INSTRUMENTS AND METHODS OF INVESTIGATION

PACS numbers: **06.20.–f**, **06.30.–k**, **07.20.–n**

Physical and technical aspects of radiometric thermometry from the standpoint of a new definition of the temperature unit

V P Khodunkov

DOI: https://doi.org/10.3367/UFNe.2023.10.039571

Contents

1. **Introduction** 706
2. **Purpose of the study** 707
3. **Physical and technical aspects of implementation of absolute method of measuring thermodynamic temperature** 707
   3.1 Physical foundations of measurement method; 3.2 Development of theory and practice of the method; 3.3 Equation of measurement method; 3.4 On the use of effective and central wavelengths; 3.5 Error estimate; 3.6 Discussion and recommendations
4. **Conclusions** 715
   References 715

Abstract. **We consider the physical and technical aspects of the transition to a new definition of the temperature unit as a physical quantity, reveal the features of the implementation of an absolute method for its reproduction in radiometric thermometry, and outline the physical foundations of the method and its theoretical substantiation. A comprehensive analysis of the procedure for reproducing the temperature unit according to its new definition is given, features of the implementation of the method in reference radiometric thermometry are shown, and an estimate of the measurement error is given.**

Keywords: thermodynamic temperature, absolute method, Planck radiation, radiometric thermometry, measurement, error, asymmetry, spectra

1. Introduction

Since 2019, the entire world of measurements has switched to new definitions of the basic units of physical quantities (length, mass, time, electric current, thermodynamic temperature, amount of substance, light intensity), which have become related to fixed values of fundamental physical constants. At the same time, the relation to any material standards completely disappeared from the previous definitions of these physical quantities. The redefinition of the basic units of the International System of Units (SI) entailed a number of scientific and technical problems, primarily related to the reproduction and transmission of units of these physical quantities according to their new definition. The ultimate goal of this reform of the International System of Units is to ensure uniformity of measurements of physical quantities and simultaneously increase their accuracy.

This paper discusses the features of the transition to a new definition of the unit of one of the basic physical quantities — temperature (kelvin) — and provides a solution to the accompanying scientific problems, which turned out to be very significant and required serious physical scrutiny.

According to the new definition of the unit of thermodynamic temperature, "Kelvin, symbol K, is a unit of thermodynamic temperature T, which is defined by fixing the numerical value of Boltzmann's constant k to 1.380649×10^{-23} J K^{-1}." The new definition of kelvin is based on assigning a fixed value to the fundamental physical Boltzmann constant, which is the coefficient relating a unit of temperature to a unit of thermal energy. The quantity kT present in the equations of state is the characteristic energy that determines the energy distribution between the particles of a system in thermal equilibrium.

It is noteworthy that the main advantage of introducing a new definition of kelvin is the increase in the accuracy of temperature measurements in the temperature range distant from the triple point of water, and primarily in the high temperature range. As a result, it is possible to use absolute radiation thermometers without relying on the triple point of water, which should ultimately lead to an increase in the accuracy of the temperature scale and an expansion of its range without any serious organizational or economic consequences.

In what follows, we present the basic physical principles of reproducing and measuring the unit of temperature according to its new definition. The theoretical foundations of a fundamentally new approach to reproducing the temperature unit for the high temperature region (range from 1357 K to 3200 K) are considered as a probable alternative to the method of conventional primary thermometry [1] used as the basic principle in the State primary standard of the temperature unit GET 34-2020.

V P Khodunkov
D I Mendeleev Institute for Metrology,
Moskovskii prosp. 19, 190005 St. Petersburg, Russian Federation
E-mail: walkerearth@mail.ru

Received 21 June 2023, revised 23 August 2023
Uspekhi Fizicheskikh Nauk **194** (7) 753 – 764 (2024)
Translated by E N Ragozin

The fundamental worldwide guideline for the implementation of the absolute method in radiometric thermometry is the "Mise-en-pratique" guideline developed by the Bureau International des Poids et Mesures (BIPM) for the new definition of kelvin. Document [1], which is a supplement to the Mise-en-pratique, outlines the theory of the absolute method, the measurement equation, gives practical recommendations for constructing a measuring system, and provides an estimate of the expected error.

A detailed analysis of this document suggests that it provides only general recommendations, and the features and subtleties of the implementation of the method are not sufficiently disclosed. So, the paper discusses the most important, in the author's opinion, physicotechnical and applied aspects of the implementation of this method and proposes a number of measures for its development and improvement.

It is believed that the metrological characteristics of the new Russian standard GET 34-2020 today make it possible to meet the needs of science and industry, and that its equivalence ranks with the best national standards of the most developed countries [2]. At the same time, it cannot be concluded that the limit of accuracy capabilities of the standard has been reached; therefore, its condition is permanently analyzed, ways of improvement are sought, and new technical solutions are developed, which, in fact, is the concern of this paper.

It is pertinent to note that the procedure for reproducing a temperature unit using the absolute method is extremely complex, is implemented with a large number of pieces of high-precision measuring equipment, and, in the author's opinion, can and should be optimized. Furthermore, the issues of improving the accuracy of measurements of a number of physical quantities remain insufficiently elaborated: the spectral emissivity of the cavity of the black body model and the quantum efficiency of the infrared radiation detector (they are parts of the GET 34-2020 standard), their accurate knowledge being indispensable in the implementation of this absolute method.

The present investigation takes into account to the maximum extent the achievements and developments of world science in this area of measurements: Refs [3–25] in relation to the reproduction of the temperature unit, known methods for measuring the emissivity of solids [26–36], and methods for measuring the quantum efficiency of photodiode detectors [37–47] are analyzed.

2. Purpose of the study

The purpose of the research undertaken is the physical substantiation of the measurement method and a comprehensive analysis of the procedure for reproducing the temperature unit according to its new definition, assessment of the actual accuracy achieved, and the development of new technical solutions aimed at its further improvement.

3. Physical and technical aspects of implementation of absolute method of measuring thermodynamic temperature

As adopted in 2019, in absolute measurements, the temperature as a measured quantity must be connected to the Boltzmann and Planck fundamental physical constants.

The main physical principles of measuring thermodynamic temperature by the absolute method are prescribed by the primary document, Mise-en-pratique. According to this document, the measurement of thermodynamic temperature by the absolute method is based on an accurate measurement of the optical power emitted in a given spectral band and a known solid angle by an isothermal cavity with a known emissivity. It is prescribed to use as such an isothermal cavity the model of an absolutely black body (ABBM) with a so-called ampoule of a temperature reference point placed in it, in which a phase transition of the pure substance or eutectic mixture contained in it occurs. Ampoules for the implementation of reference points of the temperature scale are intended to reproduce the temperature of phase transitions of melting or solidification of pure metals and eutectic mixtures and are working temperature standards of the 0th category in accordance with GOST 8.558-2009.

The use of such ampoules for the purposes of reference radiometric thermometry is justified by the extremely high constancy of the phase transition temperature and its long duration, which is sufficient for verification and calibration of high-temperature measuring instruments.

Reference point ampoules used in radiometric thermometry, as a rule, are two coaxially arranged graphite cylinders. A reference substance is placed between them, whose phase transition is used for the purposes of the specified calibration and verification (Fig. 1). The inner cylindrical cavity is the radiating cavity, has a high emissivity close to unity, and exhibits the highest isothermicity and temporal stability of the radiation power during the phase transition. For example, ensured for the phase transition of copper (melting, solidification) at a temperature of this phase transition of 1357.77 K is temporary stability of the radiation power characterized by temperature instability of the emitting cavity of the ampoule of the order of 12 mK, i.e., only 8.8×10^{-6} rel. units, or 0.0008%. Figure 2 shows a real example of the phase transition of pure copper (solidification), achieved using the state standard temperature unit GET 34-2020. From Fig. 2, one can see that the duration of the phase transition (solidification) of copper is approximately 900 s (15 min).

Structurally, the reference point ampoule is placed in a special graphite container (see Fig. 1), which in turn is placed inside a cylindrical electric furnace. Figure 3 shows a photo of the front flange of the electric furnace, with the radiating cavity of the ampoule being visible at its center. This design is referred to as the black body model. We emphasize the following. Owing to the design of the ampoule and the use

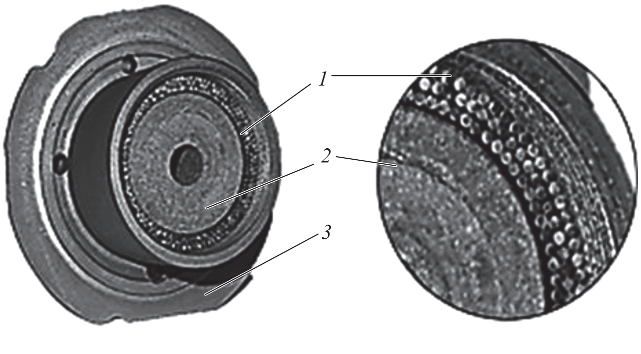

Figure 1. External view of 'reference point ampoule' assembly: leveling centering shell (*1*), ampoule (*2*), ampoule container (*3*).

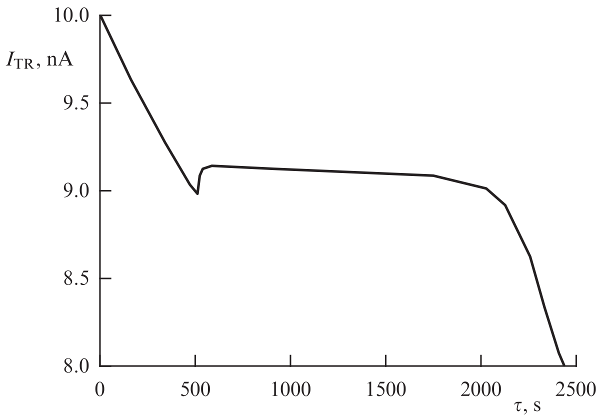

Figure 2. Phase transition of pure copper (solidification).

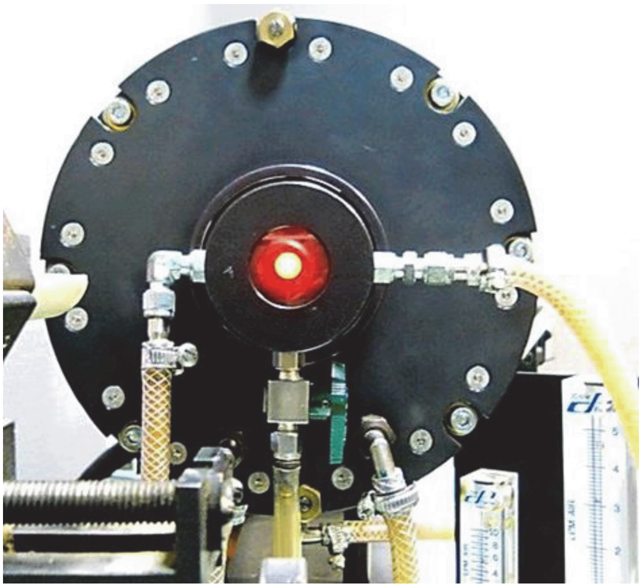

Figure 3. Front flange of blackbody model.

of special aperture stops, the emissivity of its cavity is close to unity and amounts to $\varepsilon \approx 0.9997$, and therefore it is generally accepted that the radiation of such a cavity is identical with a high degree of probability ($P = 0.99$) to the radiation of an absolutely black body (ABB), and this, in fact, is what radiometric thermometry and reference metrology make use of.

In the reproduction of the temperature unit using the absolute method, an absolute filter radiometer or, as it is otherwise called, an absolute radiation thermometer, is used as a means of measuring optical power.

As is presently accepted by all national metrology institutes, the filter radiometer includes a photodetector, a spectral selective filter, and a geometric/optical system with two specified apertures. According to the Mise-en-Pratique, the measurement of thermodynamic temperature can be carried out on the basis of four equivalent physical principles, which differ only in the method of calibration of the filter radiometer.

The main principles are as follows.

(1) *Principle of measuring spectral power*. The filter radiometer is calibrated in power sensitivity and is used to measure the blackbody radiance along with the detector and source apertures.

(2) *Principle of measuring irradiance*. The filter radiometer is calibrated in sensitivity to irradiance and is used to measure the radiance of an absolutely black body along with the source aperture.

(3) *Hybrid measurement principle*. The filter radiometer is calibrated in sensitivity to irradiance and is used to measure blackbody radiance in concert with a single simple lens and lens aperture.

(4) *Principle of measuring radiance*. The radiometer is calibrated in sensitivity to radiance and consists of a filter radiometer built into an optical system consisting of several lenses and screens.

Furthermore, other measurement principles may be used. Figure 4 shows, by way of example, a setup for calibrating a radiation thermometer using a supercontinuum laser for radiation thermometry purposes, used at the Laboratoire national de métrologie et d'essais, LNE, France.

3.1 Physical foundations of measurement method

Regardless of the calibration principle used, the general measurement equation of the absolute method is of the form

$$I = \int_0^\infty \varepsilon(\lambda) F_{\mathrm{FR}} S_\lambda(\lambda) L_{\mathrm{b},\lambda}(\lambda, T)\, \mathrm{d}\lambda\,, \tag{1}$$

where I is the photocurrent generated by the filter radiometer from the radiation of the ABBM, [A]; F_{FR} is the area of the acceptance surface of the filter radiometer, [m^2]; $S_\lambda(\lambda)$ is the spectral sensitivity of the filter radiometer to the radiance at a given wavelength λ, [A W^{-1}]; $L_{\mathrm{b},\lambda}(\lambda, T)$ is the spectral density of the radiance of the radiating cavity of an absolutely black body, [W m^{-3}]; and T is the thermodynamic temperature of the radiating cavity of the ABBM, [K]. It is believed that the radiation of the ABBM is equivalent to the emission of an absolutely black body and is characterized by the emissivity $\varepsilon(\lambda) = 0.9997$ [14, 15].

As is well known, the spectral emission of an absolutely black body obeys Planck's law, which in formulaic expression is of the form

$$L_{\mathrm{b},\lambda}(\lambda, T) = \frac{c_1}{\lambda^5} \frac{1}{\exp(c_2/\lambda T) - 1}\,, \tag{2}$$

where $c_1 = 2\pi hc^2 = 3.741771851 \times 10^{-16}$ is the first radiation constant, [W m^2]; $c_2 = hc/k = 1.4387769 \times 10^{-2}$ is the second radiation constant, [m K]; $h = 6.62607015 \times 10^{-34}$ is the physical fundamental Planck constant, [J s]; $e = 1.602176634 \times 10^{-19}$ is the elementary charge, [C] or [A s]; $c = 2.99792458 \times 10^8$ is the speed of light in a vacuum, [m s^{-1}]; and $k = 1.380649 \times 10^{-23}$ is the physical fundamental Boltzmann constant, [J K^{-1}].

We consider the procedure of measuring the thermodynamic temperature of the radiating cavity of the ABBM implemented according to the principle of measuring energy brightness (4th principle), according to which the temperature unit is reproduced with the State primary standard of the temperature unit GET 34-2020 in Russia. In Ref. [2], a complex of equipment is presented in detail that makes it possible to reproduce kelvin in accordance with this new definition by the method of absolute primary radiometric thermometry.

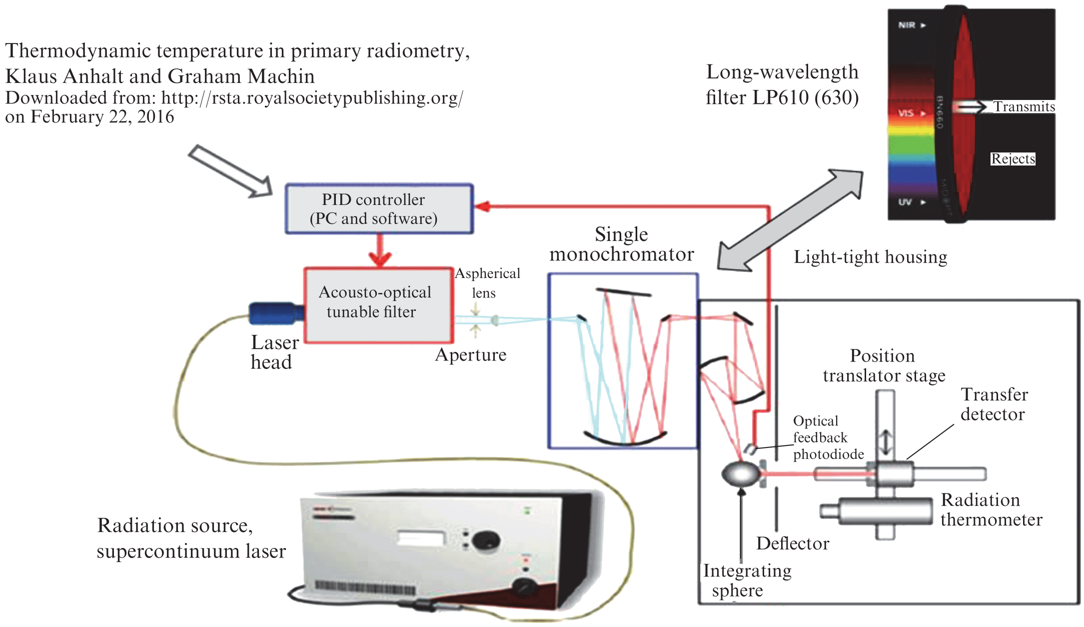

Figure 4. Schematic diagram of facility for calibrating a radiation thermometer using a supercontinuum laser for radiation thermometry purposes.

The procedure consists of four sequential stages:

Stage 1: measurement of the spectral sensitivity $S_\lambda(\lambda)$ of the trap detector using an absolute cryogenic radiometer and a tunable laser (or a monochromator and a broadband source);

Stage 2: measurement of the emission power of the integrating sphere using an infrared radiation detector — a quantum trap detector — in a given spectral band;

Stage 3: calibration of the filter radiometer according to the radiation of the integrating sphere in a given spectral band carried out according to one of the four principles listed above;

Stage 4: measurement of the thermodynamic temperature of the radiating cavity of the ABBM using a calibrated filter radiometer.

With the measurement algorithm recommended by document [1], the total error is formed due to the measurement errors of the following parameters: wavelength, spectral sensitivity of the trap detector, photocurrent, geometric factor, and emissivity of the ABBM cavity, as well as the contribution of other factors: out-of-band radiation and absorption of the spectral selective filter, radiation beam divergence, diffraction by apertures, instability of laser radiation, source size effect, temporal instability of the filter radiometer, and ABBM.

3.2 Development of theory and practice of the method

In order to improve the accuracy of measurements, it is proposed to change the algorithm for measuring the thermodynamic temperature of the ABBM. The proposed correction to the algorithm is to use the operation of comparing the energy brightnesses of the ABBM cavity and the integrating sphere.

It should be noted here that the use of an integrating sphere ensures spatially uniform dispersion of infrared radiation into its output ports, making it possible to accurately measure the luminous flux and radiation power. The integrating sphere, also referred to as an Ulbricht sphere, is an optical component consisting of a hollow sphere with a diffuse white reflective coating inside and small holes for input and output ports.

The operation of comparing the energy brightnesses of the ABBM cavity and the integrating sphere is performed by jointly using a brightness comparator and a trap detector equipped with a bandpass optical filter, which is a kind of analogue of a filter radiometer. It is appropriate to note that this technical solution is not new; however, in the author's opinion, it has been undeservedly ignored. In this regard, document [1] directly states: ...*Monochromator-based radiation comparators can be absolute-calibrated against a reference trap detector, but this calibration can spontaneously change over long periods due to insufficient monochromator stability*... .

...*Consequently, brightness measurement will be a part of the measurement procedure in all cases. This method has been used in the past to measure the thermodynamic solidification temperature of copper* [4, 5]. *This is a direct brightness measurement method using a tunable laser and an integrating sphere*... .

It is hard to disagree with the statement regarding the stability of the monochromator. However, if in the existing measuring setup the monochromator is replaced with a narrow-band optical interference filter, which has a high quality factor and much better temporal stability of spectral transmission, then this drawback can be considered eliminated, and the method is even more effective than that recommended by document [1].

In this case, the procedure of measuring the thermodynamic temperature of the ABBM cavity using a quantum trap detector, in contrast to the generally accepted approach, is significantly simplified and reduced in time. In this case, it is assumed that the following parameters have been preliminarily measured only once:

— spectral sensitivity of the quantum trap detector, which remains unchanged for all subsequent temperature measurements;

— ratio of photocurrents of a quantum trap detector generated by direct and focused radiation from the integrat-

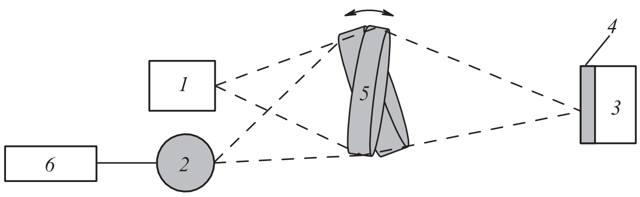

Figure 5. Block diagram of thermodynamic temperature measurements: *1* — radiating cavity of ABBM; *2* — integrating sphere; *3* — photodiode trap detector; *4* — narrow-band interference filter; *5* — optical focusing system; *6* — continuous (white) laser.

ing sphere, which is also considered unchanged for all subsequent temperature measurements.

Figure 5 shows a schematic diagram of measurements that implement the proposed algorithm. Measuring the thermodynamic temperature of the ABBM cavity is performed by measuring the trap detector photocurrent generated by focused radiation from the ABBM cavity and then calculating its value using the measurement equation. The comparison operation involves comparing the photocurrents of the trap detector generated by focused radiation from the ABBM and the integrating sphere, while the previously obtained photocurrent ratio coefficient is used to calculate the temperature.

3.3 Equation of measurement method

Planck's radiation law (2) underlies the measurement equation of the absolute method. It is well known that any heated body (hereinafter, it is referred to as an object) emits some energy into the environment with power, which can be measured, for example, using a photodiode detector. In such measurements, the photodiode detector generates a photocurrent directly proportional to the specified power. In the general case, the equation for calculating the radiation power from the measured photocurrent is of the form

$$P = \frac{I_{\mathrm{FD}}}{S_\lambda}\,, \tag{3}$$

where P is radiation power, [W]; I_{FD} is measured photocurrent of the photodiode detector, [A]; and S_λ is spectral sensitivity of the photodiode detector, [A W^{-1}].

In turn, the photocurrent generated by the photodiode detector due to the radiation of an object in a fixed spectral band, set, for example, using a bandpass optical filter, can be calculated using the also known relationship

$$I_{\mathrm{FD}}^{(1)} = F\varepsilon \int_{\lambda_1}^{\lambda_2} S_\lambda L_\lambda^{(1)} \tau_\lambda \, \mathrm{d}\lambda\,, \tag{4}$$

where $I_{\mathrm{FD}}^{(1)}$ is the photocurrent generated by the photodiode detector due to the spectral radiation of the object, [A]; $L_\lambda^{(1)}$ is the spectral energy brightness of the object, [W m^{-3}]; λ_1, λ_2 are the boundaries of a given spectral radiation band, [m]; τ_λ is the spectral transmittance of the bandpass optical filter; ε is the emissivity for the system of bodies object–photodiode detector; and F is a geometric factor that is calculated using the known relationship

$$F = \frac{2\pi r_1^2 r_2^2}{(r_1^2 + r_2^2 + d^2) + \sqrt{(r_1^2 + r_2^2 + d^2)^2 - 4 r_1^2 r_2^2}}\,, \tag{5}$$

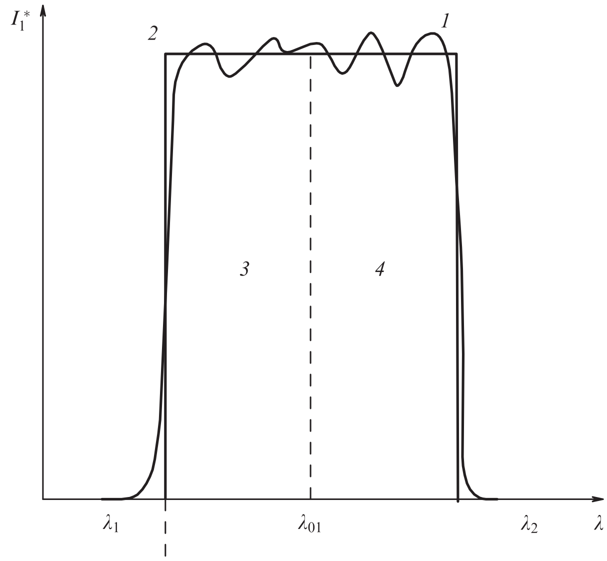

Figure 6. Emission spectrum of a real object.

where r_1 is the radius of the output window of the radiation source, [m]; r_2 is the radius of the input window of the photodiode detector, [m]; and d is the distance between the output window of the radiation source and the input window of the photodiode detector, [m]. In the special case when the photodiode detector is placed close to the output window of the radiation source and $r_2 > r_1$, the geometric factor is equal to the area of the output window of the radiation source.

The integrand in relation (4) $I_1^* = S_\lambda L_\lambda^{(1)} \tau_\lambda$ is spectral-dependent: its graphic example is plotted in Fig. 6 (line *1*). The area under this graphic dependence is nothing more than the value of the integral in relation (4), which, in turn, is equal to the product of the radiance of the object $L_{\mathrm{e}}^{(1)}$ and the integral photodiode detector sensitivity $S_{\lambda 0}$ averaged over a given spectral band, i.e.,

$$\int_{\lambda_1}^{\lambda_2} S_\lambda L_\lambda^{(1)} \tau_\lambda \, \mathrm{d}\lambda = L_{\mathrm{e}}^{(1)} S_{\lambda 0}\,. \tag{6}$$

It should be noted here that replacing the integral over a spectral band in expression (6) with the product of the radiance of an object in this band and the average sensitivity of a photodiode detector integrated over a given band always leads to some error. Due to what? This does not happen due to the spectral sensitivity of the photodetector; the dependence is strictly linear (see relation (21) below). The error is due solely to the nonlinearity of the spectral radiance. When replacing it, it is assumed that the spectral radiance is linear, but in fact it varies parabolically across the spectrum. And only in a very narrow spectral range can it be considered linear. This replacement begins to have an effect only at temperatures below 1000 K and at a spectral interval width of more than 10 nm. For temperatures above 1000 K, which are of greatest interest, such a replacement entails an error that is an order of magnitude smaller than the error due to the uncertainty in knowing the emissivity of the ABBM.

To obtain the measurement equation, we replace the spectral-dependent integrand I^* with an area-equivalent spectral-independent function with an effective spectral bandwidth equal to $\Delta\lambda_1$ and symmetric with respect to the effective wavelength λ_{01} (line *2*, Fig. 6). The equity of the areas under the indicated spectral dependences (lines *1*, *2*, Fig. 6) means that the following condition is met:

$$S_{\lambda_{01}} L^{(1)}_{\lambda_{01}} \Delta\lambda_1 = \int_{\lambda_1}^{\lambda_2} S_\lambda L^{(1)}_\lambda \tau_\lambda \,\mathrm{d}\lambda\,, \tag{7}$$

where the values of the spectral photodiode sensitivity $S_{\lambda_{01}}$ and the spectral radiance $L^{(1)}_{\lambda_{01}}$ of the object are taken for the effective wavelength λ_{01}, and the transmittance of the bandpass optical filter (τ_λ) is taken equal to unity. In this case, the effective wavelength λ_{01} is understood as a wavelength for which the condition of equity of the following integrals is satisfied (equity of the areas of sections *3*, *4* in Fig. 6):

$$\int_{\lambda_1}^{\lambda_{01}} S_\lambda L^{(1)}_\lambda \tau_\lambda \,\mathrm{d}\lambda = \int_{\lambda_{01}}^{\lambda_2} S_\lambda L^{(1)}_\lambda \tau_\lambda \,\mathrm{d}\lambda\,. \tag{8}$$

In view of condition (7), relation (4) assumes the form

$$I^{(1)}_{\mathrm{FD}} = F\varepsilon S_{\lambda_{01}} L^{(1)}_{\lambda_{01}} \Delta\lambda_1\,. \tag{9}$$

Reasoning and mathematical operations similar to those presented above in (3)–(9) are completely valid and can be applied to a reference radiation source made on the basis of an integrating sphere and a laser emitter. Figures 7 and 8 show the real measured spectrum of a laser emitter — a continuous Supercontinuum SUPER K EVO-04-type laser with a series-connected tunable acousto-optical filter of the SuperK Varia type, used in the GET 34-2020 standard.

The integrand $I_2^* = S_\lambda L^{(2)}_\lambda \tau_\lambda$ for such a reference source is shown in Fig. 9 (line *1*); line *2* is a spectral-independent and

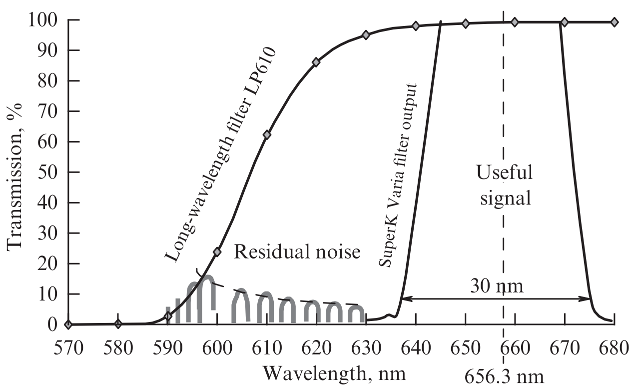

Figure 7. Transmission spectrum of a tunable acousto-optical filter of the SuperK Varia type for radiation from a continuous Supercontinuum Super K EVO-04-type laser.

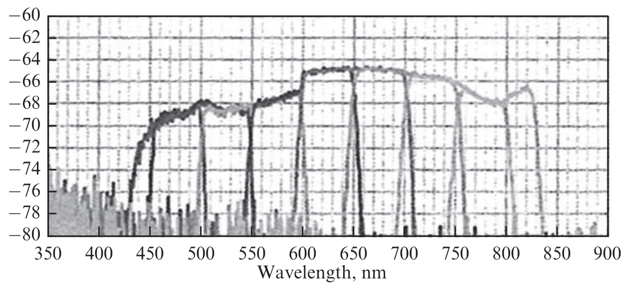

Figure 8. Spectral transmittance of SuperK Varia acousto-optical filter.

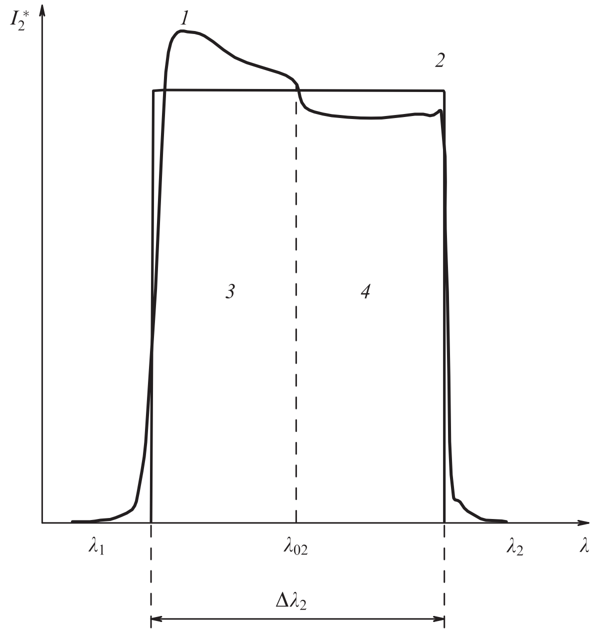

Figure 9. Emission spectrum of reference source.

area-equivalent function. In this case, the photocurrent generated by the photodiode from the source radiation can be calculated using a relationship similar to that obtained for the object, and has the form

$$I^{(2)}_{\mathrm{FD}} = F\varepsilon S_{\lambda_{02}} L^{(2)}_{\lambda_{02}} \Delta\lambda_2\,, \tag{10}$$

where $I^{(2)}_{\mathrm{FD}}$ is the photocurrent generated in the photodiode by radiation from the reference source, [A].

In relation (10), the transmittance of the bandpass optical filter (τ_λ) is absent, since its value is taken equal to unity, and the values of the spectral photodiode sensitivity (S_λ) and the spectral radiance $(L^{(2)}_\lambda)$ of the reference source are taken for the effective wavelength λ_{02}, which is calculated using a relationship similar to expression (8):

$$\int_{\lambda_1}^{\lambda_{02}} S_\lambda L^{(2)}_\lambda \tau_\lambda \,\mathrm{d}\lambda = \int_{\lambda_{02}}^{\lambda_2} S_\lambda L^{(2)}_\lambda \tau_\lambda \,\mathrm{d}\lambda\,. \tag{11}$$

We apply similar reasoning and mathematical operations to an absolutely black body. The integrand $I_3^* = S_\lambda L_{\mathrm{b},\lambda}(\lambda, T)\tau_\lambda$ for the absolutely black body is shown in Fig. 10 (line *1*); line *2* is a spectral-independent and area-equivalent function. In this case, the photocurrent generated by the photodiode detector from the radiation of the black body can be calculated using a relationship similar to that obtained for the object and for the reference source, and is of the form

$$I^{(\mathrm{ABB})}_{\mathrm{FD}} = F\varepsilon S_{\lambda_{03}} L_{\mathrm{b},\lambda}(\lambda_{03}, T)\Delta\lambda_3\,, \tag{12}$$

where $I^{(\mathrm{ABB})}_{\mathrm{FD}}$ is the photocurrent generated by the photodiode from the radiation of the black body and $L_{\mathrm{b},\lambda}(\lambda_{03}, T)$ is the radiance of an absolutely black body at effective wavelength λ_{03}, which is described by Planck's formula (2).

In relation (12), the value of the transmittance of the bandpass optical filter (τ_λ) is also taken equal to unity, and the values of the spectral photodiode sensitivity (S_λ) and the spectral radiance of the black body $(L_{\mathrm{b},\lambda})$ are taken for the

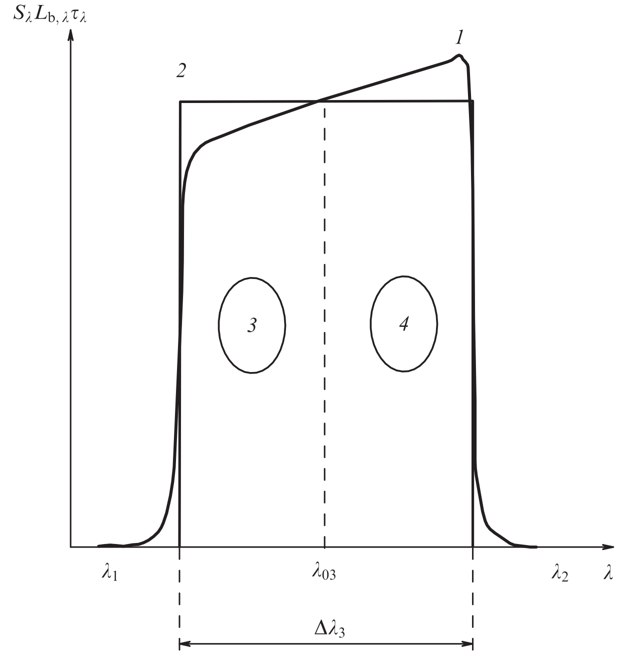

Figure 10. Emission spectrum of an absolutely black body.

effective wavelength λ_{03}, which is calculated by a relation similar to expression (8):

$$\int_{\lambda_1}^{\lambda_{03}} S_\lambda L_{\mathrm{b},\lambda}(\lambda, T)\tau_\lambda \,\mathrm{d}\lambda = \int_{\lambda_{03}}^{\lambda_2} S_\lambda L_{\mathrm{b},\lambda}(\lambda, T)\tau_\lambda \,\mathrm{d}\lambda \,. \qquad (13)$$

For the same given spectral band, if the radiances of the object under study and the reference source are equal, the measured photocurrents are also equal, i.e.,

$$I_{\mathrm{FD}}^{(1)} = I_{\mathrm{FD}}^{(2)} = F\varepsilon S_{\lambda_{01}} L_{\lambda_{01}}^{(1)} \Delta\lambda_1 = F\varepsilon S_{\lambda_{02}} L_{\lambda_{02}}^{(2)} \Delta\lambda_2 \,. \qquad (14)$$

We represent the object's radiance $L_{\lambda_{01}}^{(1)}$ at effective wavelength λ_{01} as the product of the radiance of an ideal black body at effective wavelength λ_{03} and a coefficient taking into account the difference between the emission spectra of the real object and the ABB, i.e.,

$$L_{\lambda_{01}}^{(1)} = k_{\mathrm{NES1}} L_{\mathrm{b},\lambda}(\lambda_{03}, T) \,, \qquad (15)$$

where k_{NES1} is the coefficient of reduction of the emission spectrum of an object to the emission spectrum of a perfect black body, calculated from the relation

$$k_{\mathrm{NES1}} = \int_{\lambda_1}^{\lambda_2} \frac{S_\lambda L_\lambda^{(1)} \tau_\lambda \,\mathrm{d}\lambda}{L_{\lambda_{01}}^{(1)}} \left(\int_{\lambda_1}^{\lambda_2} \frac{S_\lambda L_{\mathrm{b},\lambda}(\lambda, T)\tau_\lambda \,\mathrm{d}\lambda}{L_{\mathrm{b},\lambda}(\lambda_{03}, T)} \right)^{-1} , \qquad (16)$$

which, in view of condition (7) extended to an ABB, takes on the form

$$k_{\mathrm{NES1}} = \frac{S_{\lambda_{01}} \Delta\lambda_1}{S_{\lambda_{03}} \Delta\lambda_3} \,. \qquad (17)$$

We substitute relations (17), (15) into relation (9) to obtain

$$\frac{S_{\lambda_{01}} \Delta\lambda_1}{S_{\lambda_{03}} \Delta\lambda_3} L_{\mathrm{b},\lambda}(\lambda_{03}, T) \Delta\lambda_1 F\varepsilon S_{\lambda_{01}} = I_{\mathrm{FD}}^{(1)} = I_{\mathrm{FD}}^{(2)} \,. \qquad (18)$$

Similar actions performed in relation to the reference source lead to the following relationships:

— for the coefficient of reduction of the emission spectrum of the reference source to the emission spectrum of an ideal black body k_{NES2}:

$$k_{\mathrm{NES2}} = \int_{\lambda_1}^{\lambda_2} \frac{S_\lambda L_\lambda^{(2)} \tau_\lambda \,\mathrm{d}\lambda}{L_{\lambda_{02}}^{(2)}} \left(\int_{\lambda_1}^{\lambda_2} \frac{S_\lambda L_{\mathrm{b},\lambda}(\lambda, T)\tau_\lambda \,\mathrm{d}\lambda}{L_{\mathrm{b},\lambda}(\lambda_{03}, T)} \right)^{-1}$$
$$= \frac{S_{\lambda_{02}} \Delta\lambda_2}{S_{\lambda_{03}} \Delta\lambda_3} \,, \qquad (19)$$

— for photocurrent:

$$\frac{S_{\lambda_{02}} \Delta\lambda_2}{S_{\lambda_{03}} \Delta\lambda_3} L_{\mathrm{b},\lambda}(\lambda_{03}, T) \Delta\lambda_2 F\varepsilon S_{\lambda_{02}} = I_{\mathrm{FD}}^{(2)} = I_{\mathrm{FD}}^{(1)} \,. \qquad (20)$$

Relations (18) and (20) can be taken as the basis for the measurement equation with equal grounds. In this case, depending on the choice made, it is necessary to either measure the emission spectrum of the object and use relation (18), or use a pre-measured emission spectrum of a calibrated reference source and use relation (20).

We choose relation (20) as the basis for the measurement equation. Let's modify this equation, for which we use the known calculated relationship for the spectral sensitivity of the photodiode, which has the form

$$S_\lambda = \eta \, \frac{\lambda e}{hc} \,, \qquad (21)$$

where η is the quantum efficiency of the photodiode, λ is the wavelength, h is Planck's constant, e is the elementary charge, and c is the speed of light.

Since the quantities h, e, c are physical constants, and the quantum efficiency of a photodiode detector in a rather narrow spectral band (width of 10–20 nm, which is usually used in the measurements) is also a constant value, the ratio of the spectral sensitivities of the same photodiode detector for different wavelengths can be written as

$$\frac{S_{\lambda_{02}}}{S_{\lambda_{03}}} = \frac{\lambda_{02}}{\lambda_{03}} \,. \qquad (22)$$

In view of formula (22), relation (20) takes on the form

$$\frac{\lambda_{02}}{\lambda_{03}} \frac{\Delta\lambda_2}{\Delta\lambda_3} L_{\mathrm{b},\lambda}(\lambda_{03}, T) \Delta\lambda_2 F\varepsilon S_{\lambda_{02}} = k_{\mathrm{NES2}} L_{\mathrm{b},\lambda}(\lambda_{03}, T) \Delta\lambda_2 F\varepsilon S_{\lambda_{02}} \,,$$
$$k_{\mathrm{NES2}} L_{\mathrm{b},\lambda}(\lambda_{03}, T) \Delta\lambda_2 F\varepsilon S_{\lambda_{02}} = I_{\mathrm{FD}}^{(2)} \,. \qquad (23)$$

On solving system (23) together with Planck's formula (2) and performing the logarithm operation, we obtain the required equation for measuring the thermodynamic temperature of the radiating cavity of the ABBM:

$$L_{\mathrm{b},\lambda}(\lambda_{03}, T) = \frac{c_1}{n^3 \lambda_{03}^5} \frac{1}{\exp\,(c_2/n\lambda_{03} T) - 1} \,,$$
$$L_{\mathrm{b},\lambda}(\lambda_{03}, T) = \frac{I_{\mathrm{FD}}^{(2)}}{k_{\mathrm{NES2}} \, \Delta\lambda_2 F\varepsilon S_{\lambda_{03}}} \,, \qquad (24)$$

where n is the refractive index of the air. It should be noted here that the refractive index in formulas (24) is used or not used depending on the conditions under which the photodiode detector was calibrated. If the calibration was carried out in a vacuum, then the refractive index in formulas (24) is required. If the calibration was performed under real atmospheric conditions, then the refractive index is not used in formulas (24), since, during calibration, the refractive index is already taken into account in the measured signal.

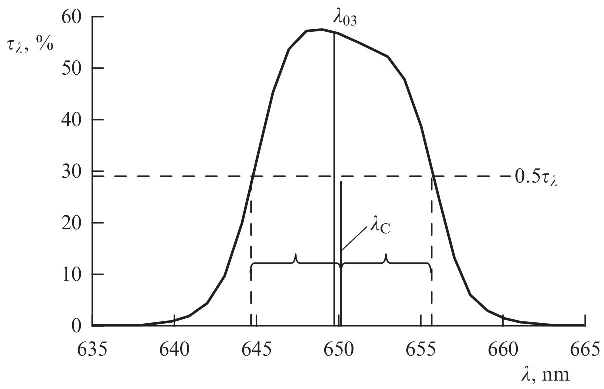

Figure 11. Transmittance of FBH 650-10 bandpass optical filter from Thorlabs (USA).

3.4 On the use of effective and central wavelengths

In world practice, it is generally accepted to use the concept of central wavelength λ_C in a radiation band of a given width. It is for this wavelength that the radiance of the black body is traditionally calculated, from which the desired thermodynamic temperature is then found. In this case, the value of the central wavelength λ_C is determined from the actual radiation spectrum proceeding from the level of 50% of the maximum radiation power. Since the real radiation spectrum is formed by a bandpass optical filter, in practice the central wavelength is found from its transmission spectrum proceeding from the level of 50% of the maximum transmission ($0.5\tau_\lambda$). Figure 11 clearly demonstrates how the center wavelength is determined and how it actually relates to the effective wavelength λ_{03} by the example of the transmission spectrum of an FBH 650-10 bandpass optical filter with the central wavelength equal to 650 nm.

Particular attention should be paid to the fact that the use of the central wavelength in calculations is incorrect, since the transmission spectra of almost all optical filters, as well as filter radiometers, are asymmetric, and therefore it is correct to use the effective wavelength, as was done above when deriving the measurement equation. As already mentioned, by the effective wavelength λ_{03} is meant the wavelength for which condition (13) is satisfied, and by the effective bandwidth of the bandpass filter $\Delta\lambda_2$ here is meant the width of the spectrum-independent filter transmission function, which is equivalent in area to the real spectrum-dependent transmission function (see Fig. 9).

For real-world practice, it is important that the effective wavelength be different for different temperatures if there is even a slight asymmetry in the spectral transmittance function $\tau_\lambda(\lambda)$ of the optical bandpass filter. This fact must be taken into account when processing the results and calculating the thermodynamic temperature. By way of example, Table 1 shows the effective wavelength values for the optical bandpass filter FBH 650-10 from Thorlabs (USA) that we employ, which were calculated for different temperatures using relation (8).

Table 1.

Phase transition	Thermodynamic temperature, K	λ_{03}, nm	λ_C, nm
Cu	1357.78	648.18	650
Co–C	1597.4	648.14	650
Re–C	2747.8	648.03	650
WC–C	3020.9	648.01	650

As follows from Table 1, for the specific FBH 650-10 filter used in the GET 34-2020 standard, in the temperature range of 1357–3021 K, the effective wavelength shifts by 1.82–1.99 nm relative to the central wavelength, which must be taken into account in calculations.

To estimate the error that can result from incorrect use of the central wavelength, Table 2 shows the values of thermodynamic temperatures calculated using Planck's formula (2) for the central and effective wavelengths.

The calculations were performed for temperatures and effective wavelengths indicated in Table 1 for the central wavelength $\lambda_C = 650$ nm at the same spectral radiance and a specific thermodynamic temperature.

As regards the effective wavelength, another feature should also be noted, which is the following. Since, initially, in measurements by the absolute method, the thermodynamic temperature of the ABBM is a priori unknown, it is not possible to straightforwardly calculate the value of the effective wavelength using relation (8). To get out of this situation, it is necessary to use the so-called iteration method, according to which, at the first iteration step, a reasonable temperature value is set, close to the expected temperature of the ABBM, and the calculation of λ_{03} and the temperature is performed for it. Then, at the 2nd iteration step, new values of λ_{03} and the temperature are calculated, using the temperature value obtained at the previous iteration step, etc. As practice has shown, 4–5 iteration steps are sufficient.

3.5 Error estimate

The relative nonexcluded systematic error (NSE) of thermodynamic temperature measurements using the absolute method is calculated proceeding from the measurement equation (24) according to the relation

$$\delta T = \frac{\Delta T}{T} = \Bigg\{ (\delta\lambda_{03})^2 \left[(\ln c_1)^2 + (5 \ln \lambda_{03})^2 + 25 \right] + \lambda_{03}^2 \left[(\delta I_{\mathrm{FD}}^{(2)})^2 + (\delta F)^2 + (\delta\varepsilon)^2 \right] + \lambda_{03}^2 \left[(\delta S_\lambda)^2 + \big(\delta(\Delta\lambda_2)\big)^2 \right] \Bigg\}^{0.5} \times \left\{ \lambda_{03} \left[\ln c_1 - 5 \ln \lambda_{03} - \ln \left(\frac{I_{\mathrm{FD}}^{(2)}}{F S_\lambda} \right) + \ln(\Delta\lambda_2) \right] \right\}^{-1} .$$

Table 2.

$T(\lambda_{03})$, K	$T(\lambda_C)$, K	$L_{b,\lambda}$, W m^{-3}	$\Delta T = T(\lambda_C) - T(\lambda_{03})$, K	$100\Delta T/T(\lambda_{03})$, %
1357.78	1360.42	2.68214×10^8	2.64	0.19
1597.4	1600.3	3.09407×10^9	2.9	0.18
2747.8	2750.9	1.02369×10^{12}	3.1	0.11
3020.9	3023.8	2.12125×10^{12}	2.9	0.09

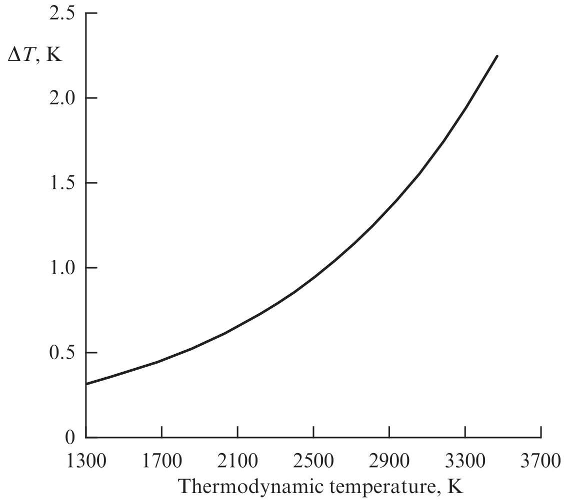

Figure 12. Dependence of absolute nonexcluded systematic error on temperature level.

For the specific measuring system we are using, the following approximating equation for the temperature dependence of the absolute NSE was obtained: $\Delta T = 0.0991 \exp(0.0009T)$, from which, for example, it follows that for thermodynamic temperature $T = 3473.16$ K (3200 °C) the NSE is $\Delta T = 2.26$ K. Figure 12 shows a graphical dependence of the indicated NSE on the temperature level.

When calculating the error, the following numerical parameter values were used:

$$c_1 = 3.74177118 \times 10^{-16} \text{ W m}^{-2},$$
$$S_\lambda = 0.5246 \text{ A W}^{-1}, \quad \lambda_{03} = 6.48 \times 10^{-7} \text{ m},$$
$$\Delta\lambda_2 = 1.542 \times 10^{-8} \text{ m}, \quad F = 2.8493 \times 10^{-10} \text{ m}^2,$$
$$I_{\text{FD}}^{(2)} = 6 \times 10^{-10} - 3.9 \times 10^{-6} \text{ A}, \quad \varepsilon = 0.9994.$$

The absolute values of the NSE are taken based on the best capabilities of the available measuring equipment and are $\Delta\lambda_{03} = 0.02$ nm; $I_{\text{FD}}^{(2)} = 0.0005 I_{\text{TR}}$ A; $\Delta F = 0.0015F$ m^2; $\Delta S_\lambda = 2 \times 10^{-4} S_\lambda$ A W^{-1}; $\Delta\varepsilon = 0.0005$.

Collected in Table 3 are the contributions of the components of the relative error of temperature measurement by the direct (absolute) method implemented according to the proposed algorithm.

Table 3.

Relative measurement error	Value
Effective wavelength	30 ppm
Emission spectrum bandwidth	30 ppm
Trap-detector photocurrent	0.05%
Spectral sensitivity of trap detector	0.02%
Geometric factor	0.15%

3.6 Discussion and recommendations

Analysis of the contribution of the error components shows that the most significant one is the influence of the error in measuring the effective wavelength, which is quite logical and explicitly follows from Planck's formula, in which the wavelength appears to the 5th power. It is significant that the document [1, p. 50] presents an absolute error in wavelength measurement equal to $\Delta\lambda = 0.1$ pm $= 10^{-13}$ m. In this connection, it is quite natural to ask the document developers whether such a recommended error value is practically achievable. The point is that, to measure thermodynamic temperature, one has to know exactly the transmission spectrum of a filter radiometer or, as in our case, a bandpass optical filter. The specified spectrum can only be measured using spectrum analyzers, the most precise of which have a wavelength measurement uncertainty of no better than $\Delta\lambda = 0.01$ nm $= 10^{-11}$ m. For example, here are the technical characteristics of the AQ6370D optical spectrum analyzer (Yokogawa Electric CIS):

— wavelength measurement range: 600–1700 nm;

— absolute uncertainty of wavelength measurement: 0.01 nm;

— resolution limit: 0.02 nm.

The value $\Delta\lambda = 0.1$ pm indicated in the document can indeed be achieved using precision wavelength meters, for example, meters manufactured by Bristol Instruments (USA), model 671A. However, such meters are not spectrum analyzers, but merely measure the wavelength of laser radiation, and therefore do not solve the problem of measuring the transmission spectrum of the filter. Therefore, the uncertainty declared in document [1] raises well-founded doubts.

Furthermore, it is noteworthy that, even if all other parameters of the measurement equation are perfectly determined (with zero uncertainty), then in this case the error in knowing the wavelength, equal to $\Delta\lambda = 0.01$ nm $= 10^{-11}$ m, leads to an error in temperature measurement $u_{k=1} = 0.10$ K for the phase transition of copper and $u_{k=1} = 0.15$ K for the phase transition of the tungsten-based eutectic WC–C mixture. For real errors in measuring other parameters, the indicated error in measuring wavelength leads to an error in measuring temperature $u_{k=1} = 0.15$ K for the phase transition of copper and $u_{k=1} = 0.74$ K for the phase transition of a tungsten-based eutectic mixture.

In this connection, the overly accurate measurement data presented by some national metrological institutes, for example, in Ref. [5], appear to be somewhat strange. Therefore, this issue needs serious verification and clarification. As our research shows, to ensure the specified temperature measurement accuracy (expanded error $u_{k=2} = 2$ K at a temperature $T = 3473$ K), the relative error in measuring the effective wavelength must be no worse than $\delta\lambda_{03} = 30$ ppm, or $\Delta\lambda = 0.02$ nm for $\lambda_{03} = 650$ nm. We emphasize once again that the indicated error is at the limit of accuracy of existing optical spectrum analyzers.

Another significant constituent of the error is the uncertainty in measuring the effective bandwidth $\Delta\lambda_2$ of the optical filter. The operation of replacing the actual transmittance spectrum of a bandpass optical filter with an equivalent spectrum-independent function of effective bandwidth $\Delta\lambda_2$ has the following disadvantage. The real spectrum, taking into account the out-of-band transmission of the filter, extends to the entire wavelength dynamic range of the quantum trap detector (400–900 nm), and the given equivalent spectrum extends to only a narrow wavelength range encompassing the effective wavelength. In this case, since the spectral sensitivity of the trap detector in the range of 400–900 nm varies nonlinearly in relation to the wavelength, an error is inevitable in the transition to the equivalent function. This is mainly due to the out-of-band transmission of the filter. It seems logical that this drawback can be eliminated by

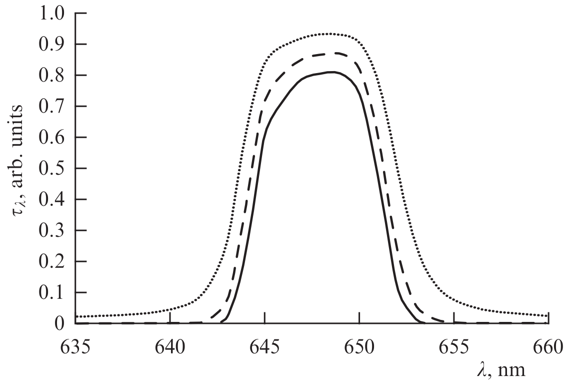

Figure 13. Transmission spectra: three-stage filter — solid line; two-stage filter — dashed line; single-stage filter — dotted line.

using a multistage bandpass filter consisting of a few (2–3 pcs.) nominally similar bandpass filters located sequentially along the optical radiation path, thereby minimizing the influence of out-of-band radiation on the measurement result. By way of example, Fig. 13 shows the transmission spectra of one-, two-, and three-stage filters composed of the nominally same FBH650-10 filters from Thorlabs (USA).

One can clearly see from Fig. 13 that the use of a three-stage filter completely eliminates the problem. It is noteworthy that increasing the number of filter stages significantly reduces the effective bandwidth; for example, for a single-stage filter, its value is $\Delta\lambda_2 = 15.57$ nm, for a two-stage filter, $\Delta\lambda_2 = 7.44$ nm, and for a three-stage filter, $\Delta\lambda_2 = 6.61$ nm. It is therefore obvious that, to reduce the contribution of $\Delta\lambda_2$ to the uncertainty of temperature measurement, it is advisable to use a filter with a broader bandwidth, for example, with a width of 20 nm, then the relative uncertainty is indeed significantly reduced, which is also a big advantage of this technical solution. In particular, broadening the effective filter bandwidth $\Delta\lambda_2$ while keeping the effective wavelength λ_{03} constant (symmetrical broadening) increases the magnitude of the measured current $I_{\mathrm{FD}}^{(2)}$, which allows it to be measured with even higher accuracy. At the same time, it would seem that the error due to averaging of the spectral sensitivity of the photodiode detector $S_{\lambda_{03}}$ should increase as the used part of the spectrum expands. However, this will not happen, since the spectral dependence of $S_{\lambda_{03}}$ is linear (see relation (21)), and the increase in the spectrum width is symmetric. The remaining physical quantities included in Eqns (24) do not depend in any way on the spectrum width. Thus, broadening the effective filter bandwidth only leads to improved accuracy. In this case, of course, it is necessary to take into account that it is impossible to infinitely broaden the spectrum width, since a specific thermodynamic temperature T in Eqns (24) is tied to a specific effective wavelength λ_{03}, so there must be a reasonable balance here. As we have determined, the optimal filter transmission spectrum width is ≈ 20 nm.

Since the effective bandwidth when using a multistage filter is no greater than 10–20 nm, we can safely assume that the spectral sensitivity of the trap detector in a given spectral band varies strictly according to a linear law. In this case, for measurements, it is enough to know the spectral sensitivity of the trap detector only in a very narrow spectral band. This is also a significant advantage of the method compared to the methodology recommended by the primary document [1, p. 22], where out-of-band transmission is taken into account by introducing a special correction factor, whose calculation requires accurate knowledge of the spectral sensitivity of the trap detector over its entire spectral range (400–900 nm). Of course, this significantly increases the measurement error. The measure we propose, on the contrary, significantly reduces the measurement error.

4. Conclusions

In this study, the influence of nonequivalence and asymmetry of the emission spectra of reference radiation sources on the accuracy of reproducing the temperature unit was estimated and taken into account for the first time. These technical solutions provide the required high accuracy of reproduction of the temperature unit (the achieved relative error is within 5×10^{-4}).

The possibility of optimizing the composition of a modern temperature unit standard is shown and substantiated; in particular, the feasibility of replacing a bulky monochromator with a high-quality bandpass optical filter built according to the stage principle is indicated. This makes it possible to significantly simplify the composition and volume of the state primary standard of the temperature unit while simultaneously increasing its accuracy.

The consequences of incorrect use of the central wavelength instead of the effective wavelength in calculations were shown and assessed for the first time.

It is shown that foreign experts somewhat overestimate the accuracy of their measurements, and in doing so do not take into account either the real capabilities of the existing measuring equipment or a number of factors that are highlighted in this article and which make a significant contribution to the overall error.

An analysis of the measurement error has been carried out, from which it can be safely concluded that the achieved accuracy of reproducing the temperature unit on the Russian State primary standard of the temperature unit GET 34-2020 is currently the limit for the present-day level of measuring technology.

The paper analyzes only some of the basic physical, technical, and applied aspects related to the implementation of the direct (absolute) method in reference radiometric thermometry. The first results of using the standard suggest that this methodological approach is indeed promising and provides the expected accuracy.

References

1. “Mise en pratique for the definition of the kelvin in the Si”, BIPM, SI Brochure, 9th ed. (Sèvres Cedex, France: Bureau Intern. des Poides et Mesures, 2019)
2. Pokhodun A I et al. *Meas. Tech.* **64** 541 (2021) https://doi.org/10.1007/s11018-021-01970-w; *Izmerit. Tekh.* (7) 13 (2021) https://doi.org/10.32446/0368-1025it.2021-7-13-21
3. Saunders P et al. “Uncertainty estimation in primary radiometric temperature measurement”, Under the auspices of the Consultative Committee for Thermometry (2018) 70 pp.
4. Anderson V E, Fox N P, Nettleton D H *Appl. Opt.* **31** 536 (1992)
5. Brown S W, Eppeldauer G P, Lykke K R *Metrologia* **37** 579 (2000)
6. Khodunkov V P, Pokhodun A I, RF Patent No. 2697429, 14.08.2019; *Izobret. Polezn. Modeli* (23) (2019)
7. Khodunkov V P, RF Patent No. 2739731, 28.12.2020; *Izobret. Polezn. Modeli* (1) (2021)
8. Zhernovoi A I, RF Patent No. 2452940, 10.06.2012; *Izobret. Polezn. Modeli* (16) (2012)

9. Cao Bolin et al., RF Patent No. 2523775, 20.07.2014; *Izobret. Polezn. Modeli* (20) (2014)
10. Figaro V A et al., RF Patent No. 2381463, 10.02.2010; *Izobret. Polezn. Modeli* (4) (2010)
11. Brown S W, Eppeldauer G P, Lykke K R *Appl. Opt.* **45** 8218 (2006)
12. Eppeldauer G P (Ed.) ''Optical radiation measurements based on detector standards'', NIST Technical Note No. 1621 (Gaithersburg, MD: National Institute of Standards and Technology, 2009); https://tsapps.nist.gov/publication/get_pdf.cfm?pub_id = 901369
13. Anhalt K, Machin G *Philos. Trans. R. Soc. A* **374** 20150041 (2016)
14. Khlevnoi B B et al., RF Patent No. 2148801, 10.05.2000; *Izobret. Polezn. Modeli* (13) (2000)
15. Ogarev S A et al. *Meas. Tech.* **58** 1255 (2016); *Izmerit. Tekh.* (11) 51 (2015)
16. ''Gosudarstvennaya sistema obespecheniya edinstva izmerenii. Gosudarstvennaya proverochnaya skhema dlya sredstv izmerenii temperatury'' (''State system for ensuring the uniformity of measurements. State verification scheme for temperature measuring means''), GOST (State Standard) 8.558-2009 (Moscow: Standartinform, 2019)
17. ''Gosudarstvennaya sistema obespecheniya edinstva izmerenii. Izluchateli v vide modelei absolyutno chernogo tela. Metodika proverki i kalibrovki'' (''State system for ensuring the uniformity of measurements. Emitters in the form of blackbody models. Test and calibration procedure''), GOST (State Standard) 8.566-2012 (Moscow: Standartinform, 2019)
18. Savvatimskiy A I *Plavlenie Grafita i Svoistva Zhidkogo Ugleroda* (Melting of Graphite and the Properties of Liquid Carbon) (Moscow: Fizmatkniga, 2014)
19. Pronin A N *Glavn. Metrolog* (1) 32 (2019)
20. Khodunkov V P, in *Sovremennye Metody i Sredstva Issledovanii Teplofizicheskikh Svoistv Veshchestv. Mezhdunarodnaya Nauchno-Tekhnicheskaya Konf., 23–24 Maya 2019 Goda.* (Modern Methods and Means of Measuring the Thermophysical Properties of Substances. Internat. Scientific and Technical Conf., May 23–24, 2019) (St. Petersburg: ITMO Univ., 2019) p. 27
21. Khodunkov V P, Zarichnyak Yu P, in *Sovremennye Problemy Teplofiziki i Energetiki. Materialy III Mezhdunarodnoi Konf., 19–23 Oktyabrya 2020* (Modern Problems of Thermophysics and Energy Industry. Proc. of the III Intern. Conf.) (Moscow: Izd. MEI, 2020) p. 458
22. Sildoja M et al. *Metrologia* **50** 385 (2013)
23. Martín M J et al. *Int. J. Thermophys.* **38** 138 (2017)
24. Eppeldauer G P et al., NIST Technical Note No. 1621 (Gaithersburg, MD: National Institute of Standards and Technology, 2009) p. 21
25. Mantilla M J *Ukr. Metrol. J.* (2017) http://doi.org/10.24027/2306-7039.1A.2017.100389
26. Linka S ''Untersuchung der Eigenschaften von Schlacken und Schmelzen in technischen Feuerungen'', Dissertation zur Erlangung des Grades Doktor-Ingenieur (Bochum: Fakultät für Maschinenbau der Ruhr–Univ. Bochum, 2003)
27. Brodnikov A F, Vikhareva N A, Cherepanov V Ya *Izmereniya i Etalony Teplovykh Velichin* (Measurements and Standards of Thermal Quantities) (Novosibirsk: Novosibirskii Filial ASMS (Akad. Standartizatsii, Metrologii i Sertifikatsii), 2017)
28. Svet D Ya, RF Patent No. 2255312, 19.08.2003; *Izobret. Polezn. Modeli* (6) (2003)
29. Mukhamedyarov R D, Kharisov R I, RF Patent No. 2086935, 10.01.1994; *Izobret. Polezn. Modeli* (1) (1994)
30. Sirenko A V et al., RF Patent No. 2617725, 26.04.2017; *Izobret. Polezn. Modeli* (12) (2017)
31. Mukhamedyarov R D, Kharisov R I, RF Patent No. 2382994, 27.02.2010; *Izobret. Polezn. Modeli* (6) (2010)
32. Khodunkov V P, RF Patent No. 2685548, 22.04.2019; *Izobret. Polezn. Modeli* (12) (2019)
33. Rumer Yu B, Ryvkin M Sh *Thermodynamics, Statistical Physics, and Kinetics* (Moscow: Mir Publ., 1980); Translated from Russian: *Termodinamika, Statisticheskaya Fizika i Kinetika* (Moscow: Nauka, 1977)
34. Khodunkov V P *Izv. Vyssh. Uchebn. Zaved. Priborostroenie* **62** 1015 (2019) https://doi.org/10.17586/0021-3454-2019-62-11-1015-1021
35. Pokhodun A I *Mir Izmerenii* (1) 32 (2011)
36. Kotov D V, Surzhikov S T *High Temp.* **45** 807 (2007); *Teplofiz. Vys. Temp.* **45** 885 (2007)
37. Khodunkov V P, RF Patent No. 2727347, 21.07.2020; *Izobret. Polezn. Modeli* (21) (2020)
38. Khodunkov V P, RF Patent No. 2746699, 19.04.2021; *Izobret. Polezn. Modeli* (11) (2021)
39. Manasson V A et al., USSR Patent No. 1562711, 07.05.1990; *Byull. Izobret.* (17) (1990)
40. Ivanovskii V V et al., USSR Patent No. 1257412, 15.09.1986; *Byull. Izobret.* (34) (1986)
41. Tolubenskii A G, USSR Patent No. 1758446, 30.08.1992; *Byull. Izobret.* (32) (1992)
42. Dunaev A Yu et al., in *Metrologicheskoe Obespechenie Fotoniki. Vserossiiskaya Nauchno-Tekhnicheskaya Konf., 14–17 Aprelya 2015 Goda, Moskva. Tezisy Dokladov* (Metrological Support for Photonics. All-Russian Scientific and Technical Conf., 14–17 April 2015, Moscow. Abstracts of Reports) (Moscow: VNIIOFI, 2015) p. 27
43. Kvochka V I, Minaeva O A, USSR Patent No. 1314237, 30.05.1987; *Byull. Izobret.* (20) (1987)
44. Khodunkov V P, RF Patent No. 2739724, 28.12.2020; *Izobret. Polezn. Modeli* (1) (2021)
45. Karshenboim S G *Phys. Usp.* **48** 255 (2005); *Usp. Fiz. Nauk* **175** 271 (2005)
46. Dul'nev G N *Teoriya Teplo- i Massoobmena* (Theory of Heat and Mass Transfer) (St. Petersburg: NIU ITMO, 2012)
47. Khodunkov V P, RF Patent No. 2727340, 21.07.2020; *Izobret. Polezn. Modeli* (21) (2020)