\parallel$ and $P_\perp$ are transverse and longitudinal pressure and the 4-vector Z^μ defines the longitudinal direction, so that in the Landau rest frame $Z^\mu = (0, 0, 0, 1)$.

The most developed approach to constructing hydrodynamics using the energy-momentum tensor (5) is that based on kinetic theory, where it is assumed that a distribution function of particles in the matter under consideration has the form

$$f(x,p) = f_{\rm iso}\left(\frac{\sqrt{p^{\mu}\Xi_{\mu\nu}p^{\nu}}}{\Lambda(x)}\right), \tag{6}$$

where in the Landau frame $p^{\mu}\Xi_{\mu\nu}p^{\nu} = \mathbf{p}^2 + \xi(x)p_{\|}^2$, ξ is an anisotropy parameter that, generically, depends on the coordinates x, and $\Lambda(x)$ is the effective temperature of the medium. Using (6), one gets the following expressions for the components of the energy-momentum tensor:

$$\begin{aligned} &\varepsilon(\Lambda,\xi) = R(\xi)\varepsilon_{\rm iso}(\Lambda)\,, \qquad n(\Lambda,\xi) = \frac{n_{\rm iso}(\Lambda)}{\sqrt{1+\xi}}\,, \\ &P_{\|}(\Lambda,\xi) = R_{\|}(\xi)P_{\rm iso}(\Lambda)\,, \qquad P_{\perp}(\Lambda,\xi) = R_{\perp}(\xi)P_{\rm iso}(\Lambda)\,, \end{aligned} \tag{7}$$

where $R(\xi)$ and $R_{\|,\perp}(\xi)$ are analytically computable functions of ξ.

The presence of pressure anisotropy leads to anisotropy in the propagation of sound, i.e., to the appearance of transverse $c_{s\|}$ and longitudinal $c_{s\perp}$ speeds of sound. The corresponding sound equation describing the evolution of density fluctuations δn reads [41]

$$\partial_t^2\delta n = \left(\frac{R_{\perp}}{R}\,\partial_{\perp}^2 + \frac{R_{\|}}{R}\,\partial_{\|}^2\right)\delta n \;\leftrightarrow\; c_{s\perp}^2 = \frac{R_{\perp}}{R}\,, \quad c_{s\|}^2 = \frac{R_{\|}}{R}\,. \tag{8}$$

The effects of anisotropy in sound propagation are reflected, in particular, in the asymmetry of the Mach cone [41] and the dependence of the properties of shock waves on the direction of their propagation [39].

Because of the realization in an ideal liquid of the regime of infinitely strong coupling, the usage of the Boltzmann distribution containing an effective temperature and assuming additivity in energy for in-matter particles is a hypothesis that is hard to defend. In this respect, it is natural to consider a generalization of anisotropic hydrodynamics to a non-additive case in which the Boltzmann distribution is replaced by the Tsallis one. Such a generalization was considered in [40] where, notably, the effects of non-additivity on the evolution of entropy were considered.

2.3 Physics of quark-gluon jets

The main component of gauge theory of strong interactions, quantum chromodynamics is a QCD-parton model describing the quark-gluon structure of hadrons as well as that of matter created in high energy collisions. The QCD parton model arises as a result of analysis of QCD equations in light-cone gauges in which the process of parton creation can be given a description in terms of a stochastic branching process. A description of QCD as a theory of strong interactions and, in particular, in application to a description of multiparticle production, is given in book [47].

Let us consider for definiteness the gluon distribution function $g(x,p_{\perp}^2)$ describing the probability of observing a gluon with a fraction of longitudinal momentum $x = p_{\|}/p_{\|h}$ on the transverse momentum scale $p_{\perp}^2$. In the leading logarithmic approximation in $p_{\perp}^2$, the evolution of gluon distribution is described by the Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) equation [44–46] [2]

$$\frac{\partial g(x,p_{\perp}^2)}{\partial \ln p_{\perp}^2} = \int_x^1 \frac{{\rm d}z}{z}\,p^{gg}\left(\frac{x}{z}\right)g(z,p_{\perp}^2) - g(x,p_{\perp}^2)\int_0^x \frac{{\rm d}z}{z}\,p^{gg}\left(\frac{z}{x}\right), \tag{9}$$

where the kernel

$$p^{gg}\left(y = \frac{x}{z}\right) = N_{\rm c}\left[\frac{1-y}{y} + \frac{y}{1-y} + y(1-y)\right] \tag{10}$$

reflects the probability of creating with a gluon with the fraction of longitudinal momentum z a gluon with the fraction of longitudinal momentum x. Equation (9) is nothing but a kinetic equation on $g(x,p_{\perp}^2)$ in which evolution in 'time'

$$Y = \frac{1}{2\pi b}\ln\left(\frac{\alpha_s(p_{\perp 0}^2)}{\alpha_s(p_{\perp}^2)}\right), \qquad b = \frac{11N_{\rm c}}{12\pi} \tag{11}$$

is defined by balancing between incoming (radiation) and outgoing (radiation and virtual corrections) probability fluxes.

Let us consider for definiteness the physics of gluon jets. An analogy with branching stochastic process allows us to write down quite generic evolution equations for the generating functional $G(\{u\},Y)$ for multiparticle gluon distributions [42, 47]. In the general case,

$$G(\{u\},Y) = \sum_n \int {\rm d}^3k_1\ldots{\rm d}^3k_n\,u(k_1)\ldots u(k_n)\,p_n(k_1,\ldots,k_n|Y)\,, \tag{12}$$

where $u(k)$ is an auxiliary function and $p_n(k_1,\ldots,k_n|Y)$ is a probability density of creation of particles with the momenta $k_1,\ldots,k_n$ on the (transverse) evolution scale Y. In this formalism, one can derive evolution equations for correlation functions of arbitrary order and ensure the validity of conservation laws [42] as well as that of the effects of coherence of radiation in the modified leading logarithmic approximation. Let us mention that work on taking into account the radiation coherence effects is to a significant extent based on the work of E L Feinberg [5, 6], in which the process of formation of the proper field of particles in the process of scattering in quantum electrodynamics was considered.

Below, we will use a simplified version of (12) for which $u(k) = {\rm const}$, so that expression (12) turns into a generating function for the multiplicity distribution $p_n(Y)$

$$G(u,Y) = \sum_{n=0}^{\infty} u^n p_n(Y)\,. \tag{13}$$

The evolution equation for generating function (13) reads

$$\begin{aligned}\frac{\partial G}{\partial Y} = \frac{2N_{\rm c}\alpha_s}{\pi}\int_0^1 {\rm d}x\,p^{gg}(x)\Big[G\big(Y+\ln(1-x)\big) \\ + G(Y+\ln x) - G(Y)\Big]\,.\end{aligned} \tag{14}$$

Evolution equation (14) allows various characteristics of a multiparticle production process to be studied in terms of the cascade multiplication of gluons [10, 48].

[2] Here and in what follows, we, for simplicity, disregard the contribution of quark degrees of freedom.

From equation (14), there follows, especially, an equation for the mean multiplicity of gluons in a gluon jet $\langle n_{\rm g}(Y)\rangle$

$$\frac{\partial\langle n_{\rm g}(Y)\rangle}{\partial Y} = \frac{2N_{\rm c}\alpha_{\rm s}}{\pi}\int_0^1 {\rm d}x\, p^{gg}(x)\Big[\big\langle n_{\rm g}(Y)(Y+\ln x)\big\rangle + \big\langle n_{\rm g}(Y)\big(Y+\ln(1-x)\big)\big\rangle - \big\langle n_{\rm g}(Y)\big\rangle\Big]\,, \tag{15}$$

which can be easily generalized by taking into account the quark degrees of freedom. The equation allows us, in particular, to analyze the contribution of higher order in the coupling constant to the mean multiplicities of quark and gluon jets [49–51].

Equation (14) also allows a detailed analysis of the properties of multiplicity distributions. Notably, in [52], the properties of multiplicity distributions in gluon cascades in small rapidity windows were studied.

Important characteristics of the multiplicity distribution in multiparticle production processes are factorial F_q and cumulant K_q moments

$$F_q = \frac{1}{\langle n\rangle^q}\frac{{\rm d}^q G(u)}{{\rm d}u^q}\bigg|_{u=1}\,, \quad K_q = \frac{1}{\langle n\rangle^q}\frac{{\rm d}^q \ln G(u)}{{\rm d}u^q}\bigg|_{u=1}\,. \tag{16}$$

Of special interest is an analysis of the ratio of factorial and cumulant moments [53]

$$H_q = \frac{F_q}{K_q}\,, \tag{17}$$

for which the perturbative QCD predicts the existence of a minimum at some q. A detailed analysis of the properties of factorial and cumulant moments in QCD jets was carried out in papers [54–57].

Of significant interest is also an analysis of the evolution of quark-gluon jets in the medium. In particular, in [58], the effects of coherence breaking in QCD cascades in dense matter were studied.

A review of the modern state of the description of multiparticle production processes from the point of view of perturbative QCD is given in reviews [13, 60].

2.4 Color glass condensate. Evolution equations

One of the fundamental challenges for a theory of strong interactions is to uncover degrees of freedom responsible for the formation of inelastic cross sections in high energy interactions of hadrons and nuclei. Within the gauge theory of strong interactions, the main focus is on the analysis of the leading logarithmic asymptotic form in energy complementing the analysis of the leading logarithmic asymptotic form in the transverse momentum leading to the DGLAP equations (9). In this (former) approximation, one sums all the contributions proportional to

$$\alpha_{\rm s}\ln\left(\frac{\Lambda^+}{P^+}\right) = \alpha_{\rm s}\ln\left(\frac{1}{\sqrt{2}}\frac{P_{\rm h}^0+P_{\rm h}^3}{\Lambda^+}\right) = \alpha_{\rm s}\ln\frac{1}{x}\,, \tag{18}$$

where $P_{\rm h}$ is the 4-momentum of the hadron/nucleus and Λ^+ is a longitudinal scale characterizing the parton modes involved in the scattering process.

Let us consider for definiteness the corresponding asymptotic form of the so-called unintegrated structure function $\varphi(y,\mathbf{k}_\perp^2)$ defined by the following relation:

$$xg(x,Q^2) = \int_0^{Q^2}\frac{{\rm d}^2k_\perp}{\mathbf{k}_\perp^2}\,\varphi(y,\mathbf{k}_\perp^2)\,, \tag{19}$$

where $y=\ln(1/x)$. The corresponding linear Balitsky–Fadin–Kuraev–Lipatov (BFKL) equation derived using an assumption of reggeization of gluonic degrees of freedom in the t-channel reads [61–63]

$$\frac{\partial\varphi(y,\mathbf{k}_\perp^2)}{\partial y} = \frac{\alpha_{\rm s}N_{\rm c}}{\pi^2}\int {\rm d}^2p_\perp\,\frac{\mathbf{k}_\perp^2}{\mathbf{p}_\perp^2(\mathbf{k}_\perp-\mathbf{p}_\perp)^2}\,\varphi(y,\mathbf{p}_\perp^2) - \frac{1}{2}\frac{\alpha_{\rm s}N_{\rm c}}{\pi^2}\,\varphi(y,\mathbf{k}_\perp^2)\int {\rm d}^2p_\perp\,\frac{\mathbf{k}_\perp^2}{\mathbf{p}_\perp^2(\mathbf{k}_\perp-\mathbf{p}_\perp)^2}\,. \tag{20}$$

In the limit of the asymptotically large energies $x\to 0$,

$$\varphi(x,\mathbf{k}_\perp^2)\sim\left(\frac{1}{x}\right)^{4\bar\alpha_{\rm s}\ln 2}\,. \tag{21}$$

From equation (21), it follows that the function $\varphi(y,\mathbf{k}_\perp^2)$ and, therefore, cross sections of interactions in which the hadron/nucleus under consideration takes place, are characterized by power-like growth with energy, which contradicts unitarity. It follows then that equation (20), which was expected to describe the asymptotic behavior of QCD at high energies, does not deliver its consistent description.

Further analysis has shown that the main problem with equation (20) is that it corresponds to a limit of small gluon density, whereas, to construct a consistent description of the QCD asymptotic form at high energies, one needs, in radical distinction from the asymptotic form in transverse momenta, to take into account the contributions from all orders in gluon density.

The key idea that allowed constructing a consistent asymptotic form of QCD at high energies is the analysis of saturation of gluon density due to nonlinear effects, which, in the considered regime of large occupation numbers of gluon modes, can be considered quasiclassically [64]. It is interesting that on dimensional grounds a characteristic transverse momentum for a system with high density (saturation momentum $Q_{\rm s}$) is large. Therefore, the coupling constant $\alpha_{\rm s}(Q_{\rm s}^2)$ is small, and consideration of a high energy asymptotic form of QCD can be performed in the framework of perturbation theory.

The first step in constructing a consistent theory of evolution of correlators in the leading logarithmic approximation in energy was a derivation of the BFKL equation (20) for the small density regime [65] based on using the effective action in which the hypothesis of gluon reggeization is not used.

The general idea of considering quantum corrections is that at some scale Λ^+ fast partons with $p^+>\Lambda^+$ form a classical source with density $\rho_a(x)$ localized at scales $\Delta x^-\lesssim 1/\Lambda^+$, which, in turn, creates a soft classical gluon field $A_a^\mu(x)$ with $k^+\lesssim\Lambda^+$ solving the Yang–Mills equation

$$(D_\nu G^{\mu\nu})_a(x) = \delta^{\mu +}\rho_a(x)\,. \tag{22}$$

Because soft gluons are short-lived, $\Delta x^+\sim 1/k^-\sim k^+\sim x$, the source $\rho_a(x)$ is effectively static. Its properties are described by a state-dependent functional $W_\lambda[\rho]$. Correlators of the gluon field can be calculated using formulas of the following form:

$$\big\langle A_a^i(x^+,\mathbf{x})A_b^j(x^+,\mathbf{y})\ldots\big\rangle_\Lambda = \int\mathcal{D}\rho\, W_\lambda[\rho]\mathcal{A}_a^i(\mathbf{x})\mathcal{A}_b^j(\mathbf{y})\ldots\,, \tag{23}$$

where $\mathcal{A}_a^i[\rho]$ solves equations (22) in the Landau gauge. The theory formulated in these terms is usually named the theory of color glass condensate.

Equations of quantum evolution are most naturally written like those for the evolution of the functional $W_\lambda[\rho]$ with changing scale $\Lambda \to b\Lambda$. The corresponding nonlinear evolution equation in $y = \ln(1/b)$, taking into account all orders in the gluon density and correctly describing the leading logarithmic asymptotic of QCD at high energies, is the Jalilian–Marian–Iancu–McLerran–Weigert–Leonidov–Kovner (JIMWLK) equation [66–69],

$$\frac{\partial W_y[\rho]}{\partial y} = \alpha_s \left\{ \frac{1}{2} \frac{\delta^2}{\delta\rho_y^a(x_\perp)\delta\rho_y^b(x_\perp)} \left[W_y \chi_{xy}^{ab} \right] - \frac{\delta}{\delta\rho_y^a(x_\perp)} \left[W_y \sigma_x^a \right] \right\}, \tag{24}$$

where

$$\begin{aligned} &\alpha_s \ln \frac{1}{x}\, \sigma_a(x_\perp) = \int \mathrm{d}x^- \left\langle \delta\rho_a(x) \right\rangle , \\ &\alpha_s \ln \frac{1}{x}\, \chi_{ab}(x_\perp, y_\perp) = \int \mathrm{d}x^- \int \mathrm{d}y^- \left\langle \delta\rho_a(x^+, \mathbf{x})\, \delta\rho_a(x^+, \mathbf{y}) \right\rangle . \end{aligned} \tag{25}$$

Let us note that equation (24) provides only a partial solution to the unitarity problem, ensuring unitarity only at a fixed scattering parameter. To fully restore unitarity, one probably needs to consider nonperturbative effects responsible for the appearance of the infrared scale in the t-channel. Another important unsolved problem related to equation (24) is the dependence of its form on the treatment of nonperturbative zero modes.

The physics of color glass condensate is described, inter alia, in reviews [14, 17].

2.5 Phenomenology of confinement

One of the central problems with describing the physics of strong interactions in the framework of QCD is the theoretical description of the confinement of quarks and gluons as well as the development of the phenomenology of this phenomenon.

At a fundamental level, the properties of QCD at large distances should be described through the construction of effective Lagrangians reflecting the corresponding relevant degrees of freedom. An expression for the effective Lagrangian for the simplest gluonic order parameter $\chi \sim (T_\mu^\mu)_G$ was calculated in [70]:

$$\mathcal{L}_{\text{eff}} = \frac{1}{2\eta^2} (\partial_\mu \chi)^2 - \frac{1}{4} \frac{9}{32\pi^2} \chi^4 \left(\ln \frac{\chi^4}{\chi_v^4} - 1 \right), \tag{26}$$

where η is a glueball mass and χ_v is the vacuum average of the effective scalar field χ. The corresponding generalization taking into account the quark degrees of freedom was studied in [71].

At a fundamental level, consideration of nonperturbative effects in QCD is, notably, related to the analysis of the properties of Wilson loops. In [72], properties of a Wilson loop in a vacuum filled with stochastic background fields were considered. In another paper [73], the properties of a spaghetti vacuum formed by chromomagnetic flux tubes were analyzed. It was shown that only tubes with a finite length are stable. In [74], possible analogies of QCD spaghetti vacuum in solid state theory were analyzed.

One of the most popular topics related to the phenomenology of confinement is consideration of a potential model of heavy quarkonia. Due to confinement, the potential of interaction between a quark and antiquark at large distances has the form

$$V(r)\Big|_{r\to\infty} \simeq \sigma r - c\, \frac{\pi}{12} \frac{1}{r}, \tag{27}$$

where σ is the QCD string tension and the Coulomb term reflects quantum fluctuations of the string. The coefficient c depends on an effective theory describing the QCD string and has the meaning of a central charge per degree of freedom. One of the ways of constructing the static potential of the quark-antiquark pair is to use an interpolation of the QCD β-function linking regimes of small and large distances [75]. Let us note that, from a consideration of the potential model using the Dirac equation, it follows in [76] that, to ensure the stability of the bound states, the linearly growing part of the potential should be Lorentz-scalar.

To develop a quantitative description of spectra and decay widths of heavy quarkonia, one needs to configure the potential $V(r)$ at all distances. One option is to construct this interpolation using that of the QCD beta function that, in particular, takes into account the Coulomb correction at large distances in (27). A detailed review of results on the potential model of heavy quarkonia can be found in [7].

As was already noted, there are several variants of development of an effective theory of the QCD string, resulting in different values of the coefficient c in (27). The possible values of c and restriction on them following from experimental data on energy spectra and decay widths of heavy quarkonia were discussed in [77]. The main result of the analysis in [77] is the restriction on c of the form $c < 1$.

An interesting possibility of formation of hadronic molecules based on consideration of pentaquarks was studied in [78].

One more effect, deceleration of colored particles by vacuum condensate gluon fields, was discussed in [79].

Another research topic was a phenomenology of nonperturbative effects in quark-gluon jets. Specifically, in [80], equations modifying equations (9) through considering interactions with soft degrees of freedom, by analogy with the theory of inelastic ionization losses in electromagnetic showers in matter. A detailed analysis of the solutions of the modified evolution equations was conducted in [81]. Dissipative effects in QCD jets, taking into account coherence, were explored in [82].

An important aspect of confinement phenomenology is a discussion of the QCD phase diagram. Universal scaling characterizing the critical hadronic matter described in terms of a Hagedorn gas was studied in [83]. A description of the QCD phase diagram under the assumption of the existence of a separate constituent quark phase is given in [15].

2.6 Electromagnetic processes in hot strongly interacting medium

An important avenue of research in the sector/laboratory was studying electromagnetic signals, i.e., photons and dilepton pairs generated in dense hot matter created at the early stages of relativistic heavy ion collisions. The foundational work in this area in high energy physics was papers [84, 85], in which a quantitative theory of photon and dilepton pair emission by quark-gluon plasma based on medium-related averaging of the corresponding correlators of electromagnetic currents was developed. As an example, let us show the result for the spectrum of invariant masses M of dilepton pairs [85]:

$$\frac{\mathrm{d}N_{l\bar{l}}}{\mathrm{d}M} \sim \exp\left(-\frac{M}{T}\right) M^{3/2} T^{3/2}\,\mathrm{d}V\,\mathrm{d}t\,, \tag{28}$$

where T is the plasma temperature.

Development of a theory of an early stage of ultrarelativistic heavy ion collisions led to the appearance of its description in terms of glasma, a state of gluonic matter in which the dominant objects are chromoelectric and chromomagnetic tubes formed by chromoelectric and chromomagnetic fields (see, e.g., [17]). Emission of dilepton pairs in glasma was studied in [86]. As an example, we show an expression for the photon spectrum

$$\frac{\mathrm{d}N_\gamma}{\mathrm{d}y\,\mathrm{d}^2k_\perp} \sim \left(\frac{Q_{\mathrm{sat}}}{k_\perp}\right)^\eta, \qquad \eta \in \left[9, \frac{24}{5}\right]. \tag{29}$$

Emission of photons and dileptons at the point of a phase transition between hadronic and quark-gluon matter was considered in [87]. This analysis was based on using the Hagedorn description of the hadronic gas augmented restrictions following from the quark-hadron duality borrowed from the consideration of electron-positron annihilation to hadrons. As a result, it was shown that, at the phase transition point, the spectra of the hadronic gas and quark-gluon plasma coincide. The result can be interpreted as 'letting loose' the quark-antiquark currents at the phase transition point.

Emission of dileptons related to the existence of a hypothetical constituent quark phase was studied in [88]. Let us also mention an analysis of two-photon correlations in electromagnetic radiation generated by quark-gluon plasma in [89].

2.7 Coherent radiation in strongly interacting medium

A natural direction towards developing a quantitative description of phenomena related to the passage of fast particles through a hot dense medium is a description of the properties of quantum modes using the notion of (chromo)electric permeability encoding the relevant properties of the scattering amplitude of a fast particle on the particles in the medium. The discovery of ring-like events in the scattering of a hadron in cosmic rays on nuclei at super-high energies led to a hypothesis on Cherenkov radiation of gluons as a basic mechanism generating such events [90]. A theoretical description of such a mechanism suggested in [91] was based on introducing the medium refraction index $n(\omega)$ for radiation with the frequency ω of the following form:

$$n(\omega) = 1 + \frac{2\pi\nu}{\omega^2}\, T_{\mathrm{f}}(\omega)\,, \tag{30}$$

where ν is the density of scatterers in the medium and $T_{\mathrm{f}}(\omega)$ is a forward amplitude of radiation scattering at the medium constituents. From the optical theorem, it follows that the imaginary part of $T_{\mathrm{f}}(\omega)$ can be expressed through the total cross section of radiation scattering on medium constituents:

$$\mathrm{Im}\, T_{\mathrm{f}}(\omega) = \frac{\omega}{4\pi}\,\sigma(\omega)\,. \tag{31}$$

Standard considerations lead to the expression for the Cherenkov angle θ_{c} of the form

$$\theta_{\mathrm{c}} = \sqrt{\frac{\nu}{\omega}\left(\frac{\mathrm{Re}\, T_{\mathrm{f}}(\omega)}{\mathrm{Im}\, T_{\mathrm{f}}(\omega)}\right)}\,. \tag{32}$$

A condition for the Cherenkov radiation to appear is $\mathrm{Re}\, T_{\mathrm{f}}(\omega)/\mathrm{Im}\, T_{\mathrm{f}}(\omega) > 0$, which does indeed hold in a certain energy range for hadron-hadron scattering.

Ring-like events can alternatively be described in terms of the formation of a Mach cone. A comparative analysis of the Cherenkov mechanism and that related to the formation of the Mach cone was presented in [92]. A detailed analysis of the Cherenkov mechanism of gluon radiation in relation to experimental data at RHIC and LHC was done in [93–95].

For a realistic description of Cherenkov gluon radiation in a medium, one has to take into account its absorbing properties. To this end, it is necessary to study the case of a complex medium permeability $\epsilon(\omega) = \epsilon_1(\omega) + \mathrm{i}\epsilon_2(\omega)$. The corresponding analysis in [96] led to the following expression for the spectrum of Cherenkov radiation per unit path length:

$$\frac{1}{\omega}\,\frac{\mathrm{d}W}{\mathrm{d}l\,\mathrm{d}\omega\,\mathrm{d}\phi\,\mathrm{d}\cos\theta} = \frac{4\alpha_{\mathrm{s}} C_{V(F)}}{\pi}\,\frac{\cos\theta(1-\cos^2\theta)\Gamma(\omega)}{\left(\cos^2\theta - \zeta(\omega)\right)^2 + \Gamma^2(\omega)}\,, \tag{33}$$

where $C_{V(F)}$ are Casimir invariants in the vector and fundamental representations of the color group and

$$\zeta(\omega) = \frac{\epsilon_1(\omega)}{\epsilon_1^2(\omega) + \epsilon_2^2(\omega)}\,, \quad \Gamma(\omega) = \frac{\epsilon_2(\omega)}{\epsilon_1^2(\omega) + \epsilon_2^2(\omega)}\,. \tag{34}$$

A detailed comparison with the RHIC experimental data taking into account the rescattering of radiating particles was performed in [96].

A fundamental distinguishing feature of the in-medium quantum chromodynamics is the possibility of Cherenkov radiation by gluon currents, including the possibility of radiating a pair of Cherenkov gluons. The corresponding calculations were performed in [98]. Here, we show an expression for the production rate of a single Cherenkov gluon with a frequency ω in the medium with chromoelectric permittivity $\epsilon(\omega)$ by the gluon current with energy E:

$$\begin{aligned}\gamma(\omega|E) = \alpha_{\mathrm{s}} N_{\mathrm{c}}&\left(1 - \frac{1}{\epsilon}\right)\left(1 - \frac{\omega}{E} - \frac{\epsilon - 1}{4}\,\frac{\omega^2}{E^2}\right)\\ &\times\left[1 + \frac{1}{2}\left(\epsilon + \frac{1+\epsilon}{1-\omega/E} + \frac{\epsilon}{(1-\omega/E)^2}\right)\frac{\omega^2}{E^2}\right.\\ &\left.+ \frac{(1+\epsilon)^2}{8(1-\omega/E)^2}\,\frac{\omega^4}{E^4}\right].\end{aligned} \tag{35}$$

Another fundamental phenomenon related to coherent radiation is the Landau–Pomeranchuk effect. A detailed consideration of this effect using the diagrammatic technique in the Abelian case was described in [99].

2.8 Nonequilibrium quantum field theory

One of the most fundamental issues in developing a quantitative theory of multiparticle production concerns the true quantum field-theoretic nature of a system born in high energy collisions of hadrons and nuclei. It is clear that the exponential spectra do not, in general, present proof of the fact that, in the system, a conventional scheme of establishing a local thermodynamic equilibrium is realized. In the discussed situation, we are possibly dealing with much more complex dynamics related to the quantum nature of the evolution. In review [37], one finds a reference to the pioneering paper [102] in which quantum mechanisms lead to the observed 'thermalization' but are not related to standard mechanisms of equilibration in kinetic theory. After many decades, mechanisms describing entropy generation in relation, in particular, to quantum entanglement, are considered to be foundational in the modern understanding of the nature of multiparticle production processes and, in this sense, the prophetic ideas discussed in [102] and [37, 100, 101] are realized at this new stage of development of a theory of multiparticle production at high energies.

A systematic development of related issues requires the usage/development of methods of nonequilibrium quantum field theory. A development of this approach in application to describing the physics of hadrons and nuclei is discussed in [103–106], in which a model problem of nonequilibrium evolution of a scalar field is considered. Among the most important results in this area are establishing a relation between quasiclassical expansion and the Keldysh diagram technique [105] and derivation of a general expression for shear viscosity in [106].

3. Conclusions

In the decades of research in the sector/at the laboratory, much has been accomplished. Many results that to a significant extent shaped the high energy physics of strong interactions were obtained. At the same time, the construction of a consistent theory in this domain is not complete, so the research continues.

Feinberg, Evgeny L'vovich (27.06.2012 – 10.12.2005) in 1959 founded the Sector of High Energy Physics in the Theoretical Physics Department of the P N Lebedev Physical Institute and actively worked in it until the last days of his life: (a) in the office in 1973; (b) at a seminar with I E Tamm; (c) with RAS President Yu S Osipov (center) and V L Ginzburg at the banquet celebrating V L Ginsburg's 2003 Nobel Prize (Central House of Scientists, January 19, 2004); (d) in the office in 2000.

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The staff of the High Energy Physics Sector/Laboratory of the I E Tamm Theoretical Physics Department of the P N Lebedev Physical Institute, whose papers are covered in the review: (a) Feinberg, Evgeny L'vovich (a founder of the Sector/Laboratory); (b) Andreev, Igor Vasil'evich; (c) Royzen, Ilya Isaevich; (d) Mironov, Andrey Dmitrievich; (e) Chernavsky, Dmitry Sergeevich; (f) Dremin, Igor Mikhailovich, head of laboratory since 1988; (g) Nechitailo, Vladimir Aleksandrovich; (h) Radovskaya, Anna Aleksandrovna; (i) Alfimov, Mikhail Nikolaevich; (j) Leonidov, Andrey Vladimirovich (author of the present review).

this review the portraits of colleagues from her remarkable photo archive.

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