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Nonlinear dynamics of creative thinking. Multimodal processes and the interaction of heteroclinic structures

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Contents

Abstract. **The dynamic processes of creative thinking, as confirmed by recent studies using electroencephalography (EEG) and functional magnetic resonance imaging (fMRI), are interactions among three main components: the originality of the author, the author's autobiographical memory, and the purpose or stimulus of the process. Different stimuli initiate the excitation of different memory components and, accordingly, different neuronal clusters and brain networks. New data make it possible to build a model of the birth and development of creative thinking, i.e., to elaborate a theory of a process that is itself, by definition, indefinitely structured and unpredictable, with the use of a structurally organized mathematical approach — nonlinear dynamics. The following key concept is discussed: the evolution of thought or the essence of other human creative activity is a dynamic process characterized by internal instability leading to the generation of new information. To construct a nonlinear dynamic model of human creativity, the following ideas common to most mental processes are used: (1) a mathematical model should be based on variables representing the evolution of brain elements in their temporal coherence and should have solutions corresponding to metastable patterns (blocks of knowledge) in the brain, (2) the model is based on competitive dynamics without a winner, i.e., a nonlinear process of interaction of many information elements or spatio-temporal modes, which guarantees sequential switching between metastable states and, as a result, a certain stability of the dynamics of creativity, (3) the model is an open dissipative system in which inhibition is balanced by excitation; as a result, being close to the boundary of instability, it turns out to be extremely sensitive to information influences.**

Keywords: nonlinear dynamics, neural networks, creative thinking, models of brain functioning

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The poet's eye, in fine frenzy rolling,
Doth glance from heaven to earth, from earth to heaven.
And as imagination bodies forth
The forms of things unknown, the poet's pen
Turns them to shapes, and gives to airy nothing
A local habitation and a name.
Shakespeare, *A Midsummer Night's Dream*, Act 5, Scene 1

1. Introduction. The brain and consciousness. The essential components of creative thinking

1.1 The brain and the mind

Speaking about an unexpected decision taken suddenly, we often say: "It's only a matter of chance." If we only knew how close we are to the truth: it is exactly the accidentally discovered connections coming from above between absolutely different, totally unrelated objects, concepts, and ideas that give rise to original approaches that were impossible to predict based on previous experience. In fact, a flash of insight known as an *aha*! or *eureka*! moment is just the tip of the iceberg of a dynamic process called creativity.

Creativity is essentially a process of generating new and at the same time useful ideas usually viewed as the interaction of many informational components or modalities of brain activity. This can be a free wandering among the ideas produced by an idle (resting) brain when neither external nor internal signals influence it, the application of standard approaches not for their intended purpose but in unexpected ways, the use of random associations.

In 1908, at a meeting of the Psychological Society of Paris, Henri Poincaré gave a lecture, Mathematical Creativity, that, unfortunately, was not written down. In his report, Poincaré emphasized that a scientific discovery is preceded by a long period of work done partly consciously and partly subconsciously after sufficient information has been accumulated and necessary efforts made. Then, a revealing insight occurs when the pieces of the puzzle suddenly come together and — eureka! — fall into place. Since the time of Poincaré, the creative process has been divided into several stages [1]: (1) preparation, (2) comprehension (focusing attention on difficult moments), (3) revelation (insight), formulation, and understanding of the new idea, and, finally, (4) finding the place and the role of the result in life. Almost all of these processes are controlled by cognizance.

Note that revelation is often unpredictable: "The one who writes a poem writes it above all because the language prompts, or simply dictates, the next line. Beginning a poem, the poet as a rule does not know the way it is going to come out, and at times he is very surprised by the way it turns out, since often it turns out better than he expected, often his thought carries further than he reckoned. And that is the moment when the future of language invades its present. There are, as we know, three modes of cognition: analytical, intuitive, and the mode that was known to the Biblical prophets, revelation. What distinguishes poetry from other forms of literature is that it uses all three of them at once (gravitating primarily toward the second and the third), for all three of them are given in the language and there are times when, by means of a single word, a single rhyme, the writer of a poem manages to find himself where no one has ever been before him, further, perhaps, than he himself would have wished for — verse writing is an extraordinary accelerator of conscience, of thinking, of comprehending the universe. Having experienced this acceleration once, one is no longer capable of abandoning the chance to repeat this experience..." (I Brodsky, *The Nobel Lecture*. *Works*, vol. 1, Pushkin House).

Here, one is faced with the multimodality of creative processes. At least three cognitive modalities are involved in poetry writing, viz. logical, emotionally intuitive, and what is called insight or, in the formal language, useful fluctuations. To describe creative processes, we use a basic dynamic model in the form of a system of nonlinear equations in ordinary derivatives for variables that represent the intensity of space-time modes of the corresponding brain networks.

The objects of interest to us in the phase space of such models are metastable states (saddle points or saddle limiting cycles) representing individual semantic images and heteroclinic trajectories, i.e., separatrices connecting them into a sequential chain describing the generation and processing of an information flow (Fig. 1).

By considering various kinds of instabilities in models with several modalities, it is possible to describe both the birth

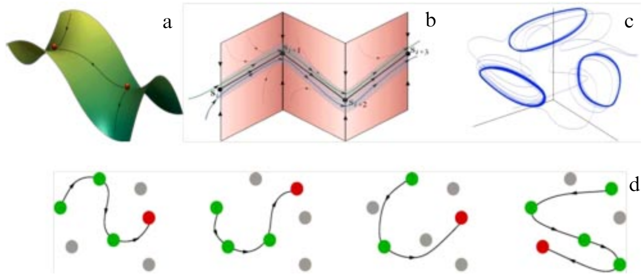


Figure 1. (Color online.) (a) Heteroclinic chain composed of two metastable states (saddles), (b) stable heteroclinic channel, (c) heteroclinic channel consisting of saddle cycles, (d) different activated heteroclinic channels in an ensemble of seven metastable states. (Adapted from [3, 4].)

of a thought and bifurcations of an insight. Here, one must accept one more source of multidimensionality of the creative process, namely the communication of the author with the reader or the viewer whose image the poet holds in their imagination. Such communication influences the creative process from the very beginning, increasing its complexity and unpredictability. This is how Anna Andreevna Akhmatova put it:

About some things they rebuke me,
About others they agree.
And so it goes, like a silent confession,
The flow of our warm exchange.
("The Reader," 1959)

Since each modality is generated by its own neural network of the brain, the description of the creative process in terms of nonlinear dynamics requires an analysis of the interaction of such networks as reflected in the title of this review.

Speaking about the author's thought, which, even at the moment of creating a verse (sculpture, music, etc.) belongs not only to that author but also to the reader, listener, etc., one often thinks of how the thought to be conveyed to the recipient is formulated in the brain, where it comes from, how a small amount of gray matter evokes sensations and fantasies. It often seems unbelievable that the fluid of material processes in the brain can give rise to the 'wine of consciousness.' This is the main problem, or rather a puzzle, that arises when considering the relationship between the mind and the brain (or between the mind and the body). The inability to achieve a consistent solution of the mind–body problem remains at the core of psychology and is the key issue in the 'unified theory of psychology' (UTP) [2].

Here, it is necessary to discuss the meaning of the word 'mind' and what people actually mean when they use this particular term. Most often they think of where consciousness dwells, i.e., appeal to the sense of self that appears to be a guide of thoughts somehow connected with the brain (body), but at the same time apparently separable from it. The idea of 'life after death' is intuitively acceptable to many because our intellectual life seems entirely unrelated to our body, so that we can imagine them as independent of each other. This leads to the dualism of common sense, which is a part of religious worldviews. Naturally, self-consciousness is one of the functions of the mind.

The famous cognitive revolution of the 1960s originated from the interrelationship among studies on the information theory, artificial intelligence, and cybernetics. It gave rise to the computational theory of the mind, offering solutions to most of the questions related to the dynamics of mind–brain interplay. The computational theory of consciousness holds that the neuronal system is an information processing system. It translates the measured activity of the body and the environment into the mind's informational language.

From the formal point of view, the 'space of the mind' is a functional space where mathematical images of various mental processes in the brain are presented as they develop in time. The dimensionality of the space depends on the complexity of the mathematical models that describe these processes. When many mental modalities are represented in such a space, it has a hierarchical structure.

The computational theory of consciousness was a huge breakthrough, because it allowed for the first time separating in principle, i.e., ideologically, the mind from the body–brain. While the cognitive revolution was a big step forward, it posed questions issuing from the infatuation with artificial algorithmic problems weakly connected with other elements of psychological phenomena, such as conscious experience, culture, and autobiographical memory. Today, the situation with cognitive dynamics is qualitatively different. As Richard Feynman put it: "If you want to really understand how something works, create it."

As far as the human mind is concerned, science is steadily moving in this direction, although the goal is yet to be achieved, and the engineering problem of modeling consciousness is still indefinably far from solution.

The objective of this review is to give an idea of the multimodal theory of human creative thinking based on widely known examples.

1.2 Identification of creative thinking at the level of brain physiology

1.2.1 A free search. Obviously, the need for creative thinking is a response to a situation that gives the author a certain freedom, i.e., it is not limited by a rigid framework and often has no clearly fixed goals. On the contrary, a noncreative regime suggests movement along a certain predetermined path. Usually, it is the 'path of least resistance' with an expected and effective result (variational principles often work here, e.g. the free energy principle [5]). To date, much is known about brain function dynamics in this routine mode.

In contrast, the main features of the *dynamics of creative thinking* are due to a wide range of possibilities for the participation of various brain regions in specific aspects of creativity, such as polysemantic understanding, imagery, associative thinking, and the ability to be self-expressive. In other words, the creative brain must be organized slightly differently than the ordinary one.

It is believed that three key neuronal networks are involved in the creative thought process: (1) the default mode network deactivated during goal-oriented activity is responsible for rapid episodic spontaneous thinking and proneness to fantasy; (2) the salience network chooses the most salient information to address the task in question and participates in such efficient problem-solving process as 'brainstorming'; however, it does not guarantee a continuous generation of creative ideas if the personal experience of a human is poor and their memory keeps no memories of unique or nonstandard situations; (3) the executive control network finds out important information in the environment or in the autobiographical memory that can be useful for solving the problem at hand. When it comes to the creative process, it is the executive control network that is responsible for sorting out ideas generated in the default mode network.

To begin with, here is just one example. The authors of a Harvard experiment asked 163 volunteers to solve a creative problem when their brain activity was being scanned with the use of the functional magnetic resonance imaging (fMRI) technique. While the apparatus was recording brain impulses of the participants, each of them had 12 seconds to propose the most creative handling of a simple object shown to them on the screen. As is known, one of the obstacles to creative thinking is the ease with which ordinary habitual solutions impose themselves on the human mind. Some participants in the experiment could not resist such a temptation. For instance, when asked about the creative use of a sock, soap, and gum wrapper, less creative people proposed to 'cover up feet,' 'make bubbles,' and 'wrap up chewing gum,' respectively. More creative individuals suggested quite different applications of the same objects, such as to filter water, seal envelopes, and use as a radio receiver antenna [6].

The analysis of fMRI data was focused on the peculiarities of brain activity. The individually-minded participants exhibited a close connection between the three aforementioned neural networks: (a) the default mode network or imagination network responsible for spontaneous thinking and thought wandering in the absence of a formulated goal and external sensory excitations; (b) the executive or central attention network involved in focusing attention and making decisions based on previously obtained information; (c) the salience network evaluating the saliency of information that comes from the environment and coordinating it with what the autobiographical memory says. The first two of these networks tend to compete with each other, i.e., one alternately suppresses the other. However, scanning data give evidence that the brain of creative subjects is able to use both these networks simultaneously, i.e., the creative brain uses their information resources more efficiently (Fig. 2).

1.2.2 Separation of functions between hemispheres. Debunking the myth. We are all familiar with the classic dogma that the left hemisphere is responsible for solving analytical and logical problems, while the right one controls emotions and creativity. Modern experiments show that this is not true. It turns out that creativity depends on the activity of the three global networks described in a preceding paragraph and that they are present in both hemispheres (Fig. 3). These networks include interacting brain blocks functionally integrated into modes closely coordinated with one another to perform specific creative tasks. These tasks are discussed in Section 4 below.

Results of these studies are presented here in parallel to illustrate the stability and reliability of fMRI studies of functional connections between large-scale brain networks carried out recently by different groups of experimenters (see Fig. 3). It can be concluded that this approach is quite credible, despite the doubts of some researchers.

If creativity is defined in terms of the quality of the product (song, invention, poem, or painting), then the key role is given to the left hemisphere [9]. However, if creativity is understood as the person's ability to find unexpected solutions in difficult situations or to analyze on the go, the right hemisphere takes on the leading role. These inferences are illustrated by an experiment, the results of which are shown in Fig. 4 (see also [10]).

Such understanding is consistent with the results of previous studies showing that the main episodic memory network and imagination are involved together in divergent

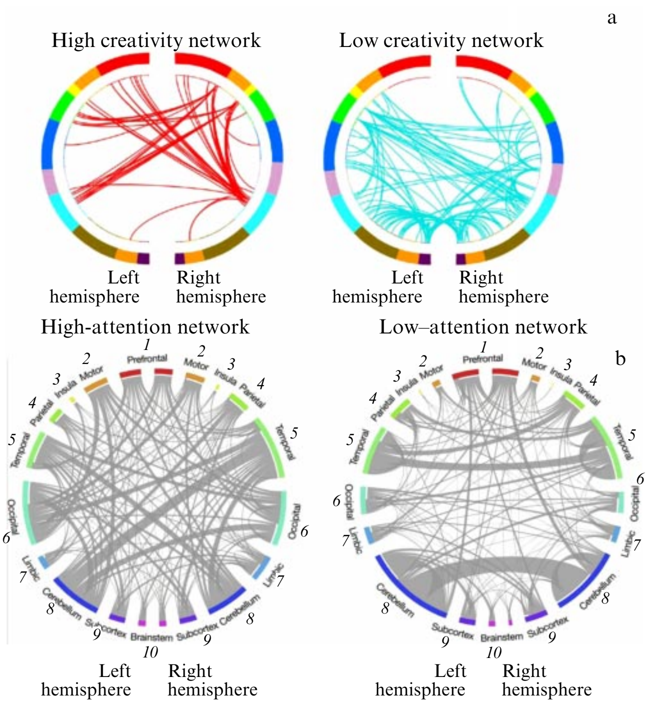


Figure 2. (Color online.) Functional connections in creativity-associated networks illustrating the interaction between the two cerebral hemispheres. (Modified from [6, 7].) The results presented in Figs a and b were obtained by a detailed treatment of a wealth of tomographic data (fMRI analysis) in experiments involving many participants performed by different research teams for different purposes. Figure a presents the structure of functional connections between the large-scale brain networks responsible for creativity [6], whereas Fig. b analyzes similar connections with regard to a more specialized problem of predicting the level of attention (see [7]). Figure b: *1* — prefrontal cortex, *2* — motor cortex, *3* — insular lobe, *4* — parietal lobe, *5* — temporal lobe, *6* — occipital lobe, *7* — limbic system, *8* — cerebellum, *9* — subcortical structures, *10* — brain stem.

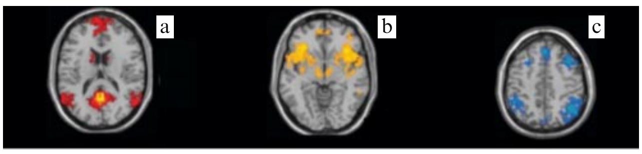


Figure 3. (Color online.) Portraits of activity of the three main participants in the creative process: imagination network (a), attention and control network (b), realization network (c). Taken together, they are assumed to generate creative thought. (Modified from [8].)

thinking [6, 13], with the left inferior parietal lobe being critical for the generation of new ideas [14].

These data help to clarify the role of mental modeling of individual episodes in creative cognition on the assumption that they occur at the level of basic constructive processes, such as a conceptual combination of notions [15].

1.2.3 Associations. One of the key mechanisms underlying creative thinking is informational associations supported by autobiographical memory. Importantly, the associations can form not only between two information objects but also among a large number of them. Biassociations are distinguished among others. They differ from a set of simple associations by a stronger activation in the hippocampus (Fig. 5) and the inferior parietal lobe (e.g., the angular gyrus) representing central network nodes critical for episodic

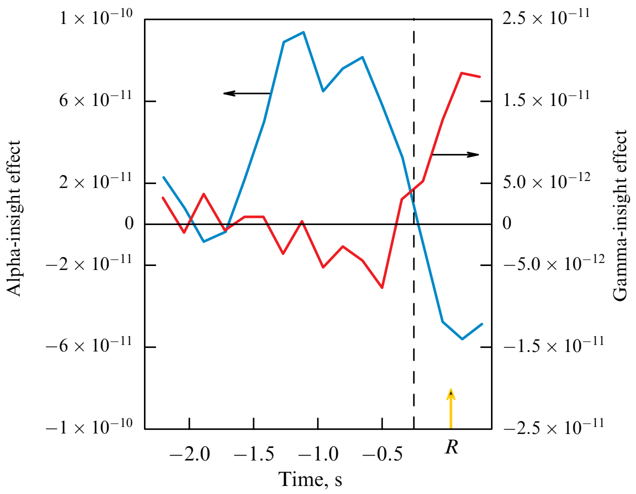


Figure 4. (Color online.) Analysis of EEGs taken during solution of creative problems demonstrates various organizations of brain network excitation depending on whether the solution is obtained intuitively or by analysis: the intuitive solution is preceded by an increase in activity in the alpha band range (10 Hz) in the occipital region of the brain, whereas the analytical solution obtained by means of analysis is immediately followed by a burst of high-frequency activity in the gamma range in the right temporal lobe. (Modified from [11]; see also [12].)

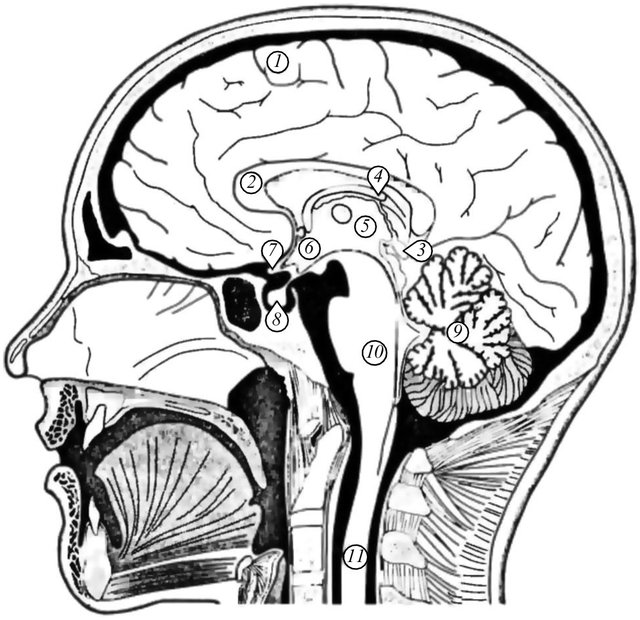


Figure 5. Schematic representation of the brain (longitudinal section): *1* — cerebral cortex with convolutions; *2* — corpus callosum, the connection between left and right hemispheres; *3* — pineal gland (epiphysis), producing the sleep hormone melatonin at night that delays the onset of puberty in children; *4* — fornix of the telencephalon, which transports memory information from the hippocampus to the mastoid body (corpus mamillare) in the posterior part of the hypothalamus whence the information goes to the thalamus and cerebral cortex; *5* — thalamus, whither the information comes from sensory organs and memory; *6* — hypothalamus, which is critical for the survival of the individual and the species as a whole; *7* — optic chiasm; *8* — hypophysis; *9* — cerebellum; *10* — brain stem; *11* — spinal cord [17].

memory and fantasy [16]. This fact suggests that the integration of two unrelated concepts supports the processes of generation of new episodes corresponding to these signals. It follows from experiment that the generation of double associations is not a mere generalized version of generation of an association but is also a new dynamic process underlain by qualitatively different cognitive mechanisms for the achievement of an effective creative activity by simultaneously using both memory of the past and information about the imaginary future.

It is natural to assume that the multimodality of the creative process promotes complex associations. In this sense, participation of a 'second self,' i.e., an *alter ego*, in the process may prove highly fruitful.

ALTER EGO

To Joseph Brodsky

While creating
when you write
you disrupt
the uniformity of lines
reflecting turns
Leaps are tagged
with verses' rhymes
Schoenberg's music
and Stravinsky's work
And bliss
that delights your soul
The alter ego that
so gracefully
ascends to the skies
For years and centuries.

M.I.R.*

The 'second self' takes an active part in the creative process inherent in artistic people; it makes its dynamics multimodal. This section can be concluded with the words of Marc Chagall: "We want to be attracted by the invisible side of form and spirit without which external life is not complete" (Fig. 6).

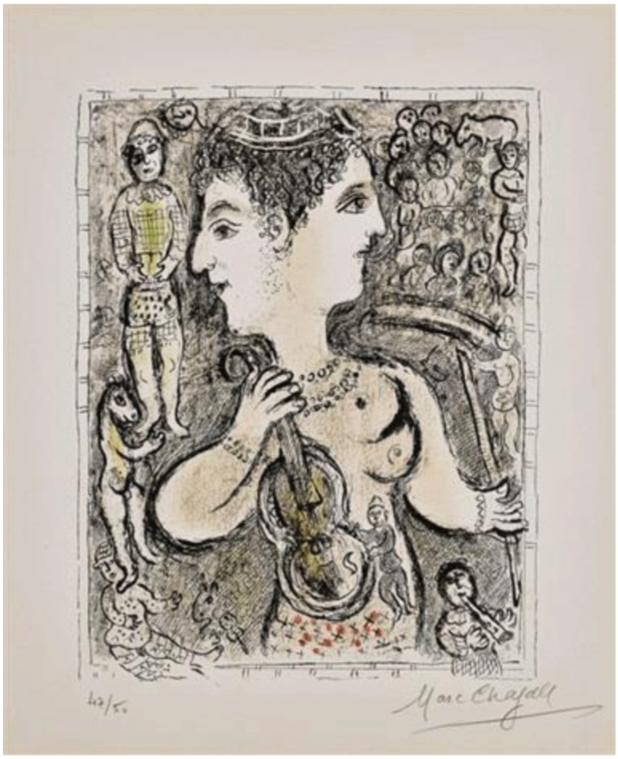


Figure 6. Marc Chagall, "Double Face" (color lithograph, 1978).

* M.I.R. — the literary pseudonym of ***M***ikhail ***I***zrailevich ***R***abinovich. (*Editor's note.*)

2. Mathematics of thinking

What is important is not what is precise,
but what is true.
A N Kolmogorov

2.1 Universal dynamic models of the processes of birth, coding, and reproduction of information

2.1.1 Kinetic equations. It was mentioned in the Introduction that the brain presents perceived information about processes in the environment and its own activity in the form of chains (or 'bunches' in the case of multimodal processes) consisting of metastable states that form heteroclinic networks in the phase space of the corresponding model. Such chains or networks represent the processes of generation (creation), treatment (encoding), and reproduction (memory) of information.

It is noteworthy from the standpoint of information dynamics that the processes of interest in an analysis of thinking are very similar to those of RNA self-replication (building the RNA world). It is no coincidence that they are described by similar kinetic equations, such as the equations of autocatalytic reactions (RNA reproduction) [18] and Lotka–Volterra equations that came from ecology [19, 20]. For ease of reading, the main model equations of the theory of thinking are collected in Table 1. The main feature of kinetic equations is that they simultaneously describe the processes of information creation, coding, and reproduction, i.e., memory. The above equations are relatively simple insofar as we are interested in processes near the stability boundaries. In fact, these are normal forms for such processes.

2.1.2 Consciousness as a multimodal chain process of interaction of information flows. Human intellectual activity is a discrete transient process, and at least its evolutionary aspect can be described by dynamic equations. Such an assertion with respect to consciousness is not so obvious. It is widely believed that consciousness is an intriguing, but too complex a phenomenon and as such is not easy to describe mathematically. Nevertheless, the mutually consistent data from experiments on the perception of oneself and the outside world suggest that all components or modules of consciousness, including autobiographical memory, attention, learning, and thought generation, are sequential dynamic processes. In these sequential processes, some cognitive objects temporarily prevail over others. Such a switching mechanism functions in accordance with the winnerless competition (WLC) principle [21–25] realized in the brain's neural networks due to the mutual asymmetric inhibition of network modes.

Mathematically, this can be described as a transition among the vicinities of various metastable states, i.e., semi-attractors, formed in the functional hierarchical networks of the brain through learning and self-improvement [26, 27]. The informational content of such metastable states depends on a function performed by the brain networks under consideration within a given time period. For example, it might be engrams, i.e., memory traces (stable combinations of activated networks in the episodic memory of a person representing what he or she has seen) or fragments of semantic data (mathematical theorems, favorite verses, etc.) that are contained in the semantic memory.

Those seeking to create a dynamic theory of such processes have to find answers to at least the following

Table 1. Basic model equations of the theory of thinking.*

Kinetic equations*
$\tau_i^m \dfrac{\mathrm{d}x_i^m}{\mathrm{d}t} = x_i^m \left(\sigma_i^m (R^m, S^m, C^m) - \sum_{j=1}^{K^m} \rho_{ij}^m x_j^m - \sum_{k=1}^{M} \sum_{j=1}^{K^m} \xi_{ij}^m x_j^k \right)$
Chunking
$\dfrac{\mathrm{d}X_i^{lk}}{\mathrm{d}t} = X_i^{lk} \left(\sigma_i^{lk}(S, C) Y^{lk} - \sum_{j}^{N^{lk}} \rho_{ij}^{lk}(S, C) X_j^{lk} \right)$
$\tau \dfrac{\mathrm{d}Y^{lk}}{\mathrm{d}t} = Y^{lk} \left(\left(V^l - \beta(C) \sum_{i}^{N^{lk}} X_i^{lk} \right) - Z^{lk} \right)$
$\theta(C) \dfrac{\mathrm{d}Z^{lk}}{\mathrm{d}t} = \sum_{m}^{M} \xi_l^{km}(S, C) Y^{lm} - Z^{lk}$
$T \dfrac{\mathrm{d}V^l}{\mathrm{d}t} = V^l \left(\left(1 - \delta(C) \sum_{j}^{M^l} Y^{lj} \right) - W^l \right)$
$\Theta(C) \dfrac{\mathrm{d}W^l}{\mathrm{d}t} = \sum_{q}^{P} \zeta^{lq}(S, C) V^q - W^l$
Binding
$\tau_{X_i}^m \dfrac{\mathrm{d}X_i^{km}}{\mathrm{d}t} = X_i^{km} \left(\sigma_i^m - \sum_{j=1}^{N_{\text{events}}} \rho_{ij}^{km} X_j^{km} - \sum_{k=1}^{N_{\text{events}}} \sum_{j=1}^{M} \xi_{ij}^{kml} X_j^{kl} \right)$
* *Stability conditions*. To analyze the stability of a heteroclinic sequence, one can consider metastable states, e.g., the equilibrium states $S_1, \ldots, S_n$ of an autonomous system. Let the eigenvalues $\lambda_1^{(i)}, \ldots, \lambda_n^{(i)}$ of the matrix of the system linearized near S_i be located so that $\lambda_1^{(i)} > \ldots \geqslant \operatorname{Re} \lambda_{m_i}^{(i)} > 0 > \operatorname{Re} \lambda_{m_i+1}^{(i)} \geqslant \ldots \geqslant \operatorname{Re} \lambda_n^{(i)}$. Then, the m_i-dimensional unstable manifold W_i^u is a strictly unstable one-dimensional manifold W_i^{uu} tangential to the eigenvector corresponding to $\lambda_1^{(i)}$. A heteroclinic channel (or cycle) can serve as an attractor if the so-called saddle value satisfies the condition $-(\operatorname{Re} \lambda_{m_i+1}^{(i)})/\lambda_1^{(i)} = \nu_i > 1$ for all i.

questions: how does the brain hierarchically organize sequential switching between engrams in order to maintain it stable and reproducible, and how many engrams can simultaneously participate in the switchings without a loss of information? The last question is obviously related to the information capacity of the corresponding cognitive process. If the process is unreliable, a mental disorder can be suspected. Sequential dynamics that are too stable and from which there is no way out can also lead to pathological behavior.

Creative people are characterized by flexible and highly variable dynamics of brain neuronal networks. Three groups of thought processes associated with network interactions during creative work are presently recognized as the most contrasting ones: the purposeful extraction of information from the memory, suppression of dominant (trivial) reactions, and focusing attention on the analysis of one's own self.

2.2 Multimodality and interactions in the hierarchy of heteroclinic networks

The process of competition without winners (WLC) or with temporary winners, to be precise, not only ensures the existence of a functional heteroclinic chain but also leads to stable transient dynamics and reproducibility of such sequential switchings as the initial conditions change. The reliability of a heteroclinic channel stems from the fact that the trajectories in the vicinity of a separatrix chain do not leave it till the end of the channel is reached (see Fig. 1). Usually, this is a simple attractor, i.e., a stable point, or a limiting cycle.

Various modalities that form consciousness modulate each other. Mathematically, this can be described within the framework of a canonical model if the inhibition resulting in chains of metastable states is not too strong and the metastable states may not have a single separatrix, but several unstable separatrices (i.e., be characterized by several positive indices; see Tables 1 and 2) which connect various metastable states and form interconnected heteroclinic channels in the phase space of the corresponding model. In this case, the joint dynamics may prove rather complicated and even chaotic [3]. Figure 7 illustrates, by way of example, the effect of one modality periodically changing in time on another described by a heteroclinic limiting cycle (Fig. 7a).

2.3 Creativity and pathologies: dynamic models

2.3.1 Processes of creativity. It is generally accepted that a creative idea or action should be original, unexpected, and at the same time useful (or pleasant). Creativity arises from the dynamic interaction among global brain networks and is not a specific function of a concrete brain region. From the dynamic point of view, a critical stage in creative processes can be regarded as a random walk among many different information patterns, the randomness of which can be quantitatively characterized by the value of the Kolmogorov–Sinai entropy [35]. Once this stage ends, the system shifts from an irregular search mode to a new metastable mode which uses feedback among all stages, including the working memory.

In humans, creative processes of various natures, such as writing poetry, musical improvisation, or painting, are supported by the same brain networks that interact during any conscious creative activity. Such universality is naturally consistent with the idea that the mathematical structure of

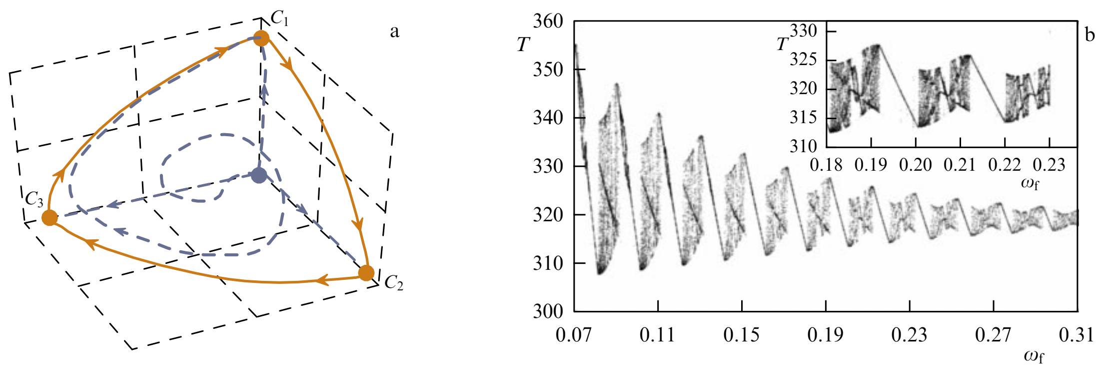


Figure 7. Heterocyclic contour. (Modified from [28].)

creativity in a dynamic model should be invariant for different classes of creativity. It is reasonable to assume that the principle of invariance is equally applicable to mental disorders with the same theoretical formalism based on the dynamics of metastable states and heteroclinic structures.

2.3.2 Obsessive-compulsive disorder. Here is an example of a dynamic analysis of an obsessive-compulsive disorder (OCD) which can be characterized by intrusive thoughts or images leading to sequential compulsive rituals. An emerging new field of *nonlinear dynamic psychiatry* offers computational methods as well as phenomenological insights for a better than static treatment of OCD. The dynamic interruption of everyday mental activity by the ritual can be represented in the phase space by the interaction between the modalities corresponding to these processes, i.e., two heteroclinic channels (Fig. 8). Cooperative mental-emotional dynamics controlled by this interaction is often chaotically repeated. The corresponding bifurcations in the framework of a dynamic model are determined by changes in functional connections, i.e., the values of coupling matrix elements, in the basic model. The models under discussion can show how to maintain the stability of mental characteristics by breaking the dynamic stability of the ritual. For example, this can be done by controlling the level of inhibition using an external stimulus delivered at the right time.

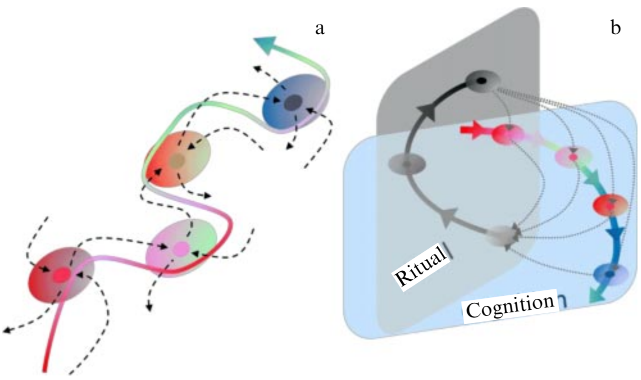


Figure 8. (Color online.) OCD. The picture shows that the ritual can interrupt the cognitive process at any step. Upon completion of the ritual, the system returns to mental work, but usually under different conditions. (Modified from [36].)

It is natural to expect that interactive approaches currently being developed based on dynamic models will provide new treatment and rehabilitation procedures, including cognitive interaction between artificial intelligence and humans.

2.4 Heteroclinic networks

2.4.1 Representation of the thinking process. The sequential process of thought is represented in the phase space by a network of heteroclinic channels. Each channel corresponds to its own modality and is connected with other unstable separatrices. The information capacity of working memory, i.e., the number of patterns a human can recall without error in order is finite and usually not very large. However, one can successfully recollect more informative sequences by creating groups of simple elements (engrams), i.e., *blocks*, often called *chunks*. Such a hierarchy can be further expanded to produce blocks from blocks (superchunks), etc. A good example is a written text where a block is a sentence and a superblock is a paragraph. An overview of other heteroclinic networks is presented in Table. 2.

2.4.2 Modes of multimodal brain networks. The playwright George Bernard Shaw once noted: ''Imagination is the beginning of creation. You imagine what you desire, you will what you imagine, and at last, you create what you will.'' Strikingly, the modern model of creativity in neurobiology is perfectly consistent with this maxim even 75 years after it was penned. When we think about creativity, arts often come to mind. Most people agree that all writers, artists, and actors are creative.

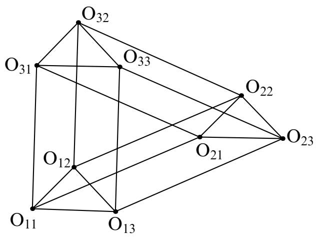
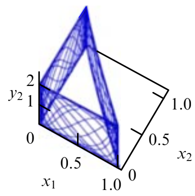
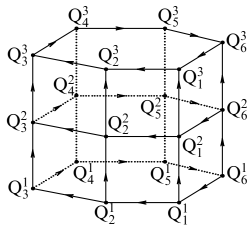
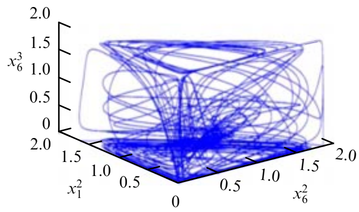
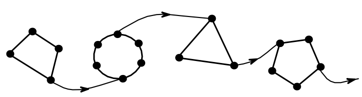
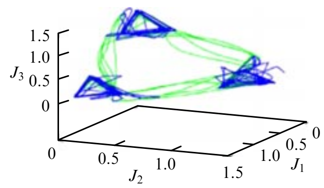
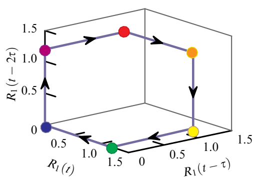
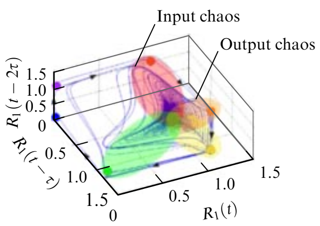
But where do creative ideas come from, and what makes some people more creative than others? Contrary to the romantic notion of a purely spontaneous process, growing evidence from experiments in psychology and neurophysiology indicates that creativity requires a cognitive effort, partly to overcome distractions and the 'stickiness' of prior knowledge (just remember that many people tend to first of all choose a stereotype approach when asked to create or develop something). In light of these results, general creative thinking can be viewed as a dynamic interaction between autobiographical memory and control systems. Without memory, our mind would be a blank slate not conducive to creativity, which requires knowledge and experience. But without mental control, we would not be able to push our thinking in new directions and would remain with what we already know.

Here, it is necessary to make a digression concerning the working memory, which is able to temporarily store and then reproduce sequences of information elements or blocks. They may be poetic images in a stanza, intermediate thoughts that form a conceptual chain of mathematical proof, etc. Naturally, for a creative process to be effective, the original order of the images in the sequence must be preserved when the information sequence is reproduced. This order is maintained by the coupling matrix of the corresponding heteroclinic channel formed in the process of memorizing (learning). The strength of the inhibitory bonds connecting metastable states is limited from above by their self-inhibition and cannot be too large. Therefore, winnerless competition ensures stability of only relatively short sequences [37].

Creative thinking is supported in part by the natural ability of humans to envision the future, i.e., anticipate events that have not yet occurred. When planning an event, from supper to an upcoming vacation, one usually relies on the imagination to see what the future might look like. Curiously, the same region of the brain (hippocampus) which enables people to imagine the future is involved in remembering the past. The hippocampus is a seahorse-shaped structure embedded deep in the parietal lobe of the brain. It plays a key role in bringing together details of individual experience, including people, places, objects, and actions, to accurately recreate past events and impressively design possible future events. Early studies involving patients with amnesia provided clear evidence of the role of the hippocampus in remembering and imagining; they showed that patients presenting with hippocampal lesions experience problems with both. Since then, experimenters have used fMRI to study how the brain remembers and imagines.

2.4.3 Wandering thoughts. For remembering past experiences and envisioning the future, a branched network of the cortex

Table 2. Heteroclinic networks in phase space.

Thinking dynamics that can be explained by heteroclinic networks			
Cognitive phenomena	Heteroclinic diagram	Phase portrait	Literature
Heteroclinic synchronization			[28, 29]
Binding of modalities			[30, 31]
Chunking			[32]
Chimeras			[33, 34]

mentioned in the foregoing (the default mode network) is of special importance. This structure got its name from early brain imaging studies that demonstrated that the areas it connects, viz. the medial prefrontal cortex, posterior cingulate gyrus, bilateral inferior parietal lobes, and middle temporal lobes, tend to be activated 'by default' when study subjects relax and the tomograph no longer monitors the performance of a specific task, leaving the person alone with their spontaneous thoughts, which are sometimes called wandering thoughts; usually, they are memories of the past or anticipations of the future. Both involve memory and imagination, ensuring a flexible recombination of details, such as people, places, and events encountered by the participants in the study. This phenomenon is not infrequently referred to as constructive episodic simulation. The involvement of the hippocampus is of primary importance. The flexible nature of episodic perception seems to be especially useful for creative thinking.

Can creativity be improved, and if so, how? This question has long attracted the attention of researchers. It was shown that neuroscience tools can be used to predict people's ability to think creatively based on the strength of their brain network connections [38]. But it is not yet clear how these connections can be strengthened.

2.5 Role of drugs in creativity management

Clinical trials are currently underway using psychedelics. Such studies were resumed after they had been completely discontinued in the 1960s. One of the many advantages of these investigations is that they provide a deeper insight into consciousness.

In 2016, researchers at the Imperial College, London, were the first to use fMRI technology to elucidate how LSD (from German Lysergsaurediethylamid) alters brain function. One of the main findings was that LSD somewhat disorganizes activity of the cerebral cortex and thereby allows the brain to operate more freely, i.e., with a smaller number of restrictions. It turns out that psychedelics promote communication among different parts of the brain which only weakly interact under normal conditions; on the contrary, they impair communication between actively interacting regions. This fact relates to the 'dissolution of one's own self,' described by many authors as the loss of the normal sense of 'self.' Instead, there is often a feeling of connectedness with other people and with nature, i.e., broadly speaking, with the outside world.

"LSD blurs the line between the experience of our own self and others during social interactions. Using MRI, we were able to show that this happens because the drug affects those parts of the brain that are responsible for the work of self-consciousness. The same shifts in cellular activity changed the way the volunteers communicated with others," writes Katrin Preller, a neurophysiologist affiliated with the University of Zurich (Switzerland).

LSD, lysergic acid diethylamide, or simply 'acid,' was accidentally discovered by the famous chemist Albert Hoffmann in 1938 during experiments with the spores of the parasitic ergot fungus. LSD had been originally intended to be used for the treatment of schizophrenia, but the acid rapidly became a popular psychedelic drug among young people in the 1960s. In recent years, as K Preller notes, a renewed interest in LSD and other psychedelic drugs has arisen among physicians and neurophysiologists. By observing how these drugs act on the brain of volunteers, scientists are trying to understand how exactly they change the work of the psyche and consciousness in order to assess the limits of their use as pharmaceutical products.

Convergent classical hallucinogens or psychedelics induce an altered state of consciousness characterized by changes in mood, sensory perception, thinking, and a sense of individuality. In other words, psychedelics provide a unique opportunity to explore the neuropharmacological and mechanistic foundations of perception, thinking, and consciousness. Naturally, interest in psychedelic compounds continues to grow, since they open up remarkable prospects for understanding neural state fluctuations and have great potential for clinical applications.

3. Interaction of modalities

3.1 Multimodality simultaneously extends the possibilities of creative thinking and the capacity of autobiographical memory

When studying cognitive processes in the human brain, psychologists usually distinguish three types of storage systems for information coming in from outside or generated by the brain itself: sensory memory, short-term or operative memory, and long-term or life-long memory. The capacity of sensory memory, i.e., the number of units of information that it is able to capture, is practically unlimited. But this memory keeps copies of what a person saw, heard, or felt for a very short time, from 0.5 to 2 seconds.

By focusing attention, part of the information from sensory memory can be transferred to the operative memory, where its lifetime is now on the order of 1 minute. New information generated by the brain itself in the process of thinking also goes into operative memory.

If a piece of information stored in the short-term memory is regarded by the brain as important, it is sent into long-term memory. This memory is static, i.e., information is once and for all 'carved in stone' there, fixed on a 'magnetic' disk, or in genes. In contrast, operative memory is a dynamic phenomenon. Information is represented by a time-varying waveform, the order of excitation of selected neuronal groups, etc. Such 'temporal' information is stored in feedback neural circuits, which ensures its reverberation. Biological mechanisms involved in the storage of dynamic information are very interesting, but unrelated to the mechanisms responsible for the limiting capacity of operative memory. Their consideration is beyond the scope of this review.

The amount of accessible operative memory is usually insufficient. Everyone who happened to ask about the way to the hotel in an unfamiliar city knows how easy it is to forget about halfway where to go further, to the left or to the right. Likewise, we are often unable to enter a phone number in the address book without confusing the right sequence of the digits, etc.

In 1956, J Miller found out in experiments with acoustic signals that the capacity of operative memory in humans is close to seven. The same result (number 7) was obtained in experiments on memorizing visual sequences, in attempts to reproduce a phrase containing more than seven linguistic units, and in many other studies and real-life situations. Truly magic!

Let us try and give a rational explanation for the specialness of this number with respect to operative memory. First of all, let us agree that the capacity of the memory is not the number of information units sent to it but their number retrieved from the memory in the correct time order (critical for both remembering the route and saving the phone number). In other words, when the operative memory cooperates with the brain centers that sequentially utilize the stored information to perform some cognitive or behavioral functions, the information units must go to the 'user' in a certain sequence.

Suppose that one wants to utter a newly invented phrase, e.g., "Our desires are destiny, intentions are more important than luck." The phrase contains ten words, and its meaning is determined by their serial number in the chain. The reproduction of one word is associated with the activation of a group of brain neurons (cluster) responsible for its storage. To avoid premature emergence of other words breaking the order, the activity of the corresponding clusters must be suppressed by inhibitory bonds between clusters. Only then will the reproduction of the phrase be stable and the meaning of the utterance preserved.

A mathematical analysis of the stability conditions for similar dynamic chains with competing elements (given that it is a winnerless competition [24]) showed that the reproduction is not disturbed if the strength of the inhibitory connections between the clusters increases exponentially (!) with increasing number of information elements in the operative memory. In other words, if the reproduction of a sequence with 7 or 8 information units requires an inhibitory bond strength on the order of 15 (in relative units), a bond strength of around 50 or 200 units is needed to reproduce 10 or 13 elements, respectively, i.e., it must be much greater than the 'magic' number, which is wholly unrealistic from the biological point of view (see below).

Psychologists and psychiatrists are well aware that the capacity of the short-term or operative memory depends on the level of intelligence. To confirm this, Matzel and coworkers from Rutgers University, USA, conducted experiments using a large group of mice ($n = 60$) as a model. It turned out that animals with a recent experience of passing through one maze moved to another of a similar design much faster than untrained mice. Other aspects of intelligence have also been tested. The results confirmed that intellectual exercises increasing the capacity of operative memory (without the involvement of long-term memory) enhance cognitive abilities.

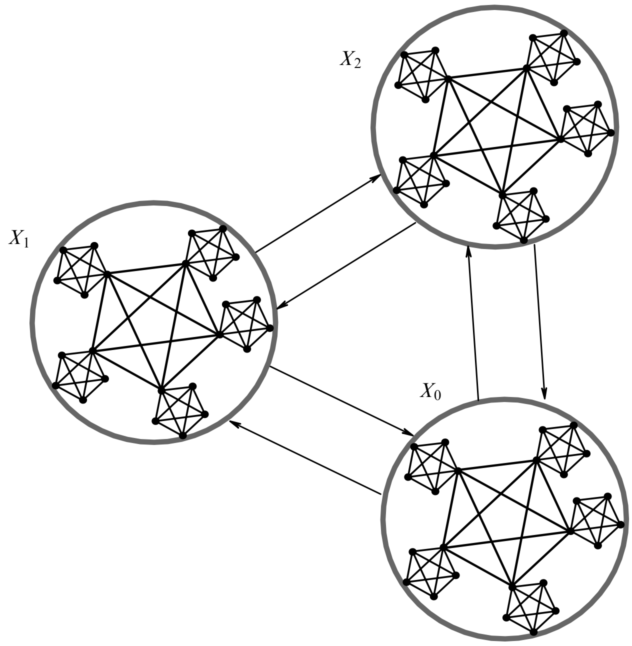


Figure 9. Hierarchical organization of large-scale brain networks involved in the creative process.

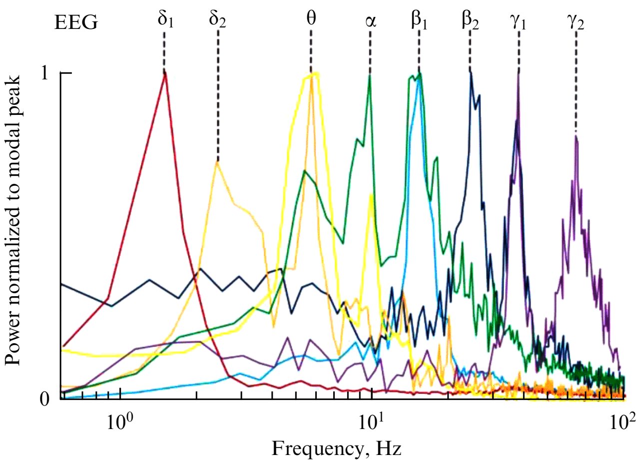


Figure 10. (Color online.) Several modal peak frequencies of constant rhythms generated in an isolated neocortex *in vitro*. All rhythms were generated in secondary somatosensory (parietal) cortical sections supported in artificial cerebrospinal fluid (aCSF). The rhythms were recorded as local field potentials (LFPs), and the resulting spectra (from 60-s data periods) were plotted with powers normalized to the modal peak. (Modified from [40].)

It should be emphasized that, as J Miller noted, the magic number seven appears when only one-sided or one-dimensional information, e.g. acoustic, visual, or tactile, is handled. However, the capacity of operative memory may increase significantly when factors related to interaction among these forms of information — and even more so to the association of, say, a text and music — stored in the long-term memory operate. For example, if the phrase cited above is associated with the melody of a song (one of Okudzhava's songs will do), then the operative memory is quite capable of reproducing the full stanza: "Our desires are destiny. Intentions are more important than luck. Like shooting at targets rushing, the

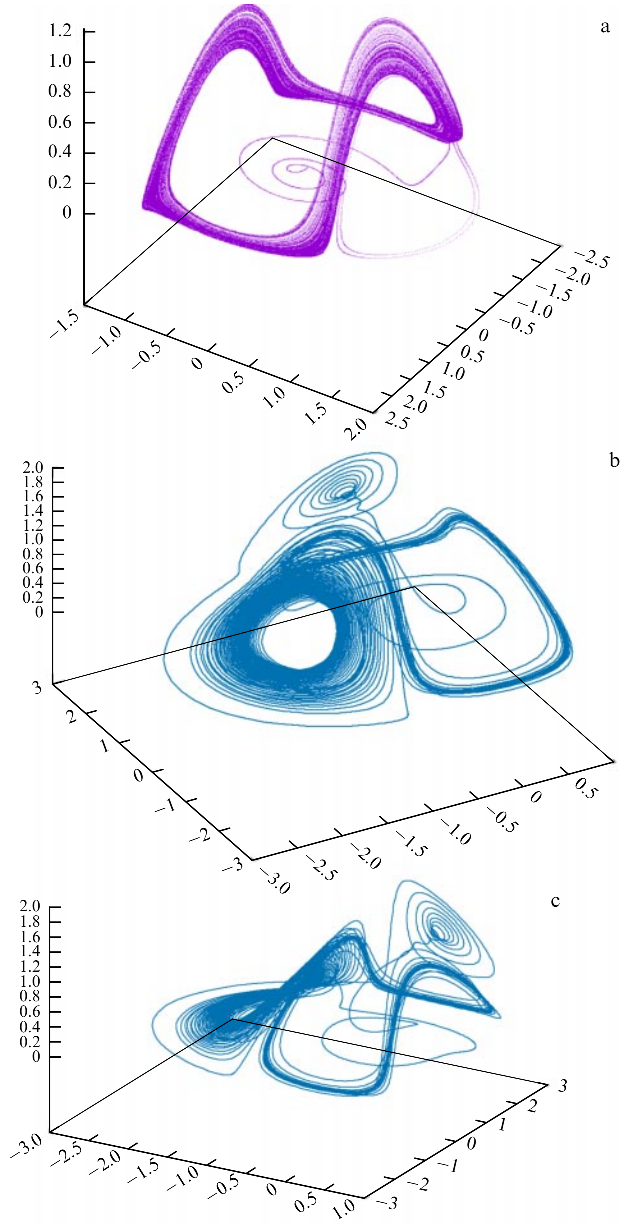


Figure 11. (Color online.) RF attractor for parameter values $\gamma = 0.1$, $v = 0.2715$ (a) and $\gamma = 0.1$, $v = 0.14$ (b, c).

unequal struggle with oneself is marked with the seal of chance." There are 27 words instead of 10.

The capacity of operative memory differs in patients with various brain diseases. For example, the connections between brain regions in dyslexia are weak, and the capacity of the operative memory is significantly lower than the average. In autism, on the other hand, the strength of connections and their number can be much greater. This accounts for the ability of certain subjects with autism to reproduce a given sequence of as many as a hundred random numbers. An amazing phenomenon was demonstrated in October 2009 by the autistic artist Stefan Viltmer. He proved capable of reproducing in pencil, structure by structure, Rockefeller Center, the Empire State Building, nearby skyscrapers, stadiums, and harbors in Manhattan on a five-meter panel after a 20-min helicopter flight over New York. Interestingly, Viltmer listened to familiar musical compositions reproduced sequentially while he was viewing the panorama.

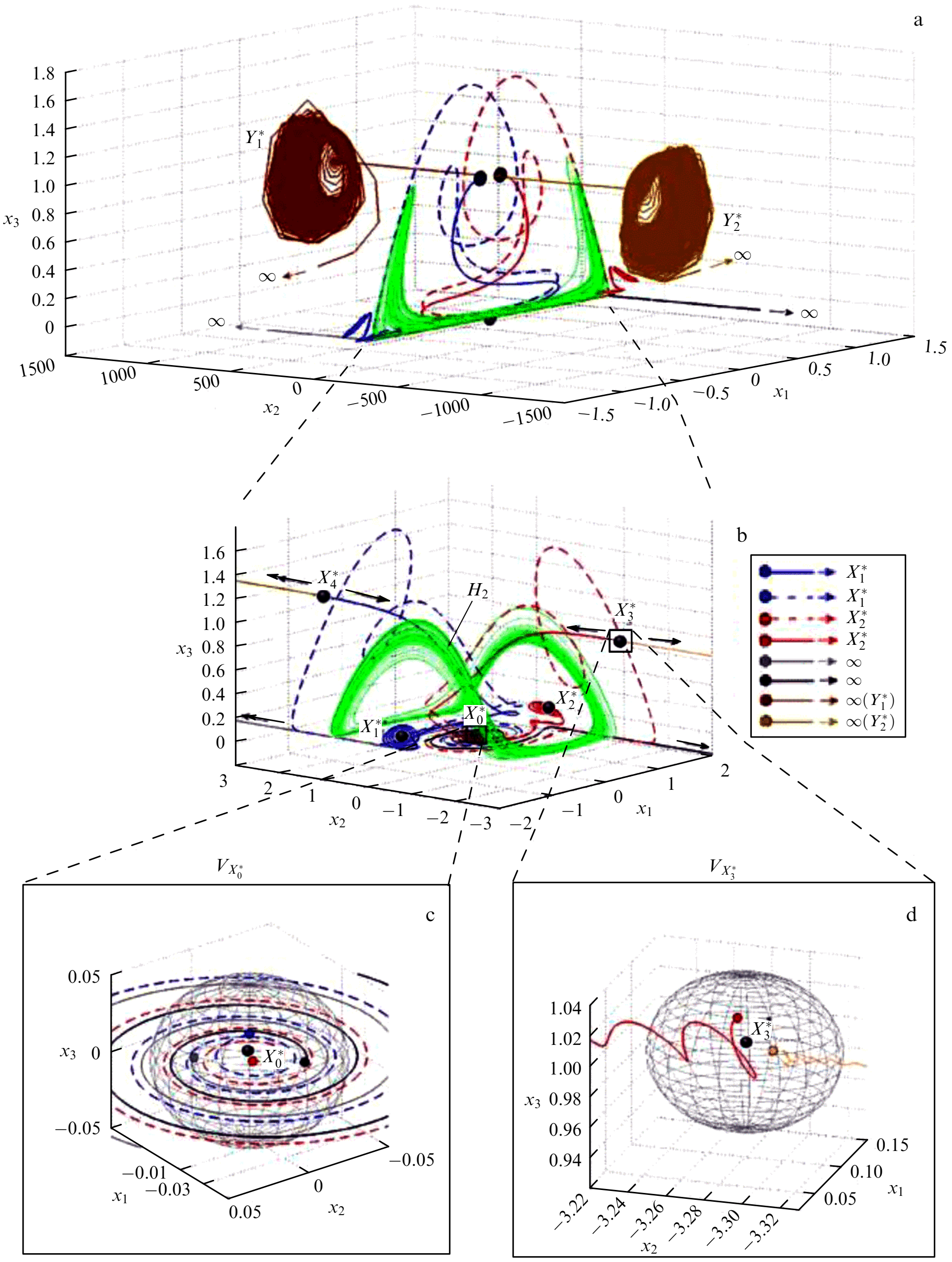


Figure 12. (Color online.) Hidden chaotic attractor. Hidden attractor is highlighted in green with part of the trajectories going away from it; remaining trajectories tend to stable equilibria. (Modified from [44].)

3.2 Resonant interaction of modalities

As mentioned earlier, creative thinking involves the activity of at least three key brain networks which cooperate or sequentially suppress each other during their joint work. Such interaction throughout the creative process can be either regular or chaotic in time. The speed of the switching also plays a role in the creative process, which employs the large amount of possible combinations in the switching among these cognitive networks and, in particular, between the decision-oriented and execution networks.

Everything depends on the strength and architectonics of connections between the above key networks. The coordination network is responsible for the identification of a network most suitable for prompt processing of ideas and switching to another network as and when necessary. For example, to compose a new guitar solo, one has first of all to increase the activity of the imagination network in order to induce enhanced emotional activity and run the feedback needed to create a new melody. However, as soon as creative zeal arises, the coordination network shifts activity to the executive attention network to create a working memory of the transitional melody and focus on coding the music into a memorable sequence.

These qualitative representations can be transformed into a mathematical theory. If one focuses on the formulation of principles and finding new dynamic images, it is possible to imagine the specificity of creative thinking at the phenomenological level of interaction of the spatio-temporal excitation modes in the cerebral cortex that generally modulate each other's activity in an excitable neural environment.

We schematize the model to consider three large-scale networks, viz. attention, memory, and silence (free thinking) networks, which interact with each other through mutual inhibitory connections and thereby create a global 'creative network' (Fig. 9). For each of the generating networks, EEG reveals a specific brain rhythm synchronization band. Let the frequency corresponding to the maximum activity of the attention network be designated as ω_0, that of the memory network as ω_1, and that of the rest network as ω_2. In the model kinetic equations, the respective intensities are represented by the variables $X_0(t)$, $X_1(t)$, and $X_2(t)$, $X_j(t) \approx A_j(t) \exp[i(\omega_j t + \varphi_j)]$. When the resonance condition $2\omega_0 = \omega_1 + \omega_2 + \Delta\omega$ is satisfied, the model equations can be written, after some simplifications, in the canonical form as the Rabinovich–Fabrikant equations (RF)

$$\dot{x} = y(z-1+x^2)+\gamma x\,,$$
$$\dot{y} = x(3z+1-x^2)+\gamma y\,,$$
$$\dot{z} = -2z(v+xy)\,.$$

The RF model [41] illustrates the birth of various chaotic regimes of three resonantly coupled modalities (see Fig. 9). The phase space of this model is organized in an extremely complicated manner due to the presence of many metastable states, i.e., saddle equilibria and unstable cycles. The richness of the dynamics is illustrated by Figs 10–12. A change in the initial conditions leads to different chaotic structures even if the parameters are preserved (see Figs 11 and 12). Here, $\gamma = 0.1$ and $v = 0.2715$.

Considering such models with reference to the comprehension of brain creative activity, it should be noted that they cannot be overly simple; for the richness of the dynamics, it is necessary to take account of the mutual influence of phase relations and intensities of different modalities. In the RF model, this is ensured by cubic nonlinearity, due to which 'virtual' saddles appear [42, 43].

3.3 Collective creativity

All of us, especially people in the creative professions, value the opinion of others. It creates a mood which often determines the result when it comes to collective creativity. In the formal language, it means that the level of functional connections among the creative networks of the brain becomes dependent on the parameters, the level and sign of self-esteem, as well as the mutual evaluation of the partners. It can contribute to both the collective creative upsurge and the loss of collective inspiration or courage. Below, we shall consider the interaction of creative objects when modeling jazz improvisation.

3.4 We are from jazz

When modeling the collective improvisation of a group of soloists (Fig. 13), the dynamics of the creative ensemble was described by six families of canonical Lotka–Volterra equations. A shift from one soloist to another was performed by an additional modality (attention). The number of components in each family was determined by the number of different themes. This is the number of metastable states for each heteroclinic channel. To assess the level of improvisation, the Kaplan–York dimension (KYD) was used (see [25]). The results are shown in Fig. 14.

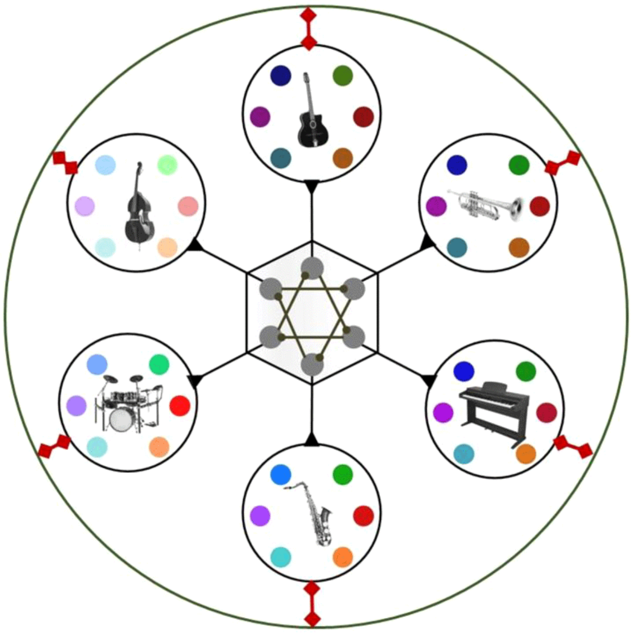


Figure 13. (Color online.) Collective creativity of a group of soloists (hexagonal organization) is realized due to the fact that they take turns soloing in the presence of each other and, therefore, under the influence of each other. (Modified from [3].)

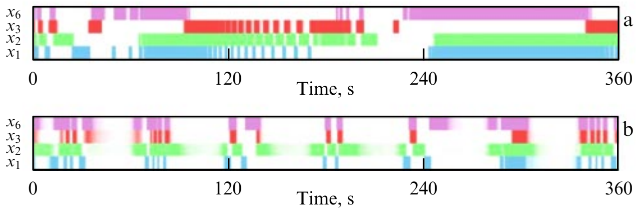


Figure 14. (Color online.) (a) Jazz musician plays independently. Originality parameter KYD = 4.05. (b) The same soloist in a chain of soloists successively replacing each other; creativity parameter KYD = 4.63. (Modified from [3].)

4. Discussion

At the very beginning of our discussion about the creative thinking processes (see the epigraph), the question arose: "Do they affect the subconscious mind or is there no creation without consciousness?" We now know the answer. In short, some parts of the creative process *can be subconscious*. Music, paintings, sculptures, poems, and other artwork produced as a result of creative processes enter into a dialogue with the viewer. They are outcomes of personal experience for the artist while he or she is creating them and for the viewer enjoying them. In this context, the subconscious mind naturally plays a fundamental role.

Thus, not only the process of creation itself but also its dynamic perception is largely subconscious. This should be regarded as an experimental fact to be taken into account in building a mathematical model of a seemingly unformalizable existential creative processes. Here are two examples: paintings by Jackson Pollock and the poetry of O E Mandelstam.

4.1 Pollock — an existentialist painter

Existence precedes essence.
Jean-Paul Sartre

Pollock did not copy nature: he invented a dynamic technique to generate images that can exist in nature. Walking around

Figure 15. (Color online.) Pollock's painting "Alchemy" (1947).

the perimeter of a huge canvases spread on the floor of his barn, Pollock covered them with drops of paint in one or more layers (Fig. 15).

Although this new technique (dripping) was recognized as a major advance in the evolution of modern art, the significance and emotional impact of the patterns created proved ambiguous, and Pollock often destroyed his paintings. An analysis of Pollock's images shows that, first, their geometric structure is composed of fractals reminiscent of pictures of nature, and, second, the dimension of the fractals increased with the artist's experience [45].

4.2 Mandelstam — a poet of metaphor

Mandelstam was a poet of metaphor, i.e., the transformation of different modalities of being: natural and portrait, visual and musical, etc., not with the aim of conveying a certain message to the reader, but of creating a feeling in them akin to that experienced during hypnosis. A wonderful example is his *Eight-Line Stanzas*. In his essay on poetry "Conversation about Dante," Mandelstam explains that the meaningful component in his works is not a word or a sentence, but a certain image that is not rigidly connected with the form: "In poetry only the executory understanding has any importance, and not the passive, the reproducing, the paraphrasing understanding." According to literary scholars,[1] the mystery of Mandelstam's poetic speech, its magic, lies in the fact that he communicates with the reader relying on the subconscious more than on consciousness. The hypnotic nature of his poetry seems obvious to them.

Here, we are interested in neurophysiological mechanisms and the analysis of the dynamics of the corresponding creative activity of the brain, i.e., the interaction of conscious and unconscious subprocesses.

4.3 Heteroclinic chimeras are one of the possible mathematical images of the creative process

Recent studies (EEG, fMRI) have shown that, from the standpoint of brain anatomy, the same neural networks are responsible for its conscious and subconscious activity. They are (1) the default mode network, (2) the executive attention network, and (3) the silence network (Fig. 16).

[1] See, for example: A V Khlystova *The Magic of O Mandelstam's Poetics* (Moscow: RUDN, 2009); E Yu Glazova *Metamorphosis of the Word. Osip Mandelstam's Theoretical Thought* (Moscow: YaSK Publishing House, 2019); E Glazova, M Glazova *Prompted by Dante. On the Poetics and Poetry of Mandelstam* (Kiev: Dukh i Litera, 2012). (*Editor's note.*)

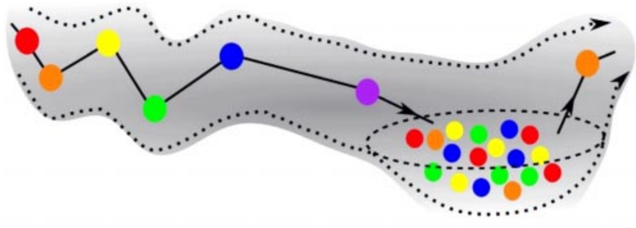


Figure 16. (Color online.) Chimera is a large-particle representation of a complex trajectory (see, for example, review [46]). After a regular segment, the trajectory enters a chaotic ravine, then exits from there switching to a stable heteroclinic channel. (Modified from [33].)

5. Conclusion

To conclude this article, it is necessary to mention the problem of interaction between human and computer creativity, i.e., between the brain and artificial intelligence (AI) [47–49]. In this area, one encounters extremely attractive and at the same time perplexing problems, such as how to control AI creativity. In particular, which observables, the temporal dynamics of which contains information about inconvenient (dangerous) intentions for humans, should be chosen. One can never be sure that creative self-learning will not enable AI to break the hierarchy and organize itself to communicate with the brain 'on an equal footing.'

Here, of course, the focus should be on the architecture of an integrated creative network, which must include networks of attention, critical appraisal, joint decision-making under uncertainty, and collective episodic and semantic memory. Naturally, the dynamics of the corresponding process must be stable. At present, such studies are just beginning to gain momentum [50–54].

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