

METHODOLOGICAL NOTES

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On the question of measurement in quantum mechanics

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Contents

Abstract. The probabilistic nature of measurements in quantum mechanics can be interpreted as a consequence of information loss arising from the chaotic dynamics of measuring instruments.

Keywords: wave function collapse, reduction postulate, quantum chaos, decoherence, causality

1. Introduction

The question of the boundary between the 'quantum' and 'classical' worlds has been in the spotlight since virtually the advent of quantum physics, but today this area still contains many open questions on which there is no consensus opinion of the community. Probably the most well-known here is the problem of measurement: how the rules that determine the behavior of macroscopic ('classical') instruments in the measurement of the properties of microscopic ('quantum') systems follow from the equations of quantum mechanics (and whether they follow at all).

First of all, it is necessary to make a caveat that the terminology adopted in quantum theory differs essentially from the general physical one. Usually in physics (and in everyday life) measurement is understood as a comparison between certain physical quantities and reference values, which is carried out using a measuring device. In this case, the measurement error is determined, as a rule, by the imperfection of the equipment, and not by the properties of the system under study, and can be reduced by improving the instruments and the measurement procedure. In quantum theory, we are dealing with a situation where the uncertainty of the measurement result arises in an inevitable way, due to the laws of nature. One may argue whether the reason for this uncertainty lie with the properties of the system, the device, or the transformation of quantum information into classical information — in any case, no improvement in measuring equipment will make it possible to get rid of the source of uncertainty under discussion.

Next, it is necessary to separate questions about the possibility of obtaining information based on the average results of measurements carried out on an ensemble of independent implementations of the system and within the framework of measurements carried out on one specific implementation. 'Ensemble' measurements make it possible, in principle, to characterize a system with an arbitrarily high accuracy. Although, for example, the quantum uncertainty in the energy of atomic levels leads to a uniform broadening of the spectral lines, multiple measurements make it possible to determine the position of the lines with an accuracy exceeding their natural width. A similar situation is observed in relation to the measurement of the energies of the excited states of nuclei by means of gamma spectroscopy, the displacements of atoms by means of tunneling or electron microscopy, and in general in all cases when the experimental technique allows multiple measurements over an ensemble of system realizations. Moreover, quantum tomography protocols make it possible, through multiple measurements over an ensemble of realizations, to fully characterize not only an arbitrary observable but also the wave function (or density matrix). However, when we are dealing with a single measurement event (say, with the frequency measurement of a single photon emitted by an excited atom), then even complete information about the object under study and the use of arbitrarily advanced instruments do not allow predicting the measurement result with arbitrary accuracy. Furthermore, it turns out that the measurement inevitably affects the object under study, and its final state is related to the result of the measurement. Hereinafter, the problem of measurement in quantum theory is understood as the question of the origination of quantum uncertainty and the influence of measurement results on the system under study, which take place in each specific implementation of the experiment.

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The results of measuring the state of a quantum system are fundamentally probabilistic. Attempts to abandon the probabilistic interpretation of quantum mechanics by introducing local hidden parameters were refuted in the course of experimental verification [1–3] of Bell's inequalities [4]. Generally speaking, quantum mechanics can be reproduced at the level of nonlocal deterministic theories with hidden parameters, which have their origin in the work of Bohm [5]. The construction of such models continues to the present: mention can be made, for example, of Refs [6–8]. Without digressing into a detailed analysis of the listed approaches, which are not directly related to the subject of this work, we note that nonlocal interpretations do not provide additional information about observables in comparison with the standard apparatus of quantum theory, i.e., from an operational viewpoint, the measurement results in these theories remain probabilistic.

From the viewpoint of information theory, the appearance of a probabilistic choice means an increase in entropy, i.e., loss of information about the system. On the other hand, the equations of quantum mechanics are linear; they do not contain either dissipative or Langevin terms — it would seem that the dynamics they describe are completely reversible and there can be no talk of information loss in this case. The complex of issues that arise around this contradiction are the problem of measurement. It seems necessary to establish some boundary separating the quantum and classical worlds — information must be lost at this boundary — but the physical nature of such a boundary is not entirely clear.

Apparently, a possible solution was formulated for the first time by Werner Heisenberg in his discussions with Niels Bohr [9]: in his formulation, it is necessary to use the concept of a classical meter, i.e., the boundary mentioned above is quite sharp and passes between quantum microobjects and a classical device. Subsequently, this procedure was introduced by von Neumann as an axiom of the mathematical apparatus of quantum mechanics [10]. Within the framework of this approach, known as the Copenhagen interpretation, an additional postulate is introduced in relation to the equations of quantum mechanics: reduction occurs during measurement. The system under study instantly finds itself in one of the eigenstates of the measured operator; the probability of choosing each of the eigenstates is determined by the square of the modulus of the product of the wave functions before and after the measurement. A similar viewpoint was held, for example, by V A Fock [11], who considered it fundamental to divide the experiment into several separate stages: preparatory, working, and measuring, of which the first and third should be described at the level of classical physics.

The head of the Copenhagen school himself, Niels Bohr, however, was inclined to talk about the quantum nature of the entire installation as a whole; in this case, the device should be arranged in such a way that its readings could be interpreted in classical terms [12]. The followers of this point of view, in particular C F Weizsäcker [13], repeatedly emphasized that such properties are necessarily associated with the loss of information in quantum systems.

Within the framework of the many-world interpretation [14], a probabilistic choice is not made. It is assumed that the Universe remains in a superposition state, in which there are all possible outcomes of all measurements. However, our consciousness is arranged in such a way that it 'sees' only one of the possible realizations. This concept does not contain internal contradictions, but it seems to the author to be 'super-redundant': when constructing a theory, one would like to confine oneself to the single — observable — reality.

There is a viewpoint according to which the world as a whole is 'quantum', but only our consciousness perceives it as classical. Then, the choice naturally occurs at the moment when the experimenter realizes the measurement results. Relatively recently, M B Menskii and other authors published a number of papers on this subject on the pages of *Phys. Usp.* (see Review Ref. [15] and references cited therein). In this connection, we can also mention the concept of 'logical inference' [16], which explains quantum mechanics by appealing not to the properties of the physical world but rather to its possible perception by our consciousness. The author, however, would like to think that the surrounding reality is objective. As noted by V L Ginzburg concerning the preface to the mentioned article by M B Menskii, it is hard to believe that blackening on a photographic emulsion does not exist with certainty until a person has looked at it.

The paradigm we adhere to is closest to the views of Bohr and C F Weizsäcker. Seeming desirable is a picture whereby the system under study, the macroscopic device, and the observer would be described by the same laws. (It is difficult to refrain from quoting Margaret Thatcher's phrase, even if it was said on a completely different occasion: "There is no such thing as public money; there is only taxpayers' money.") In this formulation, the device, being a completely quantum object, must nevertheless ensure the transformation of quantum information into classical information, and it is necessary to indicate some physical property inherent in all (or at least many) devices that provides the possibility of such a transformation.

Below, we present the view that the probabilistic choice in quantum physics appears naturally and inevitably, since quantum information in macroscopic measuring instruments is inevitably lost due to the chaotic dynamics of their multiple degrees of freedom. In order to make the text consistent and understandable, including to interested students, we start with a presentation of fairly well-known basic concepts and illustrate them using the example of the simplest models. What follows will require the involvement of more modern concepts related to ergodicity in quantum systems; in doing so, we will also try to avoid complex formal constructions as much as possible. For definiteness, we note that we restrict ourselves to the analysis of the measurement problem in the original formulation, and do not touch, for example, on weak measurements [17].

At the end of the introductory part, it is pertinent to note that the measurement problem is part of a broader issue about the properties of open quantum systems — compact quantum objects interacting with macroscopic systems that play the role of a thermostat. W Zurek developed the idea that essentially nonclassical (and, in particular, entangled) states of quantum systems in such a general case turn out to be unstable under the influence of the environment [18]. In the process of evolution, only so-called pointer states — those close to classical ones — survive. For measuring devices, such stable states should correspond to the classical measurement results. However, it is still unclear for which class of physical systems this approach is justified. In the context of the theory of open systems, in the problem of measurement, the case in point is a special choice of interaction between a quantum system and its environment. Such a choice defines a natural basis, and there is no need to specifically search for the pointer state. In this sense, analysis of the measurement problem is simpler than that of a generic open system.

2. Reduction or entanglement?

To clarify the physical meaning of the measurement procedure, we first consider how information about the state of a quantum system can be transferred to another quantum object. Let us address the problem of measuring the projection of the spin of a particle with spin 1/2. Let us add another particle (playing the role of a measuring instrument) whose spin is initially oriented downwards, so that the two-particle wave function of the system is of the form $|\Psi\rangle(t=0) = |\psi^{(1)}, \downarrow\rangle$.

Suppose that we have somehow organized the interaction between two spins, which is characterized by the Hamiltonian $\hat{H} = 2(\hat{s}_z^{(1)} + 1/2)\hat{s}_x^{(2)}$, where $\hat{s}_z^{(1)}$ and $\hat{s}_x^{(2)}$ are the operators of the z- and x-projections of the spins of the first and second particles, respectively. The interaction is turned on at the initial moment of time and is valid for the period $t = \pi$, after which it is turned off, and subsequently the systems do not interact. Solving the Schrödinger equation, one can verify that the wave function of the system after switching off the interaction has the form $|\Psi\rangle(t=\pi) = \psi_\uparrow|\uparrow,\uparrow\rangle + \psi_\downarrow|\downarrow,\downarrow\rangle$ (in the language of quantum computing, the described process corresponds to the action of the CNOT operator). Next, a third particle can be added and, acting in a similar way, we get the state

$$|\Psi\rangle(t=\pi) = \psi_\uparrow|\uparrow,\uparrow\uparrow\rangle + \psi_\downarrow|\downarrow,\downarrow\downarrow\rangle\,. \qquad (1)$$

If the system under study is known in advance to be in one of the eigenstates of the measured quantity — in our case, in the state $|\uparrow\rangle$ or $|\downarrow\rangle$ — such a procedure is a measurement in the usual, classical sense: the wave function of the system remains invariable, and the information about it can be copied into an arbitrary (including macroscopic) number of degrees of freedom, which can be identified with the operation of a classical device (although the whole procedure is completely quantum). *It is measurements of this type that we will consider a* '*bridge*' *between the quantum and classical worlds*: *we will reduce any* '*classical*' *information about the system to measurements on systems that are in the eigenstates of the measured operators*.

In the case of an arbitrary position, wave function (1) describes the state of 'Schrödinger's cat' — a coherent superposition of two different states of many-particle systems. Suppose now that we have lost access to the third particle. A complete description of the possible evolution of the remaining two in this case is given by the density matrix

$$\rho = |\psi_\uparrow|^2|\uparrow,\uparrow\rangle\langle\uparrow,\uparrow| + |\psi_\downarrow|^2|\downarrow,\downarrow\rangle\langle\downarrow\downarrow|\,, \qquad (2)$$

which is determined by taking a partial trace over the projections of the inaccessible third spin.

The result obtained can be easily correlated with the axiom of measurement. Indeed, it is well known that the density matrix can be introduced in two ways: either as done above, by taking a partial trace for a system in a state of quantum entanglement by its environment, or as an average $\overline{|\Psi\rangle\langle\Psi|}$ over a random ensemble in which each realization of a quantum system has its own wave function. These two methods lead to completely equivalent results in the sense that, based on the density matrix, it is impossible to determine its origin. The behavior of a set of systems characterized by a given density matrix will be the same, regardless of whether we are dealing with many identical copies of a system entangled with the environment or an ensemble of systems, each of which is in a randomly selected pure state. It is easily comprehended that the von Neumann postulate corresponds to an ensemble in which the states $|\uparrow\rangle$ and $|\downarrow\rangle$ appear with probabilities $|\psi_\uparrow|^2$, $|\psi_\downarrow|^2$, and that this ensemble corresponds to the density matrix (2). In addition, since in the ensemble interpretation for each implementation the wave function is an eigen one of the observable operator, it allows a classical measurement, in accordance with what was said above. Therefore, from a formal point of view, the measurement axiom only sets the statistical ensemble corresponding to the required density matrix. Nevertheless, several physically significant issues call for discussion.

One can raise the question of the reality of the reduction (collapse) of the wave function. Is it true, as von Neumann postulates, that for each specific implementation of a quantum system we are dealing with an eigenstate of the measured quantity? First of all, we note the incorrectness of such a formulation of the question: for one specific implementation, there is no way to find out what quantum state it is in (to avoid misunderstandings, we note that quantum tomography allows us to determine the wave function only if there is not a unique implementation, but an ensemble of identical systems). It is possible to unambiguously judge the wave function of a given quantum system only in the presence of additional classical information, for example, about the presence of the system in the eigenstate of some observable. Next, it is quite obvious that, in the example under consideration, the reduction did not occur: we just took the third particle out of the scope of consideration; the whole system is in reality still in an entangled state. We come to the conclusion that real reduction, if it takes place, is associated with a complete and irreversible loss of information about quantum entanglement, instead of which classical information about the state of the system should appear.

So, is the loss of information described above only a consequence of our unwillingness to monitor the necessary degrees of freedom, or in certain cases are we talking about processes that are irreversible in the true sense? Of course, if we specifically want to get rid of quantum information, this is not difficult to do: it would suffice to organize the mixing of the system under study with a photon and send it into space — it will no longer be possible to catch up with a photon moving at the speed of light, which means that for an earthly observer such information is irretrievably lost. Similarly, quantum coherence can 'leak' into the excitations of the medium with which the quantum system interacts: B B Kadomtsev [19, 20], considering some gas as a thermostat, wrote about the emergence of an irreversibility front, behind which there is a propagation of a set of wave packets that uncontrollably carry information into the surrounding world. A similar approach also helps in modeling quantum systems: the researcher considers a set of electromagnetic field modes as a thermostat, in which, once inside, information is irreversibly lost [21]. But one can always object to such arguments by saying that in fact there was no collapse of the wave function or emergence of a statistical ensemble, and the Universe is still in a single entangled state. In fact, such a picture underlies the multi-world interpretation of quantum mechanics. In any case, talking about the wave function of the Universe is completely speculative, since it is not clear whether quantum mechanics applies to the Universe as a whole. For us, it seems more important that real measuring instruments do not contain any specially organized channels for information output. On the other hand, one can hardly expect that, by isolating a macroscopic measuring device well from the rest of the world, we will achieve some fundamental differences from the case when this device simply stands on a shelf in the laboratory (in fact, Schrödinger wrote about this, ruling out the possibility of a superposition of a living and dead cat enclosed in a sealed container). A more acceptable view would be that, both in a 'normal' and even in a very well isolated macroscopic system, information is somehow lost, so that instead of a superposition we are dealing with a mixed state.

So, there is good reason to understand: can information be lost and, in particular, the collapse of the wave function take place in large but isolated physical systems? The initial answer to this question is negative: the evolution of a finite quantum system is completely reversible in time, and the loss of information inevitably means irreversibility. However, such a formulation of the question is too formal: absolutely isolated physical objects do not exist. It should be asked whether a significant loss of information can take place in systems whose communication with the outside world is very weak.

Recent decades have seen the appearance of a number of papers devoted to the search for systems that are prone to wave function collapse, even with a weak interaction with the environment. For example, Zimanyi and Vladar [22] came up with the idea of simulating a classical measuring device using a system prone to spontaneous symmetry breaking. The simplest example of such a system is the Ising model, a set of spins with an interaction of the form $\sum_{ij} J_{ij}\hat{s}_i^z\hat{s}_j^z$, $J_{ij} < 0$. In the limit of an infinitely large number of spins, the model has two equivalent ground states, $|\uparrow\uparrow\ldots\uparrow\rangle$ and $|\downarrow\downarrow\ldots\downarrow\rangle$. It is hypothesized that a large but finite system, initially placed in a state of superposition between these states, will be 'forced' to choose only one of them under even a very weak interaction with the environment, which will correspond to a projection during measurement. An attempt to implement such a layout by performing a direct calculation for a specific model [23], however, was not entirely successful: information about the measurement results was present in the system while the measured object interacted with the device, but was instantly forgotten as soon as the interaction was turned off (real devices, of course, store information even on completion of the measurement procedure). In addition, it is significant that spontaneous symmetry breaking does not actually underlie the measuring instruments actually used by physicists (at least in the vast majority of cases).

Apparently, the most common device in quantum measurements is the *detector* — a device that triggers when a particle is detected in a given state (the emulsion grain turns black when a photon hits a given point, the photodetector produces a current pulse, etc.), and does not trigger in the opposite case. For example, Fioroni and Immirzi [24] talk about a first-order phase transition or about the decay of a metastable state when a measuring device is triggered. However, they do not analyze the mechanism of information loss, restricting themselves to the statement that "the evolution of the order parameter is irreversible." Nakajima [25] considers a meter constructed similarly to a photomultiplier and also talks about the 'obviously' irreversible nature of the considered processes, without analyzing the fundamental sources of this irreversibility. We will present a simple microscopic model corresponding to a measurement with the use of a detector in which the source of information loss is the chaotic dynamics of the device.

3. Chaos

The question of the relationship between random and predetermined in physics had long been discussed before the formulation of quantum theory. In this context, it is appropriate to distinguish between truly random and pseudo-random variables: the value of the former cannot be determined in advance *in principle*, while the latter could be predicted if the necessary information about the system and perfect computing facilities were available. For a long time, it seemed that there was no place for true randomness in an isolated conservative system: solving equations of motion is a purely technical task; however, from a mathematical point of view, the trajectory of the particles that make up the system is uniquely determined by the law of their interaction and initial conditions. This view of things since the end of the nineteenth century was one of the main arguments of critics of Boltzmann's statistical theory — why describe in terms of statistical ensembles a system that is not really random?

A complete understanding of the fundamental inevitability of the emergence of random variables in classical mechanics was achieved only in the middle of the 20th century, with the development of the concept of dynamic chaos. If a conservative dynamical system is nonlinear and is far from the equilibrium point, then its dynamics in many cases turn out to be chaotic, i.e., the trajectory of its motion turns out to be unstable with respect to small changes in the initial conditions. If we consider the distance between two trajectories that differ, for example, by a small shift in the initial coordinates of the particles, it turns out that the specified distance increases exponentially fast with time:

$$\delta x(t) \sim \delta x(0) \exp\left(\frac{t}{t_{\mathrm{L}}}\right),$$

where $\lambda = 1/t_{\mathrm{L}}$ is the Lyapunov exponent. In fact, this means that it is possible to calculate the evolution of the system under given initial conditions only over an interval of several ten t_{L}: for long times, even a very small error in the initial conditions will change the trajectory beyond recognition. The converse statement is also true: looking at the state of the system at a given point in time, it is possible to understand from what point in the phase space the evolution began only if the evolution under consideration did not last very long — the exponential divergence of trajectories in the phase space means that the system 'forgets' the initial conditions. Under these conditions, the evolution of the system should be considered a *random process*. Only a few quantities associated with the integrals of motion, for example, the total energy of the system, remain defined. One of the traditional examples of systems with chaos is dynamical billiards, systems in which a material point moves on a plane inside a closed figure of a given shape, elastically reflecting from its walls. In many cases (in particular, when the billiard has both corners and curvilinear sections, as in Sinai billiards), the motion in such a system turns out to be chaotic. After sufficient time, the distribution function of such a system turns out to be uniform: a particle can be found with equal probability at any point in the phase space with a given energy. (Of course, here, we deliberately simplify the presentation and sacrifice many details important for the study of specific dynamical systems at the expense of general clarity of the text.)

We now turn to quantum systems. An important property of quantum mechanics is its linearity with respect to initial conditions (while Newton's equations are nonlinear). Indeed, the solution of the Schrödinger equation with a time-independent Hamiltonian is of the form

$$|\Psi(t)\rangle = \exp\left(-\frac{\mathrm{i}\hat{H}t}{\hbar}\right)|\Psi_0\rangle\,,$$

where $\exp(-\mathrm{i}\hat{H}t/\hbar)$ is the unitary evolution operator. A small variation in the initial wave function by the time t will lead to the variation $|\delta\Psi(t)\rangle = \exp(-\mathrm{i}\hat{H}t/\hbar)|\delta\Psi_0\rangle$. Since unitary operators conserve the norm of vectors, $||\delta\Psi(t)|| = ||\delta\Psi_0||$, small variations of the initial wave function certainly do not lead to a significant change in the result of evolution.

Nevertheless, quantum systems exhibit chaotic behavior to the full extent. To see it, instead of small variations in the initial conditions, we should consider *small variations in the Hamiltonian*, on which the evolution operator depends in a nonlinear way. With regard to billiards, the transition to quantum mechanics means that a wave packet rather than a material point should be placed on the billiard. In this case, exponential instability manifests itself not with respect to a change in the initial characteristics of the wave packet but with respect to small variations in the shape of the billiard (classical trajectories actually show instability to variations of both the initial conditions and the Hamiltonian).

As a quantitative measure of the sensitivity of a quantum system to variations in its parameters, advantage can be taken of the Loschmidt echo — the square of the modulus of the scalar product of wave functions corresponding to evolution with slightly different Hamiltonians: $|\langle\Psi|\Psi'\rangle|^2$, where $|\Psi\rangle = \exp(-\mathrm{i}\hat{H}t/\hbar)|\Psi_0\rangle$ and $|\Psi'\rangle = \exp(-\mathrm{i}\hat{H}'t/\hbar)|\Psi_0\rangle$. Quantum chaos in large (i.e., containing many particles and/or large compared to their de Broglie wavelength) systems manifests itself in the exponential decay of the Loschmidt echo with time [26, 27]. The difference between quantum and classical chaotic systems lies primarily in the fact that the Loschmidt echo in the quantum case begins to noticeably decrease not immediately but some time after the beginning of evolution. For us, this difference is not significant: in any case, after a sufficient lapse of time, the calculation of evolution for a specific Hamiltonian of a quantum system with chaos loses its meaning and it is necessary to consider an ensemble of quantum systems, i.e., move from the wave function to the formalism of the density matrix. The role of the uniform distribution function is played by the diagonal density matrix $f(E)\sum|j\rangle\langle j|$, where the envelope $f(E)$ cuts out states belonging to a certain energy interval of width δE. As is easy to see, the entropy of the system, initially equal to zero (since the system had a certain wave function), eventually turns out to be equal to $\ln(A_E\,\delta E)$, where A_E is the density of states. The generation of nonzero entropy characterizes the loss of quantum information.

Let us formulate once again the source of randomness origination in chaotic systems. In our world, both the initial conditions and the Hamiltonians of the systems under study are never known exactly, both because of the imperfection of measurement methods and because there is no way to completely eliminate the influence of external factors. Perhaps, for a precise formulation of the theory, we should introduce the definition of a physically closed system — such a system whose interaction W with the external world can be considered small, while $\ln^{-1} W$ is not small compared to t_{L}/t. A physically closed system does not exchange energy or particles with the outside world. Also, changes in the parameters of a physically closed system are so small that they cannot be detected by direct measurements. However, in the case of chaotic systems, the uncertainty of the system parameters leads to an exponentially growing uncertainty of the result of evolution with time. In this paradigm, the modeling of any physically closed system must involve the analysis of an *ensemble* of systems with slightly different parameters. It is significant that no improvement in isolation from the outside world will lead to a significant change in the situation due to the exponential nature of the instability. Therefore, the inability to calculate a specific trajectory which a chaotic system describes or to determine the initial conditions that brought it to a given state should be attributed to the fundamental properties of the system itself, and not to the imperfection of our description. In precisely the same way, NP-hard (i.e., requiring an exponentially large number of operations) problems should be considered fundamentally unsolvable using a classical computer, and the impossibility of solving them must not be attributed to the imperfection of existing computing systems. To state it in different terms, the loss of information in chaotic systems is fundamentally irreversible, in contrast to the case when the information carrier is removed from the region accessible to the experimenter, but, in principle, continues to exist. The result of evolution of a chaotic system, both in the classical and in the quantum case, is truly random.

The inevitability of information loss in physically closed systems with chaos seems to be important for substantiating the basic principles of physics related to the (ir)reversibility of the flow of time, such as the principle of causality and the second law of thermodynamics. The equations of motion are symmetrical with respect to time reversal, and it is not clear why, in accordance with the second law of thermodynamics, the entropy of a closed system increases when moving from the past to the future and decreases when moving backwards. The standard explanation relates this asymmetry to the fact that information about the system is given at the initial, and not at the final, moment of its evolution. With this approach, the direction of the flow of time is actually imposed axiomatically (time in a closed system flows in the same way as in the rest of the world, which is expressed at the instant the initial conditions are set). The introduction of such 'superfluous' axioms can be avoided if one bears in mind that the term 'closed' actually means a physically closed system. In this case, small fluctuations of random forces from the environment during the evolution of the system will lead to a loss of information, i.e., an increase in entropy, both for forward and reverse time.

If we consider an ensemble of identical physically closed billiards, then, after a sufficiently long evolution, on average, for half of them, a particle will be found in its left half, and for the other half, in its right, but there is no way to predict the result of evolution for a given implementation. For now, we are abstracting from what the term 'discovered' means in relation to a quantum system, but in any case it is clear that the mechanism for randomness origination under discussion has nothing to do with von Neumann's postulate: quantum and classical billiards behave in a very similar way; only the language differs — a description in terms of distribution functions and the density matrix, respectively.

Let us show that the concept of information loss in quantum chaotic systems may be used to understand the nature of measurements in quantum theory.

4. Quantum chaotic system as a measuring device

Let us still measure the spin state of a particle with spin $1/2$, and assume that the particle itself is localized in a single level of the discrete spectrum of a potential well located near a quantum billiard (i.e., in fact, another well with 'vertical' walls, much larger than the de Broglie wavelength of the particle and having an irregular shape). We assume that the barrier between the wells depends on the particle spin: for the state $|\downarrow\rangle$, the barrier is too high for tunneling to be noticeable, but for a particle in the state $|\uparrow\rangle$, tunneling is possible (i.e., the system is described by the Hamiltonian $\hat{p}^2/2m + U_W(\hat{r}) + U_B(\hat{r}) + V(\hat{r})s_z$, where U_W and U_B are potentials of the well and billiard, respectively, and V describes the spin-dependent part of the potential in the space between the wells). Then, the coordinate part of the wave function of the particle actually serves to measure its spin projection: when the spin is directed downward, the particle remains on a localized level, and when the spin is directed upward, it obviously tunnels into the billiard. If we start with the spin part of the wave function of the general form, $\psi_\uparrow|\uparrow\rangle + \psi_\downarrow|\downarrow\rangle$, immediately after tunneling, the spin and coordinate parts of the wave function are mixed, and information about the initial phase between $\psi_\uparrow$ and $\psi_\downarrow$ is contained in the total wave function of the system $\psi_\uparrow|\phi_W(r),\uparrow\rangle + \psi_\downarrow|\phi_B(r),\downarrow\rangle$. Here $\phi_{W,B}$ are the spatial components of the particle, respectively, remaining at the bound level in the well and having tunneled into the billiard. Component ϕ_B corresponds to some wave packet that begins to evolve, reflecting off the walls of the billiard. However, if the quantum dynamics of the system is chaotic, then, after a sufficient time due to small uncertainties in the position of the walls, decoherence will occur: we will have to go over from a single wave function ϕ_B to some ensemble ϕ_B^j. Eventually, the particle can be detected at any point in the billiard with equal probability, and all phase relations will be lost. The system as a whole, in full accordance with the postulate of reduction, will be a statistical ensemble with two types of states: the spin directed down, the particle in a localized level; and the spin directed up, the particle in the billiard. One can see that, in the example considered, the coordinate part of the particle wave function plays the role of a measuring device for the spin component. Therefore, the quantum information associated with entanglement is destroyed, and the state of the detector is described by a single classical bit that determines whether the particle is on the table or not. Fundamentally, quantum billiards should be large enough: only in this case, quantum chaos and loss of phase information are possible in the system. It can be said that the large size and chaotic dynamics provide a transition to the classical limit for the measuring device.

We come to the conclusion that measurement in quantum physics can be described as follows. At the initial point in time after the measurement, the wave functions of the quantum object and the device become entangled. In the total 'object + device' system, quantum information is not lost, so von Neumann's postulate is satisfied only if we artificially exclude from consideration some of the information about the wave function of the device. However, a macroscopic device containing a large number of interacting particles is a quantum chaotic system. There is no way to completely isolate it from the surrounding world, and arbitrarily weak variations in the instrument's Hamiltonian associated with such an interaction will be exponentially enhanced. Therefore, after several ten (or hundred, if the device is isolated very well) characteristic times t_L, for the 'object + device' system, what is relevant (and, in fact, the only thing possible!) is the description in terms of a random ensemble, in each of the implementations of which the object is located in one of the eigenstates of the observable and the device is in a state with the corresponding integrals of motion and random values of the remaining quantum numbers. In terms of Schrödinger's thought experiment, instead of a nonphysical superposition of a living and dead cat we, as one would expect, will obtain a random process, in each of the implementations of which the cat is either alive or dead.

In our reasoning, we have so far considered the forces acting from the external world as classical. If we assume that the degrees of freedom external to the device are quantum, then the system turns out to be completely linear. It would seem that in this case its full Hamiltonian is known exactly, and the description in terms of a single pure state remains valid. However, the effects of quantum chaotic dynamics in this case lead to difficulties of a different kind: as can be verified, the above reasoning for the 'fully quantum' picture of the world means that, even with very weak interaction, the wave functions of the device will be significantly entangled with the variables of the external world. Since even a very weak interaction of the external world with a chaotic system is significant, the number of variables involved in such entanglement turns out to be very large (on the order of Avogadro's number). We can say that, instead of the accuracy of measurements and computations necessary for calculations, the number of task variables 'explodes.' There is no physical difference: quantum information 'dissolves' without a trace in the Universe (in contrast to Everett's interpretation, in which information leaves the field of view of the observer, but in principle persists). The meaningful results for the quantum and classical descriptions of the environment, of course, coincide: there is a transition of a physically closed system from a pure to a mixed state.

5. Conclusions

- The randomness and irreversibility of evolution in physically closed systems emerge as a consequence of chaotic dynamics and are associated with the fundamental impossibility of absolutely exact determination of both the initial conditions (for classical systems) and the Hamiltonian parameters (in both the classical and quantum cases). The Hamiltonian parameters cannot be considered exactly known because absolutely closed systems do not exist, and any arbitrarily small perturbation of a chaotized system from the outside world will be exponentially enhanced.
- The measurement of the state of a quantum object can be split into two phases: the formation of an entangled state of the object and the device and the subsequent irreversible loss of information associated with entanglement due to the chaotic dynamics of the degrees of freedom of the device.

• Since the loss of information and the emergence of randomness are the same thing, we conclude that the random nature of the results of measurements of quantum quantities (and, in general, the phenomenon of wave function collapse) is associated with a special case of information loss in a chaotic quantum system, which is an integral part of the measuring device. The collapse of the wave function in this case occurs not at some selected point in time but continuously over a time determined by the Lyapunov exponent of the chaotic system.

The postulates made call for direct verification. The most obvious is a numerical experiment simulating a physically closed quantum meter with chaotic dynamics. Known results [27] indicate that quantum information is indeed lost in such systems. It is necessary, however, to determine how the loss rate depends on the size of the system: it must be verified that a chaotic system of macroscopic size does lose coherence when there is very weak interaction with the external world. We hope to carry out a simulation of this kind in the future.

To emphasize the point, the main conclusion can be formulated as follows. The axiom of measurement is not an axiom; it can be deduced by treating the measuring device as a physically closed chaotic system.

There is no quantum randomness, there is only the randomness of evolution in the presence of chaos.

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