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METHODOLOGICAL NOTES

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Application of the modified Duguay method for measuring the Lorentz contraction of a moving body length

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Abstract. **According to Lorentz transformations, for a stationary observer, time in a moving inertial reference frame slows down, while the linear dimensions are reduced. While the first effect was observed more than 80 years ago, the second one has not been directly observed so far. The modified Duguay method is proposed in this paper for measuring the Lorentz contraction of a moving body length using the propagation of light pulses in an optical liquid medium. Three variants of the measurement scheme are considered: with a 'light square' in an optical medium, with a 'light ruler' in two optical media with different refractive indices, and with two relativistic electron bunches in a vacuum. It is shown that the classical effect of compression of spatial intervals between light pulses in an optical medium, which was not considered earlier, considerably reduces the measurement accuracy. It is also shown that the distortion of the sides of a light square oriented orthogonal to the movement direction caused by the different delays of light from different parts of a moving body also reduces the measurement accuracy of the light square method.**

Keywords: Lorentz transformations, relativistic length contraction, light pulses

1. Introduction

It is known that Einstein's special theory of relativity (STR) [1] predicts that, due to Lorentz transformations (LTs) for an observer at rest in inertial reference frame (IRF) K, the linear dimensions of bodies in IRF K' moving at velocity v with respect to the observer are reduced by a factor of γ along the movement direction, while the time in IRF K' slows down by a factor of γ (where $\gamma = (1 - v^2/c^2)^{-1/2}$ is the so-called gamma factor or the Lorentz factor, and c is the speed of light in a vacuum).

The Lorentz time dilation by the Lorentz factor γ was observed only in the late 1930s–mid-1940s in experimental studies [2–4] of the transverse Doppler effect (TDE) predicted in [1] in 1905 and measurements of an increase in the lifetime of muons [5–7]. At present, the TDE is manifested in various precision physical measurements and often introduces fundamentally unremovable errors. For example, in gas lasers with a nonlinearly absorbing cell (NAC) used as quantum frequency standards [8–11], the TDE appearing due to Maxwell's velocity distribution for the thermal motion of molecules in absorbing gas in the direction perpendicular to the laser beam inside the NAC fundamentally restricts the width of ultra narrow absorption resonances. Because of the thermal dispersion of absolute velocities of molecules, each of the molecules will have a resonance frequency dependent on its velocity. As a result, the absorption line of an ensemble of particles will have additional broadening [12]. The TDE also causes a shift of the resonance center [13]. Already in the mid-1970s, the frequency stability and reproducibility of lasers with a methane (CH_4) NAC were of the order of 10^{-15} and 10^{-14}, respectively, of the laser frequency [8]. The TDE can shift this frequency to $10^{-12} - 10^{-13}$ [13]. The TDE influence can be reduced by cooling the absorbing gas or replacing methane by a gas of heavier molecules, in particular, OsO_4.

However, the Lorentz contraction of a length by γ has not been directly observed so far.

It is assumed that this effect is confirmed by Michelson–Morley (MM) experiments [14, 15]. Indeed, results obtained in [14, 15] are excellently explained by the Lorentz contraction of the length. However, it was pointed out in [16] that numerous 'alternative explanations' of the results of MM experiments exist. The Lorentz contraction of a length is also confirmed by the existence of the relativistic velocity addition law [16], which has been verified many times in measurements

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in accelerators. However, in this case, numerous 'alternative explanations' also exist. One of them (approximate) was proposed back in 1818 by Fresnel [17] — the so-called theory of a luminiferous aether partially carried away by a moving medium. The list of such examples could go on.

Of course, both alternative explanations mentioned above and the alternative explanations of a number of other STR effects not based on the LT application are incorrect. In particular, as shown in [16], they cannot explain all of the existing classical optical experiments. Nevertheless, it is important to measure directly the Lorentz contraction of a length.

Note that, from various observations of astronomical objects moving at near light velocities, because of the Terrell–Penrose effect [18, 19] (considered in Section 2), it is also impossible to make a conclusion about the existence of the Lorentz contraction of a length.

The aim of this paper is to propose a few variants of simple direct measurements of the Lorentz contraction of a moving object length using some modification of the known Duguay experiments [20–22] on photographing a 'light dumbbell' consisting of two picosecond light pulses.

2. Duguay's experiments

In 1969–1971, Duguay performed his famous experiments [20–22], which could have led to, but unfortunately failed to lead to, registration of the Lorentz contraction of the linear dimensions of a moving object in the movement direction. By omitting the technical details of experiments [20–22], note that a picosecond laser pulse propagated through an optical cell with water. A small amount of milk added to the water ensured the observation of the laser beam by its scattering in the cell and its being photographed using an ultrafast Kerr switch, described in detail in [23]. Because the water cell was very short (~ 15 cm), light pulses had no time to spread due to chromatic dispersion of the water.

In the USSR at that time, Duguay's experiments attracted great interest. His study [22] was translated into Russian and published in *Physics–Uspekhi*. Ugarov wrote three articles about these experiments [24–26] ([24] was written together with Smorodinskii). Malov's paper [27] describing Duguay's experiments was even published in the journal *Kvant* for school children.

However, Duguay [20–22] measured only a purely classical light delay effect: two simultaneously emitted light pulses in a cell propagated (it was one light pulse split on a mirror, the so-called light dumbbell in Duguay's terminology), which were located in a plane perpendicular to the path of the light pulses. Due to the delay of light, the image of the light pulse on a photographic film that was further from the photographic camera was delayed compared to the light pulse that was closer to the camera. During measurements, Duguay changed the angular orientation of the light dumbbell in a plane orthogonal to the path of the light pulses, thereby varying the delay of light and thus the delay of the remote light pulse.

If Duguay had transmitted through a cell a so-called light square — four light pulses in a plane orthogonal to the optical axis of the photographic camera — then the sides of the light square would have been distorted differently: the sides of the square oriented in the movement direction should experience the Lorentz contraction, while the sides of the square orthogonal to this direction should not. If Duguay had transmitted through a cell a light cube consisting of eight light pulses, he could have detected the Terrell–Penrose effect [18, 19], according to which, for the case of a cube moving at a relativistic velocity, an observer at rest sees a reversed cube image.

Unfortunately, Duguay did not do either of these things. To understand why this happened, we present briefly his statements.

"A light packet moves at the relativistic velocity, however, it does not experience the Lorentz contraction if a laser emitting it is at rest. Therefore, the image of the formed light pulse will be affected only by the wave delay effect" [22].

"This shift effect can be explained by the wave propagation effect without using the special theory of relativity; this does not contradict Terrell's theory because the latter rigorously deals with material objects" [21].

It follows from Duguay's first statement that he probably was a supporter of the ballistic hypothesis by Ritz [28, 29], according to which the speed of light is added to the velocity of a radiation source. In reality, the movement of a laser could change the laser light frequency only due to the Doppler effect. Duguay's second statement suggests that during some of his measurements he concluded that effects of the contraction of dimensions of a material body and the light image of the same body during its relativistic movement in an optical medium are substantially different. This question will be considered in detail in Section 4.

To detect the Lorentz length contraction effect, it was proposed in [24] to photograph in one frame not only a moving light ruler but also simultaneously a material ruler at rest whose dimensions were not reduced.

Modern lasers emitting femtosecond (and even attosecond) pulses [30–34] provide such measurements with a considerably better accuracy.

We will consider in Section 4 several possible methods for detecting the Lorentz contraction of a length and will also point out some sources of regular measurement error.

3. Classical effect of the compression of interval lengths between moving light pulses in an optical medium. A difference between material and light rulers. The *n* factor

If a laser emits two light pulses with the time interval τ, then in an optical medium with the refractive index n the interval length between moving light pulses becomes n times shorter independently of the Lorentz length contraction by γ. This phenomenon is very simply explained: up to the moment the next light pulse enters at the input of the optical medium, the previous pulse has time to propagate in the medium over a smaller distance than in a vacuum. Similarly to the term 'γ factor', we will call the contraction of the length of the interval between moving light pulses by n times the n factor.

Thus, the length of a light ruler oriented along the movement direction is contracted by $n\gamma$ times. Because $v = c/n$ in an optical medium, $\gamma = 1/\sqrt{1 - 1/n^2}$, and the real length between light pulses observed in IRF K is

$$L_{\mathrm{contr}}^{\mathrm{real}} = \frac{L}{n\gamma} = \frac{L\sqrt{1 - 1/n^2}}{n} \, . \tag{1}$$

Expression (1) should be taken into account by processing photographs of a light square or a light cube.

Consider a simple mechanical analogy of the classical effect of a decrease in the length between light pulses in an optical medium. Let a great number of streetcars move at the same velocity of 40 km h^{-1} with an interval of 4 m over the horizontal part of the streetcar rails. When some of these streetcars begin to move over a rising part of the route, their velocity decreases to 20 km h^{-1} and the interval between them decreases to 2 m. This occurs because, at the moment when the next streetcar begins to move over the rise, the previous streetcar has time to cover in the rise half the distance on the horizontal part of the route.

This analogy allows one to explain simply the difference between light and material rulers. If the streetcars are connected to each other by steel hitches 4 m in length, then a streetcar beginning to move over the rise cannot lose its velocity, because the next streetcars will push it via the hitches. In this case, the interval between streetcars is also preserved, because it is fixed by the hitches.

Thus, the analog of the relativistic movement of a light ruler in an optical medium is a train consisting of independent streetcars, while the analog of a material ruler is a train of streetcars connected with hitches. Therefore, for a relativistic material ruler, only the Lorentz contraction of a length will be manifested. Unfortunately, a material ruler, unlike a light ruler, is in fact impossible to accelerate to relativistic velocities.

4. Methods for detecting the Lorentz contraction of a length

Modified Duguay method with a light square. Because the sides of a square oriented in the movement direction are contracted by $n\gamma$, while the other two sides are not contracted, then the latter two sides, which are transferred from moving IRF K' to IRF K at rest without changing, are the length standard of the square side L. The first two square sides in IRF K will have length $L_{\text{contr}} = L/n\gamma$. However, here, phenomena take place that do not allow us to use automatically the equality $\gamma = L/L_{\text{contr}}$.

As shown in [35], an observer in IRF K sees this side of the square turned by α and sees that the rod length increased (see Fig. 1 [35]). The expression for the angle α can be readily obtained from (20) in [35]:

$$\alpha = \arctan\left(\frac{v}{c}\right)\frac{\sin\theta}{1 - v/c\cos\theta}\,, \tag{2}$$

where θ is the angle between a camera and the light square velocity. Correspondingly, the length of this side of the light square increases by $1/\cos\alpha$ times. If $\theta = 90°$, then $\alpha = 0$, and the side of the light square oriented orthogonally to the movement direction does not 'turn' and its length does not increase. Obviously, the condition $\theta = 90°$ cannot be satisfied simultaneously for both sides. For example, if $\theta = 90° \pm 10°$, the slope will be $\alpha \simeq 2.5°$, while the elongation will be $\simeq 11\%$ of the initial length.

Thus, in processing measurements performed by the modified Duguay method with a light square, it is necessary to introduce corrections for the results of expressions (1) and (2).

Method of a light ruler moving in two different liquids. Expression (1) allows finding the length of a light ruler consisting of two pulses and oriented in the movement direction in IRF K. If we have two different liquids with different refractive indices n_1 and n_2, the ratio of the lengths $L^{\text{real}}_{\text{contr1}}$ and $L^{\text{real}}_{\text{contr2}}$ of light rulers observed in IRS K will be

$$\frac{L^{\text{real}}_{\text{contr1}}}{L^{\text{real}}_{\text{contr2}}} = \frac{n_2\sqrt{1 - 1/n_1^2}}{n_1\sqrt{1 - 1/n_2^2}}\,. \tag{3}$$

Among all the existing liquids, water has the lowest refractive index at room temperature ($n = 1.3330$), while carbon disulfide has the highest refractive index ($n = 1.6277$). By measuring lengths $L^{\text{real}}_{\text{contr1}}$ and $L^{\text{real}}_{\text{contr2}}$, we can obtain from (3) the experimental value of the Lorentz contraction of the light ruler length.

However, a serious problem exists in this case as well. According to expression (1), the real contraction of a light ruler in water is 0.4960, and for carbon disulfide, 0.4847. This is a very small difference, and the use of expression (3) will result in a very low accuracy of calculating the experimental value of the Lorentz contraction of the light ruler length. The reason is that, as the refractive index of a liquid increases, the Lorentz length contraction decreases, while the classical effect of decreasing a length between light pulse increases, and one effect partially compensates the other. Figure 1 shows the theoretical dependence of the light ruler length $L^{\text{real}}_{\text{contr}}$ observed in IRF K on the refractive index n. One can see from Fig. 1 that, in the range of refractive indices of existing liquids ($n = 1.33-1.63$), the value of $L^{\text{real}}_{\text{contr}}$ is in fact independent of n.

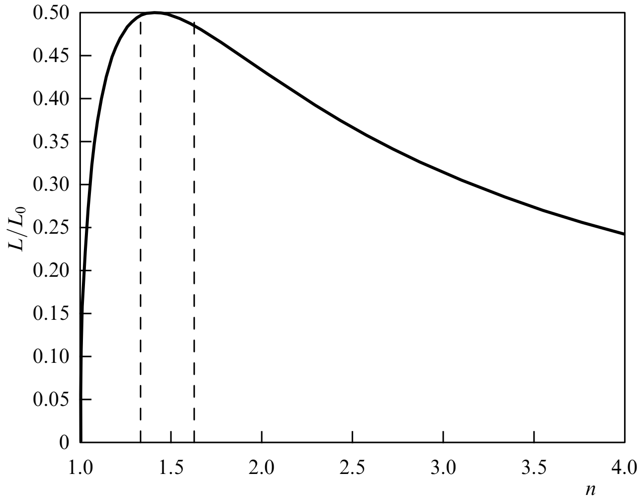

Figure 1. Dependence of the total action of Lorentz contraction of a length and classical effect of a decrease in length between light pulses in an optical medium on refractive index n. Vertical dashed lines show the range of n for actually existing liquids.

Note that all the above concerns a light square, more exactly, its two sides oriented in the movement direction because they are light rulers.

Method of a light ruler and a light square with pulses moving in gases. In principle, the method of a light ruler moving in two different gases could be used. The lowest refractive index at atmospheric pressure belongs to helium ($n = 1.000035$), and the highest, to xenon ($n = 1.000702$). In this case, the values of n factors weakly differ from 1 and do not affect the results of measurements. However, γ factors in this case are large: $\gamma \sim 120$ for helium and $\gamma \sim 30$ for xenon. Such a large contraction of a light ruler makes impossible accurate measurements, because the light ruler will be transformed into a point. Similarly, the light square will be transformed into a vertical light line.

Note that attempting to detect the Terrell–Penrose effect [18, 19] during the movement of eight light pulses (a light cube) in gas is also quite difficult because, for such large values of the γ factor, the light cube will turn almost $90°$, and an observer at rest will see a square image rather than the cube.

In addition, in gases, nonlinear effects appear already at low light intensities, and therefore the measurements in them are unpromising.

Method for detecting the Lorentz contraction of a length using relativistic electron bunches. Let two electron bunches exist at the output of a linear accelerator, which are moving by inertia in a vacuum tube at a relativistic or nonrelativistic velocity. Because the distance between bunches is small, about a few tens of centimeters, they have no time to 'spread.' Using special sensors, it is possible to determine the distance between bunches for both cases and, comparing these distances, to determine the Lorentz length contraction. Because bunches move in a vacuum, the classical effect of the decrease in length between bunches will not take place in principle.

Because the distance between bunches is small, the influence of the hypothetical effect of the anisotropy of the speed of light [36] (not exceeding 10^{-15} by recent estimates) can be disregarded. Because moving bunches are located during measurements exactly opposite sensors at rest, the influence of the effect of the nonsimultaneousness of events in different IRFs [16] on sensor responses can also be disregarded.

5. Conclusions

The question is quite often discussed as to whether the time dilation and the contraction of the linear dimensions of bodies in moving IRFs are real or not for an observer in an IRF at rest. Consider the classical Doppler effect using the example of experiments performed by Buys Ballot 175 years ago [37]. Two trumpeters on a moving railroad platform played the same note in turn without interruption. When the train was moving away, the pitch of the sound lowered and when the train was approaching, the pitch became higher. But what occurred 'in reality'? For an observer on the platform, the sound had a certain frequency, while, for an observer on the ground, the sound had a different frequency. All this occurred 'in reality.' During experiments [37], musicians and listeners interchanged their places, and, as expected, the Doppler effect proved to be commutative. The STR effects should be treated in the same way: in all IRFs, all physical phenomena occur identically, but, for other IRFs, the length and the time of events from the first IRF change according to the LTs.

The STR and LTs have been considered in thousands, if not tens of thousands, of scientific papers. However, the experimental study of the LT consequences is occuring very slowly, with a long delay. The example of detecting the relativistic time dilation performed in 1938–1947 is presented in Section 1. Let us present another example. Ritz proposed in 1908 the so-called ballistic hypothesis [28, 29], according to which the speed of light is added to the velocity of a radiation source. This hypothesis was proposed to explain within the framework of classical physics a number of problems appearing in physics at that time. In particular, the ballistic Ritz hypothesis explained MM experiments [14, 15] (see, for example, [38]). Many attempts were made to refute experimentally the Ritz hypothesis (these papers are cited in [39, 40]); however, either the accuracy of these studies was insufficient or objections were raised as to the experimental schemes used in them [16, 39]. Only in 2011 did Aleksandrov with coauthors show [41, 42] with a high accuracy in synchrotron experiments that the speed of light is independent of the radiation source velocity.

Finally, the main conclusions of this paper are:

(i) The modified Duguay method is proposed for measuring the Lorentz contraction of the length of a moving body using the propagation of optical pulses or electron bunches. Three variants of the measurement scheme are considered: with a light square in an optical medium, with a light ruler in two optical media with different refractive indices n, and with two relativistic electron bunches in a vacuum.

(ii) It is shown that the classical effect of compression of spatial intervals between light pulses in an optical medium with a refractive index (the n factor) considerably reduces the accuracy of measurements by the first two methods, because the dependence of the n factor on the refractive index n almost completely compensates the dependence of the γ factor on n.

(iii) It is shown that the distortion of the sides of a light square directed orthogonally to the movement direction caused by the differences in delays of light from different parts of a moving body also reduces the accuracy of measurements by the light square method.

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