 

*METHODOLOGICAL NOTES*

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# Kerr–Newman solution unites gravitation with quantum theory

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## Contents



**Abstract. The model of the Kerr–Newman (KN) electron generated by the superrotating gravitational field of a black hole is modified to a radiating black–white hole that both absorbs and emits electromagnetic waves. Following quantum electrodynamics (QED), we consider the KN solution to be either a bare electron model or a dressed electron model, where the bare electron forms a classical massless relativistic string responsible for the wave properties of the electron as a quantum particle, while the dressed electron forms a heavy electron–positron vacuum core dressed by the KN gravitational field due to the formation of electron and positron Wilson loops. Within the framework of the Kerr–Schild formalism, a class of radiating KN solutions is considered, whereby the electromagnetic field is absorbed by the black hole and at the same time radiated away by its white side.**

**Keywords:** semiclassical gravity, electron as a black hole, Wilson loops, radiating Kerr solution, QED, supersymmetry, Kerr–Newman electrons, classical relativistic strings, bare electron, dressed electron

## 1. Introduction

Uniting gravity and quantum theory is probably the main unsolved problem of modern theoretical physics.

A revolutionary step towards unifying quantum theory with gravity was made in *superstring theory*, which considered particles as extended strings. A counterapproach — to consider black holes as elementary particles — has been repeatedly proposed since 1980, and since the 1990s it has also attracted attention in superstring theory. However, as John Schwartz, one of the founders of superstring theory, wrote, "...since 1974, superstring theory has ceased to be considered particle physics..." and "...a complete and realistic model of elementary particles still appears to be a distant dream..." [1].

Interest in the connections among black holes, strings, and elementary particles arose soon after the discovery of the Kerr and Kerr–Newman (KN) solutions [2–5].

An alternative to superstring theory is loop quantum gravity.

These two approaches to the problem of the interaction of gravity and quantum theory are well known and have been widely discussed, although they are criticized, because they are not confirmed experimentally and do not yield new results.

Meanwhile, a third area of research related to the KN solution has also been discussed for more than 50 years [6–8] and has recently attracted attention as a development of a model of an elementary quantum particle (electron) in the form of an over-rotating Kerr–Newman black hole [3, 9–19, 21, 22].

Following the arguments of Roger Penrose [23], this third line of research, in contrast to the traditional directions of 'quantization of gravity,' could be called 'gravitization of quantum mechanics,' and recently a series of studies have appeared in which different versions of such gravitization are proposed [24, 25]. The closest approach to the KN black-hole modification in question is discussed in [26] and related papers [27, 28].

One of the most important issues in the new interpretation of a quantum particle as an over-rotating KN black hole is again the string model, but this is a classical relativistic string that arises in four dimensions as a classical solution of the Einstein–Maxwell equations rather than superstring theory of multidimensional quantum gravity.

The formation of black holes is associated with the gravitational effect of space dragging [29], which has never been considered before in particle physics. This effect turns out to be truly nontrivial and very important for understanding the governing physics behind the interaction of gravity and quantum theory, because it endows an electron with additional magnetic mass-energy generated by Wilson

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loops related to the gravitational dragging of space in a rotating KN black hole.

Electromagnetic excitations of a ring string are a source of an electron wave function, while a gravitational field interacting with a wave solves one of the main controversial problems of quantum theory — wave function reduction at the end point of its linear evolution [23].

The contradiction between quantum theory and gravity is most acute in the theory of the electron. Dirac's quantum theory presents an electron as a point mathematical object, a hybrid of a wave and a particle, while gravity requires an extended distribution of matter in space and time.

The assumption that particles are black holes was first independently expressed by some famous physicists (Nobel laureates): Abdus Salam, Frank Wilczek, and Gerard't Hooft. These early ideas concerned only the Schwarzschild solution, whose properties are very far from those of the rotating Kerr solution, and had almost nothing to do with the model of a rotating KN black hole.

The consideration of a Kerr black hole as a model of an electron begins with the work of Carter [3] (1968), who found that the Kerr–Newman solution (charged Kerr metric) has the same gyromagnetic ratio ($g = 2$) as that of the Dirac electron.

In contrast to the gravitational radius of the Schwarzschild solution, $l_{\mathrm{S}} = gm/c^2$, the effective zone of gravitational interaction in the KN solution is determined by the radius of the Kerr singular ring,

$$a = \frac{L}{mc}\,, \tag{1}$$

which is inversely proportional to the mass $m$ and directly proportional to the angular momentum $L$. For the parameters of an electron with mass $m$ and spin $L = \hbar/2$, the rotation parameter ($a = \hbar/2mc$) is equal to half the Compton wavelength, and the usual arguments about the exclusive role of the Planck length (see, for example, [30]) turn out to be invalid.

The 'nonpointness' of the electron as a model of the Kerr–Newman black hole is an important surprise that connects the KN electron with relativistic string models. In the Kerr–Schild coordinates $x^\mu = (t, x, y, z)$, $\mu = (0, i)$, and $i = (1, 2, 3)$ [10], related to the auxiliary asymptotically flat Minkowski space

$$\eta_{\mu\nu} = \mathrm{diag}\,(-1, 1, 1, 1)\,, \tag{2}$$

the KN solution describes the classical gravitational field of a ring string of half the Compton radius $a$ (Figs 1 and 2), which rotates relativistically, dragging light cones in the principal null (light) direction of a twisted Kerr congruence.

The Compton size of the electron was also noted by Israel [12], and later by López [13] and others, and this is not at all a harmless fact, since the Compton scale of $10^{-11}$ cm, being a natural scale for particle physics, is 22 orders of magnitude greater than the traditional Planck scale of $10^{-33}$ cm, on which both quantum loop gravity and superstring theory are based.

Following Carter, the KN electron model was considered in detail in the fundamental work of Debney, Kerr, and Schild (DKS) [10] and then in important studies by Israel [12] and López [13], as well as in the models [14, 15] based on Wheeler's idea of 'mass without mass' and the analogy of a singular Kerr ring with a classical Nielsen–Olesen string [31].

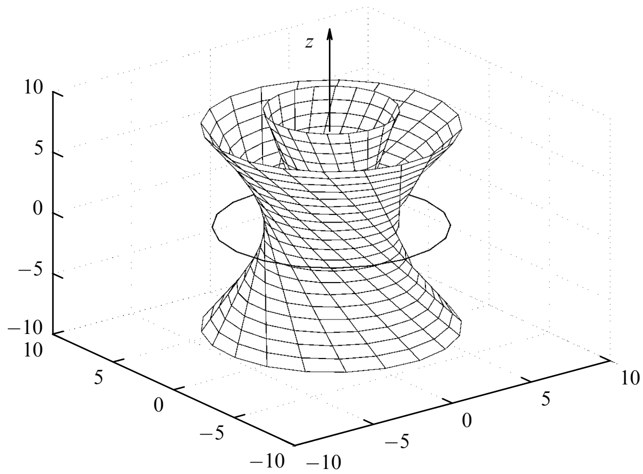


**Figure 1.** Singular ring of KN solution as a string and as a focusing line of light congruence with a twist, which analytically passes from a positive sheet $r > 0$ to a negative sheet $r < 0$ of KN solution.

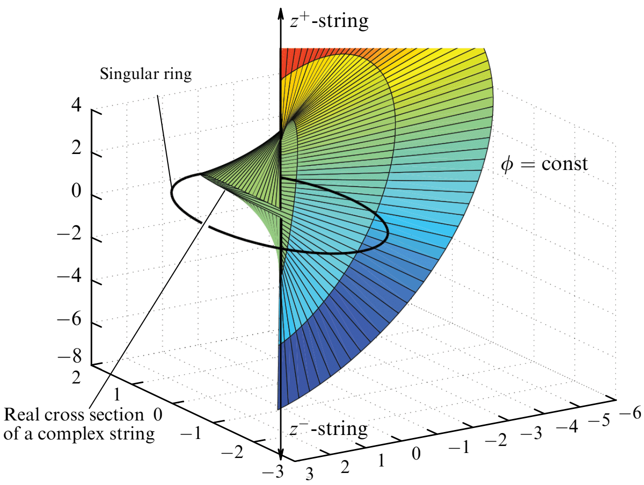


**Figure 2.** Distortion of angular coordinate of KN solution caused by gravitational dragging of space near singular Kerr ring.

Results of these studies were taken into account in a series of papers [17–19], where the electron model was considered a superconducting 'bag' (by analogy with MIT- and SLAC-bag models), having the shape of a very thin superconducting Kerr disk with a thickness of $\approx a/137$ and a radius $a$ equal to half the Compton wavelength of the electron (Figs 3 and 4).

The Kerr disk produced a vacuum core of an electron bordered by a Wilson loop, which is formed by gravitational dragging of the electromagnetic potential (frame-dragging) near the singular ring. (See also Figs 5 and 6 illustrating the dragging of space by the tilt of light cones in the direction of disk rotation [32].) By its nature, the KN electron model is consistent with classical gravity as an exact solution to the Einstein–Maxwell system of equations for interacting gravitational and electromagnetic fields:

$$R_{\mu\nu} = -8\pi T_{\mu\nu}\,, \tag{3}$$

$$T_{\mu\nu} = \frac{1}{4\pi}\left(F_{\mu\lambda}\,F^{\lambda}_{\nu} - \frac{1}{4}\,g_{\mu\nu}\,F_{\lambda\varkappa}\,F^{\lambda\varkappa}\right), \tag{4}$$

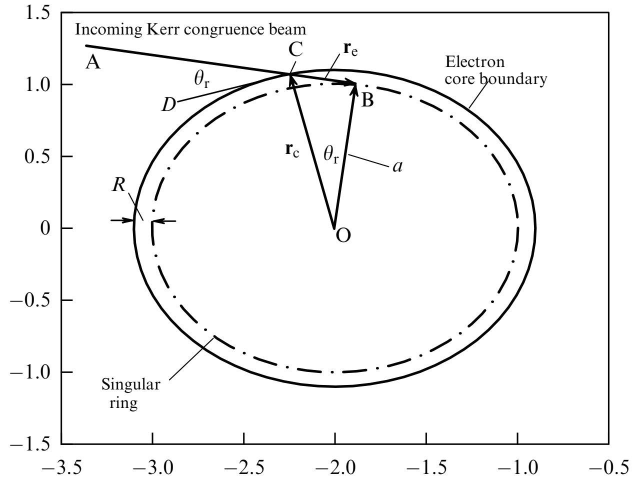


**Figure 3.** Gravitational dragging of space in López model of an electron. A relativistic ring string is formed on boundary of Kerr disk.

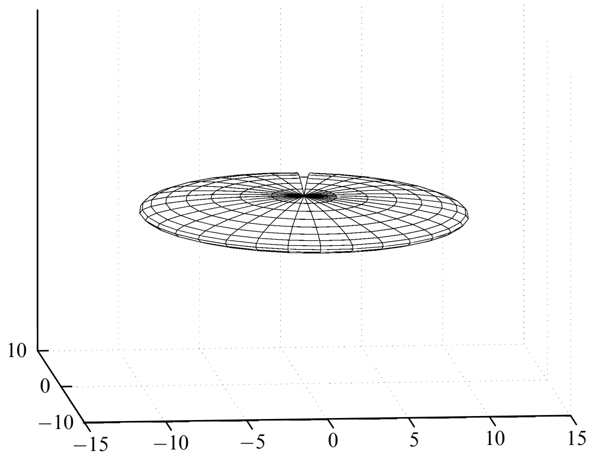

**Figure 4.** Thin López disk as a vacuum core of the electron.

and the exact solution of this system of equations in the form of a ring relativistic KN string in Kerr–Schild coordinates [10] (hereinafter referred to as DKS) resolves the seemingly insurmountable contradiction between gravity and quantum theory:

(1) a point-like structureless electron of quantum theory is *incompatible* with gravity;

(2) an extended gravitating electron is *incompatible* with quantum theory.

However, a massless ring relativistic string represented in the Kerr–Schild *world* coordinates $(x, y, z, t)$ as a state vector in the Heisenberg picture removes this contradiction. During relativistic rotation of the Kerr disk, the string is compressed into a quantum dot under the action of Lorentz transformations, generating a wave function attached to a point electron in the Schrödinger picture, in accordance with the interpretation of quantum theory in Bohm–de Broglie's pilot wave model [33].

The gravitational KN field turns out to be the field of a relativistically rotating disk [12, 34], which has long attracted attention as a field "...arising during the transition to a uniformly rotating reference system..." [35, § 89] and as a field "...in which dynamical properties of a 'particle' with space-time coordinates depends on a proper-time parameter..." [36]. The process of experimental determination of the coordinates and size of a relativistically rotating disk,

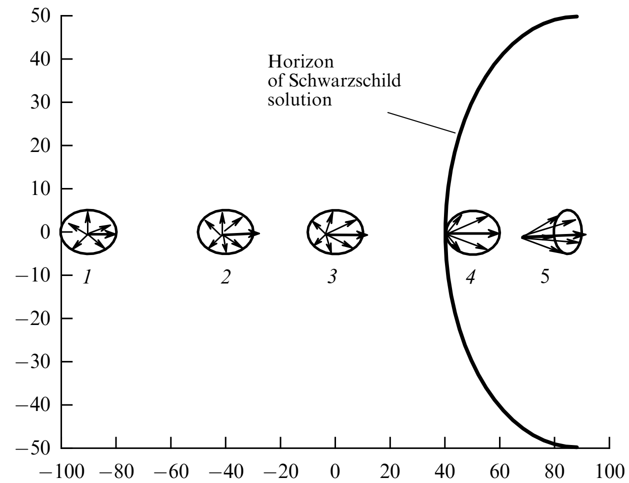


**Figure 5.** Collapse in Schwarzschild solution. Dragging of light cones *1*, *2*, and *3* into a black hole as they approach horizon of Schwarzschild solution. Thick arrows indicate direction of inclination of cones. Vertex of cone *4* is located on horizon, and cone *5* is located below horizon.

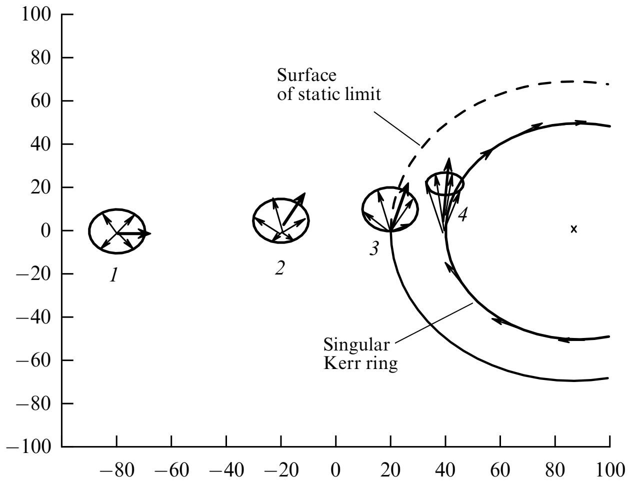


**Figure 6.** Ultrarelativistic rotation of Kerr solution. Circular dragging of light cones *1*, *2*, and *3* in the equatorial plane as they approach the Kerr singular ring. Bold arrows indicate direction of inclination of cones. Cone *3* is located on surface of static limit, and cone *4* is tangent to singular ring.

carried out by light scattering (photons), will not correspond to the theoretical description of the disk boundary in world coordinates $(x, y, z, t)$, since it is strongly distorted by the influence of *proper time* $\tau = ds/c$.

**The regularized López electron** forms a vacuum disk (bubble) of thickness $2r_e$ and radius $a$, equal to half the Compton wavelength (1) (see Figs. 3 and 4). The flat core of the López electron preserves the *external* gravitational and electromagnetic field of the KN solution, acquiring an internal metric of Minkowski space at $r < r_e = e^2/2m$. The boundary of a regularized Kerr disk is the line of the Ginzburg–Landau phase transition [37], interpolating between the outer classical region of the gravitational and electromagnetic fields and the superconducting electron core.

The López regularization actually defines the boundary between the zone of action of classical and quantum physics as a line of the phase transition of the Higgs mechanism [40] to the supersymmetric vacuum state inside the electron core. The vector potential, which forms the ring string and the Wilson

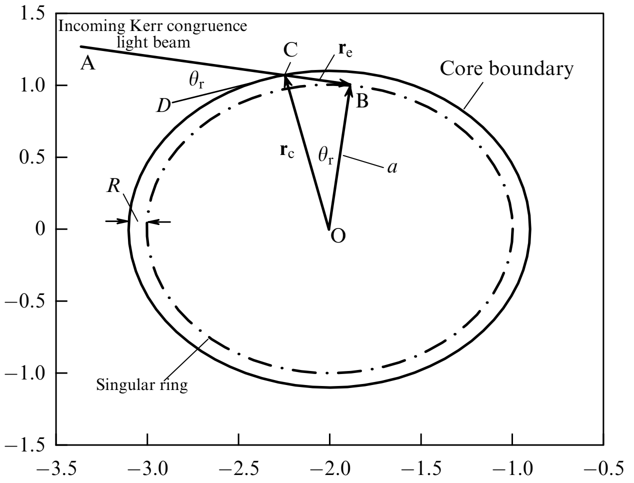


**Figure 7.** Black, positron, side of Kerr disk with incoming Kerr congruence $k^-$.

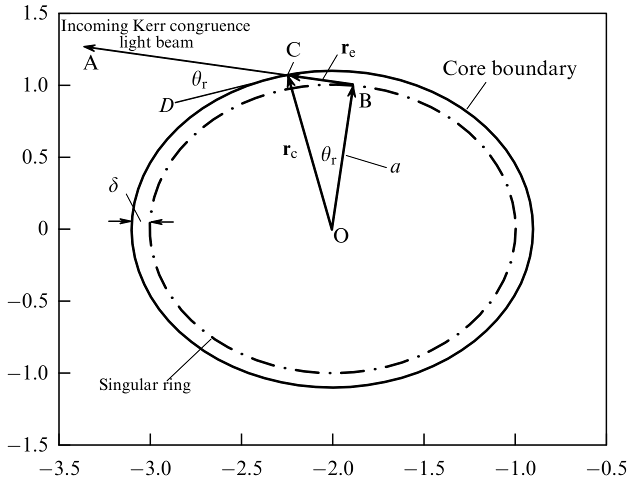


**Figure 8.** White, electron, side of Kerr disk with outgoing Kerr congruence $k^+$.

loop discovered in [38, 39], dragged by the gravitational KN field, is also concentrated at the López boundary.

In our paper [19], we first noted that the supersymmetric phase transition that forms the superconducting vacuum state is associated with two boundary surfaces of the Kerr disk $r_\mathrm{e}^{\pm}$ rather than with one, which can be interpreted as electron and positron boundaries (Figs 7 and 8). This clearly indicated a connection between the KN electron and quantum electrodynamics and was confirmed by the supersymmetric form of the Higgs mechanism [40], which necessarily arises during the formation of the vacuum core of the electron.

As was shown later in [7, 8], regularization of the supersymmetric KN solution forms two boundaries of the vacuum core — electron and positron boundary, while two corresponding Wilson loops produce a monopole–antimonopole pair, generating a strong magnetic coupling between the electron and positron sides of the Kerr disk.

On the other hand, the connection of the KN solution with quantum electrodynamics (QED), which is suggested by the effectiveness of the supersymmetry mechanism in the structure of the KN solution, assumes that QED-like separation of the electron mass formation mechanism into a bare electron mass and a dressed electron mass may prove fruitful in the structure of the KN solution as well. Indeed, we find that the mass generated by the classical relativistic string can be identified as the bare electron mass, while the mass generated by supersymmetry and Higgs fields requires renormalization according to QED. Thus, we find that the boundary between bare and dressed electrons is closely related to the one between the classical and quantum theories.

*The bare electron* is produced by classical Kerr–Schild gravity as a singular massless ring string in the Heisenberg representation. The string acquires mass under relativistic rotation by a unitary factor, and is compressed into a point electron corresponding to the wave function in the Schrödinger representation.

*The dressed electron* is produced by supersymmetric Higgs fields, which generate a phase transition consistent with QED [40] from the classical external field to the supersymmetric vacuum state of the KN electron core [19], which also implements the Ginzburg–Landau phase transition [37] to the superconducting state inside the electron core.

The mass-energy of the dressed electron is produced by the infinite gravitational and electromagnetic energy of a singular Kerr ring, in which the López regularization gives rise to electron and positron boundaries $r_\mathrm{e}^{\pm}$, and two Wilson loops formed on these boundaries transform the singular excess of gravitational and electromagnetic energy into the energy of the bound monopole and antimonopole by forming a strong magnetic interaction of the electron and positron vacuum.

## 2. Kerr–Newman metric as a black-white hole

The singular Kerr ring is a line of branching space into two sheets, and the KN solution with electron parameters *is not really a black hole*, because, for typical parameters of elementary particles, the superextremality condition $a^2 \gg e^2 + m^2$ is satisfied, under which the horizons of the black hole disappear and the singular ring of the Kerr–Newman black hole turns out to be naked, not closed by a horizon.

Even before Carter's first paper on the KN solution as a model of an electron [3], the two-sheet topology of the KN solution was criticized by Newman and Janis [11], and in the subsequent paper by Israel [12], the second sheet of the KN solution was cut off and replaced by a special distribution of matter chosen in accordance with Einstein's equations (3). The KN solution free of the second sheet was then considered by López [13] and later by Hamity [34] and many other researchers.

This slowed down the development of the KN electron model, and only in our papers of 2022–2023 [7, 8, 41] did we come to the conclusion that the second sheet of the KN solution is a positron sheet, forming an electron–positron vacuum in accordance with the QED structure. The modified KN solution should not be considered a black hole, but rather a combination of a WHITE and a BLACK hole in the form of an 'Einstein–Rosen bridge' [42], i.e., a solution, whereby the incoming electromagnetic (EM) field is not only absorbed by a BLACK hole (see Fig. 7), but also radiated away by its WHITE side (see Fig. 8). The KN solution must become radiating and not only absorb the incoming EM field, but also resonate with it, generating the Schrödinger equation wave function, and then radiate it away as the outgoing EM field of the WHITE hole.

The ‘negative’ sheet of the KN solution, cut off by Israel and López, *is not superfluous*, and in the Cartesian coordinates of the auxiliary Minkowski space $x^{\mu} = (t, x, y, z)$ (with the signature $- + ++$), the Kerr–Schild metric [10] takes a modified form: *

$$g^{\pm}_{\mu\nu} = \eta_{\mu\nu} + 2Hk^{\pm}_{\mu}k^{\pm}_{\nu}\,, \tag{5}$$

in which the scalar function $H$ of the KN solution has the form

$$H_{\mathrm{KN}} = \frac{mr - e^2/2}{r^2 + a^2\cos^2\theta}\,. \tag{6}$$

The Kerr congruence $k_{\mu}$ described by the null (light-like) vector field $k_{\mu}(x)$ ($k_{\mu}k^{\mu} = 0$) splits into two directions $k^{\pm}_{\mu}$, allowing both the ‘past’ (minus sign) and the ‘future’ (plus sign) space-time to be described as functions of the direction of the Kerr congruence: $k^{\pm}_{\mu}$ for the BLACK sheet of the metric (see Fig. 7), absorbing the incoming radiation, and the outgoing Kerr congruence for the WHITE radiating sheet of the modified KN solution (see Fig. 8).

The vector potential of the regularized KN solution depends on the direction of the Kerr congruence $k^{\pm}_{\mu}(x^{\mu})$:

$$A^{\pm}_{\mu} = \frac{\pm er}{r^2 + a^2\cos^2\theta}\,k^{\pm}_{\mu}\,, \tag{7}$$

as well as on the position of the core boundary $r^{+}_{\mathrm{e}} = e^2/2m$ (or $r^{-}_{\mathrm{e}} = -e^2/2m$) and the sign of the charge $e$.

The angular Kerr coordinates $(r, \theta, \phi_{\mathrm{K}})$ related to the Cartesian coordinates of the auxiliary Minkowski space $(t, x, y, z)$ by expressions corresponding to an oblate spheroidal coordinate system [10] are also split into relations for the incoming and outgoing solution:

$$\begin{aligned} &x \pm \mathrm{i}y = (r \pm \mathrm{i}a)\exp\{\mathrm{i}\phi^{\pm}_{\mathrm{K}}\}\sin\theta,\\ &z = r\cos\theta, \qquad \rho^{\pm} = \pm r - t\,, \end{aligned} \tag{8}$$

where the coordinate $\rho^{+} = r - t$ is the ‘retarded’ time coordinate, and the coordinate $\rho^{-} = -r - t$ is the ‘advanced’ time coordinate.

A topologically nontrivial *two-sheet* KN solution is formed by the analytical transition of the Kerr congruence from the negative sheet of the KN metric through the singular Kerr ring $r = 0$ to the positive sheet (see Fig. 1), $r^{+} \to r^{-}$ through the throat $r = 0$, and we associate the positive sheet $r = r^{+}$ and the negative sheet $r = r^{-}$ with the electron field and the positron field, respectively.

The vector potential $A^{+}_{\mu}$ of the regularized KN solution takes a maximum value $A^{\max +}_{\mu}$ in the equatorial plane $\cos\theta = 0$ and is dragged by the gravitational field in the direction of the outgoing congruence $k^{+}$, forming a WHITE (electron) Wilson loop $\phi_{\mathrm{K}} \in [0, 2\pi]$, while the potential $A^{-}_{\mu}$, oppositely charged and dragged by the incoming congruence $k^{-}$ in the opposite direction, forms a BLACK (positron) Wilson loop with the opposite magnetic effect (see Figs 7 and 8). In [43, 44], the black and white boundaries located in the equatorial plane of the Kerr disk are considered to be two half-strings of a single ring string system.

As will be shown in Section 4, black and white Wilson loops produce a gravitationally dressed vacuum state of the KN electron, which forms a strongly coupled magnetic pair of a monopole and an antimonopole.

The electrostatic energy of the Wilson loops is partially reduced, and the uncompensated remainder in the form of the electrostatic component of the retarded potential $A^{+}_{t}(r)$ forms the mass-energy of the electron in accordance with the well-known relation [35, 45]

$$m = U = \frac{e}{2}\int_{r_{\mathrm{e}}}^{\infty} A_t(r)\,\mathrm{d}r\,. \tag{9}$$

The exact KN solution was obtained by Debney, Kerr, and Schild [10], who described the Kerr congruence by the differential form

$$k_{\mu}\,\mathrm{d}x^{\mu} = \mathrm{d}u + \bar{Y}\mathrm{d}\zeta + Y\mathrm{d}\bar{\zeta} - Y\bar{Y}\mathrm{d}u \tag{10}$$

in the null Cartesian coordinates of Minkowski space:

$$\begin{aligned} &2^{1/2}\zeta = x + \mathrm{i}y\,, \quad 2^{1/2}\bar{\zeta} = x - \mathrm{i}y\,,\\ &2^{1/2}u = z + t\,, \quad 2^{1/2}v = z - t\,. \end{aligned} \tag{11}$$

The function $Y(x)(x \in \mathbb{M}^4)$ plays a central role in [10] and is determined by the Kerr theorem (see Section 5). [1]

The two solutions $Y(x)^{\pm}$ determined by the Kerr theorem yield two Kerr congruences: outgoing $k^{+}$, associated with emission by the white (electron) side in the KN solution, and incoming $k^{-}$, associated with absorption of emission by the black (positron) side in the KN solution.

The one-forms $k^{\pm}$ for the angular Kerr coordinates take the form

$$k^{\pm}_{\mu}\,\mathrm{d}x^{\mu} = \mp\mathrm{d}r - \mathrm{d}t \pm a\sin^2\theta\,\mathrm{d}\phi^{\pm}_{\mathrm{K}}\,, \tag{12}$$

in which $k^{+}$ and $k^{-}$ are related by spatial reflection $a \to -a$ with a change in the angular direction $\phi_{\mathrm{K}} \to -\phi_{\mathrm{K}}$, and the corresponding potentials of the electron and positron fields $A^{+}$ and $A^{-}$ are related by an additional change of the sign of the charge $e \to -e$.

The relativistic ring string, which is the main feature of the KN solution, is formed by a simple algebraic transformation (‘trick,’ as described in [48]) of the classical spherically symmetric harmonic solution, in particular for the coupled system of gravitational and electromagnetic fields. This ‘trick’ transforms the *point* harmonic solution, represented in the Cartesian coordinate system $\phi(t, x, y, z)$ by a *complex coordinate shift* $\phi(t, x, y, z) \to \phi(t, x, y, z + \mathrm{i}a)$, *to a new solution in the form of a relativistic ring string*. It was discovered by P Appell and described in a very short note [49] back in 1887, long before the discovery of the KN solution (see also [41]). The Appell complex shift forms a complex radial coordinate $\tilde{r} = r + \mathrm{i}a\cos\theta$, which defines the relationship between the spheroidal Kerr–Schild coordinates $r_{\mathrm{K}}$, $\theta_{\mathrm{K}}$, $\phi_{\mathrm{K}}$ and the Cartesian coordinates $(t, x, y, z)$, and also

* The Kerr solution was first obtained by Roy Kerr in 1963 in a dramatical process of the investigation the twisted and shear-free metrics of type D of the Petrov-Pirani classification (see R.P. Kerr, 2008, Discovering the Kerr and Kerr–Schild metrics, arXiv:0706.1109v2 [gr-qc]). The obtained by Newman et al. charged version of this solution, interpreted as field of a rotating and charged mass, was criticized because of its two-sheeted structure, see end note in [11]. Carter noted in [3] that the type D metrics contains the incoming and outgoing congruences, allowing us to consider the ‘past’ and ‘future’ metrics separately. (*Author’s note to English proof.*)

[1] Although the function $Y(x)$ runs through the entire text of Ref. [10], there is no mention of the Kerr theorem. It is formulated as “the Kerr Theorem” by Penrose [46] (see also [41]).

defines a complex conjugate structure by the transformation $\phi(t,x,y,z) \to \phi(t,x,y,z-\mathrm{i}a)$, describing both the incoming and outgoing fields of the black and white sides in the KN solution.

The positron sheet of the metric $g^-_{\mu\nu}$ carries a large negative energy of the positron vacuum, including the Dirac sea of states with $E<0$, interacting with the gravitational KN field. This energy, identified with the quantum Casimir effect, is compensated for by the vacuum energy of the electron (states with $E>0$). Thus, the formation of the vacuum state of the KN solution is not a purely classical gravitational effect, but rather a consequence of semiclassical gravity, in which Einstein's equations (3) are classical, but the energy-momentum tensor (4) includes the quantum radiation of the Casimir effect [26].

Vacuum excitations are compensated for in the KN solution as a strong magnetic coupling of the electron sheet with the positron one, resulting from the formation of a monopole–antimonopole pair by Wilson loops (see Section 4, as well as paper [47], in which the vacuum polarization and the production of virtual pairs are interpreted from the point of view of dark matter).

The KN solution for a black hole corresponds to the direction of the congruence into the black hole, while, in the new electron model, both congruences, incoming $k^-$ and outgoing $k^+$, play an important role. In accordance with the doubling of the Appell complex structure in two conjugate directions of the complex shift, $x^\mu \to x^\mu \pm \mathrm{i}az$, two metrics appear: incoming and outgoing.

## 3. Bare electron as a relativistic string and a quantum particle

Israel's electron model is a classical stationary solution of the Einstein–Maxwell equations in the form of a singular ring of half the Compton radius, written in the Kerr–Schild coordinate system at a fixed time $t=t_0=\mathrm{const}$.

Hamity [34] demonstrated that the core of the Israel electron rotates relativistically with an angular velocity $1/a$, and the edge of the Kerr disk moves with the speed of light, realizing the well-known Landau–Lifshitz model [35, § 89] of the generation of a gravitational field on a uniformly rotating disk. Therefore, the singular ring string forming the Kerr disk must be a massless relativistic string, which, during relativistic rotation, must contract and turn into a point under the action of the Lorentz contraction [35].

In fact, Appell's algebraic 'trick' is a realization of the physical model of the classical KN solution and the geometry of the relativistically rotating Landau–Lifshitz disk, which connects the extended classical relativistic string with the quantum point particle of the Schrödinger equation. We assume that the classical relativistic string has no mechanical mass ($m_{\mathrm{str}}=0$), and the mass-energy of the KN electron is produced by the time component of the retarded potential (9), while the spatial components of the potential $A_i$ $(i=1,2,3)$ are dragged by the gravitational field, forming a Wilson loop, which contracts to a point in the 'proper-time' system, in accordance with [35].

Thus, the extended string structure of the Israel electron does not contradict the point electron of quantum theory, eliminating the main obstacle in the problem of their unification (see points 1 and 2 in the Introduction).

While the massless Kerr ring string satisfies the Nambu–Goto equations as usual, it differs markedly from the strings used in the well-known superstring theory in that *it generates mass by rotation*, i.e., by longitudinal modes that are not allowed in superstring theory. Such strings were considered in [50, 51] as noncritical models of the classical Nambu–Goto string.

The electron state vector $|\mathrm{bra}\rangle$ is formed as an axial vector in the Heisenberg representation with a fixed axis $\mathbf{n}_z$, orthogonal to the equatorial plane of the KN solution, parameterized by the Kerr angular coordinate $\phi_{\mathrm{K}} \in [0, 2\pi]$.

The momentum operator $\mathbf{p} = \mathbf{p}^{(\mathrm{tr})} + \mathbf{p}^{(\mathrm{s})}$ is decomposed into a translational part $\mathbf{p}^{(\mathrm{tr})}$, associated with the translation of the electron as a whole, and an angular momentum operator $\mathbf{p}^{(\mathrm{s})} = \mathbf{n}_z\, \partial/\partial\phi_{\mathrm{K}}$, associated with the rotation of the ring string in the angular direction $\phi_{\mathrm{K}}$.

The state vectors in the Heisenberg and Schrödinger pictures are related by a unitary transformation [52]

$$|\Psi_{\mathrm{S}}(x,t)\rangle = \exp(-\mathrm{i}Ht)|\Psi_{\mathrm{H}}(\phi,t_0)\rangle\,, \tag{13}$$

which shows that the wave function of the Schrödinger equation is generated by the unitary rotation operator $U=\exp(-\mathrm{i}Ht)$ acting on the *static* state of the ring string $|\Psi_{\mathrm{H}}(\phi,t_0)\rangle$, which, in the Heisenberg representation, corresponds to a fixed time $t=t_0$.

Assuming the translational part of the moment to be zero, $\mathbf{p}^{(\mathrm{tr})}=0$, and leaving only the longitudinal component $p^{(\mathrm{s})}$ associated with the rotation of the string, $H=E=p^{(\mathrm{s})}$, we find that the unitary factor $U=\exp(-\mathrm{i}Ht)=\exp(-\mathrm{i}p^{(\mathrm{s})}t)$ acting on the static state of the ring Kerr string corresponds to the kinetic energy of its rotation, and the waves — excitations of the ring Kerr string — turn out to be tied to the point state of the Schrödinger electron, realizing Bohm–de Broglie's pilot wave model [33] for interpreting quantum theory.

*Dirac equation.* Following the standard derivation of the Dirac equation,[2] we linearize the Hamiltonian $H=[(\mathbf{p})^2+m^2_{\mathrm{str}}]^{1/2}$ for a massless string, $m_{\mathrm{str}}=0$. In the basis of $\gamma$ matrices $\gamma^\mu=(\gamma^0,\gamma^i)$, $i=1,2,3$, associated with the auxiliary Minkowski space (2), the Hamiltonian splits into positive- and negative-frequency parts,

$$H = \pm p_i \gamma^i\,. \tag{14}$$

Interaction with the electromagnetic field is introduced by the gauge-invariant replacement $p^\mu \to p^\mu - eA^\mu$, under which the expression for the Hamiltonian takes the form

$$H = (\pm p_i + eA_i^{\mp})\gamma^i - m_{\mathrm{str}}\gamma^0\,. \tag{15}$$

The mass term $m_{\mathrm{str}}$ is replaced by the electrostatic field of the ring string located at the edge of the white (radiating) side in the KN solution, corresponding to the field of the retarded potential $A^+$ (9).

Since the negative sign in (14) is related to negative frequencies of the wave function, it must correspond to the covector of the state of the conjugate positron:

$$\langle\Psi_{\mathrm{H}}(\phi,t_0)|\exp(\mathrm{i}Ht) = \langle\Psi_{\mathrm{S}}(x,t)|\,. \tag{16}$$

The Dirac equation takes the operator form

$$\mathrm{i}\hbar\,\frac{\partial}{\partial t}\,\psi(\phi_{\mathrm{K}},t) = (\gamma^x\mathbf{n}_x + \gamma^y\mathbf{n}_y)\,\frac{\partial}{\partial\phi_{\mathrm{K}}}\,\psi(\phi_{\mathrm{K}},t)\,, \tag{17}$$

[2] We assume that $c=1$.

in which the matrices $\gamma^\mu = (1, \gamma^i)$ are given in Cartesian coordinates $x^\mu = (t, x, y, z)$ of the auxiliary Minkowski space.

The spinor $\psi_p$ satisfies the Dirac equation

$$\left[\gamma^\mu\left(\frac{\partial}{\partial x_\mu} - eA_\mu\right) + m^{\mathrm{Dir}}\right]\psi_p = 0\,. \tag{18}$$

In the Weyl basis, the Dirac matrices have the form

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \tag{19}$$

where the Dirac spinor contains two Weyl spinors:

$$\psi^{\mathrm{D}}(\phi_{\mathrm{K}}, t) = \begin{pmatrix} \chi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}.$$

The two-component matrices $\gamma^\mu$ in the Weyl representation [40] allow one to split the Dirac equation and express the solution in terms of two Weyl spinors $\chi_\alpha(\phi_{\mathrm{K}}^+)$ and $\bar{\psi}^{\dot{\alpha}}(\phi_{\mathrm{K}}^-)$, which are parameterized by the Kerr angular coordinate $\phi_{\mathrm{K}}$ and form left $|\mathrm{left}\rangle$ and right $\langle\mathrm{right}|$ half-strings of opposite helicity in the Heisenberg representation.

The Dirac equation is divided into the positive-frequency electron equation

$$\hat{p}_\mu \sigma^\mu_{\alpha\dot{\alpha}} \chi^{\dot{\alpha}} = -m^{\mathrm{Dir}} \psi_\alpha \tag{20}$$

and the negative-frequency positron equation

$$\hat{p}_\mu \sigma^{\mu\dot{\alpha}\alpha} \psi_\alpha = -m^{\mathrm{Dir}} \chi^{\dot{\alpha}}\,, \tag{21}$$

in which the mass term $m^{\mathrm{Dir}}$ is zero, because the mass-energy of the KN electron is generated by a massless relativistic string, additionally loaded with gravity, and the 'mechanical' contribution to the mass is absent.

Thus, the ring KN string is formed as the sum of the left (electron) state vector $|\Psi_{\mathrm{H}}(\phi_{\mathrm{K}}, t_0)\rangle$ and the right (positron) covector $\langle\Psi_{\mathrm{H}}(\phi_{\mathrm{K}}, t_0)|$, which together give two full turns of the Kerr angular coordinate: the left turn $\phi_{\mathrm{K}}^+ \in [0, 2\pi]$ and the oppositely oriented right turn $\phi_{\mathrm{K}}^- \in [-2\pi, 0]$ of the closed ring string. The left and right half-strings are synchronized by the orientifold structure [43, 54] (Fig. 9), forming together a single electron–positron vacuum state.

*Formation of an orientifold.* The closed four-dimensional string $X^\mu(t, \sigma) = X_{\mathrm{L}}^\mu(t+\sigma) + X_{\mathrm{R}}^\mu(t-\sigma)$, defined on the segment $\Sigma = [0, 2\pi]$, is added and represented as the sum of two open half-strings, in which the left modes $X_{\mathrm{L}}^\mu$ are defined on the segment $\sigma \in [0, \pi]$, and the right modes $X_{\mathrm{R}}^\mu$ are defined on the oppositely oriented segment $\sigma \in [\pi, 2\pi]$, with the identification

$$X_{\mathrm{R}}^\mu(\sigma + \pi) = X_{\mathrm{L}}^\mu(\sigma)\,, \quad X_{\mathrm{L}}^\mu(\sigma + \pi) = X_{\mathrm{R}}^\mu(\sigma)\,. \tag{22}$$

In this case, the projection $\sigma \to a\phi_{\mathrm{K}}/2$ maps the left interval $\sigma_{\mathrm{L}} \in [0, \pi]$ onto the + boundary of the white Kerr disk $\sigma_{\mathrm{L}} \to \phi_{\mathrm{K}} \in [0, 2\pi]$, corresponding to the electron half-string, and maps the right interval $\sigma_{\mathrm{R}} \in [\pi, 2\pi]$ onto the oppositely oriented boundary of the black Kerr disk $\sigma_{\mathrm{R}} \to \phi_{\mathrm{K}} \in [-2\pi, 0]$, which forms the positron half-string of the electron-positron vacuum core of the electron.

Thus, the Dirac equation is mapped onto right and left ring half-strings, parameterized by the Kerr angular coordinates $\phi_{\mathrm{K}}^+$ and $\phi_{\mathrm{K}}^-$, as was assumed in [44].

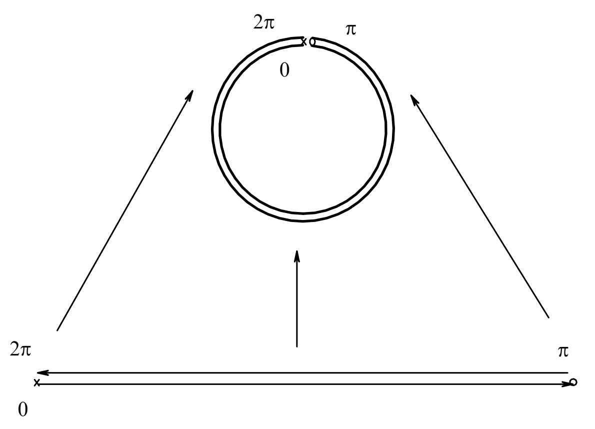


**Figure 9.** Formation of an orientifold: a straight line segment is projected onto edge of the Kerr disk, covering it twice.

## 4. Wilson loops and generation of strong magnetic coupling

In the regularized KN solution, the flat superconducting core of the KN electron is formed by two surfaces $B^\pm$ (white and black), which are fixed by the López cutoff parameter $r = r_{\mathrm{e}}$ and separate the vacuum core zone from the external gravitational and electromagnetic fields. At the same time, at the edge of the Kerr disk, the potential of the electromagnetic field (7) near the regularized singular ring increases dramatically and is simultaneously dragged by a strong gravitational field, forming two loops $C^\pm$ (electron and positron) dragged along the disk boundary and which are located in the equatorial plane $\cos\theta = 0$ at a small distance $r = r^\pm$ from the regularized singular ring.

The potential dragged by the light direction $k_\mu^+$ at the boundary $r = r^+$ takes the value

$$A_\mu^+ = -\frac{er}{r^2 + a^2\cos^2\theta}\, k_\mu^+\,, \tag{23}$$

and its angular component increases, taking the form of a closed loop,

$$C^+ : \{\phi_{\mathrm{K}} \in [0, 2\pi]\}\,, \tag{24}$$

in the form of a $\delta$ function extended along the loop:

$$A_\mu^{\max}\,\mathrm{d}x^\mu = -\frac{2m}{e}\,(-\mathrm{d}t - \mathrm{d}\phi_{\mathrm{K}})\,. \tag{25}$$

When integrating over the contour $C^+$, the Wilson loop $W(C^+) = \exp\{e \oint_{C^+} A_\mu^{+\max}\,\mathrm{d}x^\mu\}$ yields a phase shift

$$\delta\phi = e\oint_{C^+} A_{\phi_{\mathrm{K}}}^{+\max}\,\mathrm{d}\phi_{\mathrm{K}} = 4\pi m a\,, \tag{26}$$

which, in accordance with the main relation for the parameters of the Kerr solution,

$$J = ma = \frac{\hbar}{2}\,, \tag{27}$$

yields a phase shift of $2\pi\hbar$.

We see that the phase shift along the Wilson loops is quantized like the phase of the wave function, making a quantum contribution to the classical action through the 'minimal' coupling $p_\mu \to p_\mu - eA_\mu$.

To uniquely determine $W(C^+)$, we need to set $\delta\phi = 2\pi$, and we find that the Wilson loop yields the angular momentum quantization condition $J = 1/2$ [53, 55], and also has additional quantum solutions, $\delta\phi = n2\pi$, with $n = 2, 3 \ldots$.

According to Stokes's theorem, the Wilson loop $C^+$ must generate a magnetic flux

$$\Phi = \oint_{C^+} A_{\phi_{\mathrm{K}}}^{+\max} \, \mathrm{d}\phi_{\mathrm{K}} = 4\pi ma = \frac{4\pi\hbar}{2} = \frac{h}{e}\,, \tag{28}$$

equal to half a quantum of the magnetic field $\Phi_0 = h/2e$, and, therefore, the loop $C^+$ gives rise to a Dirac monopole.

The monopole carries infinite energy and cannot be produced alone; therefore, the second Wilson loop with the contour $C^-$ must generate an antimonopole.

Indeed, the potential $A_\mu^{-\max}\, \mathrm{d}x^\mu$ associated with the incoming Kerr congruence $k_\mu^-$, concentrating on the 'mirror' boundary $r = r^- = -e^2/2m$, takes the value

$$A_\mu^{-\max}\, \mathrm{d}x^\mu = \frac{2m}{e}\,(-\mathrm{d}t + \mathrm{d}\phi_{\mathrm{K}})\,, \tag{29}$$

forming a ring string along the loop $C^-$ (the potential takes into account the change of the sign of the charge during the transition $r \to -r$).

Integrating the loop $W(C^-) = \exp\{-e\oint_C A_\mu^{-\max}\,\mathrm{d}x^\mu\}$ with the opposite orientation of the contour $C^-$, we obtain the opposite phase shift

$$-\delta\phi = -e\oint_{C^-} A_{\phi_{\mathrm{K}}}^{-\max}\, \mathrm{d}\phi_{\mathrm{K}} = -4\pi ma = -4\pi J\,, \tag{30}$$

and the energy contribution of the Wilson loop at the boundary $C^-$ almost completely cancels the contribution of the loop at $C^+$, with the exception of *an important asymmetry*, which makes a finite contribution. Integration over the boundary $C^+$ is associated with a retarded vector potential, i.e., with the outgoing 'base' congruence that generates the gravitational KN field, while the boundary $C^-$ is associated with the incoming vector potential $A_\mu^{-\max}\,\mathrm{d}x^\mu$, which does not contain an electrostatic component:

$$A_t = \frac{-er}{r^2 + a^2\cos^2\theta}\,\mathrm{d}t\,. \tag{31}$$

This component is usually related to the electron mass [45], appearing as the mass term $m$ in Dirac equation (18) and in expression (6) for the total gravitational mass in the KN solution.

Therefore, strong total interaction between the gravitational KN field and the two Wilson loops $C^\pm$ does not manifest itself directly in the value of $m$ for the total mass, but acts nonlinearly, increasing the particle mass with decreasing radius $a$ in the main Kerr relation (1).

Thus, two Wilson loops located at the white and black boundaries of the Kerr disk, $C^+$ and $C^-$, generate a magnetically coupled pair consisting of a Dirac monopole and an antimonopole, giving rise to a vacuum state in the superconducting core of the electron and the Dirac current as a superconducting surface current.

## 5. Radiating Kerr–Newman solution and Kerr theorem

It is known that, for both the Schwarzschild solution and the Reissner–Nordström solution, their radiative generalizations, such as the Vaidya solution for a luminous star and the Kinnersley solution, known as the photon rocket solution,[3] were subsequently obtained. Similar attempts to find corresponding generalizations for the Kerr and KN solutions have failed, and the main obstacle in this regard is the rotation-related twist of the Kerr metric, which makes the solution chiral, losing the symmetry of complex conjugation.

However, this problem can be solved for the dual over-rotating KN solution, containing both radiation and absorption of mass-energy, i.e., both black and white sides, and this has a direct bearing on the coupled electron-positron vacuum state in the KN solution.

An analysis of the most detailed derivation of the KN solution by DKS in their fundamental paper [10] showed that the solution is not complete and contains only a chiral, radiationless part, since the system of DKS equations was integrated to its final form only under the additional condition[4]

$$\gamma = 0\,, \tag{32}$$

which preserves strong magnetic coupling between the incoming and outgoing electromagnetic fields of the electron-positron vacuum state and eliminates the electromagnetic radiation that connects them, forming states $|\mathrm{bra}\rangle$ and $\langle\mathrm{ket}|$ as a single string system.

The approach to radiative generalizations of KN solutions was considered in our papers published in 2002–2004 [56–58], in which the DKS equations were re-integrated, and, in paper [57], we obtained the matching conditions for incoming and outgoing EM excitations.

The central role in Ref. [10] is played by the *Kerr theorem*, which defines the complex functions $Y(x^\mu)$, $x^\mu = (t, x, y, z)$, allowing us to fix chiral outgoing (or antichiral incoming) Kerr congruences $k^\pm$ in the null Cartesian coordinates $(u, v, \zeta, \bar\zeta)$ of Minkowski space.

*The Kerr theorem* defines $Y(x)$ as a holomorphic solution of the algebraic equation $F = 0$, where the generating function $F(Y, \lambda_1, \lambda_2)$ can be an arbitrary holomorphic function of three projective twistor coordinates:

$$(Y, \quad \lambda_1 = \zeta - Yv, \quad \lambda_2 = u + Y\bar\zeta)\,. \tag{33}$$

While congruences (12) are determined by two complex conjugate functions $Y$ and $\bar Y$, *the theory is chiral*, since $Y$ and $\bar Y$ are considered to be independent variables[5] during the integration process.

For the KN solution, the function $F$ is quadratic in $Y$, and the equation $F = 0$ has two roots $Y^\pm$, which are related to each other by an antipodal correspondence[6] (see [61]):

$$Y^+ = -\frac{1}{\bar Y^-}\,. \tag{34}$$

Thus, the second root of the Kerr theorem $Y^- = -1/\bar Y^+$ transforms the chiral solution into an antichiral one, replacing the incoming EM field with the outgoing EM radiation.

Two solutions $Y^\pm$ yield two Kerr congruences: electron (white) $k^+(x)$ and positron (black) $k^-(x)$.

[3] Vaidya P C *Phys. Rev.* **83** 10 (1951); Kinnersley W *Phys. Rev.* **186** 1335 (1955).
[4] See equation (5.51) in [10].
[5] See note to equation (5.79) in [10].
[6] Both solutions $Y^+$ and $Y^-$ should have the same Killing symmetry.

In terms of the Penrose twistor theory [46, 62], they yield two projective spinor fields,

$$Y^+ = \frac{\xi^{\mathrm{i}}}{\xi^{\dot 0}}, \quad Y^- = \frac{\eta_1}{\eta_0}, \tag{35}$$

which are antipodally conjugate,

$$Y^- = -\frac{1}{\bar Y^+} = -\frac{\bar\xi^1}{\bar\xi^0}, \tag{36}$$

and correspond to two Weyl spinors $\xi^{\dot\alpha}$ and $\eta_\alpha$, parameterizing two Kerr congruences (incoming and outgoing):

$$k^{\mu+} = \bar\xi^{\alpha}\sigma^{\mu}_{\alpha\dot\alpha}\xi^{\dot\alpha}, \quad k^{\mu-} = \bar\eta_{\dot\alpha}\bar\sigma^{\mu\dot\alpha\alpha}\eta_\alpha, \tag{37}$$

tangent to the white and black sides of the singular ring in the KN solution.

As was shown in Section 3, the Dirac equation is split and parameterized by two Weyl spinors — $\xi^{\dot\alpha}(\phi_{\mathrm K})$ and $\eta_\alpha(\phi_{\mathrm K})$ — depending on the Kerr angular coordinate $\phi_{\mathrm K}$, and is mapped by a spinor string located in the form of two half-strings (incoming and outgoing) on the edge of the Kerr disk (see Figs 4 and 5).

The differential of the function $F$ determines Appell's *complex radial distance* [49]

$$\frac{\mathrm dF}{\mathrm dY} = \frac{P}{Z} = r + \mathrm i a\cos\theta. \tag{38}$$

The chiral electromagnetic field radiated along $k^+$ was represented by DKS in terms of the components of the null tetrad $\mathcal F_{12} = A^+ Z^2$ and $\mathcal F_{31} = \gamma^+ Z - (A^+ Z)_1$, where the plus sign denotes the components of the field emitted along the $k^+$ direction.

The tetrad components of the incoming field along the $k^-$ direction can be obtained by transmutations of the tetrad indices $12 \to 21$ and $31 \to 42$ in combination with the complex conjugation $Y \to \bar Y$. The corresponding relations are $\mathcal F_{21} = A^- \bar Z^2$ and $\mathcal F_{42} = \gamma^- \bar Z - (A^- \bar Z)_2$.

The presence of black, incoming, radiation was not considered at all by DKS, and the electromagnetic field of the electron was studied using the example of a single solution $A = \psi(Y)P^{-2}$, and the analysis of the electron field was carried out using an even simpler solution $\psi = -e = \text{const}$.

In this case, the solution $A^+ = \psi(Y)P^{-2}$ with the analytical dependence $\psi = \psi(Y)$ was discussed by DKS as a general solution that is consistent with the required Killing symmetry,[7] and the complex conjugate solution $A^- = \bar\psi(\bar Y)P^{-2}$ is also consistent with this Killing symmetry.

In our papers [56, 58], we showed that radiating KN solutions exist but require additional consistency conditions. First, the solution must be doubled and have both a radiative part $A^+$ and a radiation-receiving part $A^-$, and second, these solutions must be complex conjugate and depend on complex conjugate time-retarded parameters $\tau$ and $\bar\tau$:

$$A^+ = \psi(Y,\tau)\,P^{-2}, \quad A^- = \bar\psi(\bar Y,\bar\tau)\,P^{-2}. \tag{39}$$

These conditions are met in the KN solutions initiated by the construction of retarded Lind–Newman potentials [63], and their consistency is determined by the compatibility of the parameters $\tau$ and $\bar\tau$, which must satisfy the orientifold symmetry discussed in Section 3 [43], formulated for two open half-strings (electron and positron) defined on intervals $\sigma_{\mathrm L} \in [0,\pi]$ and $\sigma_{\mathrm L} \in [\pi, 2\pi]$.

The fields $A^-$, $\bar\psi$, and $\gamma^-$ represent radiation received by the positron (black) half-string from the $k^-$ direction, and the fields $A^+$, $\psi$, and $\gamma^+$ form the radiation produced by the 'left' electron (white) half-string from the $k^+$ direction. Their compatibility means the formation of a single resonant system relating the incoming and outgoing radiation.

Alternatively, solutions for the vector potential (39) were also obtained by DKS as the sum of an analytic 'left' one-form

$$\alpha_{\mathrm L} = -\frac12\left(\frac{\psi}{P^2}\,\frac{Z}{P}\,k^+ - \chi\,\mathrm d\bar Y\right) \tag{40}$$

and a complex conjugate 'right' one-form

$$\alpha_{\mathrm R} = \frac12\left(\frac{\bar\psi}{P^2}\,\frac{\bar Z}{P}\,k^- - \bar\chi\,\mathrm dY\right).$$

Using (38), the left form is expressed as

$$\alpha_{\mathrm L} = -\frac12\,A^+(r^+ + \mathrm i a\cos\theta)^{-1}\,k^+ - \frac12\,\chi\,\mathrm d\bar Y, \tag{41}$$

where $\chi$ has an important feature: integration

$$\chi = \int_{|\bar Y| = \mathrm{const}} P^{-2}\psi\,\mathrm dY \tag{42}$$

along the chiral loop forms a hook that hooks the antichiral direction $\bar Y$. The retarded-time construction combines the solutions $A^+$ and $A^-$ into a single correlated excitation of a coupled electron–positron string provided by the *orientifold structure* [57, 59] described in Section 3.

The mapping $\sigma \to a\phi_{\mathrm K}/2$ projects the imaginary white interval of retarded time $\sigma_{\mathrm L} \in [0,\pi]$ onto the entire region of the Kerr disk boundary $\sigma_{\mathrm L} \to \phi_{\mathrm K} \in [0, 2\pi]$, forming an electron (white) half-string, and the positron (black) interval of retarded time is projected onto the oppositely oriented boundary of the Kerr disk $\sigma_{\mathrm R} \to \phi_{\mathrm K} \in [\pi, 2\pi]$, forming a positron (black) half-string of the polarized electron–positron vacuum core of the electron.

The compatibility of the 'left–right' structures makes the KN solution radiating and leads to the emergence of an additional axial singular string located orthogonally to the left and right ring strings, as shown in Figs 10 and 11. This

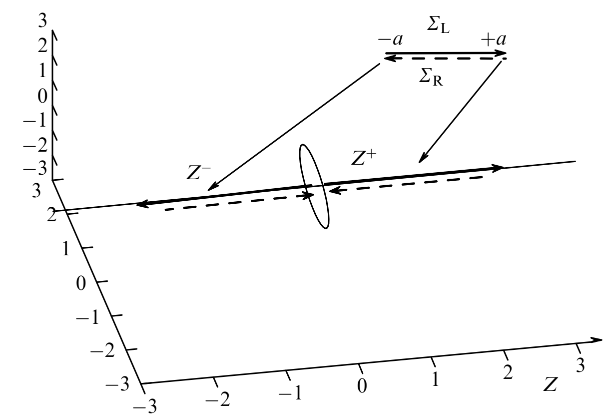


**Figure 10.** Correspondence between complex retarded time and real Kerr geometry.

[7] See equation (5.59) in [10].

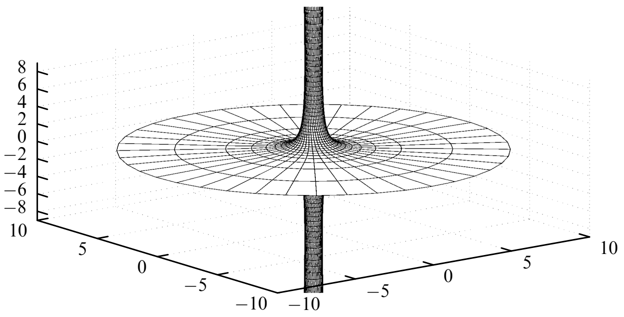


**Figure 11.** Emergence of a singular axis concentrating incoming and outgoing radiation in radiative KN solutions.

axial system synchronizes the left and right excitations, turning them into a single common resonant system. An important example of a solution of this type was given in [14, 18] as the sum of the constant charge in the KN solution and wave excitations:

$$\psi\,(Y,\tau) = -e\left(1 + \frac{1}{Y}\exp\,(\mathrm{i}\omega\tau)\right),$$

$$\bar{\psi}\,(\bar{Y},\bar{t}) = -e\left(1 + \frac{1}{\bar{Y}}\exp\,(-\mathrm{i}\omega\tau)\right).$$

In particular, the basic radiationless KN solution $\psi\,(Y,\tau) = -e$ forms a loop of the vector potential along the disk boundary in the $k^+$ direction and, acquiring here an additional singular contribution $(1/\bar{Y}) \times \exp\,(-\mathrm{i}\omega\tau)$, forms an additional imaginary shift from the disk boundary to the singular field at the disk center.

## 6. Conclusions

We come to the conclusion that the Kerr–Newman electron model — initiated by Carter [3] and then developed in the work of Debney, Kerr, and Schild [10]; Israel [12]; Namity [34]; and López [13] — represents an important step in the formation of a nonperturbative model of an electron interacting with gravity by forming an extended model of an electron in the form of a classical ring string contracting into a quantum dot under relativistic rotation. The new radiative KN solution, answering such questions as "What is an electron?" and "How to unite gravity with particle physics?" gives an answer on new positions, which, following the ideas of Penrose [23], can be interpreted as "gravitization of quantum theory," contrary to the widespread approach of "quantization of gravity."

The main feature of the KN electron model is the representation of the KN solution in the Kerr–Schild world coordinates $(x, y, z, t)$, in which the geometry of the electron is described by the well-known model of a relativistically rotating disk (Landau–Lifshitz [35]), where a nonperturbative electron forms a classical relativistic ring string, associated with the quantum Heisenberg state vector at a fixed moment of *world time t*. At the same time, the string is parameterized by *its proper time s* in the Schrödinger representation, where it contracts to a point under the action of the Lorentz contraction.

The static description of an electron in the world coordinates $(x, y, z, t)$ as a singular Kerr ring is distorted by the Lorentz contraction, and for an observer who determines the shape and size of the string by reflected light (photons), the electron will appear as a point, in accordance with the quantum Schrödinger picture. This fact, which follows trivially from the geometry of a relativistic rotating disk, reveals the main reason for the fundamental mystery of quantum mechanics — *the problem of wave–particle duality*.

The wave function of the Schrödinger equation is generated by the unitary string rotation operator $U = \exp\,(-\mathrm{i}Ht)$, which acts on the static description of the ring string in the Heisenberg picture $|\Psi_{\mathrm{H}}(\phi, t_0)\rangle$ as a relativistic rotation operator, transforming the string into a quantum dot of the Schrödinger picture, with an associated wave function $|\Psi_{\mathrm{S}}(x, t)\rangle = \exp\,(-\mathrm{i}Ht)|\Psi_{\mathrm{H}}(\phi, t_0)\rangle$.

The black KN hole is formed by an EM and gravitational field vortex, which is dragged by the tilt of the light cones (frame-dragging [29]) in the direction of the black hole rotation and forms a singular ring string in the equatorial plane of the KN solution in the form of a classical solution to the Einstein–Maxwell equations, with consistent directions of propagation of the EM and gravitational fields along the principal null Kerr congruence $k_\mu$.

For the superextremal KN solution with electron parameters (mass $m$, charge $e$, angular momentum $L = am = \hbar/2$, and magnetic moment $\mu = ae$), the relation $e^2 + a^2 \gg m^2$ is satisfied, at which the horizons disappear and the singular Kerr ring turns out to be bare. As a result, the bare singular ring forms a branching line of space and forms an Einstein–Rosen bridge, which contains, along with the incoming vortex, another gravitational and EM field vortex leaving the KN solution in the form of 'white' EM and gravitational radiation.

In this case, in the KN electron model regularized by López [13], a vacuum core of the electron is formed with two boundaries $r_{\mathrm{e}}^+$ and $r_{\mathrm{e}}^-$ (electron and positron), and at the sharp edges of the boundaries the vector potential is concentrated in the form of two Wilson loops $C^+$ and $C^-$, which form a gravitationally dressed 'heavy' electron with strong magnetic coupling of the electron and positron loop due to the formation of a monopole–antimonopole pair.

This model is consistent with QED and requires a separate consideration of bare and dressed electrons, where a bare electron is purely classical and is based on the model of the classical relativistic Nambu–Goto string.

Unlike the known strings of superstring theory, a classical Kerr string does not put the quantization problem at the forefront, and the main role in the new model is played by longitudinal modes of string excitation, which turn out to be admissible, as was shown in [50, 51]; most importantly, they solve the main problem of quantum theory, naturally linking the Heisenberg and Schrödinger representations.

The Kerr–Newman solution is two-sheeted and actually describes an electron and a positron as a single particle of the electron–positron solution.

The radiation model of the KN electron not only absorbs energy like a black hole, but also resonates with the received radiation and radiates the EM field away, acting as a radiating white hole.

A two-sheeted singular Kerr ring formed by two interacting half-strings — electron and positron — is ideal for forming an elementary adaptive system that moves according to the principle of least action, correcting motion by comparing received and reflected signals.

Analyzing the exact solution in [10], we find that it is not complete. The authors begin to calculate the emitted EM field, but bring the calculations to the final result only under the assumption $\gamma = 0$, which describes the formation of two strongly coupled EM fields (electron and positron), but does not consider the transfer of wave excitation between them. As a result, white and black half-strings of the KN solution do not form a single string system.

The wave properties of the KN electron, described in [10], require additional refinement of its string structure. We analyze the class of corresponding solutions for radiating black holes. The condition for their compatibility is the formation of a single string system consistent with the orientifold symmetry.

One of the main general problems of quantum gravity is the supposed weakness of gravitational interaction. In fact, this problem is not related to the KN solution. The ring KN string depends on an additional regularization parameter, and the proximity of the cutoff parameter to the singular Kerr ring allows regulating the energy and strength of the gravitational interaction in unlimited limits. The excess energy is then damped by the mutual cancellation of the mass-energy contributions coming from the strong magnetic coupling of the electron and positron Wilson loops, and the final part turns out to be consistent with the standard relation (1) for the parameters of the Kerr solution.

The hidden vacuum energy–mass generated by the strong magnetic coupling of a monopole with an antimonopole leads to a revision of the traditional point of view on the weakness of the gravitational interaction. The gravitational effect of space dragging, realized in the form of Wilson loops, leads to a shift in the scale of gravitational interaction from the Planck scale ($\sim 10^{-33}$ cm) to the Compton scale ($\sim 10^{-11}$ cm).

Finally, let us dwell on recent work on a topic closely related to that discussed in this paper. The authors of Refs. [26–28, 60] analyze the semi-classical interaction of gravity with the quantum theory of black holes, in which the gravitational field is classical, and the material fields are considered quantum and are determined by the mathematical expectation of the energy-momentum tensor:

$$\langle\psi|T|\psi\rangle = \frac{\beta}{2}\,C^2 - \frac{\alpha}{2}\,\Gamma\,, \tag{43}$$

where $C^2 = C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$ and $\Gamma$ is the Gauss–Bonnet scalar.

As is known, this semi-classical approach leads to the emergence of a gravitational anomaly — the energy-momentum tensor acquires a nonzero trace $\langle\psi|T|\psi\rangle = g^{\mu\nu}\langle T_{\mu\nu}\rangle$, which can have a macroscopic effect, in particular, the appearance of 'white' radiation of the EM field emerging from the black KN hole [26]. The presence of a vacuum (zero) EM field in our space is confirmed by all existing radio engineering systems, as well as the well-known semi-classical Casimir effect.

Thus, an electron as a radiating black KN hole absorbs and emits a vacuum EM field and moves according to the principle of least action [59], being an elementary adaptive system that receives information from a reflected signal probing the surrounding space, allowing one to explain the well-known quantum experiment with two slits.

## 7. Appendix. Supersymmetry, superconductivity, and Higgs field

Frank Wilczek writes in his Nobel lecture that the space–time of the standard model "...is permeated by one or more (quantum) fields that spoil the full symmetry of the primary equations," and further continues that "...supersymmetry, for example, requires at least five 'Higgs particles'..." [66].

The Higgs field displaces the electromagnetic and gravitational field from the superconducting core, generating the Dirac current as the surface current of the superconducting disk.

The Wilson loop is formed at a fixed moment of world time $t = t_0 = \text{const}$ by the electromagnetic field, $A_\mu(t_0)$, dragged by a strong gravitational field, in the form of a loop on the boundary of the Kerr disk, where the Higgs field $\Phi(x) = |\Phi|\exp(i\chi)$ displaces the electromagnetic field from the superconductor.

The supersymmetry formalism contains two superfields $\Phi_+$ and $\Phi_-$, the bosonic part of which is interpreted as two Higgs fields $H_\pm = |H_\pm|\exp(i\chi_\pm)$ containing two mutually correlating phases $\chi^\pm = 2m(t \pm a\phi)$, in which the angular parameter $\pm a\phi$ corresponds to two boundaries of the Kerr disk $\pm a\phi_K$, associated with the parametrization of two Wilson loops. The correct phase transition to the description of QED requires two charges $e^\pm$, two currents $J^\pm$, and supersymmetry with five chiral fields on two boundaries.

Two surface currents are given by two equations for the vanishing fields and currents inside the superconducting Kerr disk $J^\pm_\mu = 0$, implying $\chi^\pm_\mu + eA^\pm_\mu = 0$. These equations describe the surface currents [17, 19]

$$\chi^\pm_\phi + eA^\pm_\phi = 0\,, \quad \chi^\pm_t + eA^\pm_t = 0\,. \tag{44}$$

They are easily integrated and determine the phase dependences of two Higgs fields, $H^+ = |H^+|\exp(i\chi^+)$ and $H^- = |H^-|\exp(i\chi^-)$, on time and angular coordinates of the Kerr disk $(t, \phi_K)$.

In 1950, Ginzburg and Landau in their paper "Towards a Theory of Superconductivity" [37] were the first to establish a connection between superconductivity and the quantum wave function of the Schrödinger equation for the simplest case of a flat boundary between a superconductor and an electromagnetic field. This work was a harbinger of the fact that superconductivity would play a central role in the structure of elementary particles and, in particular, in the structure of the electron. The idea turned out to be extremely fruitful and was later developed in the form of the supersymmetric theory of the Ginzburg–Landau phase transition in more complex models.

According to the Wess–Zumino model [40], the supersymmetric version of quantum electrodynamics is described by two Higgs fields, $\Phi^+$ and $\Phi^-$, and equation (44) makes it possible to relate the phases of the corresponding Higgs fields $\chi^+$ and $\chi^-$ to the potentials $A^-_\mu$ and $A^+_\mu$ at the two boundaries of the disk: $r = r^+$ and $r = r^-$.

For $r = r^+$, we have the potential $eA_0 = 2m$, $eA_{\phi_K} = 2ma$, and for $r = r^-$, we have the potential $eA_0 = -2m$, $eA_{\phi_K} = -2ma$.

Applying this to the outgoing vector field $A^+_\mu(r^+_e)$, which forms a closed Wilson loop along the contour $C^+ : t = \text{const}$, $r = r^+_e$, we find that a change in the potential $A^+_\mu$ along this loop is controlled by the phase shift of the Higgs field:

$$\chi^+|_{r^+} = 2m\,(t^+ + a\phi^+_K)\,. \tag{45}$$

Similarly, the potential $A^-_\mu$ acting on the boundary $r^-$ yields

$$\chi^-|_{r^-} = 2m\,(t^- - a\phi^-_K)\,. \tag{46}$$

Note that the presence of an antiboundary was first noticed in [19], where, along with the superbag domain wall (DW boundary), an anti-DW boundary also arose, which made exactly the same contribution to the total mass of the solution, but with the opposite sign (see also breather-type solutions in [44, 64]).

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