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Vertical refraction of light in the atmospheric surface layer: traditional problems of determining refraction and new technical achievements

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Abstract. **Almost all the methods of determining refraction are based on the theory created by Newton. Studies performed by Newton, Euler, Oriani, Bernoulli, et al. involved the determination of the geometrical path of a light ray. However, the refraction of light propagating in the atmospheric surface layer cannot be calculated reliably because of numerous conditions determining the propagation of light in the layer. Modern methods for determining the refraction of light providing optical measurements with an instrumental accuracy of the devices used are analyzed.**

Keywords: temperature gradient, air refractive index gradient, air refractive index pulsations, angle of incidence of a laser beam, unstable temperature stratification of the atmosphere, statistical characteristics of a wave, turbulence, wind velocity, light curve radius, refractive index, atmospheric surface layer

1. Introduction

Already in ancient times a light propagation path was represented by a straight line, a fundamental concept in geometry. The physical denchmark of a straight line is a light beam. However, light propagates rectilinearly only in a homogeneous isotropic medium. Most optical observations are performed in the atmosphere, which is as inhomogeneous as any real medium. A light beam passing from one inhomogeneity to another changes its direction in the general case. The deviation of a light beam from a rectilinear propagation direction is called *refraction* (from *refractio* (late Latin). Refraction was already known in ancient times. The first written mentions about the refraction of light appeared in the 1st century AD (Cleomedes, *Cyclic Theory of Meteors*). Somewhat later (2nd century AD), Ptolemy pointed out that refraction should be absent for light beams coming from an object located at the zenith and should increase as the object approaches the horizon line (i.e., with increasing zenith distance).

In the 11th century, the Arabian scientist Ibn al-Haythan noted that the duration of the day part of a twenty-four-hour day increased due to the refraction of light. The Danish astronomer Tycho Brahe composed the first tables of refraction values using the results of astronomical observations.

The refraction theory was first developed by the famous scientist Johannes Kepler. In Kepler's time, the air was assumed weightless. Using the law of light refraction at the boundary of two media, Kepler calculated the refraction angle only for a light beam refracted on the upper boundary of the air layer. He assumed that light propagates in the atmosphere itself rectilinearly. As a result, Kepler obtained the value of the maximum refraction angle for an object located near the horizon 4′ smaller than would be expected based on experimental data known at that time.

The most important contribution to the refraction theory was made by the great British scientist Isaac Newton [1].

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Almost all classical methods presently used to determine refraction are based on Newton's theory. According to this theory, to estimate the refraction value, which depends on the gradient of the air refractive index, one should know the gradients of metrological quantities (air temperature, pressure, and humidity). The gradients are determined by direct or indirect measurements in the atmosphere. In 1935, the Soviet scientist A N Krylov reproduced conclusions and proofs of the great scientist and presented them in the book *Newton's Theory of Astronomical Refraction* [1]. He wrote that, by comparing Newton's refraction theory with modern theories, "one will see at once how simple and natural Newton's consideration is and how little was added in essence to it for 240 years." This statement is valid at present as well.

Theoretical studies by Newton, Euler, Oriani, Bernoulli, et al. were devoted to the determination of the geometrical path of a light beam. In reality, the atmosphere always experiences turbulent motion and its optical properties are always changing in time. However, these circumstances are neglected in the determination of refraction by classical methods. This is the main reason, in the opinion of many specialists, that "not accidentally, many years of studies did not give the final solution to formulated problems, although vertical refraction was investigated for more than 300 years" [2]. Classical methods for determining refraction assume implicitly that the atmosphere is in a statistically stable state during measurements. Because real conditions differ from the conditions assumed, the refraction errors are 10–15 times greater than the instrumental errors of measuring devices [2].

Vertical refraction introduces significant distortions into angular optical measurements in the surface air. Because most geodesic and astronomical measurements are performed with ground-based optical instruments, the development of methods for efficient measurements of refraction and accounting for it is quite urgent. The study of refraction requires the solution to many problems of propagation of electromagnetic waves in the atmosphere in the fields of communication, radiolocation, geodesy, navigation, and astronomy and of inverse problems concerning the determination of the height profiles of the refractive index, temperature, pressure, and humidity of air from refraction measurements.

The refraction theory is considered one of the most difficult problems. It is being studied at present by many researchers in Russia and abroad. Thus, at the L'vov Polytechnic Institute, several methods for determining refraction were proposed and more than 1000 papers were published in this field [3]. Nevertheless, progress in studying refraction is negligible. Only in recent years, based on the development of methods for studying the optics of the turbulent atmosphere, have new radiophysical (turbulent) methods been developed for accurate measurement of refraction [4–24]. As pointed out in papers by researchers at the Zuev Institute of Atmospheric Optics (IAO), Siberian Division, RAS, random refraction, unlike regular refraction, is not described by geometrical optics, but in the general case by the theory of propagation of waves in turbulent media taking diffraction into account. Methods based on this concept are called radiophysical. Somewhat later, A L Ostrovskii proposed calling them turbulent, because refraction is calculated in these methods taking into account the oscillations of a collimating ray caused by turbulence [2, 3].

These methods were developed based on theoretical studies of the propagation of waves in turbulent media performed at the Zuev IAO, SD, RAS and the Obukhov Institute of Atmospheric Physics (IAP), RAS [25–32]. The efficiency of the theory was confirmed by numerous experiments performed with the help of robotized tachymeters in different regions at different times of the year.

The aim of this paper is to analyze and choose methods and means for efficient optical measurements of refraction under real conditions in the turbulent atmosphere with the instrumental accuracy of geodesic precision instruments.

2. Vertical refraction and determining it using classical methods

There are several types of refraction. The international terminology proposed by G Teleki [33] and accepted at present includes:

- astronomical refraction: an object is outside Earth's atmosphere, in most cases at large distances, while an observer is on the surface of Earth (Fig. 1);
- atmospheric refraction: an object is in Earth's atmosphere, while an observer is on the surface of Earth;
- Earth refraction: both object and observer are on the surface of Earth (Fig. 2);
- photogrammetric refraction: an object is on the surface of Earth, while an observer is in Earth's atmosphere;
- cosmic refraction: an object is on the surface of Earth, while an observer is outside Earth's atmosphere.

Earth refraction is usually called geodesic refraction (see Fig. 2).

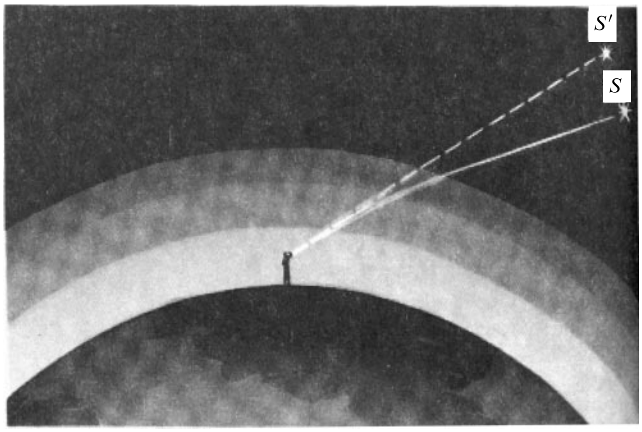

Figure 1. Bending of a light ray due to refraction.

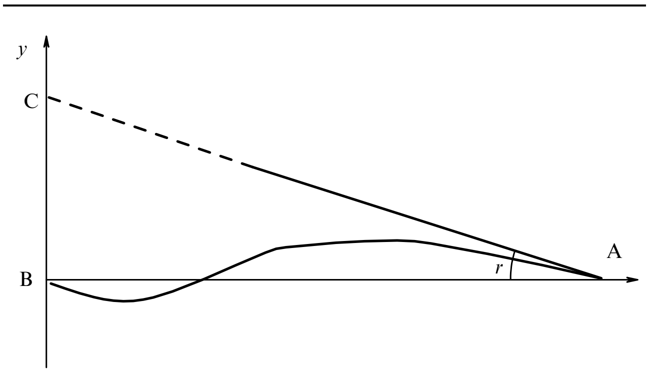

Figure 2. Refraction angle r between tangent AC to the line AB (ray path) at the receiving point A and a chord connecting points A and B.

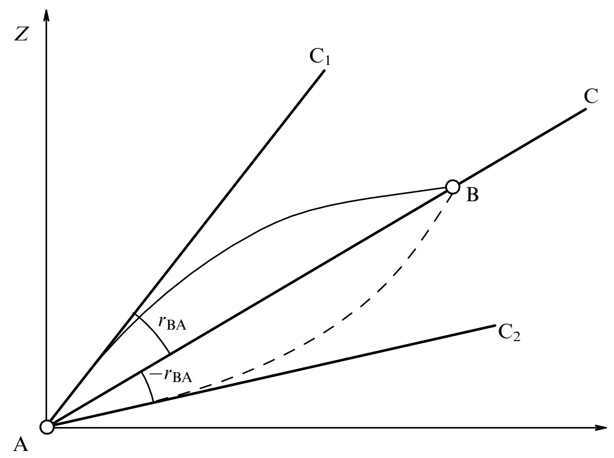

Figure 3. Influence of positive, $r_{\rm BA}$, and negative, $-r_{\rm BA}$, vertical refraction on measured zenith distances.

The rest of the refraction types concern a more detailed classification of astronomical refraction, depending on accepted models of the atmosphere based on calculations used or on atmospheric conditions of observations [34].

The refraction angle in ground-based observations is the angle r between directions to the apparent and real positions of an object observed (Fig. 3), i.e., the angle r between tangent AC_1 (AC_2) to line AB at the reception point and a chord connecting the light detector A to the light source B.

When the refraction properties of a medium change gradually without abrupt jumps, a light ray is curved smoothly, and its path represents a spatial curve with a double curvature.

Projections of a spatial curve (light path) on vertical and horizontal planes give the vertical and horizontal components of this curve. Deviations of a light ray in vertical and horizontal planes are called the vertical and side refraction, respectively. In observations on ground-based paths, vertical refraction distorts vertical angles, while side refraction distorts horizontal angles.

In estimating the refraction, it is necessary to take into account that the atmosphere is inhomogeneous, and its density changes with altitude and by a much smaller degree in the horizontal direction.

For this reason, vertical refraction is usually 3–40 times larger than side refraction [35]. When the convexity of the light-curve projection on the vertical plane is directed upwards, vertical refraction is positive (see Fig. 3), and if the light-curve convexity is directed downwards, the refraction is negative. The refraction sign and value for the same direction can change in twenty-four hours.

The radius of curvature ρ of a light beam at a point deviated due to vertical refraction is determined from the expression [2]

$$\rho = -\frac{n}{{\rm d}n/{\rm d}h \sin i}\,, \qquad (1)$$

where n is the refractive index of air for wavelength λ, ${\rm d}n/{\rm d}h$ is the refractive index gradient (called the refraction vector) at height h of the propagating light beam, characterizing the rate of change of the refractive index with height, and i is the angle of incidence of rays ($i \approx 90°$ for horizontal paths).

The vertical gradient ${\rm d}n/{\rm d}h$ of the refractive index of air is usually calculated in practice from the expression [2]

$$\frac{{\rm d}n}{{\rm d}h} = -\frac{78.85P}{T^2}\,10^{-5}\left(0.0342 + \frac{{\rm d}T}{{\rm d}h}\right), \qquad (2)$$

in which temperature T is presented in degrees Kelvin, the temperature gradient ${\rm d}T/{\rm d}h$, in units [K m^{-1}], and pressure P, in mbar. The minus sign in (2) means that the refractive index decreases with increasing height.

Vertical refraction is often characterized by the point refraction k coefficient equal to the ratio of the radius of Earth $R_{\rm E}$ to the radius of curvature ρ of the refraction curve at a particular point,

$$k = \frac{R_{\rm E}}{\rho}\,, \qquad (3)$$

and the angle of vertical refraction r is calculated from the approximate expression

$$r = \frac{kL}{2R_{\rm E}}\,, \qquad (4)$$

where L is the distance between a source and detector along the Earth sphere arc.

Errors caused by refraction in the surface atmospheric layer greatly increase with decreasing collimating ray height. For example, in 1977, E Hübner observed on sunny days the refraction coefficient in the range from -8 to 16 [36] at a height of the collimating ray of about 50 cm, which corresponds to the deviation of the ray in the vertical plane from $-129.6''$ to $259.2''$ per kilometer. Similar results were obtained in [37].

The temperature gradient is found from the relation

$$\frac{{\rm d}T}{{\rm d}h} \approx \frac{\Delta T}{\Delta h}\,, \qquad (5)$$

where ΔT and Δh are the differences in temperature and height between locations of temperature sensors.

Different expressions for calculating refraction based on different models of the atmosphere for determining the vertical gradients of the air temperature and refractive index were proposed in [38–45]. All these models introduce errors into measurements, because any model is only an approximation of the real atmosphere, whose optical properties rapidly change. At the same time, to measure refraction accurately, information is required on meteorological elements at many points of a beam path. Such measurements are expensive and impossible, in fact, and, as a result, field measurements are usually performed only at one to two points of the path. For this reason, the search for means and methods for efficient measurements and accounting for refraction has continued for many years. However, "the problem of accounting for refraction in geodesic measurements has not been solved in practice so far" [46].

2.1 Metrological method for determining vertical refraction

The metrological method is one of the known methods for determining refraction, and we consider it in more detail. This method is used, as a rule, in scientific studies. The method for determining refraction by measuring metrological elements was first proposed in 1843 by V Ya Struve [47], who introduced the empirical dependence of the refraction coefficient k on air temperature T, pressure P, and height h

of the collimating ray over Earth's surface. However, the dependence on gradients of metrological quantities was neglected, as was pointed out by N Ya Tsinger in 1873 [48]. Much later, it was proposed in [44] to calculate the temperature gradient by measuring the air temperature difference at a height of 2 m and at the 'roughness height' of the underlying surface and to use these data to calculate the average vertical temperature gradient and refraction for the entire path. It was then proposed in [44, 45] to determine the path-averaged temperature gradient $\bar{\gamma}$ based on gradient measurements of γ_1 at a height of 1 m using the expression

$$\bar{\gamma} = \frac{a\gamma_1^{0.2}}{h_{\mathrm{e}}}\,, \tag{6}$$

where a is a coefficient found in experiments and h_{e} is the equivalent height of the path.

In [42], a more complex approach to solving this problem was used. To improve the quality of estimates, the author proposed to take into account as much as possible factors affecting the propagation path of electromagnetic waves in the atmosphere.

Note that in [49] a strict formula was proposed for the calculation of the point coefficient k of vertical refraction using the temperature gradient $\mathrm{d}T/\mathrm{d}h$ and air pressure P and temperature T:

$$k = 502.4\,\frac{P}{T^2}\left(0.0342 + \frac{\mathrm{d}T}{\mathrm{d}h}\right). \tag{7}$$

This expression gives quite accurately the refraction coefficient for a homogenous atmosphere.

In reality, the atmosphere is inhomogeneous, and therefore the calculation of refraction for the whole path from metrological elements measured at one point can cause significant distortions.

Of great interest are studies performed at the IAO, SD, RAS [10–14], in which the programmed correction of random refraction using meteorological data was applied. The temperature stratification of the atmosphere was taken into account. Various conditions of the atmospheric instability affecting the propagation of optical radiation were considered. These studies were performed for the development of methods for controlling powerful earth-moving machinery (in planning extended surfaces for aerodromes in agriculture, melioration, and construction) using laser support systems (LSSs) specifying directions or a plane. These studies present the results of experiments on extended atmospheric paths. In [11], two variants of accounting for refraction for LSS operation are proposed. In the first one, an algorithm is proposed for the programmable variation of the beam axis position by introducing express metrological data. The second variant involves the adaptive correction of distortions by the instantaneous monitoring of special reference sources [14]. In the opinion of the authors of these papers, the first variant is difficult to realize because the data obtained "can be used only to estimate the possible dispersion of a random component" [11], which strongly changes rapidly. In addition, the refraction of laser radiation on short ground-based paths has characteristic features and requires further studies [13].

The difficulty of using the meteorological method of *corrections for refraction* is also confirmed in later paper [50], and by unsuccessful attempts to apply this method with the SKP and SAUL laser systems manufactured in the USSR for the vertical planning of aerodromes, paddy fields, etc. and also at present using the newest commercial LEICA iCON grade 2D devices (Switzerland), etc. [51]. These newest Swiss laser systems are widely used, for example, for the automatic control of asphalt coverings with an accuracy of the order of 1 mm. Thus, the introduction of corrections for refraction in such work is rather urgent. However, so far, to diminish the effect of refraction, it is recommended to perform studies only with short rays, no longer than 50 m. This is explained by the fact that the real atmosphere is inhomogeneous, and therefore the calculation of refraction for the entire path by measuring meteorological elements only at one point of the path can lead to considerable distortions of the results obtained. According to [52], for paths at heights of 2–5 m in the surface atmospheric layer, neither theoretical nor empirical expressions for the correction caused by refraction can be obtained. In this case, the refraction coefficient changes in a broad range from 4.28 to -4.40, having, as a rule, a minus sign.

Attempts to use such methods for refraction correction with the help of meteorological data were made for many decades [2, 3, 35, 38–49, 52–66]. First of all, almost all the researchers point out the difficulty in applying these methods, because the estimate of refraction with an accuracy of $1''$ on a path 1 km in length requires the determination of the vertical temperature gradient with an accuracy of $0.01\,^{\circ}\mathrm{C\,m^{-1}}$ [42]. In real conditions, the accuracy of measuring the temperature gradient of the order of $0.01\,^{\circ}\mathrm{C\,m^{-1}}$ cannot be achieved, in fact, because the temperature field is inhomogeneous, and temperature pulsations observed in the surface air can reach $1\,^{\circ}\mathrm{C}$ at each point of the field [26]. In addition, the estimate of temperature for the entire path with an accuracy of a few hundredths of a degree is rather difficult technically. It is known that the vertical temperature gradient of air can often drastically change, because it does not continuously depend on the height, but changes abruptly at the interfaces of layers [2, 52–54], so that meteorological methods are not efficient enough. Bahnert [64] asserts that metrological methods of determining the refraction coefficient give less accurate values of k than geodesic methods. It was concluded that the introduction of meteorological corrections for refraction is unreasonable [65].

Thus, most researchers point out the inconsistency of the real coefficient of vertical refraction with its value calculated from meteorological measurements, and, therefore, meteorological methods of determining corrections for refraction do not find broad applications in optical observations in the surface atmospheric layer.

2.2 Determination of vertical refraction by measuring reciprocal zenith distances

The method most often used in practice is based on the determination of refraction coefficient k from simultaneous measurements of direct, z_1, and reverse, z_2, zenith distances [2, 67, 68]:

$$k = 1 - \left\{\frac{R_{\mathrm{E}}}{L\rho''}\left[(z_1 + z_2) - 180^{\circ}\right] + \frac{R_{\mathrm{E}}}{L^2}\left[(l_1 + l_2) - (i_1 + i_2)\right]\right\}, \tag{8}$$

where i_1, l_1 and i_2, l_2 are the heights of a device and a sighting target, respectively, located at points 1 and 2 at the path ends, R_{E} is Earth's radius, L is the distance between points, and $\rho'' = 206265''$.

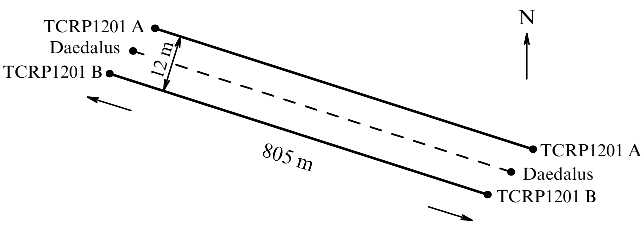

Figure 4. Layout of the installation of robotized electronic tachometers at the ends of three parallel tracks [69].

Figure 5. Robotized Leica TCRP1201 electronic tachometer with a corner reflector mounted on a horizontal plate at instrument height [69].

Studies have shown that the method does not provide a high measurement accuracy, because the values of k change in a broad range, not only with direction but also with time.

An analysis of many results of trigonometric leveling [68] has shown that "the estimate of the difference of 'forward-backward' excesses can be used only for references to find rough errors (mistakes)." However, despite such important practical conclusions, this method is used fairly broadly because of its simplicity. The method began to attract special interest with the advent of robotized electronic tachometers performing automated searches for a target and aiming at it [69].

To verify the accuracy of this method, synchronous measurements of reciprocal zenith distances were performed on three homogeneous parallel paths 805 m and 100 km in length to the south of Hamburg (Germany). The distance between adjacent parallel paths was 6 m (Fig. 4). It was assumed that, due to synchronous measurements with tachometers located at the ends of the paths, the possibility of reducing errors in the refraction measurement caused by rapid random variations in refraction was verified. Studies were performed simultaneously on three paths, both in sunny and cloudy weather for three days. At the ends of two external paths, high-precision robotized Leica TCRP1201 (Fig. 5) electronic tachometers providing an angular measurement accuracy of 1$''$ were located. At the ends of the middle path, Daedalus devices with the same instrumental accuracy were located. Altogether, six devices were used. Near each tachometer, an angular reflector was mounted on a horizontal plate at the instrument height.

Figure 6 shows the daytime dependence of the refraction angle obtained by this method. The experiments showed that the accuracy of refraction measurements with the help of two synchronized electronic tachometers increases somewhat,

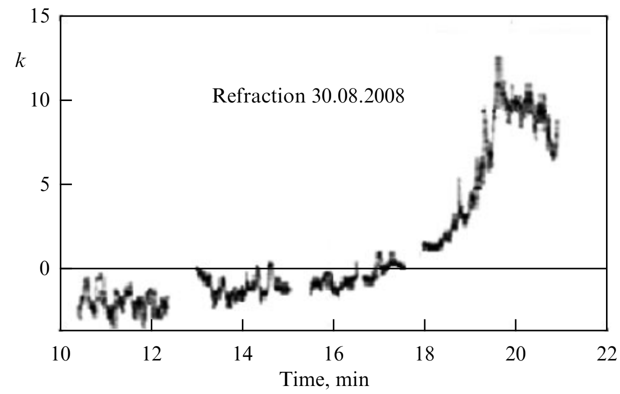

Figure 6. Time dependence of refraction on tracks A and B in clear weather [69].

although it remains considerably lower than the instrumental accuracy of the devices used.

The amplitude of fluctuations of the refraction angle in sunny weather was greater than in cloudy weather. This method does not provide high measurement accuracy, because the direct and reverse refraction angles cannot be assumed identical, since the fluctuation frequency of the refraction angle can be several ten hertz, while the synchronization accuracy of two electronic tachometers is of the order of 2 s. In addition, the heating of the underlying surface by solar rays should occur identically, which is impossible, in fact, at least because shadows from clouds can fall on different parts of the path.

Thus, errors in measurements of refraction by this method, especially at large distances, can be rather large.

2.3 Izvekov's method

Vertical refraction coefficient k can be estimated by a method based on the assumption that, during the measurement of zenith distances, the value of k at the observation point is, in fact, the same for all directions. This method is completely based on the use of only trigonometric leveling results.

The method is proposed for the case when a triangulation network exists with reciprocal zenith distances measured along all the network sides [70]. In the triangle $P_1P_2P_3$ (Fig. 7), reciprocal zenith distances z are measured. The lengths L of the triangle sides are known. Excesses h over the side P_1P_2 are described by the expressions

$$H_1 - H_2 = h_{1,2} + L_{1,2} f_1 \,, \tag{9}$$

$$H_2 - H_1 = h_{2,1} + L_{1,2} f_2 \,, \tag{10}$$

where $f_i = (1 - k_i^2)/(2R_E)$, $h_{1,2}$ and $h_{2,1}$ are excesses calculated from measured zenith distances z, and k_i is the refraction coefficient.

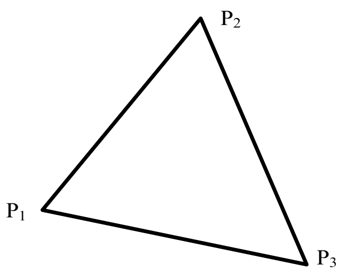

Figure 7. Triangle of a triangulation network.

By solving system of equations (9), (10), we obtain

$$f_1 - f_2 = \frac{h_{1,2} - h_{2,1}}{L_{1,2}} . \tag{11}$$

By composing equations for other sides of the triangle, we find the values of f_1, f_2, and f_3. These data can be used to calculate the three vales of k. Therefore, if a given point is a vertex of the n-th triangle, n vales of f can be obtained. In the case of redundant measurements, the problem should be solved by the method of least squares.

This method of determining refraction does not provide the required measurement accuracy for the following reasons:

— the method is based on the incorrect assumption that the refraction coefficient at a given point is practically the same for all directions during the time of measuring zenith distances. In reality, the atmosphere is inhomogeneous, and its optical properties change continuously, which causes the inequality of refraction coefficients in different directions;

— refraction coefficients obtained from measurements of direct and reverse zenith distances are nonrepresentative, because they concern only a certain observation moment and certain points of the path. Therefore, we cannot assume that the same light propagation conditions will be observed for other paths and at other moments of time, or even for the same path but at another time.

2.4 Refractive basis method. Method of geodesic refraction service

Work is sometimes performed using the refractive basis method proposed in [71]. To use this method, it is necessary to have the marks of two points in a network, for example, 1 and 2 (Fig. 8), determined from geometrical leveling, while the rest of the marks of points in the network are determined accounting for refraction from trigonometric leveling. These two points of the network are used as a refractive basis. Measurements are performed from station 1 over all directions, including point 2, whose mark, as the station mark, is determined from the geometrical leveling. The refraction coefficient for the network is determined on the basis 1, 2 by measuring reciprocal zenith distances and is then used to determine corrections for each measured direction of the network.

The refractive basis method does not provide a high accuracy either, because it has the same disadvantages as Izvekov's method (see Section 2.3). Despite its disadvantages, this method is sometimes applied. For example, it was used in observations on paths a few kilometers in length in Antarctica [67]. Somewhat later, the refractive basis method was developed to the method of geodesic refraction service [2, 66].

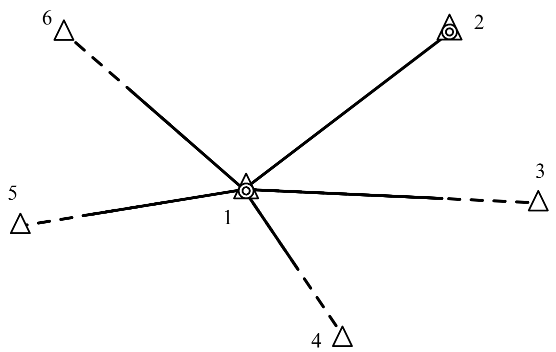

Figure 8. Determining refraction by the refraction basis method [71].

The method of geodesic refraction service is analogous to the refractive basis method, but, in this case, a separate team simultaneously performs the trigonometric leveling of the geodesic network over a large area and a set of observations in a specially chosen region most typical of the entire network. At the central point, zenith distances should be measured simultaneously with measurements of the air temperature and pressure. Based on these data, the equivalent vertical temperature gradients γ_e are calculated over all directions coming from the central point.

The refraction coefficient is determined using the values of γ_{ej} calculated for each direction of the network. This method has not found practical applications because of its low accuracy and complexity.

2.5 Method of accounting for refraction in periods of quiet images

To reduce the influence of refraction on the accuracy of measuring zenith distances, Russian scientists Struve, Tsinger, and Pomerantsev recommended that observations always be performed in isothermal periods (i.e., periods of quiet images). The value of the vertical refraction in this period is close to its 'normal' value. This method of accounting for refraction was studied by many scientists: Yakovlev, Maslich, Ostrovskii, Dzhuman, Bahnert, Brunner, et al. [2, 13, 22, 55, 72–77].

The temperature regime in low atmospheric layers is mainly determined by thermal exchange between the underlying surface and air. Vertical temperature gradients are higher in dry regions than in moist regions and are smaller over plant covers (forest, meadow) than over bare soil.

Periods of indifferent temperature stratification in the surface air, when the vertical temperature gradient is close to the adiabatic one, in the middle zone of Russia are short (a few ten minutes) and begin when the Sun is at a height of about 15° over the horizon. The Sun is at such a height in the summertime approximately 2–3 hours before sunset and 2–3 hours after sunrise. This time is recommended for performing angular measurements. In cloudy weather, the temperature gradient oscillates much more slowly, and the time duration convenient for measuring vertical angles increases.

Above the surface atmospheric layer (more than 100 m above Earth's surface), superadiabatic temperature gradients appear very rarely: as they appear, air is mixed and a state close to indifferent stratification appears. For this reason, the refraction coefficient in high-mountain regions is close to normal. The duration of such a period increases in cloudy and windy weather.

At heights in the air layer from 100 to 700 m, a 'calm' convection region, in which the vertical gradient of the wind velocity is negligibly small and stratification is close to indifferent, exists for a long time. The onset time of this period can be determined from empirical formulas proposed in [74–76].

To increase the accuracy of determining the onset of the isothermal state, it was proposed to use simultaneous two-sided trigonometric leveling by calculating refraction coefficients at the ends of the observation line. According to [77, 78], the isothermal state appears when direct and reverse refraction coefficients are equal. The latest studies have shown that even a high degree of synchronization of observations of reciprocal zenith distances does not rule out the appearance of significant errors due to inhomogeneous conditions of the propagation of light in the path.

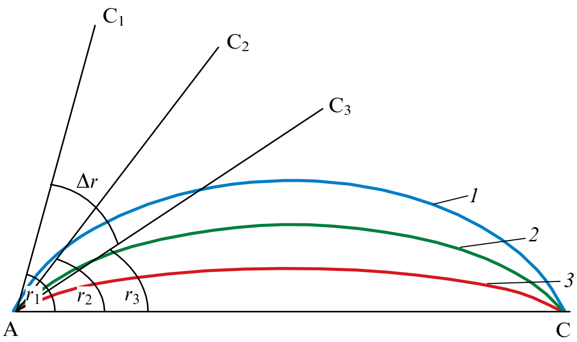

Figure 9. (Color online.) Refraction of light in the atmosphere for different wavelengths: (*1*) blue, (*2*) green, (*3*) red [81].

2.6 Dispersion method for determining refraction

The dispersion method for determining refraction was proposed in 1924 by the German geodesist Nebauer and was later developed by many researchers in [79–86]. The method is based on the dependence of the refractive index of air on the wavelength of light propagating in the atmosphere. The refraction of light at different wavelengths is shown schematically in Fig. 9.

If a light source emits two monochromatic beams — a blue beam at wavelength $\lambda_{\rm b}$ and a red one at wavelength $\lambda_{\rm r}$ — refraction angles for these beams will be different [81]:

$$r_{\rm b} = \frac{\rho''}{L}\,(n_{\rm b} - 1)\int_0^L \left(\frac{{\rm d}P}{{\rm d}h}\,\frac{1}{P} - \frac{{\rm d}T}{{\rm d}h}\,\frac{1}{T}\right) x\,{\rm d}x\,, \qquad (12)$$

$$r_{\rm r} = \frac{\rho''}{L}\,(n_{\rm r} - 1)\int_0^L \left(\frac{{\rm d}P}{{\rm d}h}\,\frac{1}{P} - \frac{{\rm d}T}{{\rm d}h}\,\frac{1}{T}\right) x\,{\rm d}x\,, \qquad (13)$$

where $r_{\rm b}$ and $r_{\rm r}$ are refraction angles for wavelengths $\lambda_{\rm b}$ and $\lambda_{\rm r}$, respectively, L is the path length, $\rho'' = 206265''$, $n_{\rm b}$ and $n_{\rm r}$ are the average refractive indices of air on the path for wavelengths $\lambda_{\rm b}$ and $\lambda_{\rm r}$, respectively, and x is the beam current coordinates.

If the difference between refraction angles (refraction dispersion) $\Delta r = r_{\rm b} - r_{\rm r}$ is measured, the refraction angle r_λ at the wavelength λ can be obtained:

$$r_\lambda = \frac{n_\lambda - 1}{n_{\rm b} - n_{\rm r}}\,\Delta r\,, \qquad (14)$$

where n_λ is the average refractive index of air at the average wavelength λ.

The refraction angle is calculated by using the values of $n_\lambda - 1$ and $n_{\rm b} - n_{\rm r}$ obtained from meteorological measurements at the final points of the path. If Δr is determined using the wavelengths at the ends of the light range for which $n_{\rm b} - n_{\rm r} = 6 \times 10^{-6}$, the error m_r of measuring the refraction angle can be approximately 50 times higher than the error $m_{\Delta r}$ of measuring the dispersion refraction angle:

$$m_r = 46 m_{\Delta r}\,.$$

If the refraction dispersion Δr is not measured accurately enough, the value of the required refraction angle r can be considerably distorted. For this reason, to measure refraction with an accuracy of $0.1''$, the dispersion angle should be measured with an accuracy of $0.002''$, which is a very difficult technological problem in the dispersion method of determining refraction.

In this method, lasers emitting at different wavelengths are commonly used. The angular dispersion for two wavelengths is determined by the diffraction or interference method. In the diffraction method, diffraction images of light sources are produced. The angular dispersion is determined visually by photographic or photoelectric methods from the shift of energy centers of diffraction images of light sources. It is known that the angular size ϕ of a diffraction image (Airy circle) slightly exceeds angular dispersion Δr and depends on diameter D of the objective of a receiving tube and wavelength λ:

$$\phi = 1.22\,\frac{\lambda}{D}\,. \qquad (15)$$

Because of its low accuracy, the visual method of determining the relative position of energy centers has limited applications. As a rule, photoelectric detection with CCDs is used [81]. A birefringent optical crystal is efficiently applied as an angular sensitive element of a diffraction refractometer [79, 82].

Such a refractometer was developed at the Central Research Institute of Geodesy, Aerial Photography and Cartography [79]. The accuracy of measuring the refraction dispersion with this instrument is 0.02–$0.03''$. Note that the dispersion method of measuring the vertical refraction has not so far found broad applications in geodesic practice mainly because of the considerable difficulties in measuring the refraction dispersion angle in the turbulent atmosphere [82]. In addition, the air turbulence strongly distorts the interference pattern already over a distance of several ten meters, making virtually impossible the reading of information in such conditions because of fluctuations in the refractive index of air in the light beam path.

As mentioned above, to determine the refraction angle $0.1''$, the dispersion angle should be measured with an accuracy of about $0.002''$, which is impossible in a turbulent atmosphere [82]. In addition, fluctuations of the arrival angle of a light beam over a path ~ 1 km in length can be several ten arc seconds at a frequency of several ten or even hundreds of hertz [82].

Thus, the refraction value randomly changes under real conditions for a very short time by two orders of magnitude greater than the refraction dispersion. For this reason, dispersion refractometers are not used for field measurements: they are still at the stage of laboratory development.

By now, more than 100 methods have been proposed for determining and accounting for vertical refraction [3]. However, they all have one disadvantage or another, and therefore most of them are not used in practice. Note that, even if one of these methods gave an accurate value of refraction at a certain moment for a given path, the value of refraction after several seconds could considerably differ from the value obtained earlier.

Experimental studies performed on ground-based paths with collimating ray heights up to several ten meters confirm that the character of the daily variation in refraction is not repeatable in detail but is somewhat similar for different times of year and for paths of different lengths. Therefore, the introduction of refraction corrections based on measurements performed on the same path, but in another time, is incorrect, because it can deteriorate the results obtained. Therefore, the determination of refraction corrections for ground-based paths based on the results of long observations of refraction in a given region and even on particular paths makes no sense.

Thus, classical methods for determining refraction cannot be widely used in precision optical measurements on paths 300–400 m in length, because they cannot provide reliable estimates of distortions caused by refraction during measurements on a longer path.

Note that even the use of electronic tachometers with automatic target pointing for simultaneous measurements of reciprocal zenith distances does not completely exclude the influence of refraction for the reasons mentioned above. Therefore, of all the methods accounting for refraction, as mentioned above, the most efficient is the one based on the choice of the observation time in periods of indifferent temperature stratification of the atmosphere, when oscillations of the sighting target image are absent and the refraction value is minimal. A considerable disadvantage of this method is the rather short operation period of the indifferent temperature stratification of the atmosphere (as a rule, shorter than 1 h) and the difficulty in determining temporal boundaries of this period, which are not constant and where the accuracy of determining them is rather subjective. The method does not require, in fact, any additional measurements and is based on the fact that, for a temperature gradient close to the dry-adiabatic one, refraction is small and virtually constant. For this reason, this method is the most in demand. However, it is necessary to take into account that the isothermal period begins at different points of the path at different times.

Based on analyses of results of numerous studies performed at different times, the following important conclusions were made in [52]:

(1) The generalization of light propagation conditions in higher atmospheric layers and the Earth surface layer is erroneous. The introduction of correction for the combined influence of Earth's curvature and refraction by the usual formula accepted in practice is inconsistent.

(2) For paths in the surface atmospheric layer at heights of 2–5 m, neither theoretical nor empirical comprehensive corrections for refraction can be obtained. In this case, the daily change in the refraction coefficient is significant, from 4.28 to − 4.40, and has, as a rule, a minus sign in the daytime.

Thus, a review of the most requested classical methods for measuring refraction showed that "the problem of accounting for refraction in geodesic measurements was not solved in practice" [46], which is confirmed by almost all researchers.

3. Radiophysical (turbulent) methods for determining refraction

3.1 Theoretical fundamentals of the method for determining refraction by fluctuations in light-wave parameters

All the methods for measuring refraction used at present are based on the theory created by Newton for the static atmosphere. The atmosphere is considered in the form of statically stable air layers with density decreasing with height. As mentioned above, most of the methods for measuring refraction are considered in known monographs and reviews. As a rule, almost all the methods for determining refraction assume by default that the atmosphere is in a statically stable state and its optical properties do not change, at least while measurements are being taken, and therefore they can be either measured with a certain accuracy or calculated, thereby estimating refraction. Such an approach to the solution to the refraction problem can be called 'classical,' and methods developed for determining refraction based on the Newtonian theory are called 'statistic.' However, using modern classical methods to determine refraction, it is rather difficult to ascertain with a high accuracy corrections to refraction by 'static' methods [2]. This is explained by the fact that refraction is a random rapid process depending on many unknown factors, because the atmosphere, like any real medium, is inhomogeneous and is always in turbulent motion, so that its optical properties randomly change [2, 4–29]. Therefore, refraction changes in the same way, as illustrated by the fine structure of the time dependence of the refraction angle obtained using a modern electronic tachometer with automatic pointing to a sighting target. Figure 10 demonstrates random variations in this angle with a frequency of the order of 1 Hz or lower [87].

By measuring the distance between points with marks determined from precision leveling, the heights of the instrument and target, and slope angles with a discreteness of 1 s, we can obtain the time dependence of refraction presented in Fig. 10. It can be seen that refraction can change by several ten arc seconds for several seconds.

Figure 10 shows that refraction rapidly changes randomly, and it is difficult to estimate it using any classical methods during measurements, because human vision is inertial with a high-frequency limit of about 10 Hz. If the oscillation frequency is higher, the target image becomes shapeless or completely disappears, although reliable visual observations are very complicated, even at frequencies of about several hertz. In this case, the moments of target pointing and reading and of measurements required for determining the refraction value are uncertain. In addition, the visual pointing makes uncertain the estimate of the target position, which always oscillates.

Therefore, it is rather difficult to obtain accurately enough the time dependence of real refraction by classical methods considered earlier. Periods of refraction determination can differ with the time of measurements, whose results should contain corrections for refraction. For this reason, in particular, the values of corrections for refraction obtained can significantly differ from real corrections. Thus, measurements should be performed in real time at a particular place.

The plot in Fig. 10 clearly shows that refraction at the moment a reading is taken can significantly differ from that obtained at a different instant. According to [42], "All the methods accounting for the influence of the atmosphere applied in the geodesic production are based on the statistical

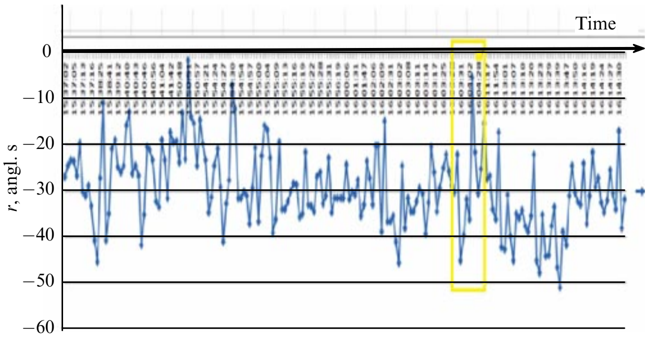

Figure 10. (Color online.) Time dependence of vertical refraction angle r obtained by comparing the results of high-precision geometrical leveling with trigonometric leveling with a discreteness of 1 s [87].

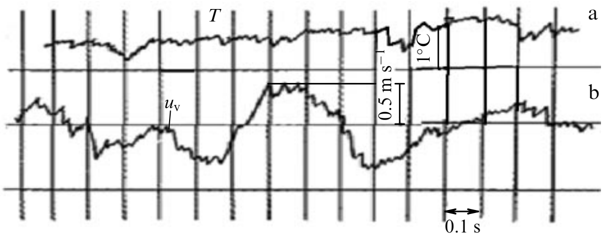

Figure 11. Sections of the synchronous recording of (a) air temperature T and (b) vertical component u_v of the wind velocity [26].

spatial and time distributions of atmospheric parameters. They cannot take into account the possible changes in the state of the atmosphere along the distance measured.'' This explains the difficulty in solving the refraction problem, which has already been studied by many researchers for a few centuries. Back in 1975, Izotov pointed out that ''the influence of turbulent and fluctuation processes in the atmosphere on geodesic observations can be completely eliminated only by direct measurements of the refraction angle at the observation moment'' [88].

To obtain information on the propagation of light during measurements, the real optical properties of the atmosphere need to be found.

According to experimental data, at two points located at the same height h separated by a distance of $0.4h$, pulsations of the vertical wind velocity can reach 1 m s^{-1} and the temperature pulsations can be 1 °C for 1 s (Fig. 11) [26], which in turn leads to significant random distortions of a light wave.

Therefore, to solve the refraction problem, a fundamentally new approach is required, which is completely independent of 'classical' methods of determining refraction accepted earlier and still used. The classical solution to the problem of measuring refraction, when the propagation of light in the static atmosphere is considered, is incorrect for Earth's real atmosphere.

The propagation of light can be studied by measuring distortions in an electromagnetic (sound) wave during its propagation [25–29]. Turbulent methods for determining refraction are based on the early unknown physical dependence between fluctuations in parameters of a light wave propagating through a layer of the turbulent atmosphere and the determinated refraction effect in the temperature gradient field. Based on these studies, radiophysical methods were developed for determining refraction by measuring statistical characteristics Ψ of an electromagnetic wave, and an algorithm was also proposed for determining the refraction angle $r(t)$ based on the semiempirical theory of atmospheric turbulence [25–29, 89–91]. According to radiophysical methods, the current value of the vertical refraction angle $r(t)$ is determined by measuring statistical characteristics Ψ of optical radiation propagating through a layer of the turbulent atmosphere and also temperature T and pressure P on the path by performing the following sequence of transformations:

$$\to C_n \to C_T \to \frac{\mathrm{d}T}{\mathrm{d}h} \to \underset{\substack{\nearrow \\ P,\,T}}{\frac{\mathrm{d}n}{\mathrm{d}h}} \to r_v\,. \qquad (16)$$

The main problem in determining refraction r_v by the turbulent (dynamic) method is how to estimate the structural characteristic of the field of the refractive index of the atmosphere — the quantity $\bar{C}_n$ averaged over the entire path — which is the most important parameter of the optical field of a turbulent atmosphere and is related to Ψ [15].

The structural characteristic of the temperature field C_T^2 in the surface air can be related to the vertical temperature gradient $\mathrm{d}T/\mathrm{d}h$ by the expression [25–29]

$$C_T^2 = C^2 a^2(\mathrm{Ri})(\chi h)^{4/3}\left(\frac{\mathrm{d}T}{\mathrm{d}h} - \gamma_a\right)^2. \qquad (17)$$

Here, $C^2 = 2.8$, $\chi = 0.4$ is the Karman constant, h is the height at which the temperature gradient is measured, $\gamma_a = -0.98°/100$ m is the adiabatic gradient, $a^2(\mathrm{Ri})$ is the universal function, and Ri is the Richardson number characterizing the degree of temperature stratification of the atmosphere.

The relation between the structural characteristic of the temperature field C_T^2 and the structural characteristic of the field of the refractive index of the atmosphere C_n^2 is described by the expression

$$C_n = \frac{10^{-6} K(\lambda)\langle P\rangle}{\langle T\rangle^2}\, C_T\,, \qquad (18)$$

obtained from theoretical and experimental studies, where $\langle T\rangle$, $\langle P\rangle$ are the average values of temperature and pressure, C_T is expressed in [K cm$^{-1/3}$], C_n, in [cm$^{-1/3}$], and P, in [mbar]. The quantity $K(\lambda)$ is expressed in [K mbar^{-1}] and is

$$K(\lambda) = N_\lambda \frac{T_0}{P_0}\,. \qquad (19)$$

Here, $N_\lambda = 77.6P/T + 0.584P/(T\lambda^2 - 0.06e)$, $T_0 = 288$ K, $P_0 = 1013.25$ mbar, and $e = 16$ mbar is the partial pressure of water vapor.

According to experiments in the surface air layer at heights up to 10 m, typical values of C_n^2, depending on observation conditions, are [26]

- the best — 5.20×10^{-16} cm$^{-2/3}$;
- middle — 7.21×10^{-15} cm$^{-2/3}$;
- the worst — 1.00×10^{-13} cm$^{-2/3}$.

Note that, under the worst observation conditions, i.e., when $C_n^2 = 1.00 \times 10^{-13}$ cm$^{-2/3}$ or C_n^2 is close to this value, the observation of sighting targets is already impossible, in fact, at distances of several hundred meters, because their images are highly blurred, shake, transform into a blurred spot, and even totally disappear. Such conditions are observed almost daily in the summer in a hot climate.

The relation between the vertical temperature gradient and the structural characteristic of the field of the refractive index is described by the expression [25, 26]

$$\frac{\mathrm{d}T}{\mathrm{d}h} = \pm\frac{10^6 C_n\langle T\rangle^2}{K(\lambda)\langle P\rangle C a(\mathrm{Ri})(\chi h)^{2/3}} + \gamma_a = \pm\frac{B(\lambda) C_n\langle T\rangle^2}{\langle P\rangle h^{2/3} a(\mathrm{Ri})} + \gamma_a\,, \qquad (20)$$

where $\mathrm{d}T/\mathrm{d}h$ is the vertical temperature gradient, T is temperature [K], P is pressure [mbar], $C = 1.67$ is the universal constant, $a(\mathrm{Ri})$ is a universal function which can be chosen depending on the temperature stratification of the atmosphere, $\chi = 0.4$ is the Karman constant, h is the equivalent height [cm], $\gamma_a = -0.98°/100$ m is the dry-adiabatic gradient, and

$$B(\lambda) = \frac{10^6}{K(\lambda) C \chi^{2/3}} \ [\text{mbar K}^{-1}] . \quad (21)$$

Therefore, measuring refraction is possible only in real time, which can be performed by measuring statistical characteristics of a wave propagating in a turbulent path. Taking into account the atmospheric dynamics, the vertical refraction angle r_d can be calculated from the expression

$$r_d = r_n + r_{an} = 8.13L\left(0.000244\,\frac{P}{T^2} \pm \frac{B(\lambda)\bar{C}_n}{h^{2/3} a(\text{Ri})}\right) . \quad (22)$$

Here, $r_n = 8.13L \times 0.000244P/T^2$ is the normal component of the refraction angle, $r_{an} = \pm 8.13L\bar{C}_n B(\lambda)/(h^{2/3} a(\text{Ri}))$ is the anomalous component of the refraction angle, where L is the path length [cm], P is pressure [mbar], T is the air temperature [K], $B(\lambda)$ [mbar K^{-1}] depends on the wavelength λ, $\bar{C}_n$ is the structural characteristic [m$^{-1/3}$] of the refractive index field, h is the equivalent path height [cm], and $a(\text{Ri})$ is the universal dimensionless function [25, 26] depending on the temperature stratification of the atmosphere.

3.2 Theoretical fundamentals of radiophysical (turbulent) methods for determining refraction by light-beam oscillations

The normal component r_n of the refraction angle in the surface air in the flat terrain of the middle zone of Russia is usually of the order of $+2.5''$ per km of the path [35]. During the day, in periods of stable and unstable temperature stratification of the atmosphere, the anomalous component r_{an} of the refraction angle can exceed in modulus the value of r_n by an order of magnitude or more.

The dependence of $\bar{C}_n^2$ on the square of fluctuations of the angle of beam arrival Δ^2 has the form [29]

$$\overline{C}_n^2 = 0.088 \langle \Delta^2 \rangle_{\text{refl}}\, \varphi^{-1}(\alpha_R) L^{-1} (2R)^{1/3} . \quad (23)$$

Here, L is the path length, $2R$ is the diameter of the receiving aperture, and $\varphi(\alpha_R)$ is a numerical function characterizing the jitter dispersion of a light spot taking into account the averaging action of the objective [26].

The image jitter and fluctuations of the angle of beam arrival are caused by phase fluctuations of the light wave [25, 26]. A random turn of the wave front through angle α causes the phase difference [90, 91]

$$\Delta S = kb \sin\alpha \approx kb\alpha ,$$

where b is the distance between two points of light-wave reception (or base) and k is the wave number.

This gives $\alpha = \Delta S/(kb)$, and the average square of fluctuations of the angle of arrival α is described by the expression

$$\langle \Delta^2 \rangle = \Delta^2 = \frac{\langle \Delta S^2 \rangle}{k^2 b^2} . \quad (24)$$

If R is the radius of a receiving objective, we can assume that $b \approx 2R$, and therefore

$$\langle \Delta^2 \rangle = \Delta^2 \approx \frac{\langle \Delta S^2 \rangle}{k^2 (2R)^2} . \quad (25)$$

The turbulent method for measuring refraction by fluctuations of the angle of arrival was tested many times on different geodesic areas in Moscow and in an industrial site in the European region of Russia (beyond the Arctic Circle) [15, 92–100].

It is known that isothermal periods (indifferent temperature stratification of the atmosphere) are the most favorable in the surface layer of the atmosphere. The refraction coefficient in such periods is very small and, as a rule, corresponds to its normal value, which can be calculated from the expression [2]

$$k = 17.24\,\frac{P}{T^2} , \quad (26)$$

where P is pressure [mbar] and T is temperature [K].

During the rest of the time of the day, the refraction coefficient can differ from its normal value and depends on observation conditions. Therefore, of practical interest is the estimate of the possibility of applying the method for measuring refraction under different conditions of temperature stratification, first and foremost, in the daytime, i.e., for indifferent and unstable temperature stratification of the atmosphere.

3.3 Experimental studies of the method for determining refraction from fluctuations in the angles of arrival for indifferent temperature stratification of the atmosphere

Studies were performed on paths of different lengths from 460 to 760 m in different seasons of year. Trimble Navigation SX8, SF9, SX10 and Leica electronic tachometers with an instrumental accuracy of $1''$ were used. The accuracy of measuring refraction by the turbulent method was estimated using zenith distances z_0 as reference values, which were determined by class II geometric leveling.

The real zenith distance z_0 was compared with the measured value z_m to obtain the real value of refraction angle r on the given path for the corresponding instant of measurements:

$$r = z_0 - z_m . \quad (27)$$

Later, the values of r obtained in experiments were used as reference values to estimate the accuracy of measuring refraction by the turbulent method. Refraction angle r_d was calculated from (22). By using Δr values characterizing deviations of refraction values r_d obtained by the turbulent method from real refraction values r, i.e.,

$$\Delta r = r - r_d , \quad (28)$$

we can calculate the root-mean-square error of point refraction measurements.

For example, on October 1, 2018, using a robotized Trimble SX9 electronic tachometer, refraction was studied by fluctuations of the angle of arrival in the isothermal period, i.e., when oscillations of a sighting target were not visually observed [92]. The length of the horizontal path was 741.42 m, temperature was $T = 1\,°\text{C}$, wind velocity was 0.5 m s^{-1}, and pressure was $P = 1025.8$ mbar. Because in this case the observation conditions were favorable, i.e., close to the indifferent temperature stratification of the atmosphere, one can assume that the anomalous vertical temperature gradient was ~ 0 and, therefore, the normal component of the

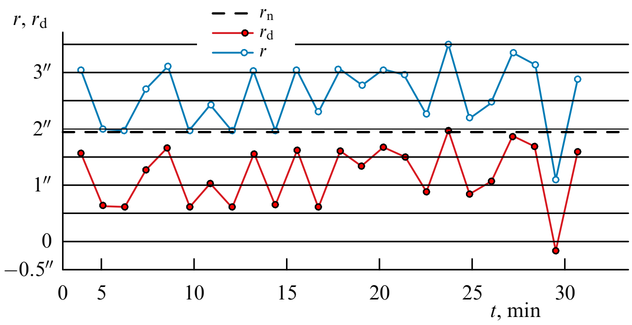

Figure 12. (Color online.) Time dependence of refraction on a flat horizontal track 741.42 m in length (October 1, 2018) [92].

refraction angle can be estimated from the expression [2]

$$r_n = 0.198\,\frac{P}{T^2}\,L\,. \qquad (29)$$

Using these observations on the path ($P = 1025.8$ mbar and $T = 274.15$ K), we obtain the normal value $r_n = 2.00''$ of refraction for indifferent temperature stratification at the moment of observation, which corresponds to the adiabatic normal gradient of the air temperature, i.e., $\gamma_a = -0.0098$ K. In this case, the root-mean-square error of measuring refraction by the dynamic method in the isothermal period was $m_d = 1.41''$.

The time dependence of the refraction angle obtained by the turbulent method is almost completely analogous to the real time dependence of refraction, but shifted by a value of no more than 1.5″ (Fig. 12).

Because the real values of the refraction angle are determined with an accuracy of about 2.2″, as mentioned above, we can assume that the accuracy of the dynamic method of measuring refraction in the case of indifferent temperature stratification of the atmosphere corresponds to the instrumental accuracy of the robotized tachometer used.

This tachometer can provide an instrumental accuracy of 1″ of measuring directions with a frequency of 2.5 Hz and also excludes, in fact, subjective errors in pointing to a sighting target in the measurement of vertical angles, because the search for a target and pointing to it are performed automatically without an operator. It is necessary to take into account that, even in the isothermal period, temperature stratification in the real atmosphere can never accurately correspond for a long time to the indifferent temperature stratification of the atmosphere, i.e., the temperature gradient during reading always, in fact, differs at least from adiabatic because of the turbulent motion of air.

For the indifferent temperature stratification of the atmosphere, the Richardson number is quite small and lies in the range $-0.05 < \mathrm{Ri} < 0.05$. These boundary values of Ri correspond to the vertical gradients of the air temperature in the range $-0.034 < \mathrm{d}T/\mathrm{d}h < +0.02$ grad m^{-1}, i.e., they differ somewhat from adiabatic. This leads to the appearance of weak fluctuations in the refractive index of air, which are difficult to estimate in this period visually by image oscillations. Angle r of vertical refraction in the observation moment was positive. The values of refraction angle r obtained at the current instant of measurements (by comparing measured zenith distances with the real zenith distance) somewhat differ from the preliminarily calculated value of the normal refraction angle $r_n = 2.00''$ for neutral temperature stratification. A comparison of r_d and r shows that the deviation in refraction values from real values obtained by the turbulent method does not exceed 1.5″. Because real values of the refraction angle are determined with an accuracy of about 2.2″, we can conclude that the accuracy of the turbulent method of measuring refraction for indifferent temperature stratification of the atmosphere approximately corresponds to the accuracy of the instrument used.

Experiments performed confirm again that the most favorable time for geodesic measurements corresponds to conditions of the indifferent temperature stratification of the atmosphere, when the angle of vertical refraction is close to zero.

3.4 Experimental studies of the turbulent method for determining refraction for the unstable temperature stratification of the atmosphere

Of practical interest are the results of studying refraction for unstable temperature stratification. Such studies were performed in 2018 and 2019 in the summertime, i.e., for the strong unstable temperature stratification of the atmosphere [98]. In 2018, several horizontal tracks of different lengths were chosen, which were in fact aligned parallel to the Skhodnya canal. The tracks had an underlying surface — a meadow covered with grass and a walking path passing along a canal (Fig. 13). For comparison, Fig. 14 [92] shows the time dependences of refraction r and r_d. The root-mean-square of the refraction angle was no more than $\sim 2''$, corresponding to the instrumental accuracy of the device used.

One can see that these plots, in fact, coincide, being shifted with respect to each other by only 1.5″. Note that the values of refraction accepted as a reference are not reliable enough because of errors in the pointing, measurements of the instrument height, and the sign and class II leveling and determination of the zero position (M0).

The time dependence of refraction in Fig. 15 was obtained in hotter weather. The plot demonstrates the fine structure of random variations in the vertical refraction angle in the daytime on 6.08.2019 from 15:37:02 to 16:13:50. The air temperature was $T = 27\,^{\circ}$C, pressure $P = 767$ mm Hg, and south wind velocity was 1.7 m s^{-1} on a sunny, low-cloudy day. Observations were performed with a robotized Trimble SX10 electronic tachometer at the GIS company group site (Moscow) [97]. The track length was 759.67 m, the under-

Figure 13. (Color online.) Robotized Trimble SX10 electronic tachometer installed on a track passing parallel to the Skhodnya Canal (left of the track) [98].

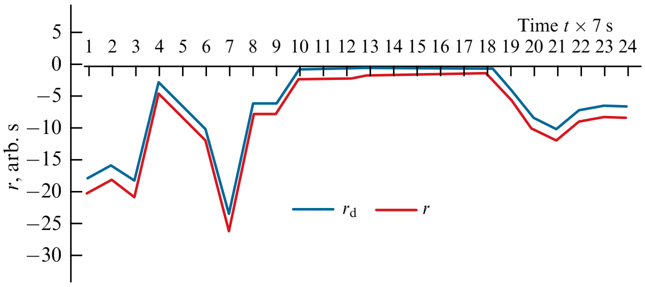

Figure 14. (Color online.) Daily time dependence of refraction at the GIS site ($L = 625.22$ m), August 13, 2018 [92].

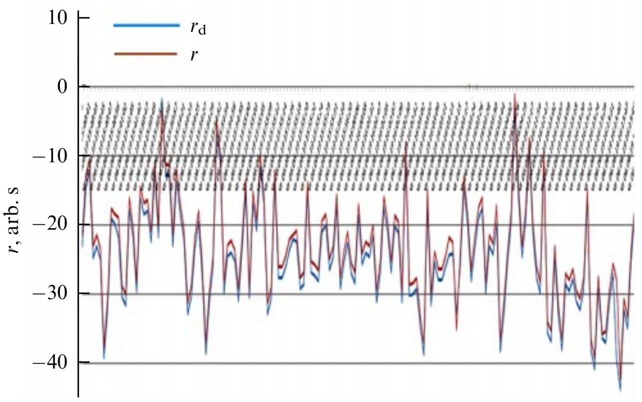

Figure 15. (Color online.) Daily time dependence of refraction at the GIS site ($L = 759.67$ m), June 6, 2019 [100].

lying surface was grass cover of the plain bank of the Skhodnya Canal. The instrumental accuracy of the tachometer was about 1″. The duty cycle of readings was 1 s and was increased sometimes, depending on the time of the reflector overlap by passing pedestrians.

Tracking-mode observations were performed by measuring vertical directions on a reflector placed at the end of the track. Figure 15 shows that refraction can change by 35″ for 2 s, which is very difficult to determine with an accuracy of 1–2″ with conventional optical instruments without automatic pointing to a sighting target. This circumstance allows one to conclude once more that the use of even high-precision instruments with visual pointing during unstable temperature stratification of the atmosphere does not provide high-precision results of observations because of rapid random variations in refraction.

At present, it is assumed that the results of measuring refraction obtained on one track can be used after some transformation on another track or even in another region. It is assumed that this area of study can be used for the development of additional ways to solve the refraction problem, which is rather difficult.

Figure 15 [100] visually confirms all the difficulty of measuring refraction by classical methods, because it is difficult to assign the results obtained by them to the instant of observation of one direction or another with the required accuracy. Thus, we can assume that, to achieve the accuracy of measuring refraction corresponding to the instrumental accuracy of modern high-precision instruments, it is necessary to measure refraction simultaneously with measurements of directions for which the correction for refraction is intended. If such measurements are not performed simultaneously, the introduction of correction for refraction found at a different moment of time can cause significant distortions.

A similar conclusion can also be made for the case of determining corrections using the refraction basis or etalon refraction values obtained in another region of measurements, etc. Thus, the situation completely corresponds to the saying of the ancient Greek philosopher Heraclitus of Ephesus (544–480 B.C.): *You can't step into the same river twice*.

Somewhat later, visual turbulent methods were developed to determine refraction from the amplitude of oscillations of a sighting target image [2, 16–24, 101–103] and by estimating a blur of the image of a regular mira [104–107]. The significant advantages of these methods are their simplicity and the absence of the necessity to use any additional instruments.

The method of determining refraction by the oscillation amplitude of a sighting target was proposed in 1978 [16]. The working formula for calculating refraction angle r has the form

$$r = r_{\mathrm{n}} + 0.05\sigma_{\max} L^{1/2} h_{\mathrm{e}}^{-1/2}\,, \tag{30}$$

where $\sigma_{\max}$ is the maximum amplitude of oscillations of a sighting target, r_{n} is the normal refraction value, L is the track length, and h_{e} is the equivalent height of a sighting beam.

As mentioned above, somewhat later, another method was proposed for determining refraction by the visual estimate of the mira blur, which was developed at the Nizhnii Novgorod Polytechnic Institute beginning in the mid-1980s [104–107]. The refraction value was determined from an estimate of the mira blur by first determining the averaged structural characteristic of the field of refractive index $\bar{C}_n$ of the atmosphere from a visual estimate of the blur of the regular mira image. Then, by using the value of $\bar{C}_n$ found, the refraction is calculated. These visual turbulent methods are at the stage of laboratory investigations. The problems with their application are related to the subjective qualities of human vision, because significant errors in measurements of refraction appear due to the accuracy and time in measuring the oscillation amplitude or an estimate of the mira period.

Figure 15 shows that refraction changes randomly and rapidly, whereas the eye of an observer reacts to these changes with a delay due to the inertia of its response. Therefore, the reading time and result obtained, which should be used to introduce a correction for refraction, cannot be accurately determined. Hence, the correction obtained cannot be accurately assigned to a certain direction and cannot be used to correct the observed direction reading.

In recent years, turbulent methods for measuring refraction have attracted certain interest in connection industrial problems related to work involved in the complex AlpTransit project on the development of the world's longest railway tunnel, 57 km in length, in the Alps. The building of this tunnel required the precise optical navigation of tunneling machines. Refraction is the main source of systematic errors in determining distances and directions. In addition, demand is increasing for precise measurements in geodesic monitoring, industrial metrology and aviation industrial lines, problems of precisely aligning magnets in accelerators, etc.

Researchers at the Institute of Atmospheric Optics, Siberian Division, RAS published paper [108] at this time presenting a short review of radiophysical methods in the regular geodesic refraction of optical radiation. The authors

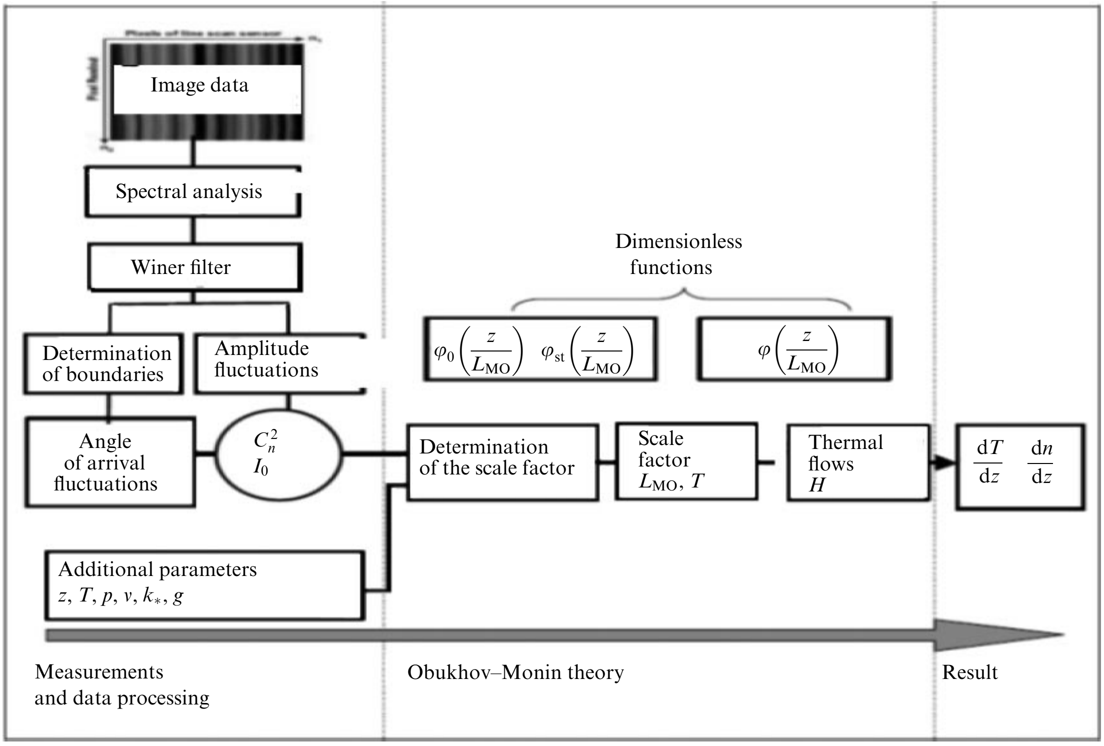

Figure 16. Block diagram for measuring turbulence, theory and calculations for determining the gradient of the refractive index of the atmosphere [111].

devoted considerable attention to the method of measuring the angles of regular refraction of optical radiation propagating in the surface atmospheric layer [108, Ch. 5] along homogeneous and inhomogeneous tracks. In addition, methods of measuring refraction angles in reflection schemes were considered.

According to work performed at the Institute of Geodesy and Photogrammetry, Federal Institute of Technology, Zürich, Switzerland) [69, 86, 109–113], great attention is devoted to turbulent methods of determining refraction and to studies of image distortions and jitter produced by intensity fluctuations by using geodesic instruments equipped with CCDs or scintillators. To study these effects, methods for determining and correcting for refraction were extensively studied in recent years at the Institute of Geodesy and Photogrammetry, ETH Zürich [109–113]. One of the approaches involves the determination of the refractive index in measurements of air turbulence using scintillometers or digital image processing. Based on the Monin–Obukhov Similarity Theory (MOST), we can find turbulent sensitive thermal flow H and turbulent pulsed flow M by using the structural constant C_T and the dissipation rate of turbulent kinetic energy ε. These operations can be performed using dimensionless equations for C_T^2 and ε, which lead by the numerical iteration scheme to the Obukhov scale L, which characterizes atmospheric stability and the turbulence of flows. The sensitive thermal flow can give the temperature gradient $\mathrm{d}T/\mathrm{d}z$ and the refractive index gradient $\mathrm{d}n/\mathrm{d}z$ for correction of atmospheric deviations in precise geodesic measurements. The sequence of operations is shown schematically in Fig. 16. It is assumed that the advantage of such measurements is the determination of averaged parameters of the surface atmospheric layer turbulence for a given track.

Methods accounting for turbulence effects have been tested several times [111]. A comparison of the values of C_n^2 obtained from measurements with a scintillator and using the method of image analysis demonstrated their good agreement. Simultaneously, the temperatures of temperature sensors and fluctuations in signal intensity were measured, which were used to calculate the vertical gradient of the refractive index of air.

Disadvantages of the turbulent methods considered above are the difficulties in applying them and the low accuracy in measuring refraction. Significant problems are also caused by the necessity of calculating many parameters characterizing the atmospheric turbulence, which have uncertain scales difficult to account for. For this reason, the relation between random distortions of a light-wave front and refraction has not been found, although this dependence was obtained empirically based on experimental studies.

Investigations performed for many years at the Obukhov Institute of Atmospheric Physics Optics, RAS showed that the determination of C_n^2 by measuring fluctuations in the signal amplitude is reasonable only at short distances (up to ~ 200 m), because saturation appears upon increasing the track length and the measured value of C_n^2 is distorted [26] and therefore cannot be used to estimate refraction.

4. Conclusions

(1) Refraction was already known in ancient times, because people observed this phenomenon all their lives. The first written mention about the refraction of light is related to the 1st century of our era (Cleomedes). The greatest contribution to the development of the refraction theory was made by Isaac Newton. The refraction theory developed by Newton has not los its significance, even now.

(2) The refraction theory is one of the complicated problems of modernity. Theoretical studies by Newton, Euler, Oriani, Bernoulli, et al. were reduced to the determina-

tion of the geometrical path of a light ray. Theoretical calculations cannot give reliable values of local vertical temperature gradients, air pressure, or humidity, and therefore the ray path, which is due to the numerous variable conditions of light propagation in the surface layer of the atmosphere.

(3) Because most geodesic and astronomic measurements are performed with ground-based optical instruments, the search for methods of determining refraction and accounting for it in the surface atmospheric layer is quite relevant. In addition, the solution to problems of precise pointing, location, navigation, and communication requires knowledge of the refraction value at the current instant of time. The problem of measuring refraction has been discussed in many books and studied in a few hundred papers and dissertations. For example, more than 1000 papers in this field have been published in recent years alone and more than 100 methods proposed for determining refraction [3]. It is pointed out in [114] that "we can only admire how insistently geodesists tried to find methods to account for refraction. The number of formulas proposed is no smaller than the number of formulas correcting for refraction. However, the problem of refraction correction was not finally solved" because "studies for many years did not lead to a final solution to the problem, although vertical refraction was investigated for more than 300 years" [2]. A N Krylov also commented in his time about weak progress in studies of refraction [1].

(4) To solve successfully the refraction problem, a fundamentally new approach is required, one that is completely independent of earlier accepted and still used 'classical' methods for measuring refraction. Such a solution can be found if the real optical properties of the atmosphere and the refraction value are determined using information on the propagation of light in a turbulent atmosphere. The classical solution to the problem of determining refraction, when the propagation of light in a static atmosphere is considered, is incorrect for Earth's real atmosphere for most of the day. The modern solution to the refraction problem is based on papers written at the Zuev Institute of Atmospheric Optics, SD RAS and the Obukhov Institute of Atmospheric Physics, RAS, where the propagation of light in a turbulent atmosphere was studied.

(5) Studies of radiophysical methods show that most of the optical measurements in the surface atmospheric layer can be performed in the nearest future without considerable distortions of the results by refraction [87, 92–100]. At present, the turbulent method of measuring refraction is already applied in practice. The use of electronic tachometers with automatic pointing to a sighting target in different climatic zones in different seasons of the year demonstrates the high efficiency of the method of determining refraction by fluctuations of the angle of arrival, its simplicity, and its high accuracy corresponding to the instrumental accuracy of the device used.

(6) The radiophysical (turbulent) method of determining refraction provides measurements in unfavorable observation conditions, which expands the temporal boundaries of optical observations without reducing their quality [87, 92–100]. Thus, recent developments in the radiophysical methods used in recent studies have shown that these methods are more efficient than classical methods based on Newton's theory.

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