METHODOLOGICAL NOTES

PACS numbers: **01.65.+g,** 05.30.Jp, **67.25.–k**

Notes on the properties of helium

S M Stishov

DOI: https://doi.org/10.3367/UFNe.2024.02.039650

Contents

Abstract. **Historical aspects of the specific properties of the melting thermodynamics and superfluid state of helium are discussed.**

Keywords: helium, melting, superfluidity

1. Melting of helium

As is well known, liquid helium (^{3}He and ^{4}He) does not crystallize at atmospheric pressure. To obtain solid helium, it is necessary to apply some pressure, as shown in Fig. 1. To explain this situation, it is commonly, although not quite correctly, stated that the crystal lattice of helium is destroyed by 'zero' vibrations. In fact, the 'zero' energy increases the equilibrium volume of helium so significantly that the existence of crystalline helium at atmospheric pressure becomes energy unfavorable (a detailed discussion of this issue is given in the book by F London [1]). Let us consider plot *2* in Fig. 1, representing the melting curve of ^{3}He. We recall that the observed minimum of pressure was predicted by I Pomeranchuk [2] as resulting from the entropy loss in liquid helium when ordering nuclear spins. This effect manifests itself in negative values of the $\mathrm{d}T/\mathrm{d}P$ derivative observed at the lowest temperatures down to < 1 mK, at which nuclear spin ordering occurs in solid helium. However, the ^{3}He melting curve is plotted on a logarithmic scale (Fig. 2), which masks the true nature of its behavior.

Here, it is worth noting the poorly known feature of the ^{4}He melting curve (Fig. 3), where a hardly noticeable pressure minimum is observed, related probably to the specific behavior of the entropy in superfluid helium (see Ref. [9] on this issue). Figure 4 also demonstrates the poorly known details of the behavior of the ^{3}He melting curve. It is seen that, at a temperature of the order of 1 mK, the curve slope experiences a jump, apparently related to a magnetic phase transition in the solid medium. Figure 5 shows the above behavior of the derivative in more detail. We emphasize that the following relation is apparently valid: $\mathrm{d}P/\mathrm{d}T \to 0$, $T \to 0$, i.e., according to the Nernst theorem, turning $\mathrm{d}P/\mathrm{d}T$ into zero should be expected exactly at $T = 0$ (which is known to

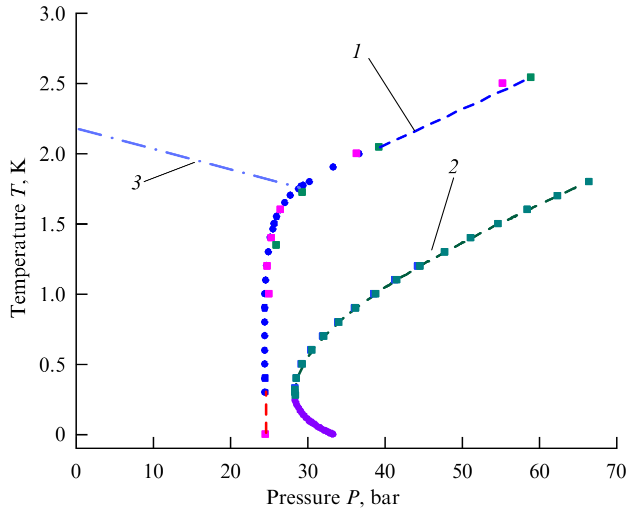

Figure 1. Melting curves for ^{3}He and ^{4}He from data from Refs [3–8]. *1* — ^{4}He, *2* — ^{3}He, *3* — phase transition to superfluid state in ^{4}He.

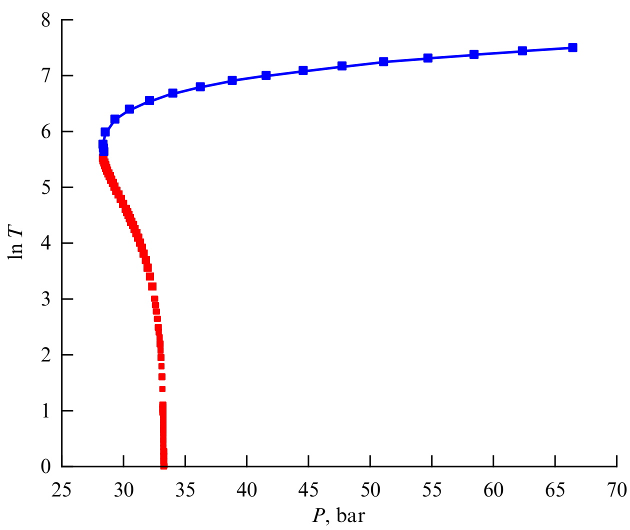

Figure 2. Logarithm of ^{3}He melting temperature as a function of pressure (plotted using data from Refs [6, 8]).

S M Stishov. Lebedev Physical Institute, Russian Academy of Sciences, Leninskii prosp. 53, 119991 Moscow, Russian Federation
E-mail: stishovsm@lebedev.ru

Received 5 October 2023, revised 13 February 2024
Uspekhi Fizicheskikh Nauk **194** (4) 360 – 364 (2024)
Translated by V L Derbov

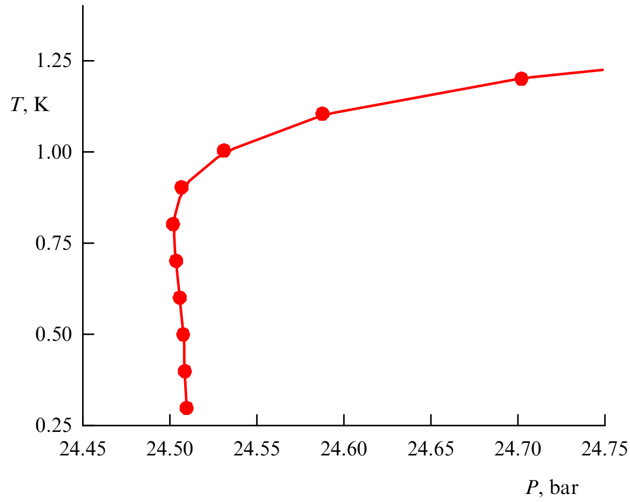

Figure 3. Melting curve for ^{4}He (plotted using high-resolution data from Ref. [7]).

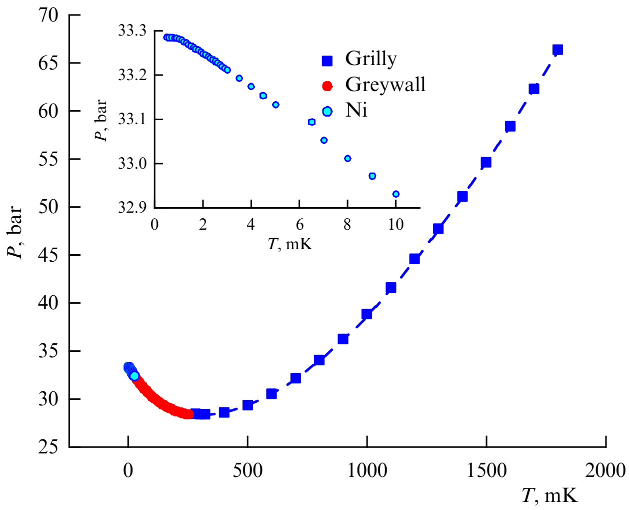

Figure 4. Melting curve of ^{3}He from data from Refs [6, 8, 10]. Inset shows low temperature part of the curve, testifying to a phase transition in solid ^{3}He.

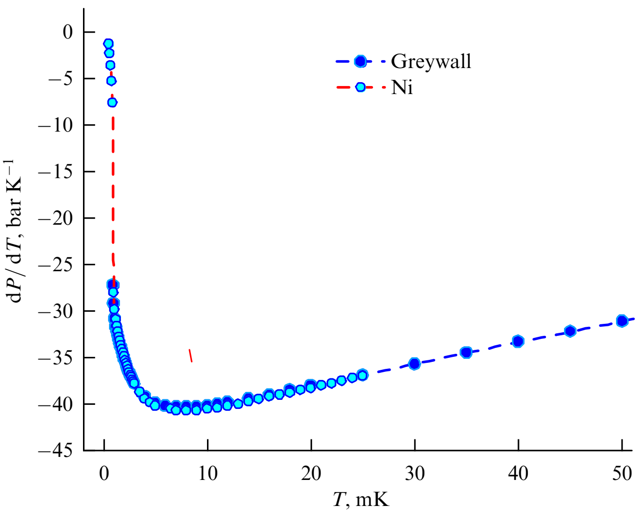

Figure 5. Temperature dependence of derivative of melting pressure of ^{3}He $\mathrm{d}P/\mathrm{d}T$ according to data from Refs [8, 9]. Sharp discontinuity of the derivative is seen at ~ 1 mK. Apparently, derivative $\mathrm{d}P/\mathrm{d}T \to 0$ exactly at $T \to 0$.

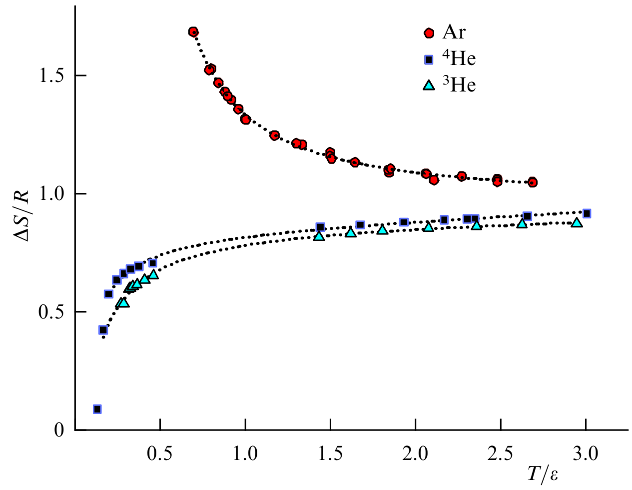

Figure 6. Melting entropy of argon and isotopes of helium as a function of reduced melting temperature from data of Refs [4, 11]. Here, $\varepsilon_{\mathrm{Ar}} = 119.3$ K, $\varepsilon_{\mathrm{He}} = 10.22$ K are minimum values of the pair interaction potential for Ar and He, respectively.

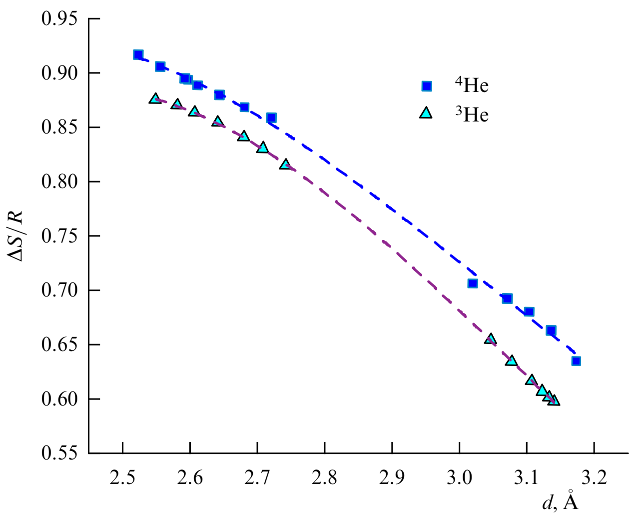

Figure 7. Dependence of melting entropy of helium isotopes on interatomic distance $d = (V/N)^{1/3}$ of solid helium along the melting curve. (Calculated using data from [4].)

be impossible), and before that the slope of the ^{3}He melting curve is always finite.

Figure 6 presents the dependences of melting entropies for helium and argon isotopes as functions of the dimensionless melting temperature. It is seen that in the classical limit at $T \to \infty$ the melting entropies of Ar and He take the same value, whereas at low temperatures the melting entropy of helium is always smaller than that of argon. The next two figures at least partially clarify this problem. Figure 7 shows the melting entropies of helium isotopes as functions of their mean interatomic distance along the melting curve. The substantial difference in the melting entropy values in isotopes reduced to the same volume is obvious. In this connection, note that, in the case of helium isotopes, the effects of quantum statistics decay rather rapidly with temperature, and in the region of temperatures considered, quantum effects manifest themselves only as a consequence of space quantization, written in the known expression for the classical partition function back by Sackur and Tetrode. As a

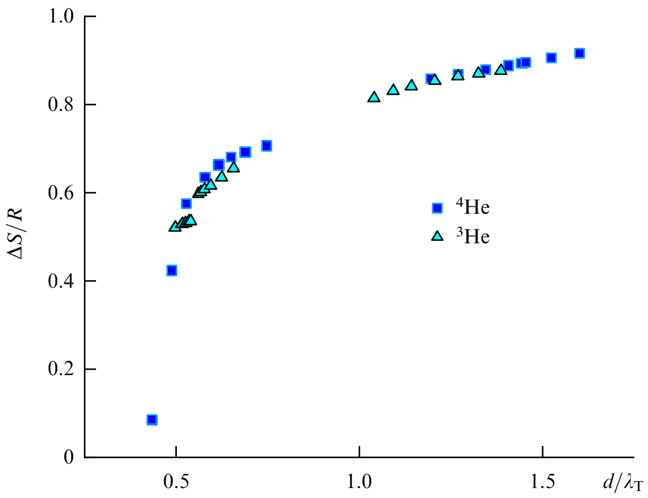

Figure 8. Dependence of the melting entropy of helium isotopes on reduced mean interatomic distance d/λ_T, where $d = (V/N)^{1/3}$, $\lambda_T = h/(2\pi mT)^{1/2}$ is the thermal de Broglie wavelength for solid helium along the melting curve (calculated using data from [4]). It is interesting that the dependence of melting entropy of helium isotopes on interatomic distance is scaled using the thermal de Broglie wavelength in correspondence with the Sackur–Tetrode formula.

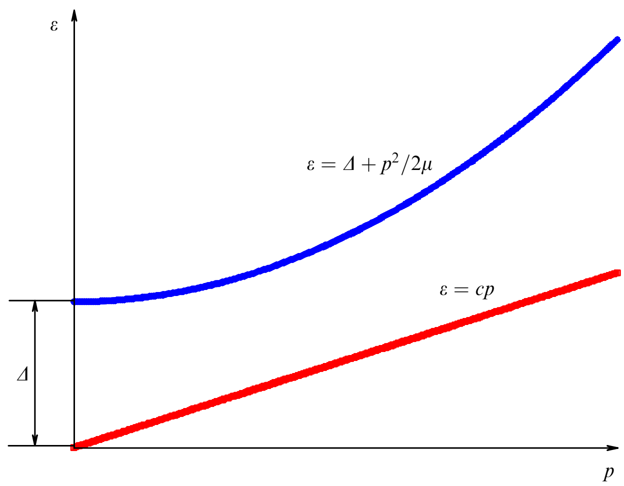

Figure 9. Spectrum of elementary excitations in superfluid helium, initially proposed by Landau [17].

result, if we scale the interatomic distances to the thermal de Broglie wavelength $\lambda_T = h/(2\pi mT)^{1/2}$, then, as is seen from Fig. 8, the melting entropies of He isotopes become practically undistinguishable. It is useful to write the known expression for the entropy of a classical ideal gas:

$$\frac{S}{kN} = \ln\left(\frac{V}{N\lambda_T^3}\right) + \frac{5}{2}\,, \tag{1}$$

or

$$\frac{S}{kN} = 3\ln\left(\frac{d}{\lambda_T}\right) + \frac{5}{2}\,, \tag{2}$$

meaning that the thermal wavelength λ_T in this case plays the role of a scaling factor or, if you wish, a reduction parameter. It is obvious that λ_T maintains this role in the case of interacting systems as well, which finds its implementation in the data presented in Figs 6 and 8.

2. On the superfluidity of HeII

In 1938, two papers were published in the journal *Nature*: "The Viscosity of Liquid Helium Below the λ-Point" by P Kapitza [12] and "Liquid Helium Flow II" by J F Allen and A D Misener [13], which reported the discovery of a new physical phenomenon — superfluidity in the low-temperature phase of liquid helium. This discovery and the subsequent experimental and theoretical studies gave rise to general revolutionary concepts in physics. It was exactly this circumstance that impelled the Nobel Committee to award the Nobel Prize to Pyotr Kapitza in 1978, 40 years after the publication. And what about Allen and Misener? They are not among the laureates. There is evidence that Kapitza refused to receive the prize together with Allen and Misener, and this is why the process of awarding the prize took so much time. The events related to the discovery of superfluidity and the publication and awarding of the Nobel Prize are described by many authors (see, e.g., [14–16]). Here, it is important to note that it was P L Kapitza who proposed the very term 'superfluidity,' which prophetically combined the newly discovered phenomenon with another 'superproperty' of matter, superconductivity. Concerning the paper by Allen and Misener, Griffin wrote [16]: "Without the stimulus of the earlier work of both Allen and Misener, it is doubtful whether Kapitza would ever have become interested in measuring the viscosity of helium-II or have received the Nobel prize." Indeed, Kapitza new about the work of Allen and Misener, including from his correspondence with Rutherford on the studies of liquid helium, carried out in the Mond Laboratory, and understood their significance. Kapitza himself said [14]: "For the first time in my life I managed to find such a fundamental property of matter. I carried out many experiments in various fields, but this was already a matter of good or bad luck. When such an opportunity comes up, you can't miss it." It is difficult to interpret Kapitza's words, especially in connection with Griffin's statement above, but one thing is clear: Kapitza did not miss his chance.

Next, it remained to construct a theory of helium superfluidity. To begin with, Kapitza achieved the miraculous release of Lev Landau from the hands of the Chekists (for details about Landau's release, see [14]). Landau got to work, and in 1941 published an article entitled "The Theory of Helium Superfluidity" [17]. In parallel, F London and L Tisza, Landau's former postdoc from his Kharkov days, worked on this problem. It was Tisza who proposed the two-fluid model of superfluid helium. Both London and Tisza considered Bose–Einstein (BE) condensation to be the main factor determining superfluidity [1]. Landau reformulated Tisza's two-fluid theory, removing from it the analysis of the behavior of individual helium atoms, operating instead with the elementary excitations of a macroscopic system — phonons and rotons. Landau completely ignored the possible effect of BE condensation and believed, following Kapitza, that since 'superproperties' are found both in Bose and in Fermi systems, then, apparently, these properties are not determined by statistics. To explain the phenomenon of helium superfluidity, Landau postulated the existence of a special spectrum of excitations, consisting of two branches with an energy gap between them (Fig. 9), separating the states with potential and vortex motions of the liquid.

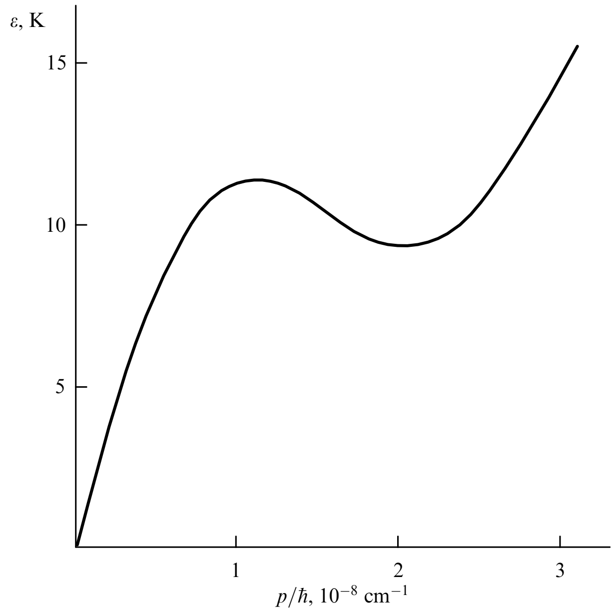

Figure 10. Spectrum of elementary excitations in superfluid helium proposed by Landau in Ref. [20].

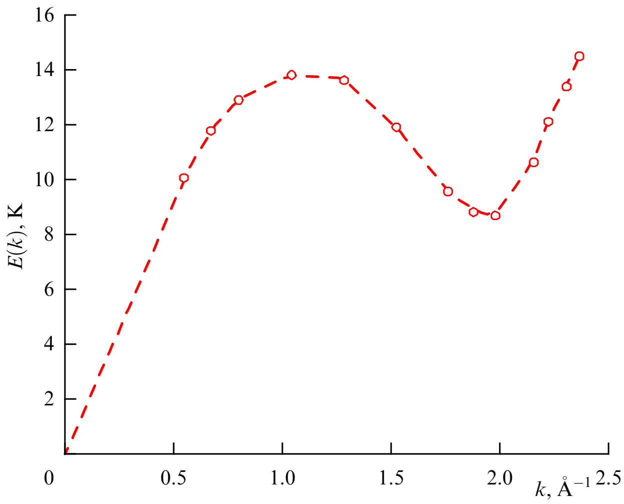

Figure 11. First experimental confirmation of existence of rotons in spectrum of elementary excitations, following from data on neutron inelastic scattering [21].

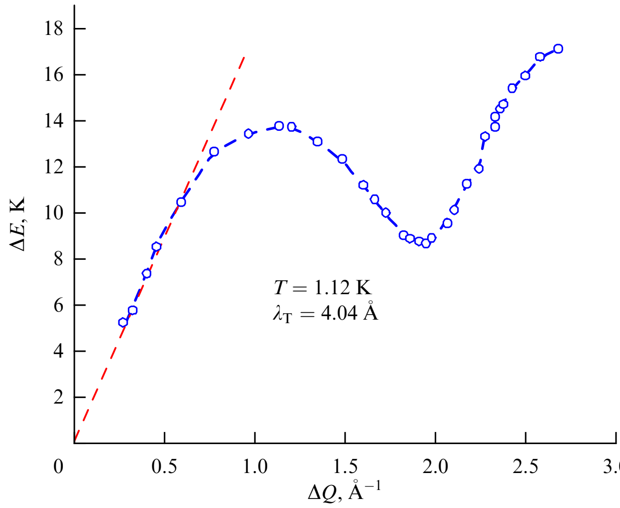

Figure 12. Results of detailed studies of excitation spectrum of superfluid helium [22].

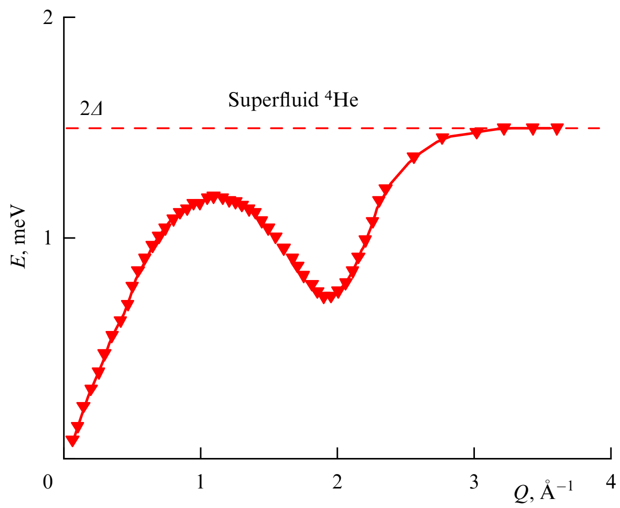

Figure 13. Results of modern studies of phonon-roton mode in excitation spectrum of superfluid helium [25].

However, on October 21, 1946, at the general meeting of the Physics and Mathematics Department of the USSR Academy of Sciences, N N Bogolyubov reported his work entitled "To the Theory of Superfluidity," where he constructed a microscopic theory of a weakly nonideal Bose gas possessing the property of superfluidity [18].

Judging by some evidence, this report by Bogoliubov was sharply criticized by Landau (see, e.g., [19]). Nevertheless, after a very short time, Landau published a brief report [20], where he actually used the results of Bogoliubov's work (Fig. 10). Subsequently, the shape of the spectrum of elementary excitations in helium, proposed in the papers by Landau and Bogoliubov, found complete experimental confirmation in the results of neutron studies (Figs 11–13). Theoretical studies of the spectrum of excitations in a Bose liquid carried out by L Pitaevskii [23] showed that at large momenta the phonon-roton (PR) excitation should decay into two excitations, each with energy Δ, and, therefore, the curve $\varepsilon(p)$ tends to the limiting value 2Δ, which has been experimentally confirmed [24].

Although the issue of the BE condensation role was almost forgotten due to the total success of Landau's theory of superfluidity, the problem remained unresolved. However, in 2012, an article appeared with the intriguing title, "Phonon-roton modes in liquid ^{4}He coincide with Bose–Einstein Condensation" [26], concluding that the phonon-roton mode in the excitation spectrum of liquid helium arises only with Bose–Einstein condensation. Indeed, the width of the scattering function in the normal phase of liquid helium is extremely large; the full width at half maximum (FWHM) of the function $S(Q, \omega)$ in the normal phase of helium is 2000–3000 times greater than the width of the phonon-roton mode at 1 K (Fig. 14), which corresponds to a very short lifetime of excitations. The existence of a rotonic peak, albeit a strongly broadened one, does not mean anything, since this feature is observed in many classical liquids and is due to fairly universal interparticle interactions. Let us recall here the well-known Feynman formula [27], relating the frequency

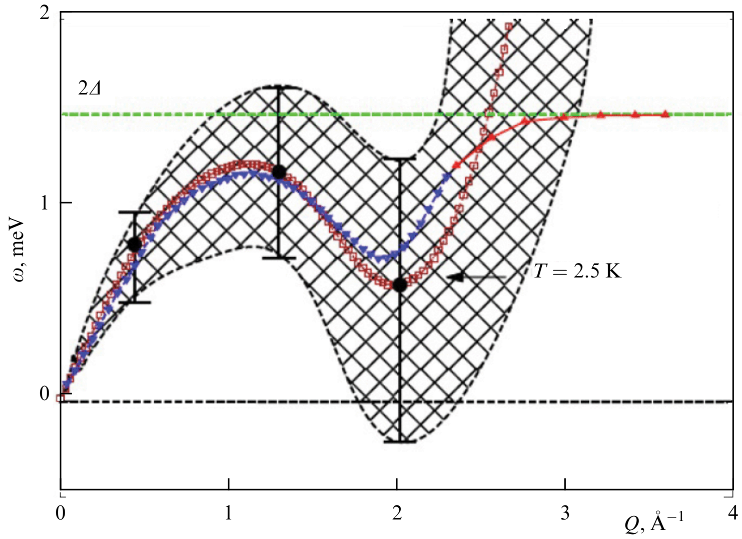

Figure 14. Dispersion of the center and half-width of function $S(Q, \omega)$ of liquid helium at $T = 2.5$ K and saturated vapor pressure (squares). Phonon-roton dispersion curve at low temperature (triangles) [28].

spectrum and the structure factor: $E(k) = \hbar^2 k^2/2mS(k)$, or $\omega(k) = \hbar^2 k^2/2mS(k)$. As can be seen, the roton minimum $\omega(k)$ corresponds to the maximum of the static structure factor, which, in turn, means the existence of short-range structural order.

References

1. London F *Superfluids* Vol. 2 (New York: Wiley, 1954)
2. Pomeranchuk I *Zh. Eksp. Teor. Fiz.* **20** 919 (1950)
3. Domb C, Dugdale J S, in *Progress in Low Temperature Physics* Vol. 2 (Ed. C J Gorter) (Amsterdam: North-Holland, 1957) p. 338
4. Grilly E R, Mills R L *Ann. Physics* **8** 1 (1959)
5. Mills R L, Grilly E R, Sydoriaik S G *Ann. Physics* **12** 41 (1961)
6. Grilly E R *J. Low Temp. Phys.* **4** 615 (1971)
7. Grilly E R *J. Low Temp. Phys.* **11** 33 (1973)
8. Greywall D S *Phys. Rev. B* **33** 7520 (1986)
9. Goldstein L *Phys. Rev.* **122** 726 (1961)
10. Ni W et al. *J. Low Temp. Phys.* **99** 167 (1995)
11. Stishov S M *Sov. Phys. Usp.* **18** 625 (1975); *Usp. Fiz. Nauk* **114** 3 (1974)
12. Kapitza P *Nature* **141** 74 (1938)
13. Allen J F, Misener A D *Nature* **142** 643 (1938)
14. Rubinin P E *Phys. Usp.* **40** 1249 (1997); *Usp. Fiz. Nauk* **167** 1349 (1997)
15. Balibar S *J. Low Temp. Phys.* **146** 441 (2007)
16. Griffin A *Phys. World* **21** (08) 27 (2008) http://doi.org/10.1088/2058-7058/21/08/36
17. Landau L D *J. Phys. USSR* **5** 71 (1941); *Zh. Eksp. Teor. Fiz.* **11** 592 (1941)
18. Bogolubov N *J. Phys. USSR* **11** 23 (1947); *Izv. Akad. Nauk SSSR. Ser. Fiz.* **11** (1) 77 (1947); *Usp. Fiz. Nauk* **93** 552 (1967)
19. Shirkov D, Dubna. JINR Periodical (31–32) 3 (2009)
20. Landau L *J. Phys. USSR* **11** 91 (1947)
21. Yarnell J L et al. *Phys. Rev. Lett.* **1** 9 (1958)
22. Henshow D G, Woods A D B *Phys. Rev.* **121** 1266 (1961)
23. Pitaevskii L P *Sov. Phys. JETP* **9** 830 (1959); *Zh. Eksp. Teor. Fiz.* **36** 1168 (1959)
24. Glyde H R et al. *Europhys. Lett.* **43** 422 (1998)
25. Diallo S O et al. *Phys. Rev. Lett.* **113** 215302 (2014)
26. Bossy J et al. *Europhys. Lett.* **98** 56008 (2012)
27. Feynman R P *Phys. Rev.* **94** 262 (1954)
28. Glyde H R *Rep. Prog. Phys.* **81** 014501 (2018)