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Quantum entanglement, teleportation, and randomness: Nobel Prize in Physics 2022

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Abstract. Precise control for individual quantum systems, such as individual photons, atoms, or ions, opens the door to a range of quantum technologies. The goal of this concept is to create devices that, due to quantum effects, will be able to solve problems of data processing and secure information transfer and high-precision measurements of parameters of the surrounding world more effectively than existing approaches do. The key step in the advent of quantum technologies was the pioneering work of the second half of the twentieth century, which, first, showed the paradoxical nature and correctness of the quantum mechanical description of nature and, second, laid down and introduced the basic experimental approaches that became the basis of modern quantum technologies. The Nobel Prize in Physics 2022 was awarded to Alain Aspect, John Clauser, and Anton Zeilinger for their experiments with entangled photons, establishing the violation of Bell inequalities, and pioneering quantum information science.

Keywords: EPR paradox, quantum correlations, entanglement, teleportation, quantum communications, quantum computing, quantum sensors, quantum metrology

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1. Introduction

About a century ago, events were taking place in physics that marked the emergence of quantum mechanics. The concept of individual 'quanta,' which Max Planck introduced in 1900, not only helped solve the problem of the ultraviolet catastrophe [1] but had much more far-reaching consequences for physics. Born in heated debates was a deeply unintuitive science not amenable to the usual 'Newtonian' description for humans, based, on the one hand, on relatively simple mathematics, and, on the other hand, on such new concepts as the wave properties of particles, probability amplitude, superposition, and entanglement. Specifically, in 1913, Niels Bohr formulated his famous model using experimental data on the spectra of the hydrogen atom [2–4]. The model is based on the concept of quantization: for stationary electron orbits, the angular momentum multiplicity is $\hbar = h/2\pi$, where $h \approx 6.626\times 10^{-34}$ J s is the famous Planck constant. The works of Max Planck, Paul Dirac, Wolfgang Pauli, Max Born, John von Neumann, Arnold Sommerfeld, and Albert Einstein, the experiments of Stern and Gerlach, and the theory of Louis de Broglie expand the understanding of the quantum world, allowing us to interpret accumulated experimental data and predict the behavior of elementary systems: the structure of the atom, the structure of spectra, properties of radiation, the interaction of light and matter, wave properties of particles, and much more. The year 1925 saw Heisenberg's work with the fundamental formulations of quantum mechanics and reasoning about the relationship between quantities fundamentally accessible to observation in experiment (which in 1927 led him to the formulation of the uncertainty principle) [5]. And, of course, there was Schrödinger's surprisingly correct formulation of his famous equation [6]. The new 'quantum world' had fully come into its own.

Quantum mechanics gave humankind a powerful tool for describing the interaction of physical systems with radiation, and its mathematical apparatus was actively developing, integrating relativism, the concept of spins, the physical vacuum, other types of interaction, and much more. For instance, quantum electrodynamics — one of the areas of development of quantum mechanics — even today retains the position of the most accurate predictive science, allowing calculations of the properties of elementary particles with an error of up to the 10th decimal place (for example, the g-factor of the electron [1]). At the same time, many formulations and concepts of quantum theory have caused, and often continue to cause, controversy and misunderstanding. Even the usual concept of superposition, written through the amplitude coefficients of the wave function $\psi = \alpha|0\rangle + \beta|1\rangle$ ($|\alpha|^2 + |\beta|^2 = 1$), where $|0\rangle$ and $|1\rangle$ are some orthogonal states, led to the formulation of the well-known paradox of 'Schrödinger's cat,' which is simultaneously in the living and dead states [7]. The lack of direct experiments and the lack of technical ability to study single quantum systems did not allow us to make a clear choice in favor of one interpretation or another. Furthermore, the postulates of quantum mechanics themselves opened up the broadest opportunity for discussing various 'Gedankenexperiments' (thought experiments), which caused heated discussions among great scientists. Uncertainty persisted for decades, which predictably increased the intrigue. In this sense, the situation was reminiscent of the story of the special theory of relativity, also rich in thought experiments and their seemingly paradoxical interpretation.

One of the cornerstones for discussions was the concept of entanglement and the inextricably linked Einstein–Podolsky–Rosen paradox (EPR paradox, 1935), which attracted the close attention of the scientific (and not only scientific) community. The formulation of the paradox is quite simple [8] (see also Fock's introductory article [9]). It was proposed to consider the state of two particles A and B, which is simultaneously an eigenstate for the sum of momenta operator $\hat{p}_+ = \hat{p}_{\rm A} + \hat{p}_{\rm B}$ and the coordinate difference operator $\hat{x}_- = \hat{x}_{\rm A} - \hat{x}_{\rm B}$. Note that, although the individual position and momentum operators of each particle do not commute ($[\hat{x}_{\rm A}, \hat{p}_{\rm A}] = [\hat{x}_{\rm B}, \hat{p}_{\rm B}] = {\rm i}\hbar$), the commutator $[\hat{x}_-, \hat{p}_+] = 0$, and therefore a quantum state with deterministic values p_+ for the observable $\hat{p}_+$ and x_- for the observable $\hat{x}_-$ exists (from the point of view of quantum optics, this state corresponds to a two-mode compressed state with an infinite value of the compression parameter). By measuring the coordinate or momentum of particle A and obtaining the values $x_{\rm A}$ or $p_{\rm A}$, respectively, one can calculate the coordinate or momentum of particle B: $x_{\rm B} = x_- + x_{\rm A}$, $p_{\rm B} = p_+ - p_{\rm A}$. Based on the principle of locality, it can be argued that the direct choice of the type of measurement ($\hat{p}_{\rm A}$ or $\hat{x}_{\rm A}$) for particle A should not affect the 'physical reality' for particle B. Therefore, the conclusion is drawn that both observables ($\hat{x}_{\rm B}$ and $\hat{p}_{\rm B}$) should simultaneously have some definite 'real' values of $p_{\rm B}$ and $x_{\rm B}$, even if not predicted by the existing 'incomplete' formalism of quantum mechanics. However, in principle, one can expect that the apparatus of quantum mechanics can be supplemented by a method for predicting the values of noncommuting observables.

A somewhat more visual development of the EPR paradox is Bohm's thought experiment, formulated by him in 1951 [10]. He considered a particle with spin 0 decaying into two particles A and B with spin 1/2, and the orientations of the spins of the daughter particles should be opposite. The wave function of the resulting system can be written as $|\Psi^-\rangle_{\rm AB} = 1/\sqrt{2}(|\downarrow\rangle_{\rm A}|\uparrow\rangle_{\rm B} - |\uparrow\rangle_{\rm A}|\downarrow\rangle_{\rm B})$, where $|\uparrow\rangle$ and $|\downarrow\rangle$ denote the eigenstates of the spin projection onto the z quantization axis. This entangled state is now commonly referred to as the Bell state, named after John Bell. The specificity of this state is that the first and second particles are anticorrelated: this state is an eigenvector of the operator $\hat{\sigma}(\mathbf{n})_{\rm A}\hat{\sigma}(\mathbf{n})_{\rm B}$ with an eigenvalue equal to -1, where $\hat{\sigma}(\mathbf{n})$ denotes the spin projection operator onto the axis defined by the three-dimensional unit vector $\mathbf{n}$. Accordingly, measuring the spin $\hat{\sigma}(\mathbf{n})_{\rm A}$ of one particle uniquely determines the result of measuring the second spin $\hat{\sigma}(\mathbf{n})_{\rm B}$, and it does not matter at what distance the particles are from each other. From a classical point of view, it would seem, what is surprising here? After all, if you take a box with a pair of shoes, cut it in half and send the halves to different cities, then finding the right shoe in Moscow will definitely mean that the left shoe will be found in St. Petersburg. However, for a quantum system, everything turns out to be much more complicated, since anti-correlation remains valid, for example, for mutually orthogonal axes $\mathbf{n}_1$ and $\mathbf{n}_2$, for which the corresponding operators $\hat{\sigma}(\mathbf{n}_1)$ and $\hat{\sigma}(\mathbf{n}_2)$ do not commute, and therefore, from the point of view of the axiomatics of quantum mechanics, cannot have certain values at the same time.

Albert Einstein, being a proponent of determinism, proposed a possible solution to the paradoxes of quantum mechanics using the idea of so-called 'hidden variables' (this was happening in the late 1930s; the theory was subsequently developed by de Broglie and Bohm). According to Einstein, it was precisely such hypothetical variables, inaccessible to direct observation, that should have unambiguously determined the outcome of the measurement of quantum mechanical quantities, and the seemingly random result to the observer should have in fact been predetermined in advance. A clear example is the Stern–Gerlach experiment, in which spin $-1/2$, described by the wave function $\Psi = \cos(\theta/2)|\uparrow\rangle + \sin(\theta/2)|\downarrow\rangle$ (θ is the angle to the z-axis), is projected onto the z-axis. From the point of view of the postulates of quantum mechanics, the measurement result is random, and the probability of detecting a spin direction co-directed with z is $p = \cos^2(\theta/2)$. From the point of view of the theory of 'hidden variables,' it can be assumed that there is some hidden deterministic parameter u, inaccessible to the observer, that predetermines the result of spin measurement. And if the probability distribution of the parameter is described by the $\cos^2(\theta'/2)$ function (θ' is uniformly distributed from 0 to π), then the measurement result will not differ from the quantum mechanical interpretation with a random outcome. An argument in favor of the theory of 'hidden variables' was also the fact that nature is replete with examples where processes that seem random (the behavior of a gas at the microscopic level, strange attractors, etc.) are actually determined through dynamic equations. However, counterarguments were also expressed: for example, John von Neumann came up with a proof [11] that quantum mechanics supplemented with hidden variables will not predict the results of experiments.

A further development of the theory of hidden variables was the interpretation of David Bohm (1952), who formulated the theory of nonlocal hidden parameters [12]. The result of quantum mechanical experiments in his theory is determined by a hidden 'guiding wave,' which significantly expand de Broglie's original theory and at the same time made it possible to interpret the results of experiments, such as electron diffraction in Young's experiment. In fact, Bohm

[1] https://physics.nist.gov/cgi-bin/cuu/Value?gem.

purposefully developed a very specific theory that made predictions identical to traditional quantum mechanics. Therefore, Bohm showed that the existence of full-fledged theories with hidden parameters is indeed possible.

Inspired by ideas about various interpretations of quantum mechanics and Bohm's theory, in 1964, the Irishman John Stewart Bell formulated his famous theorem [13]. He showed that the results of observations of particles whose behavior is described by an arbitrary hidden parameter theory must satisfy certain relationships, now known as Bell inequalities. At the same time, the axiomatics of quantum mechanics predict the violation of these inequalities for experiments on entangled particles. Therefore, discussions at this point had continued for more than 30 years, and there were still 10 years to come before the first experimental work of John Clauser, who first tested Bell inequality. It is pertinent to note that Bell also complemented von Neumann's proof by pointing out inaccuracies in his original work.

In the 1970s, the development of laser light sources, high-sensitivity detectors, and high-speed electronics opened up possibilities for working with single quantum systems (primarily photons) and experimental verification of Bell inequalities. Alain Aspect, John Clauser, and Anton Zeilinger were the first to conduct experiments that were important for the entire quantum mechanical science. The work of Clauser and Aspect showed a violation of Bell inequalities, which resolved a half-century dispute among adherents of different interpretations of quantum mechanics. Zeilinger's research demonstrated that, in addition to the importance of entangled photon states for fundamental quantum mechanics, they could provide the basis for new ideas, such as demonstrating the transfer of quantum states over distance — quantum teleportation and quantum communications. In the first two decades of the 21st century, the consequences of the scientific experiments of Clauser, Aspect, and Zeilinger became clear: their results formed the basis for quantum technologies — instruments and devices based on the control of individual quantum states. Technologies such as quantum random number generators, quantum key distribution devices (quantum cryptography), and prototype quantum computers are already entering our lives. Alain Aspect, John Clauser, and Anton Zeilinger were awarded the 2022 Nobel Prize in Physics for their experiments with entangled photons, establishing violations of Bell inequalities, and pioneering work in quantum information science.

The field of quantum information processing has already been the subject of several reviews in the journal *Uspekhi Fizicheskikh Nauk* (*Physics – Uspekhi*). A number of issues of quantum computing were considered in Refs [14–17], various aspects of quantum key distribution were considered in Refs [18, 19], and issues of quantum metrology were considered in Refs [16, 17]. The EPR paradox and Bell inequalities in the context of quantum optics were discussed in the studies by Klyshko [20, 21] and Klyshko together with Belinsky [22] (see also the review by Sokolov [23]). The possible role of entangled states for studies of living systems is discussed in review Ref. [24].

2. Bell inequality (original version), experiments of Nobel laureates, Greenberger–Horne–Zeilinger states

The remarkable result obtained by Bell [13] made it possible to demonstrate that no theory based on local hidden variables could explain the results of quantum mechanics. In essence, Bell inequalities are constraints that must be satisfied by the results predicted by an arbitrary local hidden variable theory, but which are violated by the predictions of quantum mechanics. The most famous example of Bell inequalities is the Clauser–Horn–Shimony–Holt (CHSH) inequality, which is convenient in that it can be verified using pairs of photons entangled in polarization. The heart of this inequality is as follows (see review [22]).

Let two remote participants, Alice and Bob, have certain devices at their disposal. On each of the two devices there are two keys, conventionally designated X and Y, as well as a screen on which the values $+1$ or -1 appear after each press of one of the two keys. Each of the participants N times presses one of two keys, and for each of the N presses, the choice of the key X or Y by Alice and Bob is made independently and randomly. The results read from the screen are entered into a table by Alice and Bob. We will denote by $X_i^{\mathrm{A}}(X_i^{\mathrm{B}})$ and $Y_i^{\mathrm{A}}(Y_i^{\mathrm{B}})$ the results read from the screen after the i-th pressing of one of the two keys X or Y by Alice (Bob), respectively. It is important to note that, for each value $i = 1, \ldots, N$, Alice and Bob receive only one of two values: X_i^{A} or Y_i^{A} on Alice's side and X_i^{B} or Y_i^{B} on Bob's side. Therefore, if, for example, for $i = 5$, Alice pressed X and Bob pressed Y, then the 'complementary' values of Y_5^{A} and X_5^{B} remain unknown.

However, when 1) the devices in question are fundamentally unable to interact with each other at the moment of keypressing, for example, if the events of keypressing and the appearance of results on the screens are separated by a space-like interval, 2) the choice in keypressing by Alice and Bob is not fundamentally deterministic or is determined by factors independent of the 'inside' of the devices, then, from the point of view of the complete theory that describes the response of these devices to an arbitrary choice of keystrokes, we can talk about the existence of all four values X_i^{A}, Y_i^{A}, X_i^{B}, Y_i^{B} for each i. It is easy to see that these values satisfy the inequality

$$\left| X_i^{\mathrm{A}}(X_i^{\mathrm{B}} + Y_i^{\mathrm{B}}) + Y_i^{\mathrm{A}}(X_i^{\mathrm{B}} - Y_i^{\mathrm{B}}) \right| \leqslant 2$$

due to the fact that all values belong to the set $-1, +1$. Then, it is obvious that the modulus of the average value of $S_i \equiv X_i^{\mathrm{A}}(X_i^{\mathrm{B}} + Y_i^{\mathrm{B}}) + Y_i^{\mathrm{A}}(X_i^{\mathrm{B}} - Y_i^{\mathrm{B}})$ is also limited to a maximum value of 2:

$$|\overline{S}| = \left| \frac{1}{N} \sum_i S_i \right| \leqslant \frac{1}{N} \sum_i |S_i| \leqslant 2\,.$$

(It can be easily shown that this inequality holds for any values of X_i^{A}, Y_i^{A}, X_i^{B}, Y_i^{B} not exceeding 1 in modulus [25].) On the other hand, the average value of $\overline{S}$ is the sum of four correlation functions

$$\overline{S} = \overline{X^{\mathrm{A}} X^{\mathrm{B}}} + \overline{X^{\mathrm{A}} Y^{\mathrm{B}}} + \overline{Y^{\mathrm{A}} X^{\mathrm{B}}} - \overline{Y^{\mathrm{A}} Y^{\mathrm{B}}}\,.$$

Although Alice and Bob cannot calculate $\overline{S}$ exactly, each of the four terms can be estimated to within $1/\sqrt{N}$ of the given data (for example, to estimate $\overline{Y^{\mathrm{A}} X^{\mathrm{B}}}$, Alice and Bob can calculate the average of the product of the instrument readings in $\approx N/4$ cases where Alice pressed Y and Bob pressed X). Note that the crucial assumption here is that the choice of keys by Alice and Bob is independent of the 'internal gears' that determine the functioning of the devices.

We can therefore expect that the average values observed by Alice and Bob for $N \to \infty$ will also be limited

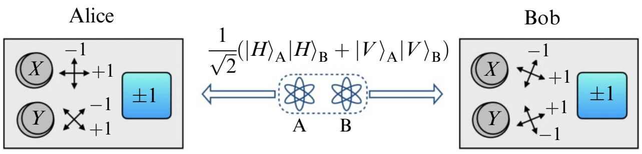

Figure 1. Schematic diagram of Bell inequality verification using entangled photons.

to a value of 2:

$$|\langle S\rangle| \equiv \left|\langle X^A X^B\rangle + \langle X^A Y^B\rangle + \langle Y^A X^B\rangle - \langle Y^A Y^B\rangle\right| \leqslant 2\,, \quad (1)$$

where angle brackets correspond to averaging over available data.

We next consider a specific version of constructing devices based on a pair of entangled photons, whose polarization degrees of freedom are described by the vector

$$|\Phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|H\rangle_A|H\rangle_B + |V\rangle_A|V\rangle_B\right),$$

where $|H\rangle$ and $|V\rangle$ denote the basis vectors of horizontal (H) and vertical (V) polarization, respectively (see also Fig. 1). Let particle A be given to Alice and B to Bob.

We introduce an observable

$$\hat{\sigma}(\theta) = |\psi(\theta)\rangle\langle\psi(\theta)| - \left|\psi\left(\theta + \frac{\pi}{2}\right)\right\rangle\left\langle\psi\left(\theta + \frac{\pi}{2}\right)\right|,$$

where $|\psi(\theta)\rangle = \cos\theta|H\rangle + \sin\theta|V\rangle$, corresponding to the measurement of the photon polarization with respect to two orthogonal axes obtained by rotating the H and V axes by an angle θ. Note that the eigenvalues, i.e., the experimentally observed values of operator $\hat{\sigma}(\theta)$, are ± 1. The average value of the product of observables $\hat{\sigma}(\theta_1)_A \hat{\sigma}(\theta_2)_B$ for the state $|\Phi^+\rangle_{AB}$ is of the form

$$\begin{aligned}\left\langle \hat{\sigma}(\theta_1)_A \hat{\sigma}(\theta_2)_B \right\rangle &= \langle \Phi^+|_{AB} \hat{\sigma}(\theta_1)_A \hat{\sigma}(\theta_2)_B |\Phi^+\rangle_{AB} \\ &= \cos\left[2(\theta_1 - \theta_2)\right]. \end{aligned} \quad (2)$$

Assume that, when pressing the X and Y keys on Alice's side, the observables $\hat{\sigma}(0)_A$ and $\hat{\sigma}(\pi/4)_A$ are measured, and when the X and Y keys are pressed on Bob's side, the observables $\hat{\sigma}(\pi/8)_B$ and $\hat{\sigma}(-\pi/8)_B$ are measured. Then, in accordance with expression (2), we have

$$\langle X^A X^B\rangle = \langle X^A Y^B\rangle = \langle Y^A X^B\rangle = -\langle Y^A Y^B\rangle = \frac{1}{\sqrt{2}}\,,$$

which leads to

$$|\langle S\rangle| = 2\sqrt{2} > 2\,.$$

Therefore, the predictions of quantum mechanics are inconsistent with the limitations that arise in a theory operating with arbitrary local hidden variables.

We emphasize once again that, in the above reasoning, a fundamental role is played by the initial premise that there is no functional dependence of the observed values on Bob's side on Alice's key choice (and vice versa). Otherwise, one can, for example, propose a scheme in which $X_i^B := X_i^A$, and also $Y_i^B := X_i^A$ if Alice pressed X, and $Y_i^B := -Y_i^A$ if Alice pressed Y, and get $|\langle S\rangle| = 4$ for arbitrary $X_i^A, Y_i^A = \pm 1$. For similar reasons, the types of measurements (choice of keys) in a full-fledged test for violation of Bell inequalities must be random: otherwise, having information about the protocol of a future experiment can allow Alice and Bob to program devices so that the observed value $\langle S\rangle$ takes almost any value from -4 to 4. Such deliberate 'preprogramming' is difficult to imagine in the case of a real physical experiment, but it can take place in industrial devices that are reported to use entangled states.

The first experimental demonstration of violation of inequality type (1) was demonstrated in the work of Friedman and Clauser [26]. It considered the production of a pair of entangled photons in $|\Phi^+\rangle_{AB} = 1/\sqrt{2}(|+\rangle_A|+\rangle_B + |-\rangle_A|-\rangle_B)$, where $+$ and $-$ denote the helicity of the photons. Photon pairs at wavelengths of 551 nm and 423 nm, respectively, were produced in the $4p^2 6^1S_0 \to 4p4s4^1P_1 \to 4s^2 4^1S_0$ cascade transition in calcium. Photons were detected by single-photon detectors, with polarizers placed in front of them. A gradual change in the relative angle between the polarizers was carried out, and the ratio of the frequency of coincidences of readings on detectors in the presence and absence of polarizers was considered the main observed quantity. The experimental results obtained from the data of 200 hours of observations were fully consistent with the predictions of quantum mechanics and contradicted the CHSH inequality (specially formulated for observable quantities).

Like any mathematical result, Bell inequalities rely on certain assumptions. The most important assumption underlying the proof of the impossibility of the existence of the theory of hidden variables is *the independence of the choice of measurement types* by Alice and Bob, as well as the fundamental impossibility of the influence of the choice of measurement of one of the parties on the measurement results of the other party. To ensure that such influence is not possible, Alice's (Bob's) measurement type selection and Bob's (Alice's) measurement must be separated by a space-like interval. Otherwise, a 'locality loophole' arises, which makes it possible to potentially explain the violation of Bell inequality in a classical way.

Important steps to close the locality loophole, as well as improve the efficiency of the experiment to demonstrate violations of Bell inequalities, were made by Aspect et al. in 1981–1982 [27, 28] (see also Aspect [29]). The work that attracted the major attention was Ref. [29], in which, thanks to the use of acousto-optical devices, it was possible to design the installation in such a way that the switching of photon measurement bases was carried out in less than 20 ns — the time required for photons to travel a distance of 6 m between the source and the detection devices.

However, the setup used still left the possibility of a locality loophole. It took more than 15 years to completely close the locality loophole: the decisive contribution to this was made by the results of the Zeilinger group [30–34]. Note that in Ref. [30] the detecting facilities were 400 m apart.

Another technical obstacle to the correct proof of Bell inequalities is the so-called 'detection loophole,' which arises in real experiments with the use of photodetectors with finite efficiency. The initial reasoning behind Bell inequalities assumes that participants will always get a result of $+1$ or -1 after choosing the type of measurement. In a real experiment, however, it may happen that the photon will not be detected by one or both parties due to the finite efficiency of the detectors. One can naively assume that in the analysis of experimental results it would suffice to limit

Table. Firmware layout for Alice and Bob's devices that ensures violation of Bell inequalities when excluding records with a detection error when calculating the value of $\langle S \rangle$.

Experiment number i	X_i^{A}	Y_i^{A}	X_i^{B}	Y_i^{B}
1	+1	+1	+1	'error'
2	−1	+1	'error'	−1
3	−1	'error'	−1	−1
4	'error'	−1	−1	+1
...	...	...	...	...

oneself to a subsample of experimental results in which both participants, Alice and Bob, successfully detected a photon. However, in this case, it is possible to propose the design of devices whose readings violate Bell inequality when the devices, in addition to results $+1$ and -1, can produce a third result ('detection error'). The table shows a variant of the 'pre-programmed firmware' of devices to ensure that the $\langle S \rangle$ value reaches the maximum value of 4.

To close the detection loophole, one can randomly select the value $+1$ or -1 in case the device itself produces an error (this approach, in particular, is used in quantum cryptography protocols), but this can significantly reduce the final value of quantity $\langle S \rangle$. Another way is to use delayed measurements, implemented in experimental work on NV centers in diamonds [35].

A much more nontrivial problem is the assumption of random independent choice of the type of measurements. Existing random number generators are usually pseudorandom, i.e., use special deterministic mathematical functions to obtain long sequences of bits from finite, so-called random seed sequences. A stochastic physical process can be used to generate the 'initial entropy,' but at the microscopic level this process can potentially be described by deterministic equations, which introduces difficulty in rigorously justifying the independence of measurement choices. An elegant approach to this problem was considered in the work of A Zeilinger's group [34], where signals obtained from astronomical observations were used as sources of randomness for choosing the types of measurements. From an analysis of the light cones of the stellar sources of randomness used, there follows the conclusion: to explain the violation of Bell inequalities in a classical way, it is necessary to assume that the stars 'conspired' about the necessary correlation of their behavior ~ 600 years before the implementation of the experiment by scientists on Earth.

Another elegant demonstration of the impossibility of describing the results of quantum mechanics by the theory of local hidden variables can be constructed on the basis of the three-qubit entangled Greenberger–Horne–Zeilinger state (GHZ) [36], which is of the form

$$|\mathrm{GHZ}\rangle_{\mathrm{ABC}} = \frac{1}{\sqrt{2}}\left(|0\rangle_{\mathrm{A}}|0\rangle_{\mathrm{B}}|0\rangle_{\mathrm{C}} + |1\rangle_{\mathrm{A}}|1\rangle_{\mathrm{B}}|1\rangle_{\mathrm{C}}\right).$$

From this point on, the basic states of qubits correspond to the vectors

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

In a particular case, these states can be realized as photon polarization states $|0\rangle \equiv |H\rangle$, $|1\rangle \equiv |V\rangle$.

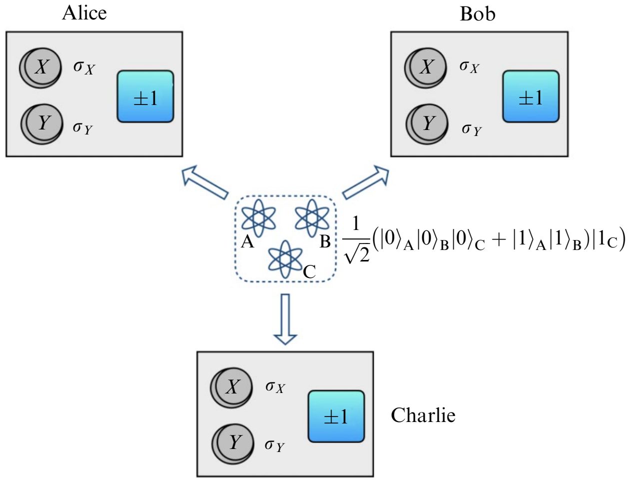

Figure 2. Diagram demonstrating the impossibility of describing the results of quantum mechanics by the theory of local hidden variables using a three-qubit GHZ state.

Let particles A, B, and C be respectively transmitted from the source to three participants — Alice, Bob, and Charlie (Fig. 2). At each round, each participant can perform one of two measurements described by the operators

$$\hat{\sigma}_X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad \hat{\sigma}_Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},$$

after which the procedure is repeated, and the participants receive a fresh GHZ state. We will denote the measurement results $\hat{\sigma}_X$ and $\hat{\sigma}_Y$ at the ith round as X_i^P and Y_i^P, respectively, where $P = \mathrm{A}, \mathrm{B}, \mathrm{C}$ denotes the particle being measured. It is easy to see that, in accordance with the axiomatics of quantum mechanics, the relation

$$X_i^{\mathrm{A}} Y_i^{\mathrm{B}} Y_i^{\mathrm{C}} = Y_i^{\mathrm{A}} X_i^{\mathrm{B}} Y_i^{\mathrm{C}} = Y_i^{\mathrm{A}} Y_i^{\mathrm{B}} X_i^{\mathrm{C}} = -1 \tag{3}$$

must be fulfilled, because $|\mathrm{GHZ}\rangle$ is an eigenvector (with eigenvalue -1) for the corresponding triple tensor products of the operators $\hat{\sigma}_X$ and $\hat{\sigma}_Y$. From equalities (3), we can conclude that

$$(X_i^{\mathrm{A}} Y_i^{\mathrm{B}} Y_i^{\mathrm{C}})(Y_i^{\mathrm{A}} X_i^{\mathrm{B}} Y_i^{\mathrm{C}})(Y_i^{\mathrm{A}} Y_i^{\mathrm{B}} X_i^{\mathrm{C}}) = X_i^{\mathrm{A}} X_i^{\mathrm{B}} X_i^{\mathrm{C}} = -1.$$

However, the $|\mathrm{GHZ}\rangle$ vector is an eigenvector of the operator $\hat{\sigma}_X \otimes \hat{\sigma}_X \otimes \hat{\sigma}_X$ with eigenvalue $+1$, which corresponds to

$$X_i^{\mathrm{A}} X_i^{\mathrm{B}} X_i^{\mathrm{C}} = 1.$$

Therefore, the assumption that for each particle $P = \mathrm{A}, \mathrm{B}, \mathrm{C}$ at each round i both values of X_i^P and Y_i^P are determined, although simultaneously unknown, is incompatible with local hidden variable models. Experimental confirmation of the validity of the predictions of quantum mechanics for the GHZ state of polarization qubits was demonstrated in Ref. [36].

3. Development of Bell inequalities: quantum entanglement as a resource and quantum teleportation

The next step in the development of the study of quantum-mechanical systems and violations of Bell inequalities was the question: how could this be used? A modern view of this issue

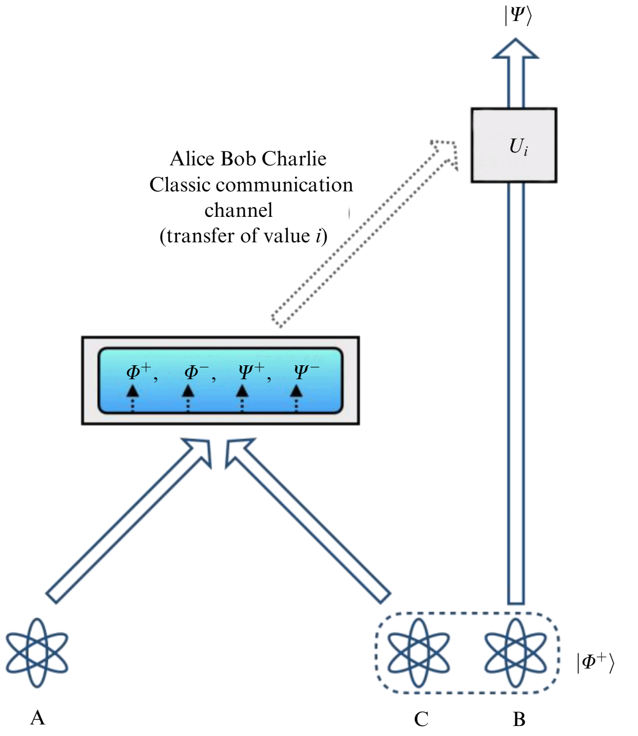

Figure 3. Quantum teleportation of a single-qubit state.

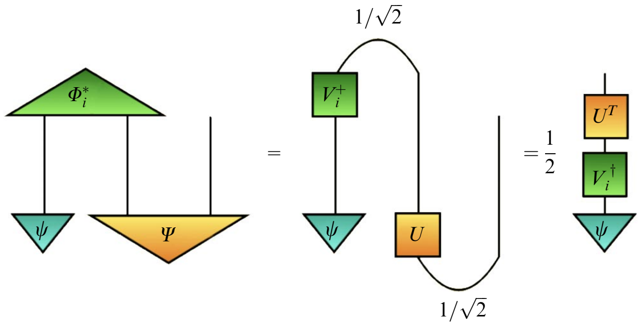

Figure 4. Tensor diagram of Bob's state before he performs a unitary transformation in the single-qubit quantum teleportation protocol using a maximally entangled state $|\Psi\rangle$ in the case of the result $|\Phi_i\rangle$ on Alice's side. The complex conjugation of bra vectors is indicated explicitly.

is presented in Section 4. One of the central ideas was the teleportation of quantum states [37] — the transfer of a quantum state over a distance (this does not imply physical movement from one place to another). Recall that the theorem on the prohibition of copying ('cloning') prohibits the possibility of producing an ideal copy of a previously unknown quantum state [38, 39]: the presence of such an operation would contradict the linearity of quantum mechanics. However, if the original state is destroyed, an ideal copy of it can be recreated elsewhere. This concept formed the basis of the quantum teleportation protocol, which was first demonstrated in 1997 by the groups of Anton Zeilinger [40] and Sandu Popescu [41].

Recall that the basis of the quantum teleportation protocol for some single-qubit state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, where α and β are arbitrary complex numbers that satisfy the normalization condition $|\alpha|^2 + |\beta|^2 = 1$, and $|0\rangle$, $|1\rangle$ denote the basic states of a two-level system, is the use of the maximum entangled state, for example, one of the four Bell states

$$|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle|0\rangle \pm |1\rangle|1\rangle\big)\,, \quad |\Psi^\pm\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle|1\rangle \pm |1\rangle|0\rangle\big)$$

(see Fig. 3).

Let the sender Alice possess particle A in state $|\psi\rangle_{\rm A}$ as well as particle C from the entangled pair $|\Phi^+\rangle_{\rm CB}$, and the recipient Bob possesses the second particle from this pair — B. If Alice performs a joint measurement of particles A and C in the basis of Bell states, with equal probabilities of $1/4$, she will have one of four outcomes: $|\Phi^+\rangle_{\rm AC}$, $|\Phi^-\rangle_{\rm AC}$, $|\Psi^+\rangle_{\rm AC}$, $|\Psi^-\rangle_{\rm AC}$. In this case, Bob's B particle will appear, respectively, in one of four states $|\psi_1\rangle_{\rm B} = \alpha|0\rangle_{\rm B} + \beta|1\rangle_{\rm B}$, $|\psi_2\rangle_{\rm B} = \alpha|0\rangle_{\rm B} - \beta|1\rangle_{\rm B}$, $|\psi_3\rangle_{\rm B} = \alpha|1\rangle_{\rm B} + \beta|0\rangle_{\rm B}$, $|\psi_4\rangle_{\rm B} = \alpha|1\rangle_{\rm B} - \beta|0\rangle_{\rm B}$. Note that an equally weighted ensemble of these four states corresponds to a maximally mixed state, which is also obtained by taking a partial trace of the state $|\Phi^+\rangle_{\rm CB}\langle\Phi^+|$ over particle C. Thus, in the absence of information about the result of Alice's measurement, the reality of the measurement itself does not in any way affect the possible predictions of the results of measurement by Bob of his particle B (from this point of view, the collapse of the state is 'insensitive' for Bob). However, if Alice transmits the result of her measurement (one of four possible outcomes), encoded by two bits of classical information, to Bob, then he will know what specific pure state $|\psi_i\rangle$ he has and will be able to recreate the state $|\psi\rangle$ from $|\psi_i\rangle$ by applying to B the appropriate local operator U_i, where $U_1 = I$, $U_2 = \sigma_Z$, $U_3 = \sigma_X$, $U_4 = \sigma_Y$ (use is made of standard notation for Pauli operators). As a result, state $|\psi\rangle$ will pass from particle A to particle B without interaction, even indirect, of particles A and B with each other (note that the preparation of particle A in state $|\psi\rangle_{\rm A}$ can be carried out after the separation of particles from the pair $|\Phi^+\rangle_{\rm CB}$).

In essence, to transfer the state $|\psi\rangle$ from particle A to particle C, two channels are used: the classical channel, through which the result of Alice's Bell measurement is transmitted, and the 'quantum channel' implemented by the state $|\Phi^+\rangle_{\rm BC}$. Consider the transfer of a single-qubit quantum state $|\psi\rangle$ via a maximally entangled state in more detail. To do this, consider a more general situation when Alice and Bob initially own some maximally entangled state $|\Psi\rangle_{\rm BC}$ and Alice makes a measurement on her side in a basis of four maximally entangled states $|\Phi_i\rangle_{\rm AB}$, $i = 1, 2, 3, 4$. According to the axiomatics of quantum mechanics, Bob's state after Alice obtains the result $|\Phi_i\rangle_{\rm AC}$, up to normalization, assumes the form $|\psi_i\rangle_{\rm C} = |\Psi\rangle_{\rm BC}\langle\Phi_i|_{\rm AB}|\psi\rangle_{\rm A}$. Next, we take advantage of the fact that any maximally entangled state can be obtained from any other maximally entangled state by applying a local unitary transformation to one of the subsystems. In particular, $|\Phi_i\rangle_{\rm AB} = V_{i,{\rm A}}|\Phi_i\rangle_{\rm AB}$ and $|\Psi\rangle_{\rm BC} = U_{\rm B}|\Phi^+\rangle_{\rm BC}$ for some unitary operators V_i and U acting on particles A and B, respectively. Then, the state of Bob's particle can be written in the form $|\psi_i\rangle = U^T V_i^+|\psi\rangle/2$, whose corresponding tensor diagram is plotted in Fig. 4. Note that in the construction of the diagram we took advantage of the fact that the $|\Phi^+\rangle$-state tensor coincides, up to a constant, with the unit transformation tensor, and also that the square of the factor $1/2$ corresponds to the probability of the realization of this outcome.

According to the constructed tensor diagram, which is also analyzed in detail in Refs [42, 43], quantum teleportation can effectively be considered as the propagation of the state $|\psi\rangle$ along the space-time trajectories of particles A, B, and C: at the instant of Alice's measurement, the state $|\psi\rangle$ passes from particle A to particle B, while transforming into $V_i^+|\psi\rangle$, and begins to propagate back in time on particle B to the moment of initialization of the state $|\Psi\rangle_{\mathrm{BC}}$. Next, the state $V_i^+|\psi\rangle$ is transformed into $U^T V_i^+|\psi\rangle$ when moving from particle B to particle C and begins to move forward in time until the moment Bob applies the inverse transformation $U_i = (U^T V_i^+)^+$ and particle C ends up in state $|\psi\rangle$. This interpretation resolves the issue of the 'nonlocal' propagation of the $|\psi\rangle$ state from Alice to Bob, but raises new questions related to the reverse-time transfer of information and possible violations of the principle of causality. It turns out that the actuality of 'absolute randomness' of transformation type i, due to the absolute randomness of the result of Bell's measurement, removes the possibility of paradoxes associated with an apparent violation of causality. Note again that the $1/4\sum_{i=1}^{4} U_i^+|\psi\rangle\langle\psi|U_i$ state, which corresponds to applying one of the four transformations U_i^+ to $|\psi\rangle$ with equal probabilities, coincides with the maximally mixed state obtained by taking a partial trace of the maximally entangled state.

A theoretical and experimental study of the time-reversal formalism as applied to maximally entangled states is presented in Ref. [44]. The use of this formalism is in good agreement with the interpretation of experiments on delayed entanglement swapping [45, 46] (research in Ref. [45] was carried out by Zeilinger's group). The post-selective closed time-like trajectories that arise in the considered context have been studied theoretically and experimentally in Refs [47, 48]. Discussed in Ref. [47], in particular, is the issue of the relation between this behavior of quantum states and the general theory of relativity. Issues related to potential causality paradoxes in experiments with entangled states are discussed in detail in Ref. [49]. The possibility of experimentally observing the backward-time propagation of the $|\psi\rangle$ state on particle B was recently demonstrated using a superconducting 7-qubit processor available in the cloud in Ref. [50]. Experimental observation of quantum effects in spacetime associated with post-selection with available quantum processors is presented in recent paper [51]. We also note that the Bell state can serve as a channel for transmitting two single-qubit states in opposite directions as part of bidirectional teleportation [52]. Finally, we emphasize that experiments on quantum teleportation have been demonstrated, among other things, for two macroscopic atomic ensembles [53, 54].

4. Entanglement in quantum computing, quantum communications, and quantum metrology

Quantum entanglement at the level of two or more particles is now seen as a key 'resource' for quantum technologies. In the context of quantum computing, it is the many-body nature of entanglement between qubits (the basic elements of quantum computers, described by states of the form $\alpha|0\rangle + \beta|1\rangle$ $(|\alpha|^2 + |\beta|^2 = 1)$) that leads to the fact that describing a register of N qubits may require about 2^N complex numbers (taking into account the normalization condition and insensitivity to the phase, the N-qubit state is generally specified by $2(2^N - 1)$ real numbers). It is hardly possible to describe such systems for more than 50 qubits using classical computing technologies. The complexity of quantum mechanical descriptions of multiqubit entangled states is the key argument for why quantum systems can accelerate the solution to computational problems [55]. At the current stage of development, it is not yet clear which of the physical systems will make it possible to create a sufficiently large multi-qubit state with the ability to highly accurately perform quantum logical operations on it (so-called quantum gates). Possible candidates include superconducting chains [56, 57], semiconductor quantum dots [58–60], optical systems [61, 62], neutral atoms [63–66], and ions in traps [67–70] (these experimental systems are also being actively developed by domestic scientific groups, for example, in the field of trapped ions [71–73] (Fig. 5), superconductors [74, 75], atoms [76, 77] and photons [78, 79]).

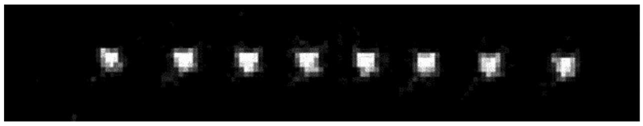

Figure 5. Photograph of a crystal of eight $^{171}\mathrm{Yb}^+$ ytterbium ions captured in a linear trap. Each ion plays the role of a ququart — a qudit of dimension $d = 4$.

It is pertinent to note that the representation of such systems as qubits (two-level systems) is a certain idealization. In fact, considering, say, an ytterbium ion [72, 73], we do well to bear in mind that such a system allows the control of a much larger number of levels. This opens up the possibility of making qudit quantum processors (i.e., processors operating multilevel quantum systems with the dimension of Hilbert space $d > 2$) [80–83]. In 2021, a four-qubit quantum processor based on two ion qudits was demonstrated [74]. Qudit manipulation opens up new possibilities for studying more complex entanglement structures and analyzing fundamental questions in quantum information theory [80–84].

In the case of quantum key distribution, on the face of it, it is possible to do without quantum entanglement. The first quantum key distribution protocol, proposed in 1984 by Charles Bennett and Gilles Brassard [84], does not use the phenomenon of quantum entanglement. It would be sufficient to encode information into single photons, and the measurement procedure proposed by the protocol makes it impossible for a potential attacker to carry out measurements without the legitimate parties of communications finding out about it. However, back in 1991, Arthur Eckert independently came up with a quantum key distribution protocol based on entangled photon pairs and Bell inequalities [85]. This protocol has been demonstrated experimentally many times; however, in the development of industrial devices for quantum key distribution, preference is given to the Bennett and Brassard protocol.

However, an important limitation for modern quantum key distribution systems is the problem of distance. Due to losses in state transfer (for example, in fiber optic communication channels), the rate of generation of a cryptographic key by two parties lowers with increasing distance. For example, industrial quantum cryptography systems that use fiber optic cables as a photon propagation medium are designed for 120–200 km [86], while the record transmission length is 830 km [87] (in this case, superconducting single photon detectors were used to detect photons [88]). It is worth noting that classical data transmission makes use of amplifiers, but in the quantum case this is impossible.

There are two fundamental approaches to increasing the distance: making trusted nodes and making intermediate untrusted nodes based on quantum memory elements that record the quantum state of the light field (see review Ref. [18]). The first method involves the use of 'classic' trusted nodes — intermediate points connecting two sites. Let's assume that Party A and Party C want to distribute a key using a trusted node (Party B). In this case, Party A and Party B distribute their key k_{AB}, and then key k_{BC} is distributed. Party B, therefore, knows both keys: k_{AB} and k_{BC}, then Party B transfers to Party C the key k_{AB}, encrypting it with the key k_{BC} using the one-time pad cipher [19]. In this case, the requirement of trust in the specified node is fundamental, since at this point the quantum-distributed keys of two sections are known. Such a system is scalable, so modern extensive quantum key distribution networks operate on its basis, for example, in Russia, China and the European Union. In the context of the Nobel Prize, it is worth noting that Zeilinger took part in the implementation of experiments on quantum key distribution for secure videoconferencing between Beijing and Vienna [89] using the Micius satellite (it can be considered a trusted node), which made it possible to transfer quantum states over a distance of more than 7000 km [89–92].

Is it possible to make intermediate nodes untrusted? Yes: the second approach suggests that this is possible, but it is technically much more difficult. The basis of untrusted nodes is Bell state measurements [93]. One of the protocols might look like this. Party A and Party B send single photons to Party C, which is located between them. Party C carries out Bell measurements of states and announces the results. In this case, there are no requirements for trust in Party C. It is worth noting that, for the effective implementation of such a protocol, subject to the presence of losses in state transfer channels, Party C must, however, use quantum memory in which one of the photons can be stored until the arrival of the second photon. The fundamental possibility of implementing such protocols with sufficient efficiency was shown in 2020 [94] (an entanglement swap mechanism is also a possible method [93]).

The possibility of using various objects as quantum memory cells is being considered (see [95, 96]), in particular, single atoms and ions [97–98], color centers [94, 100], superconducting chains [101, 102], as well as collective spin systems, for example, doped crystals at low temperatures (see Ref. [103]). The successful implementation of quantum memory cells and their integration with fiber lines of quantum communications will make it possible to create quantum key transmission systems with a length of about 1000 km, protected from hacking at the level of the laws of physics [18, 19]. To solve this problem, it is necessary to fulfill a number of requirements: ensuring a strong coupling between the photonic mode and the quantum memory element and moving to the telecommunications range (1.5 μm) where losses in the quartz optical fiber are minimal.

An integral element of quantum key distribution devices is quantum random number generators — devices capable of generating sequences whose randomness is guaranteed by fundamental physical laws. How could this be strictly demonstrated (or, as is referred to in the literature, certified)? To do this, one can use the violation of Bell inequalities: if Bell inequalities are violated, the source of randomness is of a quantum nature [104–107]. In practice, however, weaker randomness criteria are used for quantum generators [108].

Quantum entanglement can also improve the limits of measurement of physical quantities (see, for example, Refs [109–111]), which is important in the context of a number of applications, for example, global positioning systems.

5. Development of the field: experiments of C Monroe and H Weinfurter: on the way to the quantum Internet

Experiments demonstrating violations of Bell inequalities and quantum teleportation employed primarily photons as the object of research. At the same time, as we know, light, both classical and quantum, can be used for communications — the transfer of quantum states, in other words, as a resource for distributing entanglement between other objects. Over the past decades, experiments have been carried out on the distribution of quantum entanglement over a distance between, for example, atoms or ions. Being especially interesting in the context of the difficulty of scaling quantum processors without losing the quality of their control, this idea underlies the concept of the quantum Internet [112]. Then, instead of scaling the number of quantum objects in one quantum processor, it is possible to move to a quantum analogue of a 'many-core' architecture, in which different cores are connected using quantum communications [113]. Striking experimental results were obtained by Christopher Monroe's group working on quantum entanglement transfer between spatially separated ions [114], which can be considered the first step to link ion traps and create controlled entanglement transfer between them. Similar work is being carried out by groups from the United Kingdom [115] and Austria [116], where entanglement of two distant ions (at a distance up to 230 m) was demonstrated using a photonic interface.

Probably the most striking experimental work aimed at testing Bell inequalities and analyzing quantum correlations using entangled atomic systems has been carried out in recent years by the group of Harald Weinfurter (Germany). Starting with the entanglement of two rubidium atoms separated by a distance of 20 meters [117], in 2017 they experimentally proved the elimination of locality and detection loopholes on atoms separated by a distance of 398 m [118]. In 2022, his group demonstrated entanglement between two atoms separated by a 33-kilometer-long optical fiber, the next step toward the quantum Internet and distributed quantum computing [119]. These studies represent the development of the idea of entanglement transfer for practically important problems of quantum technologies.

6. Conclusions

In 2022, the Nobel Committee especially drew attention to the fact that the prize was awarded precisely for the experimental achievements of the laureates: work on Bell inequalities by Aspect and Clauser, as well as research on quantum data transfer and quantum teleportation by Zeilinger. As this was taking place, the experimental work was accompanied by conceptualization and interpretation of the philosophical foundations of quantum physics, without which the results of the experiments would not be so clear to the general scientific community. Here, it is especially worth noting the work of Aspect (see, for example, Ref. [29]), who, inheriting the tradition of disputes between Einstein and Bohr, supplemented scientific articles with relevant reflections and explanations in an attempt to answer the important question: What do these results mean? The achievements of the laureates demonstrate that the humorous approach to the perception of quantum mechanics, 'Shut up and calculate,'[2] although it can lead to practical results, does not always lead to a Nobel Prize.

The studies by Aspect and Clauser also show that there are no local hidden variables in quantum mechanics. Could nonlocal hidden variables be present and play a role? Despite significant progress in this area (see, for example, the study of nonlocal hidden parameters in the context of quantum computing [120]), a complete answer will only be obtained in future theoretical and experimental work.

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[2] Among physicists, a 'none' interpretation of quantum mechanics is widespread, succinctly expressed in David Mermin's aphorism: 'Shut up and calculate!.'

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