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METHODOLOGICAL NOTES

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Measuring the inclusive cross section of e^+e^- annihilation into hadrons in the pre-asymptotic energy range

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Contents

Abstract. **Among the observed variety of the natural phenomena reflected in experimental results, there are just a few physical features offering a view of a given fragment of the mosaic of physical laws in its entirety. One such remarkable fundamental feature is R, the ratio of the inclusive cross section of electron–positron annihilation into hadrons to the cross section of muon pair production in the Born approximation. We discuss experiments on measuring the $R(s)$ ratio in the energy range 1.84–3.72 GeV.**

Keywords: measurement of R, e^+e^- annihilation, inclusive hadronic cross section

1. Introduction

Modern physical concepts regarding the laws of nature are based on the Standard Model, which describes the electromagnetic, weak, and strong interactions of elementary particles. Part and parcel of the Standard Model is quantum chromodynamics (QCD), currently a unique theory whose rightful claim to fame is the description of strong interactions. A milestone in the emergence of QCD was the simple and elegant quark model, first proposed in 1964 by Gell-Mann and Zweig [1, 2], who hypothesized that hadrons are not elementary objects but are made up of fundamental particles, quarks. The quark model took a while to be accepted. As noted by Richter in his Nobel lecture [3], "the situation that prevailed in the Summer of 1974 [was] vast confusion," and there were about two dozen models predicting the value of R, the ratio of the inclusive cross section of e^+e^- annihilation to the cross section of muon pair production in the Born approximation.

The quark model was triumphantly validated when the J/ψ resonance was discovered 45 years ago [4, 5]. This opened a new era in high-energy physics and gave impetus to the vigorous development of the theoretical and experimental physics of elementary particles. It is therefore entirely appropriate that the problem of measuring the inclusive hadron production cross section is still relevant. For instance, the precision measurement of R is decisive for determining the muon anomalous magnetic moment a_μ, the fine structure constant $\alpha(M_Z^2)$ in the range of the Z^0-boson peak [6–8], the strong coupling constant $\alpha_s(s)$ [9], and the heavy-quark masses [10–12]. Theoretical calculations of these quantities require the exact determination of the contribution of the vacuum polarization by hadrons, which can be obtained only in experiments on measuring R.

This paper is devoted to the problems of measuring the inclusive hadron production cross section in the process of e^+e^- annihilation in the energy range of 1.84–3.72 GeV. This range of energies is notoriously difficult for studies, because investigating the inclusive cross section requires having an adequate model for generating hadronic events, which is unavailable for that energy range. The QCD perturbation theory constructed similarly to what is done in quantum electrodynamics is only applicable in the domain of large transferred momenta or, in other words, only at short distances, where the coupling constant of the strong interaction α_s is small, which is certainly not the case for the energy range of interest. The option to determine R from the sum of all possible exclusive cross sections is not practically feasible either, because it requires studying several dozen hadronic processes already in the 2-GeV energy range, and the number of possible decays goes well into the hundreds as the energy increases.

In that energy range, numerous measurements of R have been made, starting in the 1970s [13–26]. The historical trajectory traced by experiments on measuring the cross

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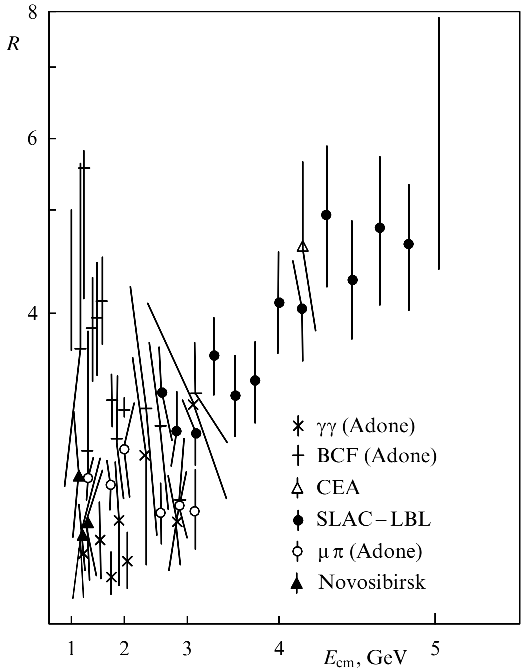

Figure 1. Results of experiments on measuring R depending on the center-of-mass energy $E_{\rm cm}$ (figure taken from the talk by Richter, "The ratio R as of July 1974," at a 1974 conference in London [27]). Adone: the electron–positron collider in operation in 1969–1993 at the National Institute for Nuclear Physics (Istituto Nazionale di Fisica Nucleare, INFN) (Italy); BCF: Bologna–CERN–Frascati; CEA: Cambridge Electron Accelerator; SLAC–LBL: Stanford Linear Accelerator Center–Lawrence Berkeley Laboratory; Novosibirsk: VEPP-2 (Colliding Electron–Positron Beams), one of the first electron–positron colliders built at the Institute for Nuclear Physics, Siberian Branch of the USSR Academy of Sciences (Novosibirsk) in 1965.

section of electron–positron annihilation into hadrons is well illustrated by Figs 1 and 2. Both show the results of experiments on determining the inclusive hadron cross section of e^+e^- annihilation; Fig. 1 was provided by Richter [27] at a 1974 conference prior to the discovery of J/ψ resonance, and Fig. 2 is given by the Particle Data Group (PDG) [28] as of 2018.

In the past decades of investigations of hadronic processes with colliding electron–positron beams, measuring the value of R has become a classic experiment. The measurement precision has dramatically improved, and the scope of research has increased manifoldly. Still, in determining the fundamental features of QCD mentioned above, perturbative QCD (pQCD) calculations are often used, because systematic uncertainties of experimental results have been quite large until recently.

Leaving the problem of resonance hadron production aside, the observed dependence of the inclusive cross section of the e^+e^- pair annihilation into hadrons is the most intuitive result ensuing from QCD, which by no means removes the hurdles to making precise experimental measurements of R, nor to deriving this feature of QCD from theoretical calculations of ever increasing complexity needed to improve the precision. The current level of systematic uncertainty in experiments on measuring R in the energy range of interest has come close to 2%. A further increase in precision is a complicated experimental task, and we briefly discuss some prospects of its solution in Section 3.

2. Annihilation of e^+e^- pairs into hadrons

2.1 The quantity *R*

Theoretical questions of the calculation of the inclusive hadron production cross section in e^+e^- annihilation process are surveyed, e.g., in [29, 30]. But in order to fully demonstrate the motivation for the experimental work on measuring R, it is useful to briefly discuss the fundamental theoretical concepts here.

In the lowest order of the perturbation theory, the amplitude of a $e^+ + e^- \to X$ process, where X is a hadronic state, has the form

$$\mathrm{i}\mathcal{M} = \frac{e}{q^2}\,\overline{v}(p_2)\gamma_\mu u(p_1)\langle X|J^\mu|0\rangle\,, \tag{1}$$

where p_1 and p_2 are the 4-momenta of the electron and positron, $u(p_1)$ and $\overline{v}(p_2)$ are bispinor amplitudes of their wave functions, $q = p_1 + p_2$, and J^μ is the electromagnetic hadron current, which can be represented as

$$J^\mu = (-\mathrm{i}e)\sum_f Q_f\overline{\psi}\gamma^\mu\psi\,. \tag{2}$$

Here, the sum is taken over the flavors of quarks, whose charges Q_f are expressed in units of $|e|$. The calculation then leads to a formula for the total cross section of hadron annihilation for nonpolarized beams:

$$\sigma^{e^+e^-\to\,\rm hadrons}(s) = -\frac{4\pi\alpha}{s\sqrt{1-4m_e^2/s}}\left(1+\frac{2m_e^2}{s}\right)\mathrm{Im_h}\,\Pi(s)\,. \tag{3}$$

The quantity $\mathrm{Im_h}\,\Pi$ is the imaginary part of the polarization operator, corresponding to the intermediate hadronic state.

Replacing the hadron current with the electromagnetic current of muons leads to the muon analogue of the function $\Pi(s)$:

$$\mathrm{Im}\,\Pi^{\mu^+\mu^-}(s) = -\frac{\alpha}{3}\left(1+\frac{2m_\mu^2}{s}\right)\sqrt{1-\frac{4m_\mu^2}{s}}\,\theta(s-4m_\mu^2)\,, \tag{4}$$

where $\theta(x)$ is the Heaviside step function. In the energy range that we consider, $s \gg 4m_\mu^2$. Using (3), we then obtain the muon pair production cross section in the Born approximation as

$$\sigma_0^{e^+e^-\to\mu^+\mu^-}(s) = \frac{4\pi\alpha^2}{3s}\,. \tag{5}$$

Hence, it is natural to define R as the ratio

$$R \stackrel{\rm def}{\equiv} \frac{\sigma^{e^+e^-\to\,\rm hadrons}(s)}{\sigma_0^{e^+e^-\to\mu^+\mu^-}(s)}\,. \tag{6}$$

It follows from (3) and (6) that

$$R = -\frac{3}{\alpha}\,\mathrm{Im_h}\,\Pi(s)\,. \tag{7}$$

Replacing m_μ in (4) with the quark mass m_f and multiplying by $3Q_f^2$, we can represent the lowest-order QCD contribution to R from each flavor as

$$R_f^{(0)}(s) = 3Q_f^2\left(1+\frac{2m_f^2}{s}\right)\sqrt{1-\frac{4m_f^2}{s}}\times\theta(s-4m_f^2) \underset{s\gg 4m_f^2}{\simeq} 3Q_f^2\,. \tag{8}$$

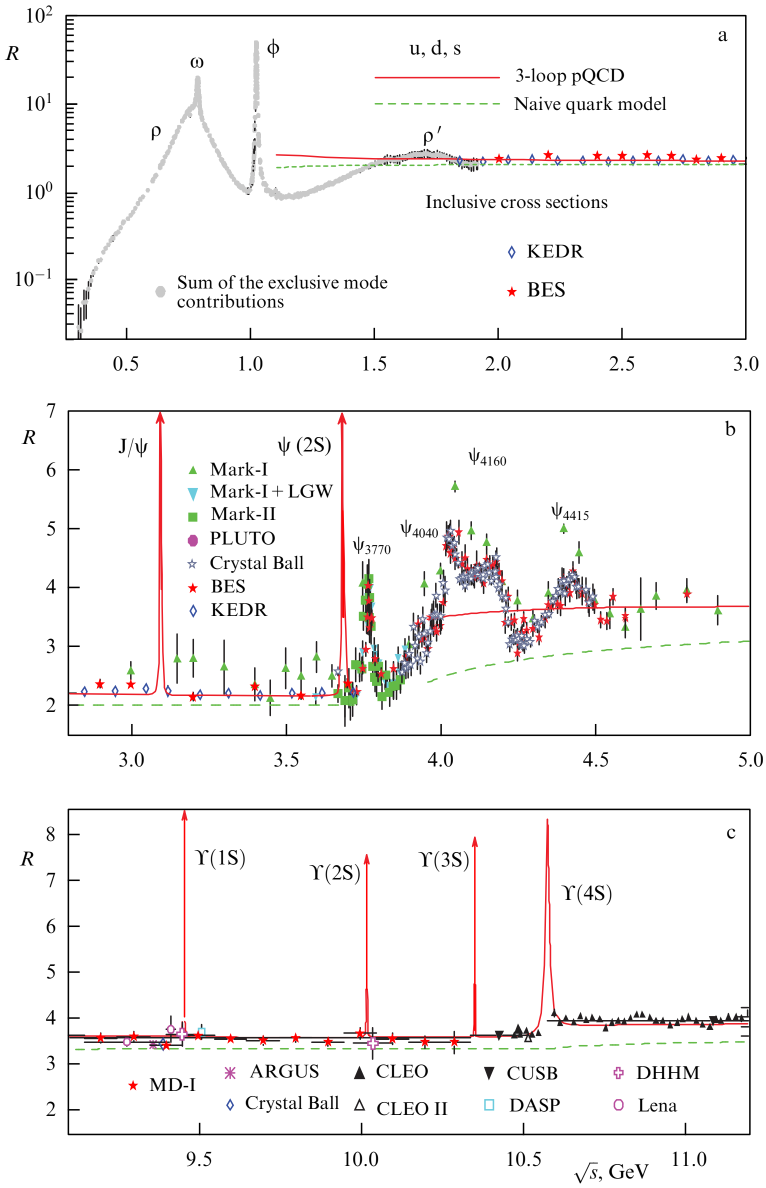

Figure 2. (Color online.) Results of experiments on measuring R depending on energy. Experimental data and calculation results as of 2018 are given in accordance with the PDG [28]. Top: the red line shows the result of a calculation within pQCD in the three-loop approximation; the green dashed line is the result of a calculation with the naive quark model. Grey symbols show the sum of the contributions to R of the exclusive decay modes measured experimentally. (Abbreviations: LGW: Lead Glass Wall; ARGUS: A Russian–German–United States–Swedish; CUSB: Columbia University–Stony Brook; DHHM: DESY–Hamburg–Heidelberg–München; DASP: Double-Arm Spectrometer.) Lena is a VEPP-4 experiment with the Lena detector.

The factor 3 in (8) occurs in summing over the three color states of the quarks. Summing over all flavors available at the specified energy, i.e., including only quarks whose mass is less than half the center-of-mass energy, we obtain the full R in the zeroth order of QCD:

$$R^{(0)} = 3 \sum_f Q_f^2 \,. \tag{9}$$

In an experiment where the characteristic energy scale is much less than the Z^0-boson mass, the quantity R evaluated within pQCD in the approximation of massless quarks can be written through the fourth order in the QCD running coupling constant α_s. The corresponding expression has the form [31–33]

$$R(s) = 3 \sum_f Q_f^2 \left(1 + \frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2 \left(\frac{365}{24} - 11\,\zeta_3 - \frac{11}{12}\, n_f \right.\right.$$
$$\left. + \frac{2}{3}\,\zeta_3 n_f \right) + \left(\frac{\alpha_s}{\pi}\right)^3 \left[n_f^2 \left(\frac{151}{162} - \frac{1}{108}\,\pi^2 - \frac{19}{27}\,\zeta_3 \right) \right.$$
$$+ n_f \left(-\frac{7847}{216} + \frac{11}{36}\,\pi^2 + \frac{262}{9}\,\zeta_3 - \frac{25}{9}\,\zeta_5 \right)$$
$$\left. + \frac{87029}{288} - \frac{121}{48}\,\pi^2 - \frac{1103}{4}\,\zeta_3 + \frac{275}{6}\,\zeta_5 \right] + \left(\frac{\alpha_s}{\pi}\right)^4$$
$$\times \left[n_f^3 \left(-\frac{6131}{5832} + \frac{11}{432}\,\pi^2 + \frac{203}{324}\,\zeta_3 - \frac{1}{54}\,\pi^2 \zeta_3 + \frac{5}{18}\,\zeta_5 \right) \right.$$

$$+ n_f^2\left(\frac{1045381}{15552}-\frac{593}{432}\pi^2-\frac{40655}{864}\zeta_3+\frac{11}{12}\pi^2\zeta_3+\frac{5}{6}\zeta_3^2 -\frac{260}{27}\zeta_5\right)+n_f\left(-\frac{13044007}{10368}+\frac{2263}{96}\pi^2+\frac{12205}{12}\zeta_3 -\frac{121}{8}\pi^2\zeta_3-55\zeta_3^2+\frac{29675}{432}\zeta_5+\frac{665}{72}\zeta_7\right)$$
$$+\frac{144939499}{20736}-\frac{49775}{384}\pi^2-\frac{5693495}{864}\zeta_3+\frac{1331}{16}\pi^2\zeta_3 +\frac{5445}{8}\zeta_3^2+\frac{65945}{288}\zeta_5-\frac{7315}{48}\zeta_7\Bigg]\Bigg)$$
$$+\left(\sum_f Q_f\right)^2\left(\left(\frac{\alpha_s}{\pi}\right)^3\left(\frac{55}{62}-\frac{5}{3}\zeta_3\right) +\left(\frac{\alpha_s}{\pi}\right)^4\left[n_f\left(-\frac{745}{432}+\frac{65}{24}\zeta_3+\frac{5}{6}\zeta_3^2-\frac{25}{12}\zeta_5\right) +\left(\frac{5795}{192}-\frac{8245}{144}\zeta_3-\frac{55}{4}\zeta_3^2+\frac{2825}{72}\zeta_5\right)\right]\right)+\mathcal{O}(\alpha_s^5)\,, \quad (10)$$

where α_s is the running coupling constant of strong interaction, n_f is the number of quark flavors available at a given energy, and ζ_n is the Euler–Riemann zeta function. Corrections to R due to the finite mass of the quarks at a sufficient distance from the production threshold of the corresponding quarks are suppressed as $m_f^2(s)/s$ when viewed outside the resonance domain. In the energy range under consideration, the contribution of electroweak interactions to R stays within 0.02%, which is negligibly small compared with systematic uncertainties of experimental data. Both effects are discussed in [34–36].

Let us digress to discuss an important point. In QCD, the coupling constant α_s tends to zero at large transferred momenta — a property known as asymptotic freedom. Another important property of QCD is the appearance of the mass parameter Λ. Because the original QCD Lagrangian does not contain mass parameters, this effect is called dimensional transmutation. Asymptotic freedom allows an exact description of strong interactions for sufficiently large transferred momenta. Within the QCD perturbation theory in the $\overline{\text{MS}}$ regularization scheme (the modified minimal subtraction scheme), analytic solutions have been found for the renormalization group equations, which allow writing an analytic approximation for α_s at the level of five-loop diagrams [37]. In theoretical calculations, the $\alpha_s(s)$ dependence can be described in a broad energy range using, e.g., the RunDec software packet [38, 39], in which the changes in both $\alpha_s(s)$ and the parameter Λ with the number of quark flavors are taken into account. The value of Λ at a given energy is set in accordance with the number of allowed quark flavors, so as to ensure the continuity of $\alpha_s(s)$.

The contribution of processes in which the transferred momentum is of the order of the QCD mass parameter Λ are no longer amenable to pQCD because of the infrared divergence of α_s; the problem is not solved by considering any finite number of multiloop contributions. Perturbative methods are applicable for $s \gg \Lambda^2$, whereas at the energy $s = (2\ \text{GeV})^2$, when $\alpha_s \simeq 0.3$, the possible corrections to R cannot a priori be assumed to be small. In that case, nonperturbative methods are used in calculations whenever possible; these methods include sum rules and operator product expansion (OPE), first proposed by Wilson [40]. In recent decades, the computational methods of the analytic QCD perturbation theory have gained popularity. A detailed review of the results obtained following this thread of theoretical physics can be found in [41, 42]. One recent paper, [43], is directly devoted to the exact calculation of R. Still, due to color confinement, the precision of QCD calculations is not on par with the precision of calculations in quantum electrodynamics, and this is one of the reasons why measuring R is an important experimental issue.

We note that in some cases, instead of R, it is useful to work with the quantity R_{exp} [44], which takes the vacuum polarization effects due to a virtual photon into account:

$$R_{\text{exp}} \stackrel{\text{def}}{\equiv} \frac{\sigma_{\text{exp}}^{e^+e^-\to\text{hadrons}}(s)}{\sigma_0^{e^+e^-\to\mu^+\mu^-}(s)}\,, \quad (11)$$

where $\sigma_{\text{exp}}^{e^+e^-\to\text{hadrons}}(s) = \sigma^{e^+e^-\to\text{hadrons}}(s)/|1-\Pi(s)|^2$. This results in the cancelation of a number of systematic uncertainties, for example, in calculating the anomalous magnetic moment of the muon.

2.2 Procedure for determining *R*

We discuss the procedure for measuring R, as used in experiments, by scanning the center-of-mass energy of the colliding e^+e^- beams.

The observable hadron cross section $\sigma_{\text{mh}}^{\text{obs}}(s)$ is given by

$$\sigma_{\text{mh}}^{\text{obs}}(s) = \frac{N_{\text{mh}} - N_{\text{res.bg}}}{\int \mathcal{L}\,\mathrm{d}t}\,, \quad (12)$$

where N_{mh} is the number of hadron events that have passed the selection, $N_{\text{res.bg}}$ is the number of beam background events under the chosen selection conditions,[1] and $\int \mathcal{L}\,\mathrm{d}t$ is the integrated luminosity.

The quantity R corresponding to the measured cross section is evaluated as

$$R = \frac{\sigma_{\text{mh}}^{\text{obs}}(s) - \sum \varepsilon_{\text{bg}}(s)\,\sigma_{\text{bg}}(s) - \sum \varepsilon_{\psi}(s)\,\sigma_{\psi}(s)}{\varepsilon(s)\big(1+\delta(s)\big)\,\sigma_0^{e^+e^-\to\mu^+\mu^-}(s)}\,, \quad (13)$$

where $\sigma_0^{e^+e^-\to\mu^+\mu^-}(s)$ is the Born cross section of the process $e^+e^- \to \mu^+\mu^-$, and $\varepsilon(s)$ is the efficiency of registering one-photon annihilation of an electron–positron pair into hadrons. The second term in the numerator contains contributions to the observed cross section coming from the lepton pair production processes l^+l^- $(l = e, \mu, \tau)$ and the two-photon production processes. The registration efficiencies of hadron events ε and of background processes ε_{bg} are to be found from Monte Carlo simulation.

The third term in the numerator in (13) includes contributions of the radiation tails of the J/ψ and $\psi(2S)$ resonances. This term is directly taken into account in calculating the cross section if the fitting of the resonances is done. In the absence of precise measurements of the collider energy, the contribution of resonances is regarded as part of the radiation correction δ.

We emphasize that the contribution $\varepsilon_\psi \sigma_\psi$ of the radiation tail of the resonance to the observed cross section depends on the combination $\varepsilon_\psi \Gamma_{\text{ee}} \mathcal{B}_{\text{h}}$ and, where the resonance parameters are determined by fitting the observed cross section, is insensitive to the individual values of the electron width Γ_{ee}, the probability of decay into hadrons $\mathcal{B}_{\text{h}}$, and the efficiency

[1] The subscripts obs, mh, and res.bg are derived from observed, multihadron, and residual background; they are used rather frequently in experimental work.

ε_ψ. In the fitting method, just the combination $\varepsilon_\psi \Gamma_{ee} \mathcal{B}_h$ is important, which characterizes the area under the resonance cross section observed in experiment.

One of the methods for determining $1+\delta(s)$ is to calculate the integral

$$1+\delta(s) = \int \frac{dx}{1-x} \frac{\mathcal{F}(s,x)}{\left|1-\tilde{\Pi}\big((1-x)s\big)\right|^2} \frac{\tilde{R}\big((1-x)s\big)\,\varepsilon\big((1-x)s\big)}{R(s)\,\varepsilon(s)}, \tag{14}$$

where $\mathcal{F}(s,x)$ is a function of radiation corrections, derived within the approach of structure functions in the classic work by Kuraev and Fadin [45]. This method was used in analyzing the data of the KEDR experiment [24–26]. In the work of the BES (Beijing Spectrometer) collaboration and in earlier experiments, the procedure for calculating the radiation correction was based on a systematic consideration of Feynman diagrams following the strategy described in [46–49].

When the resonance contribution is straightforwardly subtracted, the vacuum polarization operator $\tilde{\Pi}$ and the quantity $\tilde{R}$ do not include contributions from the J/ψ and $\psi(2S)$ resonances; otherwise, both quantities feature in the calculation in the original form. In the first case, the above procedure for analyzing experimental data directly yields the value of R_{uds}, and adding the terms corresponding to the resonances then gives R.

In experiments on measuring R at energies below the J/ψ meson production threshold, relation (13) is evidently simplified, and the resonances do not have to be considered unless the domain near the J/ψ resonance is considered, where taking interference into account is important.

2.3 Experiments on measuring *R*

As noted in the Introduction, experiments on measuring R in the energy range of 1.84–3.72 GeV have been numerous. A detailed survey of the results can be found in [50]. We discuss a number of characteristic features inherent in such measurements.

One of the first experiments where R was measured in the energy range of 2.6–7.8 GeV was the Mark-I experiment [19]. The Mark-I experimental data were analyzed without the fine tuning of the simulated e^+e^- annihilation into hadrons. The method of component decomposition was used to find the registration efficiency.

The method assumes the decomposition of the entire collection of hadron events into classes containing events with a certain multiplicity of charged particles. We suppose that a somewhat imprecise modeling is given, which does not reproduce the experimental distribution over the multiplicities of charged particles. Still, this modeling allows obtaining the matrix ε_{ij} that contains the registration efficiencies of events with a definite multiplicity j, for which i tracks can be reconstructed. The procedure for finding the initial multiplicity distribution N_j from the observed distribution of the N_i^{obs} amounts to minimizing the value of χ^2, evaluated as

$$\chi^2 = \sum_i \frac{N_i^{obs} - \sum_j \varepsilon_{ij} N_j}{N_i^{obs}}. \tag{15}$$

The reconstructed distribution of the N_j allows finding the correction to the registration efficiency of a hadron event identified in Monte Carlo simulation. This correction is an estimate of the systematic error. This approach is not free of drawbacks, in particular, it takes only the distribution over charged particle multiplicities into account and does not allow considering other features of multihadron events, resulting in an underestimation of the possible systematic uncertainties.

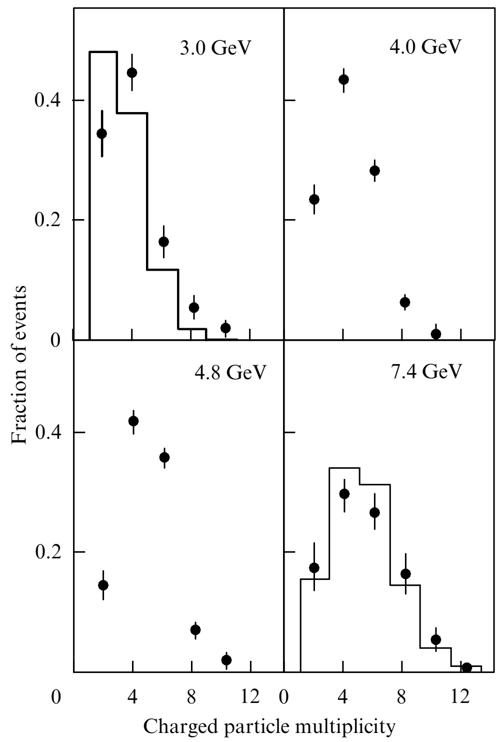

Figure 3. Distribution over multiplicities of charged particles in the Mark-I experiment [19]. Dots correspond to experimental data and the histograms show the modeling results.

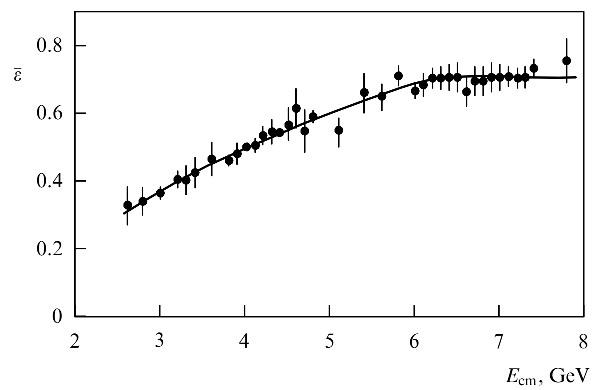

Figure 4. Registration efficiency of hadronic events $\bar{\varepsilon}$ as a function of energy in the Mark-I experiment [19].

In Fig. 3, we compare the modeled and experimental distributions over the charged particle multiplicity.

The contribution of the J/ψ and $\psi(2S)$ mesons was subtracted analytically at fixed parameters of the resonances. The registration efficiency of hadron events (Fig. 4) at energies below 3.72 GeV did not exceed 50%. The average precision of the Mark-I collaboration measurements was 13%, reaching 6% at some points. For a long time after the publication of the Mark-I results, no experiments on determining R in the energy range of 1.84–3.72 GeV were performed.

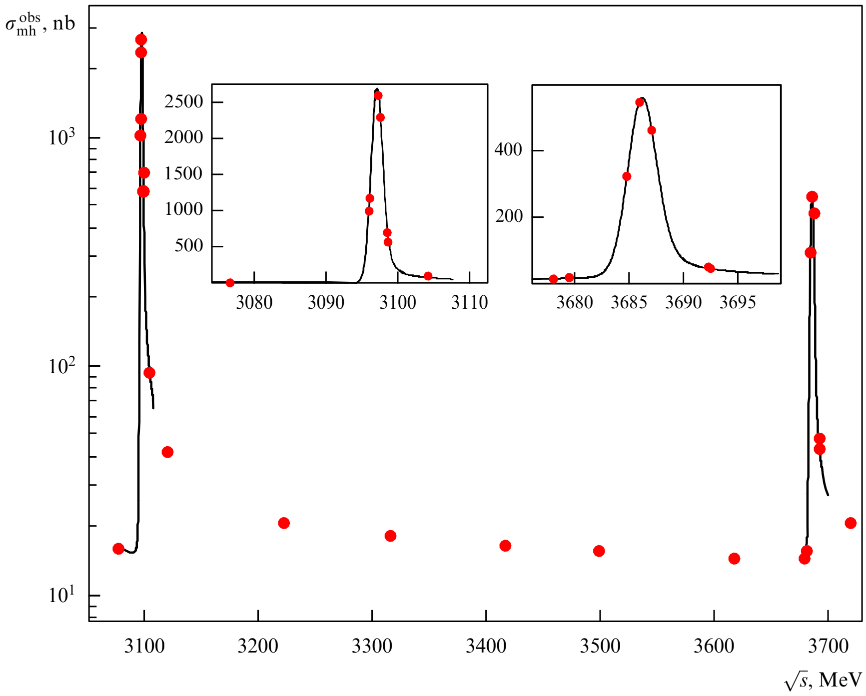

Figure 5. Observed hadronic cross section of electron–positron pair annihilation as a function of energy for the scan in [26]. The curves are the result of fitting the J/ψ and ψ(2S) resonances. The insets show enlarged parts.

Investigations in that energy range were revived by the BES collaboration [20–23]. The results for the broadest energy domain, from 2 to 5 GeV, are presented in a 2002 paper [21]. The analysis of BES II experimental data included tuning the uds-continuum modeling. The principal feature of the analysis was the use of two versions of modeling, based on two different hadron fragmentation models (see Section 3 for a more detailed description). The contribution of narrow resonances was considered jointly with the contribution of the uds-continuum cross section. In other words, the registration efficiency was found without selecting resonance components, and the analysis required several iterations. The resultant precision of the results was not better than 5.5%.

In a 2009 paper [23], the BES collaboration measured R at three energy points: 2.6, 3.07, and 3.65 GeV. The main approach of the analysis was the same as in the preceding work of the BES collaboration, but the precision of the results was improved to 3.4%, dominated by the systematic uncertainty.

Over recent years, results of the KEDR experiment at the Budker Institute for Nuclear Physics, Siberian Branch, Russian Academy of Sciences, were published [24–26], where R was measured at 22 energy points in the range of 1.84–3.72 GeV, which corresponds to the energy domain extending from the proton–antiproton pair production threshold to the production threshold of the $D\overline{D}$ pair of charmed mesons. At most of the points in the range of 1.84–3.05 GeV, R was measured with a precision of better than 3.9% with a systematic uncertainty of less than 2.4%. At most of the points in the energy range of 3.08–3.72 GeV, R was measured with a precision of better than 2.6% and with a systematic uncertainty level 1.6–2.2%.

A unique feature of the KEDR experiment was the direct determination of the contribution of the J/ψ and ψ(2S) resonances to the observed cross section. The scanning performed with the precisely measured storage energy in the immediate vicinity of the resonances allowed determining the main parameters of the resonances with high precision and finding their contribution to the observed hadron cross section. Questions pertaining to the precision fitting of the inclusive hadron production cross section are expounded in [51–53]. The result of one of the KEDR experiment scans is shown in Fig. 5 [26].

The results of measuring $R(s)$ in the energy range of 1.84–3.72 GeV are shown in Fig. 6. In modern experiments, the registration efficiency of hadronic events has spectacularly increased. Previously, it did not exceed 50% at the J/ψ resonance energy, but now, in the work of the BES collaboration [20–23] and in the KEDR experiment [24–26], it has reached and even exceeds 75%. The chosen conditions for the selection of hadronic events allow suppressing the background contributions to the observed cross section to a few percent. This largely reflects the enhanced 'leak-proof' quality of the detectors, i.e., the possibility of registering particles with high efficiency in a larger solid angle, as well as advances in the registration devices.

2.4 Applications of R measurement results

As is known, the main source of uncertainty in calculating the anomalous magnetic moment of the muon is the strong-interaction (hadronic) contribution. There are two components of the hadronic contribution. The first relates to the light-by-light scattering processes, and the second, larger, component is determined by the vacuum polarization and can be evaluated directly from experimental data.

The leading-order hadronic contribution $a_\mu^{\rm h,\,LO}$ due to polarization of the vacuum can be represented as a dispersion integral [55],

$$a_\mu^{\rm h,\,LO} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} \frac{R(s)\hat{K}(s)}{s^2}\,{\rm d}s\,, \qquad (16)$$

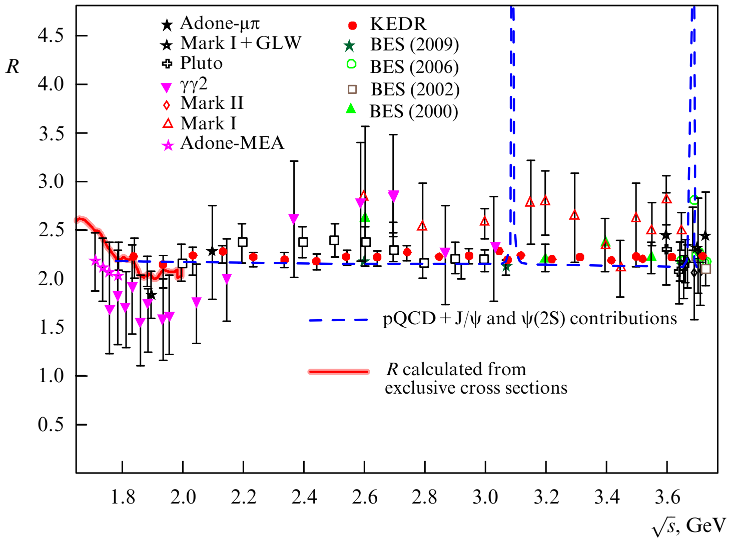

Figure 6. (Color online.) Results of experiments on measuring $R(s)$ in comparison to the calculation results with pQCD taking the contribution of narrow resonances into account (MEA: Magnete Esperienze Adone). Results of the calculation of R based on the sum of exclusive process cross sections are also given. (Data on the cross sections of exclusive channels are provided by F V Ignatov and are available in [54].)

where the kernel $\hat{K}(s)$ increases monotonically from 0.63 to 1 as s increases from $4m_\pi^2$ to $s \to \infty$.

Because of the s^2 denominator in the integrand in (16), about 91% of the magnitude of $a_\mu^{\mathrm{h,LO}}$ is determined by the energy range below 1.84 GeV. The part of the integral evaluated in the energy range above 1.84 GeV is relatively small, but it is essential for exact calculations based on experimental data. Work on the calculation of the muon anomalous magnetic moment is surveyed in more detail in review [56].

Another important application of $R(s)$ is the calculation of the fine structure constant $\alpha(M_Z^2)$ at an energy equal to the Z-boson mass. The corresponding hadronic component $\Delta\alpha_{\mathrm{h}}^{(5)}(M_Z^2)$ can be written as the dispersion relation [57–60]

$$\Delta\alpha_{\mathrm{h}}^{(5)}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi}\,\mathrm{Re}\int_{4m_\pi^2}^{\infty}\frac{R(s)}{s(s-M_Z^2-\mathrm{i}\varepsilon)}\,\mathrm{d}s\,. \qquad (17)$$

The superscript '(5)' on $\Delta\alpha_{\mathrm{h}}$ corresponds to taking five quark flavors in (17) into account. The t-quark contribution $\Delta\alpha_{\mathrm{top}} \simeq -(4\alpha/45\pi)M_Z^2/m_t^2$, suppressed by more than two orders of magnitude, is to be calculated separately. At the energies under consideration, the integrand in (17) scales as $1/s$. Thus, the contribution to $\alpha_{\mathrm{h}}^{(5)}(M_Z^2)$ associated with the discussed energy range is suppressed to a lesser degree than the corresponding hadronic contribution to the anomalous magnetic moment of the muon.

Contributions of the different energy ranges to a_μ and $\Delta\alpha_{\mathrm{h}}^{(5)}(M_Z^2)$ are illustrated in Fig. 7.

Experimental results of R measurements are also used in determining the heavy-quark masses. In [12], a characteristic systematic uncertainty of the c-quark mass related to errors in the R measurement in the energy range of 2–3.72 GeV was equal to 1.7 MeV and was determined by the precision of calculating the experimental moments M_n^{exp} given by the integrals

$$M_n^{\mathrm{exp}} = \int\frac{R(s)}{s^{n+1}}\,\mathrm{d}s\,. \qquad (18)$$

Although the other uncertainty sources are dominant in the resultant inaccuracy of the calculation, which is equal to

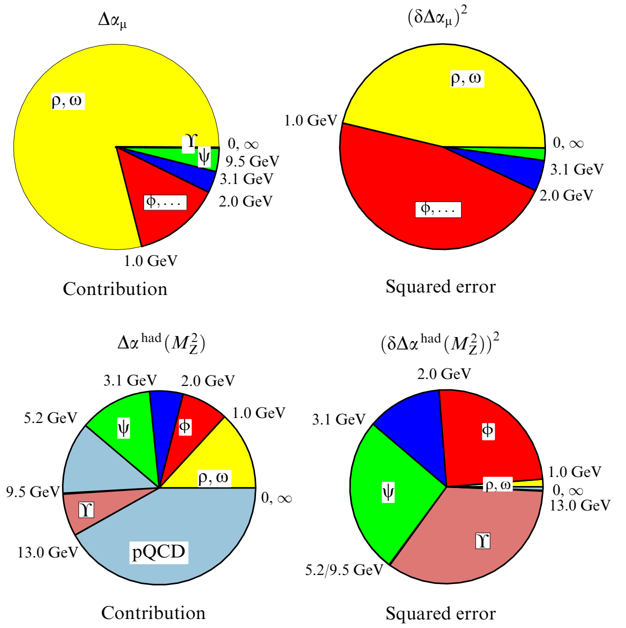

Figure 7. (Color online.) Contributions of different energy ranges to the anomalous magnetic moment of the muon and the quantity $\Delta\alpha_{\mathrm{h}}^{(5)}(M_Z^2)$, and the corresponding squared errors in the determination of these parameters. (Taken from [61].)

7.8 MeV, work on refining the inclusive hadron cross section in the energy range below 3.72 GeV remains relevant.

3. Prospects for improving the precision of measuring the inclusive hadron cross section

For modern and future experiments on measuring the inclusive hadron cross section of e$^+$e$^-$ annihilation, of primary interest is the possibility of increasing the measurement precision. What are the prerequisites for overcoming the current systematic uncertainty level of 1.6% in the energy range under consideration?

The main sources of systematic uncertainties in experiments on measuring R are related to the measurement

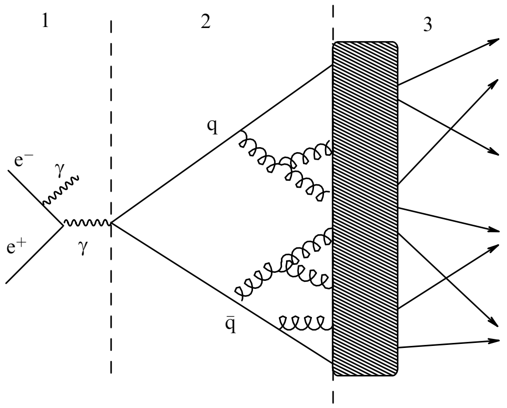

Figure 8. Principal stages of generating hadron production processes in e^+e^- annihilation.

luminosity, determination of the registration efficiency of hadronic events, calculation of the radiation correction, and estimate of the number of background events under the chosen selection conditions. From an analysis of the most recent BES and KEDR results, we can conclude that the key problem in data analysis has been, and still is, the reliable modeling of hadronic processes.

We discuss the primary modeling in more detail. Currently, there is no software for hadronic event generation that would by default guarantee the high-precision modeling of e^+e^- annihilation processes at energies below 10 GeV. Therefore, each experiment on measuring the inclusive hadron production cross section requires a tuning of the chosen event generator for primary particles and a careful analysis of systematic errors.

The main stages of the event generation procedure for hadron production processes in e^+e^- annihilation, for example, implemented in the JETSET software [62], can be seen as the following evolutionary sequence:

(1) formation of the original configuration of partons, whose description is based on electroweak theory and pQCD;

(2) radiation of hard gluons and/or their conversion into quark–antiquark pairs, described within pQCD;

(3) parton fragmentation and hadron decays, described within a phenomenological approach (nonperturbative QCD).

A lucid qualitative understanding of the sequence of the relevant processes (Fig. 8) is no guarantee against difficulties in implementing the described scheme at energies below 10 GeV.

The first stage is already marred by problems due to the appearance of soft or collinear gluons. A technical solution is to transform a three-gluon configuration into a two-gluon one. Despite the accompanying change in the event parameters, in particular, an underestimation of sphericity, this strategy generally ensures that the subsequent generation procedure is still possible. The second stage is realized using formulas representing the result of exact calculations in the second order of QCD or in the leading logarithmic approximation corresponding to parton showers.

The final stage of event generation is based on a purely phenomenological approach realized in JETSET based on the Lund string model (LSM) [63]. By default, the idea of 'longitudinal' fragmentation is used, according to which the system energy is expressed in terms of the light-cone variables $W \pm p_{\parallel}$. The fragmentation function (FF) $f(z) = (c/z)(1-z)^a \exp(bm_{\perp}^2/z)$, where a, b, and c are parameters (by default, $a = 0.3$ and $b = 0.58$ GeV^{-2}), gives the probability of a hadron acquiring the fraction z of the string energy. In the original version, the arguments of the Lund symmetric FF were not only z but also the 'transverse mass' squared, $m_{\perp}^2 = m^2 + p_{\perp}^2$, where the transverse momentum $p_{\perp}$ of the quarks produced in breaking the string has a Gaussian distribution, as is assumed in the LSM. To solve the problem of the decrease in the event sphericity at the energies of the initial system below 10 GeV, the mean square spread of the $p_{\perp}$ distribution is increased, which by default is equal to 0.36 GeV. JETSET allows using not only the original FF but also other FFs, in particular, the empirical Field–Feynman function.

Among several dozen parameters used in the JETSET event generator for tuning the modeling, we can single out the principal ones, which specify the LSM of fragmentation. For convenience of the description, we introduce the mnemonic notation $W_{\min}$, W_{stop}, δW_{stop}, $\sigma_{p_{\perp}}$, and P_{V}, corresponding to *parj*(32), *parj*(33), *parj*(37), *parj*(21), and *parj*(11) in the data set of the program. We consider the generation algorithm for S-wave meson states. The initial-state partons form color singlets, each of which is assigned a closed (with gluons at the cusps) or nonclosed (quark–antiquark) string. As the string mass exceeds the value $W_{\min}$ + quark mass, the meson formation process is initiated in the string; otherwise, a single meson is produced. The breaking of the string is implemented at each of the cusps by quark–antiquark pairs with the production of primary mesons, and only nonclosed strings therefore remain. The transverse momentum of a quark is generated in accordance with the Gaussian distribution and has the mean square spread $\sigma_{p_{\perp}}$. The nonclosed string breaking at one of the ends occurs with the formation of a meson whose energy and longitudinal momentum are calculated in accordance with the FF. The S-wave meson is a vector one with a probability P_{V} and is pseudoscalar otherwise.

The full LSM procedure includes the generation of baryons by forming colored diquarks at the break points of the string. By default, the relative formation probability of a light baryon is approximately 0.1. In addition, the algorithm envisages the production of P-wave mesons in accordance with user-defined parameters. If the invariant string mass becomes less than $W_{\min}$ + quark mass, the hadron production process terminates after the formation of a pair of the last hadrons. The algorithm also allows early termination of the fragmentation process, depending on the mass of the quarks (or diquarks) produced at the string breaking point. Such a scenario is implemented when the string energy is less than W_{stop} + mass of leading quarks + mass of the object at the breaking point. Obviously, $W_{\text{stop}} > W_{\min} + 2m_{\text{s}}$, where m_{s} is the strange-quark mass. From the values of W_{stop} and the conservation laws, the momentum of the last pair of produced hadrons can be determined. To eliminate singularities in the momentum spectrum, a parameter δW_{stop} is provided that corresponds to the spread of the W_{stop} values.

As can be seen from the description of the LSM hadron generation process implemented in the JETSET code, particle production under string breaking is independent of the number of particles that have already been formed. Terminating the fragmentation process is a complicated problem, solved under a number of assumptions, as described above. In an alternative approach implemented in the LUARLW software [64], the distribution over hadron event multiplicities is already controlled initially by parameters that determine the multiplicity distribution and specify the mean value. The parameters are extracted from the data obtained in experiment. The model in [65] assumes that the original fragmenting system can be defined in the phase space of light-cone variables as the domain that contains parts with fixed numbers of the produced hadrons. An event itself is characterized by a given probability density function represented as the product of probability densities corresponding to each of the considered parts. Parameters of the LUARLW event generator can be tuned in a broad energy range, unlike the necessarily local tuning of parameters at each energy point with the JETSET software.

Additional information on the phenomenology of nuclear interactions and popular hadronic event generators is available in reviews [66–68].

Parameter tuning for the JETSET and LUARLW event generators allows attaining satisfactory agreement between the characteristics of the events obtained from experimental data and the distributions derived from the simulation. In the discussed energy range, the registration efficiencies obtained from modeling by the LUARLW and JETSET generators agree at the level of 1% at an energy above 2.04 GeV [21, 24–26]. In [25], we considered the energy range of 1.84–3.05 GeV, but the exact tuning of the generators at an energy below 2.04 GeV was impossible due to the small number of statistics.

Despite an increase in the systematic uncertainty with the decrease in energy, there are prospects for improving the precision with increased numbers of statistics. Most probably, in the entire energy range of 1.84–3.72 GeV, the systematic uncertainty related to primary hadron event simulation can be reached at a level no worse than 1.5%, and for an energy above 2.1 GeV, 1%. This is primarily determined by the possibility of exactly tuning the inclusive hadron process generator at 1.8 GeV.

In [69], the results of simulation the uds continuum with the LUARLW and JETSET software were compared with the results obtained using an MHG2000 generator [70, 71], which models the entirety of exclusive processes known at a given energy. Multiplicity distributions of charged particles for the specified version of primary generators are given in Fig. 9. In the table, we show the differences among the registration efficiencies obtained with different initial modeling schemes.

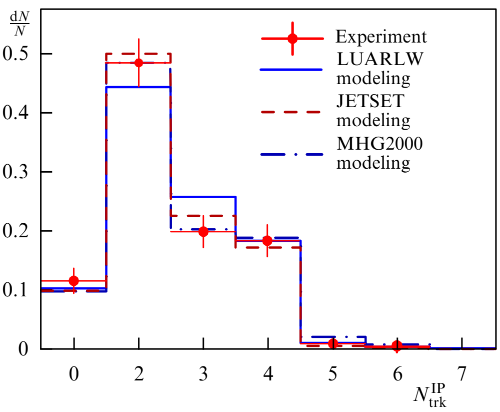

Figure 9. (Color online.) Distribution over multiplicities for charged particles $N_{\rm trk}^{\rm IP}$ for events that have passed the 'hadronic' selection, at the energy of 1.84 GeV. All distributions are normalized to unity. N is the number of events in the histogram.

Table. Relative differences among the various definitions of the registration efficiency of the uds-continuum hadronic events.

Energy, GeV	$\delta\varepsilon/\varepsilon$, %		
	LUARLW JETSET	LUARLW Decomposition method (Section 2.3)	LUARLW MHG2000
1.841	6.6	3.6	3.8
1.937 – 2.037	2.5	1.9	—
2.136 – 3.048	1.2	0.5	—

The distributions presented here show that the parameter tuning for the inclusive JETSET generator allows attaining a satisfactory agreement between the modeling and the experimental characteristics of the events. Moreover, the current level of knowledge about the entirety of the decays at 1.8 GeV suffices for the primary modeling of all exclusive processes, as implemented in the MHG2000 software.

Another strategy to model hadron processes was adopted in the BES III experiment [72, 73]. The analysis of the experiment is based on the results of simulation with a hybrid generator, given by a combination of a program for primary simulation of exclusive processes and the LUARLW program. The exclusive mode generator incorporates the PHOKHARA [74] and ConExc [75] codes, which allows considering about 50 different processes in total. The hybrid approach includes the modeling of all the known processes at a certain energy, and the remaining part of the events is simulated by an inclusive generator, such as the LUARLW software. Comparing the registration efficiencies of hadron events with the hybrid generator and the registration efficiency obtained by simulation with only the LUARLW generator yields an estimate of the systematic uncertainty of the primary simulation. It is assumed that this error must be 1.5 – 2.0%.

4. Conclusion

As follows from the results presented here, by no means have all the problems related to measuring the inclusive hadron production cross section been solved. The most complicated problem is modeling hadronic processes.

The immediate prospects for improving the precision of measuring R in the energy range discussed here are associated with the BES III experiment [72, 73]. The integrated luminosity collected at 130 points in the energy range of 2–4.59 GeV is 1.3 fb^{-1}. With this amount of data, the precision of the experiment is mainly determined by systematic errors. The analysis of data at 14 energy points in the range of 2.232–3.671 GeV is nearing completion. The respective estimates of the systematic errors associated with luminosity and the radiation correction are 0.8% and 1.0%. Given the results of hadronic decay modeling discussed in Section 3, the expected overall precision of the experiment can be better than 2.5%.

The most interesting questions are associated with the behavior of R in the energy range of 1.8–2.2 GeV; the results of experiments in that energy range are shown in Fig. 10.

For future experiments, it will be important to verify with high precision that the cross section obtained by summing the exclusive process cross sections agrees with the inclusive cross section of e^+e^- annihilation into hadrons. In the energy range of 1.84–2.0 GeV, the prediction of pQCD lies somewhat above the experimental value of R found by summing the

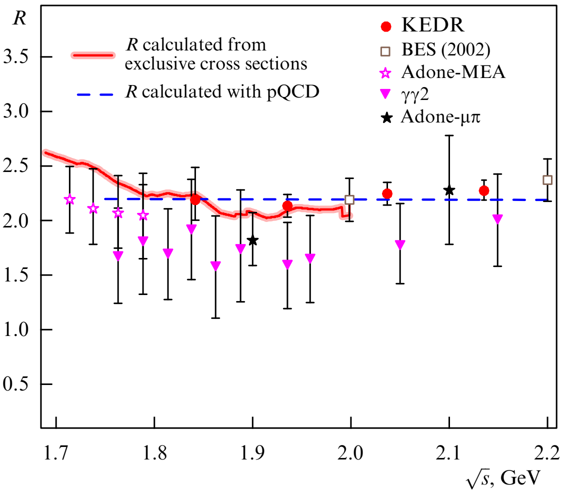

Figure 10. (Color online.) Results of experiments on measuring $R(s)$ compared with the calculation results with pQCD, in the energy range of 1.7–2.2 GeV. The results of calculating R based on the sum of exclusive process cross sections are taken from [54].

exclusive mode cross sections of hadronic decays. In the energy range above 2.0 GeV, most of the precise measurements of R in the KEDR experiment [24–26] insignificantly overshoot the calculated theoretical values. Whether this is a consequence of the systematic uncertainty of the measurement results or is caused by imprecision of the theoretical calculation can only be decided by new precision experiments.

To summarize this paper, we can say that the precision of measuring R in the energy range of 1.84–3.72 GeV is approaching the practically feasible limit of 1%, which will apparently not be attained even in the long run. Still, finding the inclusive cross section of hadron production in e^+e^- annihilation with a 1.5–2.0% precision in the entire energy range under consideration is a totally realistic task for ongoing and future experiments.

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